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**FIN STABILIZATION OF
MAGNETICALLY CONFINED PLASMAS**

Final Report

for Period September 15, 1976 - December 14, 1978

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Abstract

The effect of different outer conducting configurations on the stability of a cylindrical plasma of radius a is treated with emphasis on the $k \sim 0$, $n = 1$ mode associated with the usual surface perturbation $e^{ikz \pm in\theta}$. The conducting configuration comprises of either one or more segments of a cylindrical shell or conducting cylinder with gaps in it, or segments lying in the radial plane. The stability effects of the various geometries are compared through their resulting value for the minimum vacuum energy integral. An asymptotic technique is developed for fins of infinite extent. For a/c sufficiently small, fins in the radial planes $\theta = 0, \pi$ and $\pm\pi/2$ at a distance c from the axis, tend to have the same stabilization effect on the $n = 1$ mode as a concentric outer cylinder of radius B where $B = \sqrt{2}c$. For more general geometry including fins of finite extent, the problem is then formulated in terms of a singular differential-integral equation which is then reduced to a regular Fredholm integral equation which can be solved numerically. Results are given for a circular segment $(-\alpha < \theta < \alpha, r = A)$ for various values of the half-angle α and radius A , both in the presence of and absence of a complete circular outer conductor.

Final Report

The effect of different configurations of the outer conducting surfaces on the stability of a cylindrical plasma column of length L with circular cross section of radius a was investigated.

The study of the effect of changing the outer configuration on the stability was based upon the investigation of the corresponding change in the minimum vacuum energy integral for various perturbations of the plasma surface. This approach was taken to avoid having to consider the particular choices of internal magnetic field in the plasma, and hence obtain as general results as possible.

The choice of the outer configuration depends upon the required external magnetic field B_0 . So as to choose a configuration which disturbs the prescribed magnetic field as little as possible, the conducting surfaces were taken to be along the field lines. The external magnetic field was taken to have the form

$$B_0 = B_0 \left[\hat{z} + \frac{h}{r} \hat{\theta} \right]$$

where the constant h is related to the pitch of the field lines.

For the case where the pitch is extremely small, so that for a distance L the radial magnetic surfaces can be approximated by radial planes, the conducting configuration consist of a single or multiple set of radial fins.

Only perturbations of the plasma surface of sufficiently large wavelength such that the thickness of the fins was much less than these wavelengths were considered. Consequently, for mathematical convenience, the conducting fins or radial plane were taken to have infinitesimal or vanishing thickness.

The case of the external conductor configuration comprising a single radial fin or conducting half-plane ($\theta = 0, c \leq r < \infty$) whose edge was a distance c from the center of the plasma column was treated first. In order to get simple analytic answers when the parameter a/c is small, the conducting sheet was taken to be of infinite extent.

Using the Green's function for a half-plane, an integral equation was developed and solved. It was shown that the single fin (in plane $\theta = 0$) only effects odd perturbations and has the same effect on the odd portion ($\sin \theta$) of the $n = 1$ mode as a circular outer conductor of radius $b \sim \sqrt{8} c$. The same approach using the Green's function for two infinite half-planes was applied to multiple fins. It is shown that two fins a distance c from the center of the plasma, and on opposite sides are as effective in reducing the odd portion of the $n = 1$ mode as a circular outer conductor of radius $b = \sqrt{2} c$. Additional fins in the radial planes $\theta = \pm \pi/2$, correspondingly take care of the even portion ($\cos \theta$) of the $n = 1$ mode.

Subsequent to this, a general technique was developed which allows one to treat not only radial fins of finite extent, but configurations

composed of multiple segments of cylindrical shells of arbitrary cross-section (thickness much less than the characteristic wavelength of the perturbation to be considered).

A detailed treatment of a single cylindrical segment whose cross-section was an arc of a circle concentric to the plasma surface was carried out. The analysis included the possibility of an addition of a circular conducting cylinder of radius b enclosing the system. The parameters of the circular segment were radius A and angle 2α , the angle subtended at the axis by the edges of the arc.

In addition the technique was applied to a single radial segment of finite length ($c_1 < r < c_2$, $\theta = 0$) enclosed by a circular outer conductor of radius $b > c_2$.

The fundamental mathematical problem involved solving for the vacuum field perturbation $\delta \underline{B} = \nabla \Psi$ where

$$\Psi = e^{ikz} \psi(x, y), \quad k \sim 0.$$

The resulting problem

$$\Delta \psi - k^2 \psi = 0, \quad r > a$$

$$\frac{\partial \psi}{\partial r} = f(\theta), \quad r = a \quad (\text{plasma surface})$$

$$\frac{\partial \psi}{\partial n} = 0 \quad \text{on arc } \mathcal{C}_2,$$

$$\frac{\partial \psi}{\partial n} = 0 \quad \text{on } r = b,$$

where the arc $\mathcal{C}_2$ is the cross-section of the conducting cylindrical segment, and $f(\theta)$ is a given function describing the perturbation, is solved using the Green's function $\mathcal{G}(\underline{x}_0; \underline{x})$

$$\Delta \mathcal{G} - k^2 \mathcal{G} = \delta(\underline{x} - \underline{x}_0)$$

$$\frac{\partial \mathcal{G}}{\partial n} = 0 \quad , \quad r = a \quad \text{and} \quad r = b$$

At a point $\underline{x}_0$ in the vacuum region, the vacuum potential has the form

$$\psi(\underline{x}_0) = \int_{r=a} \mathcal{G}(\underline{x}_0, \underline{x}) f(\theta) ds - \int_{\mathcal{C}_2} \left(\frac{\partial \mathcal{G}}{\partial n} \right)_+ \phi(s) ds$$

where $\phi(s) = \psi^+(s) - \psi^-(s)$,

with $\psi^\pm(s)$ being the limits from the positive and negative sides of $\mathcal{C}_2$, and s the arc-length on the respective curves.

The boundary condition

$$\phi = 0 \quad \text{at end-points of the arc} \quad ,$$

was imposed on the unknown function ϕ . (This condition is related to a condition of finite magnetic energy).

The singular part of the Green's function as split off

$$\mathcal{G}(\underline{x}_0; \underline{x}) = \frac{1}{2\pi} \ln |\underline{x}_0 - \underline{x}| + u(\underline{x}_0, \underline{x}) \quad ,$$

then the limit $\left(\frac{\partial \psi}{\partial n} \right)_+$ of the above integral representation for $\psi(\underline{x}_0)$

was taken, and the Neumann boundary condition on $\mathcal{C}_2$ was imposed. The following singular differential - integral equation was then obtained

$$\frac{1}{\pi i} \int_{\mathcal{C}_2} \frac{\cos \alpha_0}{|\underline{x} - \underline{x}_0|} \phi'(s) ds - \int_{\mathcal{C}_2} k(\underline{x}_0, \underline{x}) \phi(s) ds = h(\underline{x}_0)$$

where $h(\underline{x}_0)$ is a known function, the kernel $k(\underline{x}_0, \underline{x})$, has at most a logarithmic singularity and α_0 is the angle between the vector $\underline{x}_0 - \underline{x}$ and the unit tangent vector $\underline{x}'(s_0)$ at $\underline{x}_0$. The Cauchy principle value of the singular integral is taken.

The singular integral equation technique of Muskhelishvili was employed to transform the above equation to an equation of the form (for a single arc)

$$\phi'(s_0) + K\phi = F(s_0) + CB(s_0)$$

where K is an integral operator, $F(s_0)$ and $B(s_0)$ are known functions, and the unknown constant C is determined through the boundary condition that ϕ vanishes at the end-points of the arc. Integration of the system yields a regular Fredholm integral equation which can be solved numerically.

The technique just described was applied in detail to a single segment, but the generalization of its applicability to multiple segments was indicated.

Computations were then performed for various values of the parameters A, α, b for a single circular segment. The resulting coefficients ψ_{mn}^e , ψ_{mn}^o (corresponding to even and odd modes) in the quadratic form representation of the minimum vacuum energy integral

$$\delta w_{\text{vac}}^{\text{min}} = \frac{-L}{2\mu_0\pi} \left\{ \sum_{n=0} \sum_{m=0} b_n^e \psi_{mn}^e b_m^e + \sum_{n=0} \sum_{m=1} b_n^o \psi_{mn}^o b_m^o \right\}$$

were then obtained. The coefficients $\psi_{11}^{e,o}$ are associated with the important kink $n = 1$ mode. The non-vanishing of the off-diagonal coefficients $\psi_{mn}^{e,o}$ ($m \neq n$) indicated the coupling between the various modes.

In addition, some calculations were performed for a single radial fin $\theta = 0$, $c_1 < r < c_2$ ($c_1 > a$) in the presence of an outer circular conductor of radius $b > c_2$.

Interpretation of the results and the comparison of the various geometries to the usual outer conducting circular cylinder on stability was presented.

The results were presented at the annual meeting of the Division of Plasma Physics, of the American Physical Society, Colorado Springs, CO, October 30 - November 3, 1978.

CYLINDRICAL SHELL
GEOMETRY

CYLINDRICAL SHELL
GEOMETRY

FIN GEOMETRY