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MASTER

THE FLORIDA STATE UNIVERSITY
COLLEGE OF ARTS AND SCIENCES

LOW-ENERGY STRUCTURE STUDIES OF ODD-ODD DEFORMED
NUCLEI AND THE CORIOLIS AND RESIDUAL INTERACTIONS

By

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A Dissertation submitted to the
Department of Chemistry
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LOW ENERGY STRUCTURE STUDIES OF ODD ODD-DEFORMED
NUCLEI AND THE CORIOLIS AND RESIDUAL INTERACTIONS

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The nuclear level structure of ^{176}Lu , ^{170}Tm , ^{166}Ho , and ^{160}Tb have been studied by means of the $^{171}\text{Hf}(t,\alpha)^{176}\text{Lu}$, $^{171}\text{Yb}(t,\alpha)^{170}\text{Tm}$, $^{167}\text{Er}(t,\alpha)^{166}\text{Ho}$, and $^{161}\text{Dy}(t,\alpha)^{160}\text{Tb}$ reactions and with the use of previously published (d,p) spectroscopy and gamma transitions from the (n, γ) reactions. The (t, α) reactions have been performed and analyzed with 17MeV tritons and the Los Alamos Q3D spectrometer. Eighty-one new rotational states in excited proton configurations or vibrational excited states have been proposed.

In addition to the previously known $[404+514_-]$ ground state configuration and $K=1^+$ $[404_-524_-]$ and $K=1^+$ $[514_-514_-]$ rotational bands, spectroscopic interpretation of the following levels in ^{176}Lu (denoted by bandhead energy in kev, spins assigned, and K^π and Nilsson configuration) has been made: 390.21, $I=1$ to 4, $1^+ [402_-514_-]$; 486, $I=8$ and 9, $8^+ [514_++514_+]$; 563.89, $I=6$ and 7, $6^+ [402_++514_+]$;

726.51, $I=4$ to 6 , $4^- [411+514+]$; 838.36, $I=5$ to 5 , $3^- [411+-514-]$; 1057, $I=0$ to 5 , $0^+ [523+-514+]$; 1273, $I=7$ to 9 , $7^+ [523++514+]$; 1390.67, $I=5$ to 7 , $5^- [411+-514-]$; and 909, $I=2$ and 3 , $K^\pi=2^-$ γ -vibrational band.

In ^{170}Tm , in addition to the previously known ground state $[411+-521-]$ configuration and the $[411+-512+]$ configuration, the following levels (denoted by bandhead energy, spin, and K^π and Nilsson configuration) have been assigned: 644, $I=4$, $4^- [404+-521+]$; 716, $I=2$, $2^- [402+-521+]$; $I=2$ level at 744, $I=2$ and 3 , $1^- [411+-521-]$; 774, $I=3$, $3^- [404+-521-]$; $I=5$ level at 829, $I=5$ and 6 , $4^+ [523++521+]$; 851, $I=2$ and 3 , $2^- [411+-521+]$; 867, $I=3$, $3^- [402+-521+]$; $I=5$ level at 891, $I=5$ and 6 , $3^+ [523+-521+]$; 1215, $I=3$ and 4 , $3^- [413+-521-]$; 1313, $I=1$ and 2 , $K^\pi=1^-$ γ -vibrational band, and 1382, $I=2$ to 4 , $2^- [413+-521-]$.

In ^{166}Ho , the $I^\pi, K=9^-, 7$ level of the previously known $K=7$ $[523+-633+]$ ground state configuration as well as the $I^\pi, K=6^+, 5$; $7^+, 5$; and $6^+, 2$ levels of the previously known $K=2$ and 5 bands of the $[411+-633+]$ configuration have been assigned. In addition, the following levels (denoted as above) have been assigned: 718, $I=3$ to 6 , $3^+ [411+-633+]$; 801, $I=0$ to 5 , $0^+ [404+-633+]$; 884, $I=4$ to 6 , $4^+ [411+-633+]$; 915, $I=7$, $7^+ [404+-633+]$; 1146, $I=1$ to 5 , $1^+ [413+-633+]$; 1272, $I=6$ and 7 , $6^+ [413++633+]$; and 1560, $I=6$ to 9 , $6^- [523+-633+]$.

In addition to the previously known $[411+-521+]$ config-

uration in ^{163}Tb , the following levels, denoted as above, were proposed: 329, $I=1$ to 3, $1^- [413+521^-]$; 524, $I=4$ and 5, $4^- [413+521^-]$; 720, $I=6$, $6^- [404+521^-]$; $I=5$ level at 895, $I=5$ to 7, $4^+ [532+521^+]$; and $I=4$ level at 1241, $I=4$ and 5, $1^+ [532+521^+]$.

An independent parameterization of the Coriolis interaction is presented, which leads to satisfactory results in reproducing experimental single-particle transfer reaction cross-sections by theoretical calculations. The anomalous population of the excited neutron configurations $[404+521^-]$ in ^{170}Lu and $[411+512^+]$ in ^{170}Tm , and the anomalously low (t, α) cross-sections of the $[411+533^+]$ configuration in ^{169}Ho are observed. Qualitative explanation of the anomalies is presented in terms of the mixing of states which satisfy the requirement $\delta I^+, I^\delta K^+, K^-$. Off-diagonal H_{INT} matrix elements are calculated, which show that the residual interaction can not be used to account for the magnitude of the cross-sections observed.

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I INTRODUCTION

The Nilsson model has had considerable success in describing intrinsic and collective rotational states in strongly deformed odd-A nuclei. It is then a natural extension of the model to describe two-quasiparticle states in deformed odd-odd nuclei in terms of Nilsson orbitals for the last odd proton and neutron. In this description the low energy spectrum of a deformed odd-odd nucleus is expected to be composed of a series of rotational bands built on intrinsic states formed by the coupling of the angular momentum of the odd neutron to that of the odd proton. Each pair of Nilsson orbitals results in two intrinsic states formed by the parallel and antiparallel coupling of the projection of the particles' angular momenta on the nuclear symmetry axis. The simple band structure is modified by perturbations of the rotational Hamiltonian. In particular the residual interaction between the unpaired neutron and proton is of interest. This interaction leads to the splitting of the two otherwise degenerate states formed by the coupling of the intrinsic odd particles. One of the important reasons for studying odd-odd nuclei is to obtain new information about the neutron-proton interaction.

Detailed study of odd-odd nuclei is usually limited or impossible using γ spectroscopy following β -decay. Generally the neighboring even-even isobars have ground states of lower energy. Where decay to the odd-odd neighbor is energetically possible, only the lowest few levels can be populated by β -decay. Further, the existence of 0^+ ground states of the even-even neighbors prevents population of high spin states in the odd-odd product. Gamma spectroscopy from (n, γ) and in-beam gamma studies is an important tool, but the high level densities in deformed odd-odd nuclides make γ -ray analysis very difficult without some a priori information about the level structure.

Single particle transfer reactions are a powerful method of studying the levels of odd-odd nuclei. High resolution magnetic spectrometers and near monoenergetic accelerators allow the study of high level density species such as these nuclei. Many single particle transfer type reactions have proven to be useful techniques for probing the single particle structure of nuclei. Single neutron transfers such as (d, t) , (d, p) , and $(^3\text{He}, \alpha)$ have been successful in investigating neutron single particle structures. More recently proton transfer reactions such as (t, α) , (α, t) , and $(^3\text{He}, d)$ have become useful to observe proton single particle states. Frequently the use of complementary reactions, i.e. (t, α) and (α, t) , has provided useful information about states populated by both reactions. Such complementary data has significantly aided researchers in recognizing and assigning single particle and two-quasiparticle states.

The objective of this project is to interpret the energy spectra of four odd-odd nuclei and to measure the experimental splitting energies of the residual interaction in several two-quasiparticle configurations. In this way the residual neutron-proton interaction can be investigated. Theoretical calculations attempting to reproduce the splitting energies using a finite range central force plus a tensor force are presented and discussed.

Levels in ^{176}Lu , ^{170}Tm , ^{166}Ho , and ^{160}Tb are studied in this work by the proton pick-up (t, α) reaction. These nuclei were selected partly because of the existence of gamma-ray data on each of them taken from (n, γ) studies. Also (d, p) reaction data was available, so at least the ground state configuration was previously known in each nucleus. This facilitated the calibration of the spectra and aided in making assignments of new levels observed.

The triton beams for the (t, α) reactions were provided by the Los Alamos Scientific Laboratory tandem Van de Graaff and the alpha particles were magnetically energy analyzed by the QSO-type spectrometer. The data are analyzed for two-quasiparticle states in terms of the collective model hamiltonian given in Chapter II. Single particle energies in neighboring odd-A isotopes and isotones are used to estimate energies of two-quasiparticle rotational bandheads. The Gallagher-Moszkowski coupling rule¹ has been used to predict which of the parallel or antiparallel coupled states of each two-quasiparticle configuration lies lower in energy. Information from angular distributions is used in transferred orbital angular momentum determination. Predicted energy

spacings and relative cross-sections within a rotational band are used to interpret peaks in the alpha-particle spectra. Configuration mixing of vibrational bands with otherwise pure two-quasiparticle bands is considered as well as perturbations to the rotational energy. An independent method of parameterization of the Coriolis interaction is introduced in Chapter IV.

Based upon the anomalous population of excited neutron configurations in ^{166}Ho and ^{170}Tm and an anomalous γ -ray transitions in data in ^{166}Ho , a separate analysis of off-diagonal matrix elements of the residual neutron-proton interaction is presented in Chapter VII. Alternatively these anomalous data are interpreted in terms of evidence of a new term in the rotational hamiltonian. An analysis and discussion is presented in terms of a perturbation of the form $\frac{\pi^2}{2I} R^4$.

II THEORETICAL NUCLEAR MODEL

(II.1) Collective Model

The nuclear shell model has been successful in accounting for the properties of many nuclei. However in the region of mass $150 \leq A \leq 190$ it has failed to account for several nuclear properties. In particular it is noted in this region that the nuclear electric quadrupole moments are much larger than predicted by the shell model, and electric quadrupole gamma transition probabilities are significantly enhanced over the shell model predictions. It was suggested by Rainwater² that collective motion of particles in the nucleus could deform the nuclear surface. The large quadrupole moments could then be explained by the resulting non-spherical charge distribution rather than being caused only by the last unpaired proton.

The low-lying states in energy spectra of deformed nuclei, $150 \leq A \leq 190$, can be explained by collective nuclear motion which leaves the intrinsic structure of the nucleus unaffected. The lowest lying states of nuclei in this region are found to closely resemble the energy spectrum of a rotating body and can be interpreted as collective rotational states built on intrinsic particle states. Another type of excited state arises from the oscillation of the nucleus about its equilibrium shape. These collective

excitations occur at energies much higher than the rotational excited states. In addition to the two modes of collective excitation, the intrinsic single particle structure of the nucleus represents a third mode of excitation.

It is a useful approximation to consider the three modes of excitation as occurring independently to zeroth order, so it is possible to write them as separate terms in the nuclear Hamiltonian. The nuclear wavefunction can therefore be written as a product of functions which are solutions of the separable terms,

$\psi_{\text{rot vib}} = D_{\text{rot}} \phi_{\text{vib}} \chi_1$. D_{rot} is the rotational wavefunction, ϕ_{vib} is the vibrational wavefunction, and the χ_i are the wavefunctions of any remaining unpaired nucleons. In the case of an odd-odd nucleus, in which this study is concerned, the entire wavefunction is $\psi_{\text{rot vib}} = D_{\text{rot}} \phi_{\text{vib}} \chi_p \chi_n$. The wavefunctions of the odd proton and neutron, χ_p and χ_n , are separable from one another. The vibrational excited states can not be directly populated in a single particle transfer reaction. The residual nuclei in this study will therefore be assumed to be always in the vibrational ground state, except where mixing of a vibrational state with a directly populated intrinsic state is allowed.

(II.2) Hamiltonian.

The Hamiltonian of an odd-odd deformed nucleus can be written

$$H = H_{\text{sp}}^{(p)} + H_{\text{sp}}^{(n)} + H_{\text{rot}} + H_{\text{vib}} + H_{\text{INT}} \quad (\text{II-1})$$

$H_p^{(p)}$ and $H_{sp}^{(n)}$ are the intrinsic single-particle proton and neutron Hamiltonians, H_{rot} is the kinetic energy operator of the rotating deformed core, H_{vib} is the vibrational kinetic energy operator, and H_{int} is the residual interaction energy between the odd neutron and odd proton. The eigenvalues of H_{vib} will not be rigorously discussed since they are collective in nature and of minor importance in single particle transfer reactions.

A diagram explaining the angular momentum coupling scheme in an axially symmetric deformed nucleus is shown in figure (II.1). The z-axis of figure (II.1) is fixed in space, and the nuclear axis of symmetry, x_3 , is fixed in the nucleus. The total angular momentum $\vec{I}$ is the vector sum of the rotational angular momentum of the core $\vec{R}$ and the intrinsic angular momentum of the unpaired particles $\vec{J}$.

$$\vec{I} = \vec{R} + \vec{J} \quad (II-2)$$

The intrinsic angular momentum is equal to the vector sum of the individual odd particles' angular momenta

$$\vec{J} = \vec{J}_p + \vec{J}_n \quad (II-3)$$

In an axially symmetric field intrinsic motion is characterized by the angular momentum component along the symmetry axis. That is to say that the intrinsic angular momenta are not good quantum numbers, but the projections of these quanta on the symmetry axis are. The components of $\vec{J}_p$ and $\vec{J}_n$ on the nuclear x_3 -axis are denoted

Fig. (II.1). Angular momentum coupling scheme for
odd-odd nuclei. Insert depicts the decomposition of
which is strictly valid only in the asymptotic limit.

figure (II.1) by Ω_p and Ω_n respectively. The total projection of the intrinsic angular momentum on the symmetry axis is equal to the sum of the individual projections, so that $\Omega = \Omega_p + \Omega_n$ in odd-odd deformed nucleus.

The rotational motion of an axially symmetric rotating system is entirely described by the quantum numbers I , M , and K . These respectively the total angular momentum, its projection on the body-fixed z -axis, and its projection on the body-fixed x_3 axis as in figure (II.1). Since rotational motion about the axis of symmetry can not be detected in quantum mechanics, the component of angular momentum is identically zero and $K = 0$. As noted the intrinsic angular momentum is not a good quantum number, and therefore the rotational angular momentum $\vec{R}$ can not be since $\vec{R} = \vec{I} - \vec{J}$. An eigenstate D_{MK}^I is then a mixture of microstates J_{TJR} . It is for this reason only that collective rotational states can be populated in single particle transfer reactions.

II.3) H_{sp} .

In order to account for apparent magic numbers of stability in nuclei there evolved a nuclear shell model Hamiltonian with a potential intermediate between an isotropic harmonic oscillator and a square well. It was necessary to subtract a scalar function of the orbital angular momentum from the harmonic oscillator potential to lift the l value degeneracy resulting from it, and to include a spin-orbit interaction to reproduce the observed magic numbers.

The resulting shell model potential is then written

$$V = \frac{1}{2} M \omega_0^2 r^2 + C \vec{L} \cdot \vec{S} + D \vec{L}^2 \quad (11-4)$$

$$\text{and} \quad H_{sp} = H_0 + C \vec{L} \cdot \vec{S} + D \vec{L}^2 \quad (11-5)$$

$$\text{where} \quad H_0 = -\frac{\hbar^2}{2M} \nabla^2 + \frac{1}{2} M \omega_0^2 r^2 \quad (11-6)$$

Constants C and D are chosen to reproduce the experimental shell model energy spacings. Shell model eigenfunctions ψ_{nlj} are found which satisfy the equations

$$H_0 \psi = (N + \frac{5}{2}) \hbar \omega_0 \psi$$

$$\vec{L}^2 \psi = l(l+1) \hbar^2 \psi \quad (11-7)$$

$$j^2 \psi = j(j+1) \hbar^2 \psi$$

and

$$N = 2(\nu + 1) + l \quad (11-8)$$

The failure of the shell model in the rare earth region indicated the nuclei in that region had deformed shapes. The first calculation of single particle energies in a deformed nucleus was made by S. G. Nilsson.³ An anisotropic harmonic oscillator potential was used with the usual $\vec{L}^2$ and $\vec{L} \cdot \vec{S}$ interactions included. This potential has had much success in predicting single particle energies and wavefunctions in the rare earth and actinide regions. Later B. Nilsson included a hexadecapole deformation in the

(11-9) as these deformations became more evident.

The single particle potential used by Nilsson⁵ was that of

where

$$H_0 = -\frac{\hbar^2}{2M} \nabla^2 + \frac{M_0}{2} (\omega_1^2 x_1^2 + x_2^2 + \omega_3^2 x_3^2) \quad (11-9)$$

for a triaxially symmetric body $\omega_1 = \omega_2 = \omega_3$. A single deformation

parameter was introduced where

$$\omega_1^2 = \omega_0^2 (1 + \frac{2}{3}\epsilon) \quad (11-10)$$

$$\omega_3^2 = \omega_0^2 (1 - \frac{4}{3}\epsilon) \quad (11-11)$$

the condition of constant volume with changing shape results

$\frac{\omega_1}{\omega_3} = \text{const.}$ and leads to

$$\omega_0(\epsilon) = \omega_0(0) (1 - \frac{4}{3}\epsilon^2 - \frac{16}{27}\epsilon^3)^{-\frac{1}{6}} \quad (11-12)$$

The potential was simplified by the transformation to the coordinate

$\frac{\sqrt{2}}{b} x_1$. H_0 can then be split into a spherical term H_0^0 and a deformed term H_δ , $H_0 = H_0^0 + H_\delta$, where

$$H_0^0 = -\frac{\hbar^2}{2M} \nabla^2 + \frac{1}{2} M \omega_0^2 r^2 \quad (11-13)$$

$$H_\delta = -\frac{4}{3} \epsilon \hbar \omega_0 \sqrt{\frac{2}{5}} r^2 Y_{20} \quad (11-14)$$

With the inclusion of a hexadecapole deformation, B. Nilsson⁴ added two new terms to (II-5), and the quadrupole distortion was defined in terms of ϵ_2 instead of β . These two parameters differ only a few percent and are related by 28) $\epsilon_2 = \beta + \frac{1}{6}\beta^2 + \frac{1}{27}\beta^3 + \dots$. Oscillator frequencies become

$$\omega_1 = \omega_0 \left(1 + \frac{1}{3}\epsilon_2\right) \quad (II-15)$$

$$\omega_3 = \omega_0 \left(1 - \frac{2}{3}\epsilon_2\right)$$

Nuclear shape expansion was made in terms of Legendre polynomials instead of the usual spherical harmonics Y_{lm} . Applying volume conservation $\omega_1^2 \omega_3 = \omega_0^3(0)$ results in

$$\omega_0(\epsilon_2, \epsilon_4 = 0) = \omega_0(0) \left[1 - \frac{1}{3}\epsilon_2^2 - \frac{2}{27}\epsilon_2^3\right]^{-1/3} \quad (II-16)$$

The first new term of B. Nilsson⁴ is $-D\langle r^2 \rangle_{\text{shell}} = -DN(N+3)/2$. This preserves the fundamental shell spacings $\hbar\omega_0$. It is necessary since the $\tilde{r}^2$ term in H_{sp} reduces the spacings with increasing N . The second new term, $\epsilon_4 \hbar\omega_0 r^2 P_4$, accounts for the hexadecapole deformation. With the introduction of parameters κ and μ

$$C = -2\hbar\omega_0(0)\kappa \quad (II-17)$$

$$D = -\hbar^2\omega_0(0)\mu\kappa$$

H_{sp} becomes

$$\left[\frac{1}{2} \hbar^2 \nabla^2 - \frac{\hbar^2}{2m} \nabla^2 - \frac{1}{2} \hbar \omega_0 \varepsilon_2 r^2 P_2 + \varepsilon_4 \hbar \omega_0 r^2 P_4 - \right. \\ \left. \hbar \omega_0(0) [\vec{L} \vec{\ell} \cdot \vec{s} + u(\vec{\ell}^2 - \langle \vec{\ell}^2 \rangle_{\text{shell}}) \right] \quad (\text{II-18})$$

A set of basis functions are chosen which are diagonal in H_0^0 , L^2 , and s_z with corresponding quantum numbers N , ℓ , Λ , and Σ . That

$$\begin{aligned} H_0^0 |\nu\rangle &= (N + \frac{3}{2}) \hbar \omega_0 |\nu\rangle \\ L^2 |\psi\rangle &= \ell(\ell+1) \hbar^2 |\psi\rangle \\ L_z |\nu\rangle &= \Lambda \hbar |\nu\rangle \\ s_z |\psi\rangle &= \Sigma \hbar |\nu\rangle \end{aligned} \quad (\text{II-19})$$

These quantum numbers are used to denote a basis set $|\nu\rangle = |N\Lambda\Sigma\rangle$.

One of the operators H_0^0 , L^2 , L_z , or s_z commutes with the total Hamiltonian H_{sp} , eq (II-18). The $\vec{L} \cdot \vec{s}$ term in (II-18) couples $\Lambda \rightarrow \Lambda \pm 1$ and $\Sigma \rightarrow \Sigma \pm 1$, and the Legendre polynomials P_λ couple $N \rightarrow N \pm 1$ and $\ell \rightarrow \ell \pm 1$. The coupling between different shells, $N \rightarrow N \pm 1$, is assumed to be negligible as the energy spacing between shells N and $N+2$ is much larger than the off-diagonal element connecting states in the two shells; therefore N is treated as a good quantum number. Also the total angular momentum projection, $\Omega = L_z + s_z = \Lambda + \Sigma$, is a good quantum number with respect to the total Hamiltonian. Eigenstates of N and Ω may then be represented as expressions of the basis set $|N\Lambda\Sigma\rangle$. That is

$$|N\Omega\rangle = \sum_{\lambda\Lambda} a_{\lambda\Lambda} |N\lambda\Lambda\Sigma\rangle \quad (11-20)$$

$$\sum_{\lambda\Lambda} \int_{-\infty}^{\infty} \langle N_j' \Omega_j' | N_i \Omega_i \rangle = \sum_{\lambda\Lambda} a_{\lambda\Lambda}^2 \delta_{N'N} \delta_{\Omega' \Omega} \delta_{j'j} \quad (11-21)$$

... $\delta_{j'i}$ is included to insure the $\langle N_j' \Omega_j' |$ and $|N_i \Omega_i\rangle$ are identical orbitals, as $N-\Omega+\frac{1}{2}+1$ orbitals exist with the quantum numbers $|N\Omega\rangle$.
 Finally,

$$\sum_{\lambda\Lambda} a_{\lambda\Lambda}^2 (N_i \Omega_i) = 1 \quad (11-22)$$

Energy eigenvalues of H_{sp} are written out by S. G. Nilsson,

11.⁵ The oscillator shell spacing is given by

$$\hbar\omega_0(0) = \hbar A^{-1/3} \text{Mev} \quad (11-23)$$

where η is usually taken to be 41.

It is customary to convert to the coupled representation $J_i = L_i + S_i$ and expand $|N\Omega\rangle$ in shell model eigenfunctions of $\bar{J}^2$. In this representation

$$|N\Omega\rangle = \sum_{J\lambda} C_{J\lambda}^{\Omega} |N\lambda J\Omega\rangle = \chi^{\Omega} \quad (11-24)$$

This means the Nilsson orbitals can be written as linear combinations of the shell model orbitals $\psi_{\nu\lambda j}$ with the coefficients $C_{J\lambda}^{\Omega}$. This is much more convenient as the eigenfunctions $\psi_{\nu\lambda j} \equiv |N\lambda J\Omega\rangle$ are exactly those excited in direct nuclear reactions. Cross-section for populating these eigenfunctions of H_{sp} (shell model) can be accurately calculated with Distorted Wave Born Approximation calculations. The

coefficients $C_{J_3}^N$ and a_{J_3} of a given eigenfunction $|ND\rangle$ are related by the Clebsch-Gordan transformation

$$C_{J_3}^N = \sum_{J_1} \langle N N_3 | J_3 \rangle a_{J_1} \quad (11-25)$$

To specify a state of given N and W additional quantum numbers are necessary. H_0 is separable in r_i so that $N = \sum_{i=1}^3 n_i$, $i = 1, 2, 3$ where n_i is the number of nodes on the i^{th} axis and n_3 is a good quantum number in H_0 . For large deformations $\bar{L} \cdot \bar{s}$ and $\bar{L}^2$ can be considered perturbations, and A and E therefore also become approximately good quantum numbers. Thus it is usual to specify the state $|ND\rangle$ in the asymptotic quantum numbers form $|ND\rangle = D^T [N N_3 A E]$ where $N_3 = n_3$. The parity π is equal to $(-1)^N$, and $D = A + E$. The quantum numbers $[N_3 A E]$ define a Nilsson orbital, which is an eigenfunction of H_{sp} . The orbitals pertinent to this study and the shell model orbitals of which they are linear combinations appear in correlation diagrams in Figures (11.2) and (11.3).

11.1 The Rotational Hamiltonian

The rotational kinetic energy for a rigid body can be written

$$T_{\text{rot}} = \sum_{k=1}^3 \frac{\vec{R}_k^2}{2I_k} \quad (11-26)$$

where I_k is the rotational moment of inertia around the k^{th} axis, and $\vec{R}_k$ is the rotational angular momentum about the k^{th} axis. For an axially symmetric nucleus $R_3 = 0$ and $I_2 = I_1 = 1$. The rotational

momentum can be replaced by $\vec{R} = \vec{I} - \vec{J}$ from figure (II.1) and (II-2), and $\vec{J} = \vec{J}_p + \vec{J}_n$. In (II-26) $\vec{R}^2 = (\vec{I} - \vec{J})^2$ so that

$$T_{\text{rot}} = \frac{\hbar^2}{2I} (\vec{I} - \vec{J})^2 = \frac{\hbar^2}{2I} (\vec{I}^2 + \vec{J}^2 - 2\vec{I} \cdot \vec{J}) \quad (\text{II-27})$$

the last two terms in (II-27) can be expanded:

$$\vec{J}^2 = (\vec{J}_p + \vec{J}_n)^2 = \vec{J}_p^2 + \vec{J}_n^2 + 2\vec{J}_p \cdot \vec{J}_n \quad (\text{II-28})$$

$$\vec{I} \cdot \vec{J} = I_1 J_1 + I_2 J_2 + I_3 J_3 = I_1 J_1 + I_2 J_2 + \hbar \quad (\text{II-29})$$

the identities for angular momentum ladder operators

$$M_+ = M_1 + iM_2 \quad (\text{II-30})$$

$$M_- = M_1 - iM_2$$

gives

$$\begin{aligned} I_1 J_1 + I_2 J_2 &= \hbar [I_+ J_- + I_- J_+] \\ &= \hbar [I_+ (J_{p-} + J_{n-}) + I_- (J_{p+} + J_{n+})] \end{aligned} \quad (\text{II-31})$$

using $\vec{J}_p \cdot \vec{J}_n$:

$$\vec{J}_p \cdot \vec{J}_n = J_{p_1} J_{n_1} + J_{p_2} J_{n_2} + J_{p_3} J_{n_3} = J_{p_1} J_{n_1} + J_{p_2} J_{n_2} + \hbar J_{p_3} J_{n_3} \quad (\text{II-32})$$

and

$$J_{p_1} J_{n_1} + J_{p_2} J_{n_2} = \hbar [J_{p_+} J_{n_-} + J_{p_-} J_{n_+}] \quad (\text{II-33})$$

Fig. (II.2). Nilsson orbitals for odd proton nuclei
in the rare earth region.

Figure (II.3). Nilsson orbitals for odd neutron nuclei in the rare earth region.

Finally the rotational kinetic energy is given by

$$T_{\text{rot}} = \frac{\hbar^2}{2I} [I^2 + J_p^2 + J_n^2 - 2Kn - (I_+ J_- + I_- J_+) + 2\Omega_p \Omega_n + (J_{p+} J_{n-} + J_{p-} J_{n+})] \quad (11-34)$$

Calling that $K = \Omega = \Omega_p + \Omega_n$, (11-34) can be rearranged to

$H = H_R + H_{\text{RPC}} + H_{\text{PP}}$ where

$$H_R = \frac{\hbar^2}{2I} [I^2 - K^2 + J_p^2 + J_n^2 - \Omega_p^2 - \Omega_n^2]$$

$$H_{\text{RPC}} = -\frac{\hbar^2}{2I} [I_+ J_- + I_- J_+] \quad (11-35)$$

$$H_{\text{PP}} = \frac{\hbar^2}{2I} [J_{p+} J_{n-} + J_{p-} J_{n+}]$$

The eigenfunctions of H_R are³⁴

$$\psi_{MK}^I(\theta_i) = \sqrt{\frac{2I+1}{8\pi}} D_{MK}^I(\theta_i) \quad (11-36)$$

where the D functions are defined by Preston⁶. In order to be properly symmetrized with respect to a rotation of 180° about the $\hat{e}_2$ axis, $\psi_{MK}^I(\theta_i)$ must be written

$$\psi_{MK}^I(\theta_i) = \frac{2I+1}{16\pi} [D_{MK}^I(\theta_i) + (-1)^{I-K} D_{M-K}^I(\theta_i)] \quad (11-37)$$

An appropriate solution of $H_R + H_{sp}^{(p)} + H_{sp}^{(n)}$ is then

$$\langle IMK\Omega_p^{\Omega} \rangle = \sqrt{\frac{2I+1}{16\pi^2}} \sum_p \sum_n C_{Jp}^{\Omega} C_{Jn}^{\Omega} (D_{MK}^I \chi_p^{\Omega} \chi_n^{\Omega}) + \quad (II-38)$$

$$(-1)^{I-J-p-n} D_{M-K}^I \chi_p^{-\Omega} \chi_n^{-\Omega}$$

Equation (II-38) has made use of the Nilsson wavefunction expansions

$$\chi^{\Omega} = \sum_J C_{J\Omega}^{\Omega} \chi_J^{\Omega} \text{ and the relation of Preston}^6$$

$$C_J^{\Omega} = \pi(-1)^{J-\frac{1}{2}} C_J^{\Omega} \quad (II-39)$$

where $\pi = (-1)^N$ is the parity of χ^{Ω} , and $K = \Omega_p + \Omega_n$ has been recalled.

In order to evaluate the matrix elements of the Hamiltonians of (II-35) it is necessary to present several general results. The orthonormality property of D functions is

$$\langle D_{M'K'}^{I'} | D_{MK}^I \rangle = \frac{8\pi^2}{2I+1} \delta_{I'I} \delta_{K'K} \delta_{M'M} \quad (II-40)$$

and for the single particle wavefunctions is

$$\langle \chi^{\Omega'} | \chi^{\Omega} \rangle = \delta_{N'N} \delta_{\Omega'\Omega} \delta_{\alpha'\alpha} \quad (II-41)$$

where α represents the asymptotic quantum numbers necessary to distinguish the states $\chi^{\Omega} = |N\Omega\rangle$. Further, the I operators act only on the D functions, and the J operators only on the χ functions. The

results of these operators are

$$\begin{aligned}
 \langle D_{MK}^I | I^2 | D_{MK}^I \rangle &= I(I+1) \\
 \langle \chi_J^2 | J^2 | \chi_J^2 \rangle &= J(J+1) \\
 \langle \chi_J^{\Omega} | J_{\pm} | \chi_J^{\Omega \pm 1} \rangle &= [(J \mp 2)(J \pm \Omega + 1)]^{\frac{1}{2}} \\
 \langle D_{MK}^I | I_{\pm} | D_{MK \pm 1}^J \rangle &= [(I \mp K)(I \mp K + 1)]^{\frac{1}{2}}
 \end{aligned}
 \tag{II-42}$$

The eigenvalues of H_R are

$$\epsilon_{IK} = \frac{\hbar^2}{2I} [I(I+1) - K^2 + J_p(J_p+1) + J_n(J_n+1) - \Omega_p^2 - \Omega_n^2]
 \tag{II-43}$$

The terms not depending on I and K are usually absorbed into the single particle Hamiltonian. This leaves for the rotational energy spacings

$$\epsilon_{I,K} - \epsilon_{K,K} = \frac{\hbar^2}{2I} [I(I+1) - K(K+1)]
 \tag{II-44}$$

Therefore for each intrinsic state $\chi_p^{\Omega_p} \chi_n^{\Omega_n}$ and $K = |\Omega_p \pm \Omega_n|$ there is a series of states with successive spins $I=K, K+1, K+2, \dots$ which form a rotational band. These states are spaced apart in energy according to the effective rotational moment of inertia $\frac{\hbar^2}{2I}$ providing $K \neq 0$. In $K = 0$ bands the energy contains a $(-1)^I$ dependence which will be discussed in the residual interaction perturbation section, (II.5).

The H_{RPC} or Coriolis coupling term in the Hamiltonian mixes states of the same spin but whose Ω_p or Ω_n differ by 1. The four terms of equation (II-51) can be rearranged to

$$H_{RPC} = -\frac{\hbar^2}{2I} [I_+ J_{p-} + I_- J_{p+} + I_+ J_{n-} + I_- J_{n+}] \quad (II-45)$$

The matrix elements of the first of these is

$$\begin{aligned} \langle IMK' \Omega' \Omega' | -\frac{\hbar^2}{2I} I_+ J_- | IMK \Omega \Omega \rangle &= -\frac{\hbar^2}{2I} \sum_{J_p J_n} C_{J_p \Omega_p}^{\Omega'} C_{J_p \Omega_p}^{\Omega} C_{J_n \Omega_n}^{\Omega'} C_{J_n \Omega_n}^{\Omega} \\ &\left\{ \begin{aligned} &[(I+K)(I+K+1)(J_p + \Omega_p)(J_p - \Omega_p + 1)]^{1/2} \delta_{K', K+1} \delta_{\Omega', \Omega} \delta_{\Omega_n', \Omega_n} \\ &+ (-1)^{I-J_p-J_n} [(I-K)(I+K+1)(J_p - \Omega_p)(J_p + \Omega_p + 1)]^{1/2} \\ &\delta_{K', K-1} \delta_{\Omega', \Omega} \delta_{\Omega_n', \Omega_n} \end{aligned} \right\} \quad (II-46) \end{aligned}$$

There are similar expressions for all four terms in H_{RPC} . Combining all four terms yields

$$\langle IMK' \Omega' \Omega' | H_{RPC} | IMK \Omega \Omega \rangle = -\frac{\hbar^2}{2I} [(I \pm K)(I \mp K + 1)]^{1/2} \delta_{K', K \mp 1} \langle K' | J_{\mp} | K \rangle \quad (II-47)$$

where

$$\langle K' | J_{\mp} | K \rangle = \sum_J \left\{ \begin{aligned} &C_{J2}^{\Omega'} C_{J2}^{\Omega} [(J \pm \Omega_p)(J \mp \Omega_p + 1)]^{1/2} \delta_{\Omega', \Omega \mp 1} \delta_{\Omega_n', \Omega_n} \\ &+ C_{J2}^{\Omega'} C_{J2}^{\Omega} [(J \pm \Omega_n)(J \mp \Omega_n + 1)]^{1/2} \delta_{\Omega', \Omega} \delta_{\Omega_n', \Omega_n \mp 1} \end{aligned} \right\} \quad (II-48)$$

expression (II-47) and (II-48) has removed all terms with K' or K'' other than zero. It is apparent from them that H_{RPC} only mixes bands when either the neutrons or the protons are in the same orbital.

Equations (II-47) and (II-48) are in the same form of the H_{RPC} off-diagonal elements obtained in the odd-A case, except that in odd-A nuclei the operator J_z has the opportunity to act on only one type of particle.

In odd-A nuclei H_{RPC} supports diagonal matrix elements when $K = 1/2$. This element can be extracted from (II-46) in the case of a single odd particle.

$$\langle IMK | H_{RPC} | IMK \rangle = \frac{\hbar^2}{2I} (-1)^{I+1/2} (I+1/2) a_{K, 1/2} \quad (II-49)$$

$$a = - \left[\left\langle C_{J2}^{1/2} \right\rangle^2 (-1)^{J+1/2} (J+1/2) \right] \quad (II-50)$$

a is called the decoupling parameter. This interaction destroys the $I(I+1)$ rotational spacing in a $K = 1/2$ band in odd-A nuclei. In the case of an odd-odd nucleus, a becomes the off-diagonal element $\langle IMK \Omega_p \Omega_n | J_z | IMK \Omega_p \Omega_n \rangle$ between two bands formed by a configuration in which either Ω_p or Ω_n is equal to $1/2$. That is

$$\langle IMK \Omega_p \Omega_n | H_{RPC} | IMK \Omega_p \Omega_n \rangle = - \frac{\hbar^2}{2I} [(I+K)(I+K+1)]^{1/2} \quad (II-51)$$

$$\times \{ a_p \delta_{K, K+1} \delta_{\Omega_p, \mp 1/2} \delta_{\Omega_n, n} + a_n \delta_{K, K-1} \delta_{\Omega_n, \mp 1/2} \delta_{\Omega_p, p} \}.$$

In the special case of an $J_p = J_n = \frac{1}{2}$ configuration the resulting i and $K = 0$ bands are coupled by more complex off-diagonal elements, both decoupling parameters a_p and a_n cause a nonvanishing element $\langle H_{RPC} | \text{IMO} \rangle$. The complete Coriolis coupling element in such a situation is

$$\begin{aligned}
 \langle H_{RPC} | \text{IMO} \rangle &= \langle \text{IMO} | H_{RPC} | \text{IMO} \rangle \\
 &= -\frac{\hbar^2}{2I} (I+1)(I-1) \left[\frac{1}{J_p} C_{J_p}^{J_p} C_{J_p}^{J_p} C_{J_p}^{J_p} C_{J_p}^{J_p} C_{J_p}^{J_p} C_{J_p}^{J_p} \right. \\
 &\quad \times \{ (J_p - \frac{1}{2})(J_p + \frac{1}{2} + 1) \}^{\frac{1}{2}} \delta_{J_p - \frac{1}{2}, J_p + \frac{1}{2} + 1} \\
 &\quad + \{ (J_n - \frac{1}{2})(J_n + \frac{1}{2} + 1) \}^{\frac{1}{2}} \delta_{J_n - \frac{1}{2}, J_n + \frac{1}{2} + 1} \\
 &\quad + (-1)^{I-J_p-J_n} \{ (J_p - \frac{1}{2})(J_p + \frac{1}{2} + 1) \}^{\frac{1}{2}} \delta_{J_p - \frac{1}{2}, J_p + \frac{1}{2} + 1} \delta_{J_n - \frac{1}{2}, J_n} \\
 &\quad + (-1)^{I-J_p-J_n} \{ (J_n - \frac{1}{2})(J_n + \frac{1}{2} + 1) \}^{\frac{1}{2}} \delta_{J_n - \frac{1}{2}, J_n + 1} \delta_{J_p - \frac{1}{2}, J_p}
 \end{aligned}
 \tag{II-52}$$

This reduces to⁷

$$\langle \text{IMO} | H_{RPC} | \text{IMO} \rangle = -\frac{\hbar^2}{2I} \sqrt{I(I+1)} [a_p + (-1)^{I+1} a_n] \tag{II-53}$$

Depending on the relative signs and magnitudes of a_p and a_n the magnitude of this element is alternately large and small with incrementing

spin in the bands.

The ultimate consequence of Coriolis coupling is that each spin member I' of a $K' = K_0 + 1$ band is mixed with its spin partner $I = I'$ in the K_0 band. The state $I = K_0$ in the K_0 band has no partner in the K' band and is thus unperturbed. Each pair of states $I' = I$ is pushed apart in energy by the perturbation. All of the I dependence of the mixing element as contained in the $[(I \pm K)(I \mp K + 1)]^{\frac{1}{2}} = [I(I+1) - K(K \pm 1)]^{\frac{1}{2}}$ factor when $K \neq 0$. Since the perturbation is related to the square of the matrix element, the $I(I+1)$ energy spacings is approximately preserved in each band. The observed effective moment of inertia becomes smaller than the unperturbed value in the lower energy band and larger in the higher energy band. In $K = 0$ bands where $\Omega_p = \Omega_n = \frac{1}{2}$ there is a $(-1)^{I-1}$ spin dependence, so the odd spin members are shifted in energy from the even spin member. This shift is also observed in the $K = 1$ band of the same configuration.

One can quickly predict where Coriolis coupling ought to be important. Considering two bands where the neutron orbitals are the same, the mixing element strongly depends on the $\sum_{J_p} \hat{C}_{J_p}^p \hat{C}_{J_p}^n$ factor. The element is large when the two $\hat{C}_{J_p}^n$'s are both large for the same value of J . This occurs when both proton orbitals originate from the same shell model state.

Finally the matrix elements of H_{pp} must be investigated. There is no spin operator in this term, and the $J_{p\pm}$ and $J_{n\pm}$ operators act only on the respective $\chi_{J_p}^p$ and $\chi_{J_n}^n$ intrinsic states. H_{pp} therefore supports off-diagonal elements of the form

$$\langle \text{IMK}^{\pm\Omega_p \pm\Omega_n} | H_{pp} | \text{IMK}^{\pm\Omega_p \pm\Omega_n} \rangle = \frac{\hbar^2}{2I} \sum_{J_p} \sum_{J_n} C_{J_p}^{\Omega_p} C_{J_p}^{\Omega_p} C_{J_n}^{\Omega_n} C_{J_n}^{\Omega_n} \\ \times [(J_p - \Omega_p)(J_p + \Omega_p + 1)(J_n - \Omega_n)(J_n + \Omega_n + 1)]^{\frac{1}{2}} \delta_{\Omega_p \pm 1} \delta_{\Omega_n \pm 1} \delta_{K-K'} \quad (\text{II-54})$$

these elements tend to be very small and typically cause perturbations on the order of 1 kev. There is no I dependence, so the perturbation causes a shift in the entire band as a unit.

As in the H_{RPC} term there is a special case of $\Omega_p = \Omega_n = \frac{1}{2}$ involving the H_{pp} term. It supports diagonal matrix elements of the form

$$\langle \text{IMK}^{\pm\Omega_p \pm\Omega_n} | H_{pp} | \text{IMK}^{\pm\Omega_p \pm\Omega_n} \rangle = \frac{\hbar^2}{2I} \sum_{J_p} \sum_{J_n} C_{J_p}^{\Omega_p} C_{J_p}^{\Omega_p} C_{J_n}^{\Omega_n} C_{J_n}^{\Omega_n} \\ \times (-1)^{I-J_p-J_n} [(J_p - \Omega_p)(J_p + \Omega_p + 1)(J_n - \Omega_n)(J_n + \Omega_n + 1)]^{\frac{1}{2}} \\ \times \delta_{K-K'} \delta_{K0} \delta_{\Omega_p \pm \frac{1}{2}} \delta_{\Omega_n \pm \frac{1}{2}} \quad (\text{II-55})$$

This reduces to⁸

$$\langle \text{IMK}^{\pm\Omega_p \pm\Omega_n} | H_{pp} | \text{IMK}^{\pm\Omega_p \pm\Omega_n} \rangle = \langle \text{IM}0^{\pm\frac{1}{2} \pm\frac{1}{2}} | H_{pp} | \text{IM}0^{\pm\frac{1}{2} \pm\frac{1}{2}} \rangle \\ = \frac{\hbar^2}{2I} (-1)^{I+1} a_p a_n \delta_{K0} \delta_{\Omega_n} |_{\Omega_n} |_{\frac{1}{2}} \quad (\text{II-56})$$

where a_p and a_n can be calculated by (II-50). This diagonal element has the effect of further shifting the odd I eigenvalues relative to

even I states in the $K = 0$ band of the special case of $\Omega_p = \Omega_n = \frac{1}{2}$.

The Residual Interaction

The final term in the Hamiltonian (II-1) is the residual interaction term, H_{INT} . This term accounts for the interaction between odd proton and odd neutron in odd-odd nuclei. Observing the interaction is one of the important objectives in studying odd-odd

The coupling of an odd proton with an odd neutron which have intrinsic angular momentum projections Ω_p and Ω_n yields two states of total projections

$$K_+ = |\Omega_p + \Omega_n| \quad (II-57)$$

$$K_- = |\Omega_p - \Omega_n|$$

Except for the residual interaction these two states are degenerate in energy. The neutron-proton interaction splits the states K_+ and K_- of an individual two-quasiparticle configuration to different energies. The largest component of H_{INT} is the intrinsic spin-spin exchange operator. Consequently, according to the Pauli repulsion of fermions, the eigenstate which is in the triplet spin state will usually lie lower in energy. This is the basis for the Gallagher-Moszkowski coupling rule¹ to which there is only one known exception.

In order to evaluate the matrix elements of H_{INT} it is useful to expand the intrinsic wavefunction $\chi_{\Omega_p \Omega_n}$ in eigenfunctions of j instead of j .

$$\chi^2 = \sum_{\lambda=1}^{\infty} a_{\lambda} \chi_{\lambda}^2 \quad (II-58)$$

is accomplished with a Clebsch-Gordan transformation on equation (4). Since H_{INT} does not operate on the collective coordinates, it is

$$\begin{aligned} \langle D_{MK}^I | H_{INT} | D_{MK}^I \rangle &= H_{INT} \frac{S+1}{2I+1} \\ \langle D_{MK}^I | H_{INT} | D_{M-K}^I \rangle &= 0 \text{ for } K \neq 0 \\ \langle D_{MK}^I | H_{INT} | D_{MK'}^I \rangle &= 0 \text{ for } K' \neq K \end{aligned} \quad (II-59)$$

Matrix elements of H_{INT} can be written⁹

$$\langle IMK \chi_p^R \chi_n^R | H_{INT} | IMK \chi_p^R \chi_n^R \rangle = A + (-1)^I \delta_{K0} \quad (II-60)$$

$$A = \langle \chi_p^R \chi_n^R | H_{INT} | \chi_p^R \chi_n^R \rangle \quad (II-61)$$

$$B = \langle \chi_p^R \chi_n^R | H_{INT} | \chi_p^{-R} \chi_n^{-R} \rangle \quad (II-62)$$

It is apparent from (II-60) that the residual interaction shifts the entire rotational band as a unit unless $K = 0$. The unperturbed rotational spacing is preserved. In $K = 0$ bands the even spin members are shifted by an amount $2B$ relative to the odd spin members. There are then effectively two rotational bands with the same rotational moment of inertia parameter in the $K = 0$ case. One band is formed

the odd members and the other from the even members.

The interaction Hamiltonian can be written with finite range central forces.⁸

$$H_{INT} = U(|\vec{r}_{pn}|) [V_1 + V_2 \langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle + V_3 \langle P_m \rangle + V_4 \langle \vec{\sigma}_p \cdot \vec{\sigma}_n P_m \rangle] \quad (II-63)$$

$U(|\vec{r}_{pn}|) = \exp(-\vec{r}/r_0)^2$. $\langle P_m \rangle$ is the space exchange operator and is the Majorana operator, $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle$ is the spin-spin force or exchange operator. The product of the two, $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n P_m \rangle$, is equivalent to a charge exchange operator. Each of the V_i are free vectors as well as r_0 . The matrix elements are all given by eq. 11.8. Experimental splitting energies allow the calculation of $\Delta A = A_{X<} - A_{X>}$. The value of V_1 is not important in calculating the experimental ΔA quantities since its matrix element is simply equal to a constant and thus does not contribute to ΔA .

The largest contributor to the H_{INT} matrix elements is $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle$. This is a scalar and has definite eigenvalues for the singlet and triplet state. If a zero-range force is used instead of a finite range force $\langle P_m \rangle \equiv 0$, and $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n P_m \rangle \equiv \langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle$. In this case (II-63) reduces to

$$H_{INT} = V(|\vec{r}_p - \vec{r}_n|) [V_1 + V_5 \langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle] \quad (II-64)$$

$V(|\vec{r}_p - \vec{r}_n|)$ can then be replaced by a delta force¹⁰

$$V(|\vec{r}_p - \vec{r}_n|) = -4\pi g \delta(\vec{r}_p - \vec{r}_n) \quad (II-65)$$

where g is the interaction parameter. Pyatov¹¹ has written the residual interaction with a zero range force as

$$H_{INT} = V(|\vec{r}_p - \vec{r}_n|) [1 - \alpha + \alpha \langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle] \quad (II-66)$$

where α is a spin-spin force parameter. It is customary to insert $\left[\frac{2v+3}{\pi} \right]^{1/2}$, where v is the quantity which appears in the expression for the shell model wavefunctions as $N = 2(v+1)+1$, into the potential (II-65) and (II-66). In this form only the parameter αW must be considered in ΔA calculations.

Calculations using (II-65) and (II-66) have successfully predicted the signs of residual interaction splitting energies in the deformed region and the order of the states formed in the vector coupling of the intrinsic state in shell model nuclei. Introducing the finite range central force has caused little improvement of the least squares deviation of the theoretical splitting values from the experimental values.¹² Equation (II-63) contains 5 adjustable parameters. The increased number of degrees of freedom does not seem to be justified by the small improvement made by their introduction. The predicted splitting energies are found to be relatively insensitive to the value of r_0 selected¹³, and best values for the parameters V_i are strongly dependent upon the set of experimental data used. However good agreement between experimental and theoretical ΔA values can be obtained using finite range calculations if one allows the parameters V_i to vary freely from one set of data to the next.

The calculation of the odd-even shift in $K = 0$ bands has met with much less success using central forces. Newby⁹ has shown that the determination of B in the triplet case is a very different problem from that in the singlet case. Expanding the intrinsic orbitals in spherical harmonics, B can be written in a form analogous to (II-62).

$$B = (-1)^{\lambda_1 + \lambda_2 + 1} \langle Y_{\lambda_1 \lambda_1}^{(1)} \chi_{\Sigma_1}^{(1)} Y_{\lambda_2 \lambda_2}^{(2)} \chi_{\Sigma_2}^{(2)} | H_{INT} |$$

(II-67)

$$Y_{\lambda_1 - \lambda_1}^{(1)} \chi_{-\Sigma_1}^{(1)} Y_{\lambda_2 - \lambda_2}^{(2)} \chi_{-\Sigma_2}^{(2)} \rangle$$

Recalling $K_1 = \lambda_1 + \Sigma_1$ and $K_1 = -K_2$, (II-67) results in singlet and triplet state elements. These are respectively for the singlet case,

$$\lambda_1 + \Sigma_2 = 0$$

$$B = (-1)^{\lambda_1 + \lambda_2 + 1} \langle Y_{\lambda_1 \lambda_1}^{(1)} \chi_{\Sigma_1}^{(1)} Y_{\lambda_2 - \lambda_1}^{(2)} \chi_{-\Sigma_1}^{(2)} | H_{INT} |$$

(II-68)

$$Y_{\lambda_1 - \lambda_1}^{(1)} \chi_{-\Sigma_1}^{(1)} Y_{\lambda_2 \lambda_1}^{(2)} \chi_{\Sigma_1}^{(2)} \rangle$$

and for the triplet case $\Sigma_1 + \Sigma_2 = 1$ ($\Sigma_1 = \Sigma_2 = \frac{1}{2}$),

$$B = (-1)^{\lambda_1 + \lambda_2 + 1} \langle Y_{\lambda_1 \lambda_1}^{(1)} \chi_{\Sigma_1}^{(1)} Y_{\lambda_2 (-\lambda_1 - 2\Sigma_1)}^{(2)} \chi_{\Sigma_1}^{(2)} | H_{INT} |$$

(II-69)

$$Y_{\lambda_1 - \lambda_1}^{(1)} \chi_{-\Sigma_1}^{(1)} Y_{\lambda_2 (\lambda_1 + 2\Sigma_1)}^{(2)} \chi_{-\Sigma_1}^{(2)} \rangle$$

The λ_2 and Σ_2 values in (II-69) come from the restrictions

$$\begin{aligned}
 -K_1 - K_2 &= 0 \\
 &= -A_1 - \bar{E}_1 - A_2 - \bar{E}_2 \\
 &= -A_1 - \bar{E}_1 - A_2 - \bar{E}_1 \\
 A_2 &= -A_1 - 2\bar{E}_1
 \end{aligned}
 \tag{II-70}$$

noted that in the triplet state the spin and orbital angular momentum projections are mixed. Since A and E can be regarded as good quantum numbers in the intrinsic orbitals, and since central forces are scalars in the spin and orbital angular momentum spaces, the matrix element in (II-69) must be zero for a central force. It thus becomes necessary to seek a tensor force interaction between the scattered proton and neutron.⁹ The necessity of a tensor interaction in odd-odd nuclei has been previously observed in the explanation of the zero quadrupole moment in the deuteron.⁶ A good explanation of tensor forces in nuclei is given on pages 77-80 of reference 6.

Adding a tensor force interaction of the form $V_T S_{12}$ to (II-66) in (II-63) where

$$S_{12} = \left[\frac{(\sigma_p \cdot r)(\sigma_n \cdot r)}{r^2} - \sigma_p \cdot \sigma_n \right]
 \tag{II-71}$$

has not significantly improved the determination of ΔA , as can be seen in references 12 and 13. However significant improvement has been obtained in predicting values of B in $K = 0$ bands.¹⁵ In particular

tensor force has significantly improved the prediction of the even shift in the triplet state, which is consistent with theory.

Mottelson has suggested a treatment including core polarization effects. This treatment derived by Boisson, et. al.⁴⁰ has changed the spin exchange operator from $\sigma_p \cdot \sigma_n$ to $(\sigma_p \cdot \sigma_n)_\parallel + (\sigma_p \cdot \sigma_n)_\perp$ corresponding to different force in the direction parallel and perpendicular to the nuclear symmetry axis, where $(\sigma_p \cdot \sigma_n)_\perp$ can be replaced by $(\sigma_p^+ - \sigma_p^-) \cdot (\sigma_n^+ - \sigma_n^-)$. This treatment has obtained significant improvement of finite range force calculations of the singlet-triplet splitting with the use of only two additional parameters. Introduction of core polarization effects in finite range central force calculations seems to have resulted in too little improvement of agreement between experiment and theory to justify use of the necessary six parameters involved.

11.6 The Complete Hamiltonian

It is now possible to write all of the matrix elements of the Hamiltonian of (II-1). When a band is not connected by any off-diagonal elements to any other band, or when the bands are far apart in energy compared to the matrix element connecting them, it (they) can be treated as a pure band(s). The energy of an unmixed state can be written as the sum of the diagonal elements obtained above.

$$E_{I,K} = \frac{\hbar^2}{2I} [I(I+1) - K^2 + (-1)^{I+1} a_p a_n \delta_{K0} \delta_{n\frac{1}{2}}] + \epsilon_{sp}(p) + \epsilon_{sp}(n) + A_K + (-1)^I B \delta_{K0} \quad (II-72)$$

and

$$E_{K,K} = \frac{\hbar^2}{2I} K(K+1) (-1)^{K+1} a_p a_n \delta_{K0} \delta_{n\frac{1}{2}} + \epsilon_{sp}(p) + \epsilon_{sp}(n) + A_K + (-1)^I B \delta_{K0} \quad (II-73)$$

that once the bandhead has been located

$$E_{K,K} = \epsilon_{K,K} + \frac{\hbar^2}{2I} [I(I+1) - K(K+1) + (1 - (-1)^I) a_p a_n \delta_{K0} \delta_{n\frac{1}{2}}] + \quad (II-74)$$

$$\epsilon_{sp}(p) + \epsilon_{sp}(n) + A_K + (-1)^{I-1} B \delta_{K0}$$

If $\epsilon_{sp}(i)$ is defined as $\langle H_{sp}(i) \rangle + \frac{\hbar^2}{2I} [J_i(J_i+1) - i(i+1)]$ as the terms in Eq. 2 of (II-43) have been absorbed into ϵ_{sp} . The terms $\epsilon_{sp}(p)$

$\epsilon_{sp}(n)$ can be estimated from the energies of single particle states in neighboring odd-A nuclei. When such information is available it

is usually more reliable than theoretical $\epsilon_{sp}(n)$ and $\epsilon_{sp}(p)$ calculations.

$\epsilon_{sp}(p)$ and $\epsilon_{sp}(n)$ can then be combined to a single term ϵ_{2sp} .

The moment of inertia parameter, $\frac{\hbar^2}{2I}$, can be calculated for a

$K \neq 0$ band from the energies of two states in the band.

$$\frac{\hbar^2}{2I} = \frac{\epsilon_{I+1,K} - \epsilon_{I,K}}{2(I+1)} \quad (II-75)$$

When $K = 0$ the terms B and $a_p a_n \frac{\hbar^2}{2I}$ contribute to the eigenvalue, and

it is necessary to know three states to calculate the effective moment of inertia and the odd-even shift terms.

$$\frac{\hbar^2}{2I} K=0 = \frac{\epsilon_{I+2,0} - \epsilon_{I,0}}{4I+6} \quad (II-76)$$

$$B = (-1)^I \frac{(e_{I,0} - e_{I+1,0})}{2} + \frac{\hbar^2}{2I} (-1)^I (I+1) a_p a_n \frac{e_{I+1,0}}{e_{I,0}}. \quad (II-77)$$

Equation (II-77) can be easily derived from (II-74), but it is more meaningful and simpler to calculate B by simple reasoning. Once the rotational kinetic energy contributed to the state ψ_{K0}^{I+1} which is contributed to the state ψ_{K0}^I has been subtracted from $e_{I+1,0}$, i.e. $e_{I+1,0} - 2(I+1)\frac{\hbar^2}{2I}$ the two states will have the same energy except for the contribution made by the odd-even shift terms. The magnitude of the difference $(e_{I+1,0} - 2(I+1)\frac{\hbar^2}{2I} - e_{I,0})$ will be equal to B. The sign of B can then be determined to consistent with (II-60) recognizing that a positive value of B will push the even states in a $K = 0$ band to higher energies and the odd states to lower energies. When $a_n = a_p = \frac{1}{2}$ in a $K = 0$ band only the quantity $B - \frac{\hbar^2}{2I} a_p a_n$ can be extracted from the experimental energies. To obtain B the decoupling parameters a_p and a_n can be taken from the corresponding observed bands in neighboring odd-A nuclei or they can be calculated relatively accurately using (II-50).

As stated in Section (II.5), for each two-quasiparticle configuration there are states formed by the parallel and antiparallel coupling of a_p and a_n . The two states have the same e_{2qp} value, but their relative energies depend on λ_K , K, and $B\delta_{K0}$. In (II-73) it is noted that the band head energy contains a zero point rotational energy of $\frac{\hbar^2}{2I} K$. Therefore the two band head energies are related by

$$\epsilon_{K_<} - \epsilon_{K_>} = A_{K_<} - A_{K_>} + \frac{\hbar^2}{2I} K_< - \frac{\hbar^2}{2I} K_> + B \delta_{K0} - \frac{\hbar^2}{2I} a_p a_n \delta_{K0} \delta_{n1} \quad (II-78)$$

where $K_<$ and $K_>$ are defined by (II-57). The difference between the residual interaction energies, $\Delta A = A_{K_<} - A_{K_>}$, is usually the dominant term in (II-78), so the residual interaction usually determines which of a single configuration lies lower.

The sign of ΔA has been predicted very accurately by the Gallagher-Moszkowski coupling rule¹. For the two intrinsic single nucleon orbitals, $\chi_i^{\pm} = R_1^{\pm} [N_1 N_{31} A_1 \Sigma_1]$, the coupling can be such that $\Sigma_p - \Sigma_n = 1$ or 0 . These are the triplet and singlet states respectively. The Gallagher-Moszkowski coupling rule states that the triplet state will lie lower in energy. The value of Σ_1 is determined by $\Sigma_1 = A_1 - A_{-1}$ and is either $\frac{1}{2}$ or $-\frac{1}{2}$. Each of the quantities Σ_1 and A_1 is always written with a positive sign. The notation used to describe a pair of bands arising from the same neutron and proton orbitals is $\left[\begin{smallmatrix} p \\ p \end{smallmatrix} N N_3 A \Sigma \right] \pm \left[\begin{smallmatrix} n \\ n \end{smallmatrix} N N_3 A \Sigma \right]$ and is referred to as a configuration.

The experimental value of ΔA can be determined by rearranging (II-78) to

$$A_{K_<} - A_{K_>} = \epsilon_{K_<} - \epsilon_{K_>} - \frac{\hbar^2}{2I} K_< + \frac{\hbar^2}{2I} K_> - B \delta_{K0} + a_p a_n \delta_{K0} \delta_{n1} \quad (II-79)$$

This value is referred to as the singlet-triplet splitting energy of the given configuration.

III SINGLE PARTICLE REACTION THEORY

III.1 DWBA Theory

The exact description of a single particle transfer stripping reaction $A + a \rightarrow B + b$ where $a = b + c$, and c is the transferred nucleon, is a three body problem and can not be readily solved. By introducing the Born approximation the problem becomes more manageable and gives results which are consistent with experiment. The direct reaction is assumed to be the overlap of two elastically scattered waves, a and b , in the optical potentials of A and B respectively with the transfer of nucleon c into a single particle orbital in the target A .

The complete Hamiltonian of the initial system can be written

$$\begin{aligned} H_a &= H_a + H_A + T_a + V_{aA} \\ &= H_{aA} + T_a + U_{aA} + V'_{aA} \\ &= H_{aA}^0 + V'_{aA} = H_a^0 + V_A' \end{aligned} \quad (\text{III-1})$$

where $H_{aA} = H_a + H_A$, T_a is the kinetic energy of a , U_{aA} is the optical potential describing the elastic scattering of a by A , and $V'_{aA} = V'_a$ is the remaining interaction of V_{aA} . A similar description can be made of the final system.

$$\begin{aligned}
H_{\pm} &= H_b + H_B + T_b + V_{bB} \\
&= H_{bB} + T_b + U_{bB} + V'_{bB} \\
&= H_{bB}^0 + V'_{bB} = H_B^0 + V_{\pm}'
\end{aligned}
\tag{III-2}$$

It is assumed that the potential $V_{\pm}' \equiv V'_{bB} = V_{bB} - U_{bB}$ can be approximated by the interaction of b with c , V_{bc} .

General scattering theory states that the cross-section can be stated in terms of a transition amplitude $T_{\alpha\beta}$ with V_{bc} considered as a perturbation.

$$\frac{d\sigma}{d\Omega} = \frac{\mu_{\alpha} \mu_{\beta}}{(2\pi \hbar^2)^2} \frac{k_{\beta}}{k_{\alpha}} |T_{\alpha\beta}|^2 \tag{14}
\tag{III-5}$$

where μ_{α} and μ_{β} are the reduced masses in the entrance and exit channels, and k_{α} and k_{β} are the corresponding wave numbers. The transition amplitude is thus given by

$$T_{\alpha\beta} = \langle \psi_{\beta}(\vec{k}_{\beta}) | V_{bc} | \psi_{\alpha}(\vec{k}_{\alpha}) \rangle
\tag{III-4}$$

where ψ_{β} and ψ_{α} are solutions to the equations

$$H_B^0 \psi_{\beta}(\vec{k}_{\beta}) = \epsilon_{\beta} \psi_{\beta}(\vec{k}_{\beta})$$

and

$$H_a^0 \psi_{\alpha}(\vec{k}_{\alpha}) = \epsilon_{\alpha} \psi_{\alpha}(\vec{k}_{\alpha}).$$

(III-5)

ψ_{α} is the exact entrance channel solution. The Born approximation substitutes this with $\psi_a(\vec{k}_{\alpha})$ where $\psi_a(\vec{k}_{\alpha})$ is the solution to

$\psi_a(\vec{k}_a) = \epsilon_a \psi_a(\vec{k}_a)$. The transition amplitude is therefore simplified by the Born approximation to

$$T_{ab} = \langle \psi_b(\vec{k}_b) | V_{bc} | \psi_a(\vec{k}_a) \rangle \quad (\text{III-6})$$

The "zero range" approximation simplifies the interaction V_{bc} to

$$V_{bc} \psi_a(\vec{r}_b - \vec{r}_c) = D_0 \delta(\vec{r}_b - \vec{r}_c) \quad (\text{III-7})$$

where D_0 is a constant. Finite range corrections can be made in the calculation of T_{ab} . The magnitude of these corrections indicate that the zero range approximation is a good approximation.

The intrinsic single-particle cross-section for transfer of a nucleon into a shell model orbital $\phi_{\alpha l j}$ is written

$$\sigma_{jl}(\theta) = (2j + 1) \left(\frac{k_B}{k_A} \right)^2 |T_{ab}|^2 \frac{\mu_A \mu_B}{(2\pi \hbar^2)^2} \quad (\text{III-8})$$

and is calculated by the computer codes DWUCK.¹⁵ In the case of a pickup reaction, such as a (t, α) reaction $\frac{\sigma_{jl}^+(\theta)}{\sigma_{jl}^-(\theta)} = \frac{k_A^2}{k_B^2}$ where

$\sigma_{jl}^+(\theta)$ is the stripping reaction and $\sigma_{jl}^-(\theta)$ is the pickup and α and β , the entrance and exit channels, are defined for $\sigma_{jl}^+(\theta)$.

III.2 Pairing Correlations

If a small perturbation G pairing is added to the Nilsson Hamiltonian of the nuclear core the wavefunction $\chi^{\hat{}}_i$ of the nuclear core are slightly mixed. G pairing is not diagonal in $\chi^{\hat{}}_i$ and therefore

mixes completely filled and completely empty Nilsson orbitals near the nuclear surface, Fermi surface. The perturbation causes the scattering of pairs of nucleons among levels near the Fermi surface and has no effect on unpaired nucleons. Any given Nilsson orbital has a finite probability of being occupied by a pair of nucleons because of the pairing interaction.

In even-even deformed nuclei the lowest lying Nilsson orbitals are each filled with pairs of particles coupled to zero spin. In the absence of a pairing force there would exist a sharp Fermi surface in which all orbitals below it would be completely filled and all those above it completely empty. The pairing force scatters pairs of nucleons among the orbitals and thus tends to make the nuclear Fermi surface diffuse. Because of this the Nilsson orbitals near the Fermi energy have occupation probabilities related to their single particle energies. The quantities U_K and V_K must be introduced, where V_K^2 is the probability of the K^{th} orbital being occupied by a pair of nucleons and U_K^2 is its emptiness probability. It readily follows that $U_K^2 + V_K^2 = 1$. Particles in an orbital K have particle properties to the extent V_K^2 and hole properties to the extent U_K^2 , as they are referred to as quasiparticles.

The pairing force causes the existence of a large energy gap for the formation of intrinsic two-quasiparticle excitations in even-even nuclei. Paired nucleons resist uncoupling and thus prefer intrinsic excitations which promote both particles to higher intrinsic

particle states. The gap is observed to be smaller for the formation of three-quasiparticle states in odd-A nuclei. The reason for this is that the odd particle in odd-A nuclei discreetly occupies a single Nilsson orbital, which is then blocked from participation in pairing correlations. Blocking an orbital from participation raises the total energy of the system just as blocking of resonance in chemical systems raises the total energy of the chemical system. The activation energy needed to uncouple a pair is therefore correspondingly larger.

The quasiparticle energy for a proton or neutron in Nilsson orbital K with the pairing perturbation included is given by

$$E_K = [(\epsilon_K - \lambda)^2 + \Delta^2]^{1/2} \quad (\text{III-9})$$

where ϵ_K is the single particle energy calculated from (II-5) and (II-7), λ is the chemical potential or Fermi surface energy, and Δ is half the energy required to break a zero coupled pair into two quasiparticles. The ground state quasiparticle energy, E_0 , of a nuclide may be slightly above or below the Fermi energy. The energy of a quasiparticle state above the ground state quasiparticle energy is given by

$$E_{ex} = |E_K - E_0| = [(\epsilon_K - \lambda)^2 + \Delta^2]^{1/2} - [(\epsilon_0 - \lambda)^2 + \Delta^2]^{1/2} \quad (\text{III-10})$$

The occupation probability of an orbital K is

$$V_K^2 = .5(1 - \frac{\epsilon_K - \lambda}{[(\epsilon_K - \lambda)^2 + \Delta^2]^{1/2}}) \quad (\text{III-II})$$

and $U_K^2 = 1 - V_K^2$. Single particle states below the Fermi energy have occupied probabilities larger than .5, which increase as ϵ_K goes to smaller values. Those states above the Fermi energy have unoccupied or emptiness probabilities greater than .5 which increase as ϵ_K goes to larger values.

In single particle transfer reactions the probability of transfer of a particle into or out of a particular Nilsson orbital is dependent upon the availability of that orbital for participation. In a stripping reaction a particle can not be transferred into an orbital which is already completely filled in the target nucleus. Therefore the probability of transferring a particle into a given Nilsson orbital i is proportional to U_i^2 . Likewise the cross-section for a pickup reaction is proportional to V_i^2 in the target nucleus. The spectroscopic factor for population of any Nilsson orbital in the residual nucleus must accordingly contain an appropriate pairing factor V_i^2 or U_i^2 .

Each of the reactions in this study is a proton pickup reaction on an even Z target. The proton core is then spread over all Nilsson orbitals near the Fermi energy of the target, and it is important to know the occupation probabilities of each of these orbitals. In order to calculate each U_K and V_K it is necessary to know the appropriate single particle energies ϵ_K , the Fermi energy, and the Δ

section for each target. The ϵ_K values can be determined from experimental energies in neighboring odd mass nuclei. The Fermi energy can be determined from an energy fit of the levels in a neighboring nucleus, and the pairing energy can be calculated from the energy of the first 2-quasiparticle state in neighboring even-even nuclei. For this work the values of each of these parameters was taken from a compilation by Immele¹⁶, where values of λ and Δ are tabulated in units of ϵ_K for rare earth nuclei, and neutron and proton single particle energies have been listed in units of $\epsilon_K/\hbar\omega$. The oscillator spacings are given by (II-23).

III.3 Cross Sections

The differential cross section for a direct single particle transfer reaction populating a state with spin (total angular momentum) I at spectrometer angle θ can be written

$$\left(\frac{d\sigma(\theta)}{d\Omega}\right)^I = N \frac{2I+1}{2I_0+1} \sum_j S_{j2}^2 \frac{\phi_{j2}(\theta)}{2j+1} \quad (III-12)$$

where $\phi_{j2}(\theta)$ is the intrinsic single particle cross-section determined from DWBA calculations, which were discussed in Section (III.1). The spectroscopic factor S_{j2}^2 is for a particle transferred with angular momentum j and orbital angular momentum 2 and contains the nuclear structure information. I_0 is the spin of the target nucleus.

The spectroscopic amplitude, S_{j2} , is given by the nuclear overlap integral. This integral is a measure of the extent to which

the residual nucleus is made up of the target core plus the transferred nucleon in some shell model orbital $\psi_{\lambda j}$. The spectroscopic amplitude or reduced width can be written¹⁸

$$S_{j\lambda} = \int \sum_{M_0} \langle I_0 M_0 j (M - M_0) | IM \rangle \langle IM K \Omega_p \Omega_n | I_0 M_0 K_0 \Omega_0 \rangle \psi_{\lambda j}(M - M_0, n) dn \quad (\text{III-13})$$

for the reaction on an odd-A nuclide to form an odd-odd nuclide, where $M - M_0 = M$, the space fixed z component of transferred angular momentum. $\psi_{\lambda j}(M - M_0, n)$ is the bound state wavefunction of the transferred particle with space fixed coordinate n . The subscripts 0 refer to the target nucleus, and M is the projection of l on the spaced fixed z -axis in the final nucleus.

If (III-13) is multiplied by $\psi_{\lambda' j'}(M - M_0, n) \langle I_0 M_0 j' (M - M_0) | IM \rangle$, summed over j' and M_0 , and integrated over all space n one obtains

$$\langle I_0 M_0 j (M - M_0) | IM \rangle^2 \langle IM K \Omega_p \Omega_n | I_0 M_0 K_0 \Omega_0 \rangle = \quad (\text{III-14})$$

$$S_{j\lambda} \psi_{\lambda j}(M - M_0, n) \langle I_0 M_0 j (M - M_0) | IM \rangle.$$

Summing over j yields

$$\langle IM K \Omega_p \Omega_n | I_0 M_0 K_0 \Omega_0 \rangle = \sum_j S_{j\lambda} \psi_{\lambda j}(M - M_0, n) \langle I_0 M_0 j (M - M_0) | IM \rangle. \quad (\text{III-15})$$

Therefore $S_{j\lambda}$ is the coefficient of

$$\psi_{\lambda j}(M - M_0, n) \langle I_0 M_0 j (M - M_0) | IM \rangle \quad (\text{III-15a})$$

in the expansion of the overlap integral (III-15).

It follows from (II-38) and the Hamiltonian for an odd-A nucleus, which is $H = H_{sp} + H_{rot} + H_{vib}$, that the wavefunction for an odd-A deformed nucleus is

$$|IMK\rangle = \sqrt{\frac{2I+1}{16\pi^2}} \sum_j C_j^I (D_{MK}^I \chi_j^I + (-1)^{I-j} D_{M-K}^I \chi_j^{-I}); \quad (III-16)$$

and the wavefunction for an odd-odd deformed nucleus has already been written in (II-38). The overlap of the initial and final states is then

$$\begin{aligned} \langle IMK_s \Omega_s | I_o M_o K_o \Omega_o \rangle &= \sqrt{\frac{(2I+1)(2I_o+1)}{(16\pi^2)^2}} \sum_{J_s J_t J_o} C_{J_s J_o}^{\Omega_s} C_{J_o J_t}^{\Omega_o} C_{J_t J_s}^{\Omega_o} \delta_{J_o J_s} \\ &\times \int \{ D_{MK}^I D_{M_o K_o}^{I_o} \chi_{J_t}^{\Omega_t} \delta_{\Omega_s \Omega_o} + (-1)^{I-J_s-J_t} \delta_{\Omega_s \Omega_o} D_{M-K}^I D_{M_o K_o}^{I_o} \chi_{J_t}^{-\Omega_t} \\ &+ (-1)^{I_o-J_o} \delta_{\Omega_s -\Omega_o} D_{MK}^I D_{M_o -K_o}^{I_o} \chi_{J_t}^{\Omega_t} \\ &+ (-1)^{I-J_s-J_t+I_o-J_o} \delta_{-\Omega_s -\Omega_o} D_{M-K}^I D_{M_o -K_o}^{I_o} \chi_{J_t}^{-\Omega_t} \} d\theta. \end{aligned} \quad (III-17)$$

The integral is over the Euler angle θ . The subscripts p and n have been replaced by s and t to indicate stationary and transferred particles. The various $\delta_{\Omega_s \Omega_o}$ and $\delta_{J_o J_s}$ arise from the requirement that the target odd particle does not change its orbital in a direct reaction. That is $\chi_{\Omega_o}^I \equiv \chi_{\Omega_s}^I$.

The wavefunction of the transferred particle must be transformed to space fixed coordinate to extract $S_{j\lambda}$ as the coefficient of (III-15a). This transformation is

$$\chi_{J_t}^{\Omega_t} = \sum_{M_t} D_{M_t \Omega_t}^{J_t} \psi_{J\lambda}(m, n) \quad (\text{III-18})$$

When this transformation is inserted in (III-17) one obtains for each term in the integral three D functions of the form $D_{MK}^I D_{M_0 K_0}^{I_0} D_{M_t K_t}^{I_t}$. These can be evaluated with

$$D_{M_1 K_1}^{I_1} D_{M_2 K_2}^{I_2} D_{M_3 K_3}^{I_3} d\theta = \frac{8\pi^2}{2I_3 + 1} \langle I_1 M_1 I_2 M_2 | I_3 M_3 \rangle \langle I_1 K_1 I_2 K_2 | I_3 K_3 \rangle \quad (\text{III-19})$$

which comes from multiplying the equation of Preston⁶

$$D_{M_1 K_1}^{I_1} D_{M_2 K_2}^{I_2} = \sum_{I, M, K} D_{MK}^I \langle I_1 M_1 I_2 M_2 | IM \rangle \langle I_1 K_1 I_2 K_2 | IK \rangle \quad (\text{III-20})$$

by $D_{M_3 K_3}^{I_3}$ and integrating over all Euler angles.

Because the stationary particle and target particle must be identical, and because of the orthonormality of the χ^{Ω} functions

$$\sum_{J_t J_s J_0} C_{J_0 \ell_0}^{\Omega_0} C_{J_t \ell_t}^{\Omega_t} C_{J_s \ell_s}^{\Omega_s} = \sum_{J_t} C_{J_t \ell_t}^{\Omega_t} \delta_{J_0 J_s}, \text{ and from the relation}$$

$$C_{J\lambda}^{-\Omega} = \pi_0 (-1)^{J-\frac{1}{2}} C_{J\lambda}^{\Omega} \quad (\text{III-21})$$

of Preston⁶ one obtains

$$\begin{aligned}
\langle \text{MKO}_t \Omega_s | \text{I}_0 \text{M}_0 \text{K}_0 \Omega_0 \rangle &= \frac{1}{2} \sqrt{\frac{2\text{I}_0 + 1}{2\text{I} + 1}} C_{J_t \lambda_t}^{\Omega_t} \langle \text{I}_0 \text{M}_0 \text{J}_t \text{M}_t | \text{IM} \rangle \psi_{J_t \lambda_t}(\text{m}_t, \text{n}) \\
&\times \{ \langle \text{I}_0 \text{K}_0 \text{J}_t \Omega_t | \text{IK} \rangle \\
&+ (-1)^{\text{I}-\text{J}_t-\frac{1}{2}} \Pi_0 \langle \text{I}_0 \text{K}_0 \text{J}_t - \Omega_t | \text{I-K} \rangle \\
&+ (-1)^{\text{I}_0-\frac{1}{2}} \Pi_0 \langle \text{I}_0 - \text{K}_0 \text{J}_t \Omega_t | \text{IK} \rangle \\
&+ (-1)^{\text{I}+\text{I}_0-\text{J}_t-1} \langle \text{I}_0 - \text{K}_0 \text{J}_t - \Omega_t | \text{J-K} \rangle \}
\end{aligned} \tag{III-22}$$

taking use of the relation of Preston⁶

$$\langle \text{J}_1 \text{K}_1 \text{J}_2 \text{K}_2 | \text{JK} \rangle = (-1)^{\text{J}_1 + \text{J}_2 - \text{J}} \langle \text{J}_1 - \text{K}_1 \text{J}_2 - \text{K}_2 | \text{J-K} \rangle \tag{III-23}$$

one obtains after performing the necessary algebra on the phase exponents

$$\langle \text{I}_0 \text{K}_0 \text{J}_t \Omega_t | \text{IK} \rangle = (-1)^{\text{I}+\text{I}_0-\text{J}_t-1} \langle \text{I}_0 - \text{K}_0 \text{J}_t - \Omega_t | \text{I-K} \rangle$$

and

$$(-1)^{\text{I}-\text{J}_t-\frac{1}{2}} \Pi_0 \langle \text{I}_0 \text{K}_0 \text{J}_t - \Omega_t | \text{IK} \rangle = (-1)^{\text{I}_0-\frac{1}{2}} \Pi_0 \langle \text{I}_0 - \text{K}_0 \text{J}_t \Omega_t | \text{I-K} \rangle \tag{III-24}$$

Finally

$$\begin{aligned}
\langle \text{MKO}_t \Omega_s | \text{I}_0 \text{M}_0 \text{K}_0 \Omega_0 \rangle &= \frac{1}{2} \sqrt{\frac{2\text{I}_0 + 1}{2\text{I} + 1}} C_{J_t \lambda_t}^{\Omega_t} \langle \text{I}_0 \text{M}_0 \text{J}_t \text{M}_t | \text{IM} \rangle \psi_{J_t \lambda_t}(\text{m}_t, \text{n}) \\
&\times \{ \langle \text{I}_0 \text{K}_0 \text{J}_t \Omega_t | \text{IK} \rangle + (-1)^{\text{I}_0-\frac{1}{2}} \langle \text{I}_0 - \text{K}_0 \text{J}_t \Omega_t | \text{IK} \rangle \}
\end{aligned} \tag{III-25}$$

The coefficient of (III-15a) can be selected in (III-25)

$$S_{J1} = \sqrt{\frac{2I_0 + 1}{2I_0 - 1}} C_{J1}^{\frac{1}{2}} [\langle I_0 K_0 J_0 | K \rangle + (-1)^{I_0 - \frac{1}{2}} \langle I_0 - K_0 J_0 | K \rangle P] \quad (\text{III-26})$$

The pairing factor P has been inserted in (III-26). This is a measure of the availability of $\frac{1}{2}$ for participation in particle transfer, and has been discussed in Section (III.2). Because P is dependent only on the orbital of the transformed particle, it will be the same for all spins I within a rotational band and for both bands formed by the same transferred particle in a given Nilsson orbital.

Recall that by (II-37) $K = |K_p \pm K_n|$. Therefore for a single allowed value of K only one of the two Clebsch-Gordan coefficients of (III-26) can be nonzero. The differential cross-section for populating a state of spin I and projection K at angle θ in the final nucleus can now be written.

$$\left(\frac{d\sigma}{d\Omega} \right) = N \sum_j [C_{j1}^{\frac{1}{2}} \langle I_0 \pm K_0 J_0 | K \rangle P]^2 \frac{F_j(\theta)}{2J+1} \quad (\text{III-27})$$

The factor N is a normalization factor. It is related to the intrinsic spins of the incoming and outgoing particles in the reaction and to the zero range approximation constant D_0 , which was introduced in the DWBA calculations.

For the transfer of a particle with quantum numbers $K^{\pi}(n, l, m)$, the value of j has an upper limit of $\frac{2K+1}{2}$. Therefore the value of

I which can be populated within a rotational band has an upper bound fixed by the maximum value of j . Because of the orthonormality of the Clebsch-Gordan coefficients, both bands of a given two-quasiparticle configuration are populated with the same total intensity when summed over all spins I . That is

$$\sum_I \frac{d\sigma}{d\Omega} = N \sum_j (C_{j\lambda}^{\Omega})^2 p^2 \frac{\phi_j(\vartheta)}{2j+1} \quad (\text{III-28})$$

If mixing effects are included the cross-section becomes

$$\left(\frac{d\sigma(\vartheta)}{d\Omega}\right)_K = N \sum_j \left[\sum_i (S_{j\lambda})_i A_i \right]^2 \frac{\phi_{j\lambda}(\vartheta)}{2j+1} \quad (\text{III-29})$$

where the A_i are the mixing amplitudes of states with the same spin.

In such a case the phases of the spectroscopic amplitudes become

important, as the sum $\sum_i (S_{j\lambda})_i A_i$ can result in constructive or destructive interference. The mixing amplitudes are determined from diagonalization of the Hamiltonian matrix as described in Section (IV.2).

IV THEORETICAL CALCULATIONS

IV.1 DWBA Calculations

The DWBA calculations were performed on the LLL CDC 7600 computer by the program DWUK72 developed by Kunc¹⁵ at the University of Colorado. The program calculates $\psi_{J\lambda}(\theta)$ of equation (III-27). The optical model potentials, U_{aA} and U_{bB} of (III-1) and (III-2), used are in the form

$$U(r) = V_R f(X_R) + iW_I f(X_I) + V_C(r, r_c) \quad (IV-1)$$

where

$$f(X_i) = [1 + \exp(\frac{r - r_i A^{1/3}}{a_i})]^{-1} \quad (IV-2)$$

and V_C is the coulomb potential of a uniformly charged sphere of radius $r_c A^{1/3}$. The subscripts R and I refer to real and imaginary terms, r_i and a_i are parameters referring to the nuclear radius and diffuseness; both have units of Fermis (10^{-15} cm). In the triton channel a spin-orbit term was added to the potential. This term is in the form

$$V_{LS}(r) = -[V_R X_R^1 f(X_{LS}) / dr] \vec{L} \cdot \vec{S} \quad (IV-3)$$

where $f(X_{LS})$ is in the same form as (IV-2). The parameters used for

Each reaction in this project are those of Lu and Alford¹⁹ with a spin-orbit potential added in the triton channel and appear in Table (IV.1).

The bound state potential has been calculated by

$$V(r) = V_c(r, r_c) - V_b[f(X_0) + \vec{L} \cdot \vec{S} \times \frac{\lambda}{45.2} \cdot \frac{1}{r} \frac{df(X_0)}{dr}] \quad (IV-4)$$

is chosen to best reproduce the spin-orbit splitting energies of the $h_{11/2}$ shell model states of the isotopic harmonic oscillator. A value of 15 has given good results.¹⁵ The parameters $r_0 = 1.25$ and $a_0 = 0.65$ were used in this work, and the potential V_b was varied by the program until the experimental proton binding energy of ${}^{166}\text{Lu}$ was reproduced.

In each reaction considered the integration for the transition amplitude determination was from 0 to 16.0F with an integration step size of 0.1F. The parameters chosen were selected after comparison of angular distributions calculated from several different parameter sets. The experimental angular distribution of the (t, α) transfer populating the ground state of ${}^{176}\text{Lu}$ was used as a standard as this is a relatively intense, pure $\ell = 4$ transition. The agreement between experiment and theory is shown in figure (IV.1).

The angular distributions calculated by DWUCK72 for the (t, α) reactions lacked appreciable structure. The intrinsic calculated cross-sections appear in Tables (IV.2)-(IV.5). Frequently distinguishing even between an experimental $\ell = 2$ transfer and an $\ell = 5$ transfer was difficult. It became necessary to rely heavily on

Table (IV.1). Optical Model parameters used in the (t, α) reactions.

	V_R	r_R	a_R	W_I	r_I	a_I	λ	r_C	b^b	V_{LS}	r_{LS}	a_{LS}
	(MeV)	(fm)	(fm)	(MeV)	(fm)	(fm)		(fm)		(MeV)	(fm)	(fm)
α	-200.	1.4	0.6	-20.	1.4	0.6	0	1.3	0.25	0	0	0
t	-200.	1.4	0.6	-50.	1.4	0.6	0	1.3	0.25	-6.0	1.15	0.74
bound state	a	1.25	0.65	0	0	0	25					

^aAdjusted to produce bound state energy

^bNonlocal range parameter

Figure (IV.1). Comparison of angular distributions calculated by DWBA using Lu and Alford optical model parameters of Table (IV.1) to the experimental angular distributions of $\lambda=2$, $\lambda=4$, and $\lambda=5$ transfers in the $^{177}\text{Hf}(t,\alpha)^{176}\text{Lu}$ reaction. The solid curves are the results of DWBA calculations.

Table (IV.2). Intrinsic single particle ($t_{1/2}$) cross-sections for shell model orbitals $\psi_{\nu j}$ in ^{177}Hf .

νj	$\left(\frac{d\sigma}{d\Omega}\right)$		$\left(\frac{1/b}{\text{sr}}\right)$				
	15°	20°	25°	30°	35°	40°	50°
$1h_{1/2}$	15.58	17.86	16.52	13.66	13.37	13.17	9.52
$1h_{3/2}$	7.03	8.05	7.45	6.17	6.03	5.93	4.50
$1g_{7/2}$	9.07	8.86	8.33	9.21	9.81	9.43	8.58
$1g_{9/2}$	4.68	4.75	4.46	4.93	5.24	5.03	4.58
$2f_{7/2}$	48.01	36.01	53.52	38.54	34.12	31.99	24.2
$2f_{5/2}$	32.20	24.08	23.77	25.78	22.87	21.42	16.2
$2d_{5/2}$	11.83	12.86	16.44	17.42	19.45	20.85	19.2
$2d_{3/2}$	8.93	9.71	12.40	13.11	14.64	15.68	14.5
$3p_{3/2}$	23.48	31.35	29.12	33.35	35.28	35.25	28.2
$3p_{1/2}$	3.80	6.77	9.82	16.72	24.84	42.34	95.8
$3s_{1/2}$	1.37	1.20	1.44	1.93	2.07	1.90	1.93

Table (IV.3). Intrinsic single particle (s.p.) cross-sections for shell model orbitals 1_{α}^{+} in ^{17}O .

α	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{lab}}$						
	15°	20°	25°	30°	35°	40°	50°
$1h_{9/2}$	18.03	20.26	18.51	15.12	14.48	13.39	9.81
$1h_{7/2}$	8.18	9.18	8.40	6.87	6.57	6.28	4.44
$1g_{7/2}$	22.77	20.56	15.87	15.23	15.10	13.02	9.72
$1g_{9/2}$	12.19	10.84	8.50	8.16	8.08	6.92	5.17
$2f_{7/2}$	56.60	41.76	39.50	42.18	37.20	24.46	24.47
$2f_{5/2}$	37.92	27.92	26.40	28.03	24.80	23.00	16.44
$2d_{5/2}$	25.53	23.38	28.23	25.28	25.57	24.00	18.87
$2d_{3/2}$	19.44	17.75	21.43	19.20	19.41	18.13	14.25
$3p_{3/2}$	25.80	35.12	32.98	36.95	36.60	37.05	29.98
$3p_{1/2}$	22.34	30.41	28.39	31.80	31.82	31.64	24.55
$3s_{1/2}$	16.63	14.58	15.04	20.22	22.89	21.08	19.27

Table (IV.4). Intrinsic single particle (t,a) cross-sections for shell model orbitals γ_{v1j} in ^{167}Er .

j	$\left(\frac{d\sigma}{d\Omega}\right) \left(\frac{\mu\text{b}}{\text{sr}}\right)$				
	25°	30°	35°	40°	50°
$1d_{5/2}$	16.42	13.01	12.07	11.80	8.12
$1d_{3/2}$	7.34	5.83	5.42	5.29	3.64
$1d_{5/2}$	14.46	13.32	13.49	11.32	8.52
$1d_{3/2}$	6.75	7.06	7.13	5.96	4.51
$2d_{5/2}$	36.72	40.08	34.56	31.58	21.75
$2d_{3/2}$	24.40	26.52	28.88	20.98	14.36
$2d_{5/2}$	29.27	25.25	24.97	23.40	17.75
$2d_{3/2}$	22.06	19.02	18.87	17.63	13.36
$3p_{3/2}$	36.72	39.52	39.30	36.22	28.45
$3p_{1/2}$	31.25	34.04	36.68	30.95	24.30
$3s_{1/2}$	18.66	22.46	23.59	23.35	18.32

Table (IV.5). Intrinsic single particle (t, α) cross-sections for shell model orbitals $\nu_{\alpha j}$ in ^{161}Dy .

$\nu_{\alpha j}$	20°	$\left(\frac{d\sigma}{d\Omega}\right) \left(\frac{\text{nb}}{\text{sr}}\right)$ 30°	40°	50°
$1h_{11/2}$	14.68	10.42	9.11	6.01
$1h_{9/2}$	14.64	10.37	9.08	5.99
$1g_{7/2}$	14.49	11.60	9.11	6.78
$1g_{9/2}$	16.44	11.56	9.07	6.75
$2f_{7/2}$	40.90	37.00	28.85	18.42
$2f_{5/2}$	40.78	36.92	28.77	18.33
$2d_{5/2}$	24.43	24.05	21.97	16.16
$2d_{3/2}$	24.47	23.95	21.92	16.10
$3s_{1/2}$	26.00	31.57	29.81	21.66

absolute energies, relative energy spacings in rotational bands, and comparison of calculated to observed cross-sections to assign states. These three are very powerful tools however. Only for the most intense peaks was it possible to determine an ℓ transfer solely by the use of angular distributions.

The determination of the normalization factor, N of (III-27), was a difficult experimental problem. For a (t, α) reaction this quantity should be in the range of 12-25²⁹, however previous experimenters have obtained values very different from that range.^{50, 51, 52} A value of 15.1 was determined for the $^{177}\text{Hf}(t, \alpha)^{176}\text{Lu}$ reaction. This is quite satisfactory agreement with theory and was found to be reproducible from one configuration to the next in the levels assigned or previously known in ^{176}Lu . In the $^{171}\text{Yb}(t, \alpha)^{170}\text{Tm}$ and $^{167}\text{Er}(t, \alpha)^{166}\text{Ho}$ reactions normalization values of 8.2 and 12.9 were observed. The small value in ^{170}Tm would seem to indicate that the ground state neutron $k_2 = [521]$ has an appreciable vibrational character. It is frequently observed to be mixed with vibrational bands in lighter odd-mass odd neutron deformed nuclei.⁵³ In ^{166}Ho some configurations other than the ground state had normalization factors lower than 12.9. Observations of this sort are indicative of other components mixing into the two-quasiparticle strength of the band being considered. These will be discussed in the interpretation of the data.

Nilsson wavefunction coefficient were calculated by the program CJL²⁰. The values of κ_p and μ_p of (II-17) used were 0.05 and 0.625

respectively. The deformation parameter ϵ_2 of (II-16) was given the value of 0.27, and ϵ_1 was set equal to 0. The program diagonalizes the single-particle Hamiltonian matrix to calculate eigenvalue of H_{sp} and $C_{J_2}^2$ coefficients. The energies calculated are of much less use than the coefficients, so the single particle energies needed for constructing predicted spectra were estimated from experimental values in adjacent odd-A nuclides. The $C_{J_2}^2$ coefficients calculated for this work appear in Tables (IV.6)-(IV.9).

To obtain estimates of the pairing factors for each Nilsson orbital the necessary single-particle energies and Fermi surface parameters, λ and Δ of (III-9), were taken from Immele¹⁶. V_{λ}^2 and U_{λ}^2 were then calculated by (III-11). The single particle energy estimates, of reference 16 were general values and meant to be used over the entire range of deformed rare earth nuclides. They nevertheless allowed the reasonably accurate prediction of the pairing factors. Where experimental energies were found to differ appreciably from the single particle estimates, the pairing factor was recalculated using the experimental energies. The pairing factors appear in Table (IV.10).

IV.2 Mixing Calculations

In the analysis of odd-odd nuclei it is important to include the effects of mixing between wavefunctions. Since there are approximately twice as many bands in odd-odd nuclei as in odd-A nuclei the opportunity for, and magnitude of, mixing effects is considerably increased.

Table (IV.6). Nilsson expansion coefficients $C_{j_k}^{\pi}$ for proton orbitals in ^{176}Lu .

Orbital	$j=1/2$	$3/2$	$5/2$	$7/2$	$9/2$	$11/2$
$\frac{1}{2}^{-}$ [411]	-.2743	.7128	.4726	-.4103	.1579	
$\frac{5}{2}^{+}$ [402]			.9457	.2488	-.2093	
$\frac{3}{2}^{+}$ [411]		-.1811	.8768	.3490	-.2768	
$\frac{7}{2}^{+}$ [404]				.9916	.1294	
$\frac{7}{2}^{-}$ [523]				-.1819	-.1376	.9736
$\frac{9}{2}^{-}$ [514]					-.01938	.9958

Table (IV.7). Nilsson expansion coefficients $C_{j_1}^j$ for proton orbitals in ^{170}Tm .

Orbital	$j=1/2$	$3/2$	$5/2$	$7/2$	$9/2$	$11/2$
$5/2^-$ [532]			.03810	.2580	-.1292	.9567
$5/2^-$ [523]				.1489	-.1266	.9807
$5/2^-$ [514]					-.09981	.9950
$5/2^-$ [541]	.2305	-.2414	.5029	-.1682	.7465	.2074
$3/2^+$ [411]		-.1178	.8361	.4544	-.2939	
$3/2^+$ [413]			-.2393	.9477	.2112	
$3/2^+$ [402]			.9464	.2763	-.1674	
$9/2^+$ [404]					1.000	
$7/2^+$ [404]				.9920	.1262	
$1/2^+$ [411]	-.3159	.6541	.4724	-.4682	-.1732	

Table (IV.8). Nilsson expansion coefficients $C_{j\lambda}^{\tau}$ for proton orbitals in ^{166}Ho .

Orbital	$j=\frac{1}{2}$	$\frac{3}{2}$	$\frac{5}{2}$	$\frac{7}{2}$	$\frac{9}{2}$	$\frac{11}{2}$
$\frac{5}{2}^-$ [532]			-.04150	.2758	-.1304	.9514
$\frac{7}{2}^-$ [525]				.1584	-.1271	.9790
$\frac{7}{2}^-$ [541]	.2712	-.3082	.5199	-.2292	.6604	.2694
$\frac{9}{2}^-$ [514]					-.09973	.9950
$\frac{3}{2}^+$ [413]			-.1936	.9609	.1980	
$\frac{3}{2}^+$ [411]		-.1567	.8646	.3882	-.2779	
$\frac{1}{2}^+$ [411]	-.2942	.6931	.4702	-.4315	-.1606	
$\frac{1}{2}^+$ [404]				.9935	.1142	

Table (IV.9). Nilsson expansion coefficients $C_{j,k}^{\tilde{n}}$ for proton orbitals in ^{160}Tb .

Orbital	j=1/2	3/2	5/2	7/2	9/2	11/2
$5/2^+$ 404					1.000	
$3/2^+$ 413			-.1687	.9667	.1903	
$1/2^+$ 411		-.1786	.8902	.3466	-.2705	
$1/2^+$ 411	-.2809	.7173	.4683	.4056	-.1516	
$5/2^-$ 532			-.0381	.2580	-.1292	.9567
$7/2^-$ 523				.1439	-.1266	.9807

These effects must be accounted for in order to extract accurate ΔA splitting energies and to accurately calculate theoretical cross-sections.

The wavefunction of a physically observed state ϕ_K^I may be written as a linear combination of unmixed two-quasiparticle wavefunctions:

$$\phi_K^I = \sum_i a_{iK} \psi_i^I \quad (IV-5)$$

where a_{iK} are the mixing amplitudes. Substituting ϕ_K^I into the Schrödinger equation yields

$$H\phi_K^I = E_K^I \phi_K^I \quad (IV-6)$$

$$\sum_i H a_{iK} \psi_i^I = E_K^I \sum_i a_{iK} \psi_i^I \quad (IV-7)$$

Multiplying by ψ_i^I , and integrating over all space yields

$$\begin{aligned} \sum_i a_{iK} \langle \psi_i^I | H | \psi_i^I \rangle &= E_K^I \sum_i a_{iK} \langle \psi_i^I | \psi_i^I \rangle \\ &= E_K^I a_{iK} \delta_{i,i} \end{aligned} \quad (IV-8)$$

The terms $\langle \psi_i^I | H | \psi_i^I \rangle$ are the matrix elements of Section (II.4).

Usually the only important off-diagonal elements are those of H_{RPC} and H_{pp} . Writing (IV-8) for each term $i' = K$ generates a series of equations $\sum_i a_{iK} [\langle \psi_K^I | H | \psi_i^I \rangle - E_K^I \delta_{Ki}] = 0$. The Hamiltonian matrix of the elements can be constructed. The equations then have

Table (IV.10). Pairing factors for proton orbitals.

Orbital	^{177}Hf		^{171}Yb		^{167}Er		^{161}Dy	
	V	U	V	U	V	U	V	U
$\frac{9}{2}^+$ [404]	.9907	.1306	.9870	.1605	.9795	.2015	.9661	.2581
$\frac{5}{2}^-$ [532]	.9840	.1781	.9751	.2216	.9546	.2978	.9113	.4117
$\frac{3}{2}^+$ [413]	.9794	.2066	.9640	.2660	.9282	.3722	.8438	.5288
$\frac{3}{2}^+$ [411]	.9671	.2543	.9388	.3444	.8639	.5037	.7153	.6988
$\frac{7}{2}^-$ [523]	.9454	.3258	.8846	.4663	.7367	.6763	.5450	.8385
$\frac{1}{2}^+$ [411]	.8743	.4854	.7161	.6981	.5074	.8617	.3675	.9300
$\frac{7}{2}^+$ [404]	.6352	.7723	.4335	.9012	.3125	.9499	.2456	.9694
$\frac{1}{2}^-$ [541]	.5456	.8381	.1962	.9806	.2087	.9780		
$\frac{5}{2}^+$ [402]	.3789	.9254	.2641	.9645	.2127	.9771		
$\frac{9}{2}^-$ [514]	.2468	.9691	.2590	.9659	.1803	.9836		

solutions when the determinants of the elements $\{\langle \psi_K^I | H | \psi_1^I \rangle - E_K^I \delta_{KI}\} = 0$. Diagonalization of the determinants gives the eigenvalue E_K^I , and the a_{IK} can be found from (IV-8) for each eigenvalue, giving the mixed wavefunctions (IV-5).

The diagonal entries E_K^I in the matrix are the experimental eigenvalues, and the diagonal entries $\langle \psi_K^I | H | \psi_1^I \rangle$ are the appropriate matrix elements of $H_{\text{rot}} + H_{\text{sp}}^{(p)} + H_{\text{sp}}^{(n)} + H_{\text{INT}}$ using unperturbed free parameters $\frac{\hbar^2}{2I}$, A , B , and ϵ_{sp} instead of the effective values observed in the experiment. In practice the elements of $H_{\text{sp}}^{(p)}$, $H_{\text{sp}}^{(n)}$, and A of H_{int} are absorbed into one parameter ϵ_{2qp} , which becomes the unperturbed bandhead energy. The unperturbed moment of inertia and B in a $K = 0$ band are then free parameters. The off-diagonal elements of H_{RPC} are determined as a function of the free parameter $\alpha = \frac{\hbar^2}{2I} \langle \psi_K^I | J_z | \psi_1^I \rangle (U_1 U_K + V_1 V_K)$. This parameter then carries the leading coefficient $-\frac{1}{2} \sqrt{(I \pm K)(I \pm K \mp 1)}$ in the determinant.

The determinant is diagonalized for each choice of the parameters, and the eigenvalues of E_K^I are calculated. The process is repeated until a satisfactory fit of E_K^I are calculated. This general procedure is also used for H_{pp} mixing, although off-diagonal elements are calculated explicitly from (II-54). Having determined the best choice of parameters $\frac{\hbar^2}{2I}_{\text{unpert}}$, B_{unpert} , α , and ϵ_{2qp} it is possible to extract true splitting energies of H_{INT} , and from them, accurate ΔA values can be determined. Knowing the a_{IK} , correct theoretical cross-sections can be calculated with (III-27).

IV.5 Experimental Coriolis Coupling Calculations

Whenever possible in this work the Coriolis interaction calculations have been performed by parameterization of terms in the Hamiltonian matrix to fit the experimental data, rather than by direct theoretical calculation of the Coriolis matrix elements by equations (II-47), (II-51), or (II-53). An example of this treatment will be given for a two by two matrix where the interaction involves only two bands which satisfy the requirement $K' = K+1$. The treatment can be expanded to the interaction of several bands K , $K+1$, $K+2$, ... by straightforward methods.

Given two bands K_1 and K_2 with $K_2 = K_1+1$ it is possible to write a Hamiltonian matrix for each state of spin I of the two bands. That is I remains a good quantum number. The matrix is written

$$\begin{pmatrix} H_{11} - \lambda & H_{12} \\ H_{12} & H_{22} - \lambda \end{pmatrix} = \begin{pmatrix} H_{2qp}^{(1)} + H_R^{(1)} - \lambda & H_{RPC} \\ H_{RPC} & H_{2qp}^{(2)} + H_R^{(2)} - \lambda \end{pmatrix} = 0 \quad (IV-9)$$

where the λ are the eigenvalues of the matrix (determinant), which have been determined by the experiment. The matrix elements of $H_{2q_i}^{(i)}$ are the unperturbed bandhead energies ϵ_{2qp} and $H_R^{(i)}$ is the unperturbed rotational energy of the I^{th} state in the i^{th} band. In a method developed by Lanier²¹ the unperturbed quantities are replaced by free parameters. ϵ_2 is the parameter $\epsilon_{2qp}^{(2)}$, and A_1 and A_2 are the parameters representing the rotational moments of inertia of bands 1 and 2

respectively. It is not necessary to parametrize $\epsilon_{2qp}^{(i)}$ since there is no spin member $I = K_1$ in band 2, so that the experimental bandhead energy of the K_1 band is the unperturbed bandhead ϵ_1 . The $I = K_1 + 1$ Hamiltonian can now be written:

$$\begin{pmatrix} \epsilon_1 + [(K_1+1)(K_1+2) - K_1(K_1+1)]A_1 - \lambda & H_{RPC} \\ H_{RPC} & \epsilon_2 - \lambda \end{pmatrix} = 0 \quad (IV-10)$$

and the $I = K_1 + 2$ matrix can be written

$$\begin{pmatrix} \epsilon_1 + [(K_1+2)(K_1+3) - K_1(K_1+1)]A_1 - \lambda & H_{RPC} \\ H_{RPC} & \epsilon_2 + [(K_2+1)(K_2+2) - K_2(K_2+1)]A_2 - \lambda \end{pmatrix} = 0 \quad (IV-11)$$

The coefficients of A_1 and A_2 are determined from equation (II-14).

In each case H_{RPC} may be replaced by $-\sqrt{(I-K_1)(I+K_1+1)} \left(\frac{\alpha}{2}\right)$ where

$$\alpha = (A_1 + A_2) \langle K_2 | j_+ | K_1 \rangle (U_1 U_2 + V_1 V_2) \quad (IV-12)$$

and is treated as a parameter, and $\langle K_2 | j_+ | K_1 \rangle$ is defined by (II-48).

The reduction factor $(U_1 U_2 + V_1 V_2)$ has been included as a result of the pairing correlation. The sign of α is determined by theory.

The objective of Coriolis calculations is to account for the experimental energies of all spin states observed and to account for

any anomalies in the observed cross-sections. It is thus necessary to determine the mixing coefficients of each two-quasiparticle band. In the treatment of the Coriolis interaction one can vary the free parameters in order to determine mixing coefficients that give best agreement with experimental cross-sections, or one can vary them in such a way to give best agreement with experimental energies and then note the perturbation this causes on the cross-section calculations. The latter course has been chosen in this work.

In a real eigenvalue problem, if the appropriate eigenvalues λ_K^I are substituted into the matrix, the sum of the diagonal elements must equal 0. That is, for the $I = K_1 + 1$ case

$$\epsilon_1 + [(K_1+1)(K_1+2) - K_1(K_1+1)]A_1 - \lambda_{K_1}^I + \epsilon_2 - \lambda_{K_2}^I = 0 \quad (IV-13)$$

Recalling that $\epsilon_1 \equiv \epsilon_{2qp}^{(1)}$ is known as well as both λ 's, it is easy to solve for A_1 in terms of ϵ_2 . Substituting either of the eigenvalues again into the determinant of I , one obtains the equation

$$(\epsilon_1 + [(K_1+1)(K_1+2) - K_1(K_1+1)]A_1 - \lambda)(\epsilon_2 - \lambda) - (I - K_1)(I + K_1 + 1) \frac{\alpha^2}{4} = 0 \quad (IV-14)$$

An equation for α^2 in terms of A_1 and ϵ_2 is obtained upon rearrangement. Therefore for a single choice of ϵ_2 it is possible to calculate A_1 and α^2 .

Progressing to the $I = K_1 + 2$ matrix, one can sum the diagonals in a similar manner to obtain

$$\begin{aligned} \epsilon_1 + [(K_1+2)(K_1+3) - K_1(K_1+1)] A_1 - \lambda_{K_1}^I + \epsilon_2 + [(K_2+1)(K_2+2) - \\ K_2(K_2+1)] A_2 - \lambda_{K_2}^I = 0 \end{aligned} \quad (\text{IV-15})$$

This yields a simple equation for A_2 in terms of the known or determined ϵ_1 , ϵ_2 , A_1 , and the λ 's. Now all parameters in the $I = K_1+2$ matrix are known, and only the coefficient of $\frac{a}{2}$ must be recalculated. Thus from the determinant of $I = K_1+2$ one obtains the equation

$$\begin{aligned} (\epsilon_1 + [(K_1+2)(K_1+3) - K_1(K_1+1)] A_1 - \lambda) (\epsilon_2 + [(K_2+1)(K_2+2) - \\ K_2(K_2+1)] A_2 - \lambda) - (1 - K_1)(1 + K_1+1) \frac{a^2}{4} = 0 \end{aligned} \quad (\text{IV-16})$$

and theoretical values for $\lambda_{K_1+2}^I$ can be calculated. These can be compared to the experimental values of $\lambda_{K_1+2}^I$ and the best choice of ϵ_2 can be made after calculation of several sets of $\lambda_{K_1+2}^I(\epsilon_2)$.

With the calculation of each of the parameters it is possible to determine the mixing coefficients as described in Section (IV.2). Then mixed cross-sections can be calculated to determine the effect Coriolis coupling has on the theoretical calculations. It is hoped that improved agreement between experiment and theory will be obtained. Also worthy of note is that the agreement between a^2 calculated by the parameterization method and a^2 calculated theoretically from the definition of a can be determined.

The use of specific examples later in this work should aid in clarifying the explanation presented here. This treatment can be expanded to 3 by 3 and 4 by 4 matrices without much change. Examples of these will be given also.

V EXPERIMENTAL

V.1 Accelerator

The experiments were performed with a 17MeV triton beam provided by the Los Alamos Scientific Laboratory Tandem Van de Graaff accelerator. Exposures were taken for the reactions $^{177}\text{Hf}(t,\alpha)^{176}\text{Lu}$ and $^{171}\text{Yb}(t,\alpha)^{170}\text{Tm}$ at spectrograph angles of 15° , 20° , 25° , 30° , 35° , 40° , and 50° ; for the reaction $^{167}\text{Er}(t,\alpha)^{166}\text{Ho}$ at angles of 25° , 30° , 35° , 40° , and 50° ; and for the reaction $^{161}\text{Dy}(t,\alpha)^{160}\text{Tb}$ at spectrograph angles of 20° , 30° , 40° , and 50° . A summary of the exposures is found in Table (V.1).

A tandem Van de Graaff requires a beam of negative particles for the initial stage acceleration. A positive beam of tritons is provided by a high-current duoplasmatron. This beam is injected into a cesium gas exchange canal where a few percent of the tritons receive a -1 charge. The negative tritons are preaccelerated to 60 keV and injected into the first stage of the accelerator by an electrostatic mirror. The negative ions are accelerated to the terminal of the Van de Graaff, which in this case was held at a potential of 8.5 MV by a charge belt system. The beam is then stripped of its negative charge, and the particles emerging with a charge of +1 are accelerated away from the 8.5 MV terminal by electrostatic repulsion. The

Table (V.1). Summary of exposures.

Exposure #	Target	Angle	Elastic Peak Integrated Counts	Dead Time Correction
1	^{177}Hf	15	165229	1.0143
2	^{177}Hf	20	164195	1.0241
3	^{177}Hf	25	161541	1.0044
4	^{177}Hf	30	163537	1.0040
5	^{177}Hf	35	161318	1.0007
6	^{177}Hf	40	161769	1.0768
7	^{177}Hf	50	166897	1.0119
8	^{171}Yb	15	365575	1.1129
9	^{171}Yb	20	366338	1.0973
10	^{171}Yb	25	368346	1.1055
11	^{171}Yb	30	363160	1.0043
12	^{171}Yb	35	371727	1.0031
13	^{171}Yb	40	367743	1.0025
14	^{171}Yb	50	369217	1.0186
15	^{167}Er	25	138106	1.0015
16	^{167}Er	30	301652	1.0015
17	^{167}Er	35	282096	1.0
18	^{167}Er	40	305597	1.1001
19	^{167}Er	50	306212	1.0006
20	^{161}Dy	20		
21	^{161}Dy	30		
22	^{161}Dy	40		
23	^{161}Dy	50		

positively charged ions exit the second stage of the Van de Graaff with a kinetic energy of $(N+1)V_{\text{terminal}}$, where N is the final positive charge of the ions. In this experiment, the tritons could obtain a charge of +1 only, and the final beam energy was 17 MeV. The energy of the beam is held constant to within a few keV by a 90° analyzing magnet and a slit feedback system. Properly placed switching magnets guide the beam into the QSD target room. A beam of approximately 150 nA was obtained on target.

For best resolution of the spectra a very thin target is desirable to reduce scattering within the target material. Targets must be thick enough however to withstand beam bombardment. The targets were prepared at Florida State University by evaporation of the enriched isotope onto an approximately $100 \mu\text{g}/\text{cm}^2$ carbon backing. Target thicknesses of about $45 \mu\text{g}/\text{cm}^2$ were obtained. Target isotopic purities of .916, .882, .872, and .875 were obtained for ^{177}Hf , ^{171}Yb , ^{167}Er , and ^{161}Dy respectively.

V.2 Spectrometer

The measurements in this work were performed with the use of the Los Alamos QSD (quadruple-three dipole) spectrometer to analyze the reaction products. These products leave the scattering chamber and enter a quadrupole focusing magnet followed by three successive dipole magnets, which bend their flight paths to a net radius of curvature uniquely determined by their energy. The QSD spectrometer has been fully described by Flynn, et. al.²², therefore only the important

features will be discussed here.

The spectrometer is operated with the three-stage Van de Graaff accelerator facility, which is capable of providing well-defined, mono-energetic triton beams. The principle features of the spectrometer are large solid angle, high resolving power, ability to correct for kinematic spreading, and large dispersion. The large solid angle of 14.7 msr is a necessity for performing experiments involving reactions with very low cross-sections, such as (t,α) reactions. Unfortunately having a large solid angle causes increased kinematic spreading, and corrections for this are needed. The energy resolution of the spectrometer has been observed to be $E/\Delta E \sim 5200$. This results in peaks in the outgoing particle spectra of approximately 11 keV FWHM. The net resolution is a composite of peak broadening due to kinematic spread, beam energy resolution, target thickness, and inherent detector resolution, and is due mostly to ΔE in the beam energy.

The Q3D requires a beam spot $\sim 3/4$ mm wide. The beam must be brought a distance of 33.2m from the Van de Graaff to the target. This is accomplished by a series of four quadrupole magnets each followed by a slit system. These quadrupole plus slit system pairs focus and trim the beam to the necessary requirements of the Q3D. The target chamber is 50 cm in diameter and is large enough to accommodate several targets mounted on a target ladder connected to a stepping motor. Targets can thus be changed without disturbing the chamber vacuum. The chamber itself can be isolated from the Q3D so that the latter need never be let up to air when it is necessary to open the

chamber. The chamber and Q3D are connected by a sliding seal, so the angle can be changed by moving the Q3D while the target chamber remains stationary. The spectrometer can be rotated through 70° of arc.

The detector system consists of a helical dE/dx proportional counter followed by a scintillator stopping detector. The stopping detector absorbs all of the energy of the product projectile and emits a total energy signal. Photomultiplier tubes are mounted on each end of the scintillator. The difference between the signals produced by these tubes is converted to a position signal indicating where the event occurred in the detector. The position is recorded as channel number in the output of the experiment. The information of total energy and dE/dx is used to discriminate between particle types. The computer system allows the setting of cursors on the dE/dx versus energy plane to select only those particles of interest for counting. Events which satisfy the dE/dx and E requirements are stored one by one in channels according to the position at which they occur in the detector system.

Inside of the target chamber a solid state silicon detector with a solid angle of 8.659×10^{-5} sr is mounted at 30° relative to the beam. This detector is used to select and count elastic scatter events. In this way the experiments are normalized from one angle to the next. This replaces the necessity of a current integrator and target thickness measurements and bypasses error introduced by non-uniformities in the target. The computer system provides computation

of dead time correction and allows the operator to set gates on up to 16 peaks during data collection. The output consists of the centroid of these gated peaks plus the resolution and area under each gated peak. This gives the operator much useful information for immediate on line analysis.

V.3 Experimental Procedure

The peaks in the alpha particle spectra were fit to Gaussian shapes by the stripping routine of the computer program STRIPDE, which had been revised to work with data in terms of channel numbers instead of distance along a photographic plate. The resulting output gives the channel number of the centroid of each peak fitted, along with peak height, area, and full width at half maximum. In addition to the unknown spectra a previously known standard was run at the same magnetic field setting for each nucleus studied. In the case of ^{176}Lu and ^{170}Tm the standard chosen was $^{195}\text{Ir}(t,\alpha)^{192}\text{Os}$, and for ^{160}Ho and ^{160}Tb the standard was $^{194}\text{Pt}(t,\alpha)^{193}\text{Ir}$. These standard spectra had been interpreted respectively by Bagnell, et. al.²⁵ and Yamazaki.²⁴ Known states in these standards were used to obtain calibration curves of excitation energy versus channel number.

After stripping the peaks in the alpha particle spectral of the appropriate standard, a linear function of excitation energy versus channel number was determined by the least squares method. A plot of the function superimposed on the experimental standard data indicated very small yet systematic errors. Therefore an experimental error

curve of calculated excitation energy minus experimental excitation energy versus channel number was made. For each standard the error curve had the form of a quadratic which is convex upward.

Since the Q-value of the reaction leading to the ground state of the standard is different from that of the unknown, the peaks representing the ground states occur at different channel numbers. That is to say that the unknown spectra are offset from the known spectrum, and this offset is different at each spectrometer angle due to the kinematic energy shift. The offset can be determined at each angle from the use of several previously known states in the nucleus being studied if the peaks representing the states can be recognized. In each of the nuclei studied in these experiments, at least both bands of the ground state configuration were previously known and were appreciably populated in the (t, α) reaction. Finally the energy of excitation for peaks in the unknown spectra were determined by

$$E_{\text{exc}} = mx + b - \text{offset} - \text{error} \quad (\text{V-1})$$

where x is the centroid of the peak in channel number units, m is the slope of the linear fit to the known spectrum, b is the intercept of excitation energy at channel number zero in the known spectrum, and the offset and error have been previously explained. The energy of any state in a nuclide was then taken to be the average of all calculated energies of that state observed at all angles, and the error was taken to be the standard deviation of those energies from the average.

Cross-sections were calculated from a method developed by Burke²⁵ using the relative efficiencies of the spectrometer and monitor detectors and the number of counts in a peak. The differential cross-section for populating a state represented by a peak of area A (number of counts) can be calculated by

$$\left(\frac{d\sigma}{d\Omega}\right) = \left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} \times \frac{(\frac{d\sigma}{d\Omega})_{\text{mon}}}{(\frac{d\sigma}{d\Omega})_{\text{QSD}}} \times \frac{\text{D.T.C.}}{N_{\text{mon}} \cdot \text{T.P.}} \times (A = \sqrt{A}) \quad (V-2)$$

where $\left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}}$ is the (t,t) elastic scattering cross-section calculated by DWUK72; $\frac{(\frac{d\sigma}{d\Omega})_{\text{mon}}}{(\frac{d\sigma}{d\Omega})_{\text{QSD}}}$ is the relative solid angle of the monitor detector to that of the QSD detector, and is equal to $\frac{8.653 \times 10^{-5}}{.0145}$; D.T.C. is the dead time correction in the detector system, typically about 1.02; N_{mon} is the number of counts in the elastic scattering peak; and T.P. is the target isotopic purity. The error was taken to be proportional to the error in the counts, which was taken to be plus or minus the square root of the number of counts.

It was very useful to identify peaks from known states of impurities existing in the target. The most obvious impurity in each target was ^{12}C , but no peaks from the $^{12}\text{C}(t,\alpha)^{11}\text{B}$ reaction could be observed in the alpha spectra as the Q-value for this reaction is much lower than that of any of the reactions studied. It was possible to identify $^{12}\text{C}(t,t)^{12}\text{C}$ elastic scattering on the monitor spectrum. Furthermore each of the targets had isotopic impurities. It was usually possible to identify at least one impurity peak resulting from the (t, α) spectrum on one of these isotopic impurities. This was done

by calculating the expected cross-section for two or three of the most intensely populated states from the (t, α) reaction on neighboring isotopes expected to be present in the target. The Q-values for these reactions were calculated with the computer code CXINEM in the Livermore tape library. It was then possible to predict at what channel number these states should occur in the spectra. To estimate the isotopic abundances of impurities it was assumed that all of the target impurity was divided between the two neighboring even-even isotopes. The distribution between these two was assumed to be proportional to their relative natural abundances. With these assumptions it was possible to predict the intensity with which the impurity states should be populated. Locating these impurity peaks with approximately the correct cross-sections was considered good confirmation of the validity of the energy calibration.

VI INTERPRETATION

(VI.1) ^{176}Lu Structure

Several excited neutron states in ^{176}Lu were proposed by Struble and Sheline³⁴ from (d,p) reaction studies. On the basis of (d,p) and (d,d') reaction spectra, high- and low-energy (n, γ) data, and γ - γ coincidences, Minor, et. al.³⁵ have proposed several bands that are now believed to be well established. That study also employed crystal-diffraction (n, γ) data of Maier³⁶. Bolodis, et. al.³⁷ have used these data, previously published (n,e⁻) data³³, and improved the (n, γ) data to construct a much more complete level scheme for ^{176}Lu . Eight rotational bands were identified including the ground state configuration and four excited neutron bands. Excited proton bands of the $\pi_{\frac{9}{2}}^{-} [514] - \nu_{\frac{7}{2}}^{-} [514]$ and $\pi_{\frac{5}{2}}^{-} [402] - \nu_{\frac{7}{2}}^{-} [514]$ configurations were proposed, the latter of which could not be unambiguously assigned. Additionally, bandheads for the $\pi_{\frac{1}{2}}^{-} [411] - \nu_{\frac{7}{2}}^{-} [514]$ and $\pi_{\frac{9}{2}}^{-} [514] - \nu_{\frac{1}{2}}^{-} [510]$ bands were proposed. A summary of the previously known bands in ^{176}Lu is shown in Figure (VI.2).

Fifty-eight peaks in the range of 0 to 1800keV have been identified in the (t, α) reaction. A representative spectrum is shown in Figure (VI.1). Table VI.1 contains a list of the

Figure (VI.1). Alpha spectrum of the $^{177}\text{Hf}(t,n)^{176}\text{Lu}$ reaction with 17.0 MeV tritons observed at 40° .

identified peaks along with their cross-sections at 40^0 and assignments wherever these were made. The predicted cross-sections excluding Coriolis coupling appear in Column 6.

The $\frac{7}{2}+[404]\pm\frac{7}{2}-[514]$ Configuration

The $K\pi = 7^-$ and 0^- bands expected from the $\frac{7}{2}+[404]\pm\frac{7}{2}-[514]$ configuration were previously known from gamma-ray studies^{34,35} and β -decay studies^{39,40} (see Figure VI.2). In the transfer reaction, the 7-state is predicted to contain virtually all of the observable cross-section in the 7-band, with population of no state higher than the 8-being allowed. The peaks at 0 and 183.5keV agree well with the energies of the 7- and 8- levels in the ground state band. Their intensities very closely resemble the intensity pattern expected. Because the $\frac{7}{2}+[404]$ proton is made from a linear combination of the $1g_{7/2}$ and $1g_{9/2}$ shell model orbitals, it must be a pure $l = 4$ orbital, and the transfer populating states in the $\frac{7}{2}+[404]\pm\frac{7}{2}-[514]$ configuration should be pure $l = 4$ transfers. The angular distribution of the $I\pi = 7^-$ ground state is shown in Figure (IV.1), and is in good agreement with the assignment.

Levels populated in the (t,α) spectrum at 123, 241, and 308keV agree well with the previously assigned $I\pi = 1^-$, $(0+3)^-$, and 2^- levels of the $K^\pi = 0^-$ band formed by the $\frac{7}{2}+[404]\pm\frac{7}{2}-[514]$ configuration. The absolute and particularly the relative intensities of these states support the assignments well. Note in Figure (VI.2) that the 0- and 3- levels lie approximately 1 keV apart and are not resolved from

Table (VI.1). Nuclear levels in ^{176}Lu populated by the (t, p) reaction.

Peak #	Previous Value (keV)	Energy (t, p) (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}^{\text{cal}}$ (mb/sr)		Assignment		Configuration
				$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}$	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{cal}}$	J ^π	K	
1	0	0	1	.0306	.0524	7 ⁻	7	$7/2^{+} [404] + 7/2^{-} [514]$
2	126.50	123	1	.0099	.0153	1 ⁻	0	$7/2^{+} [404] - 7/2^{-} [514]$
3	185.4	181	3	.0004	.0005	8 ⁻	7	$7/2^{+} [404] + 7/2^{-} [514]$
4	198.01	198	3	<.0001	--	1 ⁺	1	$7/2^{+} [404] - 9/2^{+} [624]$
5	236.75	233	4	.0003	--	2 ⁺	1	$7/2^{+} [404] - 9/2^{-} [624]$
	239.41					3 ⁻	0	$7/2^{+} [404] - 7/2^{-} [514]$
6	240.56	240	2	.0096	.0164	0 ⁻	0	$7/2^{+} [404] - 7/2^{-} [514]$
7	302.98	301	1	<.006 ^a	--	3 ⁺	1	$7/2^{+} [404] - 9/2^{+} [624]$
8	308.91	308	1	.013 ^a	.0154	2 ⁻	0	$7/2^{+} [404] - 7/2^{-} [514]$
9	376.12	376		--	--	4 ⁺	1	$7/2^{+} [404] - 9/2^{+} [624]$
10	384.98	377	4	.0041	.0021	2 ⁺	1	$9/2^{-} [514] - 7/2^{-} [514]$
11	390.21	391	3	.0106	.0106	1 ⁻	1	$5/2^{+} [402] - 7/2^{-} [514]$
12	436.66	433	1	.0118	.0101	2 ⁻	1	$5/2^{+} [402] - 7/2^{-} [514]$
13	453.73	459	7	.0049	.0039	3 ⁺	1	$9/2^{-} [514] - 7/2^{-} [514]$
14		480	3	.0085	.0054	8 ⁺	8	$9/2^{-} [514] - 7/2^{-} [514]$
15	508.49	505	2	.0069	.0059	3 ⁻	1	$5/2^{+} [402] - 7/2^{-} [514]$
16	536.70	538	3	.0023	.0039	4 ⁺	1	$9/2^{-} [514] - 7/2^{-} [514]$
17		565	3	.0321	.0287	6 ⁻	6	$5/2^{+} [402] + 7/2^{-} [514]$
18	599.35	591	3	.0028	.0023	4 ⁻	1	$5/2^{+} [402] - 7/2^{-} [514]$
19		607	10	.0021				
20		653	6	.0039				

Table (VI.1). continued

Peak #	Previous Value (keV)	Energy (1,0) (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)_{exp}^{50^\circ}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{cal}^{50^\circ}$ (mb/sr)	Assignment		Configuration
						I ^π	K	
21		683	3	.0132	.0084	9 ⁺	8	9/2 ⁺ {514} • 7/2 ⁺ {511}
22	726.51	724	2	.0730	.0869	4 ⁻	4	1/2 ⁺ {411} • 7/2 ⁻ {11}
23		757	4	.0067	.0008	7 ⁻	6	5/2 ⁺ {402} • 7/2 ⁻ {514}
24		772	8	<.002				
25		789	4	.0020				
26		830	2	.0716	.0716	3 ⁻	3	1/2 ⁺ {411} • 7/2 ⁻ {514}
27		864	6	.0363	.0485	5 ⁻	4	1/2 ⁺ {411} • 7/2 ⁻ {511}
28		889	10	<.0039				
29		909	2	.0164		{2 ⁻ }	2	-vibration}
30		945	2	.0411	.0515	4 ⁻	3	1/2 ⁺ {411} • 7/2 ⁻ {514}
31		966	3	.0094		{3 ⁻ }	2	-vibration}
32		1006	3	.0041				
33		1052	4	<.006 ^d	.0109	6 ⁻	4	1/2 ⁺ {411} • 7/2 ⁻ {511}
34		1057	8	<.013 ^d	.0020	{0 ⁺ }	0	7/2 ⁻ {523} • 7/2 ⁻ {511}
35		1074	5	.0134	.0197	5 ⁻	3	1/2 ⁺ {411} • 7/2 ⁻ {514}
36		1106	12	.0017				
37		1128	3	.0338	.0139	{2 ⁺ }	0	7/2 ⁻ {523} • 7/2 ⁻ {511}
38		1162	4	.0070				
39		1182	5	.0099				

Table (VI.1). continued

Peak #	Previous Value (keV)	Energy (I_{α}) (keV)	Error (keV)	$\left(\frac{d\alpha}{dE}\right)_{exp}^{60\theta}$ (mb/sr)	$\left(\frac{d\alpha}{dE}\right)_{cal}^{60\theta}$ (mb/sr)	Assignment I^{π} K		Configuration
40		1221	5	.011 ^a				
41		1237	4	.019 ^a				
42		1273	2	.0114	.0410	7 ⁺	7	7/2 ⁺ [523] + 7/2 ⁺ [514]
43		1294	2	.0234	.0563	(4 ⁺)	0	7/2 ⁺ [523] - 7/2 ⁺ [514]
44		1326	3	.0058				
45		1349	5	.0052				
46		1395	1	.0625	.0930	5 ⁻	5	3/2 ⁺ [411] + 7/2 ⁺ [514]
47		1426	9	.0015				
48		1462	1	.0536	.0828	8 ⁺	7	7/2 ⁺ [523] + 7/2 ⁺ [514]
49		1490	6	.001				
50		1510	2	.0134	.0365	(3 ⁺)	0	7/2 ⁺ [523] - 7/2 ⁺ [514]
51		1535	2	.0364	.0658	6 ⁻	5	5/2 ⁺ [411] + 7/2 ⁺ [514]
52		1569	5	.0082				
53		1595	9	.0058				
54		1617	5	.012				
55		1655	2	.0258	.0606	9 ⁺	7	7/2 ⁺ [523] + 7/2 ⁺ [514]
56		1679	1	.0094				
57		1689	5	.0109	.0038	7 ⁻	5	3/2 ⁺ [411] + 7/2 ⁺ [514]
58		1730	7	.0109	.0524			

^a Data solved

Figure (VI.2). Previously known levels and γ -transitions in ^{176}Lu . All energies are in keV. Spin and parity assignments lie to left of the levels; those in parentheses are considered tentative. The $\frac{5}{2}^{+}$ [402] - $\frac{7}{2}^{-}$ [514] and $\frac{7}{2}^{+}$ [404] - $\frac{5}{2}^{-}$ [512] configurations were previously believed to be interchanged from what is shown in this figure. The designations shown are based on population in the (t,1) reaction as discussed in the text.

one another in the transfer reaction. The levels at 467.40 and 440.95, which are the $I^\pi = 4^-, 0$ and $5^-, 0$ states of this band are only weakly populated, as predicted by the spectroscopic calculations, and are not clearly resolved from the 453.73 and 456.66 keV levels respectively.

The $\frac{5}{2}^+ [402] - \frac{7}{2}^+ [514]$ Configuration

Previous (n, γ) and (d,p) studies^{34,35} have resulted in the identification of a $K^\pi = 1^-$ band originating at 590.21 keV with successive spin members at 456.60, 508.49, 599.55, 719.01, and 861.55 up to the $I^\pi = 6^-$ level. This band, shown in Figure (VI.2), was believed to involve an excited neutron with configuration $\frac{5}{2}^+ [402] - \frac{5}{2}^+ [512]$. It has been populated in the (t, α) reaction up to the 4-spin member at all angles, while the $K^\pi = 1^-$ band beginning at 641.37, previously assigned as the $\frac{5}{2}^+ [402] - \frac{7}{2}^+ [514]$ band, was not populated. Spectroscopic calculations assuming the band at 590.21 keV to be the $\frac{5}{2}^+ [402] - \frac{7}{2}^+ [514]$ $K^\pi = 1^-$ excited proton two-quasiparticle band with pairing factor $V = .5944$ have agreed well with experiment. The spin 2, 3, and 4 members have cross-sections slightly larger than theoretically expected, but this can be explained by Coriolis coupling and will be discussed in Section (VI.2). The original assignments of the two $K^\pi = 1^-$ bands should be reversed, and are presented in this way in Figure (VI.2).

In accordance with the Gallagher-Moschowski coupling rule¹, the singlet state coupling of the $\frac{5}{2}^+ [402] - \frac{7}{2}^+ [514]$ configuration is

expected to lie higher in energy than the triplet state. Tensor force calculations indicate that the $K^\pi = 6^-$ bandhead of the parallel coupling should be observed at approximately 560 kev. Spectroscopic calculations predict that the 6^- level in this band should have an intensity approximately 30 times that of the 7^- level, and should be mostly an $L = 2$ transfer, while the 7^- state should be a pure $L = 4$ transfer. Previously unknown levels were seen in the (t, α) spectrum at 565 kev and 757 kev whose cross-sections agree well with the calculations. The peak at 565 kev has a good $L = 2$ angular distribution, and the 757 kev peak has an even better $L = 4$ angular distribution. This is shown in Figure (VI.5). In assigning the 565 kev level as the I^π , $K = 6^-$, 6 bandhead, it should be noted that the only transition which could be expected to depopulate this level is to the 7^- , 7 ground state. Balodis, et. al.³⁷ have observed an intense γ -ray transition of 565.89(8) kev, which is probably that transition. Accordingly, the levels at 565.89 and 757 kev have been assigned as the 6^- and 7^- members respectively of the $\frac{5}{2}^+ [402] + \frac{7}{2}^+ [514]$ band as shown in Figure (VI.5). This results in a value of $\frac{\hbar^2}{2I} = 13.8$ kev compared to 12.6 kev for the antiparallel coupling of this configuration. The singlet-triplet splitting observed is then -104.9 kev.

The $\frac{9}{2}^+ [514] + \frac{7}{2}^+ [514]$ Configuration

The $K^\pi = 1^+$ band resulting from the antiparallel coupling of the $\frac{9}{2}^+ [514] + \frac{7}{2}^+ [514]$ configuration has been previously assigned by γ -decay studies^{34, 35} up to the 6^+ spin member. Levels in order of

Figure (VI.3). Angular distributions of the $^{177}\text{Hf}(\alpha, n)$ ^{176}Lu reaction with 17.0 MeV tritons. The solid curves are the results of DWBA calculations, and the circles are experimental data points.

increasing spin are at 342.48, 384.98, 453.73, 536.70, 653.80, and 776.86 keV and are shown in Figure (VI.2). The $2+$, $3+$, and $4+$ members have been populated in the (t, α) reaction with cross-sections quite well reproduced by calculations. The 1^+ level is predicted to have a very small spectroscopic factor, and in accordance with this the 342.48 keV level was not observed in the (t, α) spectrum. The moment of inertia of this band is calculated to be $\frac{h^2}{2I} = 11.1$ keV.

The singlet state coupling of the $\frac{9}{2}[-514]\frac{7}{2}[-514]$ configuration results in a rotational band with $K = 8$ and was not previously known. The levels at 486 and 683 keV in the (t, α) spectrum have been assigned as the $8+$ and $9+$ members of this band (see Figure (VI.5)). The relative intensities of these two peaks agree very well with spectroscopic factor calculations, though the absolute cross-sections are about a factor of two larger than predicted. It has been found that the $\frac{9}{2}[-514]$ Nilsson orbital is strongly coupled to the $\frac{7}{2}[-523]$ orbital through the Coriolis interaction, and this readily accounts for the increased intensity of these two states. The transfers populating these states should be pure $\ell = 5$ transitions, and the angular distributions of both of these peaks, shown in Figure (VI.3), are in good agreement with the assignments. The calculated moment of inertia of this band is $\frac{h^2}{2I} = 11.0$ keV. Based on these two new assignments a new singlet-triplet splitting of -93.1 keV is observed. This value is in excellent agreement with the tensor force calculations, which predict the splitting to be

-83.4 kev.

The $\frac{7}{2}^- [404] - \frac{9}{2}^- [624]$ Configuration

The $\frac{7}{2}^- [404] - \frac{9}{2}^- [624]$ band is from an excited neutron configuration and to zeroth order should not be populated in a (t,x) reaction. Levels up to the $I^\pi = 6^+$ state have been assigned from the (d,p) reaction and γ -ray spectroscopy^{34,35} as shown in Figure (VI.2). The first three members of this band have almost certainly been populated in the (t,x) experiment, and the 376.12 kev 4^+ state has probably been populated at several angles. These are all very weak states, indicative of neutron excited states which gain intensity through mixing with proton states populated in zeroth order. Coriolis coupling can not be used to explain the population of any of these 1^+ band members. The most reasonable explanation comes from the mixing of this band with the $K^\pi = 1^+$, $\frac{9}{2}^- [514] - \frac{7}{2}^- [514]$ band. This mixing can be explained by a combination of the neutron-proton residual interaction and the two-particle residual interaction perturbation to the rotational Hamiltonian, H_{pp} . The latter perturbation supports matrix elements which satisfy equation (II-54).

The off-diagonal H_{pp} element calculation is outlined below. First all elements of $J_p = \frac{7}{2}$ are summed over all J_n , then all elements of $J_p = \frac{9}{2}$ over all J_n , and then all elements of $J_p = \frac{11}{2}$ over all J_n . All elements $J_p = \frac{7}{2}$ are zero since there is no $C_{J_n}^{\frac{7}{2}} = C_{J_n/2}^{\frac{7}{2}}$ for the $\frac{9}{2}^- [514]$. Similar reasoning shows that all $J_n = \frac{7}{2}$, $J_n = \frac{13}{2}$, and $J_p = \frac{11}{2}$ elements are zero. Finally nonzero

elements exist for only $J_p = \frac{9}{2}$, $J_n = \frac{9}{2}$ and $J_p = \frac{9}{2}$, $J_n = \frac{11}{2}$. These elements are

$$(.1294)(-.09121)(.1376)(.9620)\left[\left(\frac{9}{2}-\frac{7}{2}\right)\left(\frac{9}{2}+\frac{7}{2}+1\right)\left(\frac{9}{2}-\frac{9}{2}\right)\left(\frac{9}{2}-\frac{9}{2}+1\right)\right]^{\frac{1}{2}} = -.01400$$

and

$$(.1294)(-.09121)(-.09930)(.1565)\left[\left(\frac{9}{2}-\frac{7}{2}\right)\left(\frac{9}{2}+\frac{7}{2}+1\right)\left(\frac{11}{2}-\frac{9}{2}\right)\left(\frac{11}{2}-\frac{9}{2}+1\right)\right]^{\frac{1}{2}} = .002461$$

respectively. The sum is $-.01400 + .002461 = -.01154$. This must be multiplied by $\frac{\hbar^2}{2I} = 11.1$ kev, so

$$\langle \frac{7}{2}+[404]-\frac{9}{2}+[624] | H_{pp} | \frac{9}{2}-[514]-\frac{7}{2}-[514] \rangle = -.1281 \text{ kev. (VI-1)}$$

First order perturbation calculations indicate that the perturbation due to this element is less than 1 kev, and this accounts for almost no mixing of the two pure bands. The calculation of the off-diagonal elements of H_{INT} are discussed in Chapter (VII). These elements are much larger than those of H_{pp} and it is found the using the residual interaction perturbation the wavefunctions for the two $K^\pi = 1^\pi$ bands can be written as

$$\psi_+ = .0875\left(\frac{9}{2}-[514]-\frac{7}{2}-[514]\right) + .996\left(\frac{7}{2}-[404]-\frac{9}{2}+[624]\right) \text{ and}$$

$\psi_- = .996\left(\frac{9}{2}-[514]-\frac{7}{2}-[514]\right) - .0875\left(\frac{7}{2}-[404]-\frac{9}{2}+[624]\right)$ where ψ_+ is the band beginning at 198.01 kev, and ψ_- is the band beginning at 342.48 kev.

The $\frac{1}{2}^+[411]\pm\frac{7}{2}^-[514]$ Configuration

The $\frac{1}{2}^+[411]\pm\frac{7}{2}^-[514]$ configuration, previously unknown in ^{176}Lu , is the lowest-lying two-quasiparticle proton hole state, and accordingly the cross-sections populating states in this configuration should be larger than those previously discussed in this work. From γ -ray studies³⁷ an $1^\pi = 4^-$ state has been assigned at 726.51 keV. In the (t, α) spectrum intense, very good $l = 2$ transfers were observed at 725, 841, 864, and 945 keV. The angular distributions of these peaks appear in Figures (IV.1) and (VI.3). From the calculated Gallagher-Moszkowski splitting; the previously unassigned γ -transitions of 109.510(30), 111.850(30), 129.759(12), 136.09(5), 162.70(5), 221.366(25), 239.32(10), and 299.40(3) keV; the observed rotational spacings; and the (t, α) relative intensities, the partial band structure of the $\frac{1}{2}^+[411]\pm\frac{7}{2}^-[514]$ configuration shown in Figure (VI.4) has been constructed. The moments of inertia of the $K = 3$ and $K = 4$ bands are 13.292 keV and 13.609 keV respectively. The observed singlet-triplet splitting after removal of Coriolis effects is 126.6 keV, in good agreement with the calculated value of 131.0 keV.

The $\frac{3}{2}^+[411]\pm\frac{7}{2}^-[514]$ Configuration

The $\frac{3}{2}^+[411]$ proton orbital is predicted to be well below the Fermi surface. There are strong $l = 2$ transitions to levels at 1395 and 1533 keV (see Figure (VI.3)) in the (t, α) spectrum. The relative intensities and energy spacing strongly indicate that these states are the 5^- and 6^- spin members of the $K^\pi = 5^-, \frac{3}{2}^+[411]\pm\frac{7}{2}^-[514]$

band. Spectroscopic calculations predict an $l = 4$ transfer weakly populating the 7^- member of this band. The moment of inertia calculated from the energies of the 6^- and 5^- members, $\frac{h^2}{2I} = 11.5$ keV allows the prediction of the 7^- level at an energy of approximately 1695 keV. The weakly populated level at 1689 ± 7 keV has been assigned as this 7^- spin member. Intraband γ -ray transitions of 159.49(7), 140.45(14), and 299.91(3) keV can be assigned among these three levels. Also transition probably depopulating the band to the 1^+ , $K = 7^-$, 6 at 758.06 and 5^- , 4 at 863.20 keV as shown in Figure (VI.5) have been observed.

The $K^\pi = 2^-$ band resulting from the antiparallel coupling of the $\frac{5}{2}^+ [411] \pm \frac{7}{2}^- [514]$ configuration has not been identified. Spectroscopic calculations predict fairly intense peaks representing the 2^- , 3^- , 4^- , and 5^- members of this band should be observed. Tensor force singlet-triplet splitting calculations indicate that the 2^- bandhead should lie near $E_{\text{exc}} = 1270$ keV. There is an intense peak at 1293 keV in the (t, α) spectrum, which could be that state, however no other spin members can be reasonably proposed. Further, the angular distribution of this peak more closely resembles an $l = 5$ transfer than an $l = 2$, therefore the 1293 keV peak is more likely a member of the $\frac{7}{2}^- [523] \pm \frac{7}{2}^- [514]$ configuration.

Since the $\frac{3}{2}^+ [411]$ and the $\frac{5}{2}^+ [402]$ Nilsson orbitals both originate from the $2d_{5/2}$ shell model state, they can be expected to have a large Coriolis interaction. This interaction is observed in the coupling of the $\frac{5}{2}^+ [402] \pm \frac{7}{2}^- [514] K^\pi = 6^-$ band to the

Figure (VI.4). Spin and parity assignments and γ -transitions depopulating levels of the $\frac{1}{2}^+ [411] = \frac{7}{2}^- [514]$ configuration in ^{175}Lu . The $2^- = 4^-$ level at 726.51 keV and γ -transitions depopulating it were previously known.

$\frac{5}{2}^+[411]+\frac{7}{2}^-[514]K^\pi = 5^-$ band. The 7^- , 6 and 6^- , 6 states have cross-sections notably enhanced by the mixing of the $K = 5$ band with the $K = 5$ band, which has been quantitatively accounted for quite well by the Coriolis interaction. This interaction may help to explain the absence of a recognizable intensity pattern in the $K^\pi = 2^-$ band of the $\frac{5}{2}^+[411]-\frac{7}{2}^-[514]$ configuration. It is, in fact, observed that the intensity pattern of the $K^\pi = 1^- \frac{5}{2}^+[402]-\frac{7}{2}^-[514]$ band is somewhat perturbed; specifically the spin members higher than the bandhead have cross-sections slightly larger than predicted, exactly what would be expected from a large Coriolis interaction.

Several examples of the $\frac{3}{2}^+[411]$ orbital mixing with vibrational states built on the ground state or on the $\frac{1}{2}^+[411]$ orbital have been observed in odd-A nuclei.³⁵ In ^{176}Lu the $K^\pi = 2^-$, $\frac{5}{2}^+[411]-\frac{7}{2}^-[514]$ band should be able to mix with a 2^- vibrational band built on the 0^- band of the ground state configuration or on the $K^\pi = 4^- \frac{1}{2}^+[411]-\frac{7}{2}^-[514]$ band. Any such mixing would further take intensity from the $\frac{5}{2}^+[411]-\frac{7}{2}^-[514]$ band and increase the difficulty of observing it in a transfer reaction. In view of these arguments the states at 909 keV and 965 keV, which were first observed in the work, have been assigned as the 2^- and 3^- members of a $K^\pi = 2^-$ gamma-vibrational band built on the 0^- ground state configuration band. The moment of inertia of this band is then 9.45 keV. These states can be populated only to the extent that they are composed of the $\frac{5}{2}^+[411]-\frac{7}{2}^-[514]$ two-quasiparticle configuration. The angular distributions of

peaks indicate they could be $\ell = 2$ transfers and their cross-sections are approximately one fifth that which would be expected for a pure two-quasiparticle configuration.

$\frac{7}{2}^{-}[523] + \frac{7}{2}^{-}[514]$ Configuration

Peaks representing levels in the $\frac{7}{2}^{-}[523] + \frac{7}{2}^{-}[514]$ configuration are very nearly pure $\ell = 5$ transfers. The peaks at 1273, 1462, and 1515 keV are strongly populated with very good $\ell = 5$ angular distributions, as seen in Figures (IV.1) and (VI.3). The rotational band assignments are consistent with the assignment of these levels to the 7^{-} , 3^{-} and 9^{-} members of a $K^{\pi} = 7^{-}$ rotational band, and the relative intensities are well reproduced by theoretical calculations giving the band to be from the $\frac{7}{2}^{-}[523] + \frac{7}{2}^{-}[514]$ configuration. The moment of inertia is then $\frac{\hbar^2}{2I} = 11.2$ keV. Possible γ -ray transitions between these levels are the previously unassigned transitions of 166.33(7), 196.27(12), and 362.70(7) keV. However, unambiguous assignments can be made of transitions to lower levels. One should observe transitions to the 3^{+} band at 466 keV. It is probably more reasonable that neither the 7^{-} nor the 3^{+} band are populated in a thermal neutron capture experiment. The assignments to the $K^{\pi} = 7^{-}$ band appear in Figure (VI.5).

The $K^{\pi} = 0^{+}$ band expected from the $\frac{7}{2}^{-}[523] + \frac{7}{2}^{-}[514]$ configuration has not been clearly recognized. This is quite unfortunate as the energy values of a $K = 0$ band are sensitive to the residual interaction in an odd-odd nucleus. One of the interesting reasons for studying

odd-odd nuclei is to investigate the neutron-proton interaction, so assigning states in a $K = 0$ band is an important objective. Residual interaction calculations indicate that the Newby shift (odd-even shift) in the configuration is approximately -80 keV, so the even and odd spin member bands should be well separated, with the even member band lying lower. Spectroscopic calculations indicate that each of the spin states 0^+ through 7^+ should be observable in the alpha-particle spectrum. There are many moderately strong peaks in the region 1000 to 1700 keV, some of which very likely belong to the $K^\pi = 0^+$ band. The most likely candidates are the levels at 1162, 1294, 1510, and 1730 keV, all of which have reasonably good $L = 5$ angular distributions.

Very tentative assignments of the $K^\pi = 0^+$ band from $I^\pi = 0^+$ to 5^+ in order of increasing spin are 1057, 1386, 1128, 1510, 1295, and 1730 keV. The moment of inertia of the even band is 11.8 keV and that of the odd band is 12.2 keV. The singlet-triplet splitting is -7 keV, which is not in good agreement with the tensor force calculation of -99 keV though of the same sign, and the odd-even shift is -156 keV, which is in reasonable good agreement with the calculated value of -80 keV. The cross-sections fit well into the predicted intensity pattern. The 2^+ level at 1128 keV is about .020 mb/sr too large, but much of this can be accounted for, as the 1128 keV peak is largely an impurity from the $^{178}\text{Hf}(t, \alpha)^{177}\text{Lu}$ reaction populating the $\frac{3}{2}^+, \frac{1}{2}^-$ state in ^{177}Lu .

Figure (VI.5) Spin and parity assignments of the $\frac{3}{2}^+$ [411], $\frac{7}{2}^-$ [525], $\frac{5}{2}^-$ [402], and $\frac{9}{2}^-$ [514] proton orbitals coupled to the $\frac{7}{2}^-$ [514] neutron orbital in ^{176}Lu . The assignments for the $K^\pi=0^+ \frac{7}{2}^-$ [525]- $\frac{7}{2}^-$ [514] are considered as tentative. Assignments for a $K^\pi=2^-$ γ -vibrational band are included, and unassigned levels populated in the (τ, α) reaction are shown on the extreme right. All energies are in keV, and spin and parity assignments lie to the left of each level.

(VI.2) Coriolis Coupling in ^{176}Lu

In the $^{177}\text{Hf}(t,\alpha)^{176}\text{Lu}$ reaction the important positive parity proton orbitals in ^{176}Lu are the $\frac{7}{2}^+[404]$, $\frac{5}{2}^+[402]$, $\frac{5}{2}^+[411]$, and $\frac{1}{2}^+[411]$. Coupling each of these to the $\frac{7}{2}^-[514]$ neutron yields negative parity bands of $K = 0$ to $K = 7$. Each of the bands K_0 is connected to its successive K_0+1 band by Coriolis coupling. A rigorous analysis would require the diagonalization of up to an 8 by 8 matrix. It was necessary to break this up into several smaller matrices to make the problem manageable. This could be done by recognizing that certain of the interactions $\langle K_0+1 | H_{\text{RPC}} | K_0 \rangle$ caused perturbations small enough relative to the others to be ignored.

All theoretical off-diagonal matrix elements H_{RPC} were calculated explicitly by Equation (II-47) and (II-51) using the average of the effective moment of inertia of each set of two bands as the $\frac{\hbar^2}{2I}$ coefficient in (II-47) and (II-51). Then using the experimental eigenvalues as known, all H_{ii} elements were calculated for all spins $I = 1$ to 8 treating each $K_0+1 \rightarrow K_0$ mixing as a two by two matrix. The difference between H_{ii} and λ_i then gives a reasonable approximation for the perturbation. An example of this procedure is for the perturbation of the $I = 1$ matrix between the $K = 0$ band $\frac{7}{2}^-[404] - \frac{7}{2}^-[514]$ and the $K = 1$ band $\frac{5}{2}^-[402] - \frac{7}{2}^-[514]$. The H_{RPC} element was theoretically determined to be $-\sqrt{(1-0)(1+0+1)}(6.13) = -8.67$. The experimental energies are $E_{1,0} = 126.50$ and $E_{1,1} = 390.21$.

$$\begin{pmatrix} x - 126.50 & -8.67 \\ -8.67 & x - 390.21 \end{pmatrix} = 0 \quad \begin{aligned} x^2 - 516.71x + 49362 - 75.2 &= 0 \\ x &= 390.50 \text{ or } x = 126.21 \end{aligned}$$

The perturbation is of the order .29kev = $\Delta_{0,1}^I \equiv \Delta_{K_0, K_0+1}^I$. All calculated perturbations Δ_{K_0, K_0+1}^I appear in Table (VI.2). It was observed that all perturbations $\Delta_{2,3}^I$ and $\Delta_{4,5}^I$ were small relative to those flanking them in Table (VI.2). This means that the perturbations between the $K = 2$ band and $K = 3$ band and between the $K = 4$ band and $K = 5$ band are small enough to break the 3 by 3 Hamiltonian matrix into two 3 by 3 matrices involving the $K = 0, 1$, and 2 bands from the $\frac{7}{2}^+[409]$, $\frac{5}{2}^+[402]$, and $\frac{3}{2}^+[411]$ orbitals and involving the $K = 7, 6$, and 5 bands from those same orbitals, and a single 2 by 2 matrix involving the $K = 3$ and $K = 4$ bands of the $\frac{1}{2}^+[411] \pm \frac{7}{2}^-[514]$ configuration.

No approximations were necessary for the positive parity bands, as the $\frac{7}{2}^-[523]$ and $\frac{9}{2}^-[514]$ proton orbitals couple to a 2 by 2 matrix. Since the $K^\pi = 0^+$ band from the $\frac{7}{2}^-[523] - \frac{7}{2}^-[514]$ configuration was not identified with sufficient certainty, no Coriolis calculations were made on the $K = 0, K = 1$ interactions. Calculations with the $K = 7, K = 8$ interaction were performed in the manner outlined in Section (IV-3). Though those calculations are based on the parameterization method of Lanier²¹, the determination of the best set of free parameters involves a new procedure.

The states in the $K^\pi = 7^+$ band were identified at 1273, 1462, and 1655 kev; those of the $K^\pi = 8^+$ band were identified at 486 and 683 kev. It was noted that the experimental cross-sections for the

Table (VI.2). Theoretical perturbations in keV for the Coriolis interaction of negative parity bands in ^{177}Lu .

The notation Δ_{K_0, K_0+1}^I is the perturbation between $K=K_0$ and $K=K_0+1$ bands in the spin I matrix. The method of calculation is explained in the text.

I	Δ_{01}^I	Δ_{12}^I	Δ_{23}^I	Δ_{34}^I	Δ_{45}^I	Δ_{56}^I	Δ_{67}^I
1	0.29						
2	2.24	1.4					
3	1.34	3.4	0.8				
4	5.46	6.1	1.9	4.9			
5	4.00	7	5	11.3	1.1		
6	10.6	13	5.6	19.5	2.4	4.2	
7	6	23	9	26	5	9.4	0.3
8	6	39	24	52	7	16	2

K = 7 band were lower than the unmixed calculations predicted, and the opposite was observed for the K = 3 band cross-sections. The difference can be quite well accounted for by Coriolis coupling.

The I = 3 matrix can be written

$$\begin{pmatrix} \epsilon_8 - \lambda & -2\alpha \\ -2\alpha & \epsilon_7 + 16A_7 - \lambda \end{pmatrix} = 0 \quad \begin{matrix} \lambda_8^S = 486 \\ \lambda_9^S = 1462 \end{matrix}$$

where α is defined in Section (IV.3), and its coefficient is $\frac{1}{2}([1-K_+][1+K_-+1])^{\frac{1}{2}} = -2$. The negative sign was determined by theory from $\langle K=3 | J_+ | K=7 \rangle$ defined by Equation (II-48). Note that ϵ_7 is known, as the I = K = 7 state is unperturbed by Coriolis coupling. Summing the diagonals one obtains

$$\epsilon_8 - \lambda_8^S + \epsilon_7 + 16A_7 - \lambda_7^S = 0 \quad (\text{VI-2})$$

$$\epsilon_8 + 16A_7 = 675$$

Evaluating the determinant with each eigenvalue successively substituted one obtains

$$(\epsilon_8 - 486)(1273 + 16A_7 - 486) - [-2\alpha]^2 = 0 \quad (\text{VI-3})$$

$$787\epsilon_8 - 7771A_7 + 16\epsilon_8A_7 - 4\alpha^2 = 582345$$

and

$$(\epsilon_8 - 683)(1273 + 16A_7 - 683) - 4\alpha^2 = 0 \quad (\text{VI-4})$$

$$-189\epsilon_8 - 23389A_7 + 16\epsilon_8A_7 - 4\alpha^2 = -276154$$

From (VI-2) one obtains

$$A_7 = \frac{675 - \epsilon_8}{16} \quad (\text{VI-5})$$

and from (VI-3) one obtains

$$\alpha^2 = \frac{582543 - 16\epsilon_8 A_7 + 7771\epsilon_8 - 787\epsilon_8}{-4} \quad (\text{VI-6})$$

Now moving to the $I = 9$ matrix,

$$\begin{pmatrix} \epsilon_8 + 18A_8 - \lambda & -2.915\alpha \\ -2.915\alpha & \epsilon_7 + 34A_7 - \lambda \end{pmatrix} = 0 \quad \begin{aligned} \lambda_8^9 &= 683 \\ \lambda_7^9 &= 1655 \end{aligned}$$

The α coefficient is $-\frac{1}{2}\sqrt{(I-7)(I+7+1)} = -\frac{1}{2}\sqrt{34}$. Summing the diagonal yields

$$\epsilon_8 + 18A_8 - \lambda_8^9 + \epsilon_7 + 34A_7 - \lambda_7^9 = 0 \quad (\text{VI-7})$$

$$A_8 = \frac{1065 - 5A_7 - \epsilon_8}{18}$$

At this point all parameters A_7 , A_8 , and α are known in terms of ϵ_8 . The eigenvalues of the $I = 9$ determinant can be calculated by

$$(\epsilon_8 + 18A_8 - \lambda)(\epsilon_7 + 34A_7 - \lambda) - (-2.915\alpha)^2 = 0 \quad (\text{VI-8})$$

Expanding:

$$\lambda^2 - \lambda(\epsilon_8 + \epsilon_7 + 18A_8 + 34A_7) + \epsilon_7\epsilon_8 + 34\epsilon_8 A_7 + 612A_7 A_8 + 23289A_8 - 8.497\alpha^2 = 0 \quad (\text{VI-9})$$

$$\lambda = \frac{(\varepsilon_7 + \varepsilon_8 + 18A_8 - 34A_7) \pm [(-\varepsilon_8 - \varepsilon_7 - 18A_8 - 34A_7)^2 - 4(\varepsilon_7 \varepsilon_7 + 34A_8 \varepsilon_7 - 612A_7 A_8 - 23289A_7 - 9.4974^2)]^{1/2}}{2} \quad (VI-10)$$

$$= \frac{1}{2}(-b \pm \sqrt{b^2 - 4ac})$$

where expressions of a, b, and c are apparent in the quadratic equation (VI-9).

The best choice of ε_8 should produce calculated eigenvalues $\lambda_K^{i=9}$ which minimize the square of the difference between these calculated values and the experimental $\lambda_K^{i=9}$ eigenvalues. The difference Q is

$$Q = (\lambda_7^9 - 1655)^2 + (\lambda_8^9 - 683)^2 \quad (VI-11)$$

One would like to determine ε_8 such that $\frac{dQ}{d\varepsilon_8} = 0$

$$\frac{dQ}{d\varepsilon_8} = 2(\lambda_7^9 - 1655) \frac{d\lambda_7^9}{d\varepsilon_8} + (\lambda_8^9 - 683) \frac{d\lambda_8^9}{d\varepsilon_8} \quad (VI-12)$$

From (VI-10)

$$\frac{d\lambda_7^9}{d\varepsilon_8} = d \frac{-b + \sqrt{b^2 - 4ac}}{2} = -\frac{1}{2} \frac{db}{d\varepsilon_8} - \frac{2b \frac{db}{d\varepsilon_8} - 4 \frac{dc}{d\varepsilon_8}}{4(b^2 - 4ac)^{1/2}} \quad (VI-13)$$

and

$$\frac{d\lambda_8^9}{d\varepsilon_8} = \frac{1}{2} \frac{db}{d\varepsilon_8} - \frac{2b \frac{db}{d\varepsilon_8} - 4 \frac{dc}{d\varepsilon_8}}{4(b^2 - 4ac)^{1/2}} \quad (VI-14)$$

$$\frac{db}{d\epsilon_8} = \frac{d(-\epsilon_8 - \epsilon_7 - 18A_8 - 34A_7)}{d\epsilon_8} = 1 - 18 \frac{dA_8}{d\epsilon_8} - 34 \frac{dA_7}{d\epsilon_8} \quad (\text{VI-15})$$

$$\begin{aligned} \frac{dc}{d\epsilon_8} &= \frac{d(\epsilon_8 \epsilon_7 + 34A_7 \epsilon_8 + 612A_7 \epsilon_8 + 18A_8 \epsilon_7 - 8.497\epsilon_7^2 + 23289A_8)}{d\epsilon_8} \\ &= \epsilon_7 + 34A_7 + 34\epsilon_8 \frac{dA_7}{d\epsilon_8} + 612A_8 \frac{dA_7}{d\epsilon_8} + 18\epsilon_7 \frac{dA_8}{d\epsilon_8} + 612A_7 \frac{dA_8}{d\epsilon_8} \\ &\quad + 23289 \frac{dA_8}{d\epsilon_8} - 8.497 \frac{d\epsilon_7^2}{d\epsilon_8} \end{aligned} \quad (\text{VI-16})$$

$$\frac{dA_8}{d\epsilon_8} = \frac{d[1065 - 34A_7 - \epsilon_8]}{d\epsilon_8} \frac{1}{18} = -\frac{34}{18} \frac{dA_7}{d\epsilon_8} - \frac{1}{18} \quad (\text{VI-17})$$

$$\frac{dA_7}{d\epsilon_8} = \frac{d[\frac{675-68}{16}]}{d\epsilon_8} = -\frac{1}{16} \quad (\text{VI-18})$$

$$\begin{aligned} \frac{d\epsilon_7^2}{d\epsilon_8} &= \frac{d[\frac{332545 - 16\epsilon_8 A_7 + 7771A_7 - 787\epsilon_8}{-4}]}{d\epsilon_8} \\ &= 4A_7 + 4\epsilon_8 \frac{dA_7}{d\epsilon_8} - \frac{7771}{4} \frac{dA_7}{d\epsilon_8} + \frac{787}{4} \end{aligned} \quad (\text{VI-19})$$

Finally each of these can be simplified to

$$A_7 = -.0625\epsilon_8 + 42.16 \quad (\text{VI-20})$$

$$A_8 = .0625\epsilon_8 - 20.47 \quad (\text{VI-21})$$

$$\frac{dA_7}{d\epsilon_8} = -.0625 \quad (VI-22)$$

$$\frac{dA_8}{d\epsilon_8} = .0625 \quad (VI-23)$$

$$b = -2338 \quad (VI-24)$$

$$\frac{db}{d\epsilon_8} = 0 \quad (VI-25)$$

$$c = -2.391\epsilon_8^2 + 2398\epsilon_8 + 510972 \quad (VI-26)$$

$$\frac{dc}{d\epsilon_8} = -4.782\epsilon_8 + 2398 \quad (VI-27)$$

$$a^2 = -.25\epsilon_8^2 + 486.8\epsilon_8 - 177492 \quad (VI-28)$$

$$\frac{da^2}{d\epsilon_8} = -.5\epsilon_8 + 486.8 \quad (VI-29)$$

Because $\frac{db}{d\epsilon_8} = 0$ one obtains from (VI-13) and (VI-14)

$$\frac{d\lambda_7^9}{d\epsilon_8} = \frac{-\frac{dc}{d\epsilon_8}}{(b^2 - 4ac)^{1/2}} \quad (VI-30)$$

and

$$\frac{d\lambda_8^9}{d\epsilon_8} = \frac{\frac{dc}{d\epsilon_8}}{(b^2 - 4ac)^{1/2}} \quad (VI-31)$$

or

$$\frac{d\lambda_7^9}{d\epsilon_8} = -\frac{d\lambda_8^9}{d\epsilon_8} \quad (\text{VI-32})$$

Therefore

$$\frac{dQ}{d\epsilon_8} = 2[(\lambda_7^9 - 1655)\frac{d\lambda_7^9}{d\epsilon_8} - (\lambda_8^9 - 685)\frac{d\lambda_8^9}{d\epsilon_8}] = 0 \quad (\text{VI-33})$$

Dropping the factor of 2 and inserting λ_K^9 in terms of b and c

$$\begin{aligned} \frac{dQ}{d\epsilon_8} &= \frac{d\lambda_7^9}{d\epsilon_8} \left\{ \left[-\frac{b}{2} + \frac{\sqrt{b^2 - 4c}}{2} - 1655 \right] - \left[-\frac{b}{2} - \frac{\sqrt{b^2 - 4c}}{2} - 685 \right] \right\} \\ &= \frac{d\lambda_7^9}{d\epsilon_8} \{ \sqrt{b^2 - 4c} - 972 \} = 0 \end{aligned} \quad (\text{VI-34})$$

$\frac{dQ}{d\epsilon_8} = 0$ when $\epsilon_8 = 501.4$, and $\sqrt{b^2 - 4c} = 972$ does not yield a real solution in a^2 .

Below are listed values for all parameters and for Q at three values of ϵ_8 . It is noted that $\epsilon_8 = 501.4$ does appear to produce a minimum in Q.

ϵ_8	A_7	a^2	λ_8	λ_7^9	λ_8^9	Q	c
501.4	10.82	3734	10.87	1673.3	664.6	680.8	1112227
500.	10.92	3408	10.78	1673.6	664.6	691.9	1112222
502	10.73	3881	10.90	1673.2	664.4	684.6	1112226

It is interesting to determine how well α is reproduced by a theoretical calculation $\alpha_{\text{theor}} = (A_8 + A_7)^{-1} C_{J2}^{9/2} C_{J1}^{7/2} [(J+1)(J+2+1)]^{-1/2} [U_3 U_7 + V_3 V_7]$. The C_{J2}^{λ} and U_i and V_i values can be found in Tables (IV.9) and (IV.10).

$$\alpha = (10.82 + 10.87) \{ (-.09121) (-.1376) \left[\left(\frac{9}{2} - \frac{7}{2} \right) \left(\frac{9}{2} + \frac{7}{2} + 1 \right) \right]^{-1/2} + \right. \\ \left. (.9959) (.9736) \left[\left(\frac{11}{2} - \frac{7}{2} \right) \left(\frac{11}{2} + \frac{7}{2} + 1 \right) \right]^{-1/2} \right\} [U_7 U_8 + V_7 V_8] \quad (\text{VI-35})$$

$$= 52.54$$

$\alpha_{\text{exp}} = \sqrt{5754} = 61.11$. The agreement is very good.

Inserting the fixed parameters of $\epsilon_8 = 501.4$ into the Hamiltonian matrices and using the method described in Section (IV.2) it is possible to calculate the mixing coefficients. Using an obvious notation the mixed wavefunctions $\phi_K^I = a^I \psi_{K1}^I + b^I \psi_{KJ}^I$ are:

$$\begin{aligned} \phi_7^7 &= 1.00 \psi_7^7 & \phi_8^8 &= .127 \psi_7^8 + .991 \psi_8^8 \\ \phi_7^8 &= .991 \psi_7^8 - .127 \psi_8^8 & \phi_8^9 &= .179 \psi_7^9 + .984 \psi_8^9 \\ \phi_7^9 &= .984 \psi_7^9 - .179 \psi_8^9 \end{aligned}$$

The improvement in cross-section calculations using the mixed wavefunctions is shown in Table (VI.5), where the calculation is done for $(\frac{d\sigma}{dE})$ at 40° . The $K = 8$ band has calculated cross-sections significantly increased by Coriolis coupling, and much better agreement between experiment and theory has been obtained. Likewise very good improvement is noted in the $K = 7$ band.

In dealing with the $\frac{3}{2}^+[411]$, $\frac{5}{2}^+[402]$, and $\frac{7}{2}^+[404]$ proton orbitals in the $K = 2, 1$, and 0 bands of the first of the 3 by 3 matrices listed above no complete Coriolis interaction analysis can be performed because the $K^\pi = 2^-$ band was not identified. It is worthy of note however, that since the H_{RPC} off-diagonal element $\langle \frac{3}{2}^+[411] | H_{RPC} | \frac{5}{2}^+[402] \rangle$ is negative, and the amplitude S_{J2} of the pure wavefunction $\frac{3}{2}^+[411] - \frac{7}{2}^-[514]$ and $\frac{5}{2}^+[402] - \frac{7}{2}^-[514]$ are the same sign for all values of J , any mixing of these two wavefunctions would tend to enhance the cross-sections of the states $I = 2, 3$, and 4 in the $K = 1$ band. Experimentally this enhancement is observed as can be seen by noting the appropriate experimental and calculated cross-sections in Table (VI.1).

A small computer code was written to determine the effect of Coriolis coupling between the $\frac{7}{2}^+[404] - \frac{7}{2}^-[514]$ $K = 0$ and $\frac{5}{2}^+[402] - \frac{7}{2}^-[514]$ $K = 1$ bands. In this program both unperturbed band-head energies, ϵ_1 and ϵ_0 , were slowly varied over pertinent ranges, and the Coriolis mixing element, odd-even shift, and eigenvalues of the $I = 2, 3, 4$, and 5 states in both bands were calculated at each combination of ϵ_1 and ϵ_0 . This calculation was done in the same manner as the positive parity band above except that the additional odd-even shift parameter made exact calculation of all parameters from a single choice of ϵ_0 impossible. It was therefore necessary to vary ϵ_1 and ϵ_0 as free parameters. It was found that the best agreement between the calculated and experimental

Table (VI.3). Results of Coriolis band mixing for the $\frac{7}{2}^-$ [514] and $\frac{7}{2}^-$ [523] orbitals coupled to the $\frac{7}{2}^-$ [514] neutron. Unperturbed energies and cross-sections are listed with superscript u, and the mixed energies and cross-sections are listed with superscript m.

I	K	Proton Orbital	Energy (keV)			$\left(\frac{d\sigma}{d\Omega}\right)^{u,m}$ (mb/sr)		
			E^u	E^m	E^{exp}	I^u	I^m	I^{exp}
7	7	$\frac{7}{2}^-$ [523]	1273	1273	1273	.04099	.04099	.0414
8	7		1446	1462	1462	.03232	.04077	.0556
9	7		1644	1673	1655	.06061	.05523	.0253
8	8	$\frac{9}{2}^-$ [514]	501.4	485.8	486	.0054	.0112	.0085
9	8		694.1	664.6	683	.0084	.0168	.0132

eigenvalues occurred when the Coriolis coupling element between the two bands was zero. The $K = 3$ band formed from the $\frac{5}{2}^- [404] \frac{5}{2}^- [514]$ configuration is therefore a relatively pure, unmixed band.

Each of the $K = 5$, 6 , and 7 bands resulting from the coupling of the $\frac{5}{2}^- [411]$, $\frac{5}{2}^- [402]$, and $\frac{5}{2}^- [404]$ protons to the $\frac{7}{2}^- [514]$ neutron were identified in this work. It is believed a unique treatment of the 5 by 5 Hamiltonian matrix has been introduced by the following discussion. The method is similar to the 2 by 2 solution presented earlier, but does not in this case lead to results with a real physical significance. It does appear to make a worthwhile contribution to the parameterization method of Lurier²¹ and will be useful when dealing with a 4 by 4 matrix encountered later in this work.

The experimental energies of the pertinent bands appear in Figure (VI.6). The $I = 6$ Hamiltonian matrix can be written

$$\begin{pmatrix} \epsilon_5 + 12A_5 - \lambda & -\sqrt{12}a \\ -\sqrt{12}a & \epsilon_6 - \lambda \end{pmatrix} = \begin{pmatrix} a_5 - \lambda & -\sqrt{12}a \\ -\sqrt{12}a & a_6 - \lambda \end{pmatrix} = 0 \quad \begin{matrix} \lambda_5^6 = 1531.15 \\ \lambda_6^6 = 565.89 \end{matrix}$$

The $I = K = 5$ state is a pure state with respect to Coriolis mixing so one may write

$$a_5 = 1390.67 + 12A_5 \quad (\text{VI-36})$$

$$a_5 + a_6 - \lambda_5^6 - \lambda_6^6 = 0 \quad (\text{VI-37})$$

Figure (VI.6). Experimental eigenvalues $\epsilon_{\frac{1}{2}}^I$ for the $K^\pi=5^-$, 6^- , and 7^- bands of the $\frac{3}{2}^+$ [411], $\frac{5}{2}^+$ [402], and $\frac{7}{2}^+$ [404] proton orbitals coupled to the $\frac{7}{2}^-$ [514] neutron used for 3-by-3 Coriolis coupled calculations in ^{176}Lu .

$$7^- \text{-----} 490.56$$

$$6^- \text{-----} 551.5$$

$$5^- \text{-----} 390.67$$

$$\frac{1}{2} [4.1] + \frac{1}{2} [5.4]$$

$$X^- = 5^-$$

$$7^- \text{-----} 758.06$$

$$6^- \text{-----} 664.89$$

$$\frac{5}{2} [4.02] + \frac{1}{2} [5.14]$$

$$X^- = 6^-$$

$$8^- \text{-----} 935$$

$$7^- \text{-----} 0$$

$$\frac{7}{2} [4.02] + \frac{1}{2} [5.4]$$

$$X^- = 7^-$$

from summing the diagonal, and

$$(a_5 - \lambda)(a_6 - \lambda) - 12\alpha^2 = 0 \quad (\text{VI-38})$$

$$\alpha^2 = \frac{\lambda^2 - \lambda(a_5 + a_6) + a_5 a_6}{16}$$

Therefore a single choice of λ_5 uniquely determines a_5 , a_6 , and α^2 .

Progressing to the $I = 7$ matrix one has

$$\begin{pmatrix} \epsilon_5 + 26\lambda_5 - \lambda & -\sqrt{26}\alpha & 0 \\ -\sqrt{26}\alpha & \epsilon_6 + 14\lambda_6 - \lambda & -\sqrt{14}\beta \\ 0 & -\sqrt{14}\beta & \epsilon_7 - \lambda \end{pmatrix} = \begin{pmatrix} b_5 - \lambda & -\sqrt{26}\alpha & 0 \\ -\sqrt{26}\alpha & b_6 - \lambda & -\sqrt{14}\beta \\ 0 & -\sqrt{14}\beta & b_7 - \lambda \end{pmatrix} = 0$$

where $\beta = \frac{1}{2}(\lambda_6 + \lambda_7) \left[\frac{7}{2} + [404]J_+ \right] \frac{5}{2} + [402] > [U_6 U_7 + V_6 V_7]$, with the coefficient $[(I-K)(I+K+1)]^{\frac{1}{2}} = [(7-6)(7+6+1)]^{\frac{1}{2}} = \sqrt{14}$. Summing the diagonals one obtains

$$b_5 + b_6 + b_7 = \lambda_5^7 + \lambda_6^7 + \lambda_7^7 = 2449. \quad (\text{VI-39})$$

but $b_5 = 1390.67 + 26\lambda_5$ and

$$b_6 = 2449 - b_5 - b_7 \quad (\text{VI-40})$$

b_5 has already been uniquely determined by a choice of λ_5 .

Evaluating the determinant of $I = 7$ yields

$$(b_5 - \lambda)(b_6 - \lambda)(b_7 - \lambda) - [(b_5 - \lambda)(14\beta^2) + (b_7 - \lambda)(26\alpha^2)] = 0 \quad (\text{VI-41})$$

which expands to

$$\lambda^5 - \lambda^2 [b_5 + b_6 + b_7] + \lambda [b_5 b_6 + b_6 b_7 + b_5 b_7 - 14s^2 - 26a^2] + b_5 b_6 b_7 + 14s^2 b_5 + 26a^2 b_7 = 0 \quad (VI-42)$$

Substituting (VI-40) for b_6 and expanding gives

$$\lambda^5 - 2449\lambda^2 + \lambda [-b_7^2 + b_7(2449 - b_5) + 2449b_5 - b_5^2 - 14s^2 - 26a^2] + b_7(b_5^2 - 2449b_5 + 26a^2) + b_5 b_7^2 + 14s^2 b_5 = 0 \quad (VI-43)$$

This equation can be rearranged to a quadratic in b_7

$$b_7^2 (b_5 - \lambda) + b_7 (b_5^2 - 2449b_5 + 26a^2 + 2449\lambda - b_5\lambda) + \lambda^5 - 2449\lambda^2 + 2449b_5\lambda - b_5^2\lambda - 26a^2\lambda - 14s^2\lambda + 14s^2 b_5 = 0 \quad (VI-44)$$

Thus the coefficient of

$$\begin{aligned} b_7^2 & \text{ is } b_5 - \lambda \equiv X_1 \\ b_7^1 & \text{ is } b_5^2 - 2449b_5 + 26a^2 + 2449\lambda - b_5\lambda \equiv X_2 \\ b_7^0 & \text{ is } \lambda^5 - 2449\lambda^2 + 2449b_5\lambda - b_5^2\lambda - 26a^2\lambda \equiv X_3 \\ s^2 & \text{ is } 14b_5 - 14\lambda \equiv X_4 \end{aligned} \quad (VI-45)$$

Dividing (VI-44) by $(b_5 - \lambda)$ to make the coefficient of b_7^2 1 results in

$$\lambda_1 = 1, \lambda_2 = \frac{b_5^2 - 2449b_5 - 26a^2 + 2449\lambda - b_5\lambda}{b_5 - \lambda},$$

$$\lambda_3 = \frac{\lambda^3 - 2449\lambda^2 + 2449\lambda b_5 - b_5^2\lambda - 26a^2}{b_5 - \lambda}$$

$$\lambda_4 = 14.$$

Now for any two eigenvalues of λ_K^7 one obtains two characteristic equations of the form (VI-44) whose b_5^2 coefficient and λ^2 coefficient are 1 and 14 respectively. Subtracting one characteristic equation, i , from the other, j , leaves an equation of the form

$$b_7(\lambda_2^{(i)} - \lambda_2^{(j)}) + (\lambda_3^{(i)} - \lambda_3^{(j)}) = 0 \quad (\text{VI-46})$$

One can easily obtain an equation (VI-46) for any two choices of λ_K^7 from the three experimental values available. It is then a simple task to determine from a single choice of λ_5 all of the parameters a_5 , a_6 , ϵ_6 , ϵ_7 , b_5 , b_6 , b_7 , A_6 , σ^2 , and β^2 . These can be readily substituted into the $I = 8$ matrix to determine A_7 and calculate eigenvalues λ_K^8 . Unfortunately in this case only one λ_K^8 eigenvalue is known so a useful comparison of λ_K^8 calculated to λ_K^8 experimental can not be made.

If explicit calculations of α and β are made from the theoretical J_+ elements, it is possible to determine approximate values of ϵ_5 , ϵ_6 , ϵ_7 , A_5 , A_6 , and A_7 . With these the mixing coefficients can be calculated. The results of such a calculation appear in

Table (VI.4). The parameters determined were $\epsilon_5 = 1390.67$, $\epsilon_6 = 568.10$, $\epsilon_7 = 1.1$, $A_5 = 11.26$, $A_6 = 14.15$, and $A_7 = 11.66$, and the wavefunctions are

$$\begin{aligned}\psi_5^5 &= 1.00\psi_5^5 & \psi_6^6 &= +.0632\psi_5^6 + .998\psi_6^6 \\ \psi_5^6 &= .998\psi_5^6 - .0632\psi_6^6 & \psi_6^7 &= .14\psi_5^7 + .99\psi_6^7 - .017\psi_7^7 \\ \psi_5^7 &= .99\psi_5^7 - .14\psi_6^7 + .017\psi_7^7 & \psi_6^8 &= .0022\psi_5^8 + .039\psi_6^8 + 1.00\psi_7^8 \\ & & \psi_9^8 &= .0030\psi_5^8 + .050\psi_6^8 + 1.0\psi_7^8\end{aligned}$$

Considerable improvements in cross-section and energy calculation are noted in Table (VI.4).

The coupling of the $K = 3$ band to the $K = 4$ band of the $\frac{1}{2}^+ [411] \pm \frac{7}{2}^- [514]$ configuration was done just like the $\frac{7}{2}^- [523]$, $\frac{9}{2}^- [514]$ mixing 2 by 2 matrix calculations. The results are not presented as though the experimental energies were reproduced with remarkable accuracy, the calculated cross-sections in the $K = 4$ band were not altogether consisted with experiment. It appears that some experimental intensity in this level has been distributed in another band which was not identified. The parameters ϵ_3 , ϵ_4 , A_3 , and A_4 were found to be 838.36, 750.71, 12.83, and 14.36 respectively, while the decoupling factor was determined to be $\langle \frac{1}{2}^+ [411] | J_z | \frac{1}{2}^+ [411] \rangle = -.818$. Theoretically the decoupling factor has been calculated to be $a = -.794$. The final wavefunctions are

Table (VI.4). Results of Coriclis band mixing for the $\frac{5}{2}^+$ [411], $\frac{5}{2}^+$ [402], and $\frac{7}{2}^+$ [404] orbitals coupled to the $\frac{7}{2}^-$ [514] neutron. The superscript notation is explained in Table (VI.3).

I	K	Proton Orbital	Energy(keV)			I ^o	$\frac{d\sigma}{d\Omega}^{100\text{mb}} \frac{\text{mb}}{\text{sr}}$	
			E ^o	E ⁺	E ^{exp}		I ⁺	I ^{exp}
5	5	$\frac{5}{2}^+$ [411]	1391	1391	1391	.0930	.0930	.0625
6	5		1527	1534	1551	.0653	.0605	.0365
7	5		1681	1688	1691	.0033	.0035	.0109
6	6	$\frac{5}{2}^+$ [402]	568.1	563.3	563.9	.0287	.0343	.0324
7	6		766.7	757.8	758.1	.0008	.0013	.0067
7	7	$\frac{7}{2}^+$ [404]	1.1	0	0	.0524	.0414	.0306
8	7		187.7	185.6	185.4	.0005	.0004	.0004

$$\begin{aligned}
\phi_3^3 &= 1.0\psi_3^3 & \phi_4^4 &= -.14\psi_3^4 + .99\psi_4^4 \\
\phi_3^4 &= .99\psi_3^4 + .14\psi_4^4 & \phi_4^5 &= -.24\psi_3^5 + .97\psi_4^5 \\
\phi_3^5 &= .97\psi_3^5 + .24\psi_4^5 & \phi_4^6 &= -.28\psi_3^6 + .96\psi_4^6
\end{aligned}$$

(VI.5) ^{166}Ho Structure

^{166}Ho contains 67 protons and 99 neutrons, and accordingly the ground state configuration is $\frac{7}{2}^+ [523] \pm \frac{7}{2}^+ [633]$. The Gallagher-Moszkowski coupling rule¹ predicts that the I^π , $K = 7^-, 7$ state of that configuration should be the ground state, but the I^π , $K = 0^-, 0$ has been observed to be.⁴¹ This is the only known exception to the Gallagher-Moszkowski rule. The explanation can probably be presented in terms of the $\frac{\hbar^2}{2I}K$, contribution made by the zero-point rotational energy⁴² in (II-7S) of the $K = 7$ band, together with the Newby shift pushing the 0^- state down.

Studies of the (d,p) reaction⁴³ and (n, γ) and (n, e^-)³ of levels in ^{166}Ho have produced the assignments of 32 states in excited neutron bands of the $\frac{7}{2}^+ [523]$ proton coupled to the $\frac{7}{2}^+ [633]$, $\frac{1}{2}^+ [521]$, $\frac{5}{2}^+ [512]$, and $\frac{5}{2}^+ [523]$ neutron orbitals. Also 5 states in the $\frac{5}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ excited proton configuration have been assigned. This configuration appears to be mixed with the $\frac{7}{2}^+ [523] \pm \frac{3}{2}^+ [521]$ configuration. It will be interesting to observe experimental (t, α) spectroscopic factors populating these states. If the proposed admixture is correct, it will be useful to determine if the H_{INT} perturbation can explain it. A summary of several

Figure (VI.7). Previously known levels in ^{166}Ho along with γ -transitions depopulating them. All energies are in keV. The configuration designations refer only to bold face levels lying directly above the designations. The $I^\pi, K=6^+, 2$; $6^+, 5$; and $7^+, 5$ levels at 730.26, 374.9, and 514.4 keV were assigned in this study based on (t, α) reaction data.

Figure (VI.8). Alpha spectrum for the reaction
 $^{167}\text{Er}(t, \alpha)^{166}\text{Ho}$ observed at the laboratory angle of 35° .

Table (VI.5). Nuclear levels in ^{10}Be populated by the (t, α) reaction.

Peak #	Previous Value (keV)	Energy (t, α) (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}^{35^\circ}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{cal}}^{35^\circ}$ (mb/sr)	Assignment		Configuration
						J ^π	K	
1	0	0		$1.001^{+0.001}_{-0.001}$	.001	0 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
2	5	5	5	$1.024^{+0.019}_{-0.019}$	.019	7 ⁺	7	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
3	54.239	57.3	7.3	$1.0016^{+0.0056}_{-0.0056}$	.0056	2 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
4	87.470	86.5	2.5	..	.0018		0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
5	136	156.0	.7	.037	.040		7	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
6	171.074	173.1	1.4	.023	.020	3 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
7	180.467	187.3	2.1	.014	.024	4 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
8	263.787	259.3	1.2	.058	.113	5 ⁺	5	$3/2^+ \{411\} \cdot 7/2^+ \{633\}$
9	308	283.3	1.3	.027	.029	9 ⁺	7	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
10	329.775	323.4	3.9	.021	.022	5 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
11		375.3	1.4	$1.038^{+0.084}_{-0.084}$	.084	0 ⁺	5	$3/2^+ \{411\} \cdot 7/2^+ \{633\}$
12	377.806		1.4	$1.006^{+0.013}_{-0.013}$	.013	6 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
13	430.030	425	2	.017	.054	2 ⁺	2	$3/2^+ \{411\} \cdot 7/2^+ \{633\}$
14	481.845	478.4	2.7	.038	.075	3 ⁺	2	$3/2^+ \{411\} \cdot 7/2^+ \{633\}$
15		513.1	1.7	.0051	.0061	7 ⁺	5	$3/2^+ \{411\} \cdot 7/2^+ \{633\}$
16	547.956	547.0	3.3	$1.033^{+0.051}_{-0.051}$	.051	1 ⁺	2	$3/2^+ \{331\} \cdot 7/2^+ \{633\}$
17	557.688	561.6	4.0	$1.003^{+0.001}_{-0.001}$	.001	7 ⁺	0	$7/2^- \{523\} \cdot 7/2^+ \{633\}$
18		590.	1.6	.013				
19	634.310	635.7	2.9	$1.008^{+0.020}_{-0.020}$	.020	5 ⁺	2	$3/2^+ \{331\} \cdot 7/2^+ \{633\}$

Table (VI.5). continued

Peak #	Previous Value (keV)	Energy (1,2) (keV)	Error (keV)	$\left(\frac{d}{dt}\right)_{exp}^{exp}$ (mb/sec)	$\left(\frac{d}{dt}\right)_{cal}^{cal}$ (mb/sec)	$\Delta = (1,2)_{exp} - (1,2)_{cal}$	Configuration	
20		718.2	2.9	.0029	.0029	0 ⁺	3	$1/2^+ [411] + 1/2^+ [633]$
21		731.8	2.5	.0069	.0042	0 ⁺	2	$3/2^+ [433] + 1/2^+ [657]$
22		801.0	3.0	.002	.0046	0 ⁺	0	$7/2^+ [103] + 1/2^+ [633]$
23		818	3	.021	.021	0 ⁺	3	$1/2^+ [111] + 1/2^+ [633]$
24		856	6	.0011	.0038	2 ⁺	0	$1/2^+ [404] + 1/2^+ [633]$
25		884	2	.013	.036	1 ⁺	4	$1/2^+ [111] + 7/2^+ [633]$
26		915	3	.015	.015	2 ⁺	2	$7/2^+ [104] + 7/2^+ [633]$
27		946	4	.0042	.0083	5 ⁺	3	$1/2^+ [111] + 7/2^+ [633]$
28		971	9	.0021	.0038	1 ⁺	0	$1/2^+ [303] + 7/2^+ [633]$
29		1006	4	.011	.020	5 ⁺	4	$1/2^+ [111] + 7/2^+ [633]$
30		1057	2	.0081				
31		1066	5	.0019	.0024	3 ⁺	0	$7/2^+ [404] + 7/2^+ [633]$
32		1091	4	.0022	.0050	0 ⁺	3	$1/2^+ [411] + 7/2^+ [633]$
33		1126	8	.0031				
34		1146	5	.013 ⁺	.0217	0 ⁺	4	$1/2^+ [411] + 7/2^+ [633]$
35		1187	6	.011	.0048	1 ⁺	4	$5/2^+ [413] + 1/2^+ [633]$
36		1205	18	.019	.019	2 ⁺	3	$5/2^+ [315] + 7/2^+ [633]$
37		1238	3	.019	.022	3 ⁺	4	$5/2^+ [113] + 7/2^+ [633]$
38		1272	2	.032	.011	0 ⁺	6	$5/2^+ [113] + 7/2^+ [633]$

Table (VI.5). continued

Peak #	Previous Value (keV)	Energy (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)^{exp}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)^{calc}$ (mb/sr)	Assignment J ^π K		Configuration
39		1505	4	.011	.016	4 ⁺	1	5/2 ⁺ [413] + 7/2 ⁺ [633]
40		1556	5	.0077				
41		1579	14	.014	.0086	5 ⁺	1	5/2 ⁺ [413] + 7/2 ⁺ [633]
42		1417	3	.021	.031	7 ⁺	6	5/2 ⁺ [413] + 7/2 ⁺ [633]
43		1460	17	.0056				
44		1487	11	.015				
45		1519	11	.0058				
46		1560	5	.012	.024	6 ⁺	6	5/2 ⁺ [532] + 7/2 ⁺ [633]
47		1585	8	.0066				
48		1604	2	.0038				
49		1628	10					
50		1645	3	.026				
51		1692	4	.021	.054	7 ⁺	6	5/2 ⁺ [532] + 7/2 ⁺ [633]
52		1745	8	.019				
53		1790	6	.025				
54		1834		.076	.057	8 ⁺	6	5/2 ⁺ [532] + 7/2 ⁺ [633]

^aNot resolved

previously assigned states along with γ -ray transitions appears in Figure (VI.7).

Fifty-four peaks have been observed in the (t,α) spectrum of ^{166}Ho in this work. A representative spectrum is shown in Figure (VI.8). Table (VI.5) contains a list of the identified peaks along with their cross-section at 35° and assignments whenever they have been made. The predicted cross-sections excluding Coriolis coupling appear in Column 6.

$^{166}\text{Ho} \left[\frac{7}{2}^+ [523] \right] + \left[\frac{7}{2}^+ [633] \right]$ Configuration

The coupling of the $\frac{7}{2}^+ [523]$ proton to the $\frac{7}{2}^+ [633]$ neutron forms the ground state configuration in ^{166}Ho . The resulting $K^\pi = 7^-$ and 0^- bands were previously known from (n,γ) and (n, e^-) studies⁵ and from the proton spectroscopy from the $^{165}\text{Ho}(d,p)^{166}\text{Ho}$ reaction.⁴³ These two bands with spin assignments up to $I^\pi = 8^-$ and 7^- respectively are shown in Figure (VI.7). All members up to the 7^- state of the $K^\pi = 0^-$ band were populated in the (t,α) reaction with relative intensities well reproduced by the spectroscopic calculations, as can be seen in Table (VI.5). The $I^\pi, K^\pi = 6^-, 0$ state at 377.806 keV is not resolved from the very intense peak at 375 keV, which is clearly too large to be composed only of the $6^-, 0$ state.

The cross-sections populating the $K^\pi = 7^-$ band members are much larger than those of the 0^- band members. This is consistent with the spectroscopic calculations. The $I = 7, 8$, and 9 members

have been previously assigned at energies of 5 kev, 136 kev, and 308 kev.^{8,45} If the moment of inertia of this band is calculated from the 7 and 8 spin members the $I = 9$ state is then predicted to lie at 283 kev. Relatively intense peaks in the α -spectrum occur at excitation energies of 5, 136, and 283 kev. Each of these levels is populated by alphas with a very good $l = 5$ angular distribution as shown in Figure (VI.9). No peak occurs at 308 kev. The level at 283 kev is populated with an intensity of approximately .75 relative to the intensity of the level at 136 kev. Spectroscopic calculations predict the relative intensities of the $I^\pi = 9^-$ and 8^- states to be .75 and 1.0 respectively. The peak at 283 kev has been assigned to be the I^π , $K = 9^-$, 7 state in the $\frac{7}{2}^+ [525] \pm \frac{7}{2}^+ [633]$ configuration. This yields a moment of inertia of $\frac{h^2}{2I} = 8.18$ kev. The experimental normalization factor determined from the cross-sections populating states in this configuration is 12.9, consistent with theoretical predictions.²⁵

The $\frac{3}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ Configuration

The $K^\pi = 2^+$ and 5^+ bands formed from the $\frac{3}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration are shown in Figure (VI.7). The $I^\pi = 2^+$, 3^+ , 4^+ , and 5^+ members of the $K = 2$ band and the bandhead of the $K = 5$ band were previously known from (n, γ) and (n, e^-) studies⁸. The peaks in the (t, α) spectrum representing these states are all intense, excellent $l = 2$ transfers, which is consistent with the pickup of a $\frac{3}{2}^+ [411]$ proton. From the energies of states in the $K = 2$ band a

moment of inertia of 3.512 kev has been calculated. This allows the prediction of an $I = 6$ level at approximately 736 kev. This state should have a (t, α) intensity about $\frac{1}{12}$ that of the 2^+ level at 450.050 kev. The peak at 731 kev in the (t, α) spectrum has intensities at all angles consistent with this prediction. There are previously unassigned γ -rays of 102.55 and 188.98 kev which depopulate this level to the $I = 5$ and $I = 4$ levels respectively as shown in Figure (VI.7). The 1^+ , $K = 6^+$, 2 state in the

$\frac{5}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration has been assigned to 750.90 kev.

The previously unobserved level at 575 kev has a very good $L = 2$ angular distribution, as seen in Figure (VI.9), and has the correct intensity relative to the 1^+ , $K = 5^+$, 5 state at 263.787 to be the $I = 6$ state of the $K^\pi = 5^+$ band from the $\frac{5}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration. This yields a moment of inertia for the 5^+ band of 9.27 kev. There should then be a weakly populated $I = 7$ state at approximately 505 kev. A peak at 514 kev has been observed in the (t, α) spectrum with an intensity consistent with the spectroscopic calculations. The levels at 575 and 514 kev are thus assigned as the $1^+ = 6^+$ and 7^+ members of this $K^\pi = 5^+$ band.

Previous studies have observed that the $K^\pi = 2^+$ and 5^+ bands are probably a mixture of the $\frac{5}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ and $\frac{7}{2}^+ [523] \pm \frac{5}{2}^+ [531]$ configurations. As seen in Figure (VI.7) the depopulation of these two bands is predominantly to the

Figure (VI.9). Angular distributions of the $^{167}\text{Er}(t,n)$ ^{166}Ho reaction with 17.0 MeV tritons. The solid curves are the results of the DWBA calculations, and the circles are experimental data.

$\frac{7}{2}[-523] \pm \frac{1}{2}[-521]$ configuration. However only the $\frac{3}{2}[-411] \pm \frac{7}{2}[633]$ component of the aforementioned admixture can be populated in a (t, α) reaction, since the $\frac{7}{2}[-523] \pm \frac{3}{2}[-521]$ component is an excited neutron configuration. The absolute (t, α) intensities of the pertinent states indicate that the normalization factor for the $\frac{3}{2}[-411] \pm \frac{7}{2}[633]$ configuration is 6.59, about half of the value determined for the ground state configuration. The (t, α) intensities can therefore best be explained by an admixture of $.71(\frac{3}{2}[-411] \pm \frac{7}{2}[633]) + \sum_1 c_1 v_1$ where $\sum_1 c_1^2 + (.71)^2 = 1$, and the v_1 are any number of other allowed admixtures of excited neutron configuration or vibrational bands.

The $\frac{1}{2}[-411] \pm \frac{7}{2}[633]$ Configuration

The $\frac{1}{2}[-411]$ is the first excited proton particle state orbital encountered in ^{166}Ho , and accordingly should be weakly populated in a (t, α) reaction. This orbital should have almost pure $l = 2$ character. Good $l = 2$ transfers are observed in the (t, α) spectrum at 718 kev and 834 kev as seen in Figure (VI.9). The intensity of the 718 kev peak is consistent with its assignment as the $1^{\pi}, K = 3^+, 3$ member of the previously unobserved $\frac{1}{2}[-411] \pm \frac{7}{2}[633]$ configuration. The predicted cross-section is .029 mb/sr, and the experimental value is also .029 mb/sr.

H_{INT} eigenvalue calculations predict that the $1^{\pi}, K = 4^+, 4$ of the $\frac{1}{2}[-411] \pm \frac{7}{2}[633]$ configuration should lie at approximately 875 kev. That level should have a (t, α) cross-section approximately

equal to that of the 5^+ , 3 state. Based upon the predicted singlet-triplet splitting, nineteen previously unassigned γ -ray transitions shown in Figure (VI.10), the relative (t,a) intensities within each band, and the observed moments of inertia, the partial band structure of the $\frac{5}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration in ^{166}Ho has been constructed in Figure (VI.10). The observed moments of inertia of the $K = 3$ and $K = 4$ bands are 12.8 and 11.2 kev respectively. After removal of Coriolis coupling effects the experimental singlet-triplet splitting is determined to be -159 kev, in good agreement with the calculated value of -151 kev.

It is noted that the intensities populating the $K = 3$ band are considerably larger than those populating the $K = 4$ band. The difference is not predicted by theory and can be only partially explained by Coriolis coupling effects. This will be further discussed in Section (VI.4).

The $\frac{5}{2}^+ [413] \pm \frac{7}{2}^+ [633]$ Configuration

The $\frac{5}{2}^+ [413]$ proton orbital is an excited hole state in ^{166}Ho and should be strongly populated in the (t,a) reaction on ^{167}Er . Coupling this proton orbital to the $\frac{7}{2}^+ [633]$ neutron yields bands of $K^\pi = 6^+$ and 1^+ , with the 1^+ band lying lower in accordance with the Gallagher-Moszkowski coupling rule¹. The $\frac{5}{2}^+ [413]$ bandhead in ^{165}Ho lies at 995 kev⁴⁴, indicating the $\frac{5}{2}^+ [413] \pm \frac{7}{2}^+ [633]$ configuration in ^{166}Ho should occur near 1 MeV in excitation. Well resolved, moderately intense peaks are observed in the (t,a) spectrum at

Figure (VI.10). Spin and parity assignments and γ -transitions depopulating levels in the $\frac{1}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration in ^{166}Ho . All levels other than those of the $\frac{1}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration were previously known. Transitions depopulating these levels are shown to establish the absolute energy of all levels shown in the figure. All energies are in keV. Configuration designations appearing below bands refer only to bold face levels above them.

1137, 1238, 1305, 1272, and 1417 kev. The angular distributions of these peaks are all similar to one another and are reasonably good $L = 4$ transfers as seen in Figure (VI.9). This is consistent with pickup of a $\frac{5}{2}^+ [413]$ proton.

Spectroscopic factor calculations indicate the relative (t, α) intensities of the I^π , $K = 7^+$, 6; 6^+ , 6; 4^+ , 1; and 3^+ , 1 states of the $\frac{5}{2}^+ [413] \pm \frac{7}{2}^+ [633]$ configuration should be .83, 1.0, .39, and .66 respectively at 35° . The experimental relative intensities of the 1417, 1272, 1305, and 1238 kev peaks at 35° are .75, 1.0, .34, and .59. The 1272 and 1417 kev peaks in the (t, α) spectrum have been assigned as the $I^\pi = 6^+$ and 7^+ states of the $K = 6$ band of the $\frac{5}{2}^+ [413] \pm \frac{7}{2}^+ [633]$ configuration. This yields a moment of inertia of 10.4 kev. The $I^\pi = 8^+$ state of the band is expected to be only very weakly populated and has not been observed.

Assigning the 1305 and 1238 kev peaks as the 4^+ and 3^+ spin members of the $K = 1$ band of the $\frac{5}{2}^+ [413] \pm \frac{7}{2}^+ [633]$ configuration yields a moment of inertia of 8.5 kev. From this and the calculated spectroscopic factors it is possible to predict the energies and cross-sections of the remaining band members. The spin members up to the $I = 5$ state of this band have been assigned at 1150, 1134, 1235, 1303, and 1387 kev as shown in Figure (VI.11). The resulting singlet-triplet splitting is -70. kev; the value calculated is -200 kev.

It is noted in Table (VI.5) that the relative intensities of the $K^\pi = 1^+$ band are somewhat distorted in comparison to the calculated values; however the general trends are still evident. This is illustrated in Figure (VI.12), where theoretical and experimental cross-sections are compared on a bar graph. The distortion can be partly attributed to Coriolis coupling of the $\frac{5}{2}^+[413]$ orbital to the $\frac{7}{2}^+[404]$ orbital. The matrix element of the J_+ operator between these two orbitals $\langle \frac{7}{2}^+[404] | J_+ | \frac{5}{2}^+[413] \rangle$ has been calculated to be 2.622. Therefore appreciable Coriolis coupling can be expected. This will be discussed in Section (VI.4).

The absolute intensities of both the 0^+ and 1^+ bands of the $\frac{5}{2}^+[413] \pm \frac{7}{2}^+[633]$ configuration are considerably lower than that predicted by the pure two-quasiparticle calculations. Wagner, et. al.⁴⁴ have observed that the $\frac{5}{2}^+[413]$ band in ^{165}Ho is not a pure single particle band. It appears to be mixed to a considerable extent with a $K = K_0 + 2$ γ -vibrational band built on the $\frac{1}{2}^+[411]$ orbital. This is consistent with the experimental cross-section of the $\frac{5}{2}^+[413] \pm \frac{7}{2}^+[633]$ configuration in ^{166}Ho , and helps to explain the difference between the observed and calculated singlet-triplet splitting energies. Assuming the $1^\pi, K = 6^+, 6$ level at 1272 to be unperturbed by Coriolis coupling with the 7^+ band formed by the $\frac{7}{2}^+[404] \pm \frac{7}{2}^+[633]$ configuration, its experimental cross-section is found to be approximately .73 times its predicted pure two-quasiparticle cross-section. Applying that factor to all states of

Figure (VI.11). Spin and parity assignments for the $\frac{7}{2}^{+}$ [404], $\frac{5}{2}^{+}$ [413], and $\frac{5}{2}^{-}$ [532] proton orbitals coupled to the $\frac{7}{2}^{+}$ [633] neutron orbital in ^{166}Ho . Vertical lines represent γ -ray transitions assigned in this study. All energies are in keV. Configuration designations refer to those levels lying directly above the designation in the figure. The four lowest lying levels in this figure were known previous to this study.

Figure (VI.12). Comparison of experimental and theoretical (t, i) cross-sections of states in the $\frac{5}{2}^+ [413] = \frac{7}{2}^+ [633]$ configuration in ^{166}Ho . The theoretical cross-sections were determined with a normalization factor of 9.4.

the $\frac{5}{2}+[413]\pm\frac{7}{2}+[633]$ configuration greatly improves the agreement between experiment and theory, and has been included in Figure (VI.12).

The two-quasiparticle energies of proton states in several odd-odd nuclides in the region are plotted in Figure (VI.13) relative to the $\frac{3}{2}+[411]$ proton state in ^{160}Tb . The energies of each of the orbitals except the $\frac{5}{2}+[413]$ behave consistently with the Nilsson diagrams shown in Figure (II.2). The $\frac{5}{2}+[413]$ orbital has an anomalous dip at ^{166}Ho . This can be explained by the $\frac{5}{2}+[413]$ being pushed down to a deeper hole state as the mixing with the $\{\frac{1}{2}+[411]\pm\frac{7}{2}+[633]; 2+\}$ vibrational band becomes important.

The $\frac{5}{2}-[532]\pm\frac{7}{2}+[633]$ Configuration

The $\frac{7}{2}, \frac{5}{2}-[532]$ single-particle state is observed in ^{165}Ho at 1434 keV⁴⁴. The $\frac{5}{2}-[532]\pm\frac{7}{2}+[633]$ configuration should thus be expected near 1500 keV in ^{166}Ho . Observing two-quasiparticle states in odd-odd nuclei at energies of 1500 keV is difficult because of the extremely high level density. There are, however, a number of strong apparently single peaks in the (t, α) spectrum of ^{166}Ho in this energy region. Each of the first four states of the $K^\pi = 6^-$ band of the $\frac{5}{2}-[532]\pm\frac{7}{2}+[633]$ configuration should be strongly populated in a pickup reaction. The peaks at 1560, 1692, and 1834 keV have energy spacings indicative of $I = 6, 7$, and 8 spin members of a rotational band with $\frac{\hbar^2}{2I} = 9.13$ keV and are among the most intense peaks in the energy region. Assuming them to be the 6, 7, and 8

Figure (VI.13). Energies of Nilsson proton orbitals in several odd-odd nuclides in the rare earth region relative to the $\frac{3}{2}^+ [411] - \frac{3}{2}^- [521]$ configuration in ^{160}Tb . These energies were obtained from the energy of the proton orbital coupled to the ground state neutron orbital in each nuclide.

members of the $K^\pi = 6^- \frac{5}{2}^- [532] + \frac{7}{2}^- [633]$ band their relative experimental cross-sections are well reproduced by spectroscopic calculations. It is then possible to predict the $I = 9$ spin state at an energy of approximately 1998 kev. The spectrum was not fitted to such a high energy. However the peak near channel number 600 in Figure (VI.8) appears at approximately the correct position and has a large intensity as predicted by the calculations. The levels at 1560, 1692, 1834, and ~ 1998 kev thus have been assigned as the $I^\pi = 6^-, 7^-, 8^-,$ and 9^- levels of the $K^\pi = 6^-$ band of the $\frac{5}{2}^- [532] + \frac{7}{2}^- [633]$ configuration as shown in Figure (VI.11). The $K^\pi = 1^-$ band expected from this configuration should lie higher in energy. Meaningful assignments have not been made for this band.

The $\frac{7}{2}^+ [404] + \frac{7}{2}^+ [633]$ Configuration

The coupling of the $\frac{7}{2}^+ [404]$ proton orbital to the $\frac{7}{2}^+ [633]$ neutron results in bands with $K^\pi = 0^+$ and 7^+ . Identification of the 0^+ band is very difficult as the odd numbers are split from the even members by the Newby splitting term of the neutron-proton interaction in the Hamiltonian. Further the $\frac{7}{2}^+ [404]$ proton is an excited particle state and should be only weakly populated in a (t,a) reaction. The experimental odd-even shift of the $\frac{7}{2}^+ [404] + \frac{7}{2}^+ [633]$ $K = 0$ band has been observed to be -56 kev in ^{172}Lu .¹³ The odd spin members are therefore expected to be pushed to higher energies relative to the even members.

States populated by the pickup of a $\frac{7}{2}^+ [404]$ proton will be pure $2 = 4$ transfers. There are good $2 = 4$ transfers at 856 kev and

915 kev as shown in Figure (VI.9). Spectroscopic factor calculations indicate the 915 kev peak is too large to be a member of the $K = 0$ band and that the $I = 2$ and $I = 1$ states should be the most strongly populated states in the $K = 0$ band. Figure (VI.14) shows a bar graph comparing the calculated cross-sections of the $I = 0, 1, 2$, and 3 states of this band to the experimental cross-sections of the 801, 874, 886, and 1066 kev peaks respectively in the (t, α) spectrum at 35° and 40° . Based on the experimental (t, α) intensities; the observed odd-even energy shift; the experimental rotational level spacings; and on the previously unassigned γ -transitions of 430.31(18), 235.80(5), 198.31(5), 57.19(1)^a, 112.869(12), 170.09(3), 310.89(3), 368.45(16), 125.81(2), 87.195(15)^b, 211.06(6), 98.200(15)^b, 163.352(7), and 250.49(9) kev the partial bandstructure as seen in Figure (VI.11) for the $K^\pi = 0^+ \frac{7}{2}^+ [404] - \frac{7}{2}^+ [635]$ band is proposed. This results in a moment of inertia of 9.5 kev and a Newby splitting of $B = -76$ kev when Coriolis effects have been removed.

Only the $I^\pi = 7^+$ member of the $K = 7$ band formed from the $\frac{7}{2}^+ [404] - \frac{7}{2}^+ [635]$ configuration should be appreciably populated in the (t, α) reaction. According to the coupling rule of Gallagher and Moszkowski the 7^+ level should occur higher in energy than the 0^+ state of the $K = 0$ band. The cross-section of this level should be $\sim .013$ mb/sr at 35° , and the transfer should be pure $\lambda = 4$. The peak at 914 kev has a good $\lambda = 4$ angular distribution and has a

^a has been previously tentatively assigned, but is a complex transition.

^b has been previously tentatively assigned.

Figure (VI.14). Comparison of experimental and theoretical (t, α) cross-sections of states in the $\frac{7}{2}^+ [404] = \frac{7}{2}^+ [633]$ configuration in ^{165}Ho . A normalization factor of 12.9 has been used in the theoretical calculations.

cross-section at 55° of .015 mb/sr. It is thus assigned as the $I^\pi = 7^+$ bandhead of the $\frac{7}{2}^+[404] + \frac{7}{2}^-[633]$ band. This assignment is consistent with the Gallagher-Moszkowski coupling rule, however if the zero-point rotational energy is subtracted from the I^π , $K = 7^+$, 7 level using 9.5 kev as the moment of inertia and the odd-even shift is removed from the $K = 0$ band, an experimental singlet-triplet splitting of 28 kev is calculated. Theoretical calculations have predicted that splitting to be -159 kev and the odd-even shift to be -92 kev. If we accept this $I^\pi = 7^+$ experimental assignment the singlet-triplet splitting has an anomalous sign. Central plus tensor force programs have been very successful in obtaining the correct sign for the singlet-triplet splitting in odd-odd nuclei and generally good at obtaining agreement with experimental magnitudes. It thus seems probable that there may be some error in the assignments of the states in the $\frac{7}{2}^+[404] + \frac{7}{2}^-[633]$ configuration in ^{166}Ho .

(VI.4) Coriolis Coupling in ^{166}Ho

The important proton orbitals in ^{166}Ho are the $\frac{7}{2}^-[523]$, $\frac{5}{2}^-[532]$, $\frac{7}{2}^+[404]$, $\frac{5}{2}^+[413]$, $\frac{3}{2}^+[411]$, and $\frac{1}{2}^+[411]$ orbitals. Coupling each of the positive parity orbitals to the $\frac{7}{2}^-[633]$ neutron yields positive parity wave functions of $K^\pi = 0^+$ up to 7^+ . As in the ^{176}Lu case this leads to an 8 by 8 Hamiltonian matrix in Coriolis coupling calculations. All 2 by 2 perturbations within the matrix were again calculated, and it was found that mixings between the

$\frac{3}{2}+[411]$ and $\frac{5}{2}+[413]$ and between the $\frac{3}{2}+[411]$ and $\frac{1}{2}+[411]$ were significantly smaller than other mixings. This is shown in Table (VI-8) where all Δ_{K_0, K_0+1}^I values for ^{166}Ho are tabulated. With respect to proton Coriolis coupling the $\frac{3}{2}+[411]$ bands can be considered pure, and the 8 by 8 matrix reduces to three 2 by 2 matrices. These involve the $K = 0$ and $K = 1$ bands of $\frac{7}{2}+[404]$ - $\frac{7}{2}+[633]$ and $\frac{5}{2}+[413]$ - $\frac{7}{2}+[633]$, the $K = 3$ and $K = 4$ bands of $\frac{1}{2}+[411]$ - $\frac{5}{2}+[633]$, and the $K = 7$ and $K = 6$ bands of $\frac{7}{2}+[404]$ - $\frac{7}{2}+[633]$ and $\frac{5}{2}+[413]$ - $\frac{7}{2}+[633]$. No approximations were necessary for treatment of the negative parity proton orbitals $\frac{7}{2}-[523]$ and $\frac{5}{2}-[532]$.

The Coriolis interaction of the 2 by 2 matrix resulting from the $K^\pi = 6^-$ and $K^\pi = 7^-$ bands of $\frac{5}{2}-[532]$ - $\frac{7}{2}+[633]$ and $\frac{7}{2}-[523]$ - $\frac{7}{2}+[633]$ was treated as the previous 2 by 2 matrices in ^{170}Lu . The $I = 7$ matrix was written

$$\begin{pmatrix} \epsilon_6 + 14A_6 - & -\sqrt{14} \frac{a}{2} \\ -\sqrt{14} \frac{a}{2} & \epsilon_7 - \lambda \end{pmatrix} = \begin{pmatrix} \epsilon_6 + 14A_6 - \lambda & -1.871a \\ -1.871a & \epsilon_7 - \lambda \end{pmatrix} = 0 \quad \begin{matrix} \lambda_7^7 = 5 \\ \lambda_6^7 = 1092 \end{matrix}$$

and the 8 by 8 matrix was written

$$\begin{pmatrix} \epsilon_6 + 30A_6 - \lambda & -\frac{\sqrt{30}}{2}a \\ -\frac{\sqrt{30}}{2}a & \epsilon_7 + 16A_7 - \lambda \end{pmatrix} = \begin{pmatrix} \epsilon_6 + 30A_6 - \lambda & -2.759a \\ -2.759a & \epsilon_7 + 16A_7 - \lambda \end{pmatrix} = 0 \quad \begin{matrix} \lambda_7^8 = 136 \\ \lambda_6^8 = 1334 \end{matrix}$$

The ultimate equations for c and a defined similarly to these of (VI-10) and (VI-6) are

Table (VI.6). Theoretical perturbations in keV for the Coriolis interaction of positive parity bands in ^{166}Ho . The notation is explained in Table (VI.2).

I	Δ_{01}^I	Δ_{12}^I	Δ_{23}^I	Δ_{34}^I	Δ_{45}^I	Δ_{56}^I	Δ_{67}^I
1	6						
2	9	0					
3	32	1	0.6				
4		1	1.7	9.7			
5		1	2.3	21.1	0.4		
6					0.9	1	
7						1	17

$$c = -2.449\varepsilon_7^2 + 86.6\varepsilon_7 + 233880 \quad (\text{VI-47})$$

and

$$a^2 = -.2856\varepsilon_7^2 + 484.7\varepsilon_7 - 2416 \quad (\text{VI-48})$$

The minimum in Q defined by

$$Q = (\lambda_7^8(\text{calc}) - 156)^2 + (\lambda_6^8(\text{calc}) - 1834)^2$$

again arises from $\frac{dc}{d\varepsilon_9} = 0$, or when $\varepsilon_7 = 17.7$ kev. The resulting

parameters are

ε_7 (kev)	ε_6 (kev)	A_7	A_6	a^2	λ_6^8	λ_7^8	Q
17.7	1560.	8.539	8.523	6064	1843	127	162

From the definitions of $\alpha = \langle \frac{7}{2}[-523] | J_+ | \frac{5}{2}[-532] \rangle (A_6 + A_7)$

$(U_6 U_7 + V_6 V_7)$ a value of $\langle \frac{7}{2}[-523] | J_+ | \frac{5}{2}[-532] \rangle = 5.045$ can be extracted.

Theoretically this matrix element was calculated to be 5.0226,

which is extremely good agreement. The mixing coefficients were then

calculated easily. The resulting mixed wavefunctions are

$$\phi_6^6 = 1.00\psi_6^6$$

$$\phi_6^7 = -.0867\psi_7^7 + .997\psi_6^7$$

$$\phi_7^7 = .997\psi_7^7 + .0867\psi_6^7$$

$$\phi_6^8 = -.084\psi_7^8 + 1.0\psi_6^8$$

$$\phi_7^8 = 1.0\psi_7^8 + .084\psi_6^8$$

$$\phi_6^9 = -.158\psi_7^9 + .988\psi_6^9$$

$$\phi_7^9 = .988\psi_7^9 + .153\psi_6^9$$

The calculation of the $\frac{5}{2}^- [552] + \frac{7}{2}^- [633]$ cross-sections including Coriolis coupling have not been very meaningful. The incidence of improving the agreement with experiment are no more frequent than those of lessening that agreement. In the $K^\pi = 7^- \frac{7}{2}^- [552] + \frac{7}{2}^- [633]$ band the improved agreement is readily observed, though the $I = 7$ state is not well resolved enough to allow accurate determination of experimental cross-sections. It is therefore not useful to present a comparison of theoretical pure and mixed cross-sections to experimental values.

Since the $K^\pi = 1^-$ band for the $\frac{5}{2}^- [552] + \frac{7}{2}^- [633]$ configuration has not been identified, no Coriolis calculations between the $K^\pi = 1^-$ and $K^\pi = 0^-$ bands can be performed. It can be noted that no apparent anomalies in experimental cross-section exists in the $K = 0$ band. It should be expected that an even smaller perturbation in the intensities of the $K^\pi = 0^-$ band than in those of the $K^\pi = 7^-$ band will occur due to Coriolis coupling, as the $\sqrt{(I-K)(I+K+1)}$ coefficient of the coupling element is smaller for $\langle K=1 | H_{RPC} | K=0 \rangle$ than for $\langle K=7 | H_{RPC} | K=6 \rangle$. Therefore only very small corrections to the theoretical cross-sections of the $K^\pi = 0^-$ band can be expected from including Coriolis interactions. Interestingly enough no significant difference between experiment and theory exists.

The computer code for the analysis of the $K^\pi = 0^-$ to $K^\pi = 1^-$ coupling in ¹⁷⁶Lu was adapted for the coupling of the $K^\pi = 0^+ \frac{7}{2}^+ [404] + \frac{7}{2}^+ [633]$ band to the $K^\pi = 1^+ \frac{5}{2}^+ [413] + \frac{7}{2}^+ [633]$ band in

¹⁶⁶Ho. The least squares deviation of $\sum_{K=0,1} \sum_{I=2}^5 (\lambda_{K,I}^I(\text{calc}) - \lambda_{K,I}^I(\text{exp}))^2$

for these two bands occurred when

$$\sigma^2 = [(A_1 + A_0)(U_1 U_0 + V_1 V_0) \langle \frac{7}{2} [404] | J_+ | \frac{5}{2} [413] \rangle]^2 = 39. \text{ This implies } \langle \frac{7}{2} [404] | J_+ | \frac{5}{2} [413] \rangle = 1.6 \text{ compared to } 2.6 \text{ calculated by theory.}$$

Agreement is reasonably good.

The mixed wavefunctions can then be written

$$\begin{aligned} \phi_0^0 &= 1.00 \psi_0^0 \\ \phi_0^1 &= .997 \psi_0^1 + .0747 \psi_1^1 & \phi_1^1 &= -.0747 \psi_0^1 + .997 \psi_1^1 \\ \phi_0^2 &= .988 \psi_0^2 + .0691 \psi_1^2 & \phi_1^2 &= -.0691 \psi_0^2 + .988 \psi_1^2 \\ \phi_0^3 &= .983 \psi_0^3 + .185 \psi_1^3 & \phi_1^3 &= -.185 \psi_0^3 + .983 \psi_1^3 \\ \phi_0^4 &= .992 \psi_0^4 + .129 \psi_1^4 & \phi_1^4 &= -.129 \psi_0^4 + .992 \psi_1^4 \\ \phi_0^5 &= .957 \psi_0^5 + .289 \psi_1^5 & \phi_1^5 &= -.289 \psi_0^5 + .957 \psi_1^5 \end{aligned}$$

The matrix parameters ϵ_0 , ϵ_1 , A_0 , A_1 and B , the odd-even shift parameter in the $K = 0$ band, are 877 kev, 1149 kev, 9.50, 3.75, and -76 kev. Noteworthy improvement in the cross section calculations was not obtained. This is not unexpected as the $\frac{5}{2} [413] \rightarrow \frac{7}{2} [633]$ configuration has already had much of its strength removed by what is believed to be vibrational band mixing, so the amount of (t, α) intensity lost due to Coriolis coupling can not be determined. In the $K^\pi = 0^+$ band the states are weak enough such that the absolute intensity correction (increase) due to Coriolis coupling is usually

less than the error in the experimental cross-sections. These arguments will apply to the $K = 6$ and $K = 7$ bands formed from the $\frac{5}{2}^+[415]$ and $\frac{7}{2}^+[404]$ proton orbitals also.

Since the coupling of the $\frac{7}{2}^+[404]$ to $\frac{5}{2}^+[415]$ has already been calculated as $\alpha = \sqrt{89}$, it is only necessary to determine ϵ_7 and A_6 in the $I = 7$ matrix of the $K = 6$ and $K = 7$ bands of these configurations. The coefficient of α in that matrix is $-\frac{\sqrt{14}}{2}$. The results are $\epsilon_7 = 915$ kev and $A_6 = 10.3$. The mixed wavefunctions are

$$\phi_6^6 = 1.00\psi_6^6$$

$$\phi_6^7 = .998\psi_6^7 - .057\psi_7^7 \quad \phi_7^7 = .057\psi_6^7 + .998\psi_7^7$$

Mixing between the $K = 3$ band and $K = 4$ band of the $\frac{1}{2}^+[411] \pm \frac{7}{2}^+[633]$ configuration should be large. The results of such a mixing ought to make the effective moment of inertia of the $K = 4$ band greater than the $K = 3$ band. Experimentally the opposite is observed. Treating Coriolis coupling by parameterization methods in this case yields unreasonable results for the moments of inertia A_3 and A_4 . Further it has already been noted that the cross-sections of states in the $K = 4$ band are lower than predicted. It seems most reasonable to conclude there is some mixing of the $K = 4$ band with a vibrational band. This mixing ought to push the state of the $K = 4$ band down to lower energy and thus decrease the effective moment of inertia. Such mixing apparently masks the Coriolis interaction and thus renders the parameterization methods

unuseful. The same situation with the $K = 4$ cross-sections of $\frac{1}{2}^+ [411] + \frac{7}{2}^- [514]$ was noted in ^{176}Lu .

(VI.5) ^{170}Tm Structure

The nucleus ^{170}Tm has 69 protons and 101 neutrons. Therefore the ground state configuration is $\pi^{\frac{1}{2}^+} [411] \pm \nu^{\frac{1}{2}^-} [521]$. The special case of two $\Omega = \frac{1}{2}$ orbitals in a configuration presents some very interesting features. Bands of $K^\pi = 1^-$ and 0^- result from the configuration. The $K = 0$ band contains two terms which decouple the even spin members from the odd. Coriolis coupling between the two bands contains terms from both the a_p and a_n decoupling factors in Equation (II-53). This strong coupling, which contains an I dependence itself, as shown in (II-53), causes an odd-even shift in the $K^\pi = 1^-$ band also.

The study of the $^{169}\text{Tm}(d,p)^{170}\text{Tm}$ reaction along with (n,γ) and (n,e^-) spectra have resulted in the assignment of 23 states in ^{170}Tm .⁴⁵ Five of these states were tentatively assigned to γ -vibrational bands. A summary of these states appears in Figure (VI.15). Fifty-four peaks from the $^{171}\text{Yb}(t,\alpha)^{170}\text{Tm}$ reaction have been identified in this work. A representative spectrum is shown in Figure (VI.16). Table (VI.7) contains a tabulation of the identified peaks with assignments wherever possible.

The $\frac{1}{2}^+ [411] \pm \frac{1}{2}^- [521]$ Configuration

The $K^\pi = 1^-$ and 0^- bands resulting from the $\frac{1}{2}^+ [411] \pm \frac{1}{2}^- [521]$

Figure (VI.15). Previously known levels of the $\frac{1}{2}^+$ [411], $\frac{1}{2}^-$ [321] and $\frac{1}{2}^+$ [411] $\frac{5}{2}^-$ [512] configurations in ^{170}Yb with β -transitions as previously assigned. Energies in keV lie to the right of each level, and spins and parities lie to the left. The assignment in parentheses was made in this study and is considered tentative.

(5.1) — (5.2)

(5.1) — (5.2)

Figure (VI.16). Alpha spectrum for the reaction
 $^{171}\text{Yb}(t, \alpha)^{170}\text{Tm}$ observed at the laboratory angle of 30° .

Table (VI.7). Nuclear levels in ^{170}Tm populated by the (t, α) reaction.

Peak #	Previous Energy (keV)	Energy (t, α) (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)^{10^\circ}$		Assignment		Configuration
				$\left(\frac{d\sigma}{d\Omega}\right)_{\text{exp}}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{\text{cal}}$ (mb/sr)	I ^a	K	
1	0	0	2	.0069	.0115	1 ⁺	1	$1/2^+ [411] \cdot 1/2^- [521]$
2	38.712	40.0	1.8	.0042	.0227	2 ⁺	1	$1/2^+ [411] \cdot 1/2^- [521]$
3	114.543	114.7	1.8	.0046	.0125	3 ⁺	1	$1/2^+ [411] \cdot 1/2^- [521]$
4	149.721	148.4	4.1	.0022	.0028	0 ⁺	0	$1/2^+ [411] \cdot 1/2^- [521]$
5	185.580	182.0	2.8	.0051	.0037	4 ⁺	1	$1/2^+ [411] \cdot 1/2^- [521]$
6	(204.121)	194	6	--	--	(2 ⁺	2	$1/2^+ [411] \cdot 5/2^- [5121]$
7	219.712	220.1	.9	.0329	.0195	2 ⁺	0	$1/2^+ [411] \cdot 1/2^- [521]$
8	237.239	238.6	1.8	.0204	.0144	1 ⁺	0	$1/2^+ [411] \cdot 1/2^- [521]$
9	(270.550)	270.4	10	.0013	--	(3 ⁺	2	$1/2^+ [411] \cdot 5/2^- [5121]$
10		318	15	.0044	.0008	(5 ⁺	1	$1/2^+ [411] \cdot 1/2^- [521]$
11		332	5	.0050				
12	349.736	350.8	2.8	.0123	.0105	3 ⁺	0	$1/2^+ [411] \cdot 1/2^- [521]$
13	381	380.7	3.0	.0021	.0052	4 ⁺	0	$1/2^+ [411] \cdot 1/2^- [521]$
14		420.	7	a				
15	(447.078)	449	20	a	--	(3 ⁺	3	$1/2^+ [411] \cdot 5/2^- [5121]$
16		559	8	a	.0006	(5 ⁺	0	$1/2^+ [411] \cdot 1/2^- [521]$
17		605	2	.0038				
18		618	5	.0042				
19		645.8	4.7	.0191	.0083	1 ⁺	1	$7/2^+ [104] \cdot 1/2^- [521]$

Table (VI.7). Continued

Peak #	Previous Value (keV)	Energy (E _{cal}) (keV)	Error (keV)	$\left(\frac{d\sigma}{d\Omega}\right)_{exp}^{10^4}$ (mb/sr)	$\left(\frac{d\sigma}{d\Omega}\right)_{cal}^{10^4}$ (mb/sr)	Assignment l ⁺ k		Configuration
20		687.3	4.5	.0290				
21		715.5	3.7	.0107	.0072	2 ⁻	2	5/2 ⁺ [402] -1/2 ⁻ [523]
22		743.6	2.3	.0785	.0574	2 ⁻	1	3/2 ⁺ [411] -1/2 ⁻ [521]
23		774.1	4.4	.0128	.0073	3 ⁻	3	7/2 ⁺ [404] -1/2 ⁻ [521]
24		801	6	.0022	.0336	3 ⁻	1	3/2 ⁺ [411] -1/2 ⁻ [521]
25		829	7	.0178	.0108	5 ⁺	4	7/2 ⁻ [523] +1/2 ⁻ [521]
26		851	8	.0534	.0156	2 ⁻	2	3/2 ⁺ [411] -1/2 ⁻ [521]
27		867	6	.04	.0087	3 ⁻	3	5/2 ⁺ [402] +1/2 ⁻ [521]
28		891	4	.0414	.0470	5 ⁺	3	7/2 ⁻ [523] -1/2 ⁻ [521]
29		921	2	.0709	.0733	3 ⁻	2	3/2 ⁺ [411] -1/2 ⁻ [521]
30		948	4	.0603	.0521	6 ⁺	4	7/2 ⁻ [523] +1/2 ⁻ [521]
31		969	9	.0158				
32		1011	1	.0224	.0156	6 ⁺	3	7/2 ⁻ [523] -1/2 ⁻ [521]
33		1061	9	.0065				
34		1081	7	.0060				
35		1110	11	.0028				
36		1192	3	.0028				
37		1213	6	.0084	.0121	3 ⁻	3	5/2 ⁺ [413] +1/2 ⁻ [521]
38		1230	5	.0060				

Table (VI.7): continued

Peak #	Previous Value (keV)	Energy (e.v.) (keV)	Error (keV)	$\left(\frac{d\lambda}{dE}\right)_{exp}^{30^\circ}$ (mb/deg) $\left(\frac{d\lambda}{dE}\right)_{cal}^{30^\circ}$ (mb/deg)	Assignment I ^a K	Configuration
39		1291	6	.0157	4 ⁻ 3	5/2 ⁺ [413]-1/2 ⁻ [521]
40		1313	5	.0165	1 ⁻ 1	γ -vibration
41		1351	2	.0120	2 ⁻ 1	γ -vibration
42		1382	2	.0142	2 ⁻ 2	5/2 ⁺ [413]-1/2 ⁻ [521]
43		1448	1	.0150	3 ⁻ 2	5/2 ⁺ [413]-1/2 ⁻ [521]
44		1488	2	.0065		
45		1539	13	.0021	4 ⁻ 2	5/2 ⁺ [413]-1/2 ⁻ [521]
46		1566	8	.0018		
47		1587	7	.0021		
48		1640.	5	.0027		
49		1679	13	.0049		
50		1703	10	.0022		
51		1742	5	.0034		
52		1829	4	.0146		
53		1843	8	.0051		
54		1868	3	.0070		

^aUnresolved

configuration were previously known from the (d,p) reaction and (n, γ) studies⁴⁵. Levels up to the $I^\pi = 4^-$ state were identified in each band. These appear in Figure (VI.15) along with the transitions assigned. Each of these states have been populated in the (t,α) reaction with cross-section consistent with two-quasiparticle calculations. The experimental angular distributions are consistent with those expected for each state. The angular distribution of the alphas populating the $I^\pi, K = 1^-, 0$ state at 237.235 kev is representative of a good $l = 2$ transfer and is shown in Figure (VI.17).

Because the $\frac{1}{2}^+ [411] \pm \frac{1}{2}^- [521]$ configuration is composed of two $\Omega = \frac{1}{2}$ orbitals, the Coriolis interaction contains a $(-1)^{I+1}$ dependence. The interaction is of the form¹⁰ $\langle K = 1 | H_{RPC} | K = 0 \rangle = \frac{\hbar^2}{2I} \cdot \frac{1}{I(I+1)}$ $[a_n + a_p (-1)^{I+1}]$ as shown in (II-55). In addition the H_{INT} and H_{pp} perturbations produce a $(-1)^{I+1} \frac{1}{a_p a_n} + (-1)^I \frac{1}{B}$ dependence in the $K = 0$ band.⁷ These combined effects result in apparent separate odd spin member and even spin member bands within both bands of the configuration. The effective moments of inertia of these bands have been calculated to be $A_{odd}^{K=0} = 11.25$, $A_{even}^{K=0} = 11.55$, $A_{odd}^{K=1} = 11.45$, and $A_{even}^{K=1} = 10.35$. With this information it is possible to predict the energies of the $I^\pi, K = 5^-, 1$ and $5^-, 0$ states to be at approximately 321 and 552 kev respectively. Weakly populated levels were observed at 318 ± 15 kev and 359 ± 3 kev in the α particle spectrum.

Figure (VI.17). Angular distributions for the $^{171}\text{Yb}(t, \alpha)$ ^{169}Tm reaction with 17.0 MeV tritons. The solid curves are the results of DWBA calculations.

Spectroscopic factor calculations indicate these levels should be populated with cross-sections on the order of $1 \mu\text{b/sr}$, which is consistent with the experimental cross sections of the 513 keV and 559 keV peaks. These levels were assigned as the $1^{\pi}, K = 5^{-}, 0$ and $5^{-}, 0$ states respectively in this configuration.

The $\frac{1}{2}^{+}[411] \pm \frac{5}{2}^{-}[512]$ Configuration

The $\frac{1}{2}^{+}[411] \pm \frac{5}{2}^{-}[512]$ configuration is an excited neutron configuration and to zeroth order should not be populated in a (t, α) reaction. The $1^{\pi}, K = 4^{-}, 2; 3^{-}, 2; 2^{-}, 2; \text{ and } 5^{-}, 3$ states of this configuration have been previously assigned⁴⁵ and appear in Figure (A215). The states at 204.452, 270.550, and 447.078 keV appear to have been populated in this work. This introduces a very interesting feature in the spectroscopy of ^{170}Tm .

In order to populate an excited neutron level, the state must mix with a proton configuration so that that component of the state can be observed in a proton transfer reaction. Coriolis coupling can not be the active interaction in this case, as the $\frac{5}{2}^{-}[512]$ neutron can only couple to the $\frac{3}{2}^{-}[521]$ and $\frac{7}{2}^{-}[514]$ orbitals. The U_{pp} interaction can also not be the active interaction as this perturbation couples states of the form $\delta_{K;K} \delta_{p,p} \pm 1 \delta_{n,n} \mp 1$. Some component of the $\frac{1}{2}^{+}[521]$ neutron must be mixed into the $\frac{1}{2}^{+}[411] \pm \frac{5}{2}^{-}[512]$ configuration in order to explain population of these states in a (t, α) reaction. This requires a coupling containing $\delta_{K;K} \delta_{p,p} \pm 2 \delta_{n,n} \mp 2$. It is possible that the U_{INT} perturbation can

support matrix elements which satisfy this requirement, however such mixing by H_{INT} is probably very small. This type of mixing has also been observed in the $\{\frac{3}{2}+[411]\pm\frac{7}{2}+[653]\}+\{\frac{7}{2}+[523]\pm\frac{3}{2}+[521]\}$ states of ^{166}Ho . It is proposed here that the $\frac{1}{2}+[411]\pm\frac{5}{2}+[512]$ configuration is mixed with the $\frac{5}{2}+[415]\pm\frac{1}{2}+[521]$ excited proton configuration. A more rigorous discussion of this admixture and the ^{166}Ho proposed admixture is presented in Chapter VII.

The $\frac{7}{2}+[404]\pm\frac{1}{2}+[521]$ Configuration

The $\frac{7}{2}+[404]$ proton is an excited particle state and should be only weakly observed in a pickup reaction. The bandhead for this orbital is observed at 316.1 keV in ^{169}Tm , and bands of the $\frac{7}{2}+[404]\pm\frac{1}{2}+[521]$ configuration should be expected near this energy in ^{170}Tm . No candidates for these states are observed in the (t,α) spectrum below 600 keV. Using a normalization factor of 8.2 for the (t,α) cross sections (established from experimental cross-sections in the $\frac{1}{2}+[411]\pm\frac{3}{2}+[411]\pm$, and $\frac{7}{2}+[523]\pm\frac{1}{2}+[521]$ configuration) it is observed that the levels at 644 keV and 774 keV are populated with cross-sections which agree reasonably well with calculations. Their relative intensities are also in good agreement with theory. This can be seen in Table (VI.7). The agreement is improved when Coriolis coupling is included. The previously unknown levels at 644 and 774 keV have therefore been assigned as the I^π , $K = 4^-, 4$ and $5^-, 3$ states of the $\frac{7}{2}+[404]\pm\frac{1}{2}+[521]$ configuration. These bandheads are expected to contain all of the observable cross-section,

and no other states in this configuration were assigned. The angular distributions of these peaks should be pure $\lambda = 4$ transfers. The experimental angular distribution of the 774 kev peak, shown in Figure (VI.17), seems to be in good agreement with the assignment. The singlet-triplet splitting of this configuration is observed to be $\Delta A = 143$ kev. Central plus tensor force calculations reproduced that splitting well, as a value of 110 kev was calculated.

The $\frac{5}{2}^+ [402] \pm \frac{1}{2}^- [521]$ Configuration

The $\frac{5}{2}^+ [402] \pm \frac{1}{2}^- [521]$ configuration also rises from a proton excited particle state and should also be only weakly populated. The $\frac{5}{2}^+ [402]$ band in ^{169}Tm begins at 785 kev. Spectroscopic factor calculations indicate that only the $I^\pi, K = 2^-, 2$ and $3^-, 3$ bandheads of the $\frac{5}{2}^+ [402] \pm \frac{1}{2}^- [521]$ configuration should be observed in the (t, α) reaction. Transfers populating these states will be very nearly pure $\lambda = 2$ transfers. A good $\lambda = 2$ angular distribution was observed for alphas populating the peak at 715 kev, as seen in Figure (VI.18). This level has been assigned to be the $K^\pi = 2^-$ bandhead, and the peak at 867 kev has been assigned as the $K^\pi = 3^-$ bandhead of this configuration. The singlet-triplet splitting is thus -132 kev. Theoretical calculations predicted that splitting to be -154 kev.

Each of these two bandheads is populated with an anomalously large cross-section. It is expected that the Coriolis coupling element between the $\frac{5}{2}^+ [402]$ and $\frac{3}{2}^+ [411]$ orbitals should be quite large. As will be shown later, the $I^\pi, K = 3^-, 1$ level in the

$\frac{5}{2}^{+}[411]-\frac{1}{2}^{-}[521]$ antiparallel coupling was populated with a surprisingly low intensity, and a large part of this loss of intensity can be explained by Coriolis coupling. One must therefore expect that the levels in the $\frac{5}{2}^{+}[402]\pm\frac{1}{2}^{-}[521]$ configuration would have enhanced (t, α) cross-sections.

The $\frac{7}{2}^{-}[523]\pm\frac{1}{2}^{-}[521]$ Configuration

The $\frac{7}{2}^{-}[523]$ proton orbital is the first excited proton hole state encountered in ^{170}Tm . The bandhead of this orbital has been observed at 579.2 keV in ^{169}Tm . Spectroscopic calculations predict that only the $I^{\pi}, K = 6^{+}, 3$ and $5^{+}, 3$ and $I^{\pi}, K = 6^{+}, 4$ and $5^{+}, 4$ states of the $\frac{7}{2}^{-}[523]\pm\frac{1}{2}^{-}[521]$ configuration should be observable in a (t, α) reaction, and transfers populating these states should involve pure $\lambda = 5$ transfers. Very good $\lambda = 5$ angular distributions are seen at 948, 891, and 1011 keV in the α particle spectrum as seen in Figures (VI.17) and (VI.18). The intensities of these peaks indicate that they are the $I^{\pi}, K = 6^{+}, 4$; $5^{+}, 3$; and $6^{+}, 3$ states respectively of the $\frac{7}{2}^{-}[523]\pm\frac{1}{2}^{-}[521]$ configuration. The theoretical cross-section calculations reproduce the experimental intensities very well, as seen in Table (VI.7). The energies of the $I^{\pi} = 6^{+}$ and 5^{+} members of the $K = 3$ band allow the calculation of the moment of inertia to be $A = 10.0$ keV. Using this value as an approximation to the moment of inertia of the $K = 4$ band, one can predict the $I^{\pi}, K = 5^{+}, 4$ state to lie near 828 keV, and to be populated with a cross-section at 30° of approximately .011 mb/sr. A level with

Figure (VI.18). Angular distributions for the reaction $^{171}\text{Yb}(t, \alpha)^{170}\text{Tm}$ using 17.0 MeV tritons. The solid curves are the results of DWBA calculations.

$\left(\frac{d\sigma}{d\Omega}\right)_{50^\circ} = .018 \text{ mb/sr}$ has been observed at 829 kev. That level has been assigned as the $I^\pi, K = 5^+, 3$ state of this configuration. The moment of inertia of the $K = 4$ band is then $A = 9.92 \text{ kev}$, and the singlet-triplet splitting is $-10. \text{ kev}$. Theoretically that splitting was calculated to be $-70. \text{ kev}$. The assignments of the $\frac{7}{2}^+ [523] \pm \frac{1}{2}^- [521]$ configuration appear in Figure (VI.19).

The $\frac{5}{2}^+ [411] \pm \frac{1}{2}^- [521]$ Configuration

Spectroscopic calculations indicate that the $I^\pi, K = 5^-, 1; 2^-, 1; 3^-, 2; \text{ and } 2^-, 2$ states of the $\frac{5}{2}^+ [411] \pm \frac{1}{2}^- [521]$ configuration should be observable in a proton pickup reaction. Each observable state should involve a pure or very nearly pure $\lambda = 2$ transfer. Very good $\lambda = 2$ angular distributions are shown in Figures (VI.17) and (VI.18) for the alphas populating peaks seen at 851, 921, and 744 kev. The Gallagher-Moszkowski coupling rule predicts that the $K = 1$ bandhead will lie below the $K = 2$ bandhead in energy, and theoretical calculations indicate the $I^\pi, K = 2^-, 1$ and $3^-, 2$ states should be the most intensely populated in the $\frac{5}{2}^+ [411] \pm \frac{1}{2}^- [521]$ configuration. The $\frac{5}{2}^+ [411]$ band in ^{171}Tm originates at 675.9 kev. The peaks at 744 kev and 921 kev in the $^{170}\text{Tm} (t, \alpha)$ spectrum have therefore been assigned as the $I^\pi, K = 2^-, 1$ and $3^-, 2$ states of the $\frac{5}{2}^+ [411] \pm \frac{1}{2}^- [521]$ configuration, and the peak at 851 kev has been assigned as the $I^\pi, K = 2^-, 2$ state. The experimental intensities are in good agreement with theoretical predictions, as can be seen in Table (VI.7). This results in a moment of inertia of the $K = 2$ band of 11.6 kev which

Figure (VI.19). Spin and parity assignments for the $\frac{5}{2}^+$ [411], $\frac{5}{2}^+$ [413], $\frac{7}{2}^+$ [404], $\frac{5}{2}^+$ [402], and $\frac{7}{2}^-$ [523] proton orbitals coupled to the $\frac{1}{2}^-$ [521] neutron orbital in ^{170}Tm . Unassigned levels of the (t, α) reaction appear at the extreme right. Assignments to a $K^\pi=1^-$ γ -vibrational band is included. Spin and parity assignments lie to the left of each level, and energies in keV lie to the right of them.

1742
 1703
 1680
 1640
 1589
 1566
 1488

2- 1351
 1- 1313
 K=1-; K=2 y Band
 Built on K=1- Band

1229
 1192
 1100
 1081
 1061

969
 unassigned

4- 1533
 3- 1448
 2- 1382
 [413] - [521]

4- 1291
 3- 1213
 [413] + [521]

5+ 1011

3- 920.9
 2- 851.1
 [411] + [521]
 3- 774.3
 [404] - [521]
 4- 643.8
 [404] + [521]
 5+ 890.8
 [523] - [521]
 (K=3)
 2- 715.5
 [402] - [521]
 (K=1)
 3- 867.0
 [402] + [521]
 2- 800.8
 [523] + [521]
 (K=4)
 5+ 829.4

compares favorably with the moments of inertia of 12.2 kev in the $\frac{5}{2}^+ [411]$ bands in ^{169}Tm and ^{171}Tm . A somewhat smaller moment of inertia should be expected for the $K = 1$ band, due to Coriolis coupling, so the I^π , $K = 3^-, 1$ state ought to be observed near 810 kev, populated with an intensity of $(\frac{d\sigma}{d\Omega})^{30^\circ} \approx .034$ mb/sr. The level at 891 kev in the (t, α) spectrum has been assigned as the state, though it has an anomalously low intensity. This can be partially explained by Coriolis coupling with the $K = 2$ band of this configuration and with the $K = 2$ and $K = 3$ bands of the $\frac{5}{2}^+ [402] \pm \frac{1}{2}^- [521]$ configuration. The two bands of the $\frac{3}{2}^+ [411] \pm \frac{1}{2}^- [521]$ configuration appear in Figure (VI.19). The moment of inertia of the $K = 1$ band is 9.55 kev, and the singlet-triplet splitting is -147 kev. This is in reasonable agreement with the central plus tensor force calculation, which predicts that splitting to be -187 kev.

The $\frac{5}{2}^+ [413] \pm \frac{1}{2}^- [521]$ Configuration

The I^π , $K = 3^-, 3$ and $2^-, 2$ bandheads in the $\frac{5}{2}^+ [413] \pm \frac{1}{2}^- [521]$ configuration should be populated with pure $i = 2$ transfers with cross-sections at 30° of approximately .012 and .006 mb/sr respectively. Singlet-triplet splitting calculations indicate the $K = 3$ bandhead should be approximately 145 kev below the $K = 2$ bandhead. Good $i = 2$ transfers were seen at 1213 kev and 1382 kev as shown in Figure (VI.18). Experimental intensities of these levels are $(\frac{d\sigma}{d\Omega})^{30^\circ} = .008$ and .014 respectively. These levels were assigned

as the 1^π , $K = 3^-, 3$ and $2^-, 2$ states of the $\frac{5}{2}^+ [413] \pm \frac{1}{2}^- [521]$ configuration.

A moderately good $\lambda = 4$ transfer was noted at 1447 kev in the α -particle spectrum. From the relative (t, α) intensities and observed energy spacings the levels at 1291, 1448, and 1539 kev have been assigned as the $1^\pi, K = 4^-, 3; 3^-, 2$; and $4^-, 2$ states of the $\frac{5}{2}^+ [413] \pm \frac{1}{2}^- [521]$ configuration. Agreement between experimental and theoretical (t, α) intensities can be noted in Table (VI.7). The experimental intensities are consistently too low. This can be partially attributed to the proposed mixing with the $\frac{1}{2}^- [411] \pm \frac{5}{2}^- [512]$ configuration and partially attributed to Coriolis coupling to the $\frac{7}{2}^- [404] \pm \frac{1}{2}^- [521]$ configuration.

Vibrational Bands

Several vibrational bands in ^{170}Tm were tentatively proposed by Sheline, et. al.⁴⁵ built on the $\frac{1}{2}^+ [411] \pm \frac{1}{2}^- [521]$ and $\frac{1}{2}^+ [411] \pm \frac{7}{2}^- [633]$ configurations. There remain many unassigned levels in the (t, α) spectrum, as shown in Figure (VI.18), with intensities indicative of states formed from the mixing of vibrational bands with two-quasiparticle bands. Such mixing would account for the low normalization factor of 3.2 observed in the spectroscopic calculations. A $K = 2-K_0 = 1$ vibrational band has therefore been proposed built on the $K^\pi = 1^- \frac{3}{2}^+ [411] - \frac{1}{2}^- [521]$ band mixed with the $K^\pi = 1^- \frac{1}{2}^+ [411] - \frac{1}{2}^- [521]$ ground state band. The peaks at 1313 and 1351 kev are populated by alphas with angular distributions which are good $\lambda = 2$

transfers, as seen in Figure (VI.18). These levels have been assigned the $1^-, K = 1^-, 1$ and $2^-, 1$ states in the proposed vibrational band.

The moment of inertia of a vibrational band should be nearly equal to that of the band on which it is built. The $K = 1 \frac{5}{2}^+ [411] - \frac{1}{2}^+ [521]$ has a rotational moment of inertia of 9.55 kev, and that of the proposed vibrational band is 9.5 kev. The intensities of the vibrational band indicate it has an admixture of approximately 80% $\frac{1}{2}^+ [411] - \frac{1}{2}^+ [521]$.

(VI.6) Coriolis Coupling in ^{170}Tm

The important proton orbitals in ^{170}Tm are the $\frac{5}{2}^+ [413]$, $\frac{3}{2}^+ [411]$, $\frac{7}{2}^+ [523]$, $\frac{1}{2}^+ [411]$, $\frac{7}{2}^+ [404]$, and $\frac{5}{2}^+ [402]$. When each of these is coupled to the $\frac{1}{2}^+ [521]$ neutron negative parity bands of $K = 0$ through 3, two bands of $K = 1$ and three each of $K = 2$ and $K = 3$, are expected. In rigorous Coriolis calculations each of the negative parity bands is coupled to every other negative parity band. Quantitative theoretical calculations allow for the simplification of the resulting 10 by 10 matrix into a single 2 by 2 matrix and two 4 by 4 matrices. The matrices involve the $K = 0$ and 1 bands of the $\frac{1}{2}^+ [411] - \frac{1}{2}^+ [521]$ configuration, the $\frac{3}{2}^+ [411]$ and $\frac{5}{2}^+ [402]$ proton orbitals coupled to the $\frac{1}{2}^+ [521]$ neutron, and the $\frac{5}{2}^+ [413]$ and $\frac{7}{2}^+ [404]$ protons coupled to the $\frac{1}{2}^+ [521]$ neutron. The only positive parity bands are from the $\frac{7}{2}^+ [523] - \frac{1}{2}^+ [521]$ configuration. No anomalies were noted in the cross-sections of the observed states in this configuration, so no Coriolis calculations were performed.

The $\frac{1}{2}^{+}[411]\frac{1}{2}^{-}[521]$ configuration has $K = 0$ and $K = 1$ bands which are coupled to one another by both the proton and neutron orbitals since each is an $\Omega = \frac{1}{2}$ orbital. This results in two terms, a_p and a_n , in the H_{RPC} matrix element. Coriolis coupling calculations have already been performed by Jones and Shelton for this configuration.¹⁰ The mixing coefficients for each state $\frac{1}{2}_{K=0}^{+}$ appear in 10), and no significant changes in predicted cross-sections were noted when Coriolis interactions were included. It is shown in 10) that the Coriolis calculations account for the energies, in particular the apparent odd-even shift in the $K = 1$ band, of this configuration very well. It is worthy of note that the odd-even shift parameter in the $K = 0$ band contains an additional $-a_p a_n \frac{\hbar^2}{2I}$ term from the H_{pp} interaction, which has matrix elements given by (II-56). The total odd-even shift, $B - a_p a_n \frac{\hbar^2}{2I}$, is -36.5 keV. The H_{pp} term contributes a value of $a_p a_n \frac{\hbar^2}{2I} = (-1.7044)(.38031) \frac{\hbar^2}{2I} = -3.1$ keV, so that $B = -38.6$ keV.

The 4 by 4 matrix resulting from the $\frac{5}{2}^{+}[413]\frac{1}{2}^{-}[521]$ and $\frac{7}{2}^{+}[404]\frac{1}{2}^{-}[521]$ configurations presents a difficult parameterization problem because of the numerous parameters involved. The following notation has been adopted.

Configuration	K	Pure Wavefunction	Mixed Wavefunction	Moment of Inertia	Oversturbed Bandhead Energy
$\frac{5}{2}^{+}[413]\frac{1}{2}^{-}[521]$	2	ψ_{2c}	ϕ_{2c}	A_{2c}	e_{2c}
	3	ψ_{3B}	ϕ_{3B}	A_{3B}	e_{3B}
$\frac{7}{2}^{+}[404]\frac{1}{2}^{-}[521]$	3	ψ_{3c}	ϕ_{3c}	A_{3c}	e_{3c}
	4	ψ_4	ϕ_4	A_4	e_4

The $l = 3$ matrix can be written

$$\begin{pmatrix} \epsilon_{33}^{-\lambda} & H_{3B2c} & 0 \\ H_{3B2c} & \epsilon_{2c} + 6A_{2c}^{-\lambda} & H_{2c3c} \\ 0 & H_{2c3c} & \epsilon_{3c}^{-\lambda} \end{pmatrix} = \begin{pmatrix} \epsilon_{33}^{-\lambda} & -\sqrt{6}\kappa & 0 \\ -\sqrt{6}\kappa & \epsilon_{2c} + 6A_{2c}^{-\lambda} & -\sqrt{6}\mu \\ 0 & -\sqrt{6}\mu & \epsilon_{3c}^{-\lambda} \end{pmatrix} = 0$$

and the $l = 4$ matrix can be written

$$\begin{pmatrix} \epsilon_{3B} + 8A_{3B}^{-\lambda} & H_{3A2c} & 0 & H_{2c3c} \\ H_{3B2c} & \epsilon_{2c} + 14A_{2c}^{-\lambda} & H_{2c3c} & 0 \\ 0 & H_{2c3c} & \epsilon_{3c} + 8A_{3c}^{-\lambda} & H_{3c4} \\ H_{3B4} & 0 & H_{3c4} & \epsilon_4^{-\lambda} \end{pmatrix} = \begin{pmatrix} \epsilon_{3B} + 8A_{3B}^{-\lambda} & -\sqrt{14}\kappa & 0 & -\sqrt{8}\nu \\ -\sqrt{14}\kappa & \epsilon_{2c} + 14A_{2c}^{-\lambda} & -\sqrt{14}\mu & 0 \\ 0 & -\sqrt{14}\mu & \epsilon_{3c} + 8A_{3c}^{-\lambda} & -\sqrt{8}\rho \\ -\sqrt{8}\nu & 0 & -\sqrt{8}\rho & \epsilon_4^{-\lambda} \end{pmatrix}$$

where

$$\kappa = \langle \psi_{3B} | J_{\pm} | \psi_{2c} \rangle = \langle \frac{1}{2} [-51\bar{1}] | J_{\pm} | \frac{1}{2} [-52\bar{1}] \rangle (A_{2c} + A_{3B})^{\frac{1}{2}}$$

$$\mu = \langle \psi_{2c} | J_{\pm} | \psi_{3c} \rangle = \langle \frac{5}{2} [-41\bar{1}\bar{3}] | J_{\pm} | \frac{7}{2} [-40\bar{4}] \rangle (A_{2c} + A_{3c}) (U_{2c} U_{3c} + V_{2c} V_{3c})^{\frac{1}{2}} \quad (VI-47)$$

$$\nu = \langle \psi_{3B} | J_{\pm} | \psi_4 \rangle = \langle \frac{5}{2} [-41\bar{1}\bar{3}] | J_{\pm} | \frac{7}{2} [-40\bar{4}] \rangle (A_{3B} + A_4) (U_{3B} U_4 + V_{3B} V_4)^{\frac{1}{2}}$$

$$\sigma = \langle \psi_{3c} | J_{\pm} | \psi_4 \rangle = \langle \frac{1}{2} [-52\bar{1}] | J_{\pm} | \frac{1}{2} [-52\bar{1}] \rangle (A_{3c} + A_4)^{\frac{1}{2}}$$

and the $\sqrt{(I \pm K)(I \mp K + 1)}$ factor has been written as a coefficient to each off-diagonal element.

At present there are 10 variable parameters A_1 , ϵ_1 , κ , μ , ν , and σ as ϵ_{2c} is known to be 1362 kev because ϵ_{2c}^2 is a pure state. Some theoretical consideration allows the removal of several of these parameters and makes the problem manageable. Theoretically $\mu = \nu$, and $\kappa = \sigma$, and of the unperturbed rotational moments of inertia $A_{2c} = A_{3B}$ and $A_{3c} = A_4$. Making these simplifications one obtains for the $I = 4$ matrix

$$\begin{pmatrix} \epsilon_{3B} + 8A_{2c} - \lambda & -\sqrt{14} \kappa & 0 & -\sqrt{8} \mu \\ \sqrt{14} \kappa & \epsilon_{2c} + 14A_{12} - \lambda & -\sqrt{14} \mu & 0 \\ 0 & -\sqrt{14} \mu & \epsilon_{3c} + 8A_{3c} - \lambda & -\sqrt{8} \kappa \\ -\sqrt{8} \mu & 0 & -\sqrt{14} \kappa & \epsilon_4 - \lambda \end{pmatrix} =$$

$$\begin{pmatrix} \epsilon_6 - \lambda & -\sqrt{14} \kappa & 0 & -\sqrt{8} \mu \\ -\sqrt{14} \kappa & C_7 - \lambda & -\sqrt{14} \mu & 0 \\ 0 & -\sqrt{14} \mu & C_8 - \lambda & -\sqrt{8} \kappa \\ -\sqrt{8} \mu & 0 & -\sqrt{8} \kappa & C_9 - \lambda \end{pmatrix}$$

and the $I = 3$ matrix can be simplified to

$$\begin{pmatrix} \varepsilon_{3B} - \lambda & -\sqrt{6} \kappa & 0 \\ -\sqrt{6} \kappa & \varepsilon_{2C} + 6A_{2C} - \lambda & -\sqrt{6} \mu \\ 0 & -\sqrt{6} \mu & \varepsilon_{2C} - \lambda \end{pmatrix} \equiv \begin{pmatrix} b_6 - \lambda & -\sqrt{6} \kappa & 0 \\ -\sqrt{6} \kappa & b_7 - \lambda & -\sqrt{6} \mu \\ 0 & -\sqrt{6} \mu & b_8 - \lambda \end{pmatrix} = 0$$

Proceeding as with the 3 by 3 matrix in the ¹⁷⁶Lu calculations, one obtains the characteristic equations in powers of b_8 . For the $I = 3$ determinant, summing the diagonal yields

$$b_8 = 3435 - b_7 - b_6 \quad (\text{VI-48})$$

and expanding the determinant and simplifying

$$b_8^2 + \left(\frac{3434\lambda - b_6\lambda - 3435b_6 + b_6^2 + 6\kappa^2}{b_6 - \lambda} \right) b_8 + \frac{\lambda^3 - 3435\lambda^2 + 3435b_6\lambda - b_6^2\lambda - 6\kappa^2\lambda}{b_6 - \lambda} + 6\mu^2 = 0 \quad (\text{VI-49})$$

For any eigenvalue λ_k^3 (these are 1213, 1447, and 774) the coefficients of b_8^2 and μ^2 are 1 and 6 respectively. Subtracting one characteristic equation with λ_j from the characteristic equation with λ_i yields as in (VI.46), $b_8(X_1^i - X_2^i) + (X_2^i - X_3^j) = 0$. For any choice of b_6 and κ^2 one can uniquely determine b_8 and then b_7 , A_{2C} , and μ^2 .

In the $I = 4$ matrix C_6 and C_7 are directly obtainable from b_6 , b_7 , and A_{2C} . Summing the diagonal of the $I = 4$ matrix one obtains an expression for C_9 and A_{3C} .

$$C_9 + 8A_{3C} = 4332 - b_6 - 2A_{2C} - b_7 - 8A_{2C} - b_8 \quad (\text{VI-50})$$

To do this it was necessary to guess an eigenvalue for the $I = 4$ state of the $K^\pi = 5^-$ band of the $\frac{5}{2}^+ [404] \pm \frac{1}{2}^- [521]$ configuration. A value of $A_{3c}^4 = 858$ kev was guessed. Now a single choice of C_9 uniquely determines A_{3c} . Expanding the determinant of the $I = 4$ matrix one obtains

$$\begin{aligned}
 & \lambda^4 + \lambda^3 (-C_6 - C_7 - C_8 - C_9) + \lambda^2 (C_6 C_7 + C_6 C_8 + C_6 C_9 + C_7 C_8 + C_7 C_9 + C_8 C_9 - 14\mu^2 - 3\kappa^2 - 14\kappa^2 - 3\mu^2) \\
 & + \lambda (-C_6 C_7 C_8 - C_6 C_7 C_9 - C_6 C_8 C_9 - 14C_6 \mu^2 + 8C_6 \kappa^2 - C_7 C_8 C_9 + 14C_7 \mu^2 + 8C_7 \kappa^2 - 14C_8 \mu^2 \\
 & \quad + 14C_8 \kappa^2 + 8C_8 \mu^2 + 8C_7 \mu^2) \\
 & + (-C_6 C_7 C_8 C_9 - 14C_9 C_6 \mu^2 - 8C_6 C_7 \kappa^2 - 14C_8 C_9 \kappa^2 - 112\kappa^2 \mu^2 + 112\kappa^4 - 112\mu^2 \kappa^2 \\
 & \quad - 8C_7 C_8 \mu^2 + 112\mu^4) \\
 & = 0
 \end{aligned} \tag{VI-51}$$

All parameters are known from a combination of selected C_9 , b_6 , and κ^2 values. Coefficients of all powers of λ in (VI-51) are then easily obtainable.

A computer code was written to systematically vary b_6 , κ^2 , and C_9 over pertinent ranges. Any combination was rejected which resulted in $\mu^2 < 0$ or $9 \leq A_{3c} \leq 12$. Of the 10440 combinations encountered by the computer only 3 satisfied these two requirements. Those 3 combinations are labeled sets I, II, and III and are shown in Table (VI.8) along with all parameters and λ^n coefficients.

Each combination is essentially the same as only the λ^1 and λ^0 coefficients change at all. Having determined all coefficients of $(\lambda^1)^n$ it is possible to solve for $\lambda^{I=4}$ in Equation (VI-51). The calculated eigenvalues are 1533, 1297, 358, and 633 compared to the experimental values of 1539, 1291, (353), and 644. If the perturbation were "turned off" the eigenvalues of the $I = 4$ determinant would be 1514, 1252, 378, and 651. The calculated eigenvalues of the $I = 5$ determinant are 1449, 1212, and 774 compared to the experimental values of 1447, 1213, and 774. Turning the perturbation off would yield eigenvalues of 1459, 1213, and 783. From the values of ϵ^2 and ϵ^3 one can calculate $\langle \frac{1}{2}[-531] | J_z | \frac{1}{2}[-521] \rangle$ and $\langle \frac{5}{2}[413] | J_z | \frac{7}{2}[404] \rangle$. The former of these is the decoupling parameter a_n of the $\frac{1}{2}[-521]$ orbital and is equal to $\sqrt{57.2}/(9.46+11.83).5 = .710$, while theoretically that value is .8893. The latter is equal to $\sqrt{367.95}/.5(9.46+11.83)(U_{2c}U_{3c}+V_{2c}V_{3c}) = 4.44$, while theoretically that value was calculated to be 2.42.

Finally the mixed wavefunctions can be written

$I = 3$ $\psi_{3B} = 1.0\psi_{3B} + .077\psi_{2c} - .014\psi_{3c}$ $\psi_{2c} = -.078\psi_{3B} + .99\psi_{2c} - .11\psi_{3c}$ $\psi_{3c} = -.0048\psi_{3B} + .11\psi_{2c} + .99\psi_{3c}$	$I = 4$ $\psi_{3B} = .98\psi_{3B} + .11\psi_{2c} - .025\psi_{3c} - .13\psi_4$ $\psi_{2c} = -.12\psi_{3B} + .98\psi_{2c} - .17\psi_{3c} + .016\psi_4$ $\psi_{3c} = -.0087\psi_{3B} + .17\psi_{2c} + .98\psi_{3c} - .098\psi_4$ $\psi_4 = .13\psi_{3B} + .017\psi_{2c} + .094\psi_{3c} + .98\psi_4$
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and the mixed cross-sections can be calculated. The results of that calculation are listed in Table (VI.9). It is easily noted that the

Table (VI.8). Coriolis parameters and $\bar{A}$ coefficients of equation (VI-31) for the calculation of the $\frac{1}{2}^+ [411] - \frac{1}{2}^+ [321]$ and $\frac{1}{2}^+ [404] - \frac{1}{2}^+ [321]$ interaction.

Set	b_1	b_2	b_3	A_{20}	A_{22}	C_4	C_6	C_2	C_0	$\bar{A}^1$	$\bar{A}^2$
I	1213	1439	783	9.46	11.95	1039	1514	873	650	57.07	267.03
II	1213	1439	783	9.46	11.83	1039	1514	873	651	57.00	267.03
III	1213	1439	783	9.46	11.70	1039	1514	873	652	57.07	267.03
	$\bar{A}^1$	$\bar{A}^2$	$\bar{A}^3$	$\bar{A}^4$	$\bar{A}^5$	$\bar{A}^6$	$\bar{A}^7$	$\bar{A}^8$	$\bar{A}^9$	$\bar{A}^{10}$	$\bar{A}^{11}$
I	1	-4.332x10 ⁻²	6.786x10 ⁻³	-4.533x10 ⁻³	1.092x10 ⁻⁴						
II	1	-4.332x10 ⁻²	6.786x10 ⁻³	-4.533x10 ⁻³	1.092x10 ⁻⁴						
III	1	-4.332x10 ⁻²	6.786x10 ⁻³	-4.533x10 ⁻³	1.093x10 ⁻⁴						

Table (VI.9). Results of Coriolis band mixing for the $\frac{7}{2}^-$ [415], $\frac{7}{2}^+$ [404], $\frac{3}{2}^+$ [411], and $\frac{5}{2}^+$ [402] proton orbitals in ^{159}Tm coupled to the $\frac{1}{2}^-$ [521] neutron.

J	K	Proton Orbital	$\left(\frac{d\sigma}{d\Omega}\right)^{390} \left(\frac{\text{mb}}{\text{sr}}\right)$		
			Pure	Experimental	Mixed
5	3	$\frac{7}{2}^+$ [404]	.007	.013	.010
4	4		.008	.019	.011
2	2	$\frac{5}{2}^+$ [415]	.006	.014	.006
3	2		.029	.015	.025
4	2		.012	.002	.013
3	3		.012	.008	.013
4	3		.033	.016	.027
2	2	$\frac{5}{2}^+$ [402]	.007	.011	.008
3	3		.009	.040	.030
3	1	$\frac{3}{2}^+$ [411]	.034	.002	.025

Coriolis interaction improves the agreement between experiment and theory for the $\frac{7}{2}^{+}[404]$ orbital. Some improvement is noted in the $\frac{5}{2}^{+}[413]$ cross-sections, though the discrepancy between experiment and theory remains large. It must be assumed that the $\frac{5}{2}^{+}[413] \pm \frac{1}{2}^{+}[521]$ configuration is not pure. The proposed mixing between this configuration and the $\frac{1}{2}^{+}[411] \pm \frac{5}{2}^{+}[521]$ excited neutron configuration has already been discussed.

A feasible parameterization treatment of the 4 by 4 matrix involving the $\frac{5}{2}^{+}[411]$ and $\frac{5}{2}^{+}[402]$ orbitals was not possible because the levels in these bands are too close together. From explicit calculations of the off-diagonal elements from the theory of Section (II.4) the perturbations were too large to result in reasonable unperturbed moments of inertia. These values were forced to become reasonable by continuously reducing the H_{KPC} interactions. Finally only imprecise quantitative arguments could be made for the mixed cross-section calculations.

The notation

Configuration	K	Pure Wavefunction	Mixed Wavefunction	Moment of Inertia	Unperturbed Bandhead Energy
$\frac{5}{2}^{+}[402] \pm \frac{1}{2}^{+}[521]$	3	ψ_{2B}	ϕ_{2B}	A_{2B}	ϵ_{2B}
$\frac{5}{2}^{+}[402] \pm \frac{1}{2}^{+}[521]$	3	ψ_{3A}	ϕ_{3A}	A_{3A}	ϵ_{3A}
$\frac{3}{2}^{+}[411] \pm \frac{1}{2}^{+}[521]$	2	ψ_{2A}	ϕ_{2A}	A_{2A}	ϵ_{2A}
$\frac{3}{2}^{+}[411] \pm \frac{1}{2}^{+}[521]$	1	ψ_1	ϕ_1	A_1	ϵ_1

was introduced. The Hamiltonian matrix for $I = 2$ was written:

$$\begin{pmatrix} \epsilon_{2A} - \lambda & H_{2A1} & 0 \\ H_{2A1} & \epsilon_1 + 4\lambda_1 - \lambda & H_{12B} \\ 0 & H_{12B} & \epsilon_{2B} - \lambda \end{pmatrix} = 0$$

and the matrix for $I = 3$ was written:

$$\begin{pmatrix} \epsilon_{2A} + 6\lambda_{2A} - \lambda & H_{2A1} & 0 & H_{2A3A} \\ H_{2A1} & \epsilon_1 + 10\lambda_1 - \lambda & H_{12B} & 0 \\ 0 & H_{12B} & \epsilon_{2B} + 6\lambda_{2B} - \lambda & H_{2B3A} \\ H_{2A3A} & 0 & H_{2B3A} & \epsilon_{3A} - \lambda \end{pmatrix} = 0$$

The parameters were finally determined to be $\epsilon_{2A} = 350.$,

$\epsilon_{2B} = 716$, $\epsilon_{3A} = 868$, $\epsilon_1 = 693$, and $\lambda_{3A} = \lambda_{2A} = \lambda_1 = \lambda_{2B} = 13.5$.

For the $I = 3$ matrix $H_{2A1} = -14.9$, $H_{12B} = -20.3$, $H_{2B3A} = -11.5$ and for the $I = 2$ matrix $H_{2A1} = -9.4$, and $H_{12B} = -12.3$. The mixed wavefunctions were determined to be

$$\begin{aligned} \psi_{2A}^3 &= .9\psi_{2A} - .2\psi_1 + .06\psi_{2B} - .4\psi_{3A} & \psi_{2A}^2 &= .1\psi_{2A} - .08\psi_1 + .008\psi_{2B} \\ \psi_1^3 &= .1\psi_{2A} + .1\psi_1 - .2\psi_{2B} - .06\psi_{3A} & \psi_1^2 &= .07\psi_{2A} + .8\psi_1 - .6\psi_{2B} \\ \psi_{2B}^3 &= .1\psi_{2A} + .4\psi_1 + .9\psi_{2B} + .1\psi_{3A} & \psi_{2B}^2 &= .04\psi_{2A} + .6\psi_1 + .8\psi_{2B} \\ \psi_{3A}^3 &= .2\psi_{2A} + .04\psi_1 - .4\psi_{2B} + .9\psi_{3A} \end{aligned}$$

The theoretical cross-sections of the two $\frac{5}{2}^+ [402] \pm \frac{1}{2}^- [521] 1^{\mp}$, $K = 3^{\mp}, 1$ state with Coriolis coupling included are shown in Table (VI.9). Very satisfactory improvement of the $\frac{5}{2}^+ [402] \pm \frac{1}{2}^- [521]$ band-head's intensities is obtained. The agreement between experiment and theory of the $1^{\mp}$, $K = 3^{\mp}, 1$ state of the $\frac{5}{2}^+ [411] \pm \frac{1}{2}^- [521]$ is slightly improved, although the experimental intensity is still very surprisingly low.

(VI.7) ^{160}Tb Structure

A representative $^{161}\text{Dy}(\text{t}, \alpha) ^{160}\text{Tb}$ spectrum is shown in Figure (VI.20). It is clear that the level density is too high to perform an accurate analysis of ^{160}Tb with a transfer reaction on the QSD spectrometer. It is possible to identify several known peaks of the ground state configuration $\frac{5}{2}^+ [411] \pm \frac{3}{2}^- [521]$ and to make 11 tentative new assignments. Figure (VI.21) shows the levels of the ground state configuration and the new assignments made in the $\frac{5}{2}^+ [415] \pm \frac{3}{2}^- [521]$, $\frac{9}{2}^+ [404] \pm \frac{3}{2}^- [521]$, and $\frac{5}{2}^+ [532] \pm \frac{3}{2}^- [521]$ configurations of ^{160}Tb and Table (VI.10) contains a list of peaks identified. A rigorous defense of the assignments will not be presented. The assignments were made on the basis of energy spacings within rotational bands and relative intensities. Rotational moments of inertia in the $\frac{5}{2}^+ [415] \pm \frac{3}{2}^- [521] K^{\pi} = 4^{\pi}$ and 1^{π} bands are 8.7 and 13.6 keV respectively with a singlet-triplet splitting of 174 keV and in the $\frac{5}{2}^+ [532] \pm \frac{3}{2}^- [521] K^{\pi} = 4^{\pi}$ and 1^{π} bands are 9.92 and 11.3 keV respectively with a singlet-triplet splitting of 270. keV. In the

ground state configuration the previously known moments of inertia are $(\frac{A^2}{2I})_{K=0} = 10.2$ and $(\frac{A^2}{2I})_{K=3} = 9.32$ kev with $LA = 109$ kev and $B = -17.1$ kev.

(VI.8) Gallagher-Moszkowski Doublet, Newby Splitting Calculations

In this study matrix elements of $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle$, $\langle \vec{L}_n \rangle$, and $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n \vec{L}_n \rangle$ introduced in Section (II.5) were calculated with a finite range central force as well as the tensor force matrix element

$\frac{(\vec{\sigma}_n \cdot \vec{r})(\vec{r} \cdot \vec{\sigma}_p)}{r^3} \langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle$. These operations were performed by the program GMMNS of Enishi²⁶ on the Livermore CDC 7600 computer at

several values of r_0 fixed between 1.40fm and 1.95fm (see II-65).

The matrix elements were then stored, and the parameters

V_2 , V_3 , V_4 , and V_5 of (II-65) and (II-71) were varied freely to obtain a least squares fit of the calculated LA and B values to the experimental values. In this way a best value of r_0 and the best V_i were selected for each nucleus studied.

It was found that $r_0 = 1.59$ produced the best fit for the ^{170}Lu splitting in calculations not including a tensor force. The fit was determined by minimizing the sum of the squares of the differences between the experimental and calculated LA and B values. The minimum was a very shallow one, and such an extensive search was not performed on ^{160}Ho and ^{170}Tm . Finally the program GMMNS was run each of the ^{176}Lu , ^{160}Ho , and ^{170}Tm data separately to obtain best V_i values for each nucleus individually, and then it was run with all of the combined data in a single run to obtain general values for

Figure (VI.20). Alpha spectrum for the reaction
 $^{164}\text{Dy}(t, \alpha)^{160}\text{Tb}$ observed at the laboratory angle of 20° .

Table (V). (a). - Ionization levels in ^{10}B as predicted by the $3\text{P}_{1/2}$ model.

Peak #	Previous Energy (keV)	Energy (keV)	J^{π}	g_{ion}	Ionization
1	0	1.779	1^{-}	1	$3/2^{-} [3/2]^{-}$
2	90.997	1.779	1^{-}	1	$3/2^{-} [3/2]^{-}$
3	79.002	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
4	117.221	1.779	1^{-}	1	$3/2^{-} [3/2]^{-}$
5	109.671	1.779	1^{-}	1	$3/2^{-} [3/2]^{-}$
6	176.246	1.779	1^{-}	1	$3/2^{-} [3/2]^{-}$
7	206.376	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
8	279	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
9	279	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
10	279	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
11	365	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
12	365	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
13	365	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
14	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
15	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
16	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
17	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
18	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
19	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$
20	621	1.779	1^{-}	0	$3/2^{-} [3/2]^{-}$

In units of 10^4 eV .

Figure (VI.21). Assignments of levels populated in the $^{189}\text{Os}(\text{t},\text{n})^{190}\text{Tb}$ reaction. Energies in keV appear to the right of each level, and spin and parity assignments appear to the left. Levels of the $\frac{5}{2}^{+} [411] - \frac{3}{2}^{-} [521]$ configuration were previously known. All other assignments were made in this work.

$$5 \cdot \text{---} 115.4$$

$$4 \cdot \text{---} 117.31$$

$$\frac{1}{2} \cdot \{53.1\} + \frac{1}{2} \cdot \{52.0\}$$

$$N^* = 4^*$$

$$7 \cdot \text{---} 115.3$$

$$6 \cdot \text{---} 110.5$$

$$5 \cdot \text{---} 110.5$$

$$\frac{1}{2} \cdot \{53.1\} + \frac{1}{2} \cdot \{52.0\}$$

$$N^* = 4^*$$

$$6 \cdot \text{---} 112.0$$

$$\frac{1}{2} \cdot \{110.4\} + \frac{1}{2} \cdot \{52.0\}$$

$$5 \cdot \text{---} 111.0$$

$$4 \cdot \text{---} 112.4$$

$$\frac{1}{2} \cdot \{111.1\} + \frac{1}{2} \cdot \{52.0\}$$

$$5 \cdot \text{---} 111.2$$

$$7 \cdot \text{---} 111.4$$

$$6 \cdot \text{---} 111.1$$

$$\frac{1}{2} \cdot \{111.1\} + \frac{1}{2} \cdot \{52.0\}$$

$$4 \cdot \text{---} 111.4$$

$$5 \cdot \text{---} 111.1$$

$$\frac{1}{2} \cdot \{111.1\} + \frac{1}{2} \cdot \{52.0\}$$

$$6 \cdot \text{---} 111.1$$

$$\frac{1}{2} \cdot \{111.1\} + \frac{1}{2} \cdot \{52.0\}$$

$$5 \cdot \text{---} 116.154$$

$$4 \cdot \text{---} 116.154$$

$$3 \cdot \text{---} 116.154$$

$$\frac{1}{2} \cdot \{116.1\} + \frac{1}{2} \cdot \{52.0\}$$

each V_i . It was hoped that the V_i would have similar values for each of the respective terms from one nucleus to the next, and that the general values for each V_i would be similar to the V_i parameters in the individual runs. The results of the individual runs appear in Table (VI.11) for $r_0 = 1.40$ and 1.59 and of the combined run appear in Table (VI.12).

It is clear from Table (VI.11) that the potentials V_i are very sensitive to the choice of r_0 , yet the calculated ΔA and B values are quite insensitive to the choice of this parameter. Further it is noted that the potentials change greatly when going from one nucleus to another in Table (VI.11). In particular V_4 even changes sign from ^{166}Ho to ^{176}Lu and ^{170}Tm . Each of the fits is of nearly the same quality. This seems to indicate that it is physically meaningless to use an equation with as many parameters as (II-65) plus (II-71) when an apriori calculation of these parameters can not be made.

When the data of Table (VI.11) were used in a single composite run, the potentials obtained for the best fits were for $r_0 = 1.40\text{fm}$: $V_2 = -24.33$ MeV, $V_3 = -9.892$ MeV, $V_4 = -6.757$ MeV, and $V_T = -149.62$ MeV; and for $r_0 = 1.59\text{fm}$: $V_2 = -16.71$ MeV, $V_3 = -5.758$ MeV, $V_4 = -6.519$ MeV, and $V_T = -92.48$ MeV. The same relative signs were obtained when the anomalous $\Delta A_{\text{exp}} = 28.0$ kev of the $\frac{7}{2}^+ [404] \pm \frac{7}{2}^+ [633]$ configuration in ^{166}Ho was excluded from the input data. Lasijo²⁷ has performed a similar study with several previously known configurations and obtained the parameters

$V_2 = -57.62$, $V_3 = -11.21$, $V_4 = 26.85$, and $V_T = -75.88$ with $r_0 = 1.40$. Table (VI.12) shows a comparison of the calculated splitting energies and odd-even shifts obtained from the best fit parameters in the study labeled with subscript LSTMSQ, to those obtained by applying the Lasijo parameters labeled with subscript LASIJO, for the configurations of this work. The quality of these two fits are again comparable even though the V_i parameters differ greatly.

It appears that the use of four V_i parameters and a finite range parameter r_0 has little physical meaning when calculating theoretical splitting energies. It is apparently not possible to select a set of potentials which can be useful in general. The finite range parameter has little effect on selecting a set of least squares parameters as long as there are four other degrees of freedom, V_i , in the calculation. Elmore¹⁴ has calculated ΔA values obtained in 03 configurations by a zero-range force and by a finite range central force and has shown that the fits are of equal quality. No tensor force was used in that calculation. Elmore has concluded that the zero-range force is a valid approximation for ΔA calculations. Very little success was obtained by Elmore in reproducing B values without a tensor force. The present study has had much more success by including a tensor force in the theoretical calculations. This was predicted by Jones, et. al.¹³ in zero-range, finite-range, and tensor force calculations.

It is finally concluded that until some theoretical calculation of the parameters V_i can be performed to give them infinite physical

Table (VI.11). Calculated and experimental ΔA and B values for ^{166}Ho , ^{170}Tm , and ^{176}Lu at $r_0=1.40$ and 1.59 fm. The calculated values were determined from a separate selection of the best potential parameters for each of the nuclides individually as explained in the text.

^{166}Ho

$r_0=1.40$ $V_0=-10.62\text{MeV}$ $V_3=-1.592\text{MeV}$ $V_4=-22.63\text{MeV}$ $V_T=-172.76\text{MeV}$

Configuration	$\Delta A_{\text{exp}}(\text{keV})$	$\Delta A_{\text{calc}}(\text{keV})$	$B_{\text{exp}}(\text{keV})$	$B_{\text{calc}}(\text{keV})$
$\frac{1}{2}^+ [404] = \frac{1}{2}^+ [633]$	28.0	-62.4	-76.0	-63.7
$\frac{3}{2}^+ [413] = \frac{3}{2}^+ [633]$	-70.0	-84.5		
$\frac{1}{2}^+ [411] = \frac{1}{2}^+ [633]$	-159.2	-66.1		
$\frac{3}{2}^+ [411] = \frac{3}{2}^+ [633]$	197.4	74.2		
$\frac{1}{2}^- [523] = \frac{1}{2}^- [633]$	72.7	139.3	-30.6	-28.4
$r_0=1.59$ $V_0=-8.656$ $V_3=-16.26$ $V_4=-14.30$ $V_T=-97.46$				
$\frac{1}{2}^+ [404] = \frac{1}{2}^+ [633]$	28.0	-65.5	-76.0	-55.5
$\frac{3}{2}^+ [413] = \frac{3}{2}^+ [633]$	-70.0	-85.7		
$\frac{1}{2}^+ [411] = \frac{1}{2}^+ [633]$	-159.2	-67.6		
$\frac{3}{2}^+ [411] = \frac{3}{2}^+ [633]$	197.4	76.0		
$\frac{1}{2}^- [523] = \frac{1}{2}^- [633]$	72.7	139.3	-30.6	-18.7

^{170}Tm

$r_0=1.40$ $V_0=-40.56$ $V_3=-13.21$ $V_4=17.51$ $V_T=-225.42$

Configuration	ΔA_{exp}	ΔA_{calc}	B_{exp}	B_{calc}
$\frac{1}{2}^+ [411] = \frac{1}{2}^+ [521]$	191.2	209.7	-38.6	-29.0
$\frac{7}{2}^- [523] = \frac{1}{2}^- [521]$	-10.4	-77.5		

Table (VI.11). continued

Configuration	ΔA_{exp}	ΔA_{calc}	E_{exp}	E_{calc}
$\frac{1}{2}^{-} [411] \frac{1}{2}^{-} [621]$	-182.0	-182.8		
$\frac{1}{2}^{-} [411] \frac{3}{2}^{-} [612]$	-231.6	-176.1		
$\frac{1}{2}^{-} [402] \frac{1}{2}^{-} [621]$	-141.5	-146.4		
$\frac{1}{2}^{-} [404] \frac{1}{2}^{-} [611]$	140.3	119.6		
$\frac{1}{2}^{-} [403] \frac{1}{2}^{-} [621]$	176.0	152.4		
$n_1=1.59 \quad V_1=-26.04$	$V_3=-6.358$	$V_4=-4.791$	$V_5=-160.16$	
$\frac{1}{2}^{-} [411] \frac{1}{2}^{-} [621]$	191.2	196.3	-39.6	-32.6
$\frac{1}{2}^{-} [533] \frac{1}{2}^{-} [621]$	-10.4	-76.6		
$\frac{3}{2}^{-} [411] \frac{3}{2}^{-} [621]$	-132.0	-133.1		
$\frac{1}{2}^{-} [411] \frac{5}{2}^{-} [612]$	-231.6	-172.4		
$\frac{5}{2}^{-} [402] \frac{1}{2}^{-} [621]$	-141.5	-152.3		
$\frac{7}{2}^{-} [404] \frac{1}{2}^{-} [621]$	140.3	119.3		

Table (VI,11). continued

Configuration	ΔA_{exp}	ΔA_{calc}	S_{exp}	S_{calc}
$\frac{1}{2}^{+} [413] \frac{7}{2}^{-} [511]$	176.0	143.1		
$17^{+} 1_{11}$				
$r = 1.43 \quad V_2 = -31.14 \quad V_3 = -13.33 \quad V_4 = 10.35 \quad V_5 = -117.81$				
Configuration	ΔA_{exp}	ΔA_{calc}	S_{exp}	S_{calc}
$\frac{1}{2}^{+} [414] \frac{7}{2}^{-} [514]$	252.0	247.0	63.6	63.6
$\frac{3}{2}^{-} [514] \frac{7}{2}^{-} [514]$	-83.1	-83.2		
$\frac{5}{2}^{+} [402] \frac{7}{2}^{-} [514]$	-104.3	-123.7		
$\frac{5}{2}^{+} [411] \frac{7}{2}^{-} [514]$	126.6	126.4		
$r = 1.59 \quad V_2 = -20.23 \quad V_3 = -11.26 \quad V_4 = 5.396 \quad V_5 = -63.10$				
$\frac{7}{2}^{+} [404] \frac{7}{2}^{-} [514]$	252.3	245.3	63.6	63.6
$\frac{3}{2}^{-} [514] \frac{7}{2}^{-} [514]$	-83.1	-83.2		
$\frac{5}{2}^{+} [402] \frac{7}{2}^{-} [514]$	-104.3	-124.6		
$\frac{5}{2}^{+} [411] \frac{7}{2}^{-} [514]$	126.6	127.6		

Table (VI.12). Calculated and experimental ΔA and B values for the best fit potential parameters at $r_0 = 1.40$ determined with the combined data of ^{170}Lu , ^{170}Tm , and ^{166}Ho . Subscript LASIJO is calculated value by applying the Lasijo parameters to the calculated matrix elements of (II-63). Subscript LSTMSQ is calculated value using the best fit parameters of (II-63). Subscript CENT is calculated value using the best fit parameters of (VI-53) plus a tensor force matrix.

Potentials (MeV)

Parameter Set	V_2	V_3	V_4	V_{T_2}
LSTMSQ	-24.333	-9.893	-6.757	-149.02
LASIJO	-37.62	-11.21	26.85	-75.85
CENT	$U_0 = -22.628$		$V_T = -164.342$	

Nuclide	Configuration	ΔA_{exp}	ΔA_{LSTMSQ}	ΔA_{LASIJO}	ΔA_{CENT}
^{170}Tm	$\frac{1}{2}^+ [411] = \frac{1}{2}^- [521]$	191.3	165.4	146.1	172.5
^{170}Tm	$\frac{7}{2}^- [523] = \frac{1}{2}^- [521]$	-10.4	-46.2	-103.7	-59.6
^{170}Tm	$\frac{5}{2}^+ [411] = \frac{1}{2}^- [521]$	-132.0	-151.9	-136.4	-99.4
^{170}Tm	$\frac{1}{2}^+ [411] = \frac{5}{2}^- [512]$	-231.6	-151.4	-166.3	-104.5
^{170}Tm	$\frac{5}{2}^+ [413] = \frac{1}{2}^- [521]$	176.0	55.6	134.2	109.2

Table (VI.13). continued

Nuclide	Configuration	A_{exp}	A_{LSTMSO}	A_{LANSCE}	A_{BENT}
^{177}Lu	$\frac{1}{2}^+ [404] = \frac{1}{2}^- [514]$	252.9	246.3	250.4	295.3
^{178}Lu	$\frac{3}{2}^+ [514] = \frac{3}{2}^- [514]$	-83.1	-111.2	-200.1	-16.1
^{179}Lu	$\frac{5}{2}^+ [303] = \frac{5}{2}^- [514]$	-104.9	-120.1	-155.4	-92.4
^{180}Lu	$\frac{7}{2}^+ [411] = \frac{7}{2}^- [514]$	126.6	129.1	151.5	132.3
^{181}Ho	$\frac{1}{2}^+ [523] = \frac{1}{2}^- [633]$	72.7	197.8	193.4	141.4
^{182}Ho	$\frac{3}{2}^+ [411] = \frac{3}{2}^- [633]$	197.4	108.5	123.9	111.5
^{183}Ho	$\frac{5}{2}^+ [411] = \frac{5}{2}^- [633]$	-159.2	-97.9	-122.4	-84.6
^{184}Ho	$\frac{7}{2}^+ [413] = \frac{7}{2}^- [633]$	-70.0	-118.9	-159.6	-71.7
^{187}Tm	$\frac{5}{2}^+ [303] = \frac{1}{2}^- [521]$	-141.5	-191.1	-193.3	-37.9
^{188}Tm	$\frac{7}{2}^+ [404] = \frac{1}{2}^- [521]$	146.2	77.1	94.9	79.2
^{189}Lu	$\frac{1}{2}^+ [411] = \frac{1}{2}^- [514] (-134.7)$		-98.7	-144.3	-89.1
^{190}Lu	$\frac{3}{2}^+ [523] = \frac{3}{2}^- [514] (-156)$		-150.9	-197.2	-86.3
^{191}Ho	$\frac{5}{2}^+ [404] = \frac{5}{2}^- [633]$	28.0	-91.9	-127.2	-56.3

Nuclide	Configuration	B_{exp}	B_{LSTMSO}	B_{LANSCE}	B_{BENT}
^{177}Tm	$\frac{1}{2}^+ [411] = \frac{1}{2}^- [521]$	-33.6	-50.2	-51.5	-74.4
^{178}Lu	$\frac{3}{2}^+ [404] = \frac{7}{2}^- [514]$	68.6	25.3	66.3	-5.1
^{180}Ho	$\frac{7}{2}^+ [523] = \frac{7}{2}^- [633]$	-30.6	-12.1	-21.4	-36.5
^{181}Lu	$\frac{5}{2}^+ [523] = \frac{7}{2}^- [514] (-74.2)$		-109.8	-55.2	-91.4
^{182}Ho	$\frac{7}{2}^+ [404] = \frac{7}{2}^- [633]$	-76.0	-56.3	-50.7	-41.4

significance, there is little use in including three different potential parameters in the finite range force in the residual interaction calculations. If it is recognized that the use of different exchange operators does not in any way imply the existence of different potential parameters, one must ask why there are four V_1 parameters in Equation (II-63). The wavefunction of the odd neutron or proton written in terms of spherical harmonics is

$\sum_{l_1 m_1} a_{l_1 m_1} Y_{l_1 m_1} \chi_{l_1 m_1}$. One can write the product in an odd-odd nuclide as $\sum_{l_1 m_1 l_2 m_2} a_{l_1 m_1 l_2 m_2} Y_{l_1 m_1}(1) Y_{l_2 m_2}(2) \chi_{l_1 m_1}(1) \chi_{l_2 m_2}(2)$ plus terms where the spins are exchanged, plus terms where the positions are exchanged, plus terms where both are exchanged.

Expanding the wavefunction

$$H_{INT} = \sum_{\substack{l_1 m_1 \\ l_2 m_2}} \int \int a_{l_1 m_1 l_2 m_2} Y_{l_1 m_1}(1) Y_{l_2 m_2}(2) \chi_{l_1 m_1}(1) \chi_{l_2 m_2}(2) V_{np} a_{l_1 m_1 l_2 m_2} Y_{l_1 m_1}(1) Y_{l_2 m_2}(2) \chi_{l_1 m_1}(1) \chi_{l_2 m_2}(2) d\tau_1 d\tau_2 \quad (VI-12)$$

where the radial dependences are contained in the $\chi(i)$, it is noted that matrix elements for the different exchange operators will have values dependent upon the form of V_{np} , but there is no reason to expect the value of V_{np} to be dependent upon the exchange operator being considered, and certainly no reason to expect that they can have different signs. For a central force it is reasonable to expect that (II-63) takes the form

$$H_{INT} = U_0 \exp\left(-\frac{r}{r_0}\right)^2 \left[1 + \langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle + \langle P_m \rangle + \langle \vec{\sigma}_p \cdot \vec{\sigma}_n P_m \rangle \right] \quad (VI-53)$$

Each of the matrix elements $\langle \exp(-\frac{r}{r_0})^2 \rangle$ will result in scalar values, which can be summed to a single element times U_0 . Then there is a separate contribution from the tensor force. This is analogous to a zero-range interaction calculation plus a tensor force except that more than just $\langle \vec{\sigma}_p \cdot \vec{\sigma}_n \rangle$ is included in this calculation.

An attempt was made to determine a single potential parameter $V_{CENT} = U_0$ plus a potential parameter V_T . It was found that the best fit of $V_{CENT} = -12.618$ MeV and $V_T = -104.342$ for $r_0 = 1.40$ fm produced results comparable to those of using four potential parameters V_2 , V_3 , V_4 , and V_T . The eigenvalues $1A_{CENT}$ and $3A_{CENT}$ are shown in Table (VI.12). Certainly the fit for $1A_{LSTMSQ}$ and $3A_{LSTMSQ}$ is better, but considering there are two more parameters involved, that better fit is probably not meaningful. The obvious remaining problem is to perform some theoretical determination of V_{CENT} and V_T .

VII OFF-DIAGONAL i_K, j_K MATRIX ELEMENTS

The population of the excited neutron configuration $\frac{7}{2}^+ [404] = \frac{9}{2}^+ [624]$ and $\frac{1}{2}^+ [411] = \frac{5}{2}^+ [512]$ in ^{170}Lu and ^{170}Yb respectively by a (t, n) reaction, the unaccounted for intensity in populating the $\frac{5}{2}^+ [411] = \frac{7}{2}^+ [655]$ configuration in ^{170}Yb in an (t, n) reaction, and the anomalous γ -transitions depopulating that configuration in ^{164}Ho , present unique experimental evidence of appreciable off-diagonal matrix elements of the form $\langle \text{BNS} \psi_{i_K}^{\alpha} | \hat{O} | \text{BNS} \psi_{j_K}^{\beta} \rangle$, where $\hat{O}$ is some operator which supports matrix elements of the nature i_K, j_K . It is noted from the form of (1) that it is theoretically possible to couple two different pure two-particle wavefunctions which satisfy the requirement (2) of the residual interaction. However such coupling is not normally expected to be large enough to allow the observation of excited neutron configurations in proton transfer reactions. It is therefore worthwhile to perform the pertinent off-diagonal calculations of the residual interaction to attempt to account for the experimental evidence.

It is thus necessary to calculate the overlap in the initial

$$\langle \frac{1}{2}^+ | \frac{1}{r} | \frac{3}{2}^+ \rangle = 0.624, \langle \frac{1}{2}^+ | \frac{1}{r} | \frac{5}{2}^+ \rangle = 0.514, \langle \frac{3}{2}^+ | \frac{1}{r} | \frac{5}{2}^+ \rangle = 0.411 \quad (III-1)$$

$$\langle \frac{1}{2}^+ | \frac{1}{r} | \frac{5}{2}^+ \rangle = 0.555, \langle \frac{3}{2}^+ | \frac{1}{r} | \frac{5}{2}^+ \rangle = 0.523, \langle \frac{5}{2}^+ | \frac{1}{r} | \frac{5}{2}^+ \rangle = 0.411 \quad (III-2)$$

$$\langle \frac{1}{2}^+ | \frac{1}{r} | \frac{7}{2}^+ \rangle = 0.512, \langle \frac{3}{2}^+ | \frac{1}{r} | \frac{7}{2}^+ \rangle = 0.415, \langle \frac{5}{2}^+ | \frac{1}{r} | \frac{7}{2}^+ \rangle = 0.321 \quad (III-3)$$

Calculation of each of these configurations has been presented in Chapter VII. The Nilsson orbitals were extended to the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ bands, and the matrix elements of $\frac{1}{r}$ in (III-1) were calculated with a finite range central force with $r_0 = 1.60$ fm.

The results of the calculation were listed in Table III-1. However, (III-1) in $\frac{1}{2}^+$ band is -74.4eV for the $K = 1$ band, and -1.1 eV for the $K = 2$ band, and (III-2) in $\frac{3}{2}^+$ band is -1.1 eV for the $K = 1$ band, and -2.1 MeV in the $K = 2$ band. It is obvious that we have to determine the magnitude of the perturbation $\frac{1}{r}$ in terms of these off-diagonal elements, and then to determine the mixing coefficients.

If we use, for example, the eigenvalues of the unperturbed part of the $K^\pi = 1^\pi$ bands of ^{171}Lu , a perturbation $\frac{1}{r}$ can be calculated. The eigenvalues are $\lambda_1 = 12.13$ eV and $\lambda_2 = 12.53$ eV, where λ_1 and λ_2 are defined in Section III.1. The $K = 1$ band can be written

$$\begin{pmatrix} x - 12.13 & -12.53 \\ -12.53 & x - 12.53 \end{pmatrix} = 0 \quad (III-4)$$

Solving for x in the determinant results in $x = 541.59$ or 199.19 keV.

Thus the perturbation is approximately $\epsilon = -(199.19 - 197.1) =$

2.09 keV, and the mixed wavefunction is $\psi_0 = .999\psi_0 + .0175\psi_1$.

It is thus noted that the mixing is significantly larger than the mixing accounted for by the H_{PT} perturbation for mixing the ψ_0 and ψ_1 configurations. That is, the residual interaction very nearly independent of the H_{PT} interaction can be used to account for the population of the excited neutron configuration in ^{17}F .

The more interesting calculations arise from the coupling in ^{16}O and ^{17}O in ^{16}O and ^{17}O as to the contribution from H_{PT} in these cases because they are instances of $^{16}\text{O}(\psi_0) + \psi_1$ and $\psi_0 + \psi_1 = \psi_1$. Introducing the notation

Mixed wavefunction	K	ψ_0 and ψ_1
ψ_1	5	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_2	2	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_3	5	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_4	2	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_5	5	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_6	2	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_7	5	$\psi_0 = .999\psi_0 + .0175\psi_1$
ψ_8	2	$\psi_0 = .999\psi_0 + .0175\psi_1$

the mixed wavefunctions were calculated to be

$\frac{3}{2}^{-}[523] \frac{5}{2}^{-}[521]$ configuration into the $\frac{3}{2}^{-}[411] \frac{1}{2}^{-}[633]$ configuration can account for these transitions, and this component would not be populated in a (t, i) reaction.

Since only 50% of the theoretical intensity of i_1 and i_2 was observed, it is necessary to account for the loss of intensity.

There would appear to be four mechanisms to do this. They are:

1. Coriolis coupling of the $\frac{3}{2}^{-}[411]$ proton with the $\frac{1}{2}^{-}[411]$ proton.
2. Vibrational admixture in the $\frac{3}{2}^{-}[411] \frac{1}{2}^{-}[633]$ configuration.
3. Coriolis coupling of the $\frac{3}{2}^{-}[633]$ neutron with the $\frac{1}{2}^{-}[633]$ neutron.
4. Mixing of the $\frac{3}{2}^{-}[411] \frac{1}{2}^{-}[633]$ configuration with the $\frac{3}{2}^{-}[523] \frac{5}{2}^{-}[521]$ configuration as proposed above.

Coriolis coupling between the $\frac{3}{2}^{-}[411]$ and $\frac{1}{2}^{-}[411]$ proton is expected to be small, and it has been observed to be small in ^{166}Ho . Further if this were the mechanism for reducing the i_1 and i_2 intensity, the unobserved intensity should appear in the $\frac{1}{2}^{-}[411] \frac{1}{2}^{-}[633]$ configurations in ^{166}Ho . Levels in this configuration were first assigned in this work, and an enhanced intensity population then was not observed. One therefore can probably conclude mechanism 1 does not contribute significantly.

The occurrence of a large component of vibrational character in i_1 and i_2 in ^{166}Ho should also appear as a large vibrational admixture

in the $\frac{7}{2}^+ [411]$ band in ^{165}Ho . Experimentally this band has been observed to be very pure.⁴⁴ Thus mechanism 2 probably also does not contribute significantly.

With respect to mechanism 3, the matrix element $\langle \frac{7}{2}^+ [633] | J_+ | \frac{5}{2}^+ [642] \rangle$ is theoretically expected to be large, and it has been calculated by (11-47) to be 6.08. Presuming the $\frac{5}{2}^+ [411] \pm \frac{5}{2}^+ [642]$ configuration to be near 1500keV, which is consistent with $H_{sp}^{(n)}$ calculations, that J_+ matrix element was used in Coriolis calculations to predict the perturbation it causes. The perturbation was found to be approximately 50keV, which is a large Coriolis perturbation. Yet such a large perturbation mixes the wavefunctions only to the extent ψ_1 and $\psi_2 = .98(\frac{5}{2}^+ [411] \pm \frac{5}{2}^+ [633]) + .21(\frac{5}{2}^+ [411] \pm \frac{5}{2}^+ [642])$. With this admixture one should observe approximately 96% of the pure ψ_1 and ψ_2 intensities. Further one ought to observe the $\langle \frac{7}{2}^+ [633] | H_{RPC} | \frac{5}{2}^+ [642] \rangle$ coupling in all states populated in the (t,a) reaction in ^{166}Ho , and thus no loss of intensity in the ψ_1 and ψ_2 states relative to all other $J_p^{\pi} [N, \Lambda 23] \pm \frac{7}{2}^+ [633]$ configurations would be observed. This is not consistent with the experimental evidence.

It is noted that none of the first three mechanisms can explain the observed γ -ray transitions to the $\frac{7}{2}^+ [625] \pm \frac{1}{2}^+ [521]$ configuration. Also given any of the last three mechanisms, the unobserved intensity ought to be seen elsewhere in the alpha-particle spectrum. The $\frac{5}{2}^+ [642]$ and $\frac{5}{2}^+ [521]$ are each deep neutron hole states in ^{166}Ho , and occur at nearly the same energy below the Fermi surface (see

Figure (11-3) at $\epsilon = .3$), so the population of the $\frac{5}{2}^+ [411] \frac{5}{2}^+ [633]$ component in either the $\frac{5}{2}^+ [411] \frac{5}{2}^+ [642]$ or the $\frac{5}{2}^+ [523] \frac{5}{2}^+ [521]$ configuration by mechanism 3 or 4 respectively should result in peaks in the alpha spectrum near 1300 keV. There are states in the (t, α) spectrum of ^{160}Ho at 1376 and 1460. keV with intensities of 55% of (d, d, α) = .007 and .006 mb/sr respectively, which could not be otherwise assigned. It is reasonable to propose that these be the $1^-, K = 5^+, 4$ and $0^-, 1$ states of $\frac{5}{2}^+ [411] \frac{5}{2}^+ [642]$ or the $1^-, K = 5^+, 3$ and $0^-, 5$ states of $\frac{5}{2}^+ [523] \frac{5}{2}^+ [521]$. Either of these assignments yields a moment of inertia of $\frac{\hbar^2}{2I} = 8.07$ keV, which is consistent with other moments of inertia observed in ^{160}Ho . Identification of these two states in no way elucidates the mechanism of admixture mentioned above, but does lend credibility to the observed loss of (t, α) intensity in τ_1 and τ_2 .

It might be expected that the excited neutron component of τ_1 and τ_2 should have been observed in the $^{165}\text{Ho}(d, p)^{166}\text{Ho}$ reaction. However because the $\frac{5}{2}^+ [521]$ neutron orbital is a deep hole in ^{165}Ho , it would not be populated in a stripping reaction, i.e. (d, p), and states of τ_1 and τ_2 were not seen in the (d, p) reaction. It should be possible to observe them in a neutron pickup reaction, i.e. (d, t), but the necessary ^{167}Ho target is not stable.

It ought to be possible to observe these states in ^{164}Ho by the (d, t) reaction on ^{165}Ho . This reaction was performed by Jones and Shelton⁴⁷, and the $\frac{5}{2}^+ [523] \frac{5}{2}^+ [521]$ configuration was observed near 400 keV as shown in Figure (11.1). It was noted that the

Figure (VII.1). Spin and parity assignments of states in ^{163}Ho taken from Jones and Sheline.⁷⁷ All energies are in keV and appear to the right of each level. Spin and parity assignments lie to the left of each level, and configuration designations appear below each rotational band. Unassigned levels observed in the (d,t) reaction appear at the extreme right.

14.01
 11.52
 13.5

0.014

4. 0.94

0.67

5. $\frac{0.75}{[2.3] \cdot [1.5]}$

0.5

4. $\frac{0.33}{[2.3] \cdot [1.0]}$

0.18

0.19

0.25

0.25

unavailable

5. 777

4. $\frac{670}{[2.3] \cdot [1.0]}$

3. $\frac{620}{[2.3] \cdot [1.0]}$

2. $\frac{586}{[2.3] \cdot [1.0]}$

3. $\frac{343}{[2.3] \cdot [1.0]}$

4. $\frac{262}{[2.3] \cdot [1.0]}$

5. $\frac{186}{[2.3] \cdot [1.0]}$

6. $\frac{94}{[2.3] \cdot [1.0]}$

7. $\frac{87}{[2.3] \cdot [1.0]}$

8. $\frac{0}{[2.3] \cdot [1.0]}$

9. $\frac{0}{[2.3] \cdot [1.0]}$

10. $\frac{0}{[2.3] \cdot [1.0]}$

11. $\frac{0}{[2.3] \cdot [1.0]}$

12. $\frac{0}{[2.3] \cdot [1.0]}$

13. $\frac{0}{[2.3] \cdot [1.0]}$

14. $\frac{0}{[2.3] \cdot [1.0]}$

15. $\frac{0}{[2.3] \cdot [1.0]}$

16. $\frac{0}{[2.3] \cdot [1.0]}$

17. $\frac{0}{[2.3] \cdot [1.0]}$

18. $\frac{0}{[2.3] \cdot [1.0]}$

19. $\frac{0}{[2.3] \cdot [1.0]}$

20. $\frac{0}{[2.3] \cdot [1.0]}$

21. $\frac{0}{[2.3] \cdot [1.0]}$

22. $\frac{0}{[2.3] \cdot [1.0]}$

23. $\frac{0}{[2.3] \cdot [1.0]}$

$1^{\overline{7}}, K = 5^+, 5$ state at 843kev was populated with only about 75% of the theoretically predicted intensity, and other states in this configuration are too highly perturbed by Coriolis coupling to justify rigorous discussion of their intensities. If the lost intensity is real and due to an admixture of the $\frac{5}{2}^+ [411] \frac{7}{2}^+ [633]$ configuration, states of that configuration ought to be weakly populated with the remaining 25% of the intensity of the $\frac{7}{2}^+ [523] \frac{5}{2}^+ [521]$ configuration.

From α_{sp} evidence in ^{165}Dy and ^{165}Ho the $\frac{5}{2}^+ [411] \frac{7}{2}^+ [633]$ bands ought to occur near 900keV in ^{164}Ho . Population of states in this configuration ought to occur with consistent relative intensities as those of the analogous states in the $\frac{7}{2}^+ [523] \frac{5}{2}^+ [521]$ configuration. Assigning the levels at 865, 967, and 1066keV in Figure (VII.1) as the $1^{\overline{7}}, K = 5^+, 5, 6^+, 5$, and $7^{\overline{7}}, 5$ states of the $\frac{5}{2}^+ [411] \frac{7}{2}^+ [633]$ configuration is in good agreement with that prediction; $\frac{\hbar^2}{2I} = 9.77\text{keV}$. The cross-sections of these states plus those of the $K^{\overline{7}} = 5^+$ band from the $\frac{7}{2}^+ [523] \frac{5}{2}^+ [521]$ configuration are listed in Table (VII.1). Note if one takes the average of

$\frac{(d\sigma/d\Omega)_{865}}{(d\sigma/d\Omega)_{843}}$ over all three angles, he obtains $\frac{.29}{.429} \frac{.31}{.121} \frac{.46}{.111} \frac{1}{3} = .31$.

Using this relative intensity of the $\frac{5}{2}^+ [411] \frac{7}{2}^+ [633]$ configuration to the $\frac{7}{2}^+ [523] \frac{5}{2}^+ [521]$ configuration, one would predict the intensities of the 967keV state to be $.31(77) = 24.4\text{nb/sr}$ at 75° , $.31(32) = 25.4\text{nb/sr}$ at 83° , and $.31(76) = 25.4\text{nb/sr}$ at 95° . This is in excellent agreement with the observed values of 24, 23, and 30 nb/sr.

Table (VII.1). Energies and cross-sections populating the $\frac{5}{2}^+$ [111] + $\frac{7}{2}^+$ [633] and $\frac{7}{2}^-$ [523] + $\frac{5}{2}^-$ [321] configurations from the $^{164}\text{Ho}(d,t)^{163}\text{Ho}$ reaction.

Level Energy (keV)	I	K	75°	$\frac{d\sigma}{dE}$ 85	$\frac{d\sigma}{dE}$ 95
343	5	5	129	1.1	1.1
454	6	5	77	82	76
583	7	5	42	55	47
865	5	5	29	34	46
967	6	5	24	23	36
1084	7	5			

There is thus consistent evidence for the admixture $\left[\frac{3}{2}^+ [411] \pm \frac{7}{2}^+ [633]\right]$ + $\left(\frac{5}{2}^+ [513] \pm \frac{3}{2}^+ [511]\right)$ in both ^{164}Ho and ^{166}Ho . A very useful experiment might be the $^{164}\text{Dy}(d, 2n); ^{164}\text{Ho}$ reaction to observe the γ -rays depopulating the levels at 365, 967, and 1084 keV in ^{164}Ho .

Since the residual neutron-proton interaction seems unable to account for the population of the $\frac{1}{2}^+ [411] \pm \frac{5}{2}^+ [513]$ configuration in ^{170}Er and for the population of the $\frac{3}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration in ^{164}Ho and for the low experimental intensities and γ -transitions of the $\frac{3}{2}^+ [411] \pm \frac{7}{2}^+ [633]$ configuration in ^{166}Ho , it is useful to propose some operator which can specifically couple states which satisfy the requirement $\delta_{K',K} \delta_{N',N} \delta_{p',p} \delta_{n',n} \pm 1$. Recalling that the H_{pp} operator calls states of the form $\delta_{K',K} \delta_{N',N} \delta_{p',p} \delta_{n',n} \pm 1$, some operator related to the square of the H_{pp} operator should support off-diagonal matrix elements of the desired form.

A possible operator $\hat{B}$ which can couple states satisfying

$$\delta_{K',K} \delta_{N',N} \delta_{p',p} \delta_{n',n} \pm 1 \text{ is}$$

$$H_{\hat{B}} = B \left[J_{p_+} J_{p_+} J_{n_-} J_{n_-} + J_{p_-} J_{p_-} J_{n_+} J_{n_+} \right] \quad (\text{VII-5})$$

where B is a normalization factor. $H_{\hat{B}}$ would support matrix elements between the configurations of equations (VII-2) and (VII-3). The general form of the off-diagonal elements of $\hat{B}$ is

$$\begin{aligned}
\langle \text{BMK} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} | \text{BMK} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix} \rangle &= 48 \begin{smallmatrix} 7 \\ J_p \end{smallmatrix} \begin{smallmatrix} 7 \\ J_n \end{smallmatrix} \begin{smallmatrix} 7 \\ J_{p+} \end{smallmatrix} \begin{smallmatrix} 7 \\ J_{p-} \end{smallmatrix} \begin{smallmatrix} 7 \\ J_{n+} \end{smallmatrix} \begin{smallmatrix} 7 \\ J_{n-} \end{smallmatrix} \\
&\times (J_p + J_{p+} + 1)(J_p + J_{p-} + 1)(J_p + J_{p+} + 1)(J_p + J_{p-} + 1)(J_n + J_{n+} + 1)(J_n + J_{n-} + 1) \\
&\quad (J_n + J_{n+} + 1)(J_n + J_{n-} + 1) \quad (\text{VII-6}) \\
&\quad (J_n + J_{n+} + 1)(J_n + J_{n-} + 1) \quad (\text{VII-7})
\end{aligned}$$

$$\chi_{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}}^{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}}$$

This comes from, for example

$$\begin{aligned}
(J_{p+} J_{p-} J_{n+} J_{n-}) (\chi_{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}}^{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}}) &= (J_{p+} J_{p-} J_{n+} J_{n-}) (\chi_{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}}^{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}})^{\frac{1}{2}} (J_{n+} J_{n-} J_{n+} J_{n-})^{\frac{1}{2}} \\
&= (J_{p+} J_{p-}) (\chi_{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}}^{\begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 2 \\ 2 \end{smallmatrix} \begin{smallmatrix} 1 \\ 1 \end{smallmatrix}})^{\frac{1}{2}} (J_{n+} J_{n-}) (J_{n+} J_{n-})^{\frac{1}{2}} \\
&\quad (J_{n+} J_{n-} + 1)(J_{n+} J_{n-} + 1) \quad (\text{VII-7})
\end{aligned}$$

Successively applying the two $J_{p\pm}$ operators results in the term in brackets of Equation (VII-6). Each operation in (VII-7) would be preceded by $C_{J_{\pm}}$ coefficients and summed over the appropriate J 's.

When coupling the e.g. $\frac{7}{2}[633]$ to the $\frac{5}{2}[521]$, where these are denoted by χ_a^2 and χ_b^2 respectively, one first couples χ_a^2 to all orbitals with $J_n = \frac{5}{2}$, obtaining $\sum_{J_n=\frac{5}{2}} \begin{smallmatrix} 7 \\ J \end{smallmatrix} C_{J_n}^{5/2} C_{J_n}^2 (J+\frac{7}{2})(J-\frac{5}{2}+1)^{\frac{1}{2}}$. The

next application of J_{n+} takes one from any neutron orbital with $J_n = \frac{5}{2}$ to the orbital χ_b^2 . The final result of $J_{n+} J_{n-}$ is

$$\sum_{J=\frac{3}{2}}^{\frac{7}{2}} \sum_{n=\frac{3}{2}}^{\frac{5}{2}} C_{Jn}^{\frac{3}{2}} (C_{Jn}^{\frac{5}{2}})^2 C_{Jn}^{\frac{3}{2}} (J-\frac{7}{2})(J-\frac{5}{2}-1)(J-\frac{5}{2})(J-\frac{3}{2}-1) \frac{1}{2}$$

The sum $C_{Jn}^{\frac{5}{2}}$ over all $n = \frac{3}{2}$ orbitals is just equal to 1 for $N = 5$ and 1 for $N = 6$ as the C_{Jn} are orthonormal, so the sum is equal to

2. So finally for $J_n = J_{n-}$ one has

$$2 \sum_{J=\frac{3}{2}}^{\frac{7}{2}} C_{Jn}^{\frac{3}{2}} C_{Jn}^{\frac{5}{2}} (J-\frac{7}{2})(J-\frac{5}{2}-1)(J-\frac{5}{2})(J-\frac{3}{2}-1) \frac{1}{2} \quad (\text{VII-ea})$$

Similar arguments with the proton orbital result in Equation (VII-b).

The calculation of the matrix elements of

$$\frac{7}{2}[-5233] \pm \frac{5}{2}[-5211] \pm \frac{5}{2}[-4111] \pm \frac{7}{2}[-6551] \quad (\text{VII-3})$$

and

$$\frac{1}{2}[-4111] \pm \frac{5}{2}[-5111] \pm \frac{5}{2}[-4151] \pm \frac{1}{2}[-5211] \quad (\text{VII-4})$$

is performed below. The results are of course strongly dependent upon the normalization factor B , which converts the unitless matrix element of (VII-ea) to a matrix element with units of energy.

However it is noteworthy that the unitless matrix elements of $\frac{7}{2}$ are much larger than those of $H_{pp}/\frac{\hbar^2}{41}$. The value of $\frac{5}{2}$ for (VII-3) is -17.067 and for (VII-2) is 77.224.

Evaluating (VII-3), it is noted that all $J_p = \frac{3}{2}, \frac{5}{2}$, and $\frac{11}{2}$ terms are zero because the $\frac{5}{2}[-4111]$ orbital has no $J = \frac{11}{2}$ components and the $\frac{7}{2}[-5233]$ orbital has no $J = \frac{3}{2}$ or $\frac{5}{2}$ components. By similar reasoning all $J_n = \frac{3}{2}$ and $\frac{5}{2}$ terms are zero. The $C_{Jn}^{\frac{3}{2}}$

factors of (VII-6) are shown in Table (VII.2), and the individual $[J_p(J_p+1), \dots, J_n(J_n+1)]^{\frac{1}{2}}$ factors are shown in Table (VII.3). The sum in (VII-6) is finally $\{(.0013391)(84.00) + (.0074872)(168.0) - (-.0023277)(269.4) + (.000958)(168.0) - (0.0052778)(356.0) + (0.0016403)(558.8)\} = -17.067$. The calculation of (VII-8) is very similar involving terms of $J_p = \frac{5}{2}, \frac{7}{2}, \text{ and } \frac{9}{2}$ and $J_n = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \text{ and } \frac{11}{2}$.

The determination of B is dependent upon where ϵ ultimately comes from. One might expect that ϵ could come from an $\frac{\hbar^2}{2I_4} \bar{R}_4^2$ perturbation, where I_4 is the fourth rotational moment of inertia, since H_{pp} comes from the $\frac{\hbar^2}{2I} \bar{R}^2$ term ultimately. Including such a term in the nuclear Hamiltonian seems reasonable as it is frequently included in empirical equations for rotational energies.

$$\epsilon_{I,K} = \epsilon_{K,K} + A[I(I+1)] + B[I^2(I+1)^2] \quad (\text{VII-10})$$

When large contributions from the B term are noted in rotational energy bands, spectroscopists have generally explained it by including Coriolis coupling effects. However significant B term contributions occur even in even-even nuclei where Coriolis and H_{pp} perturbations are rigorously forbidden. So there would appear to be some justification for including a $\frac{\hbar^2}{2I_4} \bar{R}_4^2$ term in the rotational Hamiltonian.

If $\bar{R}_4^2 = [(I-J)^2]^2$ is expanded, one obtains $(I^2 + J^2 - 2I \cdot J)^2$. There then results a J^4 term plus many others when the latter quantity is squared. Treating this J^4 term as $J^2 J^2$, it can be expanded to

$$J^2 J^2 = (J_p^2 + J_n^2 + 2J_p \cdot J_n)^2 \quad (\text{VII-11})$$

Table (VII.2). $C_{j_p j_n}^{\bar{j}}$ factors for individual j_p, j_n terms of (VII-6) for the $\frac{7}{2}^- [523] + \frac{3}{2}^- [521] + \frac{3}{2}^+ [411] + \frac{5}{2}^- [673]$ matrix elements.

j_p	j_n	$(C_{j_p}^{\bar{j}})_2 (C_{j_n}^{\bar{j}})_0 (C_{j_n}^{\bar{j}})_{-2} (C_{j_p}^{\bar{j}})_{-2}$
$\frac{7}{2}$	$\frac{7}{2}$	.004333
$\frac{7}{2}$	$\frac{9}{2}$	-.007457
$\frac{7}{2}$	$\frac{11}{2}$	-.002327
$\frac{9}{2}$	$\frac{7}{2}$	.000233
$\frac{9}{2}$	$\frac{9}{2}$	-.003271
$\frac{9}{2}$	$\frac{11}{2}$	-.00104

Table VIII.7. Individual angular momentum eigenvalues and

lowering operator factor of (VII-1) for the

$\frac{1}{2} [323] + \frac{1}{2} [321] + \frac{3}{2} [411] + \frac{1}{2} [633]$ matrix element.

$$\frac{1}{2} j_p - \frac{1}{2} j_n = \frac{1}{2} (j_p - \frac{1}{2}) (j_p + \frac{1}{2} - 1) (j_p - \frac{1}{2} + 1) (j_p - \frac{1}{2} + 1 + 1) \dots$$

$$\times (j_n + \frac{1}{2}) (j_n - \frac{1}{2} + 1) (j_n + \frac{1}{2} - 1) (j_n + \frac{1}{2} - 1 + 1) \dots$$

$\frac{1}{2} j_p$	$\frac{1}{2} j_n$	SLC
$\frac{1}{2}$	$\frac{1}{2}$	84.0
$\frac{3}{2}$	$\frac{3}{2}$	168.0
$\frac{5}{2}$	$\frac{11}{2}$	269.4
$\frac{7}{2}$	$\frac{7}{2}$	168.0
$\frac{9}{2}$	$\frac{9}{2}$	336.0
$\frac{11}{2}$	$\frac{11}{2}$	538.8

which yields a final term of $4(J_p \cdot J_n)(J_p \cdot J_n)$ in its expansion. Disregarding all the terms except the dot product term, one obtains

$$4(J_p \cdot J_n)(J_p \cdot J_n) = 4(J_1^p J_1^n + J_2^p J_2^n + J_3^p J_3^n) \quad (\text{VII-12})$$

If (VII-12) is expanded and all terms not containing J_2^p or J_3^n are collected, it is found that the collected terms can not be replaced by an expansion of $[J_1^p J_1^n + J_2^p J_2^n + J_3^p J_3^n]$. It therefore does not appear to be possible to account for the occurrence of a perturbation δ by including an $\frac{\hbar^2}{2I_A} K^4$ term in the Hamiltonian, even though there does appear to be some evidence for such a term. Further the normalization factor δ from such a term would be very small. With that factor the matrix elements of δ would be much smaller than those of H_{INT} . It must finally be concluded that some perturbation should be included in the nuclear Hamiltonian (II-1) which can strongly couple some states which satisfy the requirement $\delta K, \delta \alpha_p, \delta \alpha_n \rightarrow \delta \alpha_p, \delta \alpha_n$, or the existing theory of evaluating the off-diagonal elements of H_{INT} must be investigated.

VIII. CONCLUSION

Eighty-one new states in excited proton configurations or in vibrational excited states have been proposed in ^{170}Lu , ^{170}Tm , ^{160}Ho , and ^{160}Tb . The Nilsson model has been found to provide a consistent and accurate description of the two-quasiparticle states in these odd-odd deformed nuclei.

Coriolis coupling calculations have provided a significant improvement of the interpretation of the nuclear data. An independent parameterization method for treatment of the Coriolis interaction has been introduced for dealing with up to a four by four Hamiltonian matrix. Results of these parameterization calculations has provided rewarding agreement with experimental and theoretical Coriolis effects.

Residual interaction calculations have been used to account for the singlet-triplet splittings and the odd-even Newby shifts observed in the data presented. Good agreement between experimental and theoretical splitting values has been noted, however the significance of finite range calculations with free potential parameters for each of the Bartlett, Majorana, and Heisenberg operators seems to be tenuous. It is concluded that theoretical calculations with the use of a tensor force plus only a single central force neutron-proton interaction potential parameter provides suitable agreement with experiment.

Evidence for large off-diagonal matrix elements between states which satisfy the relation $\delta_{K,K}$ has been observed in the (τ, κ) studies. Calculation of residual interaction perturbations has been used to explain the mixing of two $K^\pi=1^+$ bands in ^{176}Lu where the eigenstates are related by $\delta_{I,1} \delta_{K,K} \delta_{\Omega_p', \Omega_p} \pm 1$. The residual interaction has been found to be unable to account for the mixing between two $K^\pi=2^+$ and 5^+ bands in ^{166}Ho and two $K^\pi=2^-$ and 5^- bands in ^{170}Yb . The coupled states in these cases are related by

$$\delta_{I,1} \delta_{K,K} \delta_{\Omega_p', \Omega_p} \pm 2 \delta_{\Omega_n', \Omega_n} \mp 2 \quad (\text{VIII-1})$$

The existence of some new perturbation to the nuclear Hamiltonian which specifically couples states which are related by (VIII-1) has been postulated. A general form of the off-diagonal element of this perturbation has been presented and discussed.

REFERENCES

- ¹G. J. Gallagher, Jr. and S. A. Moszkowski, Phys. Rev. 117 (1958), 1282.
- ²J. Rainwater, Phys. Rev. 79 (1950) 432.
- ³S. G. Nilsson, Mat. Fys. Medd. Dan. Vid. Selsk. 29, No. 16 (1955).
- ⁴B. Nilsson, Nucl. Phys. A129 (1969) 445.
- ⁵S. G. Nilsson, C. F. Tsang, A. Sabiczewski, E. Szymanski, S. Wysocki, C. Gustafson, I. L. Lamm, P. Moller, and B. Nilsson, Nucl. Phys. A131 (1969) 1.
- ⁶M. A. Preston, Physics of the Nucleus (Addison-Wesley, Reading, 1960).
- ⁷R. K. Sheline, C. E. Watson, B. P. Maier, U. Gruber, R. H. Koch, O. W. B. Schult, H. T. Mott, E. T. Journey, G. L. Struble, T. von Egidy, Th. Elze, and E. Bieber, Phys. Rev. 115 (1966) 857.
- ⁸H. T. Mott, E. T. Journey, O. W. B. Schult, H. R. Koch, U. Gruber, B. P. Maier, H. Baader, G. L. Struble, J. Kern, R. K. Sheline, T. von Egidy, Th. Elze, E. Bieber, and A. Backlin, Phys. Rev. 155 (1967) 1265.
- ⁹N. D. Newby, Jr., Phys. Rev. 125 (1962) 2063.
- ¹⁰H. D. Jones and R. K. Sheline, Annals of Phys. 63 (1971) 28.
- ¹¹N. I. Pyatov, Izv. Akad. Nauk, SSSR, Ser. Fiz., 27 (1963) 1456 [Transl: Bull. Acad. Sci. USSR, Phys. Ser. 27 (1963) 1909].

12. T. P. Bolsson, R. Piepenbring, and W. Ogile, Phys. Letter 53D (1973) 142.
13. H. D. Jones, N. Onishi, T. Hess, and R. K. Shelton, Phys. Rev. 30 (1971) 529.
14. D. Elmore, Ph.D. Dissertation, Univ. of Rochester, 1974 (University Microfilms, Ann Arbor, Mich. 1975).
15. J. D. Hunt, Univ. of Colorado, computer program GUK72, private comm.
16. J. D. Imelo, Lawrence Berkeley Laboratory Report LBL-675, University of California (1972).
17. G. R. Satchler, Ann. Phys. 5 (1958) 275.
18. H. H. Macfarlane and J. B. French, Rev. Mod. Phys. 32 (1960) 167.
19. H. Y. Lu and W. P. Alford, Phys. Rev. C3 (1971) 1245.
20. B. Nilsson, program CJL. Private Comm.
21. R. G. Lanier, Ph.D. Dissertation, Fla. St. Univ. (1968).
22. E. R. Flynn, S. Orbesen, J. D. Sherman, J. W. Sunier, and R. Woods, Nucl. Inst. and Methods 128 (1975).
23. R. D. Bagnell, Y. Tanaka, R. K. Shelton, and J. D. Sherman, Phys. Rev. C20 (1979) 41.
24. Y. Yamazaki, R. K. Shelton, and D. G. Burke, Z. Physik A235 (1978) 191.
25. D. G. Burke and J. C. Tippet, Nucl. Inst. and Methods, 65 (1968) 353.
26. N. Onishi, computer program GMDNS, Private Comm.
27. R. Lasijo, Ph.D. Dissertation, Florida St. Univ. (1977).

- ²⁸K. E. G. Lobner, M. Vetter, and V. Honig, Nucl. Data Tables, A7 (1970) 495.
- ²⁹E. R. Flynn, R. A. Hardekopf, J. D. Sherman, J. W. Sunier, and J. C. Coffin, Nucl. Phys. A279 (1977) 399.
- ³⁰G. Lövhöiden, D. G. Burke, E. R. Flynn, and J. Sunier, unpublished data.
- ³¹D. G. Burke, G. Lövhöiden, E. R. Flynn, and J. Sunier, Phys. Rev. C18 (1978) 693.
- ³²C. R. Herring, D. G. Burke, E. R. Flynn, J. W. Sunier, P. A. Schmelzbach, and R. F. Haglund, Nucl. Phys. A287 (1977) 21.
- ³³M. E. Bunker and C. W. Reich, Rev. Mod. Phys. 43 (1971) 343.
- ³⁴G. L. Struble, Ph.D. Dissertation, Fla. St. Univ. (1964).
- ³⁵M. Minor, R. K. Shelton, E. B. Sherra, and E. T. Jurney, Phys. Rev. 187 (1969) 1516.
- ³⁶B. P. K. Maier, Z. Phys. 184 (1965) 153.
- ³⁷M. K. Balodis, J. J. Tambergs, K. J. Alsknis, P. T. Prokofjev, W. G. Vonach, H. K. Vonach, H. R. Koch, U. Gruber, B. P. K. Maier, and O. W. B. Schult, Nucl. Phys. A191 (1972) 305.
- ³⁸P. T. Prokofjev, M. K. Balodis, J. J. Bercini, V. A. Bondarenko, N. P. Kramer, E. J. Lure, G. L. Resvaja, and L. I. Siminova, Atlas of the conversion electron spectra emitted by thermal neutron capture in nuclei with 143Ac107, and the radioactive transition schemes (Finatre, Riga, 1967) p. 75.
- ³⁹C. J. Gallagher, Jr., A. Namenson, and P. C. Sims, Nucl. Phys. 49 (1965) 443.
- ⁴⁰H. Heinzelmann, Z. Physik 181 (1964) 347.

- ⁴¹L. S. Goodman, W. J. Childs, R. Marrus, E. P. E. Lindgren, and F. Cabezas, *Bull. Am. Phys. Soc.* 5 (1960) 544.
- ⁴²G. L. Struble, V. Shelton and R. K. Sheline, *Phys. Rev. Lett.* 10 (1963) 58.
- ⁴³G. L. Struble, J. Kern, and R. K. Sheline, *Phys. Rev.* 137 (1960) B772.
- ⁴⁴L. R. Wagner, D. G. Burke, H. C. Cheung, P. Kleinheins, and R. K. Sheline, *Nucl. Phys.* A246 (1975) 45.
- ⁴⁵R. K. Sheline, C. E. Watson, B. P. Maier, U. Gruber, R. H. Koch, G. W. B. Schult, H. T. Metz, E. T. Jurney, G. L. Struble, T. V. Egidy, Th. Elze, and E. Bieber, *Phys. Rev.* 143 (1966) 557.
- ⁴⁶J. P. Boisson, R. Piepenbring, and W. Ogle, *Phys. Reports* 2nd (1976) 99.
- ⁴⁷H. D. Jones and R. K. Sheline, *Nucl. Phys.* A150 (1976) 407.
- ⁴⁸Harris, *Phys. Rev.* 138 (1965) B509.

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