4391

LA-UR-90-

Received by OSTI

Conf-9009269-5

JAN 08 1991

LA-UR--90-4391

DE91 005940

Los Alamos National Laboratory is operated by the University of California for the United States Department of Energy under contract W-7405-ENG-36.

**TITLE: FLIP MHD: A PARTICLE-IN-CELL METHOD
FOR MAGNETOHYDRODYNAMICS**

AUTHOR(S): J. U. Brackbill, T-3

**SUBMITTED TO: Presented at the U.S.-Japan Workshop on Advances in Computer
Simulation Techniques Applied to Plasmas and Fusion
September 26-28, 1990
UCLA, Los Angeles, CA 90024-1547**

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

**Los Alamos National Laboratory
Los Alamos, New Mexico 87545**

FLIP MHD: A Particle-in-Cell Method for
Magnetohydrodynamics

J.U.Brackbill
Los Alamos National Laboratory
Los Alamos, NM 87545

A particle-in-cell (PIC) method, FLIP [1], is extended to magnetohydrodynamic (MHD) flow in two dimensions. Particles are used to reduce computational diffusion of the magnetic field.

FLIP is an extension of "classical" PIC [2], where particles have mass, but every other property of the fluid is stored on a grid. In FLIP, particles have every property of the fluid, so that they provide a complete Lagrangian description not only to resolve contact discontinuities but also to reduce computational diffusion of linear and angular momentum[3, 4]. The interactions among the particles are calculated on a grid, for convenience and economy. The present study extends FLIP to MHD, by including information about the magnetic field among the attributes of the particles.

To formulate FLIP-MHD, one begins with the equations for viscous, resistive MHD flow comprising a mass continuity equation, Faraday's law, a momentum equation, and an energy equation. The solenoidal condition on $\mathbf{B}$,

$$\nabla \cdot \mathbf{B} = 0,$$

is assumed as an initial condition.

To solve the MHD equations using the particle-in-cell method requires four steps.:

1. Represent the fluid and the field by assigning appropriate properties to finite-sized particles. Interpolate the particle data on to a grid to initialize the dependent variables ρ , $\mathbf{u}$, $\mathbf{B}$, and p at the grid points.
2. Solve finite-difference approximations to the MHD equations on the grid to advance the solution from t to $t+\Delta t$
3. Interpolate the solutions of MHD equations on the grid back to the particles.
4. Move the particles through the grid to model convection.

In FLIP, a particle is assigned a mass, m_p , a momentum, $m_p \mathbf{u}_p$, an energy, e_p , and a position, $\mathbf{x}_p$. The interactions among particles are calculated on a grid. The data for the grid calculation is the interpolation of the particle data on to the

grid. For example, the mass contained in a control volume V_c with centroid $\mathbf{x}_c$ is given by,

$$M_c = \sum_p m_p S^{(n)}(\mathbf{x}_c - \mathbf{x}_p; h) h$$

where $S^{(n)}$ is an n th-order B-spline and h is its support. When $|\mathbf{x}_c - \mathbf{x}_p| > (n+1)h/2$, $S^{(n)}$ is zero.

To extend FLIP to MHD, one assigns a magnetic moment, μ_p , to each particle from which one can calculate a magnetization by projection,

$$\mathbf{M}(\mathbf{x}) = \sum_p \mu_p S^{(n)}(\mathbf{x}_p - \mathbf{x}; h) \quad (1)$$

A solenoidal magnetic field is calculated from the magnetization by subtracting the gradient of a scalar potential,

$$\mathbf{B} = \mathbf{M} - \nabla \phi$$

where ϕ is given by,

$$\nabla \cdot \mathbf{M} = \nabla^2 \phi$$

Laplace's equation for ϕ is solved using a conjugate gradient iteration[5].

To advance the magnetization from t to $t + \Delta t$, evolution equations for $\mathbf{M}$ and ϕ are needed,

$$\frac{\partial \mathbf{M}}{\partial t} = \frac{\partial \mathbf{B}}{\partial t} + \nabla \frac{\partial \phi}{\partial t}$$

Assuming,

$$\frac{d\phi}{dt} = 0.$$

which gives the correct single-particle solution, one derives

$$\frac{d\mathbf{M}}{dt} = \frac{d\mathbf{B}}{dt} - (\nabla \mathbf{u}) \cdot \nabla \phi.$$

The second term on the right describes the change in $\nabla \phi$ due to strain.

At this point in a computation step, one has advanced the magnetization on the grid, but not on the particles. To do so, one must derive an evolution equation for the particle magnetic moments.

If one differentiates Eq. (1), one obtains an expression μ_p ,

$$\frac{d}{dt} \int_{V_c} \mathbf{M} dV = \sum_p \frac{d\mu_p}{dt} S_{pc} \quad (2)$$

where $\mathbf{x}_c$ is the centroid of V_c and S is defined so that it satisfies the equation,

$$\frac{dS}{dt} = 0.$$

(S_{pc} is shorthand for $S(\mathbf{x}_p - \mathbf{x}_c; h)$). It is impractical to calculate μ_p using Eq. (2), because one would have to invert a matrix for each particle. Instead, one can define an approximate inverse to Eq. (2), written,

$$\frac{d\mu_p}{dt} = \sum_c \frac{\frac{d\mathbf{M}_c V_c}{dt} S_{pc}}{\sum_{p'} S_{p'c}}$$

where $\mathbf{M}(\mathbf{x})$ has been replaced by its average value over the control volume, V_c ,

$$\mathbf{M}_c V_c = \int_{V_c} d^3\mathbf{x} \mathbf{M}(\mathbf{x})$$

This expression conserves the magnetic flux globally. However, some diffusion results from the substitution. (The magnitude of the numerical diffusion is estimated by imposing a physical resistivity that doubles the diffusion[6]. By computation, the resistivity needed to double the diffusion corresponds to a grid magnetic Reynolds number equal to 17. For an ordinary Eulerian calculation, the value of the grid Reynolds number is approximately 1.)

The magnetic field energy integral can be written,

$$E = \int dV \frac{1}{2} \mathbf{M} \cdot \mathbf{B} - \oint_s ds \mathbf{n} \cdot \mathbf{B} \phi$$

The second integral can be made zero by choosing the boundary conditions appropriately. The particle magnetic energy is given by,

$$e_p := \frac{1}{2} \mu_p \cdot \sum_c \mathbf{B}_c S_{pc}$$

The particle and grid magnetic energies are equal,

$$\sum_p e_p = \sum_p \frac{1}{2} \mu_p \cdot \sum_c \mathbf{B}_c S_{pc} = \sum_c \frac{1}{2} \mathbf{B}_c \cdot \mathbf{M}_c \mathbf{V}_c = \sum_c E_c$$

Numerical results of calculations of the evolution of contact discontinuities, the Rayleigh-Taylor instability, and the evolution of a confined eddy are shown to demonstrate the low computational diffusion, the rotational and Galilean invariance, and the conservation properties of FLIP-MHD.

References

1. J. U. Brackbill, H. M. Ruppel, FLIP: A method for adaptively zoned, particle-in-cell calculations of fluid flows in two dimensions, *J. Comput. Phys.* 314 (1986).
2. M. W. Evans, F. H. Harlow, The particle-in-cell method for hydrodynamic calculations (Los Alamos, 1957),
3. J. U. Brackbill, On modeling angular momentum and vorticity in compressible fluid flow, *Comput. Phys. Comm.* **47**, 1-16 (1987).
4. J. U. Brackbill, D. B. Kothe, H. M. Ruppel, FLIP: A low-dissipation, particle-in-cell method for fluid flow, *Comput. Phys. Comm.* **48**, 25-38 (1988).
5. T. J. Jordan, A guide to parallel computation and some CRAY-1 experiences in *Parallel Computations* G. Rodrigue, Eds. (Academic Press, Orlando, 1982), pp. 1.
6. J. U. Brackbill, FLIP MHD: A particle-in-cell method for magnetohydrodynamics, *J. Comput. Phys.* to appear, (1990).

END

DATE FILMED

01 / 22 / 91