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COMPUTER MODELING OF THE KLYSTRON*

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Summary

Although the klystron was invented over 40 years ago, it still is quite a scientific challenge to analyze this seemingly simple device. Accurate calculation of space-charge density and particle motions are required, but there are dozens of parameters that must be optimized. Thus the analysis must be inexpensive to run, because over 100 runs typically are required to optimize a particular design. A self-consistent ring model of the klystron interaction is being developed for use as a design tool, with the initial goal of improving the LAMPF klystrons to conserve electrical energy. A mathematical model that includes large-signal effects, relativity, harmonic and fundamental cavities, extended cavity fields, and self-consistent calculations of the induced currents is being developed. The model is self-consistent because the gap fields are calculated interactively, because the currents that drive the cavities are not known until after the beam transverses the cavities. Several test cases have been run, and the basic theory and results to date are discussed.

Introduction

The basic interaction model is that of Rowe,¹ modified to include relativistic effects,² with time (rather than distance) as the independent variable. Space-charge and image forces are calculated with the Green's functions for rings of charge in a cylinder of radius a . The ring fields contain singularities at the source, and several different methods of averaging these fields were tried. The most promising method is to integrate the sources radially and axially and to use a laminar-flow assumption to estimate the radial extent of the rings as the interaction proceeds. This method gives good agreement between the model's predictions and laminar theory as will be shown below. The method may be modified when the code is developed enough to calculate the performance of an entire klystron. Thus, although the particles are treated as zero-volume rings of charge in most of the dynamical calculations, the particles have finite volume for the purposes of space-charge-force calculation.

The gap fields are calculated self-consistently by calculating an induced current integral in each cavity. The induced current depends on the details of the beam motion in the gap; therefore, the gap voltage is calculated by an iterative method. An estimate, based on linear theory, is made for the voltage and phase at a gap; then, the particles are followed through the gap and an induced current integral is calculated. After all the electrons have crossed the gap, this integral is complete; its value is used to recalculate the voltage and phase at the gap. The process is repeated until successive values of voltage and phase are within a prescribed error tolerance of the past values. In actual practice, four to eight iterations are usually required through each gap.

Sometimes, an electron will be reflected at the final gap and travel to the penultimate gap, thereby altering the already computed gap voltage. In these cases, a group of several cavities is considered to be a region, and an iterative procedure is used on the entire region, rather than on each gap individually.

The Mathematical Model

The problem is analyzed in circular cylindrical coordinates r, ϕ, z . The relativistically correct force equations are derived for a particle in a solenoidal-focus field and a TM-circuit field. The normalized force and energy equations are

$$\frac{dV_r}{d\tau} = -\frac{n}{\omega\gamma} \left\{ \frac{F_r}{u_0} \left[1 - (V_r u_n)^2 \right] + V_\phi B_z - V_z (B_\phi + u_n E_z/c) \right\} + \frac{V_z^2}{R} \quad (1)$$

$$\frac{dV_z}{d\tau} = -\frac{n}{\omega\gamma} \left\{ \frac{E_z}{u_0} \left[1 - (V_z u_n)^2 \right] + V_r (B_\phi - u_n V_z F_r/c) + V_\phi B_r \right\} \quad (2)$$

$$\frac{dV_\phi}{d\tau} = -\frac{n}{\omega\gamma} \left\{ V_z (B_r - u_n V_\phi F_z/c) - V_r (B_z + u_n V_\phi F_r/c) \right\} - \frac{V_r V_z}{\rho} \quad (3)$$

$$\frac{dL}{d\tau} = \frac{nu_n}{\omega} (V_r F_r + V_z F_z) \quad (4)$$

where u_0 is electron velocity at the beginning of the problem on the axis, $V = v/u_0$, $u_n = u_0/c$, $n = Iet/m_0$, and $t = \omega t$; ρ and L are normalized dimensions; $R = \rho a/u_0$ and $Z = z a/u_0$. Equation (4) is used rather than the conservation of angular momentum to permit the model to be generalized to include quadrupolar fields at a later date. The particle energy also may be also calculated by

$$\gamma = \left[1 - u_n^2 (V_r^2 + V_\phi^2 + V_z^2) \right]^{-1/2} \quad (5)$$

and the two sets of γ 's may be compared to estimate integration errors.

The fields in Eqs. (1)-(4) are the superposition of focus, gap, and space-charge fields. The static focus fields are assumed to be piecewise linear on the axis, and are expanded to first order in r , giving B_z and B_r components.

The gap fields may be calculated in several ways in the code. The most accurate method programmed to date is to use the relations

$$E_{gz} = \frac{V_{gk} I_0(n_k r/a)}{q_k I_0(n_k r/a)} \cos^2 \left(\frac{z}{a} - \frac{t}{q_k} \right) + \cos(\phi_{0k} + n_k t) \quad (6)$$

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and

$$E_{gr} = \frac{4\pi V_{gk}}{n_k \gamma_r a_k^2} \frac{I_1(n_k \gamma_r r)}{I_0(n_k \gamma_r a)} \sin \left(4\pi \frac{z - z_{gk}}{G_k} \right) \times \cos(\phi_{ok} + n_k \tau), \quad (7)$$

where V_{gk} is the voltage of the k^{th} gap, g_k is the normalized axial length of the gap, G_k is the normalized length, z_{gk} is the axial location of the gap center, ϕ_{ok} is the phase of the gap voltage, $\gamma_r = \omega/u_0 \gamma_0$, γ_0 is the relativistic factor corresponding to u_0 , and n_k is the harmonic number of the k^{th} gap. Equations (6) and (7) correspond to an axial variation of $\cos^2(z)$ in the axial electric fields. A more accurate method has been suggested by a co-worker, K. R. Crandall. A few terms of the series

$$\Gamma_{qz}(r, z) = \sum_{m=1}^{\infty} \frac{A_m}{I_0(\gamma_m a)} \frac{I_0(\gamma_m r)}{I_0(\gamma_m a)} \cos(k_m z) \quad (8)$$

may be calculated by an rms fit between data from a Superfish calculation of the cavity fields and those coefficients used to describe the field. This procedure is used by Crandall in the PARMELA code, and will be used in the klystron code in the future.

The space-charge fields for a unit-charged ring of radius r' and location z' in a perfectly conducting cylinder of radius a are²

$$\Phi_{SC}(r, z; r', z')$$

$$= \frac{1}{2\pi} \sum_{n=1}^{\infty} \frac{J_1(v_n r)}{J_1(v_n a)} \frac{J_0(v_n r')}{J_0(v_n a)} \exp(v_n \gamma_0 |z - z'|) \quad (9)$$

$$\Phi_{SC}(r, z; r', z')$$

$$= \frac{1}{2\pi} \sum_{n=1}^{\infty} \frac{J_1(v_n r)}{J_1(v_n a)} \frac{J_0(v_n r')}{J_0(v_n a)} \exp(-v_n \gamma_0 |z - z'|) \quad (10)$$

and

$$R_{\Phi_{SC}}(r, z; r', z') = u_0 \Phi_{SC}(r, z; r', z') / r^2 \quad (11)$$

where u_0 is the ring's axial velocity, and v_n is the n^{th} solution of $J_0(v_n a) = 0$.

Equations (9)-(11) are correct for a ring of very thin radial and axial dimensions. In the model, each initial beam wavelength is simulated by n_{r1} particles, n_r radially and n_z axially, where $n_{r1} = n_r n_z$. Thus, the rings have finite dimensions. The exponential terms in Eqs. (9) and (10) assure convergence of the series for all finite values of $z - z'$; but the series diverges at $z - z' = 0$, as it should for a zero-volume source.

Equations (9)-(11) initially were used in the simulation, but maximum field errors as large as 50% were observed between the calculated radial field and

the Gauss' law field for a dc beam. The errors were largest on the inner rings. This field error caused rippling in the radial direction that was not physical. The errors are present because the beam is distributed over a finite volume and not concentrated in rings. There are only two ways to eliminate this error: one must either use many particles (perhaps several thousand) or integrate the Green's functions to find the fields for finite volume rings of charge. This latter approach is the one being used. Equations (9) and (10) are integrated radially and axially over the volume occupied by each ring, and the innermost layer is treated as a disk. The initial radial extent of the rings are known from the input routines, and a laminar-flow assumption is made to estimate the radial extent of each ring as the interaction proceeds. Currently, no estimate of the change in the ring's axial extent is made, although the length must depend on V_r/V_{z0} . This correction may be made if accelerated beams are simulated with this code.

This treatment of the mathematical model is still under development. A more complete description is contained in a status report.³

Test Results

Some dc test runs have been made with $I_0 = 24$ A and $V_0 = 86$ 000 V. The beam tunnel radius is 25.4 mm and the beam radius is 0.7 times the tunnel radius. The maximum error between the calculated space-charge radial field and Gauss' law in a balanced flow situation is 0.5% with four rings. This error occurs at the inside ring and is independent of n_z , the axial subdivision of the beam. Figure 1 is a dc test case for a Brillouin beam that enters a zero-field region at $z = 0$. The rings should follow the universal beam-spread curve,⁴ and the agreement is within 10% at $z = 5$. Most of the error is due to the first and last electrons in the wavelength. The space-charge forces are assumed periodic in z , but in reality they are not, especially with such an expanding beam as shown in Fig. 1. If the first and last electrons were omitted, the agreement will be within 2%. With the minor modification of removing the periodic statement in the space-charge subroutine, the code could be adapted to analyzing the motion of isolated bunches of charge.

Fig. 1. R versus z/Z trajectories for an unmodulated beam entering a zero-field region at $z = 0$.

Figure 2 shows the axial bunching for an ideal beam of four rings and eight axial subdivisions that hits a thin gap at $Z = 0$. The axial field is twice the Brillouin value. The plasma frequency-reduction factor predicts a maximum of $Z = 25.9$; however, the output shows a maximum $Z = 26.3$, an error of 1.5%. Tests with only one radial subdivision show better agreement with the theory. Tests with no space charge show agreement to within 1% of the location of the harmonic current's maximum.

Several test cases for a multi-cavity klystron have been made with the same dc beam parameters. Figure 3 shows the envelope of a thin gap at $Z = 0$ and $\cos^2(Z)$ gaps at $Z = 8.6, 23.2$, and 36.7 . This run simulates the LAMPF klystron second through output cavities. The subdivisions used are 1 radial and 16 axial. Small-signal theory is used to get the parameters of the second cavity. Figure 4 shows the axial bunching diagram for this interaction. The magnetic field is held fixed at 1.5 times the Brillouin value.

Fig. 2. Axial bunching diagram with $B/B_{BR} = 2.0$, for a single thin gap at $Z = 0$ and $V_0/V_{BR} = 0.12$.

Fig. 3. B versus Z for the last four cavities of the LAMPF klystron with $B/B_{BR} = 1.5$.

Fig. 4. Axial bunching diagram for the klystron of Fig. 3.

Future Plans

The goal is to develop an accurate klystron simulation code to improve the efficiency of high-power klystrons at LAMPF and other accelerators. Although many test cases have run well, and entire klystrons have been simulated, few detailed comparisons between calculations and experiments have been made. The space-charge problem has been resolved, and effort will be focused on problems that arise when the cavities are quite close together, as in the case of the LAMPF klystrons. The region convergence method does not work with three or more gaps within a wavelength, but it will be a straightforward record-keeping problem to extend this. The program also will be extended to a more sophisticated treatment of the gap fields and more general input routines, where output from a gun code can be used directly as input data. Several methods of improving the representation of the axial focus fields and the off-axis expansion also are being considered and will be implemented soon.

Before the code is used to design a new klystron, it will be tested against the LAMPF and Stanford klystrons to be certain that it can agree with experimental data. The code should be ready as a design tool before the end of the fiscal year.

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