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Resummation and the Gluon Damping Rate in Hot QCD

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ABSTRACT

At high temperature a consistent perturbative expansion requires the resummation of an infinite subset of loop corrections into an effective expansion. This effective expansion is used to compute the gluon damping rate at leading order.

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1. INTRODUCTION

In this talk I consider theories at high temperature, where the temperature is much larger than all bare masses, or any intrinsic mass scale at zero temperature. I assume that the coupling constant g is small and compute in perturbation theory.

At first this appears to be a rather prosaic limit. At temperatures well above any phase transition, at tree level all fields start out massless, with what are by definition weak interactions. Yet a careful study of these small interactions shows that hot theories exhibit a very rich structure, especially for dynamic processes involving fields with nonzero spin. The analysis of hot gauge theories¹⁻⁷ is neither elementary nor obvious. As motivation, I start with the much simpler case of a scalar field theory.

The present discussion concentrates on theory instead of possible applications to experiment. These methods have been applied to the computation of dilepton production,³ although not as yet to the production of direct photons. What is clear is that the most naive picture of the quark-gluon plasma — a nearly free gas of massless excitations — is woefully inadequate. For dynamic processes, which are what matter experimentally, all fields have momentum dependent thermal “masses”. There are also new interactions between these thermally massive fields, and new quasiparticle excitations for both quarks and gluons.

2. SCALAR EXAMPLE

Consider the action for $(\phi^4)_4$,

$$\mathcal{L} = \frac{1}{h} \int_0^{\hbar/T} d\tau \int d^3x \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{g^2}{4!} (\phi^2)^2 \right\}, \quad (1)$$

where ϕ is a spacetime scalar and an $O(N)$ isovector. The action is written in imaginary time at a temperature T , so the euclidean time τ runs from $0 \rightarrow \hbar/T$; for now I set $\hbar = 1$. The hot limit of this theory is when the temperature is much large than the mass at zero temperature, and so the mass is neglected in (1). This theory of hot scalars has been discussed previously, with similar conclusions.⁸

The first thing to do is to calculate the self energy at one loop order. This is given by a tadpole diagram, which is a constant independent of the external momentum:

$$- \Pi = \frac{g^2(N+2)}{6} T \sum_{j=-\infty}^{+\infty} \int \frac{d^3k}{(2\pi)^3} \frac{1}{K^2}. \quad (2)$$

The loop momenta $K^\mu = (k^0, \vec{k})$, with the euclidean energy $k^0 = 2\pi jT$ for integral j . The sum over k^0 can be done directly by using a contour integral representation in the standard fashion. Alternately, it is also possible to fourier transform the propagators from functions of k^0 into functions of τ .^{9,10} I term this technique the “Saclay method”, as it was introduced by Balian and De Dominicis, and it remains popular there.¹¹ For (2) it hardly matters, but the Saclay method is of great help with more complicated integrals.

After doing the sum over k^0 ,

$$- \Pi = \frac{g^2(N+2)}{6} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2k} (1 + 2n(k)) . \quad (3)$$

The factor of $1/2k$ arises from the residue of the propagator; $n(k) = 1/(\exp(k/T) - 1)$ is the usual Bose-Einstein statistical distribution function. To isolate the effects of nonzero temperature, I subtract off the value of the self energy at zero temperature. What remains is a finite integral,

$$- \delta\Pi \equiv -\Pi + \Pi_{T=0} = \frac{N+2}{72} g^2 T^2 \equiv m^2 . \quad (4)$$

At one loop order, thermal fluctuations generate a mass m for the scalar, $m \simeq gT$. This is the exact analogy of Debye screening familiar from nonrelativistic physics.

The effects of the thermal mass are incorporated by forming an effective propagator,

$$^*\Delta(K) \simeq \frac{1}{K^2 - \delta\Pi} = \frac{1}{K^2 + m^2} . \quad (5)$$

The effective propagator in (5) is only approximate. For example, if the momenta are on the order of the temperature, or “hard”, then clearly the mass term m^2 is just a perturbative correction and so negligible. On the other hand, for momenta of order gT , or “soft”, it is as large as the bare inverse propagator, K^2 , and certainly must be included.

The sign of m^2 is positive, which gives a well behaved effective propagator. If m^2 were negative, the effective propagator would have a tachyon and the theory would be thermodynamically unstable. For scalar, or gauge theories in four dimensions, m^2 is always found to be positive — as is reasonable, for these theories are thermodynamically stable. Contrary to arguments which I tried to develop in ref. (10), however, m^2 need *not* be positive. The two notable counterexamples are provided by matter in a background gravitational field,¹² and scalar theories with cubic interactions in six dimensions,¹³ $(\phi^3)_6$. For either the graviton or the scalar field in $(\phi^3)_6$, at one loop order the thermal mass squared is negative. The occurrence of $m^2 < 0$, or a “thermal tachyon”, demonstrates that these theories are thermodynamically unstable. This happens because both theories have interactions which are universally attractive, so a thermal distribution just tends to collapse upon itself. Indeed for gravity the thermal tachyon is nothing other than the usual Jeans mass. Even so, the thermal tachyon in $(\phi^3)_6$

is noteworthy, for it is well known that in $(\phi^3)_6$ one can compute with impunity at zero temperature without any sign of instability in perturbation theory. Yet perturbative instability appears for any nonzero temperature. Incidentally, the arguments of ref. (10) can be used to show that order by order in the coupling constant, the thermal m^2 is negative in $(\phi^3)_6$.

Thermal fluctuations do not significantly affect the coupling constant. As the theory is infrared free, at one loop order the four point function includes logarithms of some characteristic external momentum p , $g^4 \log(T/p)$. This $\log(T)$ is rather innocuous: even for soft $p \simeq gT$, the one loop correction is $g^2 \log(1/g)$ times smaller than the bare coupling constant, and so to leading order in g , this vertex correction can be ignored.

With these results it is simple to understand the resummation required for a scalar theory. All that is needed is to use the effective propagator of (5) instead of $1/K^2$; the vertex is the same as usual. To illustrate how this works in a simple example, I recompute the tadpole diagram of (2), using the effective instead of the bare propagator. After doing the sum over k^0 ,

$$- {}^*\Pi + {}^*\Pi_{T=0} = - \frac{g^2(N+2)}{6} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2E} 2n(E). \quad (6)$$

The only change is that the energy which enters is that appropriate for a massive instead of a massless field, $E = \sqrt{k^2 + m^2}$. To isolate the temperature dependent terms, the value of (6) at zero temperature is subtracted away.

The integral in (6) is not simple if the ratio of m/T is arbitrary. In the present instance, however, m/T is small, of order g . Then it is easy to compute the leading correction to (4). Divide the loop integral in (6) into that over hard, $k \simeq T$, and soft, $k \simeq gT$, momenta. Over hard momenta, the thermal mass in the effective propagator is small, and only generates corrections of order g^2 to (4). In the integral over soft momenta, as $E \simeq gT$, the Bose-Einstein statistical distribution function can be approximated as $n(E) \simeq T/E$. This approximation only applies at soft momenta: for example, it is not allowed in (4), where k is of order T , and the full form of $n(k)$ matters. With this approximation it is easy to compute the integral over soft momenta:

$$- {}^*\Pi + {}^*\Pi_{T=0} \simeq m^2 \left(1 - \frac{3m}{\pi T} + \dots \right) \equiv {}^*m^2. \quad (7)$$

From (7) a value for *m is extracted: this agrees with $m \simeq gT$ to leading order, but also includes corrections of order g^2T . Notice that the correction of order g^2T is given entirely by the integral over soft momenta.

The calculation of (7) illustrates the procedure required in hot gauge theories. First, all terms proportional to g^2T^2 are isolated. In a scalar theory, there is only the mass term, but in gauge theories, there are an infinite set of such terms. Secondly, an effective expansion is constructed, including all corrections proportional to g^2T^2 . Then corrections of order g , like that of (7), are found by doing (soft) loop integrals with effective propagators and vertices.

At least for quantities which are g times those at tree level, like (7), the division into hard and soft momenta is both natural and relatively obvious. What is not apparent is how one *systematically* constructs a consistent perturbative expansion at high temperature.

Indeed, even this simple example is somewhat deceptive. Computing m^2 at leading order was elementary, and possible with ordinary perturbation theory. But to obtain the leading correction in g to the thermal mass, (6) and (7), required the use of the effective propagator in (5). This represents the resummation of an *infinite* set of higher order diagrams.

The need for resummation can be understood by counting the powers of $\hbar$ at nonzero temperature. The overall $\hbar$ in front of the action in (1) can be removed by rescaling ϕ . But $\hbar$ also enters into the boundary condition on τ :

$$g^2 \rightarrow \hbar g^2 \quad , \quad T \rightarrow T/\hbar \quad . \quad (8)$$

At zero temperature powers of g^2 directly count powers of $\hbar$. At nonzero temperature $\hbar$ appears in both g^2 and T , and things are more involved.

The semiclassical limit is given by those quantities which diverge as $\hbar \rightarrow 0$. For example, momenta on the order of $T = T/\hbar$ represent classical, thermal fluctuations. But unexpectedly, the same holds for momenta of order $gT = gT/\sqrt{\hbar}$. Thus the thermal mass $m \simeq gT$ is due entirely to classical, thermal fluctuations. It is only for momenta of order g^2T , which is devoid of $\hbar$, that truly quantum effects first appear.

In quantum mechanics, the semiclassical limit of $\hbar \rightarrow 0$ at nonzero temperature has been studied by Dolan and Kiskis.¹⁴ They looked at a double well potential, and considered the two point function after analytic continuation to real time. Even for quantum mechanics, the two point function in real time is involved: e.g., the limits of $t \rightarrow \infty$ (where $t = i\tau$ is the minkowski time variable) and $\hbar \rightarrow 0$ do not commute.

In the scalar theory, the resummation discussed above must follow from the semiclassical expansion: picking out powers of $1/\sqrt{\hbar}$ is equivalent to isolating scales of order T , gT , and so on. It would be worth developing the above treatment of the scalar theory by a systematic semiclassical expansion.

3. GAUGE THEORIES

The philosophy for gauge theories is the same as for scalar theories: first, isolate what are termed *hard thermal loops*. These are diagrams which over soft momenta are as large as the corresponding diagrams at tree level. It turns out that these are only due to one loop diagrams, equal to g^2T^2 times functions of momenta.

What is most striking is the difference between euclidean and minkowski energies; for simplicity, consider the gauge self energy. In euclidean spacetime at nonzero temperature T , p^0 is an integral multiple of $2/\pi T$. For transverse fields, the bare inverse propagator is

$(p^0)^2 + p^2$; the only way for a loop correction to be comparable to this is if p^0 vanishes, and the field is static. A mass term proportional to $g^2 T^2$, at $p^0 = 0$, is then a hard thermal loop. But just on dimensional grounds, this is the only hard thermal loop possible: at $p^0 = 0$, the dependence on p is $g^2 T^2$ times a function of p/T . For soft $p \simeq gT \ll T$, the gauge self energy is well approximated by its value at the origin, $p = 0$. Calculation shows that there is such a thermal mass term in the gauge self energy.

Things are very different after analytic continuation from euclidean to minkowski energies, $p^0 = -i\omega$, with ω a real, continuous variable. The hard thermal loop in the gauge self energy was first computed by Silin in 1960.¹⁵ Silin computed the photon self energy in hot *QED* using classical kinetic theory; by the counting of powers of $\hbar$ given above, this suffices to determine quantities proportional to eT . Silin found a self energy proportional to $e^2 T^2$ times a function of ω/p . Independently, in 1981 Klimov and Weldon¹⁶ used standard field theoretic techniques to compute the hard thermal loop for the gluon self energy in hot *QCD*. This is $g^2 T^2$ times the *same* function of ω/p as Silin found in hot *QED*.

On dimensional grounds, the appearance of this function is unremarkable, for as we now have two parameters, ω and p ; we can let both be much less than T and still have a nontrivial function of their ratio. Forming the effective propagator as in (5), in euclidean spacetime static fields acquire a thermal mass proportional to gT . The dynamic propagation of fields with $\omega \neq 0$ is similar, as they also acquire a thermal “mass”. Because the hard thermal loop is a function of ω/p , this thermal “mass”, which is always of order gT , depends nontrivially upon p , which is why I refer to it in quotes.

In a gauge theory, that the hard thermal loop is a function of ω/p has profound consequences for the higher point functions. In an abbreviated notation, the loop correction to the three gluon function, $\delta\Gamma$, is related to the gluon self energy as

$$(P - Q)^\mu \Gamma^\mu = \Pi(P) - \Pi(Q) . \quad (9)$$

For static processes with $p^0 = 0$, the hard thermal loop in $\Pi(P)$ is just a constant. This constant cancels on the right hand side, so we do not expect a hard thermal loop in the static three point function, and calculation shows that there is none. For dynamic processes, however, $\Pi(P)$ is a function of ω/p ; these do not cancel on the right hand side, so the vertex must also have terms proportional to $g^2 T^2$. Thus merely the Ward identity tells us that there is a hard thermal loop in the three gluon function. To be consistent with the static limit, the hard thermal loop in the three gluon function vanishes when all external ω are zero.

Analogously, Ward identities can be used to show that there are hard thermal loops not just in the three gluon function, but in the four gluon function, and generally in the amplitude between any number of gluons. Similarly, there is hard thermal loop in the quark self energy, as computed initially by Klimov and Weldon.¹⁶ By the Ward identities, this implies that there

are hard thermal loops in the amplitudes between a quark pair and any number of gluons. These are the only hard thermal loops in a gauge theory. For example, the ghost self energy does not have a hard thermal loop; again by the Ward identities one does not expect, nor find, hard thermal loops in the amplitudes between a ghost pair and any number of gluons.

Before discussing the nature of these hard thermal loops, it is worth contrasting gauge theories with $(\phi^3)_6$. As noted before, the static thermal mass is negative, and the theory is thermodynamically unstable. Calculating the ϕ self energy anyway,¹³ one finds a hard thermal loop: if the trilinear scalar coupling is g , at one loop order the self energy has a term which is $g^2 T^2$ times a function of ω/p . Thus in $(\phi^3)_6$, the effective propagator, as in (5), has a thermal “mass” of order gT ; this “mass” is momentum dependent for dynamic propagation, $\omega \neq 0$.

The physical origin of the hard thermal loop in the self energy of $(\phi^3)_6$ is the same as for the gauge and fermion self energies in four dimensions. It is due to Landau damping,¹⁰ which occurs when a soft field scatters off of a hard field in the thermal distribution. The mechanism is always present, but it only occurs at one loop order if there are trilinear interactions. (This is why we didn’t see it in $(\phi^4)_4$ in the previous section.) With trilinear interactions, at one loop order the discontinuity of the self energy is nonzero, and has a term proportional to $g^2 T^2$. This discontinuity produces the functional dependence of the hard thermal loop on ω/p , as a cut below the light cone for $|\omega| \leq p$.

But the hard thermal loops in the three point functions are very different in gauge theories and $(\phi^3)_6$. Both have discontinuities which correspond to Landau damping, and so hard thermal loops could arise in either. In $(\phi^3)_6$, however, the power counting that demonstrates the occurrence of a hard thermal loop in the ϕ self energy can be used to show that there is *no* hard thermal loop in the one loop correction to the ϕ^3 vertex. There is of course Landau damping in this diagram: it just isn’t large enough to produce a hard thermal loop. Moreover, unlike a gauge theory there is no Ward identity which forces a hard thermal loop in the correction to the ϕ^3 vertex.

In summary, the nontrivial hard thermal loops in the gauge and fermion self energies of hot gauge theories are not due to anything subtle: like the scalar self energy in $(\phi^3)_6$, these hard thermal loops merely reflect the presence of trilinear couplings, and so of Landau damping at one loop order. In contrast, that there are an *infinite* number of hard thermal loops in the higher point functions is special to gauge theories, due in some way to the gauge symmetry, whether abelian or nonabelian.

Hard thermal loops obey several properties which greatly simplifies their computation. As discussed, they are due to one loop diagrams with discontinuities that correspond to Landau damping; the relevant momentum region is when the external momentum are soft, and the internal, loop momentum is hard.¹ They also satisfy “tree” like Ward identities.^{2,5} Braaten and I² used these tree like Ward identities can be used to show that hard thermal loops are

the same in any covariant gauge; they are also the same in any Coulomb gauge,² or, as shown by Frenkel and Taylor,⁵ in any axial gauge. This independence of gauge is most unexpected in off shell Green's functions, and certainly is special to hard thermal loops.

The tree like Ward identities enables hard thermal loops to be evaluated without much bother. Frenkel and Taylor⁵ showed that the hard thermal loops which enter into the four gluon amplitude satisfy an unexpected identity. This identity can most easily be stated in terms of the corresponding hard thermal loop in the four photon amplitude in hot *QED*: it vanishes (of course there is such an amplitude to higher order). Braaten and I⁶ were then able to generalize this to the hard thermal loops in a nonabelian gauge theory, reducing them to just angular integrals.

Most recently, Taylor and Wong⁷ have used the established gauge invariance of hard thermal loops to derive a complete generating functional for all hard thermal loops in an extremely compact and elegant form. Their result is written in coordinate space, in terms of certain path ordered exponentials.

Given the simplicity of these results, it is natural to ask whether they could be derived more directly. Following the lead of the previous section, I would start with the full one loop effective action, (3.35) of ref. (2), and expand in $\hbar$, keeping only terms proportional to $1/\sqrt{\hbar}$ to isolate the terms proportional to $g^2 T^2$. While easy to state, what I do not know how to do is to translate the diagrammatic approximations of refs. (1) — (5) into functional techniques.

There is an alternate way in which the hard thermal loops might be obtained. As demonstrated by Silin's¹⁵ calculation of the hard thermal loop in the gluon self energy, terms of order gT are calculable by means of classical kinetic theory. If all one wanted were the hard thermal loops, this might even be easier than the semiclassical expansion.

Turning to the computation of corrections of order g to hard thermal loops, this is in principle as straightforward as computing the corrections to the scalar thermal mass in (6) and (7). Effective propagators and vertices are constructed by folding the hard thermal loops for the self energies and higher point functions into the analogous bare quantities. Corrections of order g are given by integrals over soft momenta. Treating the theory to order g^2 and beyond, however, requires control over the semiclassical expansion, in powers of $1/\sqrt{\hbar}$.

I conclude this section with some technical remarks. The treatment of loop diagrams with the effective propagators and vertices is best done by using the Saclay method.⁹ In fact, a minor variant of the Saclay method is most useful for practical calculations.^{1,10} The original method of Balian and De Dominicis is essentially one of noncovariant perturbation theory: the propagators are not purely functions of momentum, p^0 and p , but mixed functions, of the euclidean time τ and the spatial momentum p . It is then necessary to keep track of time orderings and the like; see, for example, ref. (9). In the present case, it is much more efficient to stay in momentum space for all long as possible: starting with a loop integral over k^0 and k ,

first contract all indices, do any Dirac algebra, etc. After this is done, *then* fourier transform from functions of k^0 to those of τ .^{2,3} This is equivalent to computing spectral representations for the effective propagators and vertices. In fact, in the effective expansion the discontinuity of any one loop diagram can be reduced to a form where only the spectral representations of the effective propagators enter, and not those of the effective vertices.⁴ This is possible because the discontinuities of the effective propagators and vertices are both those of Landau damping.

Lastly I note that it matters that the above analysis was carried out by computing the analytic continuation of amplitudes in imaginary time. Baier et al. and Nakkagawa et al.¹⁷ have used the real time formalism to compute the one loop corrections to the three gluon vertex. They work in the static limit; in imaginary time, the one loop correction is not even $g^2 T^2$ times the term at tree level, but just $g^2 T$. Nevertheless, in the real time formalism they find terms which are $g^2 T^3$ times the tree level term — T^2 more than in the imaginary time formalism! (Powers of momenta make up the mass dimension.)

This apparent discrepancy is due entirely to the peculiarities of the real time formalism. As explained by Evans and by Kobes,¹⁸ in the real time formalism only a certain combination of diagrams correspond to the analytic continuation of the imaginary time diagram. The discrepancy arises because the real time formalism implements a Bogoliubov transformation of the fields, involving both physical and unphysical fields. Consequently, when a loop diagram is computed, in the real time formalism it is necessary to essentially undo the Bogoliubov transformation, and extract physical from unphysical parts of the diagram. The physical parts agree with the imaginary time result; the unphysical ones do not. It would be helpful if the results of Evans and Kobes could be extended to compute a physical effective action in the real time formalism.

4. THE GLUON DAMPING RATE

The description given above is extremely ahistorical. While pure thought might have revealed the necessity for resummation, in practice it was forced upon us. The problem that brought it to the fore was the question of the gluon damping rate.^{1,19--23} It is worth recounting the story, if only as a lesson in sociology.

The real part of the gluon mass is proportional to gT ; it is given by the hard thermal loop in the gluon self energy and is easy to compute.^{15,16} The damping rate for the gluon is proportional to the imaginary part of the gluon mass. This imaginary part vanishes at order gT , and does not begin until order $g^2 T$. Hence the damping rate is like the corrections to the scalar thermal mass in (7): it can only be computed *after* resummation.

In the scalar case, this was evident, for if one didn't resum, one wouldn't find any term proportional to $g^2 T$ in the scalar self energy: in (7), the terms proportional to $g(gT)^2$ vanish if

$m = 0$. For a gauge theory, however, matters are much more confused. Historically, from the first people attempted to compute the damping rate by using bare propagators and vertices inside the loop integral, but with an effective mass shell condition outside, for the external momenta. This approach, which is manifestly inconsistent, gives a nonzero damping rate.

The problem is that this damping rate doesn't make any sense. A summary to attempts to compute the gluon damping rate is given in the accompanying table. From the damping rate, γ , I define the constant "a" as

$$\gamma = + a \frac{g^2 NT}{24\pi} . \quad (10)$$

The theory is thermodynamically stable if the constant "a", and so γ , are positive; if "a" is negative the theory is unstable. The table gives the values of "a" for a gluon at rest, $p = 0$.

The first attempt to compute the gluon damping rate was in 1980, by Kalashnikov and Klimov;¹⁹ they worked in Feynman gauge, and found that the damping rate was negative. They also noted that the answer would differ in other covariant gauges. They recognized, however, that their calculation was inconsistent, and suggested that resummation, of some sort, was required. The same observations were made by Gross, myself, and Yaffe in 1981.¹⁹

The first detailed calculation to appear in the West was in 1985, by Kajantie and Kapusta.¹⁹ They computed in axial gauges, and found a positive damping rate. They noted, but did not stress, the gauge dependence of their result.

That the damping rate is (apparently) negative in covariant gauges was clear from the work of Lopez, Parikh, and Siemens²⁰ in 1985. They submitted their paper to Physical Review Letters, and unfortunately accepted the judgement of a referee. I argued that their result just had to be wrong: since there's no sign of instability *in* equilibrium, how can there possibly be any instability *near* equilibrium? I learned a lesson from this: sometimes the "wrong" papers are the most interesting, at least if they lead to the "right" result. Of course hindsight also helps.

This lesson was demonstrated in 1987 by Hansson and Zahed.²⁰ They didn't consider the gluon propagator (which in general is gauge variant) but an effective action, which is constructed to be gauge invariant. The damping rate so computed is negative. Clearly this could not be sidestepped by incanting the mysteries of gauge invariance. Thus the credit for emphasizing the significance of the problem of the gluon damping rate rests solidly with Hansson and Zahed.

Also in 1987, Kajantie returned to the problem with Heinz and Toimela;¹⁹ they noted the gauge dependence of the gluon damping rate, although they argued that the result was only meaningful in a "physical" gauge, such as Coulomb, which gave the same positive result as in axial gauge. More tellingly, however, they observed that the damping rate is sensitive to effects from higher order in the loop expansion. They proposed to overcome this by resummation,

but the resummation proposed was just a constant mass term.

These papers produced an explosion of interest in the problem.^{21,22} Most followed the lead of Hansson and Zahed in trying to develop gauge invariant probes. Generally, the gluon damping rate found was negative in covariant or background field type gauges, and positive in axial or Coulomb gauges. Thus the mystery remained.

The difficulty with the calculations in refs. (19) – (21) is that if bare propagators and vertices are used inside the loop, then the bare mass shell must be used outside the loop; at one loop order this damping rate vanishes. A nonzero result, complete to leading order in g , is possible only if resummed propagators and vertices are used throughout the loop.

I proposed this program in ref. (1). E. Braaten and I developed the general program of resummation in ref. (2), and showed how to compute the gluon damping rate to leading order in the effective expansion. The effective Ward identities were used to show that the resulting damping rate is gauge invariant, in that the result is the same in any Coulomb or covariant type gauge. The explicit calculation was carried out in ref. (4), with a result that was manifestly positive, and about $6\frac{1}{2}$ times that found in the bare expansion with Coulomb or axial gauge.

The damping rate depends nontrivially upon momentum. While “ a ” is a pure number at zero momenta, when the momenta $p \gg g^2T$, “ a ” depends upon the coupling as $\log(1/g)$. This logarithm is due to an infrared divergence from static magnetic fields. In ref. (1) I argued that this logarithmic divergence is only cut off by the “magnetic mass” of order g^2T . This conclusion is hasty. Lebedev and Smilga²³ have shown that in a self consistent treatment the presence of a nonzero damping rate, which is itself of order g^2T , cuts off the divergence regardless of the presence of a magnetic mass.

I also note that a partial resummation was developed by Kraemmer, Kreuzer, Rebhan, and Schulz.²² While their approach is ingenious, it does not include the effects of all hard thermal loops, and so their treatment is incomplete.

At the same time as our work appeared, Kobes, Kunstatter, and Rebhan²⁴ proved a complementary result. Braaten and I showed that the damping rates are gauge invariant to leading order in g . Kobes, Kunstatter, and Rebhan showed, independent of perturbation theory, that for physical excitations the position of the pole in the gauge *variant* gluon propagator has a gauge *invariant* position. This result is familiar from QED and the weak interactions: after all, while the muon and the Z^0 are gauge variant fields, their mass and lifetime are physical quantities. The proof of ref. (24) is, however, much more explicit and useful than any given previously. They begin by computing, at arbitrary momentum, the transformation of the inverse propagator under a change in the gauge fixing condition. They then observe that for a physical field, that whenever the inverse propagator vanishes (which need not be a zero), that the gauge transformation thereof does as well. This implies that the position of the pole (or

the branch point, etc.) in a physical propagator has a gauge invariant position; other features of the propagator, such as the residue at the pole, are not gauge invariant.

The results of Kobes, Kunstatter, and Rebhan show that while the damping rate can be computed from a gauge invariant effective action, this is unnecessary; the pole in the gluon propagator will do. It would be a fascinating exercise to compute the gluon damping rate from a gauge invariant effective action, but this will be far more involved than any of the calculations to date: the full effective action, including the resummation of all hard thermal loops, must be included. Perhaps this would be possible by a functional treatment of the semiclassical expansion.

To be fair I note that the results of ref. (2) are criticized by Yamagishi and Zahed.²⁵ To leading order in g , Braaten and I show that the gluon damping rate is gauge invariant by using the effective Ward identities. The term of leading order in g is isolated by integrating over a loop integral with soft momenta. Yamagishi and Zahed argue that the remaining part of the loop integral, over hard momenta, does not cancel as it should. This is mistaken on two counts. The first is because at leading order, the only contribution to the damping rate is from the integral over soft momenta. These terms, which are of order $g(gT)^2$ in the gluon self energy, arise in precisely the same fashion as those of the same order in the example of sec. 2, (6) and (7); in either case, the integral over hard momenta only gives terms of higher order, $g^2(gT)^2$. Secondly, we could simply drop the restriction of integration over soft momenta to integrate over all loop momenta; this would mix terms at order $g(gT)^2$ and $g^2(gT)^2$. Our proof would then go through without any alteration whatsoever: the effective Ward identities have *exactly* the same form as the bare Ward identities, and all we need for the proof is the form of the Ward identities, not the detailed expressions for the effective propagators and vertices.

Speaking more broadly, the significance of resummation in hot gauge theories is not merely the problems which it solves, such as that of the gluon damping rate, but the questions which it raises.

The gluon damping rate requires an understanding of hard thermal loops. For static processes the only such term is a constant mass, which is pretty boring. In contrast, for dynamic processes there is an infinite series of hard thermal loops, which can be represented by an elegant action.⁷ Surely this action can be derived more directly, through a systematic expansion of the functional integral at high temperature.

To leading order in g , the gluon damping rate is computed by the resummation of hard thermal loops. Yet the results of Kobes, Kunstatter, and Rebhan²⁴ show that this program can be extended order by order in g ; even at next to leading order, this will not be easy. Indeed, the work of ref. (24) has significance far beyond the problem at hand. At either zero or nonzero temperature, their methods could be used to compute how, in a gauge theory, n -point functions transform under a change in the gauge fixing condition; a by product would

be an alternate proof of the gauge invariance of S -matrix elements.

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TABLE OF CALCULATIONS OF THE GLUON DAMPING RATE

Gauge:	$A_0 = 0$	Coulomb	Covariant ¹	BFG ²
Kalashnikov & Klimov '80			-15 + ...	
Gross, Pisarski, & Yaffe '81			< 0 + ...	
Kajantie & Kapusta '85	+1		-5 + ...	
Lopez, Parikh, & Siemens '85			< 0 + ...	
Hansson & Zahed '87				-11 + ...
Heinz, Kajantie, & Toimela '87	+1	+1	-5 + ...	
Elze, Heinz, Kajantie, & Toimela '88	+1			-11 + ...
Kobes & Kunstatter '88				-27/4
Nadkarni '88				-11
Carrington, Hansson, Yamagishi, & Zahed '89				-11 + ...
Carrington, Yamagishi, & Zahed '89	-3			
Kobes, Kunstatter, & Mak '89	+1	+1	-5 + ...	
Kraemmer, Kreuzer, & Rebhan '89				-11
Kreuzer, Rebhan, & Schulz '89				-13/4
Kraemmer, Kreuzer, Rebhan, & Schulz '89				-13/4
Catani & d'Emilio '90	+1			
James '90	+1			
<i>Braaten & Pisarski '90</i>		$\simeq +6.635$	$\simeq +6.635$	

¹ The values quoted are in Feynman gauge, $\xi = 1$. The last entry, for Braaten & Pisarski, is the same in *any* covariant gauge. *All* other entries in the column for covariant gauge, and some in that for BFG, are functions of ξ : $a(\xi) = a(1) + c_1(\xi - 1) + c_2(\xi - 1)^2$, with c_1 and c_2 constants. This is denoted by -15 + ..., etc.

² BFG = Background Field Gauge, and other "gauge-invariant" techniques.

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