MASTER

Heat Transfer at a Beam Port Corner

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INFORMAL REPORT

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Along the general run of the vacuum chamber synchrotron radiation strikes the wall at a glancing angle of about 5.6° . The heat source is well-approximated by a ribbon of uniform power density having a small vertical height and an infinite azimuthal length. The heat transfer problem reduces to one in two-dimensions (Figure 1), and it has been considered in a previous note.¹ At the corner of a beam port (Figure 2) the angle of incidence becomes 90° , so the temperature rises much higher than elsewhere. Since the power density at the corner is not uniform in its azimuthal dependence, but is strongly peaked at the point of normal incidence (Figure 3), two-dimensional heat flow is not a good approximation. It would be difficult to consider the actual curved geometry sketched in Figure 2, and we do not attempt such a discussion. Instead we study the rectangular 3d problem shown in Figure 4. This is easily solved and we expect it yields a good first estimate of the temperature rise at the corner.

The parameters associated with the heat transfer problem of Figure 4 are:

thickness of aluminum L (cm),
distance of corner from source D (cm),
film coefficient h (watts/cm²-°C),
total power absorbed by corner P (watts),
incident power density (watts/cm²):

$$P(x,y) = \frac{P}{2\pi\sigma_1\sigma_2} e^{-\frac{1}{2}\left(\frac{x^2}{\sigma_1^2} + \frac{y^2}{\sigma_2^2}\right)} \quad (1)$$

We shall take the azimuthal width σ_1 equal to the radius R of the corner, see Figure 3. The total power absorbed by the corner can be estimated using Eq. (2) of G. K. Green's report,²

leg

Figure 1

Figure 2

Figure 3

Corner subtends θ radians of electron orbit.

Total power absorbed by corner is

$$P = 14.09 E^2 I \theta / p.$$

Gaussian vs Cosine Power Distribution

$\alpha = x/\sigma_1 = \text{arc length/radius}$

Figure 4

$$P = 0.01409 E^4 I \theta / \rho, \quad (\text{watts})$$

where I is the electron current in ma, θ is the angle of orbit subtended in mrad, E is the electron energy in GeV, and ρ is the bending radius of the dipole magnet in meters. In our case $E = 2.5$ GeV, $I = 500$ ma, $\rho = 8.17$ m, and $\theta = 2000 \sigma_1 / D$, so

$$P = (6.74 \times 10^4) \frac{\sigma_1}{D} \quad (\text{watts}). \quad (2)$$

The vertical height σ_2 can be determined from Eq. (15) of G. K. Green's report,²

$$\sigma_2 = \frac{0.608}{\gamma} D,$$

where $\gamma = 1957 E$. For $E = 2.5$ GeV,

$$\sigma_2 = (1.24 \times 10^{-4}) D.$$

Assuming the corner is not vertical, but inclined at an angle β from the horizontal, the effective source height will be increased to

$$\sigma_2 = (1.24 \times 10^{-4}) D / \sin \beta. \quad (3)$$

Let us define

$$\theta(x, y, z) = T(x, y, z) - T_w$$

to be the difference between the temperature at the point x, y, z , and the temperature of the water T_w . We take the origin of the coordinate system to be at the center of the heat source, and let the x -direction be azimuthal, the y -direction be vertical, and the z -direction be radial (into the aluminum). The maximum temperature rise on the surface of the aluminum is

$$\Delta T = \theta(0, 0, 0),$$

and the maximum heat flux into the water is

$$Q = -k \frac{\partial \theta}{\partial z}(0,0,L).$$

In the appendix we solve Laplace's equation $\nabla^2 \theta = 0$ under the appropriate boundary conditions to determine $\theta(x,y,z)$. We find

$$\Delta T = \frac{P}{\pi^2 k} \int_0^\infty \int_0^\infty \frac{d\Omega_1 d\Omega_2}{\omega} e^{-\frac{1}{2}(\sigma_1^2 \Omega_1^2 + \sigma_2^2 \Omega_2^2)} \frac{h \sinh \omega L + k_W \cosh \omega L}{h \cosh \omega L + k_W \sinh \omega L}, \quad (4)$$

$$Q = \frac{P}{\pi^2} \int_0^\infty \int_0^\infty d\Omega_1 d\Omega_2 e^{-\frac{1}{2}(\sigma_1^2 \Omega_1^2 + \sigma_2^2 \Omega_2^2)} \frac{h}{h \cosh \omega L + k_W \sinh \omega L}, \quad (5)$$

where

$$\omega = \sqrt{\Omega_1^2 + \Omega_2^2}$$

The Gaussian power density of Eq. (1) is an approximation whose validity we shall briefly discuss. The vertical distribution of the synchrotron radiation is indeed very accurately described by a Gaussian, but the azimuthal dependence of the power density at the corner is not really Gaussian. The cosine distribution $\frac{x}{2R} \cos \frac{x}{2R}$ would be more accurate, where R is the radius of the corner, but we shall take the Gaussian $\frac{1}{\sqrt{2\pi} R} \exp(-\frac{x^2}{2R^2})$ to simplify the numerical integration of Eqs. (4) and (5). A comparison of the two distributions is given in Figure 3, and we believe the Gaussian is satisfactory for a first estimate. In the future we could consider a more realistic distribution, since the solution found in the appendix is not restricted to a Gaussian.

In Figures 5 a,b,c we plot ΔT and Q vs σ_1 for distances $D = 80$ cm to 130 cm. The film coefficient is fixed at $h = 3$ watts/cm² - °C, the thickness of the aluminum is taken to be $L = 0.4$ cm, and the wall is inclined with respect to the horizontal at an angle of (a) $\beta = 90^\circ$, (b) $\beta = 30^\circ$, (c) $\beta = 15^\circ$. Since the temperature of the aluminum should not exceed 175°C, and the water temperature is about 20°C, we must require $\Delta T < 155^\circ\text{C}$. Taking into account that our calculation is only a rough estimate, let us demand

$$\Delta T < 130^{\circ}\text{C}. \quad (6)$$

The situation we are considering is one in which the water is flowing out of the paper in Figure 2. If the radius of the corner σ_1 is made too small there will be no room to place the water channel in close proximity to the corner. We shall therefore take $\sigma_1 = 0.4$ cm which is about as small as possible. Then in order to satisfy the condition in Eq. (6), it would be necessary to have the distance D between the corner and the source satisfy:

$$D > 130 \text{ cm} \quad \text{for } \beta = 90^{\circ}, \quad (7a)$$

$$D \approx 125 \text{ cm} \quad \text{for } \beta = 30^{\circ}, \quad (7b)$$

$$D \approx 110 \text{ cm} \quad \text{for } \beta = 15^{\circ}. \quad (7c)$$

The distance between the source and the wall along the general run of the vacuum chamber is only 80 cm. Since the distances in Eq. (7) are much larger than this, they will be very difficult to achieve while maintaining the smoothness required to keep the RF impedance of the vacuum chamber low.

In Figures 6 a,b,c we plot ΔT and Q vs L for film coefficients $h = 1, 3, 5, 6$, and ∞ watts/cm² - °C. We fix $\sigma_1 = 0.4$ cm and $\beta = 30^{\circ}$, and take the distance of the corner to the source to be (a) $D = 80$ cm, (b) $D = 100$ cm, and (c) $D = 120$ cm. We note that for $h \leq 6$, reducing the aluminum thickness L does not significantly reduce ΔT , and in fact may result in an increase in ΔT . The reason for this is that although decreasing L decreases the temperature drop across the aluminum, the heat flux into the water is increased, resulting in a larger temperature drop across the film. Increasing the film coefficient from $h = 1$ to $h = 3$ results in a significant decrease in temperature. However, increasing it from $h = 3$ to $h = 5$ results in a smaller change. Considering the extremely high water velocity required to realize $h = 5$, one would expect that $h = 3$ watts/cm² - °C is about as high as it is worthwhile raising the film coefficient.

$\Delta T (^{\circ}\text{C})$ $Q (\text{watts}/\text{cm}^2)$

$$h = 3 \text{ watts}/\text{cm}^2 \cdot ^{\circ}\text{C}$$

$$L = 0.4 \text{ cm}$$

$$\sigma_2 = (1.29 \times 10^{-4}) D$$

$$D = 80 \text{ cm}$$

Figure 5a

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$\Delta T(^{\circ}\text{C})$ $Q (\text{watts}/\text{cm}^2)$

$$h = 3 \text{ wth}/\text{cm}^2 \cdot ^{\circ}\text{C}$$

$$L = 0.4 \text{ cm}$$

$$\sigma_2 = (1.24 \times 10^{-4}) D / \rho \mu \beta$$

$$\beta = 30^{\circ}$$

$$D = 90 \text{ cm}$$

Figure 5b

same as wire

$$\sigma_2 = (1.24 \times 10^{-4}) D$$

240

220

200

180

160

140

120

100

90

100

110

120

130

 $\sigma_1 (\text{cm})$

0.2

0.3

0.4

0.5

 $\sigma_1 (\text{cm})$

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10 X 10 TO THE HALF INCH

SQUARE

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$\Delta T (^{\circ}\text{C})$

$$h = 3 \text{ watts/cm}^2\text{-}^{\circ}\text{C}$$

$$L = 0.14 \text{ cm}$$

$$Q_2 = (1.24 \times 10^{-4}) D / \sin \beta$$

$$\beta = 15^{\circ}$$

Figure 5c

240

220

200

180

160

140

120

100

 $D = 80 \text{ cm}$

90

100

110

0.2

0.3

0.4

0.5

 $\sigma_1 (\text{cm})$

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REFERENCES

1. S. Krinsky - "Heat Transfer in the Vacuum Chamber Wall". NSLS Technical Note 2, Brookhaven National Laboratory.
2. G. K. Green = "Beam Power in the Synchrotron Radiation". NSLS Technical Note 3, Brookhaven National Laboratory.

APPENDIX: Solution of the 3d Potential Problem

We shall not restrict our attention to the Gaussian distribution of Eq. (1), but rather consider the more general distribution

$$P(x,y) = P f(x) g(y), \quad (A1)$$

where

$$f(x) = \frac{1}{\pi} \int_0^{\infty} d\Omega_1 [F(\Omega_1) \cos \Omega_1 x + \tilde{F}(\Omega_1) \sin \Omega_1 x],$$

$$g(x) = \frac{1}{\pi} \int_0^{\infty} d\Omega_2 G(\Omega_2) \cos \Omega_2 y.$$

The normalization $F(0) = 1$ and $G(0) = 1$ assures that P is the total integrated power. We let

$$\theta(x,y) = T(x,y) - T_w$$

be the difference between the temperature at the point x,y,z and the water temperature (in $^{\circ}\text{C}$). For the geometry of Figure 4, the solution to Laplace's equation will have the form

$$\theta(x,y,z) = \int_0^{\infty} \int_0^{\infty} d\Omega_1 d\Omega_2 [\cos \Omega_1 x + C(\Omega_1) \sin \Omega_1 x] \cos \Omega_2 y$$

$$\times \{A(\Omega_1, \Omega_2) \sinh[w(L-Z)] + B(\Omega_1, \Omega_2) \cosh[w(L-Z)]\}, \quad (A2)$$

where

$$w = \sqrt{\Omega_1^2 + \Omega_2^2}.$$

For turbulent water flow, the interface between the aluminum and the water is described by a film coefficient h . The boundary condition due to the film coefficient,

$$-k \frac{\partial \theta}{\partial z}(x, y, L) = h\theta(x, y, L),$$

implies

$$k_w A(\Omega_1, \Omega_2) = hB(\Omega_1, \Omega_2). \quad (A3)$$

The boundary condition due to the incident synchrotron radiation heat flux,

$$-k \frac{\partial \theta}{\partial z}(x, y, 0) = P(x, y),$$

together with (A3) determines $\theta(x, y, Z)$,

$$\theta(x, y, Z) = \frac{P}{\pi^2 k} \int_0^\infty \int_0^\infty \frac{d\Omega_1 d\Omega_2}{\omega} [F(\Omega_1) \cos \Omega_1 x + \tilde{F}(\Omega_1) \sin \Omega_1 x] G(\Omega_2) \cos \Omega_2 y$$

$$\times \frac{h \sinh[\omega(L-Z)] + k_w \cosh[\omega(L-Z)]}{h \cosh \omega L + k_w \sinh \omega L} \quad (A4)$$

Let us first consider Eq. (A4) in the limiting case of a uniform heat source of infinite length. In this limit the heat flow becomes two-dimensional. We take

$$f_1(s) = \frac{\alpha}{2} e^{-\alpha|x|} \quad (\alpha \rightarrow 0+).$$

Then $F(\Omega_1) = \pi \alpha \delta(\Omega_1)$ and $\tilde{F}(\Omega_1) = 0$, so

$$\theta_1(0, 0, 0) = \frac{(P\alpha/2)}{\pi k} \int_0^\infty \frac{d\Omega}{\Omega} G(\Omega) \frac{h \sinh \Omega L + k \Omega \cosh \Omega L}{h \cosh \Omega L + k \Omega \sinh \Omega L}, \quad (A5)$$

which agrees with Eq. (C3) of our previous note¹, since $P\alpha/2$ is the uniform power per unit length.

A second case easily dealt with is the antisymmetric power distribution

$$f_2(x) = \begin{cases} \frac{\alpha}{2} e^{-\alpha|x|} & (-\infty < x < 0) \\ -\frac{\alpha}{2} e^{-\alpha|x|} & (0 < x < \infty). \end{cases}$$

In the limit $\alpha \rightarrow 0+$, $\tilde{F}(\Omega_1) = \frac{-\alpha}{\Omega_1}$ and $F(\Omega_1) = 0$, so

$$\theta_2(x, 0, 0) = \frac{-P\alpha}{2\pi k} \int_0^\infty \int_0^\infty \frac{d\Omega_1 d\Omega_2}{\omega} \frac{\sin \Omega_1 x}{\Omega_1} G(\Omega_2) \frac{h \sinh \omega L + k\omega \cosh \omega L}{h \cosh \omega L + k\omega \sinh \omega L} \quad (A6)$$

In the limit $x \rightarrow \pm\infty$, we expect

$$\theta_2(x, 0, 0) \rightarrow \mp \theta_1(0, 0, 0),$$

and this is the case since when $x \rightarrow \pm\infty$, one has $\sin \Omega_1 x / \Omega_1 \rightarrow \pm \pi \delta(\Omega_1)$.

Superposition of the two previously considered cases yields the solution to the problem of a half-infinite uniform heat source,

$$f_3(x) = \frac{1}{2}[f_1(x) + f_2(x)] = \begin{cases} \frac{\alpha}{2} e^{-\alpha|x|} & (-\infty < x < 0) \\ 0 & (0 < x < \infty). \end{cases}$$

In this case,

$$\theta_3(x, 0, 0) = \frac{1}{2}[\theta_1(0, 0, 0) + \theta_2(x, 0, 0)], \quad (A7)$$

and since $\theta_3(0, 0, 0) = 0$, we see that

$$\theta_3(0, 0, 0) = \frac{1}{2}\theta_1(0, 0, 0). \quad (A8)$$

At the end of the half-infinite ribbon the temperature is only one-half of its value at $x = -\infty$. However, this temperature reduction is confined to a very small region near $x = 0$. If we take $G(\Omega_2) = \exp(-\sigma_2^2 y^2 / 2)$, then

$$\theta_2(x, 0, 0) \sim \frac{P\alpha}{2\pi k} \sqrt{\frac{\pi}{2}} \left(\frac{-x}{\sigma} \right) \ln \frac{L}{2|x|} \quad (x \rightarrow 0).$$

We see that the length scale is set by the vertical height of the ribbon of synchrotron radiation, which is very small. The rapid rise in temperature is seen in Fig. 7, where we plot the result of a numerical integration of Eq. (A6) for $h = 5 \text{ watts/cm}^2 - ^\circ\text{C}$, $L = 0.4 \text{ cm}$, and $\sigma_2 = 0.03 \text{ cm}$. We see that $\theta_2(x, 0, 0)$ has already reached 75% of its limiting value (as $x \rightarrow -\infty$) at $x = 0.2 \text{ cm}$.

Figure 7

$\theta_2(x, y, z) / (p_0 / z)$

$\lambda = 5, \mu = 0.4, \sigma = 0.03$