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MICROWAVE ENERGY STORAGE IN
RESONANT CAVITIES

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MICROWAVE ENERGY STORAGE IN RESONANT CAVITIES

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ABSTRACT

One method of generating short, high-power microwave pulses is to store rf energy in a resonant cavity over a relatively long fill time and extract it rapidly. A power gain roughly equal to the ratio of fill time to extraction time can be obtained. During the filling of a resonant cavity some of the energy is lost in heating the cavity walls, and some will generally be reflected at the input coupling of the cavity. In this paper we discuss the time dependence of the stored energy and related quantities and the way in which it depends on the coupling of the source to the cavity.

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J. A. Alvarez

I. INTRODUCTION

Resonant microwave cavities have a number of applications in which their transient response, as well as steady-state properties, is important. Among these is the use of cavities, both normal and superconducting, as pulse power amplifiers.¹⁻⁷ This paper will present expressions for the time dependence of stored and dissipated energy, transmitted and reflected power, etc., during the storage and extraction of energy from a cavity; time $t = 0$ will be taken as the instant when power is initially applied, as a step function, to the cavity input port.

An ideal case will be considered:

- a) The source power, P_0 , is constant, as are its phase and frequency.
 $f \equiv \omega/2\pi$, until some cutoff time, t_c .
- b) The intrinsic quality factor, Q_0 , of the cavity is constant.
- c) The coupling, β , of the rf source to the cavity is constant.

SLATER'S FORMULA AND SOME DEFINITIONS

As a starting point Slater's formula⁸ for stored energy in a cavity will be used with a few notational changes: †

$$W(t) = \gamma W^* (1 - e^{-\alpha t})^2 \quad (1)$$

where

$$\left. \begin{aligned} W(t) &= \text{energy in EM fields in cavity at time } t. \\ W^* &= P_0 \tau_0. \\ \gamma &= 4\beta(1+\beta)^{-2}. \\ \alpha &= (1+\beta)/2\tau_0 = \kappa/\tau_0. \\ \kappa &= (1+\beta)/2. \\ \tau_0 &= Q_0/\omega. \\ \beta &= Q_0/Q_e. \end{aligned} \right\} \quad (2)$$

† Slater uses the notation Q_{ext} for the external Q of a resonant cavity. He also uses ω_a to denote the angular frequency of a cavity being excited on a resonance. In this note it is generally assumed that the cavity is being driven at resonance, so the subscript a has been dropped, for simplicity, except in Eq. (8).

The quantity β is the coupling factor for the cavity and input port; it is related to the fractions of the input power that are transmitted and reflected at the cavity input port in the steady state, as well as the VSWR (see Sec. IIIC). The quantity Q_e is known as the "external Q" of the cavity; it is the parameter describing the loss of power due to flow back out through the coupling port, and is determined by the port geometry and the spatial variation of the EM fields.

The intrinsic Q-value of a cavity, Q_0 , describes the losses due to resistive dissipation in the cavity walls (see Sec. IIIA). The overall, or loaded Q-value, Q_L , is related to Q_0 and Q_e by

$$Q_L^{-1} = Q_0^{-1} + Q_e^{-1} \quad (3)$$

If other dissipative mechanisms are present other terms ($1/Q_k$) can be added to the right-hand side of Eq. (3) to account for their effect on the loaded Q of the cavity. From Eqs. (2) and (3) one can write

$$Q_L = Q_0(1 + \beta)^{-1} \quad (4)$$

It is Q_L that determines the stored-energy decay rate for the cavity when the drive power is turned off. If P_0 is suddenly turned down to zero at time t_c , the stored energy decays according to

$$W(t') = W(t_c) \exp(-t'/\tau) \quad (5)$$

where

$$t' = t - t_c, \quad (t > t_c) \quad (6)$$

and

$$\tau = Q_L/\omega. \quad (7)$$

During the cavity fill time ($P_0 > 0$) the "reflected power" is made up of a reflected amplitude from the coupling port added coherently to a re-emitted amplitude due to stored energy reradiated through the aperture. In the steady state ($t \rightarrow \infty$) the reflected and reradiated amplitudes exactly cancel if $\beta = 1$, that is, if the coupling aperture is designed so that $Q_e = Q_0$. For $\beta \neq 1$ these amplitudes do not generally cancel; there will be a net reflected power in the steady state for either the undercoupled ($\beta < 1$) or overcoupled ($\beta > 1$) case, though its phase, relative to that of P_0 , will be different in the two cases. (The steady-state case is discussed further below.) After P_0 has been turned off, there is no cancellation of the reradiated amplitude, which results in a stored-energy loss in addition to losses due to ohmic heating of the cavity walls. In this case it is the loaded Q which determines the energy decay rate, as indicated in Eq. (7). It is also

Q_L which determines the "sharpness" of the resonance:

$$\omega_a/\Delta\omega = Q_L, \quad (8)$$

where ω_a is a resonant frequency of the cavity and $\Delta\omega$ is the resonance full width at the points where half the drive power is reflected. In the limit of very weak coupling ($\beta \rightarrow 0$) Q_L approaches Q_0 .

II. ENERGY AND POWER-FLOW RELATIONSHIPS DURING CAVITY FILL TIME

It will be convenient to express the cavity transient response in dimensionless form; thus we define

$$x \equiv t/\tau_0, \quad (9)$$

$$W^* \equiv P_0\tau_0. \quad (10)$$

W^* will be used to normalize quantities with dimensions of energy, and P_0 will be used for power normalization; ω can be replaced by kx .

A. STORED ENERGY

In terms of the normalized time, x ,

$$W(x) = W^*\gamma(1 - e^{-kx})^2, \quad (11)$$

so Eq. (1) can be written in normalized form:

$$Y(x) \equiv W(x)/W^* = \gamma(1 - e^{-kx})^2, \quad (12)$$

With a cw drive source the steady-state ($x \rightarrow \infty$) stored energy approaches $\gamma\tau_0 P_0$, which has a maximum value of $P_0\tau_0$ for $\beta=1$. Where efficiency of energy storage is important, or where the pulse length of drive power is limited, it may be advantageous to use a coupling of $\beta>1$. The issue of efficiency has been addressed by Baraev and Korovin⁹, and we shall return to this point below.

B. POWER DISSIPATION IN CAVITY WALLS

The rate of energy dissipation in the cavity walls is proportional to the amount of stored energy; it defines the intrinsic Q of the cavity, and is given by

$$\Pi \equiv W\omega/Q_0 = W/\tau_0, \quad (13)$$

or, in normalized form,

$$D \equiv \Pi/P_0 = W/W^* = Y(x). \quad (14)$$

C. RATE OF INCREASE OF STORED ENERGY

The rate at which stored energy builds up in the cavity can be obtained by differentiating Eq. 1:

$$\begin{aligned} dW/dt &= 2W^* \alpha \gamma e^{-\alpha t} (1 - e^{-\alpha t}) \\ &= (W/\tau_0) (1+\beta) e^{-\alpha t} (1 - e^{-\alpha t})^{-1}. \end{aligned} \quad (15)$$

Upon normalizing to P_0 and replacing W/W^* by the expression given by Eq. (12) one can write

$$\begin{aligned} P_w &\equiv (dW/dt)/P_0 = \gamma(1+\beta) e^{-\alpha t} (1 - e^{-\alpha t}) \\ &= \gamma(1+\beta) e^{-kx} (1 - e^{-kx}). \end{aligned} \quad (16)$$

One might notice that the function, $e^{-\alpha t} (1 - e^{-\alpha t})$ has a maximum at $t = \ln(2)/\alpha$; the build-up of stored energy is therefore increasing most rapidly at the normalized time,

$$x_w = \ln(4)/(1+\beta) \quad (17)$$

D. POWER FLOW INTO CAVITY

The net power flow into the cavity, Φ_i , must account for the rate of energy build-up plus the power dissipated in the walls:

$$\begin{aligned} \Phi_i &= (dW/dt) + \Pi \\ &= P_0 \gamma (1 - e^{-\alpha t}) (1 + \beta e^{-\alpha t}). \end{aligned}$$

or in normalized form,

$$P_i \equiv \Phi_i/P_0 = \gamma(1 - e^{-kx})(1 + \beta e^{-kx}). \quad (18)$$

For $\beta \ll 1$ the maximum power is coupled into the cavity in the asymptotic limit ($x \rightarrow \infty$); for the overcoupled case ($\beta > 1$) the power coupled into the cavity reaches a maximum ($P_i = 1$) at the finite time,

$$x_p = 2(1+\beta)^{-1} \ln[2\beta/(\beta-1)]$$

E. REFLECTED POWER

The reflected power is simply

$$\Phi_R = P_0 - \Phi_i,$$

or in normalized form,

$$P_R \equiv \Phi_R/P_0 = 1 - P_i. \quad (19)$$

At $x = 0$, $P_R = 1$ for all β . For $x \rightarrow \infty$, $P_R \rightarrow 0$ for $\beta = 1$, but remains finite for all other β . For $\beta > 1$, there is a finite time, x_p , at which $P_R = 1$ and, therefore, $P_R = 0$.

F. RATIO OF ENERGY BUILD-UP RATE TO WALL DISSIPATION RATE

At $t=0$ there is no power loss to the walls; for $t \rightarrow \infty$ the increase in stored energy approaches zero and all the power pumped into the cavity goes into wall heating. If we define $R_R(x)$ as the ratio of energy build-up rate to dissipation rate,

$$R_R = (dW/dt)/\Pi = (1+\beta)e^{-Kx}/(1-e^{-Kx}). \quad (20)$$

At the time $x_R = \kappa^{-1} \ln(\beta+2)$, $R_R = 1$.

G. INTEGRATED ENERGY DISSIPATED IN CAVITY WALLS.

The integral of power dissipated in the walls of a cavity may be particularly important in the case of pulsed superconducting cavities, because it may be the main factor determining the liquid helium boil-off rate. In dimensionless form, it is

$$E_D = \int_0^1 (\Pi/W^*) dt \\ = [4\beta/(1+\beta)^3] [(1+\beta)x - e^{-2Kx} + 4e^{-Kx} - 3]. \quad (21)$$

H. INTEGRATED ENERGY INFUX TO A CAVITY

In dimensionless form, the integrated energy input to a cavity is

$$E_i = Y + E_D. \quad (22)$$

This quantity may be of interest because it is the amount of energy per pulse that a cooling bath may have to remove if the stored energy in the cavity is also dumped into the cavity walls, as may happen, e.g. if a superconducting cavity is run until "breakdown" occurs.

I. INTEGRATED REFLECTED ENERGY

The reflected energy during a pulse may be of importance because it represents the heat that must be dissipated in the load of a circulator/isolator that must generally be provided between an rf source and a resonator to protect the former from damage due to reflected power, which, from Sec. E, is always equal to P_0 at $t=0$. This quantity can be written, in dimensionless form,

$$E_R = P_0/W^* - E_i = x - E_i. \quad (23)$$

J. GRAPHICAL REPRESENTATION OF FULL-TIME TRANSIENT EFFECTS

The time dependence of several of the quantities discussed in Secs. II(A-1) is displayed in dimensionless form in Figs. 1 and 2 for several different values of the coupling constant, β . For a given cavity of known Q_0 driven at a given frequency, ω , and power, P_0 , the transient response can be obtained from these curves. Fig. 3 demonstrates the effect of the coupling on the time dependence of stored energy. In particular, it is evident from Fig. 3 that one can increase the stored energy at finite times by increasing β , though the potential energy-storage capability is thereby reduced. Increasing β can be advantageous when the microwave driving source has a limited pulse length which would make the larger storage capability of the matched ($\beta=1$) case irrelevant. This effect is related to the optimization of efficiency, which is discussed further in Sec. VI.

It is qualitatively evident from a glance at Figs. 1 and 2 that there is generally some time, x , for a given value of β , beyond which losses due to reflected power and cavity wall heating outweigh the increase in stored energy. In particular, the ratio, R_R , of energy storage rate to wall dissipation rate is shown explicitly, for several values of β , in Fig. 4. In Fig. 5 are shown those normalized times, as a function of β , for which the energy build-up rate is maximum, the power coupled into the cavity is maximum (for $\beta>1$), and the energy buildup and dissipation rates are equal: x_w , x_p , and x_R , respectively.

III. ASYMPTOTIC LIMITS

In the following, the limiting value of a quantity, for x (or t) $\rightarrow \infty$, will be written with ∞ as an argument, e.g., $Y(\infty)$. One may also refer to these values as the "steady-state" values.

A. STORED ENERGY

$$W(\infty) = P_0 Q_0 \gamma / \omega \quad (24)$$

In Sec. II(A we pointed out the fact that $\beta=1$ (critical coupling) is a special case, for which the steady-state stored energy, $W_1(\infty)$, is

$$W_1(\infty) = P_0 Q_0 / \omega.$$

The above equation can be manipulated to give

$$Q_0 = 2\pi W_1(\infty) / P_0 T,$$

where T is the rf period.

If the above equation is interpreted according to the definition of the intrinsic Q of a cavity, namely, 2π times the ratio of stored energy to energy loss per cycle, one can see that the time-averaged power dissipation must be P_0 ; this is consistent with the fact (see below) that the asymptotic reflected power is zero, for $\beta=1$, and the entire drive power, P_0 is coupled into the cavity to make up for the ohmic losses.

B. POWER COUPLED INTO CAVITY.

$$\Phi_i(\infty) = P_0 \gamma = 4P_0 \beta / (1 + \beta)^2. \quad (25)$$

For $\beta=1$ the entire drive power couples into the cavity in the steady state.

C. REFLECTED POWER

$$\Phi_R(\infty) = P_0(1 - \gamma). \quad (26)$$

The steady-state ratio of reflected to incident power is

$$\begin{aligned} \Phi_R(\infty) / P_0 &= 1 - \gamma \\ &= (1 - \beta)^2 / (1 + \beta)^2. \end{aligned} \quad (27)$$

This steady-state reflected-power ratio can also be obtained by squaring the amplitude of the voltage reflection coefficient, ρ :

$$|\rho|^2 = (S-1)^2 / (S+1)^2, \quad (28)$$

where S is the voltage standing-wave ratio (VSWR). The VSWR is the ratio of maximum to minimum voltage amplitudes in the cavity-feed cable or waveguide; it can never be less than unity, and is related to the coupling coefficient by

$$\left. \begin{aligned} S &= \beta^{-1}, & (\beta < 1); \\ S &= \beta, & (\beta > 1). \end{aligned} \right\} \quad (29)$$

D. DISSIPATED POWER

From Eqs. 11 and 13

$$\Pi(\infty) = W(\infty) / \tau_0 = \gamma P_0. \quad (30)$$

This is equal to the power coupled into the cavity, as it must be, since, asymptotically, the stored energy is not changing.

IV. DECAY OF STORED ENERGY AFTER CUT-OFF OF DRIVE POWER

The decay of stored energy after the drive power is cut off is given by Eq. 5. The stored energy at cutoff, $W(t_c)$ can be gotten from Eq. 11, with $x = t_c / \tau_0$. In the following, the notation W_c will be used, for simplicity, to represent the quantity, $W(t_c)$.

In practice, one often monitors forward and reflected power, as well as stored energy, and it is the behavior of the "reflected" power after cutoff that will be dealt with specifically in this section. After shutoff of the drive power, it is re-emitted power from

the cavity that is seen by the reflected-power monitor. Taking the derivative of Eq. 5 and using Eq. 7 one gets

$$\begin{aligned} dW/dt &= -W_c\omega/Q_L e^{-(\omega t'/Q_L)} \\ &= -[(W_c\omega/Q_0) + (W_c\omega/Q_e)]e^{-(\omega t'/Q_L)} \end{aligned} \quad (31)$$

The first term in the brackets (multiplied by the exponential) can be interpreted as the dissipative decay of stored energy, while the second term represents re-emitted power. Immediately after P_0 is shut off, at $t'=0$, the peak magnitude of re-emitted power is

$$P_c = W_c\omega/Q_e = W_c\beta\omega/Q_0. \quad (32)$$

For the case where drive-power cutoff occurs after several time constants (τ_0), $W_c \approx W(\infty) = P_0 Q_0 \gamma / \omega$, and

$$\begin{aligned} P_c &= P_0 \beta \gamma \\ &= 4P_0 [\beta / (1+\beta)]^2. \end{aligned} \quad (33)$$

For critical coupling the initial re-emitted transient has amplitude P_0 , the same as the initial turn-on transient at $t = 0$. For $\beta > 1$, the re-emitted transient is larger than P_0 ; for $\beta < 1$, it is smaller; it approaches $4P_0$ for very large values of β .

Idealized reflected-power curves (normalized to P_0) are shown in Fig. 6 for several values of β ; drive-power shut-off is at $t=5\tau_0$.

V. USE OF CAVITIES FOR PULSED-POWER AMPLIFICATION

In the previous section it was assumed that β remained constant. In the use of cavities as pulse-power amplifiers, the principal effect involved is the rapid increase in β to allow the stored energy to be extracted much more rapidly than it is dissipated in the cavity walls. The external Q of the cavity, Q_e , is made much smaller than the initial Q_e ; generally Q_e' will dominate Q_L in the output stage.

As an example, consider the case where there is critical coupling ($\beta=1$) during the energy-storage stage, and the fill time is long enough that $W_c \approx W(\infty) = P_0 Q_0 / \omega$. The peak "reflected" power immediately after cutoff of drive power will be

$$P_c = W_c\omega/Q_e' = P_0 Q_0 / Q_e'.$$

The power gain is

$$G = P_c/P_0 = Q_0/Q_e'. \quad (34)$$

The output power may be taken out of the input port and fed to a load via a circulator (see, e.g., Ref. 5), or out of a separate port, as in Ref. 6. In the latter case, some

power will also be re-emitted from the input port, but the magnitude is determined by the original Q_0 and is usually negligible compared to the enhanced output.

VI. ENERGY-STORAGE EFFICIENCY

As noted above, the issue of energy-storage efficiency has been considered by Baraev and Korovin⁹. In this section the dependence of the efficiency on the coupling and fill time will be displayed.

The time-dependent efficiency, $F(t, \beta)$, for a cavity with coupling factor β is defined as the ratio of stored energy, at time t , to the total rf output energy from the source up to time t :

$$F(t, \beta) = W(t) / P_0 t,$$

or, in terms of normalized time, x ,

$$F(x, \beta) = W(x) / P_0 \tau_0 x = Y(x) / x. \quad (35)$$

The efficiency is shown graphically in Fig. 7, in which both the time dependence of F and its behavior as a function of β are illustrated.

One practical situation in which the filling efficiency for a cavity is important arises when the rf drive source has a limited pulse length, extending, say, from $t = 0$ to $t = t_1$ ($x = 0$ to $x = x_1 = t_1 / \tau_0$). The efficiency curves of Fig. 7 suggest that the greatest fraction of the available rf energy, $P_0 t_1$, can be stored by making x_1 much smaller than unity and using strong coupling, i.e., $\beta \gg 1$. Since t_1 and ω of the source are fixed in this situation, one must then make Q_0 of the cavity as large as possible in order to get the largest possible τ_0 and the smallest possible x_1 . (One may generally enhance the efficiency also by shortening the length of the rf pulse to reduce x_1 , but at the expense of total stored energy.)

In practice the magnitude of Q_0 may be limited by practical considerations such as size limitations or the need for extracting the stored energy quickly: the maximum practical Q_0 will therefore fix x_1 . The coupling should then be chosen to maximize $F(x_1, \beta)$.

The condition for maximum F at fixed x is

$$(\partial F / \partial \beta)_x = [(1 - \beta) / \beta(1 + \beta)] - x e^{-Kx} / (1 - e^{-Kx}) = 0. \quad (36)$$

The solution, $\beta = B(x)$, of Eq. 36 has been found numerically for $0.001 \leq x \leq 50$. For large values of x , $B(x)$ asymptotically approaches unity; for small values of x , $x \cdot B(x)$ asymptotically approaches 2.513. The product, $x \cdot B(x)$ is plotted in Fig. 8a. The corresponding maximum efficiency, $F(x, B)$ is plotted in Fig. 8b. It can be seen that the

maximum efficiency asymptotically approaches a value of approximately 0.815 for small x and zero for large x .

As an example of maximizing energy-storage efficiency, consider the case where an S-band klystron with $2\text{-}\mu\text{s}$ pulse width is to be used to store energy in a copper cavity to produce a high-power, 5-ns output pulse, using the technique described in Sec. V. Such a short output pulse requires that the cavity be in a waveguide geometry, with a length of about 50 cm. Such a cavity, made out of WR-284 waveguide, can have a Q_0 , at room temperature, of about 1.5×10^4 , giving $\tau_0 \approx 0.8\text{ }\mu\text{s}$. The normalized fill time will be $x_1 = 2.0/0.8 = 2.5$. From Fig. 8a, $x_1 \cdot B(2.5) = 3.6$; the coupling should be $\beta = B(2.5) = 1.4$; the efficiency, from Fig. 8b, will be about 36%. If the klystron has a 10-MW peak power, the total output energy in $2\text{ }\mu\text{s}$ is 20J, of which 7.2J will be available as stored electromagnetic energy in the cavity. In this case the intrinsic Q_0 of the cavity is too low to get much better storage efficiency than would be obtained with critical coupling (see Fig. 7).

If liquid-nitrogen cooling is used to enhance Q_0 and τ_0 by about a factor of three, then, with $x_1 \approx 0.83$, one can use a coupling factor of $\beta = B(0.83) \approx 3.5$ to achieve a 60% storage efficiency, compared to about 35% if the cavity were critically coupled.

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FIGURE 1 A-M

Plots of the quantities Y , E_D , E_I , and E_R as functions of normalized time $x = t/\tau_0$, for values of the cavity/feed coupling, β (BETA), ranging from $\beta = 0.010$ to $\beta = 100.0$. These four quantities each represent an energy integrated to time x ; they are normalized to $W^* = P_0\tau_0$ to make them dimensionless. Y represents stored energy, E_D is the energy dissipated in heating the cavity walls, E_I is the total energy coupled into the cavity (the sum of Y and E_D), and E_R represents the total energy reflected from the cavity.

FIG. 1A

FIG. 1B

FIG. 1C

FIG. 1D

FIG. 1E

FIG. 1F

FIG. 1G

FIG. 1H

FIG. 11

FIG. 1J

FIG. 1K

FIG. 1L

FIG. 1M

FIGURE 2 A-M

Plots of the quantities P_i , R_w , D , and P_R as functions of normalized time $x = t/\tau_0$, for values of the cavity/feed coupling, β (BETA), ranging from $\beta = 0.010$ to $\beta = 100.0$. These four quantities each represent an instantaneous power at time x ; they are normalized to the incident (source) power, P_0 , to make them dimensionless. P_i is the net power coupled into the cavity, R_w is the rate of build-up of stored energy, D represents the power dissipation due to heating of the cavity walls, and P_R is the power reflected back toward the rf source.

FIG. 2A

FIG. 2B

FIG. 2C

FIG. 2D

FIG. 2E

FIG. 2F

FIG. 2G

FIG. 2H

FIG. 21

FIG. 2J

FIG. 2K

FIG. 2L

FIG. 2M

FIGURE 3

Stored energy, Y (normalized to $W^* = P_0 \tau_0$), as a function of normalized time, x , showing the dependence on coupling, β . The solid curve is for critical coupling ($\beta = 1$); the dotted curves are for $\beta = 0.01, 0.10, 10$, and 100 ; the dashed curves are for $\beta = 0.055, 0.55, 5.5$, and 55 . Maximum stored energy for very large values of x is attained with critical coupling.

FIG. 3

FIGURE 4

Time dependence of the ratio of rate of build-up of stored energy to rate of energy dissipation: $R_R = R_w/D$. x is the normalized time: $x = t/\tau_0$. The solid curve represents the critically coupled case, $\beta = 1$. The dotted curves are for $\beta = 0.01, 0.10, 10$, and 100 ; the dashed curves are for $\beta = 0.055, 0.55, 5.5$, and 55 . For $\beta \approx 1$, heating losses begin to dominate energy build-up rate (i.e. $R_R < 1$) at times a little greater than $x = 1$. For strongly overcoupled cavities ($\beta > 1$) a break-even point occurs at earlier times; for strongly undercoupled cavities it occurs at later times.

FIG. 4

FIGURE 5

Dependence of several critical times on the cavity/feed coupling, β (BETA). x_R is the normalized time at which P_R is unity (see Fig. 4); x_w is the normalized time at which the energy build-up rate, R_w , is maximum (see Fig. 2); x_p is the normalized time at which power coupled into the cavity (P_i) is maximum and reflected power (P_R) vanishes. x_p is finite only for overcoupled cavities; for $\beta \leq 1$, P_i reaches maximum value (and $P_R = 0$) only in the limit, $x \rightarrow \infty$.

FIG. 5

FIGURE 6 A-H

Reflected power, P_R (normalized to P_D), as a function of normalized time, $x = t/\tau_0$, for several values of coupling, β (BETA). Incident power is assumed to be applied as a step function at $x = 0$ and shut off instantaneously at $x = 5$. A spike in P_R occurs at the shut-off time for values of β larger than about $1/3$, as the amplitude due to re-emitted energy appears without interference with the amplitude reflected from the coupling aperture. For critical coupling ($\beta = 1$) the peak power of the shut-off spike is equal to that of the spike at $x = 0$; for weak coupling ($\beta < 1$) it is smaller; for strong coupling ($\beta > 1$) it is larger, reaching a limiting magnitude four times that of the initial spike. Note the vanishing of P_R at a finite time for the strongly coupled cases.

FIG. 6A

FIG. 6B

FIG. 6C

FIG. 6D

FIG. 6E

FIG. 6F

FIG. 6C

FIG. 6H

FIGURE 7

A. Energy storage efficiency, F , as a function of normalized time, $x = t/\tau_0$, for several values of coupling factor, β . The solid curve represents the case with critical coupling, $\beta = 1$. The dotted curves are for $\beta = 0.01, 0.10, 10$, and 100 ; dashed curves are for $\beta = 0.025, 0.25, 2.5$, and 25 ; dash-dot curves are for $\beta = 0.055, 0.55, 5.5$, and 55 . Energy storage efficiency for the undercoupled cases is always smaller than for critical coupling, whereas it is greater at early times in the overcoupled cases. In the limit of very large times, critical coupling gives the greatest efficiency, but with a value generally far below the maximum efficiencies ($F = 0.81$) attainable with large β at small x .

B. Energy storage efficiency, F , as a function of coupling factor, β (BETA), for several values of normalized time, x . The solid curve represents the case for $\beta = 1$. The dotted lines are for $x = 0.001, 0.01, 0.10$, and 10.0 ; the dashed curves are for $x = 0.0025, 0.025, 0.25, 2.5$, and 25.0 ; the dash-dot lines are for $x = 0.0055, 0.055, 0.55, 5.5$, and 55 .

FIG. 7A

FIG. 7B

FIGURE 8

A. The product $x \cdot B$ as a function of the normalized time, x . B is that value of β (assumed constant in time) which yields the maximum stored energy at time x . $x \cdot B$ reaches an asymptotic value approximately 2.51 at small x . For large x , $x \cdot B$ approaches x from above (and $B \rightarrow 1$).

B. The maximum energy storage efficiency at normalized time x , obtained with the optimized coupling, $\beta = B$, from the curve in (A); it asymptotically approaches zero for large x and 0.815 for small x .

FIG. 8

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