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SIMULATION OF IRREVERSIBLE ROCK COMPACTION EFFECTS ON GEOPRESSURED RESERVOIR RESPONSE

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FOREWORD

This topical technical report describes work performed by S-CUBED, a Division of Maxwell Laboratories, Inc., under subcontract to the University of Texas at Austin (UTA). The research effort was performed for the U. S. Department of Energy (DOE) under its Cooperative Agreement No. DE-FC07-85NV10412 with UTA. Close liaison was maintained between the UTA Rock Mechanics Group and S-CUBED teams to ensure that the parametric reservoir simulation calculations described herein utilized the best estimates for geopressured formation rock properties that are currently available. The author is indebted to Drs. Eric Farenthold and Kenneth Gray of UTA, Dr. Sabodh K. Garg of S-CUBED and Mr. Jack Ramsthaller of EG&G for their contributions to this technical liaison. Dr. Myron H. Dorfman of UTA was overall Principal Investigator for this DOE/UTA Cooperative Agreement and Ms. Peggy A. M. Brookshier was the DOE Project Manager.

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ABSTRACT

A series of calculations are presented which quantitatively demonstrate the effects of nonlinear stress-deformation properties on the behavior of geopressured reservoirs. The range of stress-deformation parameters considered is based on information available from laboratory rock mechanics tests performed at the University of Texas at Austin and at Terra Tek, Inc. on cores recovered from geopressured wells. The effects of irreversible formation rock compaction, associated permeability reduction, and repetitive load/unload cycling are considered. The formation rock and geopressured brine properties are incorporated into an existing reservoir simulator using a bilinear model for the irreversible compaction process. Pressure drawdown and buildup testing of a well producing from the geopressured formation is simulated for a suite of calculations covering the range of formation parameters. The results are presented and discussed in terms of the inference (e.g., permeability and reservoir volume) that would be drawn from the simulated test data by an analyst using conventional methods.

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I. INTRODUCTION

Under the U. S. Department of Energy (DOE) Design Wells Program, the following four production wells have been drilled and tested:

1. Pleasant Bayou No. 2 Well, Brazoria County, Texas.
2. Amoco Fee No. 1 Well, Sweet Lake Field, Cameron Parish, Louisiana.
3. L. R. Sweezy No. 1 Well, Parcperdue Field, Vermillion Parish, Louisiana.
4. Gladys McCall No. 1 Well, Cameron Parish, Louisiana.

Sufficient data are available during the depletion phase of the production tests of the Sweezy and McCall wells to conclude that unanticipated reservoir drive mechanisms are operative; in both cases the reservoir pressure is maintained at a higher value than expected.

Mechanisms that might explain the unanticipated pressure maintenance include stress-dependent rock formation compressibility, long-term formation creep, cross-flow from overlying/underlying sands, shale-water recharge, and leakage across boundary faults. Costs of drilling/completing more than one well in a geopressured fault block preclude well interference testing which might otherwise be employed to help determine which of the candidate mechanisms is dominant.

Laboratory rock mechanics tests have been performed at the University of Texas at Austin (UTA) and Terra Tek, Inc. on small cylindrical samples cut from rock cores recovered from geopressured wells. These tests included uniaxial compaction and triaxial loading tests (both jacketed andunjacketed). Tests were conducted at simulated *in-situ* pressure and temperature conditions and the variation of rock properties with the state of stress were studied. The UTA rock mechanics group has performed tests on samples from the Pleasant Bayou and Amoco Fee wells whereas Terra Tek performed tests on core samples from the Sweezy and McCall wells. UTA also performed extensive tests on sandstone and shale cores recovered from the Hitchcock geopressured well under a Gas Research Institute program. Both groups have also performed limited studies of long-term creep of rock samples from the wells.

It is generally accepted that rock properties measured on small core samples in laboratory tests can be expected to demonstrate the qualitative behavior of the *in-situ* formation rock but quantitative agreement cannot be expected. Large variations are commonly obtained for properties measured on samples recovered from a single well; much larger variations at large lateral distances within the reservoir formation would be expected. Because of this unavoidable uncertainty, an evaluation of the potential effect of reservoir drive mechanisms using laboratory data must consider a range of rock material parameters.

This report presents the results of a series of parametric calculations to estimate the effect of irreversible rock compaction on geopressured reservoir response. The S-CUBED reservoir simulator THOR was employed in order to provide a quantitative basis for determining the range of

reservoir parameters and test conditions under which this potential reservoir drive mechanism might be significant. The effect of stress-dependent permeability is also included in the parametric calculations.

It is appropriate to briefly outline here the contents of the rest of this report. In Section II of the report the rock mechanics data available for use in the parametric calculations are described. Section III presents an interpretation of the data within a framework that accounts for rock grain deformation effects, irreversible pore volume compaction, stress-dependent permeability, and cyclical loading. In Section IV the irreversible compaction behavior is approximated by a bilinear model employed in the S-CUBED reservoir simulator THOR; extension of classical well test analysis methods to a reservoir exhibiting bilinear irreversible compaction is also discussed. Section V describes the suite of parameters employed in the simulated well test calculations. Section VI presents the results of the THOR calculations simulating a simple drawdown/buildup test. Section VII discusses how these pressure transient tests would be interpreted by an analyst using classical methods. Section VIII presents the results of THOR calculations simulating repetitive drawdown/buildup testing. Finally, Section IX describes the conclusions that may be drawn regarding geopressured reservoirs.

II. BACKGROUND - ROCK MECHANICS DATA

On April 8, 1986 representatives of S-CUBED, EG&G and UTA met in Austin, Texas to delineate a work plan to evaluate the potential effect of nonlinear formation rock material properties on geopressured reservoir response. It was agreed that sufficient laboratory data on the stress-dependent rock formation compressibility existed for an evaluation of its potential effect on geopressured reservoirs. Since the Gladys McCall well is currently being produced, it was decided to focus the study on this reservoir. There is also interest in the Pleasant Bayou reservoir since the well there is scheduled to be reopened in 1987. Rock properties were defined for which UTA agreed to supply its best estimates for field-applicable values based on available laboratory test data from both UTA and Terra Tek. S-CUBED agreed to utilize the information supplied by UTA in an existing reservoir simulator to study the effect on reservoir performance of stress-dependent rock formation compressibility.

Figure 2.1 (from Kelkar, et al., 1982) illustrates the test procedure employed in the laboratory rock mechanics tests to obtain approximately *in situ* conditions of temperature, confining pressure and pore pressure. The portion OA of the plot of axial stress versus volumetric strain represents hydrostatic loading (to the *in situ* pore pressure) under equal confining pressure and pore pressure, maintaining zero effective stress. The volumetric strain observed is due to the compression of the solid constituents of the rock matrix (effect of rock grain deformation). The portions AB of the curves show the further increase in the confining pressure up to the *in situ* reservoir conditions.

Figure 2.1. Stress-strain data for hydrostatic compaction and uniaxial compaction (loading) of a test sample (from Kelkar, *et al.*, 1982).

The pore pressure is held constant during this portion of the test but the axial stress (Σ_1 in Figure 2.1) increases as the rock undergoes skeletal compression. Point B therefore corresponds to the desired *in situ* reservoir conditions. The portions BC of the curves for the uniaxial tests show the axial compaction of the rock at constant axial stress (representing the overburden) with increasing effective stress due to pore pressure drawdown. In the triaxial tests, portions BC of the curves represent increasing axial stress while maintaining constant confining and pore pressure.

Figure 2.2 presents the results of one of the uniaxial compaction tests performed by Terra Tek on Gladys McCall core samples. The stresses and strains were continuously logged as the effective stress was increased from its *in-situ* value by pore pressure drawdown while maintaining constant axial stress and varying the confining pressure to maintain zero lateral strain. The loading (decreasing pore pressure) and unloading (increasing pore pressure) response is essentially bilinear in the range of incremental axial stress tested, i.e.

$$0 < \Delta\sigma_{eff} < 7000 \text{ psi}$$

There is unrecovered (plastic) strain upon return to the initial hydrostatic stress state.

Figure 2.3 presents the results of one of the uniaxial compaction tests performed by UTA on Pleasant Bayou core samples. During loading the rock is initially more compressible but at higher loads ($\Delta\sigma_{eff} > \sim 1000 \text{ psi}$) the loading

Figure 2.2. Effective axial stress versus volume strain for Gladys McCall No. 1 well sandstone (from Kelkar, et al., 1982).

P.B.#2(14,706 FT.): S1 VS. E1, UNIAXIAL TEST #1

Figure 2.3. Incremental axial stress versus incremental axial strain, uniaxial test (from Fahrenthold and Gray, 1985a).

response becomes essentially linear. During unloading the compressibility is at first much less than it was during the final loading stages but increases as the loading is further decreased.

Although the average compressibility of a core sample during loading is generally greater than during unloading, the measurements vary significantly between samples from the same core. Based on their measurements on core samples from the Pleasant Bayou well, however, the UTA rock mechanics group (Fahrenthold and Gray, 1985a) have proposed analytic fits to the load-deformation curves. Their analytic fits are based on the assumption of inelastic loading and elastic unloading. They represent the loading incremental effective stress-strain curves by the exponential relations

$$\sigma_1 = \alpha \epsilon_1 - \beta [1 - \exp(-\epsilon_1/\tau_1)] \quad (2.1)$$

$$\sigma_3 = 2\eta_1 \epsilon_1 - C_3 [1 - \exp(-\epsilon_1/\tau_3)] \quad (2.2)$$

for the uniaxial tests and

$$\sigma_1 = E \epsilon_1 - S_1 [1 - \exp(-\epsilon_1/\beta_1)] \quad (2.3)$$

$$\sigma_3 = 2\eta_3 \epsilon_3 - S_3 [1 - \exp(-\epsilon_3/\beta_3)] \quad (2.4)$$

for the triaxial tests. The elastic unloading incremental effective stress-strain curves were represented by

$$\sigma_1 = A \epsilon_1 + B \epsilon_1^2 \quad (2.5)$$

$$\sigma_3 = C \epsilon_1 + D \epsilon_1^2 \quad (2.6)$$

for the uniaxial tests and

$$\sigma_1 = M e_1 + N e_1^2 \quad (2.7)$$

$$\sigma_3 = Q e_3 + R e_3^2 \quad (2.8)$$

for the triaxial tests.

In relations (2.1) through (2.4)

σ_1 (σ_3) = axial (radial) incremental effective stress component.

ϵ_1 (ϵ_3) = axial (radial) incremental strain component.

In relations (2.5) through (2.8)

e_1 (e_3) = axial (radial) elastic strains.

The plastic strain is defined as the residual strain present upon completion of the loading-unloading test. The remaining parameters in relations (2.1) through (2.8) are experimentally determined material coefficients (α and β replace symbols C_m and C_1 used by Fahrenthold and Gray).

The material coefficients in relations (2.1) through (2.8) have been determined for the Pleasant Bayou core specimens (Fahrenthold and Gray, 1985a). The UTA group has supplied S-CUBED with additional information relating to both stress-dependent and long-term creep properties of geopressed formation rock. Appendix A presents the information relating to stress-dependent rock deformation and permeability; information provided for long-term creep behavior are

not included since this report does not consider the potential effect of the latter reservoir drive mechanism.

The properties listed in Section A.1 of Appendix A are based primarily on measurements made by the UTA rock mechanics group on core samples recovered from the Pleasant Bayou well. The properties listed for the Gladys McCall reservoir in Section A.2 are based on UTA'S review of the Terra Tek data report (Kelkar, et al., 1982). Since we are interested in evaluating the potential effects of irreversible rock compaction and more information is available for the Pleasant Bayou formation rock, we have relied primarily on the UTA data in designing the parametric calculations.

III. ANALYSIS OF ROCK MECHANICS DATA

In traditional treatments of fluid flow through porous media reservoir it is assumed that the rock/soil grains are incompressible even when stress-dependent rock properties are considered (Samaniego, *et al.*, 1977). The laboratory tests performed at UTA and Terra Tek on core samples from geopressured wells indicate, however, that rock grain compressibility is not negligible compared with bulk rock compressibility. It is therefore necessary to incorporate the effect of rock grain deformation in the reservoir response model which we will use for the parametric studies. In this section the poroelastic constitutive relations that include rock grain deformation effects are presented and then utilized to interpret the UTA data within this framework.

3.1 POROELASTIC CONSTITUTIVE RELATIONS

Under the conditions of uniaxial strain ($\epsilon_{11} \neq 0$, $\epsilon_{ij} = 0$ otherwise) and constant overburden ($\dot{\sigma}_{11} = 0$) that are approximated in a reservoir,

$$\dot{\epsilon}_{11} = C_m \dot{p} \quad (3.1)$$

$$\dot{\phi} = (1 - \phi_0) C' \dot{p} \quad (3.2)$$

Here p is the fluid pore pressure and ϕ the porosity. It can be shown (Garg, 1986) that the uniaxial compressibility (C_m) and the coefficient in the second relationship (C') are given by

$$C_m = \frac{1 - K/K_r}{K + (4/3) \mu_p} \quad (3.3)$$

$$C' = \left[1 - \frac{K}{(1 - \phi_0) K_r} \right] \left\{ \frac{1 + (4/3) (\mu_p/K_r)}{K + (4/3) \mu_p} \right\}, \quad (3.4)$$

where

K = Drained bulk modulus

K_r = Rock grain bulk modulus

μ_p = Shear modulus of porous rock

ϕ_0 = Initial porosity.

For finite rock grain compressibility $C' \neq C_m$; $C' = C_m$ if and only if $K \ll K_r$ and $\mu_p \ll K_r$.

It should be noted that the coefficient C' (needed input parameter for reservoir simulations) cannot, in general, be evaluated from the usual uniaxial strain laboratory rock mechanics tests since C' involves K , μ_p and K_r . Drained triaxial (K , μ_p) andunjacketed (K_r) test data are required to evaluate C' . Measured uniaxial compressibility C_m can, however, be used to evaluate the consistency of parameters (K , μ_p , K_r) measured during the drained triaxial and unjacketed tests.

3.2 INTERPRETATION OF UTA DATA

According to the UTA data input (Appendix A), rock grain compressibility is approximately constant at $0.30 \times 10^{-6} \text{ psi}^{-1}$. We therefore assume

$$K_r = 3.33 \times 10^6 \text{ psi} \quad (3.5)$$

in our analysis of the UTA uniaxial strain and triaxial test data for the Pleasant Bayou cores.

3.2.1 Uniaxial Strain Test Data

During loading (Fahrenthold and Gray, 1985a),

$$\sigma_1 = \alpha \epsilon_1 - \beta [1 - \exp (- \epsilon_1 / \tau_1)] \quad (3.6)$$

where

$$\begin{aligned} \alpha &= 3.23 \times 10^6 \text{ psi} & \tau_1 &= 0.3 \times 10^{-3} \\ \beta &= 0.90 \times 10^3 \text{ psi} \end{aligned} \quad (3.7)$$

Here σ_1 , denotes the incremental uniaxial effective stress. Because of the experimental conditions (constant uniaxial stress, changing pore pressure), it is clear that $\dot{\sigma}_1 \equiv \dot{p}$. Hence,

$$(1/C_m) = \frac{d\sigma_1}{d\epsilon_1} = \alpha - (\beta/\tau_1) \exp (- \epsilon_1 / \tau_1) \quad (3.8)$$

The computed values of $(1/C_m)$ during loading are shown in Table 3.1.

During unloading (Fahrenthold and Gray, 1985a),

$$\sigma_1 = A\epsilon_1 + B\epsilon_1^2 \quad (3.9)$$

TABLE 3.1

UNIAXIAL STRAIN TEST DATA (LOADING)

ϵ_1 (10^{-4})	σ_1 (psi)	$1/C_m$ (10^6 psi)
0	0	0.23
1	68	1.08
2	208	1.69
3	400	2.13
4	629	2.44
5	885	2.66
6	1160	2.82
8	1747	2.99
10	2362	3.12
20	5561	3.23

where e_1 is the elastic strain and

$$A = 1.73 \times 10^6 \text{ psi} \quad B = 0.85 \times 10^9 \text{ psi.} \quad (3.10)$$

In this case

$$(1/C_m) = \frac{d\sigma_1}{de_1} = A + 2 B e_1 \quad (3.11)$$

and the computed values of $(1/C_m)$ during unloading are listed in Table 3.2.

3.2.2 Triaxial Test Data

During loading, according to Section A.1 of Appendix A, Young's modulus and Poisson's ratio are given by

$$Y = E - (S_1/\beta_1) \exp(-\epsilon_1/\beta_1) \quad (3.12)$$

$$\nu = Y/[-2\eta_3 + (S_3/\beta_3) \exp(-\epsilon_3/\beta_3)] , \quad (3.13)$$

where (for Pleasant Bayou data fit)

$$\begin{aligned} E &= 2.36 \times 10^6 \text{ psi} \\ \beta_1 &= 0.42 \times 10^{-3} \\ -2\eta_3 &= 9.40 \times 10^6 \text{ psi} \\ S_1 &= -1.08 \times 10^6 \text{ psi.} \end{aligned} \quad (3.14)$$

TABLE 3.2

UNIAXIAL STRAIN TEST DATA (UNLOADING)

e_1 (10^{-4})	σ_1 (psi)	$1/C_m$ (10^6 psi)
0	0	1.73
1	182	1.90
2	380	2.07
3	596	2.24
4	828	2.41
5	1078	2.58
6	1344	2.75
8	1928	3.09
10	2580	3.43
20	6860	5.13

For uniaxial strain ($\epsilon_3 \equiv 0$) the relation for ν reduces to

$$\nu = Y/(-2\eta_3 + S_3/\beta_3).$$

The drained bulk modulus and shear modulus needed to compute C_m and C' from the triaxial data fits are given by

$$K = \frac{Y}{3(1-2\nu)} \quad \mu_p = \frac{Y}{2(1+\nu)} \quad (3.15)$$

The computed values of these material parameters during loading are presented in Table 3.3.

During unloading the triaxial data are fit by

$$Y = M + 2 N e_1 \quad (3.16)$$

$$\nu = Y/(-Q - 2Re_3), \quad (3.17)$$

where

$$M = 0.78 \times 10^6 \text{ psi} \quad Q = -8.82 \times 10^6 \text{ psi} \quad (3.18)$$

$$N = 0.92 \times 10^6 \text{ psi} \quad R = 14.0 \times 10^6 \text{ psi}$$

For uniaxial strain ($\epsilon_3 \equiv 0$) ν reduces to

$$\nu = Y/(-Q). \quad (3.19)$$

The values of the material parameters during unloading computed from the UTA data fits for specified values of the elastic strain are presented in Table 3.4.

TABLE 3.3
 TRIAXIAL TEST DATA (LOADING)

ϵ_1 (10^{-4})	γ (10^6 psi)	ν (10^{-2})	K (10^7 psi)	μ_p (10^7 psi)	$1/C_m$ (10^6 psi)	$1/C'$ (10^6 psi)
0	0.741	2.04	2.58	3.63	0.80	0.72
1	1.084	2.98	3.84	5.26	1.23	1.05
2	1.354	3.72	4.88	6.53	1.59	1.32
3	1.567	4.31	5.72	7.51	1.90	1.54
4	1.735	4.77	6.39	8.28	2.16	1.72
5	1.868	5.13	6.94	8.88	2.37	1.87
6	1.972	5.42	7.37	9.35	2.55	2.00
8	2.119	5.82	7.99	10.01	2.81	2.18
10	2.210	6.07	8.38	10.42	2.98	2.29
20	2.346	6.44	8.98	11.02	3.24	2.48

TABLE 3.4
 TRIAXIAL TEST DATA (UNLOADING)

e_1 (10^{-4})	$\bar{Y}$ (10^6 psi)	ν (10^{-1})	K (10^7 psi)	μ_p (10^7 psi)	$1/C_m$ (10^6 psi)	$1/C'$ (10^6 psi)
0	0.780	0.885	3.16	3.58	0.876	0.787
1	0.964	1.09	4.11	4.35	1.131	0.998
2	1.148	1.30	5.17	5.08	1.413	1.231
3	1.332	1.51	6.36	5.97	1.740	1.501
4	1.516	1.72	7.70	6.47	2.124	1.824
5	1.700	1.93	9.23	7.12	2.590	2.229
6	1.884	2.14	10.98	7.76	3.182	2.768
8	2.252	2.55	15.32	8.97	5.052	4.723
10	2.620	2.97	21.51	10.10	9.880	12.93
20	4.460	5.06*				

(* Not permissible)

Comparison of Tables 3.1 and 3.3 shows that the values of C_m during loading obtained from the UTA uniaxial data fits are in good agreement with those obtained from the triaxial data fits. Tables 3.2 and 3.4 show that the values of C_m during unloading obtained from the uniaxial data are qualitatively similar to those obtained from the triaxial data fits but the quantitative agreement is not good.

3.3 CYCLICAL LOADING

During production testing of a geopressured reservoir the well is subjected to multiple flow rate changes and shutin periods. The UTA and Terra Tek tests on cores from the geopressured design wells do not appear to have included cyclical load/unload test conditions simulating repetitive reservoir pressure drawdown/buildup conditions. The UTA group, however, has performed such tests on sandstones from the Hitchcock well. The results of one of their tests are shown in Figure 3.1 for illustrative purposes. The tests show irreversible compaction with hysteresis loops on cyclical loading/unloading. In general, it was found that the loading curve returned to the previous loading curve after an unloading cycle.

HITCHCOCK(9.138 FT.): S1 VS. E1, TRIAXIAL TEST (C)

Figure 3.1. Response of formation rock core from Hitchcock well under repetitive loading and unloading (from Fahrenthold and Gray, 1985b).

IV. ANALYSIS FOR BILINEAR COMPACTION MODEL

The values of $1/C'$ listed in Tables 3.3 and 3.4 confirm that the rock compressibility during loading is greater than during unloading. The net reduction in porosity as a consequence of fluid pressure drawdown/buildup will tend to maintain the pressure in a reservoir undergoing net fluid withdrawal. This consequence of the irreversible compaction mechanism can be approximated in a bilinear fashion by specializing Equation (3.2) as follows:

$$\begin{aligned}\dot{\phi} &= (1 - \phi_0) C_U \dot{p} \quad \text{if } \dot{p} > 0 \text{ and/or } p > p_{\min} \\ &= (1 - \phi_0) C_L \dot{p} \quad \text{otherwise.}\end{aligned}\quad (4.1)$$

Here ϕ is the formation porosity, ϕ_0 its initial value, and $p_{\min}$ is the minimum pore pressure experienced by the rock at the specific location up to the present time. This condition ensures that the load curve returns to the previous loading curve after an unloading cycle as observed during the UTA tests on the Hitchcock sandstone samples.

The total compressibility during loading (L) or during unloading (U) is computed from the sum of the formation rock pore volume compressibility (C_f) and fluid compressibility (C_w) in accordance with the relation

$$\begin{aligned}C_{Tj} &= C_{fj} + C_w \\ &= \frac{1-\phi}{\phi} C'_j + C_w \quad (j = L, U)\end{aligned}\quad (4.2)$$

The coefficient C_j in the pore volume compressibility term equals the associated uniaxial compressibility if and only if the rock grain compressibility is negligible (Garg, 1986).

4.1 SIMULATOR THOR

The parametric calculations employed the S-CUBED general purpose reservoir simulator THOR which treats large variations in formation compressibility by using an incremental relationship to calculate changes in porosity with pore pressure. Irreversible compaction was approximated in a bilinear fashion according to relation (4.1)

The thermomechanical properties for the brine/methane reservoir fluid at each stage of the THOR calculations were computed using the S-CUBED CH4BRN equation-of-state package (Pritchett, 1985). The CH4BRN equation of state is an update of an earlier S-CUBED software package. In particular, recent data concerning the compressibility of liquid brines described by Osif (1984) and measurements of methane solubility in brines reported by Price, *et al.* (1981) are incorporated. Since the objective of the parametric experiments is to determine the effects of irreversible formation compaction on reservoir behavior, the initial conditions and fluid properties were kept fixed for each calculation.

The initial reservoir conditions

$$p_i = 12,784 \text{ psia (88,143 kPa)}$$

$$T_i = 145.39^\circ\text{C (293.7}^\circ\text{F)}$$

and the mass fractions of the reservoir fluid

H₂O: 0.904364

CH₄: 0.003296

NaCl: 0.092340

were the same in each case treated. These values are based on the conditions measured for Sand Zone No. 8 tested by the Gladys McCall No. 1 well. The corresponding fluid compressibility (C_w) at the initial reservoir conditions computed from the CH4BRN package is

$$C_w = 2.56 \times 10^{-6} \text{ psi}^{-1} \quad (3.71201 \times 10^{-10} \text{ Pa}^{-1}).$$

The following properties of the reservoir rock were kept constant throughout all calculations:

Rock grain density = 2633 kg/m³

Initial rock porosity = 0.2

Rock grain thermal conductivity = 5.25 watts/m-°C

Rock heat capacity = 963 joules/kg-°C.

The reservoir permeability (k) and pore volume compressibility (C') were varied.

Finite changes of permeability with the changes in porosity, $k = k(\phi)$, were represented in each of the parametric calculations by a choice of one of the following relations:

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^n \quad (4.3)$$

$$\frac{k}{k_0} = 1 - N (p_i - p) \quad (4.4)$$

The second relationship can be used to treat a linear decrease in permeability with a decrease in pore fluid pressure.

4.2 EXTENSION OF CLASSICAL WELL TEST ANALYSIS

To examine the effects of irreversible compressibility (and associated variations in permeability) a series of simulated well test calculations were performed in which the well was produced at a constant rate and then shutin. These calculations will be described in a subsequent section. The pressure drawdown and buildup response of the reservoir can also to some extent be analyzed using an extension of the classical semi-log analysis methods modified for bilinear irreversible compaction (Garg, et al., 1978). The extended theory does not consider changes in permeability during drawdown or buildup. The interpretation of the pressure drawdown response of the reservoir is unchanged except the normally assumed reversible total compressibility (C_T) is replaced by C_{TL} . In analyzing the buildup data it is further assumed that the total compressibility during buildup is given by C_{TU} throughout the reservoir (this is not strictly true as pore pressure will be declining in parts of the reservoir distant from the well).

For a well in an infinite reservoir undergoing production at a constant volumetric rate, classical linear theory yields the following approximate solution for the wellbottom pressure (in oil field units),

$$p_{wf} = p_i - \frac{162.6 q\mu}{kh} \left[\log \frac{kt}{\phi\mu C_T r_w^2} - 3.2275 + 0.8686 s \right]. \quad (4.5)$$

Here the skin factor (s) is introduced to account for the pressure drop near the sandface, due to formation damage and completing the well, and

p_{wf} = flowing bottomhole pressure (psia)

p_i = initial reservoir pressure (psia)

q = wellbottom volumetric flowrate (bbbls/day)

μ = fluid dynamic viscosity (cp)

h = reservoir thickness (ft)

t = production time (hrs)

k = reservoir permeability (md)

r_w = radius of the well (ft).

For a reservoir exhibiting bilinear irreversible compaction, the slope (m_L) of the semi-log plot of p_{wf} versus t yields an estimate of the permeability during drawdown,

$$k_L = \frac{162.6 q\mu}{m_L h}. \quad (4.6)$$

The value of the skin factor can be computed from the expression

$$s_L = 1.1513 \left[\frac{p_i - p_1}{m_L} - \log \frac{k_L}{\phi \mu C_{TL} r_w^2} + 3.2275 \right] \quad (4.7)$$

If the well is shut in at time $t = t_p$ then the wellbottom buildup pressure at $t = t_p + \Delta t$ is approximated by

$$p_{ws} = p_i - \frac{162.6 q \mu}{k_U h} \left[\log \frac{t_p + \Delta t}{\Delta t} + \log \frac{C_{TU}}{C_{TL}} + 0.8686 (s_L - s_U) \right] \quad (4.8)$$

The slope of the Horner plot of the buildup data yields an estimate of the permeability,

$$k_U = \frac{162.6 q \mu}{m_U h} \quad (4.9)$$

The skin factor can be estimated from the Horner plot using the expression

$$s_U = 1.1513 \left[\frac{p_{\Delta t=1} - p_{wf}}{m_U} - \log \frac{k_U}{\phi \mu C_{TU} r_w^2} + 3.2275 \right] \quad (4.10)$$

The true skin factor due to drilling/completing the well should be the same whether estimated from drawdown or from buildup data, i.e. $s_L = s_U$. We note from Equation (4.8) therefore that as $(t_p + \Delta t)/\Delta t \rightarrow 1$, then

$$p_{ws} \rightarrow p^* = p_i + m_U \log \frac{C_{TL}}{C_{TU}} \quad (4.11)$$

For a well located near the center of a reservoir with symmetric geometry such that boundary effects do not complicate the response, Equation (4.11) can be a useful diagnostic. Irreversible compaction of the formation would result in $p_{ws} \rightarrow p^* > p_i$, whereas $p_{ws} \rightarrow p_i$ for a reservoir with linear properties.

We note from Equation (4.11) that for a given bilinear compressibility ratio C_{TL}/C_{TU} , the magnitude of $p^* - p_i$ is inversely proportional to the transmissivity (kh) of the reservoir.

V. PARAMETRIC CALCULATIONS

The one-dimensional version of THOR was employed to simulate a cylindrical reservoir with a single well located at its center. The thickness (h) and outer radius (R) of the reservoir were varied. The well is produced at a constant rate of

$$\dot{m} = 25.914 \text{ kg/s}$$

until shutin. This mass flow rate is equivalent to a well-bottom volumetric production rate of

$$q = \frac{\dot{m}}{\rho} = \frac{25.914}{1025.83} = 0.025262 \text{ m}^3/\text{s}$$

$$= 13,728 \text{ res bbls/day.}$$

This value corresponds to the average surface volumetric production rate ($q = 14,162 \text{ sep bbls/day}$) maintained during the Gladys McCall well Reservoir Limits Test of Sand Zone No. 8. The skin factor associated with formation damage during drilling and completion of the well was assumed to be negligible ($s = 0$) for all of the parametric calculations.

Table 5.1 defines the cases for which parametric calculations have been performed to simulate a simple drawdown/buildup well test. The first two calculations (Cases a and b) considered a reservoir of thickness ($h = 100 \text{ m}$) and volume ($V = \pi h R^2 = 436 \times 10^6 \text{ m}^3$) approximating the dimensions estimated for Gladys McCall Sand Zone No. 8 at

TABLE 5.1

SUITE OF PARAMETRIC CALCULATIONS PERFORMED WITH THOR

Case	h (m)	R (m)	V (10 ⁶ m ³)	k ₀ (md)	C _L (10 ⁻⁶ psi ⁻¹)	C _L /C _U	n*	N**
a	100	1178	436	140	0.5	5	1.9	—
b	100	1178	436	140	0.5	1	1.9	—
c	20	2336	343	140	0.5	5	1.9	—
d	20	2336	343	20	0.5	5	1.9	—
e	20	2336	343	20	0.5	1	1.9	—
f	20	2336	343	20	5.0	1	1.9	—
g	20	2336	343	20	5.0	5	1.9	—
h	20	2336	343	20	5.0	1	0.0	—
i	20	2336	343	20	5.0	5	0.0	—
jℓ	20	2336	343	20	0.5	1	—	1.6×10 ⁻⁴
kℓ	20	2336	343	20	5.0	1	—	1.6×10 ⁻⁴

$$* \quad \frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^n$$

$$** \quad \frac{k}{k_0} = 1 - N (p_i - p), \quad N \text{ in psi}^{-1}$$

the conclusion of the Reservoir Limits Test (Pritchett and Riney, 1985). The initial permeability ($k_0 = 140$ md) and the variation of k with ϕ assumed for Cases a and b are based on the data in Section A.1 of Appendix A. This permeability (140 md) is approximately equal to the value estimated from the Reservoir Limits Test pressure transient measurements. Five of the remaining calculations were designed to examine the effects of reservoir thickness (Case c), permeability (Cases d and e), compressibility (Cases f and g) and the dependence of permeability on either the porosity or the pore pressure (Cases j ℓ and k ℓ). The remaining two calculations with constant permeability serve as base cases for reversible (Case h) and irreversible (Case i) compaction.

During the drawdown phases of Cases a and b the formation properties were identical. The compressibility for Case b was kept the same during drawdown and buildup ($C_L = C_U = 0.5 \times 10^{-6}$ psi $^{-1}$). The pore volume compressibility during buildup for Case a, however, was reduced from its drawdown value ($C_U = 0.2$ $C_L = 0.1 \times 10^{-6}$ psi $^{-1}$). In spite of this large irreversibility in the formation compressibility, the pressure buildup at the sandface for Case a was extremely close to that calculated for Case b. The buildup responses for both cases were calculated under the assumptions that the well was shutin after producing (at $q = 13,728$ res bbls/day) for $t_p = 37$ days and $t_p = 337$ days. At corresponding buildup times the difference between the sandface pressures calculated for the two cases was always less than 5 psi.

A contributing factor to the negligible effect of irreversibility is the large transmissivity (kh) assumed for Cases a and b. This results in a small value of m_U and, consequently, the value of p^* calculated from Equation

(4.11) is only very slightly larger than p_i . Table 5.2 lists the values of m_j and $p^* - p_i$ calculated from Equation (4.11) for Cases a through i. Case c was designed to reduce the transmissivity without altering the formation properties with the expectation that the resulting higher pressure gradient would emphasize the effect of the irreversible compressibility. The pressure gradient, however, was still rather small for Case c. In the remaining parametric calculations the initial formation permeability was reduced from 140 md to 20 md and the thickness of the reservoir was set equal to 20 m.

The formation properties for Cases d and e are the same as for Cases a and b, respectively, except for the reduction in the initial permeability to $k_0 = 20$ md. Since the reservoir thickness is also reduced (from 100 m to 20 m), the initial reservoir transmissivity for Cases d and e is only one thirty-fifth of the value employed for Cases a and b.

The next four parametric calculations (Cases f through i) assumed much larger pore volume compressibility; during loading the coefficient $C_L = 5 \times 10^{-6} \text{ psi}^{-1}$. This value of C_L for the bilinear model used in THOR is close to the upper limit of the range of values measured by UTA. Since C_L is a much larger fraction of the total compressibility (C_T) under this assumption, the irreversibility of C_L will have a much greater effect than for the cases discussed above. As listed in Table 5.2, the ratio C_{TL}/C_{TU} is equal to 3.44 for Cases g and i whereas it is only 1.54 for Cases a, c and d. Cases f and g differ from Cases h and i, respectively, only by including a reduction of permeability with porosity,

TABLE 5.2
EXTRAPOLATED RESERVOIR PRESSURE INCREMENTS

Case	C_L/C_U	C_{TL}/C_{TU}	m_U (MPa/cycle)	$p^* - p_i$	
				(kPa)	(psi)
a	5	1.54	0.089	16.7	2.4
b	1	1	0.089	16.7	2.4
c	5	1.54	0.447	83.8	12
d	5	1.54	3.13	587	85
e	1	1	3.14	0	0
f	1	1	3.25	0	0
g	5	3.44	3.14	1685	244
h	1	1	3.14	0	0
i	5	3.44	3.02	1620	235

$$\frac{k}{k_0} = \left(\frac{\phi}{\phi_0} \right)^{1.9} \quad (5.1)$$

Cases jℓ and kℓ treat the case where the permeability is a strong linear function of the pore pressure. The permeability is assumed to drop to one-fifth its initial value when the pressure decreases by 5000 psi from the initial reservoir pressure,

$$\frac{k}{k_0} = 1 - 1.6 \times 10^{-4} (p_i - p), \quad \Delta p \text{ in psi} \quad (5.2)$$

This is based on the data for the Gladys McCall rock reported by Kelkar, *et al.* (1982) and listed in Section A.2 of Appendix A. The formation compressibility is assumed to be reversible with the values $C_L = C_U = 0.5 \times 10^{-6} \text{ psi}^{-1}$ for Case jℓ and $C_L = C_U = 5 \times 10^{-6} \text{ psi}^{-1}$ for Case kℓ.

VI. SIMULATION OF SIMPLE DRAWDOWN/BUILDUP TESTS

Figure 6.1 shows the reduction in the permeability of the formation near the well at production time

$$t_p = 12.25 \text{ days}$$

for all of the well tests simulated with THOR (Table 5.2). The reduction in permeability for Cases a, b and c are negligible since the reservoir transmissivity is large. The permeability is held constant ($k = k_o = 20 \text{ md}$) for Cases h and i. Even when the transmissivity employed in Cases a and b is decreased by a factor of 35 (Cases d and e) the reduction in permeability near the well is small. When the formation compressibility is also increased (Cases f and g) the permeability reduction is significant near the well.

Figure 6.2 illustrates the effect of irreversible compaction on the formation permeability for Case g. The initial ($t_p = 0$) constant reservoir permeability of 20 md monotonically decreases in the vicinity of the wellbore up to the time the well is shutin ($t_p = 12.25 \text{ days}$). As the pressure builds up after shutin the pore volume expands but much less than it would if the compressibility was not reduced during unloading ($C_u < C_i$). The associated permeability recovery is also constrained according to Equation (5.1). As shown in Figure 6.2 there is very little recovery between $\Delta t = 7.5 \text{ hrs}$ and $\Delta t = 104 \text{ days}$. The permanent reduction in permeability near the well would be interpreted as an increase in the apparent skin factor.

During drawdown testing of a well, the pressure reduction at a given location will tend to decrease the local specific internal energy because of the increase in the

Figure 6.1. Formation permeability as function of distance from sandface ($r = r_w$) of well at production time $t_p = 12.25$ days.

Figure 6.2. Formation permeability at indicated production (t_p) and shut-in ($\Delta t = t - 12.25$ days) times as function of distance from sandface ($r = r_w$) for Case g.

specific volume. The fluid movement towards the well, however, will tend to increase the specific internal energy as a result of the local viscous energy dissipation. Both the counteracting pressure work and the viscous dissipation terms have been included in the energy balance relations incorporated in THOR (Garg and Pritchett, 1977). Consequently, the simulated well tests also provide calculations of the local temperature changes in the reservoir for the assumed reservoir formation parameters.

Figure 6.3 shows the temperature distributions in the reservoir near the well at a production time of $t_p = 12.25$ days. The decrease in the specific internal energy associated with the radially expanding pressure drawdown pulse dominates at radii sufficiently large ($r/r_w > \sim 300$); the temperature is decreased slightly. Nearer the well the increasing velocity of the fluid causes the viscous dissipation to dominate for all simulations other than Cases a and b; the temperature increases significantly above its initial value ($T_o = 145.39^\circ\text{C}$). For Cases a and b the high reservoir transmissivity (kh) results in very small viscous dissipation; there is a slight temperature decrease in the produced fluid ($T_{wf} = 145.37^\circ\text{C}$ at $t_p = 12.25$ days). The temperature drops only to $T_{wf} = 144.95^\circ\text{C}$ at $t_p = 1.362$ years at which time the pressure has dropped from the initial reservoir pressure of $p_i = 88,143$ kPa (12,784 psia) to $p_{wf} = 68,387$ kPa (9,919 psia).

Figures 6.4 and 6.5 show semi-log plots of the sand-face pressures during drawdown for Cases d through k ℓ . For the case of linear reservoir properties the semi-log plot should be approximated by a straight line of slope

Figure 6.3. Reservoir temperature as function of distance from sandface ($r = r_w$) of well at production time $t_p = 12.25$ days.

Figure 6.4. Semi-log drawdown pressure plots for Cases d, e, h, i, jℓ and kℓ.

Figure 6.5. Semi-log drawdown pressure plots for Cases f, g, h and i.

$$m = \frac{162.6 \mu(qB)}{kh} = \frac{162.6 (0.26568) (13,728)}{(20) (65.617)}$$

$$= 452 \text{ psi/cycle} = 3.12 \text{ MPa/cycle} \quad (6.1)$$

In Case h the formation parameters were held constant ($k = k_0 = 20 \text{ md}$, $C_L = C_U = 0.5 \times 10^{-6} \text{ psi}^{-1}$) and the slope of the semi-log plot of the calculated drawdown pressure is in excellent agreement with the expected value. Cases h and i are identical during drawdown.

The reduction in permeability for Cases d and e is only significant near the well and it does not noticeably affect the slope of the straight line approximation to the calculated drawdown response (compare Cases d and e with Cases h and i in Figure 6.4). Its effect during drawdown shows up as a vertical displacement which would be interpreted as a difference in the skin factor.

Figure 6.5 compares the sandface pressures during drawdown of Cases f and g with the calculation performed with linear formation properties (Case h). The higher rock compressibility for Cases f and g results in a greater reduction in ϕ and consequently a larger reduction in k than is the case for the corresponding calculations using the smaller compressibility values (Cases e and d, respectively). The semi-log plots for Cases f and g infer an effective permeability reduction of about 15 percent rather than simply the presence of a nonzero skin factor as would be inferred for Cases d and e (Figure 6.4).

Figure 6.6 presents semi-log buildup plots for Cases d and e after shutting the well subsequent to production periods of $t_p = 3.92 \text{ days}$ and $t_p = 12.25 \text{ days}$. The slopes of the straight lines approximating the calculated sandface pressure are all in the range

Figure 6.6. Semi-log shutin pressure plots for Cases d and e.

$m \sim 2.99$ to 3.13 MPa/cycle.

These values agree with the drawdown slope of 3.06 MPa/cycle within the accuracy of the semi-log method. Figure 6.7 presents a Horner plot for Cases d and e for the pressure buildup after the production period $t_p = 12.25$ days. The effect of the irreversible compressibility and associated permeability changes during buildup would likely be analyzed as variations in the skin factor(s).

Figure 6.8 presents Horner plots of the calculated buildup pressures for Cases f and g. The corresponding results for the base cases (Cases h and i) are also shown; inclusion of the variation of k given by Equation (5.1) has a relatively small effect. The irreversibility (Case g) during buildup results in an apparent skin factor and a somewhat higher permeability than for the corresponding case with reversible compressibility (Case f). The permeabilities inferred from the buildup response for both Cases f and g are about 10 percent greater than inferred from the semi-log drawdown plot (Figure 6.5).

The semi-log plots of the calculated drawdown sandface pressures for Cases j_l and k_l are presented in Figure 6.4. For the less compressible rock (Case j_l) there is a greater pressure drawdown and consequently a more dramatic decrease in permeability; under this assumption the sandface pressure decreases to the hydrostatic head value very quickly and the constant flow rate cannot be sustained beyond $t_p = 10$ days. For the more compressible rock (Case k_l) there is still a dramatic decrease in permeability. The straight line approximation to the drawdown data infers an effective decrease to about 60 percent of the initial permeability, but the reduction is much greater near the well (see Figure

Figure 6.7. Horner plots for Cases d and e: $p_i = 88143$ kPa and $p^* = 88730$ kPa.

Figure 6.8. Horner plots for Cases f, g, h and i: $p_i = 88143$ kPa and $p^* = 89830$ kPa.

6.1). The falloff of the pressure below the straight line approximation in Figure 6.4 results from the increasing radius of the low-permeability zone near the well; the apparent effect is equivalent to a continuously increasing skin factor.

Figure 6.9 presents the Horner plot of the buildup for Case k_l subsequent to producing the well for $t_p = 12.25$ days. The slope of 3.56 MPa/cycle infers an effective reduction in permeability to a value about 12 percent below the initial value. The inferred permeability during buildup, however, is 50 percent higher than would be inferred from the drawdown semi-log plot for Case k_l (Figure 6.4). Horner plots of the calculated buildup pressures for Cases j_l and k_l after producing for $t_p = 4.9$ hours were also made. In both cases the calculations are approximated by the same straight line as shown in Figure 6.9.

Figure 6.9. Horner plot for Case $k\ell$.

VII. INTERPRETATION OF SIMULATED WELL TESTS

As illustrated by the Horner plots shown in Figures 6.7 and 6.8, values of $p^* > p_i$ are obtained for each of the simulated well tests in which irreversible compaction was assumed in the calculations (i.e., Cases d, g and i). On the other hand, $p^* = p_i$ is obtained for the remaining cases wherein the compressibility was the same during drawdown and buildup. The values of p^* obtained from the plots are in excellent agreement with the values calculated from Equation (4.11) and listed in Table 5.2.

Although $p^* > p_i$ is indicative of irreversible compactions for the cylindrical reservoir simulations presented here, it is not generally a useful diagnostic. The presence of nearby faults in a highly skewed reservoir configuration would preclude a straight line being established prior to the onset of boundary effects.

We now consider how the analyst would interpret the results of the simulated well tests using classical methods. In Table 7.1 we have listed the values of k and s that would be inferred from the pressure transient data if it were assumed that $C_U = C_L$. The values listed for k_L (drawdown) and k_U (buildup) are computed from Equations (4.6) and (4.9) with

$$q = 13,728 \text{ res bbls/day}$$

$$\mu = 0.2657 \text{ cp}$$

$$h = 65.617 \text{ ft (20 m)}$$

TABLE 7.1
APPARENT PERMEABILITIES AND SKIN FACTORS INFERRED
FROM CONVENTIONAL ANALYSIS OF
PARAMETRIC WELL TEST SIMULATIONS

Case	k_L (md)	k_U (md)	C_{TL} (10^{-6} psi $^{-1}$)	s_L	s_U	$s_L - s_U$
d	20.36	19.90	4.56	+0.092	+0.137	-0.045
e	20.36	19.84	4.56	+0.092	-0.072	+0.164
f	18.01	19.17	22.56	-0.374	+0.146	-0.520
g	18.01	19.84	22.56	-0.374	+0.983	-1.357
h	20.36	19.84	22.56	+0.049	-0.142	+0.191
i	20.36	20.63	22.56	+0.049	+0.715	-0.666
k ℓ	11.65	17.50	22.56	-1.496	+3.184	+4.680
g2*	18.01	19.84	22.56	+1.489	+1.016	+0.473

* See Section VIII.

and the slopes (psi/cycle) of the straight line approximations shown in Figures 6.4 through 6.9. The values for the apparent skin factor listed are computed from Equations (4.7) and (4.10) with the drawdown values of total compressibility (C_{TL}) listed in Table 7.1 used for both the drawdown (s_L) and buildup (s_U) computations.

In the parametric calculations the skin factor due to formation damage and completion of the well was set equal to zero in all cases. The values listed in Table 7.1 are apparent skin factors resulting from applying methods based on linear theory to reservoirs with nonlinear formation properties. Only Case h simulates a reservoir with linear properties; the small but nonzero values of s inferred for Case h by the classical analysis reflect inherent inaccuracies in applying graphical semi-log methods. In Case i the permeability is constant; the significant apparent skin factor estimated from the buildup data from this simulated well test results from the irreversible compressibility. In Case k₂ the compressibility is constant; the large negative (positive) apparent skin factor inferred from the drawdown (buildup) analysis results from the large variations in permeability.

Table 7.1 also lists the difference between the apparent drawdown and buildup values of the skin factor ($s_L - s_U$). Of interest is the result that the difference between the apparent skin factor values calculated from drawdown and buildup data ($s_L - s_U$) is negative for each of the three cases (Cases d, g and i) in which reservoir simulated is irreversibly compacted. Since a real skin factor due to formation damage and/or well completion would be the same during drawdown and buildup, well test data for which

classical analysis yields a significantly higher apparent skin during buildup than drawdown might indicate variable formation properties.

We note that the values for the skin estimated by Pritchett and Riney (1985) from the drawdown and buildup portions of the Reservoir Limits Test of Sand Zone No. 8 are as follows:

$$s_L = + 0.98$$

$$s_U = + 2.55$$

These values yield

$$s_L - s_U = - 1.57$$

Although this difference might appear to indicate irreversible compaction, it more likely merely reflects inaccuracies in calculating the skin factor.

For long shutin periods the effect of the reservoir's outer cylindrical boundary comes into play and the Horner plot of the buildup pressures deviate from the straight line approximation. For Cases a through g the simulations of the buildup periods were continued sufficiently long to obtain the final static reservoir pressure ($\bar{p}$). The values of the corresponding pressure drops from the initial reservoir pressure ($\Delta p = p_i - \bar{p}$) are listed in Table 7.2

We now consider how the analyst would interpret the long-term simulated buildup data using classical methods. The connected pore volume of the reservoir would be inferred from the Δp values under the assumption that $C_U = C_L$ using the relation

TABLE 7.2

APPARENT RESERVOIR PORE VOLUMES INFERRED FROM CONVENTIONAL
ANALYSIS OF PARAMETRIC WELL TEST SIMULATIONS

Case	t_p (days)	Q (res bbls)	C_{TL} (10^{-6} psi $^{-1}$)	$p_i - \bar{p}$ (psi)	Inferred V_p (10^6 res bbls)	Actual V_p (10^6 res bbls)
a	37	507,936	4.56	207.8	536	548
	337	4,626,336	4.56	1883	539	548
b	37	507,936	4.56	208.4	534	548
	337	4,626,336	4.56	1884	538	548
c	12.25	168,168	4.56	84.7	435	431
d	12.25	168,168	4.56	70.4	524	431
e	12.25	168,168	4.56	87.7	420	431
f	12.25	168,168	22.56	17.4	428	431
g	12.25	168,168	22.56	3.86	1,932	431
h	12.25	168,168	22.56	17.4	428	431
i	12.25	168,168	22.56	3.99	1,869	431
g1+g2*	25	336,336	22.56	12.9	1,151	431
g*	132.25	1,815,528	22.56	138.2	582	431
ff*	110.25	1,513,512	22.56	156.9	428	431
gg*	110.25	1,513,512	22.56	115.3	582	431

*See Section VIII.

$$V_P = \frac{Q}{C_T \Delta p} , \quad (7.1)$$

V_P = pore volume (res bbls)

Q = total fluid produced (res bbls)

C_T = total compressibility (psi^{-1})

Δp = average reservoir pressure drop (psi).

Table 7.2 lists the estimated V_P values based on Equation (7.1) and the assumption $C_T = C_{TL}$. For comparison purposes the actual V_P values employed in the simulations are also listed. The simulations representing reversible compaction behavior (Cases b, e, f and h) provide estimates in good agreement with the actual V_P values. For simulations representing irreversible compaction (Cases a, c, d, g and i) the amount by which the estimated values exceed the actual V_P values increased as the reservoir transmissivity decreased. For Cases g and i the inferred reservoir volume would be over four times the actual value. When the large transmissivity and reservoir volume estimated for Sand Zone No. 8 at the end of the Reservoir Limits Test are employed (Cases a and b), however, the effect is small.

VIII. SUCCESSIVE DRAWDOWN/BUILDUP TESTING

For a single drawdown/buildup test the pressure at a given radius decreases monotonically to a minimum value and then monotonically increases. The radius of investigation during the drawdown portion of the test can be estimated by (see e.g., Matthews and Russell, 1967)

$$r_{inv} = \sqrt{0.00105 \frac{k t_1}{\phi \mu C_{TL}}} \text{ ft,} \quad (8.1)$$

where oil field units are employed and t_1 is the duration of the drawdown period. For a formation with bilinear response, Equation (4.1), the total compressibility subsequent to the initial drawdown/buildup test is given by

$$\begin{aligned} C_T &= C_{TU} \text{ for } r < \sim r_{inv} \\ &= C_{TL} \text{ for } r > \sim r_{inv} \end{aligned} \quad (8.2)$$

If the well is subsequently reopened, the rates at which the second drawdown pressure pulse propagates radially outward will be faster than the first since $C_{TU} < C_{TL}$. The original radius of investigation will be reached at time t_2 during the second drawdown period as approximated by

$$t_2 \sim t_1 \frac{C_{TU}}{C_{TL}} \frac{k/\phi}{(k/\phi)} \quad (8.3)$$

Here $(\overline{k/\phi})$ represents an effective value of k/ϕ in the region of the formation surrounding the well ($r < r_{inv}$) which has been processed by the first drawdown/buildup test.

To demonstrate the effect of successive drawdown/buildup testing, the material parameters employed in Case g were again used in a simulation in which the flow rate of the well varied as follows ($q_o = 13,728$ res bbls/day):

$$\begin{aligned} q &= q_o & 0 &< t < 6 & \text{days} \\ &= 0 & 6 &< t < 6.25 & \text{days} \\ &= q_o & 6.25 &< t < 12.5 & \text{days} \\ &= 0 & 12.5 &< t < 37 & \text{days} \\ &= q_o & 37 &< t < 49.25 & \text{days} \\ &= 0 & 49.25 &< t & \end{aligned}$$

The results for the first 37 days of the simulation will be denoted by Case g1 whereas the results from $t > 37$ days are referred to as Case g2.

Figures 8.1 and 8.2 present semi-log plots of the bottomhole pressures for the drawdown and buildup portions of the simulation, respectively. The results for Cases g and h are also shown for comparison purposes.

For $t < 6$ days Case g1 is identical to Case g. After the brief shutin period ($6 < t < 6.25$ days) the bottomhole pressure for Case g1 is of course greater but it rapidly

Figure 8.1. Semi-log drawdown pressure plots for first (Case g₁) and second (Case g₂) drawdown periods for successive testing simulation.

Figure 8.2. Semi-log shutin pressure plots for first (Case g1) and second (Case g2) buildup periods for successive testing simulation.

approaches the drawdown curve for Case g. In plotting Case g1 for $t > 6.25$ days we have used the equivalent production time defined by

$$t_p = \frac{1}{q_o} \int q \, dt \quad (8.4)$$

The buildup portion of Case g1 ($12.5 < t < 37$ days) is seen in Figure 8.2 to be indistinguishable from Case g. At the end of this shutin period the wellbottom pressure has approached a static reservoir pressure of

$$\bar{p} \sim 88,042 \text{ kPa (12,769 psia)}.$$

The bottomhole pressure in the well during the drawdown portion ($37 < t < 49.25$ days) of the second test is shown in Figure 8.1. Although the static pressure prior to the test is only ~ 100 kPa below the initial pressure ($p_i = 88,143$ kPa), the drawdown curve for Case g2 lies below the curve for Case g by ~ 200 kPa. As listed in Table 7.1, the apparent skin factor calculated by conventional analysis for Case g2 is $s_L = + 1.489$ whereas it was $s_L = - 0.374$ for Case g. This apparent additional skin effect results from the lower permeability of the irreversibly compacted formation near the well. Also of interest to an analyst is the abrupt change in slope of the drawdown curve for Case g2. This effect results from the discontinuity in formation properties at the radius of investigation of the first drawdown test (Equation 8.1). Since k/ϕ is only about 15 percent less than k/ϕ for Case g, the time at which the effect of the formation discontinuity is observed should be approximated by

$$t_2 \sim t_1 \frac{C_{TU}}{C_{TL}} = 12.25 \frac{6.56}{22.56} = 3.56 \text{ days} \sim 3.1 \times 10^6 \text{ sec.}$$

This value is in good agreement with the calculated time of the abrupt slope change in the drawdown pressure semi-log plot (Figure 8.1).

The apparent skin factors calculated by conventional analysis from buildup data are also listed in Table 7.1. The value computed for the first ($s_U = + 0.983$) and second ($s_U = + 1.016$) simulated tests do not differ significantly.

Comparison of the buildup portions of the first and second tests in Figure 8.2 shows that the wellbottom pressure for the second buildup (Case g2) is initially about ~ 800 kPa below that for the first (Case g1) but the difference narrows as the buildup time increases. At the end of the second buildup period the wellbottom pressure has approached a static reservoir pressure of

$$\bar{p} \sim 88,053 \text{ kPa (12,771 psia)}.$$

Using Equation (7.1), conventional analysis of the total well test simulation (Case g1 plus g2) would yield an estimated reservoir pore volume of 1151×10^6 res bbls compared to an actual value of $V_p = 431 \times 10^6$ res bbls. This estimate is less than the estimate made based on Case g (see Table 7.2) since the reservoir formation near the well from which fluid is withdrawn during the second test is less compressible, after the first drawdown/buildup, than it was originally.

To further illustrate the effects of repetitive production/shutin phases that are imposed on a reservoir during long-term production, the material parameters employed in Case g were again used in a simulation in which the flow rate of the well varied periodically as follows ($q_0 = 13,728$ res bbls/day):

$$q = q_0 \quad (n-1)(t_1+t_2) < t < (n-1)(t_1+t_2) + t_1$$

$$= 0 \quad (n-1)(t_1+t_2) + t_1 < t < n(t_1+t_2).$$

Here $n = 1, \dots, 9$ and the drawdown and buildup phases in each of the nine 15-day production/shutin periods are of the following durations:

$$t_1 = 12.25 \text{ days} \quad (\text{drawdown})$$

$$t_2 = 2.75 \text{ days} \quad (\text{buildup}).$$

The flowing wellbottom pressure for this simulation (Case gg) is plotted in Figure 8.3 as a function of the equivalent production time defined by Equation (8.4).

Two additional simulations are also shown in Figure 8.3 for comparison purposes. Case ff (material parameters same as Case f) is identical to Case gg except the formation compressibility is assumed to be reversible. Cases ff and gg are identical up to the time (12.25 days) when the well is first closed. During subsequent production phases, the fluid pressure reduction pulse propagates more rapidly in the irreversibly compacted formation (Case gg) and there is an abrupt slope change at a time corresponding to the time when the front encounters the radius of investigation of the previous drawdown phase.

Figure 8.3. Cartesian plot of long-term production behavior for constant flow rate (Cases f and g) and repetitive production/shut-in phases (Cases ff and gg).

For reference the drawdown response for constant production ($q = q_0$) is also shown in Figure 8.3 for Case g (which is the same as in Case f for constant drawdown).

At the end of the production phase of the ninth cycle, the simulations of the buildup responses for Cases ff and gg were continued until the static reservoir pressure $\bar{p}$ was attained. These values and the static pressure attained for Case g after long-term constant production are listed in Table 7.2. The reservoir volume inferred by an analyst, using Equation (7.1) and the assumption $C_T = C_{TL}$, for the reservoir with reversible formation properties (Case ff) is the same as for the short time drawdown/buildup test (Case f); both estimates for the volume are in excellent agreement with the actual value. The inferred volume for the reservoir with irreversible formation properties, again using Equation (4.1) and the assumption $C_T = C_{TL}$, are the same for the two quite different long-term production histories (Cases g and gg). The inferred value significantly exceeds the actual reservoir volume but not by nearly as much as the values inferred for shorter production periods of 12.25 days (Case g) and 25 days (Case g1 plus g2).

The decrease in the value inferred for V_p from $\bar{p}$ as t_p increases, for the irreversible compaction simulations, is a consequence of Equation (4.1). The irreversible compaction model employed ensures that the load curve returns to the previous loading curve after an unloading cycle. (This property was observed during the UTA tests on the Hitchcock sandstone cores as illustrated in Figure 3.1.) As the production time increases, a greater proportion of the reservoir formation is subjected to the loading curve as the reservoir is depleted.

The buildup response for the shutin phase ($\Delta t < t_2 = 2.75$ days) following each of the nine cycles for Case gg can be used to infer the changing formation permeability as the formation is irreversibly compacted. The semi-log plot of the differential pressure

$$\Delta p_{ws} = p_{ws} \Big|_{\Delta t} - p_{wf} \Big|_{\Delta t = 0}$$

versus Δt is shown in Figure 8.4 subsequent to the first, fifth and ninth cycle. The permeabilities (inferred from the slopes of the lines) decrease only slightly as the formation loading is cycled by the repetitive production/shutin phases.

Figure 8.4. Effect of repetitive loading cycles on apparent formation permeability.

IX. CONCLUSIONS

The parametric simulations of pressure testing of a geopressured well have demonstrated the principal effects of irreversible formation rock compaction. The effect increases with formation compressibility and decreases with increased formation transmissivity. The simulations employed a bilinear model that maximizes the effect of irreversibility by using compressibility values and loading/unloading contrasts that are larger than those reported for either the Pleasant Bayou or Gladys McCall sandstones. The associated reduction in permeability was simulated using either a power dependence on the porosity (based on UTA tests on Pleasant Bayou cores) or a linear dependence on the fluid pore pressure (based on Terra Tek tests on Gladys McCall cores).

When the large transmissivity of Sand Zone No. 8 of the Gladys McCall reservoir is simulated, it is found that irreversible compaction has only a negligible effect on the simulated well test results. Both the permeability and reservoir volume inferred from the simulated pressure test data using classical analysis methods are in excellent agreement with the actual input values.

For a more compressible reservoir of lower transmissivity, simulations using the UTA expression for permeability reduction (with porosity decrease) results in significant permeability reduction near the wellbore. An analyst using conventional methods would interpret the simulated well test data as indicative of a positive skin factor. The apparent skin during buildup would appear to be larger than the

apparent skin inferred during drawdown. This feature might sometimes serve as a diagnostic for irreversible compaction in the reservoir. The effective permeabilities inferred from the simulated pressure transient data are within 15 percent of the initial permeability, even when the well is repetitively produced/shutin.

For simulations using the Terra Tek expression for permeability reduction (five-fold decrease for pore pressure reduction of 5000 psi) there is a large decrease in the inferred permeability. The assumed permeability reduction, however, is based on tests by Terra Tek on a single Gladys McCall core sample and is therefore considered questionable.

The long-term shutin pressure (p^*) inferred from the straight line portion of the Horner plots of the simulated buildup data is greater than the initial pressure (p_i) for calculations in which the formation is irreversibly compacted. Occurrence of $p^* > p_i$ might serve as a diagnostic for wells located near the center of symmetric geopressured reservoirs, but it is not useful for analyzing the Gladys McCall well test data.

The simulations were continued sufficiently long to obtain the final static reservoir pressure ($\bar{p}$). For cases in which the large transmissivity of Sand Zone No. 8 was simulated, there was negligible difference between the $\bar{p}$ values calculated whether the formation is reversible or irreversible. For the more compressible reservoir formations, however, the simulated $\bar{p}$ value is significantly increased by the irreversible formation compaction; the reservoir volume inferred using classical analysis methods is greater than the actual volume. Volumes inferred can be several times the actual reservoir volume.

Under simulated repetitive drawdown/buildup testing of a reservoir with an irreversibly compacting formation, only a small change in the permeability is inferred from the successive buildup tests. As the depletion continues a greater portion of the reservoir is controlled by the loading compressibility. The inferred value of the reservoir volume is larger than the actual value but decreases with successive drawdown/buildup phases.

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APPENDIX A

COMPACTION PROPERTIES OF GEOPRESSURED-GEOTHERMAL
RESERVOIR ROCK

Supplied to S-CUBED
by
UTA Rock Mechanics Group

A.1 GEOPRESSURED-GEOTHERMAL RESERVOIR ROCK

1. Porosity variations follow directly from the bulk deformation model and constant rock grain compressibility coefficient described below.

2. Rock grain compressibility is approximately constant at $0.30 \times 10^{-6} \text{ psi}^{-1}$.

3. Poisson's ratio is given by

$$\nu = (E - (S_1/\beta_1) \exp(-\epsilon_1/\beta_1)) \div (-2\eta_3 + (S_3/\beta_3) \exp(-\epsilon_3/\beta_3))$$

on loading and

$$\nu = (M + 2N e_1) \div (-Q - 2R e_3)$$

on unloading where

$$E = 2.36 \times 10^6 \text{ psi}$$

$$M = 0.78 \times 10^6 \text{ psi}$$

$$-2\eta_3 = 9.40 \times 10^6 \text{ psi}$$

$$N = 0.92 \times 10^9 \text{ psi}$$

$$S_1 = 0.68 \times 10^3 \text{ psi}$$

$$Q = -8.82 \times 10^6 \text{ psi}$$

$$S_3 = -1.08 \times 10^3 \text{ psi}$$

$$R = 14.00 \times 10^9 \text{ psi}$$

$$\beta_1 = 0.42 \times 10^{-3}$$

$$\beta_3 = -0.04 \times 10^{-3}$$

The shear modulus is calculated from

$$\mu = Y/(2(1 + \nu))$$

where ν is given above and

$$Y = E - (S_1/\beta_1) \exp (-\epsilon_1/\beta_1)$$

on loading and

$$Y = M + 2N \epsilon_1$$

on unloading.

The variables ϵ and ϵ_e are total and elastic strains, with the subscripts 1 and 3 denoting axial and radial strain components. These are incremental strains measured from the *in-situ* state.

4. Rock properties may vary as much as 50 percent between samples.

5. Permeability varies with porosity and effective stress according to

$$k = C/\phi^{-m}$$

where

$$-m = m_1 + m_2 P_e + m_3 P_e^2$$

$$P_e = (1/3) (\sigma_1 + \sigma_2 + \sigma_3) - P$$

The coefficients are given by

$$m_1 = -1.9064$$

$$m_2 = -0.0030 \times 10^{-3} \text{ psi}^{-1}$$

$$m_3 = 0.0010 \times 10^{-6} \text{ psi}^{-2}$$

and $C = 140 \text{ md} \times (0.20)^{-m}$ calculated at $P_e = 0$. Here P is the pore pressure and σ_1 , σ_2 and σ_3 are the principal total stresses.

A.2 PROPERTIES OF THE GLADYS McCALL RESERVOIR ROCK

1. Poisson's ratio

Loading: 0.18 ± 0.07

Unloading: 0.13 ± 0.03 .

2. Young's modulus

Loading: $4.3 \pm 0.6 \times 10^6$ psi

Unloading: $5.5 \pm 0.7 \times 10^6$ psi.

3. Uniaxial compaction coefficient:

Loading: $0.21 \pm 0.05 \times 10^{-6}$ psi⁻¹

Unloading: $0.18 \pm 0.03 \times 10^{-6}$ psi⁻¹.

4. Bulk compressibility

Loading: $1.8 \pm 0.9 \times 10^{-6}$ psi⁻¹

Unloading: $1.3 \pm 0.3 \times 10^{-6}$ psi⁻¹.

5. Grain compressibility

Loading: $2.0 \pm 1.3 \times 10^{-7}$ psi⁻¹

Unloading: $2.3 \pm 0.3 \times 10^{-7}$ psi⁻¹.

Permeability

Initial: 9.0 ± 8.0 md (pore pressure = 13,860 psi)

Final: 2.0 ± 1.7 md (pore pressure = 8,860 psi).

With the absence of any data at intermediate pore pressures, a linear variation with pore pressure could be assumed.

7. Porosity variations are calculated using the bulk and grain compressibility coefficients given above.