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WIGGLERS, BEAM EMITTANCE AND ENERGY SPREAD,

H. Wiedemann

Stanford Linear Accelerator Center
Stanford University, Stanford, California 94305

I. INTRODUCTION

Wigglers are used in storage rings¹⁾ to increase the beam emittance for maximum luminosity. If, however, a storage ring is used as a synchrotron light source a beam emittance as small as possible is desired. Also cases in high energy physics research a large luminosity achieved with wigglers may not be the optimum running configuration. Together with an increase of the beam emittance also the energy spread of the beam is increased. This increase reduces the event rate for narrow resonances like the ψ or T .

In this note we will investigate the conditions for a reduction in beam emittance and/or of the energy spread in the beam.

II. WIGGLER SCHEMES

In the following part of this note we use a wiggler unit as a series of four equal field magnets of length l each and the field polarity as $+ - +$.

For normal storage ring operation the wigglers are put in a place where the η -function is not zero. The blow up of the beam emittance and the energy spread then is given by¹⁾

$$(1) \quad \frac{\epsilon_w}{\epsilon_0} = \frac{1 + \frac{L_w}{2\pi\rho_0} \left(\frac{\rho_0}{\rho_w}\right)^3 \frac{\langle \mathcal{H}_w \rangle}{\langle \mathcal{H}_0 \rangle}}{1 + \frac{L_w}{2\pi\rho_0} \left(\frac{\rho_0}{\rho_w}\right)^2}$$

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1) E. Paterson, J. Rees, H. Wiedemann PEP Note-125

40

and

$$(2) \quad \frac{\sigma_{\epsilon_w}^2}{\sigma_{\epsilon_0}^2} = \frac{1 + \frac{L_w}{2\pi\rho_0} \left(\frac{\rho_0}{\rho_w}\right)^3}{1 + \frac{L_w}{2\pi\rho_0} \left(\frac{\rho_0}{\rho_w}\right)^2}$$

where ρ_0 is the bending radius of the main bending magnets, ρ_w the bending radius of the wiggler magnets, L_w the total length of the wiggler magnets ($L_w = 4\ell \cdot N_w$), N_w the number of wiggler units. $\langle \mathcal{H}_0' \rangle = \frac{1}{2\pi\rho} \int \mathcal{H}' ds$ without wiggler magnets, $\langle \mathcal{H}_w' \rangle = \frac{1}{L_w} \int \mathcal{H}' ds$ and $\mathcal{H}' = 1/\beta \left\{ \eta^2 + \left(\beta \eta' - \frac{1}{2} \beta' \eta \right)^2 \right\}$ with β the betatron function.

From eqs. (1) and (2) it is clear that the change in beam emittance and energy spread is about the same if $H_w \approx H_0$ like for example in PEP or any other typical ring. If, however, special high η sections are incorporated into the storage ring design²⁾ we can make $H_w \gg H_0$ and therefore the blow up in beam emittance is much larger than that of the energy spread.

What, however, can be done once a storage ring is constructed without such a high η section? In the next sections we will investigate the possibilities for that using a wiggler where the η -function and therefore H_w is zero.

III. BEAM EMITTANCE

We assume the wiggler to be located at a place where $\beta' \approx 0$ and where in the storage ring without wiggler we have $\eta_0 = \eta_0' = 0$.

Even so we have chosen a place where $\eta_0 = 0$ the quantity H_w will not be zero once the wiggler magnet are turned on because they create their own η -function.

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2) H. Wiedemann PEP Note-290

The η -function in the first two magnets of a wiggler unit are given by

$$(3) \quad \eta_1(s) = -1/2 \cdot s^2/\rho_w \quad \eta_1'(s) = -s/\rho_w$$

and

$$(4) \quad \eta_2(s) = -1/2 (l^2 + 2ls - s^2)/\rho_w \quad \eta_2'(s) = -(l-s)/\rho_w$$

where $0 \leq s \leq l$.

For all other wiggler magnets the η -function is similar to eqs. (3) and (4) for symmetry reasons.

The change in the beam emittance due to the wiggler system is now:

$$(5) \quad \varepsilon_w/\varepsilon_0 = \frac{1 + \frac{1}{H_0} \int \mathcal{H}'/\rho_w^3 ds}{1 + \frac{1}{R_0} \int \mathcal{H}'/\rho_w^2 ds}$$

where $H_0 = \int \mathcal{H}'/\rho_0^3 ds$ and $R_0 = \int \mathcal{H}'/\rho_0^2 ds$

The contributions of the first and second wiggler magnets to the integral $\int \mathcal{H}'/\rho_w^3 ds$ are

$$(6) \quad \begin{aligned} \Delta_1 \int \mathcal{H}'/\rho_w^3 ds &= \frac{l^5}{20\beta_x \rho_w^5} + \frac{\beta_x l^3}{3\rho_w^5} \approx \frac{\beta_x l^3}{3\rho_w^5} \\ \Delta_2 \int \mathcal{H}'/\rho_w^3 ds &= \frac{43}{60} \frac{l^5}{\beta_x \rho_w^5} + \frac{\beta_x l^3}{2\rho_w^5} \approx \frac{\beta_x l^3}{2\rho_w^5} \end{aligned}$$

For the whole wiggler system we have then

$$\oint \mathcal{H} / \rho_w^3 ds = N_w \frac{5}{3} \beta_x \frac{l^3}{\rho_w^2}$$

(7)

$$\oint 1 / \rho_w^2 ds = N_w \cdot 4 \cdot \frac{l}{\rho_w^2}$$

with $H_0 = \langle \mathcal{H}_0 \rangle 2\pi R / \rho_0^3$ and $R_0 = 2\pi R / \rho_0^2$ and R the average bending radius of the storage ring we finally get from eq. (5):

$$(8) \quad \epsilon_w / \epsilon_0 = \frac{1 + \frac{5}{6\pi} \frac{\beta_x}{\langle \mathcal{H}_0 \rangle} \frac{\rho_0}{R} \left(\frac{\rho_0}{\rho_w} \right)^2 N_w \Theta^3}{1 + \frac{2}{\pi} \frac{\rho_0}{R} \left(\frac{\rho_0}{\rho_w} \right) N_w \Theta}$$

where $\Theta = l / \rho_w$ is the deflection angle of one wiggler magnet. Without going much into detail we see from eq. (8) that the beam emittance indeed can be decreased by wiggler magnets if we only keep Θ small.

For a typical storage ring we can use the so called "smooth approximation" which gives $\langle \mathcal{H}_0 \rangle \approx 3/v_x \beta_x$

$$(9) \quad \epsilon_w / \epsilon_0 \approx \frac{1 + \frac{5}{6\pi} v_x^2 \frac{\rho_0}{R} \left(\frac{\rho_0}{\rho_w} \right)^2 N_w \Theta^3}{1 + \frac{2}{\pi} \frac{\rho_0}{R} \left(\frac{\rho_0}{\rho_w} \right) N_w \Theta}$$

In order to obtain a reduction of the beam emittance we arrive at the condition

$$(10) \quad 0 < \frac{5}{12} v_x^2 \frac{\rho_0}{R} \frac{\rho_0}{l} \Theta^3 < 1$$

which always can be met if we only choose Θ small enough. In practical cases it turns out that to reduce the emittance significantly a large number

of wiggler magnets are required and the bending field has to be as high as possible.

IV. ENERGY SPREAD

For the change in the energy spread due to the wiggler system we get from eq. (2) with $L_w = 4l N_w$ and $\Theta = l/\rho_w$:

$$(11) \quad (\sigma_E/\sigma_{E0})^2 = \frac{1 + \frac{2}{\pi} \left(\frac{\rho_0}{\rho_w} \right)^2 N_w \epsilon}{1 + \frac{2}{\pi} \left(\frac{\rho_0}{\rho_w} \right) N_w \epsilon}$$

The nominator and denominator in eq. (11) differ only in the power of the factor ρ_0/ρ_w . For the change in the energy spread we can immediately draw the following conclusions:

- a: $\rho_0 > \rho_w$ In this case we always have an increase in energy spread due to wiggler magnets.
- b: $\rho_0 = \rho_w$ There is no change in energy spread.
- c: $\rho_0 < \rho_w$ In this case we have a reduction of the energy spread. However, here the number of turns out to be unrealistically high for a real storage ring.

V. CONCLUSION

It has been shown that indeed the beam emittance as well as the beam energy spread can be reduced in a storage ring by the use of wiggler magnets. However, the practical implications - number of wiggler magnets

and field strength - are rather severe.

To illustrate this statement we will use wiggler magnets of $\ell = 0.01$ m and a field of 50 k G (superconducting magnets). With such magnet we get for the reduction of the beam emittance:

Storage Ring	E (GeV)	N_w	L_w (m)	ϵ_w/ϵ_o
SPEAR (ψ)	1.55	500	20.0	0.081
PEP (T)	5.0	500	20.0	0.11
	5.0	100	4.0	0.24

To reduce the energy spread in SPEAR we arrive at the following parameters:

L_w (m) :	10	100	1000
ρ_w (m) :	18.8	21.2	31.0
$\sigma_{\epsilon_w}/\sigma_{\epsilon_o}$:	.99	.93	.88

Here we have used the optimum value for ρ_w to minimize $\sigma_{\epsilon_w}/\sigma_{\epsilon_o}$. We see that the total length L_w of the wiggler required to get any appreciable reduction in energy spread is totally unrealistic.