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SEISMOLOGICAL INVESTIGATION OF CRACK FORMATION IN HYDRAULIC
ROCK FRACTURING EXPERIMENTS AND IN NATURAL GEOTHERMAL
ENVIRONMENTS

PROGRESS REPORT

For Period September 1, 1980 - August 31, 1981

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Abstract

This is a progress report for period September 1, 1980 to August 31, 1981 on the project entitled "Seismological Investigation of Crack Formation in Hydraulic Rock Fracturing Experiments and in Natural Geothermal Environments." Under this title, we have been working on the following three major problems in geothermal seismology.

The first is to define the geometry and mechanical construction of a geothermal system such as the Hot Dry Rock geothermal energy source at the Fenton Hill site. The second is to find the transport rate of geothermal fluids such as magma from observations of seismic vibrations caused by their movement. The third is to map 3-dimensional distribution of seismic velocity and attenuation for defining the extent of geothermal energy sources.

In the past year, we focused our major efforts on the following specific problems.

- (1) Interpretation of seismic data from hydraulic fracturing experiments at the Fenton Hill, New Mexico Hot Dry Rock Geothermal Site.
- (2) Interpretation of 3-D velocity anomalies in the western U.S. with special attention to geothermal areas.
- (3) Theoretical and observational studies of scattering and attenuation of high-frequency seismic waves.

- (4) Theoretical and observational studies of volcanic tremors in relation to magma transport mechanism.
- (5) Deployment and maintenance of 9 event-recorders around Mt. St. Helens in cooperation with the Oregon State University.

The past year has been an unusually productive period. Six papers were published or accepted for publication and an additional five have been submitted for publication on the subjects mentioned above. We presented the results described in these papers at various scientific meetings including the IAVCEI meeting in Tokyo which was attended by the coinvestigator. The principal investigator gave an Association Lecture at the IASPEI meeting in London, West Ontario on the subject of 3-D seismic structure in the western U.S.

Scope of Investigation

This research program has grown through participation in two major geothermal projects, namely, the Hot Dry Rock project of the Los Alamos Scientific Laboratory and the Magma Tap project of Sandia Laboratories. In order to interpret data from various seismic experiments conducted at Fenton Hill, New Mexico, and Kilauea Iki, Hawaii, we have developed theory and methods on seismic wave generation, transmission, scattering and attenuation in a medium containing a fluid-filled crack(s).

The results of interpretation are synthesized into a fluid-filled crack model for each geothermal site, and the model parameters are updated as new experimental results are analyzed. Our model is intended not only to define the geometrical and physical properties of a geothermal system, but also to estimate the mass and energy transport through measurements of seismic signals generated by the geothermal system such as volcanic tremors.

We have been developing models for the Hot Dry Rock fracture system at Fenton Hill, a magma lens in Kilauea Iki, deep and shallow tremor sources under Kilauea, and tremor sources inside Mt. St. Helens. The data needed for study are collected by a network of mobile digital seismographs of the Massachusetts Institute of Technology, as well as from the U.S. Geological Survey, Los Alamos Scientific Laboratories, Oregon State University, and University of Washington, through cooperative arrangements.

Summary of Results in the Present Reporting Period

We shall summarize the results obtained in the present reporting period in the following. Only a brief description will be given because a copy of a full paper on each subject has been submitted to the Department of Energy except for items 11, 13, 14 and 15.

(1) Interpretation of seismic data from hydraulic fracturing experiments at the Fenton Hill, New Mexico Hot Dry Rock Geothermal Site (K. Aki, M. Fehler, R.L. Aamodt, J.N. Albright, R.M. Potter, C.M. Pearson and J.W. Tester). This paper describes a synthesis of the results of various active seismic experiments conducted at Fenton Hill, and was submitted for publication in the Journal of Geophysical Research. Major conclusions about the Hot Dry Rock system are stated below with the experimental results on which each conclusion is based.

Presence of discrete planar vertical cracks: This was mainly evidenced by the strong attenuation of 12 KHz waves (by 20-30 dB) in the transmission experiments using the Dresser-Atlas piezo-electric source. The smoothness of crack plane was evidenced by the coherent arrivals of P to S and/or S to P converted waves.

Multiplicity of the vertical cracks with spacing of a few meters: the amplification of 0.5 KHz waves in the "shear-shadow" transmission experiments at line C requires at least a double crack separated by about 3.5 meters. The strong frequency dependence of the attenuation in the same transmission

experiment, if attributed to the crack, requires several parallel cracks to amplify the small effect for a single crack.

Presence of inclined joints: The S to S reflected waves observed in a transmission experiment indicates a reflector parallel to the gneiss-granodiorite contact. The radius of self-propped joints in the unpressurized condition is at least 1 meter, and that in the pressurized case may be larger than 10 - 15 meters. The latter evidence comes from the Fresnel zone radius of the S to S reflection observed at line A during the shear-shadow experiment.

Multiplicity of inclined joints: Observations of SS in the shear-shadow experiment, SS in a transmission experiment at 2586 m, PP in another experiment at a deeper location, and the shift of envelope peak from 13 to 16 msec with pressurization all indicate the presence of reflectors at various depths.

Presence of small-scale (~3 meters) cracks in an extended volume surrounding the main fracture system: This is evidenced by the broadening of the envelope of P and S wave amplitudes by pressurization observed during a transmission experiment, and by the decrease of P velocity with the decrease in temperature attributed to thermal cracking caused by heat extraction, as well as by the increased variance of P velocity fluctuation by

40% with the scale-length of 3 meters, and increased Q^{-1} of coda waves by 35% within the volume of 10-20 meters radius, both by pressurization. The strong frequency dependence of attenuation observed in the shear-shadow transmission experiments can be explained (alternatively to the one given in (2)) by the scattering effect due to inhomogeneities of scale-length, roughly 3 meters.

Seismic attenuation in the fractured region may be primarily due to scattering by small-scale cracks: The agreement between the percentage increase (35-40%) in variance for velocity fluctuation and for Q^{-1} of coda waves by pressurization suggests that scattering may be the major cause of attenuation. The agreement between the scale-length of heterogeneity estimated from the frequency dependence of attenuation and spatial fluctuation of velocity change due to pressurization also supports the scattering hypothesis.

(2) Changes in P wave velocity during operation of a Hot Dry Rock Geothermal System (M. Fehler). This paper was published in the Journal of Geophysical Research, vol. 86, Pages 2925-2928, 1981 with the following abstract.

P wave arrival times have been picked on seismograms recorded during two dual well seismic experiments, separated in time by one year, that were carried out in the Los Alamos Scientific Laboratory Hot Dry Rock Geothermal Energy Reservoir. Average in situ P wave velocity is computed from arrival times measured along segments of the borehole. Velocity decreases as temperature drops owing to heat extraction from the reservoir. The velocity changes linearly with the change in

temperature at a rate of $1.07 \text{ (km/s)/(100}^\circ\text{C)}$. The linear decrease in velocity leads to the important conclusion that P wave velocity can be used as an effective measure of change in situ temperature during operation of the geothermal system. The change in velocity is attributed to an increase in the microcrack porosity which is a result of the heat extraction from the rock.

(3) Interaction of seismic waves with a viscous liquid layer (M. Fehler). This paper was submitted to the Bulletin of Seismological Society of America.

This study is intended to provide the basis for designing active seismic experiments to detect the existence of hydraulically induced fractures in a Hot Dry Rock Geothermal energy system. We compute transmission and reflection coefficients for P, SV and SH waves incident at arbitrary angles onto a fluid layer imbedded between two homogeneous half spaces. We find that coefficients for SH waves at normal incidence can be written in terms of two dimensionless parameters. One of these parameters, I , is the ratio of acoustic impedance of the fluid to the acoustic impedance of the solid. The other dimensionless parameter, γ_d , is a measure of the amount of energy the SH wave loses during one pass through the fluid layer. We find that the amplitude of the SH wave transmitted through the fluid layer is equal to the amplitude of the incident SH wave if I is nearly 1 and γ_d is small. As γ_d increases, energy is lost due to viscous dissipation in the fluid and transmitted wave amplitude diminishes. If I is not nearly 1, some energy is reflected by the fluid layer. Three dimensionless parameters are required for solution for the case of P waves normally incident

on the fluid layer. One parameter, R is the ratio of the bulk modulus to effective shear modulus of the fluid. The second parameter is the ratio of elastic impedance of the solid to the shear impedance of the fluid and the third is γ_d . We find that the value of R is a good measure of the importance of fluid viscosity on the propagation of P waves through the fluid. Detailed studies of the effects of fluid and solid physical properties on the amplitudes of transmitted and reflected waves when waves are normally incident provide valuable insight into the importance of physical properties in the calculations for non-normal incidence. Finally, reflection and transmission coefficients are computed for P and SV waves incident at arbitrary angle on a viscous fluid layer. Physical properties of fluid and rock are chosen to be those thought appropriate for the Los Alamos National Laboratory Hot Dry Rock Geothermal test site located at Fenton Hill, New Mexico.

(4) Source and scattering effect on the spectra of small local earthquakes (K. Aki). This paper describes a basic study of small-scale heterogeneities of lithosphere using the scattering of high-frequency seismic waves. It was accepted for publication in the Bulletin of Seismological Society of America with the following abstract.

In order to separate the scattering effect from the source effect on high-frequency seismic waves, S and coda waves are studied simultaneously for 900 small earthquakes (magnitude 3 to 4) in the Kanto region, Japan. The observed S wave spectrum is the source spectrum modified by the attenuation and forward-scattering effect, while the coda spectrum is the source spectrum modified by the attenuation and back-scattering effect. From previous studies (Aki, 1980 a,b) on the same data set, we know about the attenuation effect. Comparing the S wave spectrum with the coda spectrum, we find that our data are consistent with the source spectrum with corner frequencies 3 to 6 Hz and high-frequency decay ω^{-1} to ω^{-2} , and support the hypothesis that scattering is the major cause of attenuation of S waves in the lithosphere, in which the small-scale Rayleigh scattering ($Q^{-1} \propto f^3$) applies to frequencies lower than 3 Hz and the large-scale scattering ($A^{-1} \propto f \sim -1$) to higher frequencies. The high-frequency decay of the source spectrum appears to be somewhat steeper for shallower earthquakes. We found that two areas with significantly different tectonic features shared similar attenuation and source effects but showed very different envelopes of seismograms due to different forward scattering effects. The forward scattering effect may be the most sensitive to the structural difference.

(5) Attenuation of short period seismic waves due to scattering (R.S. Wu). This paper describes a theory of seismic attenuation due to scattering. It was submitted for publication in the Geophysical Research Letters.

We point out first the inadequacy of the two widely used approaches in calculating the amplitude attenuation of seismic waves in a random medium, the formulation of mean field attenuation and that of the scattering coefficient under the single-scattering approximation. Then we calculate the average amplitude attenuation in an infinite random slab. We slice the random slab into layers of thickness a correlation length and use the Born approximation for each slice to calculate the scattered field. To include the effect of multiple scattering, we

take the back-halfspace integration of the scattered energy as the energy loss and do energy correction for each successive slice. Taking scalar wave approximation for seismic waves, we get a formula for average amplitude attenuation valid for high frequency range ($ka \gg 1$). The attenuation depends strongly on the form of correlation function of the random inhomogeneities. We derive the formulas for Gaussian, exponential and Von Karman correlation functions.

(6) Mean field attenuation and amplitude attenuation due to wave scattering (R.S. Wu). This paper expands the preceding one in more detail, and was submitted to the journal "Wave Motion".

We point out the inadequacy of two widely used approaches in formulating the amplitude attenuation of seismic waves, the formulation of mean-field attenuation and that of scattering coefficient under single scattering approximation. Using a one-dimensional layered slab, we show that the attenuation of mean field is merely a statistical effect caused by phase interference among different realizations of the random wave ensemble, and does not represent the amplitude attenuation. We will call the attenuation coefficient of mean field the randomization coefficient in order to distinguish it from the amplitude attenuation coefficient.

We also show that the scattering coefficient method leads to the same result as that by the first order approximation to the renormalized perturbation series (or the bilocal approximation to Dyson's equation) of the mean field. Therefore these two approaches are equivalent to a certain degree. This is also shown by using the one-dimensional layered slab model as an example.

After pointing out the incorrectness of comparing experiments on amplitude attenuation with mean field formulation, we suggest and discuss some methods of obtaining mean field in experiments. For one-dimensional problems, the samples must be taken along the whole propagation path in order to use spatial average to substitute ensemble average. For three-dimensional wave field, measurements over a large seismic array can be used to obtain the mean field. The data from Lasa measured by Aki are used to compare with theory; the agreement between them is well. Finally we compare the mean-field attenuation (randomization) and the amplitude attenuation under the First order multiple-scattering approximation introduced by Wu, and compare them with the measured data by Aki. The comparison shows further the inability of mean-field formulation in dealing with the problem of amplitude attenuation.

(7) The scattering attenuation of short period seismic waves in a random medium under the first order multiple-scattering

approximation (R.S. Wu). The paper further presents a theory of attenuation due to scattering and its application to various regions including the Fenton Hill geothermal site. It was submitted for publication in the Bulletin of Seismological Society of America.

After pointing out briefly the inadequacy of the two widely used approaches, the mean field formulation and the single scattering approximation, in dealing with the amplitude attenuation of short period seismic waves in random medium, we derive the amplitude attenuations from the transfer relation of power-flux in a random slab. First we define and derive the scattering intensity function of a small volume inhomogeneous medium with randomly perturbed velocities using the Born approximation and the scalar wave approximation in the high frequency range. Then we obtain the transfer equation of forward power flux in a random slab. Taking the first order multiple-scattering approximation for normally incident plane wave, we get a formula for average amplitude attenuation for random media having different forms of correlation function. The formulas for equivalent Q^{-1} of scattering attenuation for exponential, Gaussian and different order Von Karman correlation functions are deduced, the normalized curves are presented. The Q^{-1} curve has a peak. For exponential correlation function the peak occurs at $k_{0a} = 2$, where k_0 is the average wavenumber, a is the correlation length, and has a value $Q^{-1} \approx 0.86 \gamma^2$, where γ is the r.m.s. velocity maximum fluctuation of the medium.

Two examples have been calculated. The comparison between our calculation and the measured data by Aki in Kanto regions, Japan, shows the good agreement in frequency dependence. The medium parameters needed for fitting the data with theory are $\gamma \approx 7.7\%$, $a \approx 2$ km.

The theory was applied to acoustic log data from the Fenton Hill site, and it was found that the attenuation effect due to small scale heterogeneity as recorded by the log is negligibly small (Q of 8000 for 1 KHz).

The possibility of extracting information about the power spectrum and the forming process of inhomogeneities using measured frequency dependence is also discussed from the viewpoint of fractals of Mandelbrot.

(8) Seismic images of the deep structure of the San Andreas fault system, central coast ranges, California (G. Zandt). This paper was published in the Journal of Geophysical Research, Vol. 86, Pages 5039-5052, 1981. Low velocity bodies similar to those beneath geothermal areas in the western U.S. were found in the mantle to the east of the San Andreas fault. The paper is summarized in the following abstract.

Three-dimensional inversion of teleseismic P wave travel time residuals recorded at the U.S. Geological Survey central California array has resolved small-scale (~tens of kilometers) crustal and upper mantle heterogeneity down to depths of 90 km beneath the California coast ranges. Upper crustal lateral velocities are associated with thick Tertiary sedimentary

fill and higher-than-average velocities with basement exposures. Lower crustal velocity heterogeneity of $\pm 4\%$ appear to reflect crustal thickness variations. A thinner crust is indicated southwest of the San Andreas fault and northwest of San Pablo Bay. A linear zone of low-velocities (0 to -4%) subparallel to the San Andreas fault was resolved in the upper mantle. The preferred interpretation is that the low-velocities indicate a narrow upwarp of asthenosphere to unusually shallow depths (~ 45 km) beneath the coast ranges. Such an unusual upper mantle structure may have been produced by the northwestward migration along the California coast of a transiently unstable Mendocino triple junction. The inversion results also indicate the possibility of partial decoupling of the crust from the upper mantle.

(9) 3-D seismic inhomogeneities in lithosphere and asthenosphere: Evidence for decoupling in lithosphere and flow in asthenosphere (K. Aki). This is a review of numerous studies of seismic velocity anomalies obtained under a seismograph network using the 3-D inversion method developed by the principal investigator and his colleagues. It also offers interpretation of the anomalies in terms of decoupling between the brittle upper lithosphere and the ductile lower lithosphere and flow in the asthenosphere. It was presented as an Association Lecture at the IASPEI meeting at London, West Ontario and will be published in the Review of Geophysics and Space Physics.

The observed strong anomalies in asthenosphere under California were used to infer the mode of convection current in asthenosphere with the help of viscosity structure estimated from various loading-unloading responses. The result is found to depend on what is causing the velocity inhomogeneities. If Birch's law about density and velocity is applicable, we infer

a vigorous plume as implied in the Hot Spot Hypothesis of Morgan. On the other hand, if partial melts in cracks are responsible for the velocity anomaly, we infer very quiet flow as implied in Elsasser's model of driving forces. Velocity anomalies under the Roosevelt Hot Spring and Geysers-Clear Lake geothermal areas supports the decoupling between the upper brittle part and the lower ductile part of the lithosphere in the western U.S.

(10) Deep volcanic tremors and magma ascent mechanism under Kilauea, Hawaii (K. Aki and R. Koyanagi). This paper was published in the Journal of Geophysical Research, Vol. 86, 7095-7109, with the following abstract.

Deep harmonic tremor originating at depths around 40 km under Kilauea was studied using records accumulated since 1962 at the Hawaii Volcano Observatory of the U.S. Geological Survey. The deep source of the tremor was determined by onset times and confirmed by the relative amplitude across the island-wide network of seismometers. The period of tremor was conclusively shown to be determined by the source effect and not by the path or station site effect because the period would change considerably in time but maintained uniformity across the seismic net during the tremor episode. The tremor appeared to be primarily composed of P waves. We interpret the observed period and amplitude in terms of the stationary crack model of Aki et al. (1977) and find that the seismic moment rates for deep tremors are considerably larger than those for shallow-tremors suggesting more vigorous transport for the former. We propose a kinematic source model which may be more appropriate for deep tremor. According to this model, a measureable quantity called "reduced displacement" is directly proportional to the rate of magma flow.

A systematic search for deep tremor episodes was made for the period from 1962 through 1979, and the amplitude, period, and duration of the tremor were tabulated. We then constructed a cumulative reduced-displacement plot over the 18-year period. The result shows a generally steady process which does not seem to be significantly affected by major eruptions and a large earthquake near the surface. The total magma flow estimated from the reduced displacement is however, one order of magnitude smaller than that estimated by Swanson (1972). It may be that most channels transport magma aseismically, and only those with strong barriers generate tremor.

(11) A new source mechanism for volcanic tremor (D.D. Pollard and K. Aki). This paper was presented at the Spring AGU meeting, 1981 by D.D. Pollard of the U.S. Geological Survey who developed a new idea on the generation mechanism of volcanic tremor during his stay at MIT in collaboration with the principal investigator. The abstract published in the EOS, Vol. 62, p. 400, 1981 reads as follows.

An empirical correlation between volcanic tremor and igneous intrusion or eruption has encouraged the use of tremor in forecasting volcanic activity, yet the origin of this seismic signal remains obscure. The discovery of tremor at Mt. St. Helens that is similar to tremor at Kilauea casts doubt upon the general applicability of source mechanisms which rely on magma flow in cracks because viscosities of dacitic and basaltic magmas differ by many orders of magnitude. However, water is common to both volcanoes. We suggest that periodic opening and closing of a vertical crack tip containing water vapor could produce tremor. Slowly moving magma wedges open the crack tip, providing a low-pressure sink for water lodged in the surrounding rock (or perhaps a vapor exsolved from the magma). The magma supplies heat that raises vapor pressure and drives the opening phase: pressure drops and vapor expands into the dilating crack tip and heat is transferred to the crack walls. A theoretical analysis of a crack subject to linear gradients in driving pressure has identified an instability in the rate of crack dilation with change

in vapor pressure that could excite vibration of the magma reservoir. Escape of steam from ground cracks preceding fissure eruption in Kilauea and the steam eruptions and fumarolic activity at Mt. St. Helens support this mechanism. Further evidence comes from the presence of hydrothermally altered rock in cracks above basaltic dikes.

(12) Ground Motion in the near field of a fluid-driven crack and its interpretation in the study of shallow volcanic tremor (B. Chouet). This paper was published in the Journal of Geophysical Research, Vol. 86, pages 5985-6016, 1981, with the following abstract.

We present a study of the motion of the ground in the near field of a fluid-driven tensile crack embedded in a layered half space. The source that we consider is the jerky opening of a channel connecting two fluid-filled cracks, and the cause of this opening is the excess pressure of fluid in one of the cracks. We make a complete representation of the three components of ground motion in the space, time, and frequency domains and analyze the effects of fluid compressibility, source depths, and medium structure on the ground response. The calculations show the presence of a dominant frequency of motion which depends not only on the source geometry and bulk modulus of the fluid but also on medium characteristics, receiver position, and the component of motion being considered. Using this source model, we view an episode of volcanic tremor as a continuous sequence produced by numerous jerky openings of channels occurring randomly in time along a chain of cracks. Our results are applied to the October 5-6, 1963, east rift eruption of Kilauea volcano, Hawaii, and found to be compatible with available seismic data, suggesting that magma is transported through an ensemble of cracks with the area of 1×1 km, each pair of cracks constituting an individual dike segment which opens in discrete increments at a rate of $1/s$. We obtain the following parameters for each dike-segment opening: increase in cavity volume, 40 m^3 ; stress drop, 0.004 bar ; seismic moment, 10^{19} dyn cm ; and force applied by the fluid to open the dike, 10^{14} dyn . The total moment integrated over the entire duration of tremor for the eruption is $0.5 \cdot 10^{24} \text{ dyn cm}$,

roughly equivalent to a single magnitude $M = 5$ earthquake. The seismic source parameters and other field observations offer constraints on the process of mass transport and are compatible with the excess pressure and viscosity of magma of about 40 bars and 10^2 P, respectively.

(13) GROUND MOTION NEAR AN EXPANDING PREEXISTING CRACK AND SOURCE
PARAMETERS OF SEISMIC EVENTS IN THE COOLING LAVA LAKE OF
KILAUEA IKI, HAWAII (B. Chouet)

Cooling and contraction in thick lava flows often produce shrinkage cracks which may extend vertically throughout the entire thickness of the flow dividing it into pillars called columnar joints. Using the model of a preexisting crack expanding under excess tensile stress recently developed by Aki et al. (1977) we are studying the ground motion resulting from the columnar jointing activity in the cooling lava lake of Kilauea Iki, Hawaii. Our approach uses Bouchon's discrete wave number representation of elastic wave fields for an arbitrary three-dimensional seismic source (Bouchon, 1979).

The two-dimensional solution obtained by Aki et al. (1977) for a crack suddenly extending at one end in response to an excess tensile stress is reproduced in Figure 1. These results show the normal component of the particle velocities and displacements calculated at ten positions on the crack surface and given as dimensionless functions of the excess stress ΔP , lengths L and Δl of the crack and of its extension, and the compressional velocity α and rigidity μ of the solid. The grid labelled 10 is where the extension takes place. In this solution the initial static elastic field which exists prior to the crack extension is not considered. Thus the static opening depicted in the figure only represents the increment ΔV in crack volume resulting from the extension Δl of the crack. Note also that the static opening at grid 1 is

Fig. 1

not zero because this position is offset from the crack edge by half a grid owing to the discretization. This solution was derived assuming a Poisson ratio of 0.25 in the rock matrix and, according to Aki et al. (1977) it is reliable down to the wavelength $\lambda = L/2$ or wavenumber $k = 2\pi/\lambda = 4\pi/L$. The stability and dispersion of the numerical result were discussed by Madariaga (1976) and tests of the validity of the finite-difference scheme can be found in his study and in Fehler and Aki (1978).

Another representation of these results is offered in Figure 2 which shows a contour map of the quantity $10\mu V(\omega)/(\Delta P \Delta x)$ plotted as a function of dimensionless frequency and distance along the crack. As expected from the overshoot and ringing characteristics of the wall motion seen in Figure 1, the source spectrum is strongly peaked. A dominant peak at the frequency $\nu = 0.7\alpha/L$ is clearly marked near the tip being extended and several peaks of lesser amplitudes and spanning the band $\nu = 0.5\alpha/L$ to $\nu = 0.7\alpha/L$ are conspicuous along the length of the crack.

To calculate the ground motion resulting from the extension of the crack we shall use medium parameters which are compatible with the structure of Kilauea Iki determined by Aki et al. (1978). The crack is set on a rectangular vertical plane having a width of 1.5m and extending in length between the depths of 10 and 40m. In all the examples shown the crack opening function (Fig. 1) is assumed to depend on depth only.

Snapshots of the ground motion produced by the sudden

Fig. 2

extension of the bottom tip of the crack are shown in Figure 3. The medium considered is a homogeneous half space with the compressional and shear velocities $\alpha = \sqrt{3}\text{km/S}$, $\beta = 1\text{km/S}$, and a density $\rho = 2.5 \text{ g/cm}^3$. The representation of the displacements is carried out at 861 locations spaced at regular intervals of 4.5m in the directions parallel and transverse to the trace of the crack plane. Each grid point represents a station in the plots. Because of the symmetry of motion the wave field is depicted on one side of the crack only; the receiver array covers a rectangular area of 180 x 90m, the front edge of which coincides with the crack trace, the source epicenter being located midway along this segment. The u and v components represent the horizontal displacements parallel and normal to the crack, respectively, and w is the vertical component of motion. There are two axes of symmetry for horizontal motion with this source-receiver configuration, i.e., u is symmetric and v is antisymmetric about the crack here, while u is antisymmetric and v is symmetric with respect to the orthogonal axis passing through the epicenter. The vertical component, w , itself has a symmetry with respect to both axes.

The snapshots are taken every 10ms, starting 16ms after the onset of the rupture and ending 126ms later. The static displacements resulting from the opening of the crack and calculated using Maruyama's (1964) formulation are shown at the bottom of the figure for comparison. By convention the displacements outward from the source are positive. The

STATIC

sequence gives an overall picture of the dynamic ground response to the opening crack, clearly showing the onset of the disturbance and the propagation of the various wave fronts over the area. Of particular interest is the strong overshoot of the vertical and transverse horizontal displacements in relation to their static values. It is worth noting that the static vertical displacement is downward and the static horizontal displacement parallel to the crack is inward directly above the source.

For a more detailed interpretation, the seismograms of the three components of displacement obtained at 49 stations set along 7 profiles transverse to the crack are shown in Figure 4. The profiles, listed at the top of the figure, appear from left to right in constant increments of 15m at increasing distance from the trace of the plane of symmetry transverse to the source, the first profile lying in this plane and the last being located at a distance of 90m. Similarly the stations are set at distances of 0, 15, 30, 45, 60, 75 and 90m from the crack trace, shown from top to bottom for each individual component of motion. The records start with the arrival of the P wave and last 160ms from this arrival, covering most of the duration of the ground response at each location. All the displacements have been normalized by the produce $\Delta P \Delta l$ of excess stress applied on the crack times the length of the crack extension. The main characteristics of these records are the first motion which is outward everywhere, the marked oscillations in the signal

Fig. 4

associated with the ringing of the crack surfaces, the rapid damping of the vertical component of motion with range, the presence of a clear SH wave in the records obtained at the most distant stations, and the large overshoot of the ground response near the source. The peak displacement occurs at the epicenter on the vertical component and within 15m of the epicenter on the two horizontal components. The directions of profile 1 and of the crack trace are nodes for SH motion because of the antisymmetry of the problem for horizontal displacements about these two lines.

Further details on the P-SV and P-SH motions of the ground are given in Figure 5 which shows polar plots of the particle trajectories in the three orthogonal planes defined by u , v , and w . Four stations are depicted with the x, y coordinates as indicated at the top of the figure; these stations are at a distance of 90m from the crack trace and from left to right correspond to profiles 1, 3, 5 and 7. The retrograde elliptical motion appearing at the tail end of the trajectories shown in the bottom plots represents the contribution of the Rayleigh wave. The maximum amplitude of the Rayleigh motion is found along the first profile which lies on the axis of the main lobe of radiation for this wave. The character of the ground response evolves from displacements which are purely P-SV to displacements which are predominantly P-SH as we move toward the azimuth which bisects the two planes of symmetry of the source. This azimuth is the direction of maximum radiation for the far field SH wave and

(0,90)

(30,90)

(60,90)

(90,90)

SCALE
0.05 μ

$\Delta P \Delta l = 1 \text{ BAR} \cdot \text{M}$

this wave is the dominant contribution to the motion observed at the most distant station depicted on profile 7.

The next model we consider is that of a crack extending at the top. In this study the positions of the source and stations and the physical properties of the medium remain identical to those used above. The seismograms of the ground motion obtained in response to an extension of the top tip of the crack are shown in Figure 6. The close proximity of the crack tip being extended produces a twofold amplification of the displacements. The effect of the source directivity is particularly evident in the vertical component which shows a sharp pulse-like response at the epicenter, the period of which is distinctly shorter than that of the signal observed at the same location for a crack extending at the bottom. The short period content of the horizontal records is likewise enhanced. The damped sinusoidal oscillation observed in the latter part of the signal in the immediate source region reflects the ringing of the crack walls seen earlier. While this characteristic oscillation of the ground response is more strongly marked at the epicenter than in the previous model, it attenuates more rapidly with range resulting in a simpler signature away from the source.

Going back to the case of a crack extending at the bottom our final model, shown in Figure 7, studies the effect of a more complex structure on the ground response. The disposition of the source and stations is the same as in Figure 4 but the medium structure now consists of two layers

NORMALIZED DISPLACEMENT
AMPLITUDE (BAR⁻¹)

10⁻⁶
0
-10⁻⁶

Fig. 6

Fig. 7

overlying a homogeneous half space. The source is located in the topmost layer which has a thickness of 45m and whose elastic parameters are identical to those of the half space considered in our first two models. The second layer, which is 5m thick, has the compressional velocity, shear velocity, and density of 0.3 km/S, 0.2 km/S, and 2.3 g/cm³, respectively. Finally, the density in the half space is 2.4 g/cm³ and the wave velocities are 0.9 km/S for P waves and 0.5 km/S for S waves. While the pattern of radiation is analogous to that obtained in a homogeneous half space, the complexity of the signal is greatly increased by the interference of multiply-reflected P, SV, and SH waves. Close to the source the vertical displacement pulse associated with the first arrivals is followed by a lower-frequency component propagating with a low phase velocity. At increasing range in the direction transverse to the crack the vertical motion evolved into a wave train which consists primarily of inversely dispersed Rayleigh waves. As noted earlier the horizontal displacements are larger than the vertical displacements outside the epicentral region and the Love and SH waves became the dominant contributions to the motion at the most distant station. The displacements are amplified by a factor of 3/2 with respect to those obtained with a simple half space. The lateral extent and amplitude of the downward static displacement is also more pronounced owing to the softness of the deep seated structure.

Using the models of source and structure discussed above we can obtain bounds on the source parameters of the seismic events observed in Kilauea Iki. The data available to us for this purpose are the records of the vertical component of motion obtained at a station located near the center of the lake. The frequency response of the seismograph used for these recordings is shown in Figure 8 and a 6 hour-sample of the data gathered with the set up is reproduced in Figure 9. With the exception of the large event appearing near the top of the latter figure and which occurred outside the lake the seismic signatures are characterized by a sharp pulse followed by a short coda lasting less than 3s. Because of the strong attenuation of high frequency waves (>20 Hz) observed in Kilauea Iki, Chouet (1979) suggested that these shocks are representative of the local seismicity within a radius of a few tens of meters from the receiver. As accurate locations are unavailable for those events we shall assume here that the largest amplitudes seen in the figure are typical of the largest sources situated directly below the station. Although we did not take anelastic attenuation into account in our calculations this effect is probably small for a crack located within one wavelength from the receiver. In other words our model can be used to obtain an upper bound on the parameters of the largest seismic event in Kilauea Iki.

Assuming a medium consisting of two layers overlying a homogeneous half space as described in our last model we shall obtain an estimate of the product $\Delta P \Delta t$ by matching the maximum

Fig. 8

peak to peak amplitude of the synthetic records with the largest signals observed in Figure 9. An illustration of our results is provided in Figure 10 showing the records obtained by convolving the vertical displacements given in Figure 7 with the instrument response depicted in Figure 8. From left to right and from top to bottom these synthetics are obtained at intervals of 5m in the range of 0 to 85m from the epicenter in the direction of profile 1. As noted earlier the first 160ms of the signal are marked by the presence of a sharp impulse near the epicenter of the opening crack. The largest signal occurs at the epicenter itself yielding a maximum peak to peak amplitude of 61mm for an assumed value of $\Delta P \Delta l$ equal to 1 bar.m. Since this amplitude is roughly twice as large as that of the biggest events observed in Figure 9 the implied value of the product $\Delta P \Delta l$ for a 30m long crack which extends at the bottom is of the order of 0.5 bar.m. For a crack extending at the top we obtain another estimate of $\Delta P \Delta l$ of 0.1 bar.m. Finally, repeating our calculation for a 38m long crack extending between the depths of 2 and 40m we get values of $\Delta P \Delta l$ in the range of 0.05 to 0.09 bar.m. Thus for cracks ranging in length between 30 and 38m in the upper crust of Kilauea Iki, the product $\Delta P \Delta l$ may range from 0.05 to 0.5 bar.m.

To find out the implication of these results for the source parameters of the observed events we can use the data of Figure 1. From this figure we can derive the maximum static opening D , cavity volume ΔV due to the opening and

Fig. 9

TIME SCALE: SEC
0 10 20 30 40

AMPLITUDE SCALE: MM
0 10 20 30 40

Fig. 10

seismic moment M of the source as

$$\begin{aligned}
 D &= 2v = 1.8 \Delta P \Delta l / \mu \\
 V &= \int_0^L D(z_0) dz_0 \\
 M &= \Delta V \begin{pmatrix} \lambda & 0 & 0 \\ 0 & \lambda + 2\mu & 0 \\ 0 & 0 & \lambda \end{pmatrix}
 \end{aligned} \tag{1}$$

where the Lamé constants are obtained from the parameters of the topmost layer in our structural model as $\lambda = \mu = \rho \beta^2 = 2.5 \cdot 10^4$ bar. Taking $\Delta P \Delta l = 0.1$ bar.m for convenience, we get

$$\begin{aligned}
 D &= 7 \cdot 10^{-6} \text{ m} \\
 \Delta V &= 8.3 \cdot 10^{-5} \text{ m}^3 \\
 M &= 1012 \begin{pmatrix} 2 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 2 \end{pmatrix} \text{ dyn.cm}
 \end{aligned} \tag{2}$$

According to Peck and Minakami (1968) the tensile strength of basalt such as found in the Hawaiian lava lakes is of the order of 100 bar. For the value $\Delta P \Delta l = 0.1$ bar.m this would imply a length of the crack extension of the order of 1 mm. In view of the highly cracked and macroscopically heterogeneous nature of the crust of Kilauea Iki the actual strength of in situ material is probably much lower than the

above value, however. Thus the largest columns in the cooling lava lake may perhaps extend by up to a few centimeters under an excess stress of a few bars.

(14) SEISMIC EXPERIMENT AT MT. ST. HELENS

(B. Chouet and P. Roberts)

In collaboration with Dr. M. Fehler of Oregon State University and with the support of the U.S. Geological Survey in Vancouver, Washington we have deployed and are currently operating a network of 9 digital seismic event recorders on the outer flank and in the crater of St. Helens. Four of the stations are fully three-component while the remaining five record the vertical component of motion only. The instruments have a dynamic range of 72db, with automatic gain control, and record the digital data on magnetic tapes with the capacity of four event-hours. The three-component stations have a bandwidth of 0.1 to 60 Hz and the single-component units record over the band from 1 to 60 Hz. Each station has its own internal clock and absolute timing across the array is synchronized with the help of a portable master clock twice a month providing an accuracy of the order of the millisecond. The locations of the stations are given in Figure 11 and the data pertinent to each site are listed in Table 1.

The disposition of the array provides both the azimuthal coverage and aperture required for the location of seismic events and for the study of the source of these events. Our objective is to investigate the source of tremor and the seismic activity associated with magma intrusion, dome growth,

HT BT MELNS HV, VA. C. I. 79 400-11

(X) 3 CMP
X 1 CMP E

NOTE: SFT IS
SET DUE
WEST OF
MARKED
LOCATION,
AT ALTITUDE
2540'

TO PLACE ON THE PREDICTED NORTH AMERICAN DATUM 1983 MOVE
THE PROJECTION LINE 22 METERS NORTH AND 83 METERS EAST
SHOWN BY DASHED CONTOUR LINES

NT ST HELENS NW. WA. C.1.00 488 11
WSN 11 00 22.000 9-10 00
HASH, 3 ZONE — 10000 FEE
LTH ZONE 10 — 1000 REFERS
LAMB CONG CONIC PROJ
NT ST HELENS PROJ. SCALE 1:24000

Fig. 11

and steam-ash emissions within the crater of St. Helens. We shall also use the source of seismic events under St. Helens to study the anelastic and statistical properties of the volcanic medium and the mechanical properties of the magma as well as the structure of the magma feeding system.

TABLE 1. MT. ST. HELENS SEISMIC ARRAY

Station location	Station code	Component(s)	Altitude, Ft. (Approx.)
North West Dome	NWD	NS, EW, Z	6980
West Ridge	WRW	NS, EW, Z	6600
South Ridge	SRW	NS, EW, Z	6800
Shoe String Glacier	SST	Z	6160
East Dome	EDM	Z	5314
Sugar Bowl	SBW	NS, EW, Z	6120
Bread Crumbs	BCR	Z	6280
South Fork Table	SFT	Z	2540
Abraham's Plain	APL	Z	4430

(15) DISCRETE WAVE NUMBER REPRESENTATION OF THE MOTION OF AN
ARBITRARY CRACK DUE TO PRESSURE FLUCTUATIONS (B. Chouet)

To gain further insight into the effects of crack geometry and fluid properties on the observed characteristics of volcanic tremor we are developing a new model of fluid-driven crack which includes a complete representation of the wave field of fluid within the crack. The purpose of this model is to study the motion of the crack surface for an arbitrary crack shape in response to an arbitrary pressure excitation. Our approach is similar to that taken by Bouchon and Aki (1977) in their study of the radiation and scattering in the vicinity of a seismic source located in a medium with irregular interfaces. The method is based on the discrete wave number representation of the elastic wave field which results from a periodic arrangement of the source and crack interface. It is applicable down to wavelengths which are of the order of the crack width.

For simplicity the model we consider is two-dimensional and we assume that the shape of the crack as well as the pressure source within the crack are both symmetrical with respect to the crack plane. To further simplify the problem the visco-elastic fluid is treated as an elastic solid with a low rigidity. Choosing a cartesian system of coordinates x, z where the z axis is taken positive downward and the x axis is horizontal we place the tips of the crack at the locations $x = 0$ and $x = L$ on the axis $z = 0$ and assume that the shape of the crack interface is given by the arbitrary function $z = \zeta(x)$.

The source is a uniform pressure jump applied at $z = 0$ and $x = x_0$ ($0 < x_0 < L$). Taking advantage of the symmetry we consider one half of the crack only, choosing the half for which z is positive. Since the vertical displacement w and shear stress σ_{xz} are antisymmetric with respect to $z = 0$ they must vanish on this axis. This condition as well as the radiation condition lead to a system of four integral equations expressing the continuity of the displacements and stresses at the crack surface. These equations are given as

$$\int_{-\infty}^{+\infty} \left[\sum_{j=1}^4 A_j(k) g_{ij}(k, x) \right] \exp(-ikx) dk = \int_{-\infty}^{+\infty} h_i(k, x) \exp[-ik(x-x_0)] dk; \quad i = 1, 4 \quad (3)$$

where k is the horizontal wavenumber and the $g_{ij}(k, x)$ $h_i(k, x)$ are functions of x through the interface shape only. The left-hand side of (3) represents the contributions of the scattered field and the right-hand side those of the source. by solving the above equations for $A_j(k)$ we can determine the wave field in any part of the medium.

To solve this system of equations we introduce a periodicity in the source and interface $\zeta(x)$, that is, we consider an infinity of sources and cracks distributed along the x axis at equal intervals L . As shown by Bouchon and Aki (1977) this periodic assumption converts the integral equations into a system of coupled infinite-sum equations which, for a convergent series, can be replaced by the finite-sum equations:

$$\sum_{h=-N}^{+N} \sum_{j=1}^4 A_j(k_n) g_{ij}(k_n, x) \exp(-ik_n x) = \sum_{h=-N}^{+N} h_i(k_n, x) \exp[-ik_n(x-x_0)];$$

$$i = 1, 4 \quad (4)$$

where the source and scattered field are represented by the same number of discrete orders and k_n is given by

$$k_n = (2\pi/L)n \quad ; \quad n = 0, \pm 1, \pm 2, \dots \quad (5)$$

Instead of solving (4) directly in x we first take the Fourier transform of both sides by multiplying each one by the factor $(1/L)\exp(2\pi i l x/L)$ and integrating over $0 < x < L$. We thus obtain a system of $8N+4$ simultaneous linear equations

$$\sum_{h=-N}^{+N} \sum_{j=1}^4 A_j(k_n) G_{ij}(k_n, l) = \sum_{h=-N}^{+N} H_i(k_n, l); \quad i=1, 4; \quad l=-N, \dots, +N$$

$$(6)$$

with

$$G_{ij}(k_n, l) = \frac{1}{L} \int_0^L g_{ij}(k_n, x) \exp[-2\pi i(n-l)x/L] dx$$

$$H_i(k_n, l) = \frac{1}{L} \int_0^L h_i(k_n, x) \exp(2\pi i n x_0/L) \exp[-2\pi i(h-l)x/L] dx$$

$$(7)$$

The inversion of (6) is straightforward and involves no stability problem because the diagonal elements are usually large. The solution of (6) corresponds to a steady state radiation from the source. To stabilize the numerical computation a small imaginary part is given to the frequency (see Bouchon and Aki, 1977), i.e.

$$\omega = \omega_R - i\omega_I, \quad \omega_I > 0 \quad (8)$$

Using the coefficients $(A_j(k_n))$ determined by the solution of the boundary conditions (6), we finally obtain an approximate solution for the displacement field by summing over the discrete horizontal wave numbers from $n=-N$ to $n=N$. An example of our preliminary results is given in Fig. 12 showing the horizontal and vertical displacements u, w obtained at 51 points on the crack surface in response to a point source of pressure radiating 15 period waves at $x_0 = 0.9L$ on the crack axis. The calculation is made using 41 discrete wave number orders ($N=20$) for the source and scattered field. The crack has a length $L=1$ km and its shape is given by $z=\zeta(x)=W[1-\cos(2\pi x/L)]/2$ where $W=0.5$ m is the maximum width at the center of the crack. The density, compressional and shear wave velocities are $\rho=2.4$ g/cm³, $\alpha=0.5$ km/s, $\beta=0.2$ km/s in the crack and $\rho=2.7$ g/cm³, $\alpha=3$ km/s and $\beta=3$ km/s in the medium outside the crack. We are now studying the stability and convergence of the method and will attempt to synthesize individual spectral components to obtain a discrete spectrum of the motion of the crack at a later stage.

Fig. 12