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S. H. Kahana
Physics Department
Brookhaven National Laboratory
Upton, New York 11973
U S A

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Chiral Field Theories as Models for Hadron Substructure

Sidney H. Kahana

Physics Department, Brookhaven National Laboratory, Upton NY 11973
U S A

ABSTRACT

A model for the nucleon as soliton of quarks interacting with classical meson fields is described. The theory, based on the linear sigma model, is renormalizable and capable of including sea quarks straightforwardly. Application to nuclear matter is made in a Wigner-Seitz approximation.

1. THE BASIC MODEL: VALENCE QUARK SOLITONS

Several field theoretic models of the substructure of hadrons now exist, some more phenomenological than others, but most based on some effective Lagrangian. The MIT bag model [1] and the soliton model of FRIEDBERG-LEE [2,3] are the antecedents, but most of the more modern theories [4-8] incorporate the chiral symmetry which is so dominantly a part of low energy particle and nuclear physics. I wish to discuss one version of these effective field theories pursued with George RIPKA [6,9], which allows naturally for the possibility of sea quarks in the nucleon substructure. We begin with the Lagrangian for the linear sigma model [10]

$$\mathcal{L} = \bar{\psi} [i\gamma\partial - g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})] \psi + \frac{1}{2} \partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2} \partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} - F(\sigma, \vec{\pi})$$

$$F(\sigma, \vec{\pi}) = \frac{\alpha}{4f} (\sigma^2 + \pi^2 - f^2)^2, \quad (1)$$

where $\psi, \bar{\psi}, \sigma, \vec{\pi}$ are quark scalar and pseudoscalar meson fields.

Baryons appear in this model as static solutions of the hamiltonian

$$H = \int d^3r \{ \psi^\dagger [\vec{\alpha} \cdot \vec{p} + g\beta(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi})] \psi + \frac{1}{2} [(\nabla\sigma)^2 + (\nabla\vec{\pi})^2] + F(\sigma, \vec{\pi}) \} \quad (2)$$

If $\sigma, \vec{\pi}$ assume classical values through coherent states,

$$\phi^a(x) | \phi_{c1}^a \rangle = \phi_{c1}^a(x) | \phi^a \rangle, \quad \phi^a \equiv [\sigma, \pi^1],$$

then the hamiltonian can be diagonalized by seeking solutions of

$$h | \lambda \rangle = e_\lambda | \lambda \rangle$$

$$h = \vec{\alpha} \cdot \vec{p} + g\beta(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}). \quad (3)$$

The Dirac sea is constructed by filling all of the negative energy states

$$|D\rangle = \prod_{e_\lambda < 0} \alpha_\lambda^\dagger |0\rangle, \quad (4)$$

and the total energy, with some valence states occupied, can be split into a Dirac and a meson part

$$E = E_D + E_M$$

$$E_D = \sum_{\lambda} e_{\lambda}$$

$$E_M = \int d^3r \frac{1}{2} [(\nabla\sigma)^2 + (\nabla\vec{\pi})^2 + F(\sigma, \vec{\pi})] \quad (5)$$

Self-consistent solutions can be obtained by maximizing E with respect to the scalar fields $\sigma, \vec{\pi}$. The potential function $F(\sigma, \vec{\pi})$ is minimized by remaining on the "chiral circle"

$$\phi^a \phi_a = \sigma^2 + \pi^2 = f_{\pi}^2,$$

and such a choice is tantamount to the limit of large α . We work near such a limit in which case the chiral fields are collectively described by the unitary operator

$$U = \frac{1}{f_{\pi}} (\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) = e^{i\gamma_5 \vec{\tau} \cdot \vec{\theta}(\vec{r})} \quad (6)$$

However, a lowest energy is obtained for the hedgehog configuration [4,11]

$$\vec{\theta}(\vec{r}) = \frac{\pi}{r} \hat{r} \quad (7)$$

To properly solve the classical equations of motion and achieve a finite total energy, one must impose

$$\theta = \frac{1}{r^2} \rightarrow 0 \quad \text{as } r \rightarrow \infty \quad (8a)$$

$$\theta = n\pi, \quad r = 0 \quad (8b)$$

The first condition must be altered appropriately for a finite π -meson mass and implies the symmetry-breaking, but translationally invariant vacuum described by $\sigma = f_{\pi}$, $\vec{\pi} = 0$ (that is for $\theta(r) \equiv 0$) is achieved at large distance. In this latter vacuum the quark energies are given by

$$\varepsilon(k) = \sqrt{k^2 + m^2}, \quad (9)$$

and the states are solutions of

$$h_0 = \vec{\alpha} \cdot \vec{p} + \beta m.$$

Chiral symmetry is broken at the minimum in $F(\phi)$, and the fermions acquire the mass $m = gf_{\pi}$ there. States $\{\lambda\}$ are labelled by the overall spin obtained by combining the broken spin and isospin operators

$$\vec{G} = \vec{J} + \vec{T}. \quad (10)$$

The single fermion spectra obtained for a chiral angle,

$$\theta = n\pi e^{-r/R} \quad (11)$$

are displayed in Fig.1 for the "winding number", $n = 1, 2$. A localized solution of the system of $\{(2), (3), (7), (11)\}$ exists as soon as the energy ε_0 in Fig.1 drops below gf_{π} . Baryon numbers 1, 2 are associated with $n = 1, 2$ through

$$N_B = n N_C \left(\frac{1}{N_C} \right) = n 3 \left(\frac{1}{3} \right), \quad N_C \text{ the number of colours,}$$

for filling of the n lowest lying valence levels. For sufficiently large value of the dimensionless coupling or size parameter $X = gf_\pi R$, first a $G=0^+$ and then a $G=0^-$ level actually pass through zero energy joining the Dirac sea.

Figure 1

The single fermion spectrum for two cases in which (a) a single $G=0^+$ level with energy ϵ_0 drops to negative energies or (b) both a $G=0^+$ and a 0^- level drop into the Dirac sea. These correspond to winding number $n=1,2$ in Eq.(11). For small enough X these valence levels remain at positive energy.

A soliton, constructed from valence quarks only, has an energy approximately given by $E = E_{\text{valence}} + E_{\text{meson}}$,

$$\frac{E}{gf_\pi} = N_C \left(\frac{a}{X} - b \right) + \frac{4\pi^2 X}{6g^2}, \quad (12)$$

with, for example, $a = 3.12$, $b = 0.93$ for $\theta = \pi(1 - r/R)$ [6] and slightly different values for θ given by (11). Minimizing (12) to obtain the stable soliton size R

yields $E_{\text{soliton}} = N_{\text{cf}} \pi [10.6 - 0.94g]$. The stability is destroyed for sufficiently large coupling $g = 10.6/0.94$, a situation which persists when non-valence quarks are included. Of course, the soliton constructed in this way has broken isospin and is properly an intrinsic state for both the nuclear and delta. Fixing $f_\pi = 93$ MeV to the π -meson decay constant and the total soliton mass to an average between m_N and m_Δ yields a radius ≈ 0.6 fm, some 30% too small.

2. SEA QUARKS AND THE CASIMIR ENERGY

The fermion part of the vacuum energy of our soliton, i.e. the Casimir energy, is more properly evaluated relative to the translationally invariant vacuum [12]

$$E_D = \sum_{\lambda < 0} \epsilon_\lambda - \sum_{k < 0} \epsilon(k) \\ = \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \text{tr} \ln \left[1 + \frac{1}{-\gamma_5 + m} V_F \right], \quad V_F = [g(\sigma + i\gamma_5 \vec{\tau} \cdot \vec{\pi}) - gf_\pi] \quad (13)$$

Equation (13), when expanded in the fermion perturbation, $V_F = h - h_0$, constitutes the so-called quark one-loop approximation. To this latter energy we could also add the valence energies and obtain the total baryon energy, except in the case for $n = 1$ with $\epsilon(0^+) < 0$, when (13) already includes the valence energy.

As it stands (13) is divergent and must be renormalized. An alternative form for (13) can be written in terms of the one-fermion hamiltonian [13,16]

$$E_D = -\frac{1}{2} \text{tr} (\sqrt{h^2} - h) - \frac{1}{2} \text{tr} (\sqrt{h_0^2}) = -\frac{1}{2} \int_{-i\infty}^{i\infty} \frac{d\omega}{2\pi i} \text{tr} \ln (1 + G V), \quad (14)$$

where the squared hamiltonians are given by

$$h^2 = -\nabla^2 + g^2(\sigma^2 + \vec{\pi}^2) + ig\vec{\gamma} \cdot [\nabla\sigma + i\gamma_5 \vec{\tau} \cdot \nabla\vec{\pi}] \\ = h_0^2 + V = (-\nabla^2 + g^2 f_\pi^2) + V, \quad G = (h_0^2 + \omega^2)^{-1} \\ V = g^2(\sigma^2 + \vec{\pi}^2 - f_\pi^2) + ig f_\pi \vec{\gamma} \cdot \nabla U. \quad (15)$$

On the chiral circle the perturbation potential in (14) and (15) is a function only of field derivatives, leading to the derivative expansion of the energy and other observables [7,8,13,15,16]. The quark current operator is the simplest such operator expressible in these terms, and is given in the lowest order in these derivatives by

$$J^\mu(x) = \langle \bar{\psi} \gamma^\mu \psi \rangle = \frac{1}{2\pi^2} \epsilon^{\mu\alpha\beta\gamma} \epsilon_{dabc} \phi^d \partial_\alpha \phi^a \partial_\beta \phi^b \partial_\gamma \phi^c. \quad (16)$$

This yields the following approximate relation for the baryon density [15]

$$J^0(x) = -\frac{1}{2\pi r^2} \sin^2 \theta \frac{d\theta}{dr}, \quad (17)$$

and the baryon number

$$N_B = \int 4\pi r^2 J_0(x) dr = \frac{1}{\pi} \left[\theta - \frac{1}{2} \sin 2\theta \right]_0^\infty = n, \quad (18)$$

relating our model density to the anomalous density of SKYRME [11].

Figure 2

The quark density distribution in the baryon treated as soliton. The valence orbital, Dirac sea, and total densities are shown separately. For comparison, the dotted line indicates the anomalous density given in Eq. (17). If the soliton were sufficiently small then the deviation between perturbative and exact densities would be larger.

These approximate relationships become exact in the limit $X = gf_{\pi}R$. Figure 2 shows the actual densities arising from both volume and sea quarks and a comparison of the summed densities to the right hand side of (17), i.e. to the baryon density in a Skyrmeion [11,14,15]. This comparison of the derivative expanded chiral theory [7,8,16] to the Skyrme theories is extended further in the following. The contribution of sea quarks to baryon structure can clearly be appreciable. The perturbation expansion of (13) can be used to define the necessary counterterms to render E_D finite. The first two terms are in fact quadratically and logarithmically divergent in terms of a cutoff momentum and yield as counterterms

$$\Delta E_D^{(1)} = -\frac{1}{2} \int \frac{d\omega}{2\pi i} \text{tr} \, G V = -\frac{g^2}{4} \frac{2v}{\Omega} \langle \sigma^2 + \pi^2 - f_{\pi}^2 \rangle \sum_{\underline{k}} \frac{1}{\epsilon(\underline{k})} \quad (19a)$$

$$\Delta E_D^{(2)} = \frac{1}{16} \langle g^4 (\sigma^2 + \pi^2 - f_{\pi}^2)^2 + g^2 [(\nabla \sigma)^2 + (\nabla \pi)^2] \rangle \frac{2v}{\Omega} \sum_{\underline{k}} \frac{1}{\epsilon(\underline{k})^3} \quad (19b)$$

where v is the total Coulomb spin-isospin degeneracy of each quark level, and $\frac{1}{\Omega} \langle \rangle \equiv \frac{1}{\Omega} \int d^3 \underline{r}$. These counterterms are clearly proportional to field quantities already defined in the Lagrangian (1). Thus the renormalized Dirac energy is expressible as

$$E_D^R = -\frac{1}{2} \text{tr} \left(\sqrt{h_0^2 + V} - \sqrt{h_0^2} \right) + \frac{1}{4} \text{tr} \, V \frac{1}{|h_0|} - \frac{1}{16} \text{tr} \, V^2 \frac{1}{|h_0^3|} \quad (20)$$

Fig. 3 indicates the results of a detailed non-perturbative calculation of the spectrum [13,16].

Figure 3 The renormalized Dirac energy including both valence and sea contributions as calculated "exactly" from (20) by actually summing over eigenvalues of h . The calculation cannot be carried to arbitrarily large $X = g f_\pi R$. However, the derivative expansion result of (21) then applies, and the curve in this figure will drop slightly below the axis.

3. STRONG COUPLING LIMIT: DERIVATIVE EXPANSION

The use of a derivative expansion for "smooth" slowly varying chiral fields $\phi^a(x)$ and also of the limit $g \rightarrow \infty$ (or $g f_\pi R \rightarrow \infty$) have been referred to above. These limits are in effect equivalent and lead to several interesting results [13]. First, the gap between occupied negative energy levels and unoccupied valence orbits in Fig. 1 will in this limit become extremely large. In such a limit then, the present theory acquires a form of confinement. Clearly any nondegenerate level at negative energy not completely occupied by N_c quarks shall possess infinite energy in the limit $g \rightarrow \infty$.

The strong coupling limit can then be shown, by integrating out all fermion variables, to have a combined meson plus Dirac energy

$$E_{\text{TOT}} = \frac{1}{2} f_\pi^2 \langle \phi_1^2 \rangle + \frac{v N_c}{384 \pi^2} \langle 3 \phi_1^2 \phi_j^2 - 2 (\phi_1 \cdot \phi_j)^2 - 2 \phi_{ij}^2 \rangle, \quad (21)$$

with $\phi_i \cdot \phi_j = (\partial_i \phi^a) (\partial_j \phi^a)$ etc.

Two comments can be made about this limit: first, the quartic terms in (21) differ from those in the Skyrme theory; second, the theory is unstable. This can be demonstrated by explicit evaluation of the total meson (first term) plus Dirac (second term) energy in (21) using (11) for $\theta(r)$:

$$E = 30.82 \frac{f_\pi^2 R}{\pi} - \frac{0.017}{R} \quad (22)$$

The total Dirac mass acquired in symmetry breaking is more than cancelled by the binding to mesons yielding the above negative second term, which in turn yields $E \rightarrow -\infty$ as $R \rightarrow 0$.

A stable theory can be recovered by including in this strong coupling limit a vector meson with energy

$$E_{\text{vector}} = \frac{1}{2} \frac{g_\omega^2}{2\pi} \int d^3r_1 d^3r_2 n_B(\vec{r}_1) \frac{e^{-m_\omega r_{12}}}{r_{12}} n_B(\vec{r}_2) = \frac{g_\omega^2}{m_\omega R^3} . \quad (23)$$

4. APPLICATIONS

4.1 Phenomenology

The baryon phenomenology of the model presented above can be obtained only after introducing some form of symmetry restoration. D. E. KAHANA, G. RIPKA and A. JACKSON [18] have put forward an isospin cranking picture which allows the proper definition of physical variables. In this approach, for example, a quark moment of inertia,

$$I_q = \frac{2}{3} \sum_{p,h} \frac{\langle h | \vec{t} | p \rangle \cdot \langle p | \vec{t} | h \rangle}{\epsilon_p - \epsilon_h} , \quad (24a)$$

is introduced and added to the meson moment of inertia to yield a rotational spectrum for the nucleon and delta,

$$E_T = E_0 + \frac{T(T+1)}{2(I_{\text{meson}} + I_q)} \quad (24b)$$

4.2 Nuclear Matter

A second application can be made to nuclear matter [19], treating the latter as a solid or gas of solitons. A very simple approach making use of the derivative expansion, and performing the many-body calculation in a Wigner-Seitz approximation has been done. Such a limiting calculation possesses the virtue of an underlying confinement for the quarks in individual baryons.

In place of the chiral angle in (11), one uses

$$\theta(r) = \pi e^{-r/R_0} - \pi \frac{r}{R} e^{-R/R_0} , \quad (25)$$

where the soliton size parameter is R_0 , and the average Wigner-Seitz cell radius R is related to the nuclear density

$$\rho = \frac{4}{4\pi/3 R^3} .$$

At the edge of the cell $\theta(r) = 0$ in (25), guaranteeing that a single baryon is contained in each cell. By using (21) and (22) one arrives at a soliton energy, in matter, of the form

$$M_{\text{soliton}} = A R_0 - \frac{B}{R_0(R)} + \frac{C}{R_0^3(R)} ,$$

and on reminimizing obtains a relation between soliton sizes at zero and finite density

$$R_0(R) = R_0(\infty) (1 + \eta(R)) , \quad R_0(\infty) = R_0 .$$

For low density $R = 3R_0$, $\eta = -0.03$, and for near nuclear density $R \approx 2R_0$, $\eta = -0.05$. This apparent decrease in soliton size on being immersed in nuclear matter is obtained leaving out the usual decrease in f_π found in other models, but is a warning that the usual "swelling" need not obtain. The EMC effect can then be explained, for example, by allowing for an effective rest mass perhaps 5% reduced from the normal value [19].

More recent work [20] has suggested that the $N_B = 0$ sector of the above theories is unstable in the limit in which boson (quantum) fluctuations are omitted. Such a situation follows from the nature of the counterterms in (11), and may force one to use models such as defined here as the only effective low energy theories. Efforts to understand this interesting point are under way.

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