MASTER

CODE DEVELOPMENT AND ANALYSIS PROGRAM

AXISYM FINITE ELEMENT CODE:

MODIFICATIONS FOR PELLET — CLADDING MECHANICAL INTERACTION ANALYSIS

G.P. ENGELMAN

October 1978

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INTERACTION ANALYSIS

by

Gail P. Engelman

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PRELIMINARY ANALYSIS REPORT

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PRELIMINARY ANALYSIS REPORT

ABSTRACT

Local strain concentrations in nuclear fuel rods are known to be potential sites for failure initiation. Assessment of such strain concentrations requires a two-dimensional analysis of stress and strain in both the fuel and the cladding during pellet-cladding mechanical interaction. To provide such a capability in the FRAP (Fuel Rod Analysis Program) codes, the AXISYM code (a small finite element program developed at the Idaho National Engineering Laboratory) was modified to perform a detailed fuel rod deformation analysis.

This report describes the modifications which were made to the AXISYM code to adapt it for fuel rod analysis and presents comparisons made between the two-dimensional AXISYM code and the FRACAS-II code. FRACAS-II is the one-dimensional (generalized plane strain) fuel rod mechanical deformation subcode used in the FRAP codes. Predictions of these two codes should be comparable away from the fuel pellet free ends if the state of deformation at the pellet midplane is near that of generalized plane strain. The excellent agreement obtained in these comparisons checks both the correctness of the AXISYM code modifications as well as the validity of the assumption of generalized plane strain upon which the FRACAS-II subcode is based.

SUMMARY

Local strain concentrations in nuclear fuel rods are known to be potential sites for failure initiation. Assessment of such strain concentrations requires a two-dimensional analysis of stress and strain in both the fuel and the cladding during pellet-cladding mechanical interaction. To provide this capability in the FRAP (Fuel Rod Analysis Program) codes, the AXISYM code (a small finite element program developed at the Idaho National Engineering Laboratory) was modified to perform such an analysis.

The AXISYM code will be added to the FRAP codes to provide a detailed local strain analysis capability as a user's option. The AXISYM code uses constant strain, axisymmetric, triangular elements and employs a standard finite element displacement formulation. The code can accommodate temperature dependent material properties and has full elastic-plastic creep capabilities. The AXISYM code has been modified for pellet-cladding mechanical interaction (PCMI) analysis by the addition of fuel-cladding gap elements and special cladding boundary constraints.

This report describes the modifications made to the AXISYM code to adapt it for fuel rod analysis and presents comparisons made between the two-dimensional AXISYM code and the FRACAS-II code. FRACAS-II is the one-dimensional (generalized plane strain) fuel rod mechanical deformation subcode used in the FRAP codes. Predictions of these two codes should be comparable away from the fuel pellet free ends if the state of deformation at the pellet midplane is near that of generalized plane strain.

Five test problems were chosen to compare analyses by the AXISYM code and by the FRACAS-II code. These problems considered fuel pellet free thermal elastic expansion, an elastic analysis of PCMI both with and without axial lockup, an elastic-plastic analysis of PCMI, an analysis of PCMI considering fuel and cladding creep, and a PCMI analysis considering fuel hot pressing and cladding creep. The excellent agreement obtained in these comparisons checks both the correctness of the AXISYM code modifications as well as the validity of the assumption of generalized plane strain upon which the FRACAS-II subcode is based.

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AXISYM FINITE ELEMENT CODE ;
MODIFICATIONS FOR PELLET-CLADDING
MECHANICAL INTERACTION ANALYSIS

I. INTRODUCTION

Local strain concentrations in nuclear fuel rods are known to be potential sites for failure initiation. For example, strain concentrations in the cladding at pellet-to-pellet interfaces (bambooing) are thought to cause cladding failure during power ramps. In addition, cladding has been observed to collapse down onto the fuel pellets and into the space between fuel pellets ("waisting") during Power Cooling Mismatch tests^[1] at the Idaho National Engineering Laboratory. Bambooing, cladding collapse and waisting all give evidence that local stress and strain analysis capabilities are needed in a fuel rod analysis program.

The finite element code, AXISYM, was modified to analyze local strain concentration effects as part of the FRAP-S^[2] and FRAP-T^[3] nuclear fuel rod modeling effort at the Idaho National Engineering Laboratory. The FRAP codes treat, as extensively as possible, all the mutual interactions of temperature, pressure and structural response in determining fuel rod behavior. Structural response in the FRAP codes is currently analyzed by the FRACAS-II^[4] subcode. The FRACAS-II subcode predicts an averaged fuel and cladding response using a one-dimensional generalized plane strain model in cylindrical geometry to predict pellet-cladding deformation at the pellet midplane. The FRACAS-II subcode does not, however, model the effects of localized cladding deformation known to occur and from which fuel rod failure can often be traced.

The finite element method was chosen as the most appropriate method^[5] for analyzing such local strain phenomena. A code based on a two-dimensional axisymmetric finite element model of fuel and cladding would predict both radial and axial variations in stress and strain over the entire pellet and its associated cladding. Two dimensional predictions of stress and strain could also directly verify the applicability of simpler one dimensional mechanical deformation models, such as FRACAS-II.

Prior to the modification of the AXISYM code, local strain studies^[5] were begun utilizing the commercially available MARC finite element code, but this did not prove feasible. Comparisons were made between the MARC code predictions for fuel pellet free thermal elastic expansion and predictions of the original AXISYM finite element code and the FRACAS-II code. The results of these comparisons^[6] indicated that the MARC code had problems in using temperature-dependent coefficients of thermal expansion. This probable error in the MARC code, as well as a lack of adaptability in terms of coupling the MARC code with existing computer programs, indicated that the MARC code was not suitable for analysis of pellet-cladding interaction local strain effects. The decision was made at this point to adapt the AXISYM code for local strain effects studies.

The AXISYM code was originally a small, finite element program written in-house specifically to use axisymmetric triangular elements and to accommodate temperature-dependent material properties. As adapted for the local strain studies, the AXISYM code is a special purpose code which can perform the complete elastic-plastic creep analysis of a pellet and its associated cladding. To model the nonlinear geometry of a pellet-cladding

configuration, both gap elements and special boundary constraints were developed and added to the AXISYM code. The AXISYM code two-dimensional analysis of local strain concentrations will be available in the FRAP codes as a user option.

This report^[a] describes the modifications made to the AXISYM code to adapt it for fuel rod analysis and presents comparisons made between the two-dimensional AXISYM code and the FRACAS-II code. Predictions of these two codes should be comparable away from the fuel pellet free ends if the state of deformation at the pellet midplane is near that of generalized plane strain.

Five test problems were chosen to compare analyses by the AXISYM code and by the FRACAS-II code. These problems included an analysis of fuel pellet free thermal elastic expansion, an elastic analysis of PCMI both with and without axial lockup, an elastic-plastic analysis of PCMI, an analysis of PCMI considering fuel and cladding creep, and an analysis of PCMI considering fuel hot pressing and cladding creep.

A description of the AXISYM finite element program with the gap element and boundary node modifications and the AXISYM finite element geometrical model are found in Sections II and III, respectively. The five problem comparisons are presented in Section IV. Illustrations of the AXISYM code predictions of local stress and strain variations near the pellet free end are given in Section V. Recommendations for future work and conclusions are given in Sections VI and VII, respectively.

[a] This report is subsequent to an earlier report^[7] which presented AXISYM-FRACAS-II comparisons for only elastic and elastic-plastic deformation during PCMI. Problem comparisons from the earlier report are included for completeness.

II. DESCRIPTION OF THE AXISYM PROGRAM AND MODIFICATIONS

The AXISYM code is a small, finite element program written in-house specifically to use axisymmetric triangular elements and to accommodate temperature-dependent material properties. This code employs the standard finite element displacement formulation using constant strain, axisymmetric elements. It has full, elastic-plastic creep analysis capability, and because of its simplicity, can be easily modified by the user. Descriptions of the AXISYM code plasticity and creep-stress relaxation calculations are given in Appendices A and B.

Both radial and axial gap elements were added to the AXISYM code in order to model the nonlinear geometry of the PCMI problem. Tied boundary nodes with a prescribed axial load were developed to model the cladding in the region of pellet-to-pellet interface. This section presents descriptions of the special elements and boundary constraints.

1. RADIAL BILINEAR GAP ELEMENTS

In order to model the radial discontinuity of fuel pellet-cladding interaction, nonlinear geometric elements (i.e., radial gap elements) have been added to the AXISYM code. A solution method for pellet-cladding interaction problems involving both plastic and geometric nonlinearities has been developed and added to the AXISYM code. This solution method has the following advantages:

- (1) Both the bilinear geometrical problem and the nonlinear plasticity problem are solved by the method of successive approximations and are solved simultaneously
- (2) This method models the gap elements with a bilinear stress-strain relationship without changing the total stiffness matrix, resulting in a major savings in computation time.

- (3) This method allows efficient solutions to elastic problems with multiple gap elements
- (4) The gap-plasticity test problems with multiple gap elements converge at the same rate as similar problems involving only plasticity.

2. LOCKING AXIAL GAP ELEMENTS

Axial lockup in the AXISYM code is enforced by the addition between gap element nodes of very stiff springs that act only in the axial direction. It should be noted that the locking axial gap elements do not require an iterative solution as axial lockup is assumed to occur instantaneously upon radial gap closure. The advantage of using stiff springs to model rigid nodal links is that predictions of friction forces between pellet and cladding are available.

AXISYM has the capability of either allowing total slip at the pellet cladding interface or enforcing no further slip after PCMI occurs. In order to avoid iterating for gap closure with the AXISYM code, the FRACAS-II prediction for contact will be used to establish axial lockup for the AXISYM code. Using the FRACAS-II gap closure assumes that no further slip between pellet and cladding occurs after contact is made at the pellet midplane. The FRACAS-II closure prediction is valid in light of the very close agreement of the two codes, as is shown in Section IV.

3. CLADDING BOUNDARY CONSTRAINTS

Tied boundary nodes were developed to model the cladding in the region of pellet-to-pellet interface (see A in Figure 1). The free end nodes of the cladding in this region have been tied or forced to remain vertical which will allow modeling symmetry about the pellet-to-pellet

Fig. 1 AXISYM model

interface. Also, before pellet-cladding contact occurs, the integral of axial stress over the cladding cross-sectional area is forced to equal an equivalent axial force F_z which is a function of the pressure loading on the cladding, so that

$$F_z = P_g \pi r_i^2 - P_c \pi r_o^2$$

where

P_g = fission gas or internal gas pressure;

P_c = coolant or external pressure;

r_i = inside cladding radius;

r_o = outside cladding radius.

III. DESCRIPTION OF THE AXISYM FINITE ELEMENT MODEL

The finite element model of the fuel and cladding (Figure 2) for the sample problems consists of 440 4-CST (constant-strain-triangle) elements in the pellet, 88 4-CST elements in the cladding, and 12 gap elements. This model represents a radial section of one-half of a pellet and its associated cladding, assuming a solution symmetric about the midplane. Figure 1 illustrates the relation of the pellet model to a section of fuel rod.

The axial gap elements in the AXISYM code either allow total slip in the axial direction or axial lockup after PCMI occurs. A discussion of the axial gap elements is contained in Section II.2.

The cladding nodes at the far right in Figure 2 are tied boundary nodes as described in Section II.3. The first three sample problems discussed in Section IV did not use the tied nodes and leave the cladding boundary nodes as a free end. The last two sample problems modeled the cladding exactly as shown in Figure 2. A brief discussion of the boundary model options available in the FRAP codes for analysis of the pellet-to-pellet interface region is given in Section VI.

Fig. 2 AXISYM mesh and nodes

IV. SAMPLE PROBLEMS WITH AXISYM - FRACAS-II COMPARISONS

Five problem comparisons are made in this report. All are comparisons of generalized plane strain predictions to the AXISYM code predictions of stress and strain at the pellet midplane. The generalized plane strain solutions are made by either FRACAS-II or its subroutine TRANSF. TRANSF is the subroutine in FRACAS-II that solves for the elastic deformation of homogeneous, thick-walled cylinders. A brief description of the FRACAS-II code, its subroutine TRANSF, and their models for these comparisons is given in Appendix C.

All the AXISYM code comparison problems discussed in this report use parameters of a PCMI sample problem based on data for a Halden experimental fuel assembly (IFA-118). The fuel pellet geometry is shown in Figure 3. The dimensions and material properties used in the sample problems are listed in Tables I, II, and III. It should be noted that all the analyses allow for variable material properties such as temperature dependent Young's Modulus(E) and temperature dependent coefficients of thermal expansion(α). The power ramp history and temperature profiles at each power step in both the fuel and the cladding were taken from a FRAP-S^[8] run for Halden experiment IFA-118. The FRAP-S run gave temperatures at eleven points across the pellet radius and at the surfaces of the cladding. Temperatures at other points across the pellet radius were calculated by parabolic interpolation. A linear temperature distribution is assumed across the cladding for all problems.

1. PROBLEM 1 - FUEL PELLETT FREE THERMAL EXPANSION

This first problem compares the two-dimensional AXISYM code predictions of fuel pellet free thermal elastic expansion at the pellet

Fig. 3 Fuel pellet - Halden IFA 118

TABLE I

DIMENSIONS AND MATERIAL PROPERTIES, ELASTIC PROBLEMS 1 AND 2

Fuel	Geometry Material Properties	Pellet Radius $6.965 \times 10^{-3} \text{ m}$ Pellet Length $30.759 \times 10^{-3} \text{ m}$ $\alpha = 7.218 \times 10^{-6} + 5.873 \times 10^{-9} * (T - T_{\text{ref}}) \text{ K}^{-1}$ $E = 1.620 \times 10^5 - 90.638 (T - T_{\text{ref}}) \text{ MPa}$ $\nu = 0.2588$ $T_{\text{ref}} = 300.0 \text{ K}$ (T_{ref} is the stress-free reference temperature).
Cladding	Geometry Material Properties Temperature	Inside Radius $7.015 \times 10^{-3} \text{ m}$ Outside Radius $7.765 \times 10^{-3} \text{ m}$ $E = 8.066 \times 10^4 \text{ MPa}$ $\nu = 0.2613$ $\alpha = 5.22 \times 10^{-6} \text{ K}^{-1}$ $T_{\text{ref}} = 300 \text{ K}$ (T_{ref} is the stress-free reference temperature). $T = 579.22 \text{ K}$ at cladding radius $= 7.2024 \times 10^{-3} \text{ m}$ $T = 569.36 \text{ K}$ at cladding radius $= 7.5776 \times 10^{-3} \text{ m}$

TABLE II

DIMENSIONS AND MATERIAL PROPERTIES, ELASTIC-PLASTIC PROBLEM 3

Fuel	Geometry	Pellet Radius $6.965 \times 10^{-3} \text{ m}$ Pellet Length $30.759 \times 10^{-3} \text{ m}$
	Material Properties	$\alpha = 7.218 \times 10^{-6} + 5.873 \times 10^{-9} * (T - T_{\text{ref}}) \text{ K}^{-1}$ $E = 1.620 \times 10^5 - 90.638 (T - T_{\text{ref}}) \text{ MPa}$ $\nu = 0.2588$ $T_{\text{ref}} = 300.0 \text{ K}$ (T_{ref} is the stress-free reference temperature)
	Stress-Strain Relationship	From MATPRO 10 * subroutines
Cladding	Geometry	Inside Radius $6.990 \times 10^{-3} \text{ m}$ Outside Radius $7.764 \times 10^{-3} \text{ m}$
	Material Properties	$E = 8.066 \times 10^4 \text{ MPa}$ $\nu = 0.2613$ $\alpha = 5.22 \times 10^{-6} \text{ K}^{-1}$ $T_{\text{ref}} = 300 \text{ K}$ (T_{ref} is the stress- free reference temperature)
	Stress-Strain Relationship	From MATPRO 10 * subroutines
	Temperature	$T = 579.22 \text{ K}$ at cladding radius $= 7.2024 \times 10^{-3} \text{ m}$ $T = 569.36 \text{ K}$ at cladding radius $= 7.5776 \times 10^{-3} \text{ m}$

* MATPRO 10 as reported in TREE-NUREG-1180, February 1978

TABLE III

DIMENSIONS AND MATERIAL PROPERTIES, CREEP AND HOT PRESSING PROBLEMS 4 AND 5

Fuel	Geometry	Pellet Radius $6.965 \times 10^{-3} \text{ m}$ Pellet Length $30.759 \times 10^{-3} \text{ m}$
	Material Properties	(E, ν , α) From MATPRO 10 * Subroutines
	Stress-Strain Relationships	From MATPRO 10 * subroutines
	Creep Law	$\sigma^{**} = 6894.75 (\dot{\epsilon} / 1.937 \times 10^{-7})^{1.37}$ for $T < 1818 \text{ K}$ $\sigma = 6894.75 (\dot{\epsilon} / 3.742 \times 10^{-8})^{0.901}$ for $1818 \text{ K} < T < 1923 \text{ K}$ $\sigma = 6894.75 (\dot{\epsilon} / 1.271 \times 10^{-9})^{0.588}$ for $1923 \text{ K} < T < 2023 \text{ K}$ $\sigma = 6894.75 (\dot{\epsilon} / 3.149 \times 10^{-9})^{0.5682}$ for $2023 \text{ K} < T$
Cladding	Geometry	Inside Radius $6.990 \times 10^{-3} \text{ m}$ Outside Radius $7.753 \times 10^{-3} \text{ m}$
	Material Properties And Stress-Strain Relationships	(E, ν , α) From MATPRO 10 * Subroutines From MATPRO 10 * subroutines
	Creep Law	$\sigma^{**} = 6894.75 (\dot{\epsilon} / 4.0 \times 10^{-10})$

* MATPRO 10 as reported in TREE-NUREG-1180, February 1978

** σ is in N/m^2 and $\dot{\epsilon}$ is (1/h)

midplane to the FRACAS-II predictions. The temperature distribution used in this problem is shown in Figure 4. The fuel dimensions and material properties are shown in Table I. Predictions by both codes of thermally induced radial, axial and hoop stress are shown in Figures 5, 6 and 7. All three figures show stresses peaking away from the center of the fuel. This is because the high temperatures at the center of the fuel (see Figure 4) decrease the fuel strength in the hot soft pellet core. This effect is modeled by a temperature dependent Young's Modulus (see Table I) taken from the material properties package MATPRO^[9]. The excellent agreement in this problem of stress predictions by two such diverse solution methods indicates both the validity of the generalized plane strain predictions in FRACAS-II as well as the reliability of the results in both codes.

The AXISYM prediction of pellet deformation due to free thermal expansion is shown in Figure 8. The typical "hour-glassing" deformation of a finite length cylinder is observed.

INEL-A-4573

Fig. 4. Fuel pellet temperature distribution
at 53.5 kW/m

Fig. 5 Free pellet elastic thermal expansion - radial stress

Fig. 6 Free pellet elastic thermal expansion - axial stress

Fig. 7 Free pellet elastic thermal expansion - hoop stress

Fig. 8 Deformation for free pellet elastic thermal expansion

2. PROBLEM 2 - ELASTIC PCMI WITH AXIAL SLIP

The second comparison is for an elastic PCMI problem that allows total slip at the pellet-cladding interface. The temperature distribution in the fuel is shown in Figure 4. The fuel and cladding dimensions, material properties, and temperature in the cladding are shown in Table I. To account for slip at the pellet-cladding interface, the stress comparison had to be made by the TRANSF subroutine in FRACAS-II, using a discontinuity analysis^[8] (FRACAS-II assumes axial lockup to occur as soon as radial contact occurs and cannot model axial slip).

Both the AXISYM code predictions and the TRANSF code predictions of the variation in radial, axial and hoop stress across the radius are shown in Figures 9, 10 and 11. The stresses shown for the AXISYM code are the stresses for elements at the pellet midplane where the state of deformation is most nearly that of generalized plane strain. Comparisons of the AXISYM code results and the TRANSF results for this elastic PCMI problem with axial slip show that the two methods are in excellent agreement.

As in Problem 1 - Fuel Pellet Free Thermal Expansion, the maximum stresses are found away from the hot soft pellet core where the temperature dependent strength coefficient or Young's Modulus is significantly decreased. For the problem with complete slip between fuel and cladding none of the axial stress of the fuel is imparted to the cladding as shown in Figure 10. The cladding axial stress varies slightly from zero because of the small temperature gradient across the cladding.

Fig. 9 Radial stress - elastic PCMI comparison problem with axial slip

Fig. 10 Axial stress - elastic PCMI comparison problem with axial slip

Fig. 11 Hoop stress - elastic PCMI comparison problem with axial slip

3. PROBLEM 3 - PCMI ELASTIC- PLASTIC ANALYSIS

This is a PCMI problem analyzing elastic deformation in fuel and cladding, and plastic deformation in the cladding. The thermal load history used in this elastic-plastic analysis consisted of eight load steps to the maximum thermal load shown in Figure 4. The cladding maximum temperature, fuel and cladding dimensions and material properties are shown in Table II. The AXISYM model for this problem includes axial lockup. The stress predictions by the AXISYM code are the stresses for elements at the pellet midplane, where the state of deformation is most nearly that of generalized plane strain.

Figures 12, 13, and 14 show the radial, axial and hoop stresses as a function of radius predicted by both the FRACAS-II and the AXISYM codes. These predictions for a pellet centerline temperature (T_0) of 1438 K represent the loadstep just prior to the onset of plastic strain in the cladding at the pellet midplane.

Figures 15, 16, 17 show similar comparisons for the loadstep with a maximum centerline temperature of 2060 K. Inspection of the axial stress prediction at both loadsteps (Figures 13 and 16) show considerable axial stress in the cladding resulting from axial lockup between the fuel and cladding. The predictions at $T_0 = 1438$ K (Figures 12, 13 and 14) are interesting in that the maximum stresses occur at the center of the fuel. The lower centerline temperature and smaller thermal gradient have not softened the core of the fuel to the extent shown in the analyses at the maximum thermal load. It is significant that excellent agreement is found between the FRACAS-II and the AXISYM code predictions at the maximum

Fig. 12 Radial stress - elastic-plastic PCMI comparison problem with axial lockup
Centerline temperature $T_0 = 1438 \text{ K}$

INEL-A-4659

Fig. 13. Axial stress - elastic-plastic PCMI comparison problem with axial lockup
 Centerline temperature $T_0 = 1438$ K

INEL-A-4660

Fig. 14 Hoop stress - elastic-plastic PCMI comparison problem with axial lockup
Centerline temperature $T_0 = 1438$ K

INEL-A-4656

Fig. 15 Radial stress - elastic-plastic PCMI comparison problem with axial lockup
Centerline temperature $T_0 = 2060$ K

INEL-A-4658

Fig. 16 Axial stress - elastic-plastic PCMI comparison problem with axial lockup
Centerline temperature $T_0 = 2060$ K

INEL-A-4657

Fig. 17. Hoop stress - elastic-plastic PCMI comparison problem with axial lockup
Centerline temperature $T_0 = 2060$ K

loadstep as the plastic analysis indicated that considerable plastic deformation had occurred in the cladding at the pellet midplane. The agreement found between the two codes at both loadsteps, demonstrates the ability of the FRACAS-II generalized plane strain solution to predict an average fuel and cladding response even for problems considering such complex phenomena as multiaxial plasticity.

4. PROBLEM 4 - PCMI FUEL AND CLADDING CREEP

Problem 4 analyzes the two dimensional creep and stress relaxation response of the fuel and cladding after lockup but at stresses below the plastic yield. The temperature distribution in the fuel and cladding, shown in Figure 18, was reached in a number of steps. The fuel and cladding dimensions and the material properties used in this problem are shown in Table III. Again it should be noted that these analyses include the effects of variable material properties dependent on such parameters as temperature and flux. With the temperatures in the fuel and cladding held constant, the fuel and cladding were both allowed to creep for 0.5 hours. The results of the creep calculations at the pellet midplane by the AXISYM code and the FRACAS-II generalized plane strain predictions are shown in Figures 19 through 22.

The stresses and strains are shown for both $t = 0$ (which corresponds to the time at which the final temperature distribution was reached) and for $t = 0.5$ hours. The hoop stress in the fuel and cladding (Figure 19) shows only minor stress readjustment during the 0.5 hour of creep. By contrast, the axial stress (Figure 20) shows very significant stress relaxation in the hotter, central portions of the fuel pellet. This

Fig 18 Temperature Distribution,
Creep and Hot Pressing Problems

Fig 19 Hoop Stress, PCMI, Fuel and Cladding Creep Problem

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K-E 10 X 10 TO THE CENTIMETER 18 X 25 CM.
KEUFFEL & ESSER CO. MADE IN U.S.A.

Fig 20 Axial Stress, PCMI Fuel And Cladding Creep Problem

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K-E 10 X 10 TO THE CENTIMETER 18 X 25 CM
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Fig 21 Hoop Strain, PCMI Fuel And Cladding Creep Problem

Fig 22 Axial Strain, PCMI Fuel And Cladding Creep Problem

is due to the fact that the creep rate of UO_2 fuel has a thermal activation energy which gives rise to abrupt increases in creep rate for fuel temperatures above 1392.3 K. As can be seen in Figure 18, this transition temperature occurs at $r = 0.137 r_p$. Thus the most pronounced stress readjustment is expected to occur inside the fuel radius which is at the transition temperature, and this is observed in Figures 19 and 20. Once again, it can be seen that the generalized plane strain solution of FRACAS-II accurately predicts fuel behavior at the pellet midplane. The hoop and axial strains at the midplane are shown in Figures 21 and 22. The largest variation in hoop strain occurs near the center of the fuel where the hot core is experiencing the largest creep-stress relaxation. The axial strains show length decreasing in both the fuel and the cladding. In all cases, excellent agreement is found between the AXISYM and FRACAS-II predictions.

5. PROBLEM 5 - PCMI WITH FUEL HOT PRESSING AND CLADDING CREEP

Problem 5 is similar to the creep response problem in Section IV.4 in that the fuel volumetric plasticity or "hot pressing" and cladding creep are analyzed after lockup but at stresses below the plastic yield. The same temperature distribution (shown in Figure 18) as used in the creep problem was reached in a number of steps. Geometry and material properties are shown in Table III. With the temperatures in the fuel and cladding held constant, the time dependent creep and hot pressing were analyzed for 0.5 hours.

The time-dependent volumetric plasticity or "hot pressing" model used for this example problem was taken to be

$$\begin{aligned}\dot{V} &= 5.0 \times 10^{-7} \sigma_m & T > 1392.3 \text{ K} \\ \dot{V} &= 0 & T < 1392.3 \text{ K}\end{aligned}$$

where $\dot{V}$ is the time rate of change of volume (1/hour) and σ_m is the hydrostatic stress. The addition of hot pressing in the time dependent calculation is described in Appendix B. This model is a gross simplification of the actual creep and hot pressing processes taking place inside a fuel rod, as it does not reflect the experimental observation that thermal creep and hot pressing of UO_2 fuel are essentially inactive for temperatures below some threshold temperature. In addition, this model does not reflect the fact that the rate of hot pressing must decrease as the material density increases during the hot pressing process.

The results of the AXISYM predictions for this problem at the pellet midplane and the FRACAS-II generalized plane strain predictions are shown in Figures 23 through 26. Figures 23 and 24 show stress relaxation both in hoop and axial stresses at the hot center of the fuel after 0.5 hours of hot-pressing. Once again the two-dimensional analysis by the AXISYM code predicts a stress distribution at the pellet midplane identical to the one-dimensional generalized plane strain predictions of FRACAS-II.

Figures 25 and 26 compare predictions of fuel and cladding hoop and axial strains at $t = 0$ and $t = 0.5$ hours. Total axial elongation and radial displacement are seen to decrease. In Problem 4 - PCMI Fuel and Cladding Creep, the AXISYM code predicts a small decrease in fuel and cladding elongation (see Figure 22) and an increase in cladding hoop strain (Figure 21). The increase in hoop strain indicates an increase in outside radius. This is reasonable since

Fig 23 Axial Stress - PCMI With Fuel
Hot Pressing And Cladding Creep

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K&S 10 X 10 TO THE CENTIMETER 18 X 25 CM
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Fig 24 Hoop Stress, PCMI With Fuel
Hot Pressing And Cladding Creep

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K&E 10 X 10 TO THE CENTIMETER 18 X 25 CM
KEUFFEL & ESSER CO. MADE IN U.S.A.

Fig 25 Axial Strain, PCMI With Fuel
Hot Pressing And Cladding Creep

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K&E
10 X 10 TO THE CENTIMETER 18 X 25 CM
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Fig 26 Hoop Strain, PCMI With Fuel Hot Pressing And Cladding Creep

creep and plasticity theory^[10] requires that creep and plastic strains incur no volume change (i.e. the sum of the plastic or creep strain components are zero). However, fuel rod experiments have shown that when a fuel rod is held at power a significant decrease in total fuel rod elongation is observed simultaneous to a decrease in radius^[11.].

These problem predictions by the AXISYM code and the FRACAS-II code indicate that the simplified hot pressing model does allow for basic length and radius changes and for stress relaxation observed to occur in fuel rods which consideration of creep alone cannot predict.

V. AXISYM PREDICTIONS OF AXIAL VARIATION IN STRESS AND STRAIN

To illustrate the AXISYM predictions of axial variation in stress and strain, graphs were made of pellet-cladding interface pressure and hoop strains along the inner surface of the cladding. These predictions are discussed below for the elastic-plastic problems, the creep problem and the hot pressing problem (Problem 3, 4 and 5).

Local peaking of stress and strain near the pellet end is evident in all cases.

1. PROBLEM 3 - PCMI ELASTIC-PLASTIC ANALYSIS

Figures 27 and 28 show the AXISYM predictions for axial variation of pellet-cladding interface pressure. These predictions are for the same two loadsteps described in Section IV where a centerline temperature of $T_o = 1438$ K represents the loadstep just prior to the onset of plastic strain in the cladding at the pellet midplane. At $T_o = 2060$ K, considerable plastic deformation has occurred in the cladding at the pellet midplane. The generalized plane strain predictions are indicated in both figures. Both figures show that deviation from the generalized plane strain prediction of interface pressure is quite evident near the pellet end.

Figure 29 shows both the axial variation in total hoop strain and the axial variation in plastic hoop strain for the AXISYM PCMI model with axial lockup. These predictions are for the maximum thermal load with a pellet centerline temperature of 2060 K. Figure 30 shows a similar comparison but for an AXISYM model with total axial slip. The variation in the total strain predictions for the slip versus no-slip problems is only significant at the pellet free end.

Fig. 27 Pellet-cladding interface pressure for $T_0 = 1438 \text{ K}$

Fig. 28 Pellet-cladding interface pressure for $T_o = 2060$ K

Fig. 29 Plastic and total hoop strains in the cladding axial lockup model

Fig. 30 Plastic and total hoop strains in the cladding - total axial slip model

2, PROBLEMS 4 AND 5 - CREEP AND HOT PRESSING

Figure 31 compares the AXISYM predictions of total hoop strain at the cladding inner surface to the FRACAS-II generalized plane strain predictions. The AXISYM code predictions at the pellet midplane match the FRACAS-II predictions. However, the two-dimensional AXISYM solution shows a significant variation in strain away from the pellet midplane. Note that the AXISYM predictions at $t = 0.5$ hour are given both for Problem 4 - PCMI Fuel and Cladding Creep and for Problem 5 - PCMI With Fuel Hot Pressing and Cladding Creep. The fuel and cladding creep prediction with no hot pressing effect shows an increasing diameter which contradicts experimental evidence as discussed in Section IV.5.

Figure 32 compares the AXISYM code predictions of axial deformation at the pellet end with the end of displacement computed from FRACAS-II axial strain predictions. The $t = 0.5$ hour prediction is from Problem 5 - PCMI With Fuel Hot Pressing, Cladding Creep. The averaged FRACAS-II prediction for axial end displacement of the fuel is bracketed by the radially varying AXISYM predictions, giving a reasonable prediction of fuel elongation. However, AXISYM - FRACAS-II comparison for cladding elongation is dependent on pellet-to-pellet interface boundary conditions so that a comparison must wait until these boundary conditions in the AXISYM code are completed as described in Section VI.

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Fig 31 Cladding Hoop Strain Creep
And Hot Pressing Problems

Fig 32 Fuel And Cladding End Displacement

VI. FUTURE MODIFICATIONS REQUIRED FOR COUPLING AXISYM INTO THE FRAP CODES

The AXISYM code will be soft-coupled into the FRAP codes and will be available as an input-specified option. An automatic element mesh generation scheme will be used so that no additional input to the FRAP codes will be required.

There will be some changes to the AXISYM code to further optimize computation running time. Also, to model detailed local strains such as ballooning or cladding collapse effectively, more detailed boundary conditions for the pellet and the cladding must be added at the pellet-to-pellet interface. The user will be given the choice of analyzing either a pellet in contact with the two adjacent pellets or a pellet with an axial gap above and below it. These predictions will give extremes in local bending and axial cladding expansion.

VII. CONCLUSIONS

Comparisons of the AXISYM code stress predictions at the pellet midplane to the FRACAS-II generalized plane strain predictions indicate that the AXISYM code gives reliable results. It is also significant to note that all the FRACAS-II predictions duplicate the two-dimensional AXISYM solutions at the pellet midplane even for those physical phenomena (cladding plastic deformation, fuel hot pressing and fuel creep) most likely to cause the deformation in the midplane of the fuel to vary from generalized plane strain.

The PCMI sample problems analyzing the effect of fuel creep and fuel volumetric plasticity or "hot pressing" indicate that realistic modeling of fuel high temperature deformation cannot be predicted by creep considerations alone, but must also consider the volumetric plasticity or "hot pressing".

The AXISYM code local strain analysis will be available in the FRAP codes as an input-specified option. The two-dimensional AXISYM code calculations are, of course, significantly more time-consuming than the FRACAS-II subcode and for this reason will not feed back into the heat transfer-mechanical response iteration loop in FRAP-S or FRAP-T. However, the option of making rigorous, detailed local strain and stress calculations with no additional input will give the FRAP codes a unique capability for making nonroutine calculations of cladding deformation.

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APPENDIX A

PLASTICITY CALCULATIONS

APPENDIX A

PLASTICITY CALCULATIONS

The plasticity solution is obtained by the method of Successive Elastic Solutions. A full discussion of this technique is presented in Reference A-1.

Briefly, the method involves breaking the loading path up into a number of small increments. For each new increment of the loading, the solution is obtained as follows.

First, an initial estimate of the plastic strain increments, $d\epsilon_{ij}^P$, is made. Based on these values, the equations of Equilibrium, Hookes' Law, Strain-Displacement, and compatibility are solved as for any elastic problem. This is called the "Pseudo-Elastic" solution. From the stresses so obtained, the deviatoric stress, S_{ij} , may be computed. This represents one path in the computational scheme.

Independently, using the assumed $d\epsilon_{ij}^P$ values, the increment of effective plastic strain, $d\epsilon^P$, may be computed, and from this and the stress-strain curve, a value of the effective stress, σ_e , is obtained.

Finally, a new estimate of the plastic strain increments is obtained from the Prandtl-Reuss Flow Rule:

$$d\epsilon_{ij}^P = \frac{3}{2} \frac{d\epsilon^P}{\sigma_e} S_{ij} \quad (A-1)$$

and the entire process is continued until the $d\epsilon_{ij}^P$ converge. A schematic of the iteration scheme is shown in Figure A-1.

The mechanism by which improved estimates of dc_{ij}^P are obtained results from the fact that the effective stress obtained from dc^P and the stress-strain curve will not be equal to the effective stress which would be obtained from the stresses from the "elastic" solution. They will only agree when convergence is obtained.

The question of convergence is one that cannot, in general, be answered a priori. However, it can be shown [A-2] that convergence will be obtained for sufficiently small load increments. Furthermore, lack of convergence can usually be traced to the "flatness" of the stress-strain curve in the plastic region. Problems with convergence are accentuated when the material is at high temperatures (approximately 1200°F and above for zircaloy), due to the fact that work-hardening effects decrease with temperature.

MODIFIED ITERATION SCHEME

To avoid (or at least minimize) such convergence problems, an alternate iteration scheme was devised by Mendelson and his co-workers. This scheme is based on the von Mises yield function and the Prandtl-Reuss flow rule, so no new hypotheses or assumptions are involved. The iteration scheme is essentially the same as the Prandtl-Reuss iteration scheme described above, except that the iterated variables σ_e and dc_{ij}^P (which are related by the highly nonlinear stress-strain law) are replaced by a new pair of variables which are related to dc^P in a more nearly linear fashion. The new variables

Fig. A-1 Schematic of the Method of
Successive Elastic Solutions

are ϵ_{et} , the equivilent modified total strain, and e'_{ij} , the modified deviator strains, and in terms of these new variables, the Prandtl-Reuss flow rule becomes

$$d\epsilon_{ij}^P = \frac{d\epsilon^P}{\epsilon_{et}} e'_{ij} \quad (A-2)$$

Thus, ϵ_{et} plays the role of σ_e , and e'_{ij} replaces the deviator stresses, S_{ij} . At each step in the iteration, ϵ_{et} is calculated from the strains obtained from the "pseudo-elastic" solution based on the previous values of the plastic strain increments, and then $d\epsilon^P$ is related to ϵ_{et} via the uniaxial stress-strain curve by the expression

$$d\epsilon^P = \frac{\epsilon_{et} - \frac{2}{3} \left(\frac{1+\nu}{E} \right) \sigma_e^{old}}{1 + \frac{2}{3} \left(\frac{1+\nu}{E} \right) \left(\frac{d\sigma_e}{d\epsilon^P} \right)_{old}} \quad (A-3)$$

in which σ_e^{old} is the value of the effective stress at the end of the last converged load step, and $\left(\frac{d\sigma_e}{d\epsilon^P} \right)_{old}$ is the slope of the stress-plastic strain curve at the end of the last converged loadstep. Since $\frac{d\sigma_e}{d\epsilon^P}$ is usually at least an order of magnitude smaller than the elastic modulus E , it can be seen that the denominator in Equation (A-3) is nearly unity, and thus, $d\epsilon^P$ is very nearly linearly related to ϵ_{et} . Thus, the convergence difficulties associated with the nonlinearity of the stress-strain curve can be avoided.

Only the appropriate equations which are used in this "Plastic Strain-Total Strain" iteration scheme will be given here. The full derivation is presented in Mendelson's book^[A-2], Sections 7-9 and 9-1. The modified strains are obtained from

$$\epsilon'_{ij} = \epsilon_{ij}^e + d\epsilon_{ij}^P \quad (A-4)$$

in which ϵ_{ij}^e are the elastic strains defined by,

$$\epsilon_{ij}^e = \frac{1+\nu}{E} \sigma_{ij} - \frac{\nu}{E} \sigma_{kk} \delta_{ij} \quad (A-5)$$

The modified deviator strains are then obtained from Equation (A-4) as:

$$e'_{ij} = e_{ij}^e + d\epsilon_{ij}^p \quad (A-6)$$

where e_{ij}^e are the elastic deviator strains, and the fact that $d\epsilon_{ii}^p = 0$ has been used. The equivalent modified total strain is defined as

$$\epsilon_{et} = \frac{\sqrt{2}}{3} \left[(\epsilon'_{11} - \epsilon'_{22})^2 + (\epsilon'_{22} - \epsilon'_{33})^2 + (\epsilon'_{33} - \epsilon'_{11})^2 + 6(\epsilon'_{12})^2 + 6(\epsilon'_{23})^2 + 6(\epsilon'_{31})^2 \right]^{1/2} \quad (A-7)$$

A schematic outline of the Plastic Strain-Total Strain iteration scheme is shown in Figure A-2. The similarity with the previous iteration scheme based directly on the Prandtl-Reuss flow relations is evident.

At the start of each loadstep calculation, initial values of $d\epsilon_{ij}^p$ are assumed. Using these values, the "pseudo-elastic" equations are solved for new stresses and strains. From the strains so calculated, ϵ_{et} and e'_{ij} are obtained. Then, using the uniaxial stress-strain curve, a value of $d\epsilon^p$ corresponding to ϵ_{et} is obtained using Equation (A-3). Finally, new estimates of $d\epsilon_{ij}^p$ are obtained from the modified flow rule

$$d\epsilon_{ij}^p = \frac{d\epsilon^p}{\epsilon_{et}} e'_{ij}$$

and the process is repeated until convergence is obtained.

Fig. A-2 Flow Chart for the Plastic Strain-
Total Strain Iteration Scheme

This modified iteration procedure differs from the previous iteration scheme in one important aspect. In the previous scheme, if one of the nodes at which the plastic strains are being sought is actually only strained elastically, then in the course of the iteration process, the plastic strain increments will approach zero, and the correct elastic solution will be obtained automatically. In the modified iteration scheme, however, the iteration process will not converge to the appropriate elastic solution, but will, in fact, generally diverge in this case. Thus, at each step in the iteration process it is necessary to compute the effective stress and compare it with the current, work-hardened yield stress. If the yield stress has not been exceeded, then the plastic strain increments are set to zero.

Not only will the modified iteration scheme described above permit convergent plasticity calculations for higher temperatures and larger loadsteps, it also converges much more rapidly than the previous iteration scheme. Hence, the modified iteration scheme is utilized in all the (time-dependent) plasticity calculations.

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APPENDIX B

GENERAL CONSIDERATIONS IN CREEP-STRESS RELAXATION CALCULATIONS

GENERAL CONSIDERATIONS IN CREEP-STRESS RELAXATION ANALYSIS

1. UNIAXIAL BEHAVIOR

Creep can broadly be defined as the inelastic time-dependent change in stress and strain in a body under constant load. For most common structural metals, high temperatures are required. Shown in Figure B-1 are typical creep curves taken on a uniaxial specimen at constant stress for Zircaloy-2 at 300°C (572°F) taken from a correlation by Ibrahim^[B-1]

Two features are notable. First, doubling the stress more than doubles the strain. Second, the strain rate decreases as time goes on. A typical uniaxial creep test taken to failure is shown in Figure B-2. From $t = 0$ to $t = t_1$, the creep rate (slope of ϵ versus t) is continually decreasing. This is called the period of primary, or transient, creep. From time t_1 to t_2 , the creep rate is approximately constant (although it does depend on the stress level, temperature, radiation dose, etc.). The region of constant creep rate is called the region of secondary or steady state creep.

The final region is one in which the creep rate begins increasing and is called the region of tertiary creep. In this region, the specimen usually exhibits cracks or other nonuniform deformation, and hence, the usual continuum theory is not applicable. Thus, in a structural analysis, attention is restricted to the primary and secondary stages of creep.

One approximation that is often used in creep analysis is to consider only the steady state portion of creep, so that the creep rate is independent of time. This considerably simplifies any such analysis. However, as can be seen from Figure B-1 the creep strain incurred during the period of transient creep cannot be neglected in comparison to that incurred during the period of steady state creep (at least for hold times of less than 3000 hours), so that both transient and secondary creep must be included. This consideration is especially important when

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Fig B-1. Creep curves for Zircaloy-2
at 300°C (572°F)

Fig B-2 Typical creep test taken to failure

comparisons are to be made between code predictions and laboratory experiments which are usually of limited duration compared to anticipated service life.

The strains induced during creep are permanent, and it is a basic assumption of creep analysis that the constitutive law can be written

$$\epsilon_1 = \frac{1}{E} \left[\sigma_1 - \nu(\sigma_2 + \sigma_3) \right] + \epsilon_1^P + \epsilon_1^C + d\epsilon_1^C$$

where

ϵ_1^P = accumulated plastic strain

ϵ_1^C = accumulated creep strain

$d\epsilon_1^C$ = increment of creep strain.

Although both plastic strains and creep strains are permanent, there is a fundamental difference between plasticity, which does not depend on time, and creep, which is explicitly time-dependent. That is, the magnitude of the plastic strains resulting from the application of a sequence of loads to a body does not depend on the time intervals between the applications of the individual loads. The magnitude of the resulting creep strain, however, is greatly affected by the time history of the application of the loads.

Since creep is explicitly a time-dependent process, it would be most appropriate to use a theory in which the time rates of strain are related to the rates of stress, that is, a visco-elastic theory. And, in fact, there are simple, linear rheological models available which can qualitatively reproduce all the effects observed in a uniaxial creep test. However, as pointed out in regard to Figure B-1, the creep response of metals is a nonlinear function of stress. Hence, a nonlinear theory of visco-elasticity would have to be used to accurately predict creep strains. While such a nonlinear theory does exist (in the Frechet Expansion Theorem), its implementation is currently beyond the scope of present day computers. Further, the theory requires experimental data

which do not exist to date, and which would be relatively difficult to obtain. Thus, the use of a nonlinear visco-elastic theory is not within the current state of the art. For a further discussion of this, reference is made to C. E. Pugh, et al. [A-2]

2. THE EQUATION OF STATE APPROACH

A workable alternative to a visco-elastic creep analysis is to use the so-called "equation of state" approach. Here, analytical expressions are fit to the experimental uniaxial creep strain versus time curves measured at constant applied stress, giving the functional relation

$$\epsilon = f(\sigma, t, T, R) \quad (B-1)$$

where

σ = applied constant stress

t = time

T = temperature

R = irradiation flux parameter.

The creep strain rate is obtained analytically by differentiating Equation B-1 to obtain

$$\dot{\epsilon} = g(\sigma, t, T, R). \quad (B-2)$$

Now it must be kept in mind that Equations B-1 and B-2 are strictly valid for constant stress only. To extend their use to problems involving continuously varying stress, certain assumptions must be made. Consider the two creep curves in Figure B-3, obtained at two different stress levels. Suppose that creep strain has been accumulating at stress σ_1 until time t_1 , and then the stress is changed to σ_2 . The question to be answered is; at what point on the $\sigma = \sigma_2$ creep curve are the calculations for further creep analysis started? One of two hypotheses is commonly made.

Fig. B-3. Time hardening and strain hardening hypotheses for creep with variable stress

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- (1) The Time-Hardening Hypothesis. Here it is assumed that it is the elapsed time which is the determining factor, so that in making the transition from stress σ_1 to σ_2 , a move is made from point O to point A in Figure B-3. Hence, creep strain rates are determined directly from Equation B-2.
- (2) The Strain-Hardening Hypothesis. Here it is assumed that it is the accumulated creep strain which is the determining factor. Thus, a move from point O to point B in Figure B-3 is made. Analytically, this is accomplished by eliminating time, t , from Equation (B-2) by use of Equation (B-1) to obtain

$$\dot{\epsilon} = h(\sigma, \epsilon, T, R) \quad (B-3)$$

and thus, the subsequent creep strain rates are determined by the accumulated creep strain.

In general, experimental data agree better with analytical predictions based on the strain-hardening assumption, so this is the assumption which is incorporated into the FRACAS-II subcode. This is in accordance with the recommendations of Reference B-2.

When the creep function, Equation (B-1), is a complicated function of its arguments, it is often not possible to eliminate time analytically between Equations (B-1) and (B-2). In these cases it is necessary to perform the elimination of time numerically. This can be done for arbitrary functional forms of Equations (B-1) and (B-2) and thus, complete generality can be maintained. However, this is achieved at the cost of having to solve a pair of highly nonlinear, coupled algebraic equations at each step of the creep iteration scheme, which is a time-consuming procedure.

In order to optimize the computation time for creep calculations in AXISYM, it is assumed that the creep law, Equation (B-1), is such that

time can be explicitly evaluated so as to obtain Equation (B-3), and that Equation (B-3) is available to AXISYM from a subroutine denoted CREPR. Secondly, it is assumed that Equation (B-3) can be written in the form

$$\sigma = (\dot{\epsilon}, \epsilon, T) \quad (B-4)$$

and that this equation is available to AXISYM from a subroutine denoted CREEP. In the majority of cases, subroutine CREEP will use an iteration scheme to obtain σ , given $\dot{\epsilon}$ and ϵ , but this can usually be done very efficiently. For use in the FRAP codes, subroutine CREPR is obtained from the material properties subcode MATPRO^[B-3]. Subroutine CREEP uses the analytic expression for creep rate from MATPRO and the method of successive substitutions. This results in minimizing the creep calculation time. Following the same approach, most other creep functions can be incorporated by the user.

3. EXTENSION TO MULTIAXIAL CREEP PROBLEMS

All of the foregoing discussion applied to uniaxial loading behavior only. To extend this to creep and stress relaxation in multiaxial states of stress, it is possible to proceed in a fashion similar to that used in Plasticity, and introduce an effective creep strain increment defined by

$$d\epsilon^C \triangleq \sqrt{\frac{2}{3}} \sqrt{d\epsilon_{ij}^C d\epsilon_{ij}^C} \quad (B-5)$$

which is, of course, directly analogous to the effective plastic strain defined for the plasticity calculations. Then, the effective creep strain rate is computed from

$$\dot{\epsilon}^C = \frac{d\epsilon^C}{dt} = \frac{1}{\Delta t} \sqrt{\frac{2}{3}} \sqrt{d\epsilon_{ij}^C d\epsilon_{ij}^C} \quad (B-6)$$

The same effective stress is utilized:

$$\sigma_e = \sqrt{\frac{3}{2} \sigma_{ij} \sigma_{ij}} \quad (B-7)$$

It is then assumed that the effective stress, σ_e , and the effective creep strain rate, $\dot{\epsilon}^C$, are related by the uniaxial creep strain function, Equation (B-3). For any given problem, the equilibrium and kinematic equations are identical with those used in the plasticity calculations.

The external loadings (temperatures, pressures, displacements, etc.) are assumed to be applied stepwise. At each time that these loads are incremented, a (time-independent) plasticity solution is obtained, as described in the previous sections. The time periods in between (at constant load) are subdivided into a number of small time increments, Δt , and a creep analysis is performed. For each time increment Δt , the Method of Successive Elastic Solutions is applied in the usual fashion, except that the effective stress, σ_e , is obtained from the uniaxial creep curve at the iterated value of the effective creep rate, $\dot{\epsilon}^C$, instead of from the uniaxial stress-strain curve. Thus, for each time increment Δt , the creep solution is obtained as follows:

- (1) Assume the creep strain increments, $d\epsilon_{ij}^C$
- (2) Compute the effective creep strain rate

$$\dot{\epsilon}^C = \frac{1}{\Delta t} \sqrt{\frac{2}{3} d\epsilon_{ij}^C d\epsilon_{ij}^C} \quad (B-8)$$

- (3) Obtain the effective stress from the uniaxial creep curve, using the strain-hardening hypothesis

$$\sigma_e = h(\epsilon^C, \dot{\epsilon}^C, T, R) \quad (B-9)$$

where ϵ^C is the total accumulated effective creep strain, including $d\epsilon^C = \dot{\epsilon}^C \Delta t$ incurred during the current time increment

- (4) Obtain the deviator stresses from the "elastic" solution, with Hookes' Law taken as

$$\epsilon_{11} = \frac{1}{E} [\sigma_{11} - \nu(\sigma_{22} + \sigma_{33})] + \epsilon_{11}^P + \epsilon_{11}^C + d\epsilon_{11}^C \quad (B-10)$$

$$\epsilon_{12} = \frac{1+\nu}{E} \sigma_{12} + \epsilon_{12}^P + \epsilon_{12}^C + d\epsilon_{12}^C$$

etc.

where

ϵ_{ij}^P = accumulated plastic strains at end of last load step.

ϵ_{ij}^C = accumulated creep strains at end of last Δt time increment

$d\epsilon_{ij}^C$ = increments of creep strain during current Δt time increment

- (5) Obtain new estimates of the creep strain increments from the Prandtl-Reuss flow rule

$$d\epsilon_{ij}^C = \frac{3}{2} \frac{\dot{\epsilon}^C \Delta t}{\sigma_e} S_{ij} \quad (B-11)$$

- (6) Repeat until convergence.

The above equations apply in the case when creep is occurring at constant volume. If hot-pressing (stress-induced permanent volume changes) is occurring, the above method can be modified to take this into account. Let dV^P be an increment of permanent volume change, which is related to the plastic strain increments by

$$dV^P = d\epsilon_1^P + d\epsilon_2^P + d\epsilon_3^P .$$

The rate of permanent volume change is then given by

$$\dot{V}^P = \frac{dV^P}{\Delta t} .$$

It is assumed that there is available a relation between $\dot{V}^P$ and the mean stress,

$$\sigma_m = (\sigma_1 + \sigma_2 + \sigma_3) / 3$$

and that this relationship is of the form

$$\sigma_m = H(\dot{V}^P, T, \rho, \dots)$$

where T is temperature, ρ is the material density, etc. Then, in principal coordinates, the Prandtl-Reuss flow rule is taken as

$$d\epsilon_i^C = \frac{3}{2} \frac{\dot{\epsilon}^C \Delta t}{\sigma_e} S_i + \frac{\dot{V}^P \Delta t}{9} \frac{(\sigma_1 + \sigma_2 + \sigma_3)}{\sigma_m} .$$

this equation replaces Equation (B-11) in the creep strain iteration scheme described above. The σ_i are the stresses obtained from the pseudo-elastic solution, while σ_m is the mean stress obtained from the experimental hot-pressing relationship. Thus, the same creep iteration scheme can still be utilized when hot-pressing is present.

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APPENDIX C

DESCRIPTION OF THE FRACAS-II CODE

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DESCRIPTION OF THE FRACAS-II CODE

FRACAS-II^[C-1] is a subcode to the FRAP-S^[C-2] and FRAP-T^[C-3] fuel rod analysis computer codes. FRACAS-II obtains a discontinuous solution which is valid for determining the simultaneous deformation of an elastic-plastic fuel pellet and an elastic-plastic cladding.

TRANSF is that subroutine in FRACAS-II that solves for the elastic deformation of homogeneous, thick-walled cylinders. TRANSF is a finite difference program.

FRACAS-II solves for the stresses and strains in a cylinder due to elastic-plastic deformation, and is based on a finite difference discretization of the governing equations in the radial direction (only radial variations in stresses are permitted).

FRACAS-II is based on the assumption that the deformation is that of generalized plane strain. Therefore, FRACAS-II should reliably predict behavior at the midplane of the fuel pellet where the state of deformation is very nearly that of generalized plane strain and direct comparison to finite element predictions is thus possible.

The FRACAS-II comparison model for fuel pellet free thermal expansion divided the pellet radius into twenty equal increments. All PCMI models used for AXISYM code comparisons divide the pellet into ten equal increments and the cladding into four equal increments. All material properties were obtained from MATPRO^[C-4] and are the same as those used by the AXISYM code for the comparisons.

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