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MASTER

TMX⁷ UPGRADE.
X-RAY DIAGNOSTIC;
Low-Energy Temperature Determination

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May 1981

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TMX-UPGRADE

X-RAY DIAGNOSTIC

Low Energy Temperature Determination

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I. SOURCE SIMULATION PHYSICS

In order to properly design the x-ray filter set, a reasonable computational model of the plasma emission had to be developed. The radiation continuum computed consisted of two components: Bremsstrahlung and recombination radiation. The contribution of line radiation from low Z impurities was estimated to be negligible for x-ray energies above 1 keV.⁽¹⁾ The Bremsstrahlung will be dealt with first.

In the regime of interest of x-ray energies (1 keV to 20 keV), the Bremsstrahlung comes from electron-ion collisions⁽²⁾ and is given by equation (3)

$$\frac{dP}{dhf} = 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \exp(-hf/kT_e) g_{ff}(T_e, hf) / (T_e)^{1/2} \quad [1]$$

where T_e is the electron temperature in keV, Z_i is the charge of ion species i , n_i is the density of ion species i in cm^{-3} , and n_e is the density of the electrons in cm^{-3} . The term $g_{ff}(T_e, hf)$ is the Gaunt factor ⁽⁴⁾ averaged over the electron velocity space distribution. A more detailed discussion of equation 1.1 is provided in Appendix A.

The recombination radiation is given by the equation ⁽³⁾

$$\begin{aligned} \frac{dP}{dhf} &= 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \frac{\exp(-hf/kT_e)}{(T_e)^{1/2}} \\ &\times \frac{\xi \exp(-\chi_i/kT_e)}{3} = \exp(-\chi_i/kT_e) g_{fb}(T_e, hf, n) U_1(hf - \chi_i) \end{aligned} \quad [1.2]$$

where ξ is the number of electron holes in the last unfilled shell that the electron can recombine into, χ_i is the ionization potential of the ground state of the ion species i , and $g_{fb}(T_e, hf, n)$ is the Gaunt factor ⁽⁴⁾ for free-bound: i.e. recombination radiation. The term $U_1(hf - \chi_i)$ is the step function which is 1 when its argument is positive and 0 when its argument is negative. In equation 1.2, contributions from recombination into higher order shells is neglected. A more detailed discussion of recombination radiation is provided in Appendix B.

For both Bremsstrahlung and recombination radiation, a knowledge of the various impurities and their charge states was necessary in order to handle the terms Z_i , n_i , and x_i . The impurities of oxygen, nitrogen, and carbon were expected due to their presence in previous experiments on 2X11B and TMX.⁽¹⁾ Their concentrations were also estimated from these experiments⁽¹⁾ and were presumed to be

Oxygen	1.5%
Nitrogen	0.5%
Carbon	0.3%

The appropriate charge state for each impurity was estimated by use of the steady-state corona model.⁽¹⁾ A better model would of course be a time dependent one. However, the fact that a corona model was reasonable can be seen from the fact that for LTE to hold⁽⁵⁾

$$n_e \geq 8.72 \times 10^{15} x_i^3 (T_e)^{1/2} \text{ cm}^{-3} \quad [1.3]$$

where T_e is in keV and x_i is in eV. With temperatures of about 1 keV and x_i at least as large as 13.6 eV, the required n_e was $2.19 \times 10^{19} \text{ cm}^{-3}$. Since the density is TMX-U is expected to be $\sim 10^{13}$, the use of a corona model was reasonable. For a steady-state corona model, the ratio of the density of charge state Z to the density of charge state $Z + 1$ for an ion was given by⁽⁶⁾

$$n(Z,g)/n(Z+1,g) = 7.87 \times 10^{-9} \chi^2(Z) \left(\frac{\chi(Z)}{kT_e} \right)^{3/4} \exp(\chi(Z)/kT_e) \quad [1.4]$$

where $\chi(Z)$ is the ionization potential in eV's of the ground state of an ion of charge state Z .

II. SOURCE SIMULATION PROGRAM

The program incorporated the above physics along with the effects of a radial density profile, radial temperature profile, and elliptical plasma cross-section. Briefly, the program integrates the radiation emitted per volume over a volume of plasma in a designated chord which cuts across the plasma in the radial direction, as shown in Figure 1. Inputs to the program consist of

- $d\omega$ = solid angle into which the electron radiates that is detected (ster.)
- dz = length in the Z direction seen by the detectors (cm)
- line 1 xne = peak electron density (cm^{-3})
- $x1$ = distance from the top of the chord to the center line (cm)
- $x1x$ = $e \phi / kT$ = the cut-off potential divided by kT_e
- uel = upper x-ray energy limit (keV)
- $xlel$ = lower x-ray energy limit (keV)
- xn = number of points of the spectrum to be generated
- line 2 xb = distance from the bottom of the chord to the center line (cm)

FIGURE 1 Integration over the volume source

ae = major or minor 1/e Gaussian radius of the plasma cross-section in the x direction (cm)

be = major or minor 1/e Gaussian radius of the plasma cross-section in the y direction (cm)

line 3 nz = number of ion species

line 4 Z(i) = charge of ion species i

line 5 xni(i) = density of ion species i divided by the peak electron density

line 6 ei(i) = $\epsilon_{m\eta}$ = ionization energy of ion species i to produce charge state Z from charge state Z - 1

The format for all the real values is 6e10.3 while that for the integer values is i1. The results are put into an output file labeled spec. dat. This file is a listing of xn lines where typically xn = 500. Each line contains the x-ray energy in keV followed by the radiated power per energy interval corresponding to that x-ray energy. The radiated power per energy interval is in keV/keV-sec. The output file is in (t2, e10.3, 5x, e10.3) format.

III. FILTER-SCINTILLATOR-DETECTOR BASICS

The effect of the filter upon the source radiation is given by

$$S_{af} = \frac{dP}{dhf} \exp(-\mu_f(hf)t_f) \quad [3.1]$$

where S_{af} is the source after it emerges from the filter, $\mu_f(hf)$ is the attenuation coefficient of the filter material at hf in cm^{-1} and t_f is the filter thickness. The radiation is now incident upon the scintillator which converts the x-ray energy deposited in it into visible light. The power P_s deposited in the scintillator by the x-rays is given by

$$P_s = \int S_{af}(1 - \exp(-\mu_s(hf)t_s)) dhf \quad [3.2]$$

where $\mu_s(hf)$ is the attenuation coefficient for the scintillator and t_s is the thickness of the scintillator. It should be noted that the use of Equations 3.1 and 3.2 presumes that a collimated source of radiation is incident upon both filter and scintillator since a constant value of t_f and t_s is employed. Of the power deposited in the scintillator, a fixed fraction is converted to visible light output. This fraction is denoted by ϵ and it is presumed to be independent of hf . The detector is a photomultiplier tube with current amplification A and quantum efficiency η . Finally, there is a geometrical coupling factor of the scintillator light output to the photomultiplier tube which is denoted by g_c . Thus, the current produced by the tube for the incident source radiation is

$$I = \frac{A\epsilon g_c}{E_p} \int \frac{dp}{dhf} \exp(-\mu_f t_f)(1 - \exp(-\mu_s t_s)) dhf \quad [3.3]$$

where $\bar{E}_p$ is the average energy per visible photon produced by the scintillator material. Once μ_f and μ_s are known as a function of hf , it is a straight forward procedure to evaluate 3.3. Accordingly, that aspect shall not be discussed except to say that μ_f and μ_s were generated using the L-division code SIGSOK1024 with the CSLOW3/80 cross-section library.

IV. HEURISTIC DISCUSSION OF FILTERS WITH REGARD TO TEMPERATURE DETERMINATION

This discussion is not rigorous and merely serves to indicate trends that are otherwise obscured in the use of computer codes. To begin, assume that the source is approximated by

$$dP/dhf = C \exp(-hf/kT) \quad [4.1]$$

where C is a constant incorporating the electron density, ion densities, charges, etc. This can be seen from the equations 1.1 and 1.2. These equations also indicate the one gross approximation involved in the use of 4.1: the neglect of the variation of g_{ff} and g_{fb} with respect to hf .

Traditionally in plasma physics a filter material without any K or L edges in the x-ray energy regime of interest is used. (7, 8, 9). This type of material acts as a high pass filter which is approximated in the following

manner. Let E_f be the x-ray energy at which the filter transmission is $1/e$. Then below E_f the filter transmission will be set to zero while above E_f the filter transmission will be set to one. It is also assumed that the scintillator is thick enough to absorb the x-rays incident on it. Thus, the energy deposited in the scintillator, a fixed fraction of which is ultimately detected by the photomultiplier tube, is given by

$$P_s = C \int_{E_f}^{\infty} \exp(-hf/kT) dhf = CkT \exp(-E_f/kT) \quad . \quad [4.2]$$

If two such filters of $1/e$ energies E_f^1 and E_f^2 are now used and their output signals ratioed, it is seen that

$$R = CkT \exp(-E_f^1/kT) / CkT \exp(-E_f^2/kT) \quad . \quad [4.3]$$

The temperature is then given by

$$kT = (E_f^2 - E_f^1) / \ln R \quad . \quad [4.4]$$

This equation immediately demonstrates how to optimize the selection of E_f^2 and E_f^1 in order to best detect the temperature. Taking derivatives of 4.4 gives

$$dR/R = - (E_f^2 - E_f^1) dT/T^2 \quad . \quad [4.5]$$

Accordingly, for a percentage change in temperature, the percentage change in R is maximized when $E_F^2 - E_F^1$ is maximized consistent with signal detection requirements. Thus, to get a sensitive temperature sensing filter set, the signal from a very thick filter should be ratioed to the signal from a very thin filter.

Since line radiation does not share this exponential dependence on x-ray energy, regions in the spectrum that are rich in strong line radiation should be avoided. From results of a code run by A. Toor ⁽¹⁾ a spectral region that is devoid of line radiation is that region above 1.0 keV. This sets the 1/e energy of the lowest filter. There is also a possibility of using K-edge filters to get around the line radiation since the K-edge filter can act as a bandpass filter.

The use of K-edge filters, of course, has the attendant problem of the transmission of the filter rising again after the K-edge cut-off and so the filter no longer acts as a bandpass filter. This problem is the most severe for the lowest energy filter. The tail of the K-edge filter can be eliminated by using Ross filters ⁽¹⁰⁾, but this requires the dedication of two X-ray detectors in order to resolve one spectral region. The highest energy filter, on the other hand, has the problem that it cannot look at an energy region as high as that seen by a non-K-edge filter. This is due to the fact that the K-edge filter for the high X-ray energies does act as a bandpass filter and takes a narrow bite from the spectrum whereas the non-K-edge filter integrates the entire spectrum from roughly its 1/e transmission point onward. Then, for a fixed minimum detectable signal, the K-edge filter must look at a lower energy bin than the non-K-edge filter. Thus it is interesting to examine a

two channel system consisting of a low channel K-edge filter with a high channel non-K-edge filter and compare it to a two channel non-K-edge filter set. Specifically, the sensitivity to temperature determination is examined and it is shown that the K-edge-filter set has a factor of greater than 10 less sensitivity to temperature than does the corresponding non-K-edge filter set.

For calculational purposes assume that the K-edge-filter acts as a narrow bandpass filter about f and so passes a spectrum

$$P_s = CT_r \int \exp(-hf/kT_e) dhf = CT_r \exp(-hf/kT_e) \Delta(hf) \quad [4.6]$$

For ease of detection, a high value of R , the ratio of the channels, along with a high value of $\frac{1}{R} dR/dT_e$, the percentage change with temperature, is desired. Thus, examine $R \times \frac{1}{R} dR/dT_e = dR/dT_e$ as a figure of merit. Then

$$R_k = CT_{r1} \Delta(hf)_1 \exp(h(f_2 - f_1)/kT_e) / CkT_e \quad [4.7]$$

for the K-edge lower channel to non-K-edge upper channel ratio. Then

$$dR_k/dT_e = - \frac{T_{r1} \Delta(hf)_1}{T_e^2} \exp(h(f_2 - f_1)/kT_e) [1 + h(f_2 - f_1)/kT_e] \quad [4.8]$$

which should be compared with

$$dR/dT_e = -h(f_2 - f_1) \exp(h(f_2 - f_1)/kT_e)/T_e^2 \quad [4.9]$$

for the two non-K-edge channels. Equation 4.9 was derived from a rearrangement of equation 4.5. Picking nominal values of 1 keV for T_e , 5 keV for $h(f_2 - f_1)$, 0.1 for T_{r1} , and 0.3 keV for Δhf gives

$$\frac{dR_k/dT_e}{dR/dT_e} = 0.036 \quad .$$

This illustrates the lower sensitivity to temperature that is possessed by this type of K-edge filter set.

Obviously, the ratio of K-edge filter to K-edge filter also produces

$$dR/dT_e = -h(f_2 - f_1) \exp(h(f_2 - f_1)/kT_e)/T_e^2$$

for $T_{r1} = T_{r2}$ and $(hf)_1 = (hf)_2$. However, the useable value of T_e for this set is lower than that of the corresponding non-K-edge filter set since the latter integrates over all the spectrum beyond the filter's transmission point. Accordingly, for a desired minimum electrical output from an X-ray channel, f_2 for a non-K-edge filter will be greater than that for a non-K-edge channel. A non-K-edge filter set again has a superior dR/dT_e compared to a K-edge filter set.

One final item that can be approached in this section is the error developed in the temperature determination using non-K-edge filters. From equation 4.5,

$$\Delta T_e / T_e = (T_e / (E_f^2 - E_f^1)) (\Delta R / R) \quad [4.10]$$

which shows that once ΔR is determined, the error denoted by ΔT can be found. Three types of errors will be accounted for: errors in relative calibration of the two channels, errors in not viewing the same plasma volume and errors in filter thicknesses. For this last error it is assumed that

$$\mu \approx a/E^n$$

where $n \approx 2.7$ and for beryllium $a \approx 550$. Thus $E_f^1 = (at_f^1)^{1/n}$ where t_f^1 is the thickness of filter 1 and $E_f^2 = (at_f^2)^{1/n}$ where t_f^2 is the thickness of filter 2. The total error in R is roughly given by

$$\begin{aligned} \Delta R^2 = & \left(\frac{\partial R}{\partial C_1} \right)^2 (\Delta C_1)^2 + \left(\frac{\partial R}{\partial C_2} \right)^2 (\Delta C_2)^2 + \left(\frac{\partial R}{\partial t_f^1} \right)^2 (\Delta t_f^1)^2 \\ & + \left(\frac{\partial R}{\partial t_f^2} \right)^2 (\Delta t_f^2)^2 \end{aligned} \quad [4.11]$$

which works out to

$$\left(\frac{\Delta R}{R}\right)^2 = \left(\frac{\Delta C_1}{C_1}\right)^2 + \left(\frac{\Delta C_2}{C_2}\right)^2 + \left(\frac{E_f}{nI_e}\right)^2 \left(\frac{\Delta t_f'}{t_f'}\right)^2 + \left(\frac{E_f}{nI_e}\right)^2 \left(\frac{\Delta t_f''}{t_f''}\right)^2 \quad [4.12]$$

If a 10% percentage error is allowed for each quantity,

$$\left(\frac{\Delta R}{R}\right)^2 = 2 (0.1)^2 + (0.1)^2 \left[\left(\frac{E_f'}{nI_e}\right)^2 + \left(\frac{E_f''}{nI_e}\right)^2 \right] \quad [4.13]$$

When $E_f'/T_e \sim 1$ and $E_f''/T_e \sim 6$, Equation 4.13 becomes

$$\Delta R/R = 0.266$$

which produces

$$\Delta T_e/T_e = 0.0532$$

or a percentage error of only 5.32% in the temperature determination.

V. RESULTS

In this section, results for two non-K-edge filter types are shown. The two types are beryllium and paralyene N (chemical formula CH). To generate the plasma emission spectrum, the following parameters were used

$d\omega = 1.81 \times 10^{-4}$ ster
 $dz = 5.08$ cm
 $xne = 7.0 \times 10^{-12}$ cm⁻³
 $x1 = 2.54$ cm
 $x1x = 7$
 $ue1 = 50$ keV
 $x1e1 = 0.1$ keV
 $xb = -2.54$ cm
 $ae = 3.0$ cm
 $be = 20.0$ cm
 $nz = 4$ (H₂, C, N₂, O₂)
 $Z = 1.0, 6.0, 7.0, 8.0$
 $xni = 0.977, 0.003, 0.005, 0.015$
 $ei = 0.0136, 0.476, 0.643, 0.836$

The electron velocity space distribution was assumed to be a Z-cut-off Maxwellian.

The scintillator used was CaF_2 of 1 mm thickness. The energy conversion efficiency for this material was 6.5% (11). The photomultiplier quantum efficiency was presumed to be 20% and the photomultiplier amplification was presumed to be 2×10^6 . The minimum acceptable signal was taken to be 0.2 μA , and the geometrical coupling factor for the scintillator and the photomultiplier was taken as 0.5.

The results for the design of the beryllium filter material are shown in Table 1. Table 1 displays the thicknesses in cm, and the $1/e$ transmission points in keV for the four sets of five filters. Using the plasma emission simulation program, the expected signals for the four sets were computed. These results are shown in Table 2. Finally, the filter response folded in with the plasma spectrum, the integrated response of the filters, and the channel variation of the filters as referenced to filter 1 are shown in Fig. 2 through 13 for each filter set.

The results for the design of the paralyene N filter material are shown in Table 3. The expected signals for these four sets of filters are displayed in Table 4. Lastly, the set of graphs for these filter sets are shown in Fig. 14 through 25.

TABLE 1
Beryllium Filters

Temperature (in keV)	Channel Number	Filter Thickness (in cm)	1/e Transmission Point (in keV)
0.300	1	3.11×10^{-3}	1.50
0.300	2	4.64×10^{-3}	1.71
0.300	3	6.63×10^{-3}	1.93
0.300	4	9.18×10^{-3}	2.14
0.300	5	1.24×10^{-2}	2.36
0.600	1	3.11×10^{-3}	1.50
0.600	2	7.12×10^{-3}	1.97
0.600	3	1.39×10^{-2}	2.45
0.600	4	2.43×10^{-2}	2.92
0.600	5	3.91×10^{-2}	3.40
1.00	1	2.11×10^{-3}	1.50
1.00	2	1.19×10^{-2}	2.32
1.00	3	3.07×10^{-2}	3.15
1.00	4	6.44×10^{-2}	3.97
1.00	5	1.17×10^{-1}	4.80
1.50	1	3.11×10^{-3}	1.50
1.50	2	2.11×10^{-2}	2.79
1.50	3	7.03×10^{-2}	4.09
1.50	4	1.66×10^{-1}	5.38
1.50	5	3.15×10^{-1}	6.67

parallel configuration of detectors
 chan#1 chan#2 chan#3 chan#4 chan#5
 filter thicknesses
 0.011E+02 0.44E+02 0.55E+02 0.010E+02 0.10E+01
 scintillator thicknesses
 0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.10E+01

temperature in keV= 0.000E+00
 signals in cps
 0.564E+05 0.166E+05 0.700E+05 0.700E+05 0.200E+05

0.300 keV
 Filter Set

temperature in keV= 0.000E+00
 signals in cps
 0.700E+05 0.400E+05 0.200E+05 0.100E+05 0.100E+05

temperature in keV= 0.100E+01
 signals in cps
 0.100E+04 0.700E+05 0.500E+05 0.410E+05 0.010E+05

temperature in keV= 0.150E+01
 signals in cps
 0.140E+04 0.110E+04 0.000E+05 0.700E+05 0.500E+05

parallel configuration of detectors
 chan#1 chan#2 chan#3 chan#4 chan#5
 filter thicknesses
 0.011E+02 0.710E+02 0.100E+01 0.040E+01 0.000E+01
 scintillator thicknesses
 0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in keV= 0.000E+00
 signals in cps
 0.064E+05 0.600E+05 0.180E+05 0.370E+07 0.100E+07

0.600 keV
 Filter Set

temperature in keV= 0.600E+00
 signals in cps
 0.000E+05 0.000E+05 0.100E+05 0.400E+05 0.000E+05

temperature in keV= 0.100E+01
 signals in cps
 0.100E+04 0.000E+05 0.000E+05 0.100E+05 0.000E+05

temperature in keV= 0.150E+01
 signals in cps
 0.140E+04 0.040E+05 0.000E+05 0.000E+05 0.000E+05

Table 2 a
 Beryllium Filter Spectral Response

parallel configuration of detectors
chan01 chan02 chan03 chan04 chan05
filter thicknesses
0.311E-02 0.211E-01 0.703E-01 0.166E+00 0.315E+00
scintillator thicknesses
0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in Kevs 0.750E+00
signals in cps
0.100E-04 0.100E-05 0.100E-05 0.100E-05 0.100E-05

temperature in Kevs 0.800E+00
signals in cps
0.100E-05 0.100E-05 0.100E-05 0.100E-05 0.100E-05

temperature in Kevs 0.100E+01
signals in cps
0.100E-04 0.100E-05 0.100E-05 0.100E-05 0.100E-05

temperature in Kevs 0.150E+01
signals in cps
0.100E-04 0.100E-05 0.100E-05 0.100E-05 0.100E-05

1.00 KeV
Filter Set

parallel configuration of detectors
chan01 chan02 chan03 chan04 chan05
filter thicknesses
0.311E-02 0.211E-01 0.703E-01 0.166E+00 0.315E+00
scintillator thicknesses
0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in Kevs 0.750E+00
signals in cps
0.100E-04 0.100E-05 0.100E-05 0.100E-05 0.100E-05

temperature in Kevs 0.800E+00
signals in cps
0.100E-05 0.100E-05 0.100E-05 0.100E-05 0.100E-05

temperature in Kevs 0.100E+01
signals in cps
0.100E-04 0.100E-05 0.100E-05 0.100E-05 0.100E-05

temperature in Kevs 0.150E+01
signals in cps
0.100E-04 0.100E-05 0.100E-05 0.100E-05 0.100E-05

1.50 KeV
Filter Set

Table 2b
Beryllium Filter Spectral Response

Table 3
Paralyene N Filters

Temperature (in keV)	Channel Number	Filter Thickness (in cm)	1/e Transmission Point (in keV)
0.3	1	1.29×10^{-3}	1.50
0.3	2	1.92×10^{-3}	1.72
0.3	3	2.72×10^{-3}	1.95
0.3	4	3.77×10^{-3}	2.17
0.3	5	5.07×10^{-3}	2.39
0.6	1	1.29×10^{-3}	1.50
0.6	2	2.87×10^{-3}	1.98
0.6	3	5.51×10^{-3}	2.46
0.6	4	9.43×10^{-3}	2.95
0.6	5	1.50×10^{-2}	3.43
1.0	1	1.29×10^{-3}	1.50
1.0	2	4.68×10^{-3}	2.33
1.0	3	1.18×10^{-2}	3.17
1.0	4	2.42×10^{-2}	4.00
1.0	5	4.33×10^{-2}	4.84
1.5	1	1.29×10^{-3}	1.50
1.5	2	8.06×10^{-3}	2.79
1.5	3	2.58×10^{-2}	4.08
1.5	4	6.01×10^{-2}	5.38
1.5	5	1.15×10^{-1}	6.68

parallel configuration of detectors
chan01 chan02 chan03 chan04 chan05
filter, thicknesses
0.129E-02 0.102E-02 0.070E-02 0.377E-02 0.207E-02
scintillator thicknesses
0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in KeV 0.300E+00
signals in amps
0.407E-05 0.102E-05 0.342E-05 0.404E-05 0.200E-05

0.30 KeV Filter Set

temperature in KeV 0.600E+00
signals in amps
0.740E-05 0.447E-05 0.203E-05 0.101E-05 0.110E-05

temperature in KeV 0.100E+01
signals in amps
0.102E-04 0.700E-05 0.553E-05 0.407E-05 0.302E-05

temperature in KeV 0.100E+01
signals in amps
0.140E-04 0.111E-04 0.070E-05 0.601E-05 0.553E-05

parallel configuration of detectors
chan01 chan02 chan03 chan04 chan05
filter, thicknesses
0.100E-02 0.207E-02 0.551E-02 0.040E-02 0.150E-01
scintillator thicknesses
0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in KeV 0.300E+00
signals in amps
0.407E-05 0.750E-05 0.163E-05 0.401E-07 0.106E-07

0.60 KeV Filter Set

temperature in KeV 0.600E+00
signals in amps
0.740E-05 0.060E-05 0.104E-05 0.400E-05 0.201E-05

temperature in KeV 0.100E+01
signals in amps
0.102E-04 0.520E-05 0.277E-05 0.154E-05 0.006E-05

temperature in KeV 0.150E+01
signals in amps
0.147E-04 0.640E-05 0.010E-05 0.334E-05 0.200E-05

Table 4a
Paralyene N Filter Signal Responses

parallel configuration of detectors
chan#1 chan#2 chan#3 chan#4 chan#5
filter.thicknesses
0.100E+00 0.400E+00 0.100E+01 0.200E+01 0.400E+01
scintillator thicknesses
0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in KeV= 0.300E+00
signals in amps
0.400E-05 0.200E-05 0.200E-07 0.200E-07 0.200E-00

temperature in KeV= 0.300E+00
signals in amps
0.700E-05 0.100E-05 0.300E-05 0.300E-07 0.200E-07

1.0 KeV Filter Set

temperature in KeV= 0.100E+01
signals in amps
0.100E-04 0.320E-05 0.110E-05 0.470E-05 0.200E-05

temperature in KeV= 0.150E+01
signals in amps
0.140E-04 0.500E-05 0.270E-05 0.150E-05 0.720E-05

parallel configuration of detectors
chan#1 chan#2 chan#3 chan#4 chan#5
filter.thicknesses
0.100E+00 0.800E+00 0.250E+01 0.600E+01 0.110E+00
scintillator thicknesses
0.100E+00 0.100E+00 0.100E+00 0.100E+00 0.100E+00

temperature in KeV= 0.300E+00
signals in amps
0.400E-05 0.610E-07 0.180E-08 0.200E-10 0.200E-11

1.5 KeV Filter Set

temperature in KeV= 0.300E+00
signals in amps
0.700E-05 0.300E-05 0.700E-07 0.700E-07 0.700E-00

temperature in KeV= 0.400E+00
signals in amps
0.100E-04 0.100E-05 0.400E-05 0.100E-05 0.700E-07

temperature in KeV= 0.100E+01
signals in amps
0.140E-04 0.500E-05 0.100E-05 0.400E-05 0.100E-05

Table 4b
Paralyene N Filter Signal Responses

FIG. 2. Spectral Response of the 0.3 keV Beryllium Filter Set.

FIG. 3. Intergrated Response of the 0.3 keV Beryllium Filter Set.

FIG. 4. Channel Variation of the 0.3 keV Beryllium Filter Set.

CHANNEL VARIATION VERSUS CHANNEL

CHANNEL NUMBER

FIG. 5. Spectral Response of the 0.6 keV Beryllium Filter Set.

FIG. 6. Integrated Response of the 0.6 keV Beryllium Filter Set.

FIG. 7. Channel Variation of the 0.6 keV Beryllium Filter Set.

FIG. 8. Spectral Response of the 1.0 keV Beryllium Filter Set.

FIG. 9. Integrated Response of the 1.0 keV Beryllium Filter Set.

Fig. 10. Channel Variation of the 1.0 keV Beryllium Filter Set.

FIG. 11. Spectral Response of 1.5 keV Beryllium Filter Set.

FIG. 12. Integrated Response of the 1.5 keV Beryllium Filter Set.

FIG. 13. Channel Variation of the 1.5 keV Beryllium Filter Set.

CHANNEL VARIATION VERSUS CHANNEL

FIG. 14. Spectral Response of the 0.3 keV Paralyne N Filter Set.

FIG. 15. Integrated Response of the 0.3 keV Paralyene N Filter Set.

FIG. 16. Channel Variation of the 0.3 keV Paralyene N Filter Set.

FIG. 17. Spectral Response of the 0.6 keV Paralyene N Filter Set.

FIG. 18. Integrated Response of the 0.6 keV Paralyene N Filter Set.

FIG. 19. Channel Variation of the 0.6 keV Paralyene N Filter Set.

FIG. 20. Spectral Response of the 1.0 keV Paralyene N Filter Set.

FIG. 21. Integrated Response of the 1.0 keV Paralyene N Filter Set.

FIG. 22. Channel Variation of the 1.0 keV Paralyene N Filter Set.

FIG. 23. Spectral Response of the 1.5 keV Paralyene N Filter Set.

FIG. 24. Integrated Response of the 1.5 keV Paralyene N Filter Set.

FIG. 25. Channel Variation of the 1.5 keV Paralyene N Filter Set.

APPENDIX A: Bremsstrahlung

The energy radiated by a plasma due to Bremsstrahlung is given by⁽¹²⁾

$$dP/dhf = n_e n_i \langle d\sigma \rangle hf/dhf \quad [A.1]$$

where

$$d\sigma = \left(\frac{Zr_0c}{137} \right)^2 \frac{16}{3v^2} \ln \frac{v + v'}{v - v'} \frac{dhf}{hf} \quad [A.2]$$

where

$$\frac{1}{2} mv^2 - \frac{1}{2} mv'^2 = hf \quad [A.3]$$

Equation (A.3) comes about from energy conservation with hf being the x-ray photon energy.

Equation (A.2) is the Born approximation⁽¹³⁾ for the Bremsstrahlung cross-section.

$$\langle v d\sigma \rangle = \int d^3v \, v f(v) d\sigma \quad [A.4]$$

where $f(v)$ is the electron velocity distribution function. Thus, equation (A.4) becomes

$$\langle v d\sigma \rangle = \iiint d^3v f(v) \left(\frac{Zr_0 c}{137} \right)^2 \frac{16}{3v} \ln \left[\frac{v + \sqrt{v^2 - 2hf/m_e}}{v - \sqrt{v^2 - 2hf/m_e}} \right] \frac{dhf}{hf} .$$

For an isotropic distribution one can use spherical co-ordinates and integrate out the angles to give

$$\langle v d\sigma \rangle = 4\pi \left(\frac{Zr_0 c}{137} \right)^2 \frac{16}{3} \int dv f(v) v \ln \left[\frac{v + \sqrt{v^2 - 2hf/m_e}}{v - \sqrt{v^2 - 2hf/m_e}} \right] \frac{dhf}{hf} .$$

Now the lower limit on the velocity integral must correspond to the lowest electron energy that can produce an x-ray photon of energy hf and this occurs when $V' = 0$ (ie the electron stops dead). Thus the lower limit is

$$(2hf/m_e)^{1/2}$$

and $\langle v d\sigma \rangle$ can now be written as

$$\langle v d\sigma \rangle = 4\pi \frac{Zr_0 c^2}{137} \frac{16}{3} \left(\int_{(2hf/m_e)^{1/2}}^{\infty} dv f(v) v \ln \left[\frac{v + \sqrt{v^2 - 2hf/m_e}}{v - \sqrt{v^2 - 2hf/m_e}} \right] \frac{dhf}{hf} \right) .$$

Next change the integral to energy or $E = \frac{1}{2} mv^2 \dots dE = mv dv$ or

$$\int_{hf}^{\infty} \frac{dE}{m_e} f(E) \ln \left[\frac{E^{1/2} + (E-hf)^{1/2}}{hf} \right]^2$$

Change variables again to $y = (E-hf)$

$$\int_0^{\infty} \frac{dy}{m_e} f(y+hf) \ln \left[\frac{((y+hf)^{1/2} + y^{1/2})^2}{hf} \right]$$

Accordingly,

$$\langle v_{do} \rangle = 4\pi \frac{(Zr_0c)^2}{137} \frac{16}{3m_e} \int_0^{\infty} dy f(y+hf) \ln \left[\frac{((y+hf)^{1/2} + y^{1/2})^2}{hf} \right] \frac{chf}{hf} \quad [A.5]$$

Combining equations (A.1) and (A.5) gives

$$\frac{dP}{dhf} = 4\pi \frac{(Zr_0c)^2}{137} \frac{16}{3m_e} n_e n_i \int_0^{\infty} dy f(y+hf) \ln \left[\frac{((y+hf)^{1/2} + y^{1/2})^2}{hf} \right] \quad [A.6]$$

From equation (A.6) the Gaunt factor can be arrived at from

$$\frac{dP}{dhf} = 4\pi \frac{(Zr_0c)^2}{137} \frac{16}{3m_e} n_e n_i \left(\frac{m_e}{2\pi kT} \right)^{3/2} e^{-hf/kT} \int_0^{\infty} dy f(y+hf) \left(\frac{2\pi kT}{m_e} \right)^{3/2} e^{hf/kT} \ln [] .$$

Finally making one last change of variables to $y' = y/kT$ one has

$$\frac{dP}{dhf} = 4\pi \frac{(Zr_0c)^2}{137} \frac{16}{3m_e} n_e n_i \left(\frac{m_e}{2\pi kT}\right)^{3/2} kT e^{-hf/kT} \int_0^\infty dy' f(kTy' + hf) \\ \times \left(\frac{2\pi kT}{m_e}\right)^{3/2} e^{hf/kT} \ln \frac{((y' + hf/kT)^{1/2} + y'^{1/2})^2}{hf/kT} \quad \text{or}$$

$$\frac{dP}{dhf} = \frac{(Zr_0c)^2}{137} \frac{16}{3} \sqrt{\frac{2m_e}{\pi}} \frac{n_e n_i e^{-hf/kT}}{(kT)^{1/2}} \frac{\pi}{3^{1/2}} g_{ff}(hf/kT) \quad [A.7]$$

where g_{ff} is the Gaunt factor (the $\frac{\pi}{3^{1/2}}$ is convention), thus

$$g_{ff}(hf/kT) = \frac{3^{1/2}}{\pi} \left(\frac{2\pi kT}{m_e}\right)^{3/2} e^{hf/kT} \int_0^\infty dy f_E(kTy + hf) \ln \frac{((y + hf/kT)^{1/2} + y^{1/2})^2}{hf/kT} \quad [A.8]$$

(remember $f_E(y)$ is now an energy distribution function).

APPENDIX B: Recombination Radiation

The cross-section for radiative recombination is given by (14)

$$\sigma_{rec} = 1.06 \times 10^{-22} \frac{Z^2 \epsilon_{m\eta} E_H g_{fb}}{m^3 E (E + \epsilon_{m\eta})} \text{ cm}^2 \quad [B.1]$$

where $\epsilon_{m\eta}$ is the ionization energy from level m and E is the energy of the electron. The power radiated per photon energy interval is (15)

$$\frac{dP_{fb}}{d\eta f} = n_e n_i \langle \sigma_{rec} v \delta f \eta (hf - \epsilon_{m\eta} - \frac{1}{2} m_e v^2) \rangle \quad [B.2]$$

where the delta function arises from energy conservation since the radiated photon energy is given by

$$hf = \frac{1}{2} m_e v^2 + \epsilon_{m\eta} \quad [B.3]$$

The brackets denote an average over the electron velocity space distribution or

$$\frac{dP_{fb}}{d\eta f} = n_e n_i hf \int d^3 v f(v) v \delta (hf - \epsilon_{m\eta} - \frac{1}{2} m_e v^2) \sigma_{rec} \quad [B.4]$$

For calculations, the free-bound Gaunt factor g_{fb} is set equal to 1.

APPENDIX C: Different Electron Velocity Space Distributions

This appendix shows the results of equations (A.1) and (B.2) for three different electron velocity space distributions: Maxwellian, Cut-off Maxwellian, and a 2-Cut-off Maxwellian.

1. Maxwellian

a) distribution function: $f(\underline{v}) = \left(\frac{m_e}{2\pi kT_e}\right)^{3/2} \exp(-m_e v^2/2kT_e)$

b) Bremsstrahlung: $\frac{dP}{dhf} = 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \exp(-hf/kT_e) \bar{g}_{ff}(T_e)^{1/2}$

$$\bar{g}_{ff} = \frac{\sqrt{3}}{\pi} \int_0^\infty dy \exp(-y) \ln \left[\frac{((y+hf/kT_e)^{1/2} + y^{1/2})^2}{hf/kT_e} \right]$$

c) recombination radiation:

$$\frac{dP}{dhf} = 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \left(\frac{\exp(-hf/kT_e)}{(T_e)^{1/2}} \right) \frac{\xi_i}{m^3} \frac{ex_i}{kT_e} \exp(ex_i/kT_e)$$

$$\times \bar{g}_{fb} \underline{u}_i (hf - ex_i) \text{ and } \bar{g}_{fb} = 1$$

2. Cut-Off Maxwellian

a) distribution function:

$$f(v) = \frac{\left(\frac{m_e}{2\pi kT} \right)^{3/2}}{1 - \frac{2}{\sqrt{\pi}} e^{-1} \left(\frac{2}{3} \right)^{3/2} + 1^{1/2} + \frac{1}{2^{1/2}}} \left[e^{\frac{-mv^2}{2kT}} - e^{-1} \right] \frac{1}{2} \left(1 - \frac{m_e v^2}{2kT} \right)$$

where $1 = e\phi/kT$ and ϕ is the cut-off potential.

b) Bremsstrahlung:

$$dP/dhf = 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \exp(-hf/kT) \bar{g}_{ff}/T^{1/2}$$

$$\bar{g}_{ff} = \frac{\sqrt{3}}{\pi} \int_0^{1-hf/kT} [e^{-y} - e^{-(hf/kt-1)}] \ln[(y+hf/kT)^{1/2} / (hf/kT)] \frac{1}{2} \left(1 - \frac{hf}{kT} \right) dy$$

$$1 - \frac{2}{\sqrt{\pi}} e^{-1} \left(\frac{2}{3} \right)^{3/2} + 1^{1/2} + \frac{1}{2^{1/2}}$$

c) recombination radiation:

$$\frac{dP}{dhf} = 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \frac{\exp(-hf/kT)}{T^{1/2}} \frac{\xi}{m} \frac{e x_i}{kT} \exp\left(\frac{e x_i}{kT}\right) \bar{g}_{fb} \frac{1}{2} (hf - e x_i)$$

$$\overline{g_{ff}} = \frac{\left[1 - \exp\left(\frac{nf - e m r_i}{kT}\right) - 1 \right] \underline{u}_i \left(1 - \frac{nf - e m r_i}{kT} \right)}{1 - \frac{2}{\sqrt{\pi}} e^{-1} \left(\frac{2}{3} \right)^{3/2} + 1^{1/2} + \frac{1}{2T} 1^{1/2}}$$

3. Z-Cut-off Maxwellian

a) distribution function:

$$F(v) = \frac{(m/2\pi kT)^{3/2}}{1 - \frac{2e^{-1}}{\pi} \left(1^{1/2} + \frac{1}{T} - \frac{1}{2T} \right)^{3/2}} \left[\frac{-m v_z^2}{2kT} - 1 \right] \exp\left(\frac{-m v_r^2}{2kT}\right) \underline{u}_i \left(1 - \frac{m v_z^2}{2kT} \right)$$

b) Bremsstrahlung:

$$\frac{dP}{dhf} = 3 \times 10^{-15} n_e Z_i^2 n_i \frac{\exp(hf/kT)}{T^{1/2}} \overline{g_{ff}}$$

$$\overline{g_{ff}} = \frac{\frac{\sqrt{3}}{\pi} \int_0^\infty dz e^{-z} \int_0^1 dy \frac{(e^{-y} - 1)}{(Z+y+hf/kT)^{1/2}} \ln \left[\frac{(y+Z+hf/kT)^{1/2} + (y+Z)^{1/2}}{(y+Z+hf/kT)^{1/2} - (y+Z)^{1/2}} \right]}{1 - \frac{2e^{-1}}{\pi} \left(1^{1/2} + \frac{1}{T} - \frac{1}{2T} \right)^{3/2}}$$

c) recombination radiation:

$$\frac{dP}{dhf} = 3 \times 10^{-15} n_e \sum_i Z_i^2 n_i \frac{\exp(-hf/kT)}{T^{1/2}} \frac{\xi}{m^3} \frac{ex_i}{kT} \exp\left(\frac{ex_i}{kT}\right) \overline{g_{ff}} \underline{u}_i (hf - ex_i)$$

$$\frac{1}{g_{ff}} = \left(\frac{1kT}{hf - \epsilon_{m\eta}} \right)^{1/2} \frac{[1 - \frac{e^{-1}}{21^{1/2}} \int_0^1 dy e^y / y^{1/2}]}{1 - \frac{2e^{-1}}{\pi} (1^{1/2} + \frac{1}{1}^{1/2} - \frac{1}{21}^{2/3})}$$

$$\frac{1}{g_{fh}} \approx \frac{\frac{1}{2} \left(\frac{1kT}{hf - \epsilon_{m\eta}} \right)^{1/2}}{1 - \frac{2e^{-1}}{\pi} (1^{1/2} + \frac{1}{1}^{1/2} - \frac{1}{21}^{3/2})}$$

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