OPTICAL MODELS FROM LOW-ENERGY
s-, p- AND d-WAVE CROSS SECTIONSC. H. Johnson
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ABSTRACT

From transmission measurements with good resolution at low energies one can obtain data on the optical model potential (OMP) for individual partial waves by first making a multilevel analysis to isolate the partial waves and then averaging for comparison to the OMP. For each J^π the averaging yields two quantities which are related to the amplitude and phase of the OMP scattering function or, alternatively, to the volume integrals of the real and imaginary potentials. Historically, the experimental averages have been represented by the s- and p-wave strength functions, S_0 and S_1 , and the s-wave scattering radius R' . To make full use of data from modern time-of-flight facilities such as ORELA it is necessary to re-examine the averaging procedure in order to extend it upward both in energy and neutron ℓ -value. This averaging is discussed and applied to data on ^{30}Si , ^{32}S , ^{34}S , ^{40}Ca , ^{48}Ca , ^{60}Ni , ^{86}Kr and ^{208}Pb . The resulting OMP shows a systematic real potential with some indication of a parity dependence. The imaginary potential shows considerable fluctuations indicating the importance of nuclear structure at neutron energies below 1 MeV. A coupled channel OMP is also discussed for some of the nuclei.

INTRODUCTION

Most data for the neutron phenomenological optical model potential (OMP) have come from measurements at several MeV where many partial waves are effective. However, my subject is the deduction of model parameters for individual partial waves from multilevel analyses of high-resolution cross sections for neutron energies up to about 1 MeV. In 1963 Peter Moldauer wrote a paper¹ entitled "Optical Model for Low Energy Neutron Interactions with Spherical Nuclei." That paper provides valuable insight on the role of the OMP for low energy neutrons. Moldauer's primary experimental data were the s-wave and p-wave strength functions, S_0 , S_1 , and the s-wave potential scattering radius R' . In 1963 such data were obtained with lower energies and poorer resolution than are now possible. He also used total cross sections and angular distributions measured with poorer resolution at higher energies.

In the twenty years since Moldauer's paper the experimental techniques for high resolution transmission measurements have been improved such that there are now extensive tabulations² of S_0 , S_1 and R' . The use of these data for parametrizing the OMP has continued. Delaroche, Lagrange and Salvy³ refer to the use of S_0 , S_1 and R' , along with the total cross sections, as the SPRT method.

However, high resolution measurements provide far more information on the OMP than is embodied in S_0 , S_1 and R' . For each partial wave the OMP scattering function embodies two quantities, i.e., an

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amplitude and a phase. In turn, these can be related to the real and imaginary volume integrals for the given partial wave. Thus, for $s_{1/2}$, $p_{1/2}$, $p_{3/2}$, $d_{3/2}$ and $d_{5/2}$ neutrons there are altogether ten OMP quantities. In principle these should be available from an experiment that is performed with good resolution such as to isolate and measure each partial wave. In fact, S_0 and R' do give the amplitude and phase of the s-wave scattering function, but only at low energies. Clearly, we need to extend the OMP analysis to higher energies and higher partial waves. That is the purpose of this study.

EXPERIMENTAL DATA

All measurements discussed here were obtained by several researchers at the neutron time-of-flight facility ORELA, i.e., the Oak Ridge Electron Linear Accelerator. Nearly all of the data come from transmission measurements using a burst width of about 6 nsec and a 200-m flight path. The resulting energy resolution, $\Delta E/E$, is better than 10^{-3} for neutron energies up to about 1 MeV. Some of the data are supplemented by differential scattering measurements. Only zero spin targets are included.

The total cross section is a sum over partial waves. For a given partial wave

$$\sigma_T = 2\pi k^{-2} [1 - \text{Re}U(E)] \quad (1)$$

where the scattering function U is unitary for the one-channel cases studied here. We expand in the R-matrix formalism, which automatically embodies the unitary property;

$$U(E) = e^{-2i\phi(E)} \frac{1 + iP(E) R(E)}{1 - iP(E) R(E)} \quad (2)$$

where ϕ is the hard-sphere phase shift, P is the penetrability and R is the R-function. Each function depends on the chosen boundary radius a . In writing Eq. (2) we have set the boundary condition equal to the shift factor. The R-function includes a sum over the N resonances observed in the measured region from E_L to E_U plus a parametric function which accounts for all levels outside the region;

$$R(E) = R_{\text{ext}}(E) + \sum_{\lambda=1}^N \gamma_{\lambda}^2 / (E_{\lambda} - E), \quad (3)$$

where γ_{λ}^2 is a reduced neutron width and E_{λ} is a resonance energy. Both terms in Eq. (3) are important not only for the fitting but also for the eventual OMP interpretation.

Using this formalism we fit the observed transmissions with the effects of finite resolution included. For each prominent resonance the correct ℓ and J are deduced from the resonance-potential interference pattern and from the peak height. Often correct assignments

experimental scattering function for each partial wave to the OMP scattering function. Our vehicles for comparison are the compound and shape elastic cross sections. For the OMP for given partial wave we have

$$\sigma_c(E) = \pi k^{-2} g (1 - |U^{OMP}(E)|^2) \quad (7)$$

and
$$\sigma_{se}(E) = \pi k^{-2} g |1 - U^{OMP}(E)|^2. \quad (8)$$

For the corresponding experimental cross sections we replace U^{OMP} by an appropriate average, which we designate U^{OM} .

Parenthetically, we note that a poor resolution experiment with a thin sample yields an average of only the real part of U [see Eq. (1)]. Thus, that measurement does not give information on both the amplitude and phase of U^{OM} .

NUMERICAL AVERAGING OF THE SCATTERING FUNCTION

For comparison to the OMP we wish to average the observed scattering function over the resonance structure. Ideally the resonances should be closely spaced such that

$$D \ll 2I \ll (E_U - E_L) \quad (9)$$

where $2I$ is the width of the averaging function and E_L and E_U are the lower and upper energy limits of the data. However, the spacing D for a given partial wave is often so large that we satisfy only the inequality

$$D < 2I < (E_U - E_L) \quad (10)$$

Figure 2 illustrates the problem. The upper figure shows the neutron total cross section and multilevel fit⁵ for neutrons on natural sulfur (95% ^{32}S). (Resolution effects were included in the analysis but not in the plotted curve.) The lower curve shows only the s-wave component of the multilevel curve. Even though the total cross section has many resonances, the given partial wave has relatively few. That is frequently the case. Since a multilevel analysis is practical only in a region where the total number of resonances is restricted, the number for each partial wave must be relatively small.

Recently, Johnson, Larson, Mahaux and Winters⁶ (JLMW) discussed the problem of averaging. They define the

Fig. 2. Total cross section and s-wave component for sulfur (95% ^{32}S).

average of a function X to be

$$\langle X(E) \rangle_I = \int_{E_L}^{E_U} F_I(E, E') X(E') dE' \quad (11)$$

where the weighting function F is normalized to unity in the experimental region from E_L to E_U and where the average energy $\bar{E}$ is calculated for $X=E'$. JLMW compared various F but here we use only their truncated Lorentzian;

$$F_I(E, E') = \frac{f(E)}{(E-E')^2 + I^2} \quad (12)$$

where f is the normalizing factor. Figure 3 shows F for $I=0.3$ MeV, i.e., 30% of the experimental region. Such broad functions are needed to average over the sparse structure for s-waves in Fig. 2.

If the weighting function were not so broad relative to the neutron energy, we could average immediately by inserting the scattering function from Eq. (2) into the integral of Eq. (11). However, that procedure introduces a spurious absorption. To illustrate we consider a real potential (no absorption) such that the s-wave phase shift is real and given by the dashed line in Fig. 4. This curve is simply that for a hard-sphere of 5 fm radius. Averaging U , i.e., $\exp(-2i\phi)$ yields the solid curves for $I = 50$ to 400 keV. The upper figure shows the real parts of the phase shifts for $\langle U \rangle$ and the lower figure shows the strength functions calculated by Eq. (6) from $\langle U \rangle$. The S_0 are comparable to experimental values² in the valleys between size resonances, but they are spurious because there is no absorption in the model.

Figure 5 shows graphically the origin of the spurious absorption. The arrow on the circle represents the rotating unitary scattering function. Averaging over the large angle yields a vector of less than unit magnitude, as indicated by the enlarged view. Even though the difference from unity is small, it is serious when compared to the actual difference for real nuclei.

To solve the problem JLMW recommend the experimental scattering function be factored into resonant and background terms and averaged only over the resonant part

$$U_{om}(E) = U_{bkg}(E) \langle U_{res}(E) \rangle_I. \quad (13)$$

The background is reasonably written as

Fig. 3. Normalized weighting functions for $I=0.3$ MeV and $E=0, 0.5$ and 1.0 MeV. Arrows indicate the average energies.

$$U_{bkg}(E) = e^{-2i\delta(E)} \quad (14)$$

$$\text{where } \delta(E) = \phi(E) - \tan^{-1} P(E)\text{Rext}(E_m) \quad (15)$$

with Rext being evaluated at the midpoint E_m of the energy interval. (A slightly better expression for δ is obtained by replacing Rext in Eq. (15) by $\bar{R}$, as defined below.)

Fig. 4. Numerical averaging of only s-wave hard sphere scattering for various I . Upper: dashed curve for hard-sphere of 5 fm; solid for real part of phase shift of the average plotted versus $\bar{E}$. Lower: strength function from Eq. (6) for the averages.

Fig. 5. Graphical description of spurious absorption from averaging for a real potential.

To illustrate the use of Eqs. (11-15) for the definition of the average we have obtained U_{0m} by numerical integration of an s-wave scattering function which Perey et al.⁷ deduced from a multilevel analysis for 0-450 keV neutrons on ^{60}Ni . Figure 6 shows the resulting compound and shape elastic cross sections versus $\bar{E}$ for $I=20, 50$ and 90 keV. In the following discussion of this nucleus we use the curve for $2I=180$ keV, which is about 12 times the level spacing. As expected the larger values of I produce a smoothing of the resonance structure.

ANALYTICAL APPROXIMATION TO THE AVERAGE

Since the integration of the scattering function cannot be done in closed form, the foregoing integration is difficult; it must be done numerically using carefully selected quadrature schemes. Therefore JLMW sought an analytical approximation to simplify the analysis. As a starting point they considered "Brown's⁸ theorem." Brown pointed out that, if the scattering function were known for all energies from $-\infty$ to $+\infty$, then by contour integration with Lorentzian weighting one has

$$\langle U(E) \rangle_{I_\infty} = U(E+iI), \quad (16)$$

where the subscript ∞ denotes the integration limits. This theorem is a powerful tool for theoretical works but JLMW found it can be quite inaccurate for the experimental case where U is known only in a limited region. One reason⁶ for the failure is that, since the observed U senses only the effects of external poles, it can be parametrized by many forms of R_{ext} , none of which are likely to represent the actual poles and corresponding U in the external region.

To study this problem we consider a simple example, as illustrated in Fig. 7. For this hypothetical nucleus we find in the region ΔE a picket fence of 20 equally spaced levels, each with the same reduced width. The external R -function, R_{ext} , is adequately described by a continuation of the picket fence to $\pm\infty$, as illustrated in Fig. 7a. The corresponding R_{ext} is shown in Fig. 8. However, we find R_{ext} is also adequately described as in Fig. 7b with two small nearby poles and two

Fig. 6. Numerical average for s-waves on ^{60}Ni for three values of I .

(a)

(b)

(c)

Fig. 7. A picket fence model of s-wave resonances with three equivalent parametrizations of the external R -functions.

distant very large ones or, as in Fig. 7c, by only two relatively large nearby poles. The corresponding R_{ext} are shown by dashed curves in Fig. 8.

By numerical averaging of our hypothetical scattering function using Eqs. (11-15) we find, as expected, nearly the same answer for all three parametrizations of R_{ext} . However, we find three different answers by evaluating $U(E+iI)$. We do find good agreement with the numerical average if we use the parametrization of Fig. 7a and substitute $E+iI$ only in the R-function.

The justification for this success in terms of a contour integration is presented by JLMW.

For the analytical approximation JLMW concluded R_{ext} should be parametrized by an expression that Johnson and Winters used earlier⁵ for the analysis of the sulfur data in Fig. 1. It is

$$R_{ext}(E) = \tilde{R}(E) - \tilde{s} \ln [(E_u - E)/(E - E_l)] \quad (17)$$

$$\text{where } \tilde{s} = \langle \gamma^2 \rangle / D, \quad (18)$$

and where $\tilde{R}(E)$ is a simple parametric function,

$$\tilde{R}(E) = \alpha + \beta E. \quad (19)$$

The effect of the log term in Eq. (17) is equivalent to the external picket fence in Fig. 7a. A good approximation to the averaged U is then achieved by substituting $E+iI$ in the R-function, except for the slowly varying part $\tilde{R}$. Thus we have

$$U_{om}(E) = e^{-2i\phi(E)} \frac{1 + iP(E)[\tilde{R}(E) + i\pi\tilde{s} + R^f(E+iI)]}{1 - iP(E)[\tilde{R}(E) + i\pi\tilde{s} + R^f(E+iI)]} \quad (20)$$

where

$$R(E+iI) = \sum_{\lambda=1}^N \frac{\gamma_{\lambda}^2}{E_{\lambda} - E + iI} - \int_{E_l}^{E_u} \frac{\tilde{s} dE'}{E' - E + iI}. \quad (21)$$

We see that both the real and imaginary parts of the fluctuating term R^f tend to oscillate about zero and finally go to zero for large I . The net result is a smoothing similar to that shown in Fig. 6 for the numerical average.

To illustrate the success of this approximation we repeat the averaging for ^{60}Ni . In Fig. 9 the solid curve shows the original R_{ext} from Perey's⁷ multipole parametrization, the short dash curve is our new parametrization from Eq. (17) and the long dash curve is

Fig. 8. External R-function for the three parametrizations of Fig. 7: solid, Fig. 7a; short dash, Fig. 7b; long dash, Fig. 7c.

Fig. 9. External R-function and the $\tilde{R}$ (long dash) for 0-450 keV s-wave neutrons on ^{60}Ni . Boundary radius is 6 fm.

the corresponding linear $\tilde{R}$. Substituting in Eq. (20) we find an approximate U^{om} for calculating σ_c and σ_{se} . These are plotted by solid curves in Fig. 10 for $I=90$ keV. For comparison the corresponding numerical average from Fig. 6 is reproduced as a short-dash curve. The agreement is good.

The analytical approximation of Eqs. (17-21) is used for the rest of this report.

OPTICAL MODEL GEOMETRY

For the following OMP analysis we use a standard Woods-Saxon real potential with a surface derivative spin-orbit term and a Woods-Saxon derivative imaginary term. For each partial wave the experimental U^{om} or the resulting σ_c and σ_{se} can be fit by adjusting two parameters corresponding, qualitatively, to the volume integrals of the real and imaginary parts. For convenient comparison among various nuclei we fix the geometries and adjust the real and imaginary well depths, V_0 and W_0 . For $\ell > 1$ we also adjust the spin orbit depth V_{so} . Our geometry is

$$r_0 = r_{so} = r_D = 1.21 A^{1/3} \text{ fm},$$

$$a_0 = 0.66 \text{ fm and } a_D = 0.48 \text{ fm.} \quad (22)$$

Fig. 10. Compound and shape elastic cross section for s-wave neutrons on ^{60}Ni

SPHERICAL OMP FOR ^{60}Ni

In Fig. 10 the long dash curve is the fit achieved by adjusting V_0 and W_D for s-waves. We find $V_0 = 48$ MeV and $W_D = 29$ MeV, each with a ± 5 MeV uncertainty. The V_0 is consistent with other nuclei in our studies and it places the peak² of the 3s size resonances properly near $A=55$. However the W_D is much larger than any other in our studies.

Figure 11 demonstrates the roles of V_0 and W_D . The solid curves show σ_c and σ_{se} versus V_0 for various W_D at our midrange energy of 200 keV, and the dashed horizontal lines are the experimental values from Fig. 10, each with the small uncertainties indicated by the vertical bars in Fig. 10. The peak in σ_c near $V_0 = 48$ MeV is due to the $3s_{1/2}$ state. We see that σ_{se} with its small uncertainty restricts V_0 to near 48 MeV. With V_0 so restricted, the W_D has been increased to the large value of 29 MeV in order to spread the $3s_{1/2}$ state and bring σ_c down to the observed value. It might appear that a fit could be achieved with small $W_D \approx 0.5$ MeV. However, the resulting σ_{se} has a strong energy dependence, in disagreement with the data of Fig. 10.

Fig. 11. Compound and shape elastic cross section for 200 keV neutrons on ^{60}Ni . Solid lines are for the OMP and dashed lines are experimental from Fig. 10.

DEFORMED OMP FOR ^{60}Ni

The spherical model could be a poor approximation because ^{60}Ni is a vibrational nucleus. Therefore, we repeated the fit using the coupled-channel formalism.^{9,10} The target channels are the 0^+ ground state and the 2^+ state at 1.33 MeV with $\beta = 0.311$. (The 3^- state at 4.04 MeV has negligible effect.) Using this model we achieved the same fit as above with minor adjustments in the well depths, i.e., $V_0 = 50$ MeV and $W_D = 24$ MeV.

Figure 12 shows curves for this model plotted in like manner to Fig. 11. We see that the vibrational effects shift the $3s_{1/2}$ peak downward about 2 MeV and introduce two particle-core resonances. Arrows indicate the energies deduced for these resonances by simple addition of the 1.33-MeV 2^+ excitation to the bound 2d states in the potential, such as to equal the neutron energy of 200 keV.

The solid points indicate our chosen well depths. There is another solution, as indicated by the open circles, with $V_0 = 54.5$ MeV and $W_D = 1.5$ MeV. The smaller W_D is more consistent with other nuclei; however, the V_0 is too large for other considerations. Firstly, it places the $3s$ size resonance near $A=44$, in serious disagreement with the known peak² near $A=55$. Secondly, if the same well depth is assumed for p -waves, it predicts the bound $2p$ state to be about 3 MeV deeper than the known energies,¹¹ whereas the other solution ($V_0 = 50$ MeV) places the levels very close to the correct energies.

Recently, Guss et al.¹² measured neutron scattering on ^{60}Ni for 8, 10, 12 and 14 MeV neutrons and fit with a coupled-channel model including the requirements of fitting the low energy S_0 , S_1 , R' and σ_T , i.e., the "SPRT³ method." Figure 13 is plotted in like manner to Fig. 12 for their potential geometry. The essential difference from Fig. 12 is that the resonance peaks are shifted upward due to the smaller real radius ($r_0=1.165$ fm versus 1.21 fm). The solid points indicate the solution of Guss, et al. That solution was influenced considerably by the low energy scattering radius² of $R' = 6.7 \pm 0.3$ fm. The open circles represent the adjusted solution required to fit the present accurate evaluation of σ_{se} . With this solution ($V_0 = 58$ MeV) one must increase the slope for V_0 from the value of 0.46 MeV^{-1} of Guss et al. to about 0.62 MeV^{-1} to maintain consistency with their data at higher energies. Essentially this solution with their geometry¹² is the one we have discarded from Fig. 12 for our geometry. The choice is a matter of judgment. Further data on nearby nuclei are desirable.

CROSS SECTION AND OMP FOR ^{86}Kr

The total cross section of ^{86}Kr is being studied at ORELA and is the subject of a contributed paper by Carlton, Harvey and Johnson¹³. The analysis is nearly complete up to 700 keV. Since this nucleus lies near the $3p$ size resonance at $A=95$, its resonance structure is

Fig. 12. Compound and shape elastic cross sections for 200 keV neutrons on ^{60}Ni . Solid Lines are for the coupled channel model. The dashed lines and shaded areas represent the observed cross sections with uncertainties from Fig. 10. Arrows indicate unperturbed energies of particle-core states. Solid points represent the accepted well depths and open points are rejected.

dominated by p-waves. As an example Fig. 14 shows the structure for 340-440 keV neutrons. The upper figure shows the observed total cross sections with the fitted multilevel curve and the lower figure presents the p_{3/2} component without the effects of resolution broadening. Clearly p_{3/2} neutrons dominate the resonance structure.

Given the multilevel analysis we deduced average scattering functions for $\ell=0$ and 1 by use of the analytical approximation for $21=200$ keV. In Fig. 15 the solid curves show the compound and shape elastic cross sections, divided by the g-factor, for p_{1/2} and p_{3/2}. The dashed curves are preliminary fits achieved by adjusting the well depths of our standard model. (For the final fit we may increase the width of the averaging interval to give more smoothing over the p_{3/2} structure.)

The well depths are listed in the figure. We see that the imaginary potentials are about the same for the two partial waves and that the primary reason for J-dependence of the cross sections lies in the real spin-orbit term. This V_{SO} is consistent with values deduced in the literature from polarization measurements.

Fig. 14. Total cross section for ^{86}Kr . The upper figure shows the data and multilevel fit only from 340 to 440 keV and the lower curve is the p_{3/2} component.

Fig. 13. Compound and shape elastic cross sections for 200-keV neutrons on ^{60}Ni . See Fig. 12 caption. Model geometry by Guss et al., ref. 12. Solid points are their solution and open circles are adjustments to present data.

The s-wave analysis gives $V_0 = 48.5$ MeV and $W_0 = 3.5$ MeV. Thus, the imaginary potentials are about the same for all three partial waves but the real well depth is shallower for s-waves than for p-waves. This ℓ -dependence may not be very significant by itself but it is consistent with a similar trend that we observe in lighter nuclei.

CROSS SECTION AND OMP FOR ^{208}Pb

The ^{208}Pb nucleus is particularly interesting and has been studied extensively both experimentally and theoretically. At ORELA the total cross section has been measured and fitted to 1 MeV. Furthermore, angular distributions have been measured to aid in the J^π assignments. From the present viewpoint this nucleus is interesting because it is large enough to give substantial d-wave scattering. In Fig. 16 the upper curve shows the observed total cross section with the multilevel fit from 300 to 1000 keV and the lower curve shows the rather prominent $d_{5/2}$ component.

Given the multilevel scattering functions we averaged over the full 0-1 MeV region using averaging widths appropriate for each partial wave. We then fit σ_c and σ_{se} for each partial wave by adjusting the well depths for our standard geometry. These well depths are compared with the other lighter nuclei in the next section.

We also fit the cross section using the common geometry deduced by Finlay et al.¹⁴ from measurements in the 7-40 MeV region. The fixed parameters are 1.205 fm, 1.105 fm and 1.283 fm respectively for r_0 , r_{s0} and r_D ; 0.685 fm, 0.499 fm and 0.569 fm respectively for a_0 , a_{s0} and a_D and $V_{s0} = 5.75$ MeV. In Fig. 17 the X-symbols show the resulting well depths for the first five partial waves. The real well depths are in very good agreement with the known bound single-particle states of ^{209}Pb and provide evidence for a parity dependence, with the potential being about 3 MeV deeper for odd parity than even parity. Of course the observed J-dependence for $\ell=1$ could be removed by increasing V_{s0} for that partial wave. The fluctuations in W_D may result from the well-known nuclear structure effects in this doubly closed shell nucleus.

Fig. 15. Compound and shape elastic cross sections for ^{86}Kr . Solid curves are experimental for $2I = 200$ keV and dashed curves are for the model.

Fig. 16. Total cross section for ^{208}Pb . The upper figure shows the cross section and multilevel fit and the lower curve shows the $d_{5/2}$ component.

Fig. 17. Potential well depths for the model of Finlay et al., ref. 14. The X-symbols are from the present analysis for 0-1 MeV and the circles are from Finlay. The solid curves were suggested by Finlay for $E_n > 7$ MeV and have been extrapolated here to zero energy.

The open circles are from the report by Finlay et al.¹⁴ and the solid lines are their suggested curves, with further extrapolation from 7 MeV to zero energy. Overall this figure shows a consistency in OMP parameters.

PRESENT SUMMARY OF V_0 and W_D for 0-1 MEV NEUTRONS

This paper is intended to be more a progress report than a review. Some potential parameters have been published for both the spherical^{15,16} and coupled channel analysis.^{17,18} The published¹⁹ analysis for ^{206}Pb was performed with earlier techniques than reviewed here. But most of the work is still in progress and subject to modifications. Table I gives present results with appropriate footnotes.

There are some trends in Table I. For the spherical model and $A < 100$ the real well tends to be deeper for $\ell=1$ than for $\ell=0$. Coupled channels calculations remove some but not all of this difference. On the other hand the odd parity well is relatively deeper in the lead region. There is evidence for the isospin term for nuclei such as ^{48}Ca , ^{86}Kr and $^{206,208}\text{Pb}$, each of which have relatively large $N-Z$ differences. Altogether, the real well depths show consistent patterns. On the other hand the imaginary depths show large fluctuations with little apparent pattern at this time. Apparently W_D is sensitive to specific nuclear structure effects.

More data are needed with optical model analyses as reviewed in this paper.

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Table I. Potential^a Well Depths^b from High Resolution Neutron Cross Sections.

Target	ℓ	V_0 (MeV)		W_D (MeV)		V_{SO} (MeV)	
		sph	cc	sph	cc	sph	cc
³⁰ Si c,d	0	48(1.7)	49.5	2.0(1.5)	1		3.3 ^b
	1	62(2.5)	51.5	4.5(2.5)	2		3.3 ^b
³² Se, f	0	51.5(0.4)	50.8	6.0(3.0)	4.0		5.3
	1	61.4(1.1)	50.8	2.7(1.5)	0.8	11(3)	5.3
³⁴ Sc, d	0	51.5(1.1)	50.5	3.0(2.0)	2.5		3.5
	1	58.5(1.2)	51	3.5(1.9)	2.5	6(3.5)	3.5
⁴⁰ Ca g	0	53.6(1.0)	51	6.65(0.3)	7		
	1	56.1(1.0)	53	0.6 (0.3)	0.1	6(1)	4
⁴⁸ Ca h	0	48(1)		0.0			
⁶⁰ Ni g	0	48(5)	50	29(5)	24		5 ⁱ
	1		50 ⁱ		1.5		5 ⁱ
⁸⁶ Kr j	0	48.5(1)		1			
	1	51.4(1)		3.5(1)		5.5(1)	
²⁰⁶ Pb k	0	50.4		6			
	1	48.7		2		7.5	
	2	50.4		6		7.5	
²⁰⁸ Pb g	0	49.5(1)		6(2)			
	1	47(1)		1.5(1)		10(2)	
	2	49.5(1)		2-5(2)		7.5(1)	

- a. Model geometric parameters are $r_0 = r_D = 1.21$ fm, $a_0 = 0.66$ fm, $a_D = 0.48$ fm.
- b. Uncertainties for the spherical (sph) model are in parenthesis. The same uncertainties apply to the coupled-channel (cc) model.
- c. See Ref. 16 for the spherical model.
- d. See Ref. 18 for the coupled channel model.
- e. See Ref. 15 for the spherical model.
- f. See Ref. 17 for the coupled channel model.
- g. Work in progress subject to modification.
- h. Contributed paper, work in progress by Harvey, Johnson, Carlton, and Castel.
- i. Assumed well depth.
- j. Contributed paper, work in progress by Carlton, Harvey and Johnson.
- k. See Ref. 19, Analysis by earlier method.