MASTER

Princeton University

**Department of
Mechanical and
Aerospace Engineering**

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency Thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.

A THEORY OF THERMIONIC ENERGY CONVERTER WITH
VARIABLE PLASMA ELECTRON TEMPERATURE

by

Albert Tiengtsung Yen

and

Sau-Hai Lam

Princeton University
Department of Mechanical and Aerospace Engineering
Princeton, New Jersey 08540

September 1978

Prepared for the Department of Energy Under Contract AT(11-1)-2533

NOTICE

This report was prepared as an account of work sponsored by the United States Government. Neither the United States nor the United States Department of Energy, nor any of their employees, nor any of their contractors, subcontractors, or their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness or usefulness of any information, apparatus, product or process disclosed, or represents that its use would not infringe privately owned rights.

2
12

ABSTRACT

A computational procedure for calculating the C-V characteristic, the plasma arc-drop and other important parameters for a cesium vapor thermionic energy converter has been developed.

The original theory dealing with the constant electron temperature has been extended to treat the problem for the variable electron temperature. Comparison between the two cases shows that the C-V characteristic moves generally toward the left as the electron thermal conductivity decreases, but the magnitude of this degradation appears small.

LEGAL NOTICE

"This report was prepared as an account of Government-sponsored work. Neither the United States, nor the Energy Research and Development Administration nor any person acting on behalf of the Commission

- A. Makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, nor that the use of any information, apparatus, method, or process disclosed in this report may not infringe privately owned rights; or
- B. Assumes any liabilities with respect to the use of, or for damages resulting from the use of, any information, apparatus, method, or process disclosed in this report.

As used in the above, 'person acting on behalf of the Commission' includes any employee or contractor of the Administration or employee of such contractor, to the extent that such employee or contractor of the Administration employee of such contractor prepares, disseminates, or provides access to any information pursuant to his employment or contract with the Administration, or his employment with such contractor."

TABLE OF CONTENTS

	<u>Page</u>
Title Page	i
Abstract	ii
Legal Notice	iii
Table of Contents	iv
Nomenclature	v
I. Introduction	1
II. Formulation of the Physical Phenomenon	4
1. Plasma Region Analysis	4
2. Sheath Region Analysis	7
3. Fundamental Equations of Electrons in the Thermionic Diode	12
4. Boundary Conditions for the Center Plasma Region	15
III. Computational Algorithm	19
1. Computational Procedure for the Isothermal Case	21
2. Computational Procedure for the Non-isothermal Case	23
IV. Results and Comparison	27
V. References	30
VI. Description of Figures	31
Appendix A The Derivation of Schottky Effect	52
Appendix B Flow Chart for the Isothermal Case	54
Appendix C Flow Chart for the Non-isothermal Case	55
Appendix D Listing and Comments for the Computer Programs	57
1. Isothermal Case	58
2. Non-isothermal Case	72
3. Procedures to Run Additional Cases for Different Electron Thermal Conductivities	91

NOMENCLATURE

List of Symbols

E	Electric field in the x direction
ψ	Motive $\left[E \equiv \frac{d\psi}{dx} \right]$
T_E	Emitter temperature
d	Gap size
Φ_E	Work function of the emitter
Φ_C	Work function of the collector
V	Output voltage
J	Converter current density
T_e	Electron temperature
T_c	Collector temperature
n_i	Number density of ions
n_e	Number density of electrons
V_d	Converter voltage arc drop
m_e	Electron mass
k	Boltzmann's constant
e	Electron charge
h	Planck's constant
m_i	Cesium ion mass
ϵ_0	Permittivity of free space
V_{ion}	Ionization energy of the cesium atom
u_i	Electron velocity

List of Symbols

u_e	Electron velocity
χ	Dimensionless motive $\left[\chi \equiv \frac{e\psi}{kT_e} \right]$
ξ	Dimensionless motive distance $\left[\xi \equiv \frac{\chi}{d} \right]$
χ_E	Dimensionless emitter sheath motive height
χ_C	Dimensionless collector sheath motive height
$\Delta\chi_p$	Dimensionless plasma motive drop
n_p	Plasma number density
J_R	Richardson current density of the emitter [$J_R(\text{amps/cm}^2) = T_E^2 \exp(-\Phi_E)$]
a_E	Average thermal speed of the electrons at temperature T_E $\left[a_E \equiv \left(\frac{8kT_E}{\pi m_e} \right)^{1/2} \right]$
n_R	Richardson number density $\left[n_R(\text{cm}^{-3}) \equiv \frac{J_R}{ea_{E/2}} = 0.62 \times 10^{12} \left(\frac{1000}{T_E} \right)^{1/2} J_R \right]$
J_E	Net emitted current density from the emitter $\left[J_E \equiv J_R \exp(-\Delta\hat{\chi}), \Delta\hat{\chi} > 0 \right]$ $J_E = J_R$ (when no motive peak exists)
$\hat{n}_E$	Net emitted number density from the emitter $\left[\hat{n}_E \equiv n_R \exp(-\Delta\hat{\chi}), \Delta\hat{\chi} > 0 \right]$ $\hat{n}_E = n_R$ (when no motive peak exists)
j	Normalized converter current density $\left[j = \frac{J}{J_E} \right]$
τ	Dimensionless plasma electron temperature $\left[\tau \equiv \frac{T_e}{T_E} \right]$

List of Symbols

T_i	Cesium ion temperature
T_a	Cesium atom temperature
P	Cesium vapor pressure
$\lambda_i, \hat{\lambda}_i$	Mean-free-path of ions colliding with neutrals and electrons, (See Equation (2.10)).
$\lambda_e, \hat{\lambda}_e$	Mean-free-path of electrons colliding with neutrals and ions, (See Equation (2.10)).

I. INTRODUCTION

The performance of a thermionic diode is conventionally assessed by comparing the converter C-V characteristic with the ideal vacuum C-V characteristic. The main performance index of interest is the arc-drop. In the present work, the isothermal theory outlined in reference 1 is extended to the non-isothermal case. In the development of the present theory, we assume the interelectrode spacing d is relatively wide, probably 0.5 mm or larger, the output current density J is of the order of 5 amp/cm^2 - 20 amp/cm^2 and the emitter is at a temperature of 1500°K - 1800°K . In this temperature range, surface ionization will be small and positive ions must be supplied by gas phase ionization. The gap size d will be much larger than the expected ion-neutral momentum mean-free-path λ_i which is in turn much larger than the Debye length h_D . We shall be dealing with the ignited mode operations and shall assume that the electrons in the plasma region have a Maxwellian distribution.

The theoretical problem can be divided into two parts: the plasma region and the electrode sheath regions. From the plasma region analysis, a second order non-linear diffusion equation for the plasma number density can be derived. This non-linear diffusion equation involves an important parameter A which, in the isothermal case, is a constant and

determines the minimum electron temperature at which ignition occurs.¹ For the non-isothermal case, this parameter A will be a function of the dimensionless distance ξ . From the sheath region analysis, under the assumption that the emitter sheath is non-monotonic, four important parameters N_E , N_e , $\hat{\alpha}$ and χ_E can be derived. The coupling of the regions is through the matching of the solutions at the emitter sheath-plasma interface and at the collector sheath-plasma interface.

The electron continuity, energy and momentum equations will be considered in turn in Chapter II. For the energy equation, we include the thermal conduction term as well as the ionization energy loss term. The electron momentum equation shows explicitly that the arc-drop has two contributions. A small contribution comes from the randomization and scattering of hot electrons back to the emitter. The main contribution is due to the ohmic drop required to pass the converter current J through the plasma region.

For practical applications, we are only interested in the portion of the C-V curve from the ignition point to the "knee", since the arc-drop above the knee is expected to be large as the converter current saturates. In our computations, we have assumed a sufficiently low value of the emitter work function so that no saturation occurs. Thus a motive peak always exists in our solutions.

Chapter III describes the computational procedures used for both the isothermal and the non-isothermal case. The isothermal case is treated as a special case of the non-isothermal formulation by letting the electron thermal conductivity to be asymptotically large. Since the isothermal calculation is considerably easier, it is used as the

zeroth iterant in the non-isothermal iterative computational scheme, and the computation proceeds by gradually lowering the electron thermal conductivity. Because of the slow convergence of our algorithm, we have been able to obtain solutions for electron thermal conductivity about five times the theoretical value. Nevertheless, the computed results as discussed in Chapter IV show clearly that the arc-drop is increased as the electron thermal conductivity is decreased.

II. FORMULATION OF THE PHYSICAL PHENOMENON

The motive distribution in the cesium vapor thermionic converter is qualitatively sketched in Figure 1. Extremely thin collisionless sheaths exist adjacent to the electrodes and are separated by a plasma region in which quasi-neutrality holds so that we shall use n_p to denote either n_i or n_e . The edges of the emitter and the collector sheaths are located at $\xi = 0^+$ and $\xi = 1^-$, respectively. The symbols $\Delta\hat{\chi}$, χ_E , χ_C and $\Delta\chi_p$ are defined in Figure 1 from which we have the following relations

$$V_d = V - (\Phi_E - \Phi_C) - \Delta\hat{\chi}, \quad (2.1)$$

$$V_d = (\chi_C - \chi_E) - \Delta\chi_p. \quad (2.2)$$

1. Plasma Region Analysis

The continuity equations are

$$\frac{d(n_i u_i)}{dx} = \beta_r n_e (n_s^2 - n_e^2), \quad (2.3)$$

$$\frac{d(n_e u_e)}{dx} = \beta_r n_e (n_s^2 - n_e^2), \quad (2.4)$$

where $n_i u_i$ is the ion number flux,

$n_e u_e$ is the electron number flux,

β_r is the three-body recombination coefficient,

n_s is the equilibrium Saha electron number density,

$\beta_r n_e (n_s^2 - n_e^2)$ is the volume ionization rate.

Now we define a new flux Γ called the ambi-polar flux

$$\Gamma \equiv \frac{\mu_i}{\mu_e} (n_e u_e) + n_i u_i, \quad (2.5)$$

where μ_i , μ_e are the mobility coefficients.

Taking derivative of Γ with respect to x and substituting Equations (2.3)-(2.4) into Equation (2.5), we have

$$\frac{d\Gamma}{dx} = \left(1 + \frac{\mu_i}{\mu_e}\right) \beta_r n_e (n_s^2 - n_e^2) \quad (2.6)$$

The momentum equation for the electron gas has the following form

$$m_e n_p u_e \frac{du_e}{dx} = - \frac{dP_e}{dx} - en_p \frac{d\psi}{dx} - \frac{m_e n_p u_e}{t_e}, \quad (2.7)$$

where $P_e = n_p k T_e$, $t_e \equiv \frac{\lambda_e}{a_e}$, and

t_e is the momentum mean-free-time of the electrons colliding with neutrals,

λ_e is the mean-free-path of the electrons colliding with neutrals,

a_e is the average thermal speed of the electrons at temperature T_e .

By an order of magnitude analysis, we neglect the electron inertia term on the left-hand side of Equation (2.7) and obtain

$$n_p u_e = -\lambda_e \left(\frac{\pi}{8 m_e k T_e} \right)^{1/2} \left\{ k \frac{dn_p T_e}{dx} + en_p \frac{d\psi}{dx} \right\}. \quad (2.8)$$

A similar procedure can be used for the ions. We obtain

$$n_i u_i = \mu_i \left\{ -k \frac{dn_i T_i}{dx} + en_i \frac{d\psi}{dx} \right\}, \quad (2.9a)$$

$$n_e u_e = \mu_e \left\{ -k \frac{dn_e T_e}{dx} - en_e \frac{d\psi}{dx} \right\}, \quad (2.9b)$$

where μ_i , μ_e are the mobilities of ions and electrons, respectively, and are related to λ_i , λ_e (or $\hat{\lambda}_i$, $\hat{\lambda}_e$ as defined below) by:

$$\mu_e = \lambda_e \sqrt{\frac{\pi}{8 m_e k T_e}} \equiv \frac{\hat{\lambda}_e}{3 \sqrt{m_e k T_e}}, \quad (2.10a)$$

$$\mu_i = \lambda_i \sqrt{\frac{\pi}{8 m_i k T_i}} \equiv \frac{\hat{\lambda}_i}{3 \sqrt{m_i k T_i}}, \quad (2.10b)$$

$$\begin{aligned} \frac{\mu_i}{\mu_e} &= \Lambda \sqrt{\frac{m_e}{m_i}}, \\ \Lambda &\equiv \frac{\lambda_i}{\lambda_e} \sqrt{\frac{T_e}{T_i}} \end{aligned} \quad (2.10c)$$

Since $n_i = n_e = n_p$, we obtain another relation for Γ from Equations (2.9a) and (2.9b)

$$\Gamma = -k\mu_i \frac{dn_p T_i}{dx} + \frac{dn_p T_e}{dx} \quad (2.11)$$

Assuming that the temperature variation with respect to space is much smaller than the number density variation with respect to space, Equation (2.11) becomes

$$\Gamma = -D_{ambi} \frac{dn_p}{dx}, \quad (2.12)$$

$$\text{where } D_{ambi} \equiv \mu_i k (T_i + T_e). \quad (2.13)$$

Substituting Equation (2.12) into Equation (2.6) we obtain

$$-\frac{d}{dx} D_{ambi} \frac{dn_p}{dx} = \left(1 + \frac{\mu_i}{\mu_e}\right) \beta_r n_p (n_s^2 - n_p^2). \quad (2.14)$$

Since $\frac{\mu_i}{\mu_e}$ is very small, we can rewrite Equation (2.14) as

$$\frac{d}{dx} D_{ambi} \frac{dn_p}{dx} + \beta_r n_p (n_s^2 - n_p^2) = 0. \quad (2.15)$$

Equation (2.15) is the second order non-linear diffusion equation for the plasma number density n_p .

After rearranging Equation (2.15) and using dimensionless parameters, we can obtain the following simultaneous first order ordinary differential equations :

$$\frac{dC}{d\xi} = \frac{D_{\text{ambi}}(E)}{D_{\text{ambi}}(\xi)} z \quad (2.16a)$$

$$\frac{dz}{d\xi} = -A(\xi)C \left[1 - C^2 \frac{\bar{n}_s^2}{n_s^2} \right] \quad (2.16b)$$

where $C \equiv \frac{n_p}{\bar{n}_s}$, z is defined by Equation (2.16a), and

$\bar{n}_s$ is the equilibrium Saha electron number density at the emitter sheath-plasma interface,

$D_{\text{ambi}}(E)$ is the ambi-polar diffusion coefficient at the emitter sheath-plasma interface, and

$$A(\xi) = \frac{\beta_r d^2 n_s^2}{D_{\text{ambi}}(E)} \quad (2.17)$$

If the plasma electron temperature is assumed to be a constant (isothermal case), Equations (2.16)-(2.17) can be simplified to

$$\frac{dC}{d\xi} = z, \quad (2.18a)$$

$$\frac{dz}{d\xi} = -AC[1 - C^2], \quad (2.18b)$$

$$\text{where } A = \frac{\beta_r d^2 n_s^2}{D_{\text{ambi}}} \quad (2.19)$$

2. Sheath Region Analysis

In the region immediately adjacent to a solid surface, charge neutrality does not prevail. A thin sheath is expected to exist whose extent is of the order of h_D (Debye length):

$$h_D = \left(\frac{kT_E}{4\pi e^2 n_0} \right)^{1/2} = 6.9 \sqrt{\frac{T(^{\circ}\text{K})}{n_0(\text{cc}^{-1})}} \text{ cm} \quad (2.20)$$

where n_o is the unknown value of the ion density at the emitter sheath-plasma interface.

Now we define η to be the dimensionless coordinate

$$\eta = \frac{x}{h_D} \quad (2.21)$$

The emitter is located at $\eta = 0$ and the sheath-plasma interface is considered to be at $\eta = \infty$. The dimensionless motive χ is assumed to be zero at $\eta = \infty$. At some point $\eta = \hat{\eta}$, the motive distribution may have a peak with $\chi = \chi_E$. If the motive distribution is monotonic, then $\hat{\eta} = 0$. Since the sheath is very thin, there are no collisions and we can identify three classes of charged particles as follows.

2.1 The plasma ions enter the sheath with density n_o and speed u_o . Assuming these ions to be mono-energetic, we have the following energy equation:

$$\frac{1}{2}m_i u_o^2 = \frac{1}{2}m_i u_i^2 - e\phi. \quad (2.22)$$

After rearranging the above equation and using dimensionless motive notation, we obtain:

$$\frac{u_i}{u_o} = \sqrt{1 + \chi \frac{2kT_E}{m_i u_o^2}}. \quad (2.23)$$

Since $u_i n_i = u_o n_o$, then

$$\frac{n_i}{n_o} = \frac{1}{\sqrt{1 + \pi \hat{\alpha} \chi}}, \quad (2.24)$$

$$\text{where } \hat{\alpha} = \frac{2kT_E}{\pi m_i u_o^2} = \frac{(\hat{a}_E/2)^2}{u_o^2}, \quad (2.25)$$

$$\hat{a}_E = \sqrt{\frac{8kT_E}{\pi m_i}},$$

and $\hat{\alpha}$ is an unknown parameter to be determined

(2 b) The number density of the emitted electrons is denoted by n_E and can be represented by the similar relation as Equation (2.24)

First, we have

$$\frac{1}{2} m_e u_e^2 + e\phi = \frac{1}{2} m_e \hat{u}_e^2 + e\hat{\phi}. \quad (2.26)$$

After rearranging the above equation and using the motive peak notation as well as the dimensionless motive notation, we have

$$\frac{u_e^2}{\hat{u}_e^2} = 1 + (\chi_E - \chi) \frac{2kT_E}{m_e \hat{u}_e^2}. \quad (2.27)$$

Since $u_e n_E = \hat{u}_e \hat{n}_E$, we obtain

$$\frac{n_E}{\hat{n}_E} = \frac{1}{\sqrt{1 + \pi(\chi_E - \chi)}}, \quad (2.28)$$

where $\hat{n}_E$ is the unknown value of n_E at $\hat{n}$

Now we define

$$N_E \equiv \frac{\hat{n}_E}{n_o}. \quad (2.29)$$

Equation (2.28)

$$\frac{n_E}{n_o} = \frac{N_E}{\sqrt{1 + \pi(\chi_E - \chi)}} \quad (2.30)$$

(2 c) The number density of the plasma electrons is denoted by n'_e . We assume that these electrons have a Boltzmann distribution with unknown temperature T_e . Therefore,

$$n'_e = n'_{e,0} e^{-\frac{\chi}{\tau_0}} \quad (2.31)$$

where $n_{e,0}$ is the unknown value of n'_e at $\eta = \infty$,

τ_0 is the dimensionless plasma electron temperature at the emitter sheath-plasma interface.

Let's define

$$N_e \equiv \frac{n'_{e,0}}{n_0} \quad (2.32)$$

Equation (31) becomes

$$\frac{n'_e}{n_0} = N_e e^{-\frac{\chi}{\tau_0}} \quad (2.33)$$

(2 d) Poisson's equation can now be written as follows

$$\frac{d^2\psi}{dx^2} = \frac{e}{\epsilon_0} [n_i - (n_E + n'_e)] \quad (2.34)$$

After rearranging the Equation (2.34) and using the dimensionless coordinate as well as the dimensionless motive notation, we have

$$\frac{d^2\chi}{d\eta^2} = \frac{n_i}{n_0} - \left(\frac{n_E}{n_0} + \frac{n'_e}{n_0} \right) \quad (2.35)$$

a function F to represent the term on the right-hand side of Equation (35), we have

$$\frac{d^2\chi}{d\eta^2} = F(\chi; \chi_E, N_E, N_e, \hat{\alpha}, \tau_0) \quad (2.36)$$

Now two conditions can immediately be imposed on F

1. As $\eta \rightarrow \infty$, $\chi \rightarrow 0$, the plasma must become quasi-neutral

$$F(0, \chi_E, N_E, N_e, \hat{\alpha}, \tau_0) = 0.$$

So

$$N_E + (N_e - 1) (1 + \pi \chi_E)^{1/2} = 0 \quad (2.37)$$

2. The dimensionless motive χ must approach zero without oscillation. (Bohm's sheath criteria). That is

$$\frac{dF}{d\chi}(0; \chi_E, N_E, N_e, \hat{\alpha}, \tau_0) = 0.$$

So

$$(2N_e - \tau_0 \pi \hat{\alpha})(1 + \pi \chi_E)^{3/2} - N_E \pi \tau_0 = 0. \quad (2.38)$$

3. A third condition on F may be found by considering the electric field at $\hat{\eta}$.

(a) If a motive peak exists ($\hat{\eta} > 0$), the local electric field is zero. Thus we require

$$\left. \frac{d\chi}{d\eta} \right|_{\eta=\hat{\eta}} = 0.$$

That is

$$\int_0^{\chi_E} F(\chi; \chi_E, N_E, N_e, \hat{\alpha}, \tau_0) d\chi = 0.$$

So

$$2 \left[(1 + \pi \hat{\alpha} \chi_E)^{1/2} - 1 \right] + 2 \hat{\alpha} N_E \left[1 - (1 + \pi \chi_E)^{1/2} \right] + \pi \hat{\alpha} \tau_0 N_e (e^{-\chi_E/\tau_0} - 1) = 0. \quad (2.39)$$

(b) If the motive distribution is monotonic ($\hat{\eta} = 0$), the value of $\hat{n}_E$ should equal, in the absence of Schottky effect, to the Richardson electron number density n_R :

$$\hat{n}_E = n_R = \frac{1}{e} \sqrt{\frac{\pi m_e}{2 k T_e}} \left\{ 120 T_e^2 e^{-\frac{e \phi_E}{k T_e}} \right\}. \quad (2.40)$$

If Schottky effect is included, we have

$$\hat{n}_E = n_R \exp \left\{ 0.276 \left[\frac{n_o (\text{cc}^{-1})}{10^{14}} \right]^{1/4} \left[\frac{1000}{T_E (\text{°K})} \right]^{3/4} K^{1/2} \right\}, \quad (2.41)$$

$$\text{where } K = \sqrt{2} \left\{ \frac{2}{\hat{\alpha}} \left(\sqrt{1 + \pi \hat{\alpha} \chi_E} - 1 \right) + \frac{2N_E}{\pi} \left(1 - \sqrt{1 + \pi \chi_E} \right) + N_e \tau_o \left(e^{-\chi_E/\tau_o} - 1 \right) \right\}^{1/2}. \quad (2.42)$$

For the derivation of Equation (2.41) the reader is referred to Appendix A.

3. Fundamental Equations of Electrons in the Thermionic Diode

3.1 The electron continuity equation is

$$J_E = J_{BS} + J. \quad (2.43)$$

J is the electron current density and can be assumed to be a constant across the converter since the contributions from gas phase ionization and other sources are small. If we evaluate J at $\xi = 1$, we obtain

$$J = \frac{a_e n_p(1)}{4} \exp(-\chi_c/\tau_1), \quad (2.44)$$

$$\text{where } a_e \equiv \left(\frac{8kT_e}{\pi m_e} \right)^{1/2} = a_E \sqrt{\tau_1}.$$

τ_1 is the plasma electron temperature at the collector sheath-plasma interface. J_{BS} is the current density of electrons scattering at the emitter sheath edge back over the emitter motive peak into the emitter. We have

$$J_{BS} = \frac{n'_{e,0} a_e}{4} \exp(-\chi_E/\tau_o) \quad (2.45)$$

Substituting Equations (2.44)-(2.45) into Equation (2.43) and using dimensionless notation, we obtain

$$\frac{1}{j} = \left\{ 1 + \sqrt{\frac{\tau_o}{\tau_1}} \frac{N_e C_o}{C_1} \exp \left[\frac{\Delta \chi_p + V_d}{\tau_1} - \left(\frac{1}{\tau_o} - \frac{1}{\tau_1} \right) \chi_E \right] \right\}, \quad (2.46)$$

where V_d is defined in Fig. 1.

For the isothermal case, Equation(2.46) can be reduced to

$$j = \left\{ 1 + \frac{N_e C_0}{C_1} e^{(\Delta\chi_p + V_d)/\tau} \right\}^{-1} \quad (2.47)$$

3.2 The energy equation for electron can be written in the form

$$J \frac{2kT_e}{e} (\tau + \chi) - 2D_e n_p k \frac{dT_e}{dx} = J_E \left(2 + \hat{\Phi}_E \right) \frac{kT_E}{e} - (J_E - J) \left(2\tau_0 + \hat{\Phi}_E \right) \frac{kT_E}{e} + \\ + V_{ion} \int_0^{\chi} \beta_r n_p (n_s^2 - n_p^2) dx, \quad (2.48)$$

where D_e is the electron ambi-polar diffusion coefficient and has the following formula²

$$D_e = \left(\frac{8}{\pi} \right)^{1/2} \frac{\hat{\lambda}_e}{3} \sqrt{\frac{kT_e}{m_e}}, \quad (2.49)$$

$$\hat{\Phi}_E = \Phi_E + \Delta\chi.$$

If we rearrange Equation(2.48) and substitute Equations (2.16) (2.17) and (2.49) into Equation (2.48), we obtain

$$\tau_0 = 1 - \frac{1}{2} J \left(2\tau(\xi) - 2\tau_0 - \chi_E - \Delta\chi(\xi) \right) + \frac{2}{3} \sqrt{\tau(\xi)} \frac{C(\xi)}{C_0 N_E} \frac{\hat{\lambda}_e}{d} \frac{dT_e(\xi)}{d\xi} + \\ + \frac{1}{2} V_{ion} \frac{e}{kT_E} \left[\left(\frac{\pi m_e}{2kT_E} \right)^{1/2} \frac{2D_{ambi}(E)}{N_E C_0 d} - \right. \\ \left. - \frac{1}{N_E} \sqrt{\frac{m_e}{m_i}} \frac{1}{\sqrt{\hat{\alpha}}} \right], \quad (2.50)$$

where $C_0 = \frac{n_0}{n_s}$, and $\Delta\chi = \chi(0) - \chi(\xi)$.

The global electron energy equation, which shall be found useful later, is

$$J_E(2+\hat{\phi}_E)\frac{kT_E}{e} - (J_E - J)(2\tau_0 + \hat{\phi}_E)\frac{kT_E}{e} - V_{ion}\left[n_0\sqrt{\frac{2kT_E}{\pi m_i}}\frac{1}{\sqrt{\hat{\alpha}}} + n_1\sqrt{\frac{2kT_E}{\pi m_i}}\sqrt{\frac{\pi\tau_1}{2}}\right] = J(2\tau_1 + \hat{\phi}_E + V_d)\frac{kT_E}{e} \quad (2.51)$$

3.3 Electron momentum considerations

From Equation (2.7) we define the flow velocity of the electron gas u_e as

$$J \equiv n_p u_e. \quad (2.52)$$

Combining Equations (2.8) and (2.52) we have

$$J = -\frac{\lambda_e}{m_e a_e} \left[k \frac{dn_p}{dx} T_e + en_p \frac{d\psi}{dx} \right]. \quad (2.53)$$

Now dividing both sides by J_E , we get

$$j = -\frac{\pi}{4} \frac{\lambda_e}{d} \frac{1}{\sqrt{\tau_1 \hat{n}_E}} \left[\frac{d\tau n_p}{d\xi} + n_p \frac{d\chi}{d\xi} \right], \quad (2.54)$$

which is the conventional diffusion-mobility formula. We can split χ into two parts as follows³.

$$\chi = \chi_{ambi} + \chi_{ohmic}, \quad (2.55)$$

where χ_{ambi} is chosen to be

$$\tau \frac{dn_p}{d\xi} + n_p \frac{d\tau}{d\xi} + n_p \frac{d\chi_{ambi}}{d\xi} = 0. \quad (2.56)$$

After integration from $\xi = 0$ to $\xi = 1$, Equation (2.56) becomes

$$\Delta\chi_{ambi} = \tau_0 - \tau_1 - \int_0^1 \tau \frac{dn_p}{n_p}. \quad (2.57)$$

Therefore

$$\frac{d\chi_{\text{ohmic}}}{d\xi} = \frac{4d}{\pi} \frac{\sqrt{\tau}}{\lambda_e} \frac{\hat{n}_E}{n_p} j \quad (2.58)$$

Also, after integration from $\xi = 0$ to $\xi = 1$, Equation (2.58) becomes

$$\Delta\chi_{\text{ohmic}} = jR, \quad (2.59)$$

$$\text{where } R = \frac{4d}{\pi\lambda_e} \int_0^1 \sqrt{\tau} \frac{\hat{n}_E}{n_p} d\xi,$$

and is called the normalized plasma resistance. If T_e is constant across the converter (i.e. the isothermal case), Equations (2.57) and (2.60) can be reduced to

$$\Delta\chi_{\text{ambi}} = \left(\ln \frac{n_p(1)}{n_p(0)} \right) \tau_1 \quad (2.61)$$

$$R = \frac{4d}{\pi\lambda_e} \int_0^1 \sqrt{\tau} \frac{\hat{n}_E}{n_p} d\xi. \quad (2.62)$$

So the plasma motive drop can be expressed as

$$\Delta\chi_p = \Delta\chi_{\text{ambi}} + \Delta\chi_{\text{ohmic}}. \quad (2.63)$$

4. The Boundary Conditions for the Center Plasma Region

The boundary conditions for the center plasma region are supplied by matching the sheath analysis with the plasma analysis.

4.1 Emitter sheath analysis

Since $J = J_E - J_{BS}$ and $j = J/J_E$, we have

$$j = 1 - J_{BS}/J_E \quad (2.64)$$

Substituting Equation(2.45)into Equation(2.46) we obtain

$$j = 1 - \frac{N_e \sqrt{\tau_0}}{2N_E} \exp\left(-\chi_E/\tau_0\right). \quad (2.65)$$

The number fluxes for ions and electrons at the emitter sheath-plasma interface can be expressed as follows.

$$\Gamma_{i,o} = -n_o \frac{\hat{a}_E}{2} \frac{1}{\sqrt{\hat{\alpha}}} \quad , \quad \Gamma_{e,o} = \hat{n}_E \frac{a_E}{2} - n_o N_e \frac{a_e}{4} e^{-\chi_E/\tau_0}.$$

From Equation (2.5) we have

$$\Gamma_o = \Gamma_{i,o} + \frac{\mu_i}{\mu_e} \Gamma_{e,o}.$$

Therefore,

$$\Gamma_o = -n_o \sqrt{\frac{2kT_E}{\pi m_i}} \frac{1}{\sqrt{\hat{\alpha}}} + \Lambda \sqrt{\frac{m_e}{m_i}} \left[\hat{n}_E \sqrt{\frac{2kT_E}{\pi m_e}} - n_o N_e \sqrt{\frac{kT_e}{2\pi m_e}} e^{-\chi_E/\tau_0} \right] \quad (2.66)$$

$$\text{where } \Lambda = \frac{\lambda_i}{\lambda_e} \sqrt{\frac{T_e}{T_i}} = \sqrt{\tau} \frac{\lambda_i}{\lambda_e}.$$

From Equation (2.12), we also have

$$\Gamma_o = -D_{ambi} \left(\frac{dn_p}{dx} \right)_{o+} = -\frac{\lambda_i}{d} \frac{\sqrt{k}}{3} \frac{(T_i + T_e)}{\sqrt{m_i T_i}} \left(\frac{dn_p}{d\xi} \right)_{o+} \quad (2.67)$$

Equating Equation(2.66) to Equation(2.67) and using the relation of Equation(2.65) we have the following boundary condition at the emitter sheath-plasma interface.

$$n_o = \frac{\hat{\lambda}_i}{d} K_o \left(\frac{dn_p}{d\xi} \right)_o, \quad (2.68)$$

$$\text{where } K_o = \sqrt{\frac{\pi}{2}} \frac{1}{3} \frac{(1+\tau_0)}{\textcircled{H}}, \quad (2.69)$$

$$\odot = \frac{1}{\sqrt{\hat{\alpha}}} - \Lambda N_E j \quad (2.70)$$

4.2 Collector sheath analysis

We assume $N_E = 0$, $N_e = 1$ at the collector sheath-plasma interface.

From Equation (38), we obtain

$$\hat{\alpha}_1 = \frac{2}{\tau_1 \pi} . \quad (2.71)$$

So the velocity of plasma ions which are entering the collector sheath is

$$u_{i,1} = \sqrt{\frac{\tau_1 \cdot k T_E}{m_i}} .$$

The number fluxes for ions and electrons at the collector sheath-plasma interface can be expressed as follows:

$$\Gamma_{i,1} = n_{i,1} u_{i,1} , \quad \Gamma_{e,1} = \frac{n_e a_e}{4} e^{-\chi_c / \tau_1} .$$

Then, the ambi-polar flux at the collector side is in the form

$$\Gamma_1 \equiv \Gamma_{i,1} + \frac{\mu_i}{\mu_e} \Gamma_{e,1} .$$

Therefore,

$$\Gamma_1 = n_{i,1} \sqrt{\frac{2kT_e}{\pi m_i}} \left[\frac{1}{2} \Lambda e^{-\chi_c / \tau_1} + \sqrt{\frac{\pi}{2}} \right] . \quad (2.72)$$

On the other hand, from Equation (2.12) we again have

$$\Gamma_1 = D_{\text{ambi}} \left(\frac{dn_p}{dx} \right)_1 = - \frac{\lambda_i}{d} \frac{\sqrt{k}}{3} \frac{(T_i + T_e)}{\sqrt{m_i T_i}} \left(\frac{dn_p}{d\xi} \right)_1 . \quad (2.73)$$

Equating Equation(2.72) to Equation(2.73) we get the following boundary condition at the collector sheath-plasma interface

$$n_{i,1} = -\frac{\hat{\lambda}_i}{d} K_1 \left(\frac{dn_p}{d\xi} \right)_1, \quad (2.74)$$

$$\text{where } K_1 = \sqrt{\frac{\pi}{2}} \frac{1}{3} \frac{\left(\frac{T_c}{T_E} + \tau_1 \right)}{\sqrt{\frac{T_c}{T_E} \tau_1} \Omega}, \quad (2.75)$$

$$\Omega = \sqrt{\frac{\pi}{2}} + \frac{1}{2} \Lambda e^{-\chi_c/\tau_1} \quad (2.76)$$

Since

$$j = \frac{J}{J_E} = \frac{\sqrt{\frac{8kT_e}{\pi m_e}} n_p(1) \left[\exp(-\chi_c/\tau_1) \right] / 4}{\hat{n}_E \sqrt{\frac{8kT_E}{\pi m_e}} / 2},$$

or

$$j = \frac{\sqrt{\tau_1}}{N_E} \frac{C_1}{C_0} \frac{1}{2} \exp(-\chi_c/\tau_1), \quad (2.77)$$

we can also express K_1 as in the form

$$K_1 = \sqrt{\frac{\pi}{2}} \frac{1}{3} \frac{\left(\frac{T_c}{T_E} + \tau_1 \right)}{\left(\sqrt{\frac{T_c}{T_E} \tau_1} \frac{\pi}{2} + \sqrt{\frac{T_c}{T_E}} \Lambda N_E j \frac{C_0}{C_1} \right)}. \quad (2.78)$$

Equation (2.78) is preferred over equations (2.75), (2.76) since χ_c , the collector sheath height does not appear. Note that both K_0 and K_1 are dimensionless and are of order unity.

III. COMPUTATIONAL ALGORITHM

Before entering the detailed computational procedures, it will be necessary to give some important relations to express the equilibrium Saha electron number density n_s , the cesium atom number density n_a and the three-body recombination coefficient β_r as some functions of the cesium vapor pressure P , the plasma electron temperature T_e and the cesium atom temperature T_a .

We may write the Saha equation in the form⁴

$$\frac{n_e n_i}{n_a} = 2 \frac{Z_i}{Z} \left(\frac{2\pi m_e k T_e}{h^2} \right)^{3/2} e^{-V_{ion}/kT_e} \quad (3.1)$$

where Z and Z_i are the partition functions for the cesium atom and for the cesium ion, respectively. For the cesium ion C_s^+ , Z_i is approximately equal to 1 and for the cesium atom C_s , Z is approximately equal to 2 within the temperature range between 1500°K and 2500°K.

The ionization energy of cesium atom V_{ion} is 3.89 eV. The collection of constants on the right hand side of Eq. (3.1) has the value

$$\left(\frac{2\pi m_e k}{h^2} \right)^{3/2} = 2.42 \times 10^{15} \text{ cm}^{-3} \text{ } ^\circ\text{K}^{-3/2}$$

Therefore, in the plasma region, the Saha equation can be written as

$$\frac{n_s^2}{n_a} = 1.15 \times 10^{14} \left(\frac{T_e}{3000} \right)^{3/2} \exp \left[-15 \left(\frac{3000 - T_e}{T_e} \right) \right] (\text{cm}^{-3}) \quad (3.2)$$

According to Hinnoy and Hirschberg (1962)⁵, the

values of the three-body recombination coefficient can be given as follows.

$$\beta_r(T_e) = 2.46 \times 10^{-24} \left(\frac{3000}{T_e} \right)^{4.5} \text{ cm}^6 \text{ sec}^{-1}. \quad (3.3)$$

The relationship between the vapor pressure and the vapor temperature for the cesium atom has been determined in a careful experiment by Taylor and Langmuir.⁶ Their results are summarized as follows for the pressure and for the density.¹¹

$$\log_{10} P = 11.0531 - 1.35 \log_{10} T_a - \frac{4041}{T_a}, \quad (3.4)$$

$$\log_{10} n_a = 30.0379 - 2.35 \log_{10} T_a - \frac{4041}{T_a}, \quad (3.5)$$

where P is the cesium pressure in torr, T_a is the temperature of the cesium atom in degrees Kelvin and n_p is the cesium vapor density in atoms/cm³. Combining Equations (3.4) and (3.5) we obtain the following relation

$$n_a = 9.656 \times 10^{18} \frac{P}{T_a} (\text{cm}^{-3}). \quad (3.6)$$

Then Equation (80) becomes

$$n_s^2 = 1.11 \times 10^{33} \left(\frac{T_e}{3000} \right)^{1.5} \frac{P}{T_a} \exp \left[-15 \left(\frac{3000 - T_e}{T_e} \right) \right] (\text{cm}^{-6}). \quad (3.7)$$

Another important physical quantity needed to be determined for the computational work is the ambi-polar diffusion coefficient D_{ambi} .

From Equations (2.10b) and (2.13) we know

$$D_{\text{ambi}} \equiv - \frac{\hat{\lambda}_i \sqrt{k}}{3 \sqrt{m_i T_i}} (T_i + T_e)$$

where $\hat{\lambda}_1 = (\hat{Q}_{ia} n_a)^{-1}$ and $\hat{Q}_{ia}$ is the nominal cross section of cesium ions in cesium gas, which has a relation with the true cross section of cesium ions in cesium gas Q_{ia} in the following form:

$$Q_{ia} = 3\sqrt{\pi/8} \times \hat{Q}_{ia}. \quad (3.8)$$

The reason for using the nominal cross section is that, in our present computational procedures, all calculations are based on the assumed data of the nominal cross section. We assume $\hat{Q}_{ia}$ has some value around $3 \times 10^{-14} \text{ cm}^2$ and write

$$\hat{Q}_{ia} = 3 \times 10^{-14} \text{ (correction factor)}$$

where the correction factor is equal to $\hat{Q}_{ia}/(3 \times 10^{-14})$ (i.e., if $\hat{Q}_{ia} = 3 \times 10^{-14} \text{ cm}^2$, the correction factor will be unity). Then the ambi-polar diffusion coefficient can be written in the following form

$$D_{\text{ambi}} = 9.2 \times 10^{-4} \frac{\sqrt{T_a}}{P} (T_a + T_e) \times \frac{3 \times 10^{-14}}{\hat{Q}_{ia}} \quad (3.9)$$

In addition to the value of $\hat{Q}_{ia}$, we also need the value of $\hat{Q}_{ea}$, the cross section for electrons in the cesium gas. We shall simply assume $\lambda_i/\lambda_e = 2/3$. The temperature for cesium atoms and cesium ions T_a will be assumed to vary linearly along with the interelectrode space. That is

$$T_a = T_E - (T_E - T_C)\xi.$$

1. Computational Procedure for the Isothermal Case

The computational procedure for the isothermal case is relatively straightforward. Nevertheless, an iterative method is required.

Each point on the C-V characteristic is characterized by the value of τ . It can readily be shown that, in the limit of small λ_i/d and λ_e/d , Equation (2.18) has a non-trivial solution for the plasma density distribution $C(\xi)$ only if $A \geq \pi^2$. We first determine τ^* such that $A(\tau^*) = 10$ as our starting point, and consider only $\tau \geq \tau^*$. For each τ chosen for computation, the procedure is as follows.

For the plasma region, the computation of the plasma density distribution $C(\xi)$ requires the values of K_0 and K_1 which appear in Equations (2.68) and (2.74). We begin by guessing a reasonable value for j , and use the emitter sheath analysis to obtain $N_e(j, \tau)$, $N_E(j, \tau)$, $\hat{\alpha}(j, \tau)$, $\chi_E(j, \tau)$, etc. These values are used to evaluate K_0 and K_1 and enable us to generate an iterant for $C(\xi)$. Using the computed $C(\xi)$, the corresponding normalized plasma resistance R can be computed from Equation (2.62):

$$R = \frac{4d N_E C_0 \sqrt{\tau}}{\pi \lambda_e} \int_0^1 \frac{d\xi}{C(\xi)} \quad (3.10)$$

This value of R of course depends on the value of j originally chosen. To complete the iteration loop, an alternative relation between R and j (and τ) is required.

Substituting Equations (2.59), (2.61) and (2.63) into Equation (2.47), we obtain

$$\frac{1}{j} = 1 + N_e \exp \frac{V_d + jR}{\tau} \quad (3.11)$$

which can be solved for R as follows:

$$R = \frac{1}{j} \left\{ \tau \ln \left(\frac{1-j}{j N_e} \right) - V_d \right\} \quad (3.12)$$

The arc-drop V_d can be related to τ and j by the global energy equation (2.51). For the isothermal case under consideration here, we have simply

$$\tau = 1 - \frac{1}{2} j V_d \quad (3.13)$$

where the energy losses term due to ion diffusion has been neglected.

Using Equation (3.13) in Equation (3.12), we have

$$R = \frac{1}{j} \left\{ \tau \ln \left(\frac{1-j}{j N_e} \right) + \frac{2(\tau-1)}{j} \right\} \quad (3.14)$$

which relates R with (τ, j) .

Using the value of R computed from Equation (3.10) in Equation (3.14), we can solve numerically for a new value of j which is used to start the computational loop again. The iteration is considered converged when two successive values of R 's computed differ by less than 5×10^{-4} .

The flow chart of the computer program is given in Appendix B.

2. Computation Procedure for the Non-isothermal Case

The isothermal case essentially assumes that the electron thermal conductivity is infinite. It is well known⁸ that in practice the electron temperature will be variable across the plasma region because of finite electron thermal conductivity, and will substantially affect the plasma density distribution.

The calculation for the variable electron temperature case is much more difficult because of the strong sensitivity of the ionization term on the electron temperature distribution. Because of this strong coupling between the electron energy equation and the ambi-polar diffusion equation, considerable difficulty was encountered in designing a stable and efficient

iteration scheme. Our procedure is to rewrite the electron energy equation (2.50) in the following form:

$$\begin{aligned} \frac{d\tau(\xi)}{d\xi} = f \times 1.5 \frac{C_0 N_E^d}{C \lambda_e \tau^{1/2}} \left\{ \tau_0 - \left[1 - \frac{1}{2} j(2\tau - 2\tau_0 - \chi_E - \Delta\chi(\xi)) \right] - \right. \\ \left. - \left[7.264 \frac{z D_{ambi}(E)}{N_E C_0^d \tau_E^{3/2}} - \frac{45.858}{N_E \tau_E \sqrt{\alpha}} \right] \right\} \end{aligned} \quad (3.15)$$

where

$$\Delta\chi(\xi) = \tau_0 - \tau - \int_0^\xi \frac{\tau}{C} \frac{dC}{d\xi} d\xi - \frac{4}{\pi} \frac{\lambda_i}{\lambda_e} N_E K_0 j \left(\frac{dC}{d\xi} \right)_0 \int_0^\xi \frac{\sqrt{\tau}}{C} d\xi. \quad (3.16)$$

We have arbitrarily introduced a constant factor f on the right hand side. When $f = 0$ we recover the isothermal case which can be computed as outlined previously and is used as our starting iterant. Then f is slowly increased toward unity; for each value of f , a "converged" solution is computed. Our experience shows that f must be increased very slowly, resulting in considerable computing expense. The solutions presented in this thesis are for $f = 0.213$ only. Obviously, the present computational algorithm is not satisfactory and a more efficient algorithm is needed. Nevertheless the present results show clearly the effects of finite electron thermal conductivity.

For a fixed value of f , the computational procedure is as follows. The equation analogous to Equation (3.11) for the variable T_e case is Equation (2.46)

$$\frac{1}{j} = \left\{ 1 + \sqrt{\frac{\tau_0}{\tau_1}} \frac{N_e C_0}{C_1} \exp \left[\frac{\Delta\chi_p + V_d}{\tau_1} - \left(\frac{1}{\tau_0} - \frac{1}{\tau_1} \right) \chi_E \right] \right\} \quad (3.17)$$

$$\text{where } \Delta\chi_p = (\tau_1 - \tau_0) + \int_0^1 \frac{\tau}{C} \frac{dC}{d\xi} d\xi + j \frac{4}{\pi} \frac{\lambda_i}{\lambda_e} N_E K_0 \left(\frac{dC}{d\xi} \right)_0 \int_0^1 \frac{\sqrt{\tau} d\xi}{C} \quad (3.18)$$

and V_d is again related to j , τ_0 and τ_1 (analogous to Equation (3.13) by the global electron energy equation (2.51) which is rewritten in dimensionless form below:

$$\tau_0 = 1 - \frac{1}{2} j V_d + j(\tau_0 - \tau_1) - \frac{45.858}{N_E T_E} \left(\frac{1}{\sqrt{\alpha}} + \frac{C_1}{C_0} \sqrt{\frac{\pi \tau_1}{2}} \right) \quad (3.19)$$

where the last term on the right represents the energy losses due to ion diffusion. Each point on the C-V characteristic is now characterized by a value of τ_0 . Equation (3.15) is solved first for the electron temperature distribution $\tau(\xi)$ with all terms (except τ) being evaluated using the previously converged solution for a lower value of f . The integration proceeds from $\xi = 1$ with a guessed value of τ_1 and marches toward $\xi = 0$. An iteration loop is needed here to determine the value of τ_1 such that $\tau(\xi = 0) = \tau_0$ as originally specified. Once $\tau(\xi)$ is found, this electron temperature distribution is used in Equation (2.16) to compute the plasma density distribution $C(\xi)$. Using this new pair of iterants $\tau(\xi)$ and $C(\xi)$, the value of $\Delta\chi_p$ can be computed from Equation (3.17) and the value of V_d can be computed from Equation (3.18). These values, when substituted into Equation (3.15) yields a new value for j . This completes the main iteration loop, and the computation begins anew. The solution is considered converged when two successive iterants yield $\Delta\chi_p$ within a specified convergence criterion (10^{-2}).

The solving of the electron energy equation by marching from $\xi = 1$ toward $\xi = 0$ deserves further comment. It can readily be shown by a

linearized analysis that the electron energy equation is unstable for marching in the positive ξ direction; any small error will tend to grow exponentially. By marching in the negative ξ direction, the equation is stable, and any small error will damp out exponentially. When f is very small this unstable tendency in forward marching is weak, but as f increases the forward marching scheme becomes highly unstable and the backward marching scheme must be used.

The flow chart of the computer program is given in Appendix C.

IV. RESULTS AND COMPARISON

All calculations are done for $d = 1$ mm, $p(\text{cesium}) = 1$ torr and $T_E = 1500^\circ\text{K}$.

For the isothermal case, Fig. 2 shows the plasma number density distribution across the converter for various values of $1.656 \leq \tau \leq 1.966$. Since in this case the electron temperature and therefore the Saha density is a constant, the plasma density distribution is very nearly symmetrical about the centerline. For larger values of τ , $A(\tau)$ is quite large and the plasma density approaches the local Saha density in the center region. For the five highest τ cases shown, a boundary layer approximation was used to take advantage of the largeness of $A(\tau)$, resulting in a slight jump in slope at the centerline. Fig. 3 shows the plasma density distribution for the non-isothermal case for $1.781 \leq \tau_0 \leq 1.901$. The peaks of the plasma density distributions move toward the emitter as expected. Fig. 4 shows the corresponding electron temperature distribution across the converter. It is seen that τ increases slightly near the emitter and then decreases nearly linearly with distance. The sudden jump of τ at the collector is probably due to the numerical procedures employed, but further study is required. Mathematically, reasonable amount of error in τ_1 will damp out quickly as one marches backward toward the emitter; thus it is not possible to determine τ_1 accurately using the present algorithm. However, the relatively small jump does not affect the computed C-V characteristic to any significant extent. Fig. 5 shows the corresponding Saha electron number density distributions.

Figs. 6, 7, 8, 9 shows N_E , N_e , $\hat{\alpha}$ and χ_E as a function of τ_0 as obtained from the self-consistent emitter sheath analysis. It is

interesting to see that all of them are relatively constant in the range of τ_0 considered. Fig. 10 and 11 shows K_0 and K_1 which are also relatively constant. These observations shall be most useful in any later attempt to develop an approximate analytical theory. Fig. 12 shows the normalized plasma resistance for the isothermal case. It is seen that R is lowest near ignition (where τ is smallest), and increases relatively rapidly with τ_0 . Fig. 13 shows the normalized motive drop across the plasma region for the isothermal case. The increase of $\Delta\chi_p$ with τ_0 is the direct consequence of the increase in R as shown previously in Fig. 12. Fig. 14 shows the motive distribution in the plasma region for the non-isothermal case. The substantially lower plasma density near the collector side results in a favorable ambi-polar motive contribution, and it is seen that the net motive drop across the plasma region is less than the corresponding isothermal case in spite of the obviously larger plasma resistance. Fig. 15 shows the collector sheath height for the isothermal case.

Fig. 16 shows j or J/J_E as a function of the arc-drop V_d as defined by Fig. 1. It is most interesting to note that j is again practically constant. Note that j has the physical meaning of being the fraction of emitted electron current which after passing over the emitter motive peak is able to reach the collector. While j remains relatively constant, the actual dimensional converter current J is increasing rapidly with V_d (or τ_0) as can be seen from the C-V characteristic shown in Fig. 17. The sharply rising converter current J is due entirely to the lowering of the emitter motive peak by the increase of plasma density in the plasma region.

All computations presented here have been done with ϕ_E assumed to be

sufficiently low so that an emitter motive peak always exists. For fixed given ϕ_E , the saturation Richardson current can easily be computed, and the corresponding vacuum ideal C-V characteristic can easily be constructed. A sample vacuum ideal C-V characteristic with $\phi_E = 2.15$ eV is included in Fig. 17 as curve 1. The isothermal and non-isothermal C-V characteristics are shown as curves 2 and 3, respectively. For a given emitter Richardson current, the computed characteristic will cease to be valid when J is larger than about 50% of the vacuum saturation current. The arc-drop at 10 amps/cm² are 0.34 volts and 0.4 volts for the isothermal and non-isothermal case, respectively. It is important to note that the non-isothermal result is obtained by assuming an electron thermal conductivity nearly five times its actual theoretical value. Thus, the actual adverse effect of finite electron conductivity is expected to be larger than that shown in Fig. 17. However, the isothermal case did not include the energy loss term due to diffusion. Thus, part of the degradation of the non-isothermal result is due to this loss mechanism.

While the present calculation shows clearly the adverse role of finite electron thermal conductivity, the computational algorithm used remains to be improved. Because of the inefficiency of the present procedure, it would be extremely costly to increase the factor f from 0.213 toward unity. Alternative algorithms have not been exhaustively evaluated because of limitations in time. With the experience gained from the present study, particularly the observation of insensitivity of many parameters such as N_E , N_e , $\hat{\alpha}$, χ_E , j etc. to variations in τ_0 , and that the bulk of the electron temperature distribution is nearly linear, it appears likely that a much more efficient computational algorithm can be developed which could allow extensive parametric studies of the performance of thermionic energy converters.

V. REFERENCES

1. S. H. Lam, Annual Progress Report on Thermionic Energy Conversion Research Analysis, Princeton University, September 1975, p. 7.
2. F. G. Baksht and V. G. Yur'ev, Thermionic Converters and Low-temperature Plasma (English Edition Edited by L. K. Hansen, Rasor Associates, Incorporated, 1976), p. 87-91.
3. S. H. Lam, Preliminary Report on Plasma Arc-Drop In Thermionic Energy Converters, Princeton University, March 1976, p. 10.
4. M. Mitchner and C. H. Kruger, Jr., Partially Ionized Gases (Wiley, New York, 1973), p. 78.
5. Hinov, E., and J. G. Hirschberg, 1962, Electron-Ion Recombination in Dense Plasmas, Phys. Rev., vol. 125, No. 3, p. 795.
6. J. B. Taylor and I. Langmuir, Phys. Rev., 51, 753 (1937).
7. S. H. Lam, Preliminary Report on Plasma Arc-Drop In Thermionic Energy Converters, Princeton University, March 1976, p. 8.
8. F. G. Baksht and V. G. Yur'ev, Thermionic Converters and Low-temperature Plasma (English Edition Edited by L. K. Hansen, Rasor Associates, Incorporated, 1976), p. 293-296.
9. Leon Lapidus and J. H. Serifeld, Numerical Solutions of Ordinary Differential Equations (Academic Press, New York, 1971), p. 182.
10. S. M. Sze, Physics of Semiconductor Devices (Wiley, New York, 1969), p. 365.
11. L. K. Hansen, Physics of the Cesium Vapor Thermionic Diode (Atomics International, A Division of North American Rockwell Corporation, 1969), p. 24.

VI. DESCRIPTION OF FIGURES

- 1--Qualitative sketch of motive distribution. Work functions and motives are in units of $k T_E/e$.
- 2--Normalized plasma number density versus dimensionless gap size for the isothermal case. n_p denotes the plasma number density. n_s denotes the constant Saha number density. The dimensionless plasma electron temperature τ has a value of 1.656 for the lowest curve and a value of 1.966 for the highest curve. The direction of arrow denotes the increase direction in τ . For the first twelve curves the increment in τ is 0.01 and for the next four curves the increment in τ is 0.05. In our case, $d = 1\text{mm}$, $T_R = 550^\circ\text{K}$ and $T_E = 1500^\circ\text{K}$. These three values will be the same for all the following figures.
- 3--Normalized plasma number density versus dimensionless gap size for the non-isothermal case. n_p still denotes the plasma number density. $\bar{n}_s$ denotes the Saha number density at the emitter side. The dimensionless plasma electron temperature at the emitter side τ_o has a value of 1.781 for the lowest curve and a value of 1.901 for the highest curve. The direction of arrow denotes the increase direction in τ_o . The increment in τ_o between the lowest two curves is 0.02 and for the rest curves the increment in τ_o is 0.025.

- 4--Normalized plasma electron temperature distribution across the converter in the non-isothermal case. The reason for the sudden jump near the collector will be explained in detail in Chapter V.
- 5--Normalized Saha number density distribution across the converter in the non-isothermal case. For the explanation of the hook near the collector, the reader can be referred to Chapter V.
- 6-- N_E denotes the number density of the emitted electrons at the emitter motive peak normalized by the number density of the plasma ions at the edge of the sheath. The solid line represents the isothermal case and the dashed line represents the non-isothermal case.
- 7-- N_e denotes the number density of the plasma electrons which are entering the sheath normalized by the number density of the plasma ions at the edge of the sheath. The solid line represents the isothermal case and the dashed line represents the non-isothermal case.
- 8-- α_e denotes the ratio of kT_E to the kinetic energy of ions entering the sheath. The circles represent the overlap portion for both the isothermal case and the non-isothermal case.
- 9-- χ_E denotes the emitter sheath motive height. The solid line represents the isothermal case and the dashed line represents the non-isothermal case.

- 10-- K_0 is a parameter involving in the boundary condition for $n_p(\xi)$ at $\xi = 0$. The solid line represents the isothermal case and the dashed line represents the non-isothermal case.
- 11-- K_1 is a similar parameter involving in the boundary condition for $n_p(\xi)$ at $\xi = 1$. The solid line represents the isothermal case and the dashed line represents the non-isothermal case.
- 12--Normalized plasma resistance versus dimensionless plasma electron temperature for the isothermal case.
- 13--Dimensionless plasma motive drop versus dimensionless plasma electron temperature for the isothermal case
- 14--Dimensionless plasma motive distribution across the gap for the non-isothermal case. The lowest curve corresponds to τ_0 being equal to 1.781 and the highest curve corresponds to τ_0 being equal to 1.901. The situation of increment in τ_0 is the same as that in Figure 2. The detailed explanation for these curves distribution is in Chapter V.
- 15--Dimensionless collector sheath motive height versus dimensionless plasma electron temperature for the isothermal case.
- 16--Normalized C-V characteristic. The solid line represents the isothermal case and the dashed line represents the non-isothermal case.
- 17--Curves 1, 2, and 3 represent the C-V characteristics for the ideal vacuum case, the isothermal case and the non-isothermal

case, respectively. 0.34 volts and 0.4 volts are the converter arc-drops for the isothermal case and the non-isothermal case, respectively.

FIG. 1

FIG. 2

FIG. 4

FIG. 5

FIG. 6

FIG. 7

FIG. 8

FIG. 9

FIG. 10

FIG. 11

FIG. 12

FIG. 13

FIG. 14

FIG. 15

FIG. 16

FIG. 17

APPENDIX A

THE DERIVATION OF SCHOTTKY EFFECT

The lowering of the metal work function by an amount $\Delta\phi$ as a result of the image force and the electric field is called the Schottky Effect. The Schottky barrier lowering $\Delta\phi$ and the location of the lowering x_m are given as follows¹⁰.

$$x_m = \sqrt{\frac{e}{16\pi\epsilon_0 E}} \text{ cm} \quad (\text{A.1})$$

$$\Delta\phi = \sqrt{\frac{eE}{4\pi\epsilon_0}} = 2E x_m \text{ volts} \quad (\text{A.2})$$

$$E = -\frac{d\psi}{dx} = \frac{d\chi}{d\eta} \frac{kT_E}{eh_D} \quad (\text{A.3})$$

where $k = 1.381 \times 10^{-23} \text{ J/}^\circ\text{K}$ Boltzmann's constant

$$h_D = 6.9 \left(\frac{T_E(^{\circ}\text{K})}{n_0(\text{cc}^{-1})} \right)^{1/2} \text{ cm} \quad \text{Debye length}$$

Since
$$\frac{d^2\chi}{d\eta^2} = \frac{n_i}{n_0} - \left(\frac{n_E}{n_0} + \frac{n'_e}{n_0} \right)$$

or
$$\frac{d^2\chi}{d\eta^2} = \frac{1}{\sqrt{1+\pi\alpha}\chi} - \left(\frac{N_E}{\sqrt{1+\pi(\chi_E - \chi)}} + N_e e^{-\chi/\tau_0} \right) \quad (\text{A.4})$$

we have

$$\begin{aligned} \frac{d\chi}{d\eta} = \sqrt{2} \left\{ \frac{2}{\pi\alpha} \left(\sqrt{1+\pi\alpha}\chi_E - 1 \right) + \frac{2N_E}{\pi} \left(1 - \sqrt{1+\pi\chi_E} \right) \right. \\ \left. + N_e \tau_0 \left(e^{-\chi_E/\tau_0} - 1 \right) \right\}^{1/2} = K \end{aligned} \quad (\text{A.5})$$

Substituting Eq. (A.5) into Eq. (A.3), we get

$$E = K \frac{k T_E}{e h_D} \quad (A.6)$$

If Schottky Effect is included

$$\hat{n}_E = n_R \exp \left(\frac{e\Delta\phi}{k T_E} \right)$$

in which

$$\frac{e\Delta\phi}{k T_E} = \frac{e}{k T_E} \sqrt{\frac{K k T_E}{4\pi\epsilon_0 h_D}}$$

or

$$\frac{e\Delta\phi}{k T_E} = 0.276 \left[\frac{n_o (\text{cc}^{-1})}{10^{14}} \right]^{1/4} \left[\frac{1000}{T_E} \right]^{3/4} K^{1/2} \quad (A.7)$$

Finally, we get

$$\hat{n}_E = n_R \exp \left\{ 0.276 \left[\frac{n_o (\text{cc}^{-1})}{10^{14}} \right]^{1/4} \left[\frac{1000}{T_E} \right]^{3/4} K^{1/2} \right\} \quad (A.8)$$

APPENDIX B FLOW CHART FOR THE ISOTHERMAL CASE

APPENDIX C

FLOW CHART FOR THE NON-ISOTHERMAL CASE

APPENDIX D

LISTING AND COMMENTS FOR THE COMPUTER PROGRAMS

Two computer programs are listed for the isothermal case and for the non-isothermal case, respectively. The variables representing the physical quantities of the input data, common to both cases, are first described.

- DC = gap size (mm.)
- PSIE = work function of the emitter (volts)
- COWO = work function of the collector (volts)
- FI = dimensionless emitter work function ($e\Phi_E/kT_E$)
- FIW = dimensionless collector work function ($e\Phi_C/kT_C$)
- P = cesium vapor pressure (torrs.)
- TEI = emitter temperature ($^{\circ}\text{K}$)
- TC = collector temperature ($^{\circ}\text{K}$)
- KI = number of nodes across the converter
- H1 = step size for $\xi (=1/KI)$
- CQIA = correction factor for the nominal cross section of cesium ions
 in cesium gas ($\hat{Q}_{ia} = \text{CQIA} \times 3 \times 10^{-14} \text{ cm}^2$)
- CDBR = correction factor for the three-body recombination
 coefficient ($\beta_r = \text{CDBR} \times 3.45 \times 10^{-22} \text{ cm}^6/\text{sec}$)
- SLM = ratio of the mean-free-path of ions colliding with
 neutrals and electrons to the mean-free-path of electrons
 colliding with neutrals and ions

RJR = Richardson current density of the emitter (amps/cm^2)
 RNR = Richardson number density (= RJR $\times 2/a_E$)
 J = normalized converter current density

The computational procedures in the emitter sheath region are the same for both the isothermal case and the non-isothermal case. Therefore, the names of the variables used in computer programs for N_E , N_e , $\hat{\alpha}$ and χ_E are also the same and described as follows.

NE = N_E
 NEI = N_e
 R = $\hat{\alpha}$
 XE = χ_E
 ES = convergence criterion for calculating the above
 four values (normally 1×10^{-4})

The default constants for both the isothermal case and the non-isothermal case are DBR, QIA and KM2.

1. Isothermal Case

The main control parameters are as follows:

M = iteration counter for calculating the normalized plasma
 resistance. When $M > 1$, new J is obtained by interpola-
 tion
 FLAG = 0, motive peak exists
 = 1, no motive peak (modified as needed during iteration)

- ACO = index for calculating the dimensionless plasma electron temperature τ which corresponds to $A = \pi^2$ to ensure ignition, use only once
- ACOO = index for increment to τ along the C-V characteristic

The main variables used in this computer program are given as follows:

- TE = plasma electron temperature ($^{\circ}\text{K}$)
- AB = the parameter A (see Eq. (2.19))
- TAU = dimensionless plasma electron temperature ($=\text{TE}/\text{TEI}$)
- ZNZ1 = Saha number density (cc^{-1})
- CO = plasma number density normalized by ZNZ1 at the emitter sheath edge
- C = normalized plasma number density across the converter (a subscripted variable)
- CDRO = derivative of the normalized plasma number density, C, at the emitter sheath edge
- CDR = derivative of the normalized plasma number density across the converter (a subscripted variable), denoted by Z in Eq. (2.18a)
- KO = K_0 (see Eq. (2.69))
- K1 = K_1 (see Eq. (2.78))
- YI = intermediate values during the course of iteration for Z
- CI = intermediate values during the course of iteration for C

ERROR	=	convergence criterion for calculating the plasma number density distribution (normally 5×10^{-4})
RO	=	normalized plasma resistance calculated from the plasma number density distribution
RC	=	normalized plasma resistance calculated from the considerations of the global electron continuity, energy and momentum equations
ES3	=	convergence criterion for calculating the normalized plasma resistance (normally 5×10^{-4})
RNER	=	ratio of $\hat{n}_E$ to the Richardson number density
TA	=	cesium atom temperature ($^{\circ}\text{K}$)
XC	=	collection sheath motive height/ kT_E
VDE	=	converter voltage arc-drop/ kT_E
BGJ	=	converter current density (amps/cm^2)
BGV	=	sum of output voltage and work function of the collector (volts)
BGVV	=	output voltage (volts)
XP	=	$\Delta\chi_p$ (motive drop across plasma)/ KT_E

IV G LEVEL 21

MAIN

DATE = 78236

20/18/5

```

    IMPLICIT REAL*8(A-H,O-Z)
    REAL*8 NE,NEI,J,NET,NEIT,K10,K20,K11,K21,K12,K22,K13,K23,KO,K1,
    *J1,J2,J12
    DIMENSION F(4),A(4,4),CP(150),CDR(150),YI(150),CI(150),C(150),
    *VC(150)
    REAL*4 SX(100),SY(100),SX1(100),SX2(100),SX3(100),SX4(100)
    *,SY1(100),SY2(100),SY3(100),SY4(100),SY5(100),SY6(100),
    *SY7(100),SY8(100),SY9(100),SY10(100),SY11(100),SY12(100),
    *SY13(100),SVD(100),SJ(100)
    DC=1.0
    TE=2300.0
    SLM=2.0/3.0
    PSIE=1.15
    COWO=1.1
    ES=1.D-4
    P=1.0
    TC=1500.0
    TEI=1500.0
    TA=(TC+TEI)/2.0
    DBR=3.45
    QIA=3.D-14
    ES3=5.0D-4
    J=0.6
    M=1
    KI=50
    KM2=KI/2
    H1=0.02
    FLAG=0.D0
    ACO=0.D0
    ACOO=0.00
    RNER=0.D0
    DO 21 I=1,51
    SX(I)=(I-1)*0.02

    PI=4.D0*DATAN(1.D0)
    ABS=10.0/(DC*DC)
    AB=ABS*DC*DC
    A1=DSQRT(AB)
    CPO=0.D0
    CDRO=0.1*A1
    DO 150 L=1,KI
    UL=H1*L
    CP(L)=0.1*DSIN(A1*UL)
    CDR(L)=CDRO*DCOS(A1*UL)
    CONTINUE
    TET=TE
    II=0
    49 II=II+1
    PD=P/10.0
    PD1=PD*PD
    TE3=3000.0/TE
    TEE=TE3**5.5
    TER=TE/TA
    RTER=TER**1.5/(1.0+1.0/TER)
    PO=-15.0*TE3

```

21
C
C

150
50

```

DE=DEXP(PO)
CDBR=3.45/DBR
CQIA=3.0-14/QIA
AS=1.96D10*PD1*TEE*RTER*DE*CDBR/CQIA
IF(ACO.EC.1.0) GO TO 51
PA=AS-ABS
DPA=AS*(45000.0/TE**2-3.0/TE-1.0/(TE+TA))
TE=TE-PA/DPA
IF(DABS(TE-TET).GT.E5) GO TO 50
51 AB=AS*DC*DC
TCE=TC/TEI
TAU=TE/TEI
A1=DSQRT(AB)
TEO=(1000.0/TEI)**0.75
TE31=(TE/3000.0)**0.75
TE4=-15.0*(3000.0-TE)/TE
TEX=DEXP(TE4)
STEX=DSQRT(TEX)
PTA=DSQRT(P/TA)
ZNZ1=3.33D16*PTA*TE31*STEX
FI=-1.602D4*PSIE/(1.381*TEI)
FIW=1.602D4*COWO/(1.381*TEI)
IFI=DEXP(FI)
RJR=120.0*TEI*TEI*DFI
RNR=0.62D12*PJR*DSQRT(1000.0/TEI)
PRINT 140,TAU,AB,TE
140 FORMAT('1','TAU=',1P1D10.3,2X,'A=',1P1D10.3,2X,'TE=',1P1D10.3/)
SX2(II)=TAU

C
C
C
92 CAI=PI*TAU
IF(M.NE.1) GO TO 2
NE=0.627
NEI=0.792
R=0.292
XE=2.588
2 NET=NE
NEIT=NEI
XET=XE
RT=R
1 CAB=DSQRT(1.0+PI*DABS(XE))
CA=CAB**3
CAC=PI*R
CAD=DSQRT(1.0+DABS(CAC*XE))
CAE=-XET/TAU
CAF=DEXP(CAE)
IF((RNER-1.0).GT.0.D0) GO TO 40
FLAG=0.D0
F(1)=NE+(NEI-1.0)*CAB
F(2)=(2.0*NEI-CAI*R)*CA-NE*CAI
F(3)=2.0*(CAD-1.0)+2.0*R*NE*(1.0-CAB)+CAC*NEI*TAU*(CAF-1.0)
F(4)=2.0*NE*(J-1.0)+DSQRT(TAU)*CAF*NEI
5 A(1,1)=1.D0
A(1,2)=CAB
A(1,3)=0.D0

```



```

A(1,4)=PI*(NEI-1.0)/(2.0*CAB)
A(2,1)=-CAI
A(2,2)=2.0*CA
A(2,3)=-CAI*CA
A(2,4)=1.5*PI*(2.0*NEI-CAI*R)*CAB
A(3,1)=2.0*R*(1.0-CAB)
A(3,2)=CAI*R*(CAF-1.0)
A(3,3)=PI*XE/CAD+2.0*NE*(1.0-CAB)+CAI*NEI*(CAF-1.0)
A(3,4)=CAC/CAD-CAC*NE/CAB-CAC*NEI*CAF
A(4,1)=2.0*(J-1.0)
A(4,2)=J.D0
A(4,3)=0.D0
A(4,4)=-CAF/DSQRT(TAU)

```

```

CALL MATINV(A, 16, 4, JX)

```

```

DO 3 K=1,4

```

```

NE=NE-A(1,K)*F(K)

```

```

NEI=NEI-A(2,K)*F(K)

```

```

R=R-A(3,K)*F(K)

```

```

XE=XE-A(4,K)*F(K)

```

```

3 CONTINUE

```

```

IF(DABS(NE-NET).GT.E5) GO TO 2

```

```

IF(DABS(NEI-NEIT).GT.E5) GO TO 2

```

```

IF(DABS(R-RT).GT.E5) GO TO 2

```

```

IF(DABS(XE-XET).GT.E5) GO TO 2

```

```

GO TO 12

```

```

40 XK=2.0*(2.0*(CAD-1.0)/CAC-NEI*TAU*(1.0-CAF)-2.0*NE*(CAB-1.0)/PI)

```

```

DXK=DSQRT(DABS(XK))

```

```

DDXK=DSQRT(DXK)

```

```

ZNZ=CO*ZNZ1

```

```

ZNZO=(ZNZ/1.0D14)**0.25

```

```

SZTD=0.276*ZNZO*TEO*DDXK

```

```

EZTD=DEXP(SZTD)

```

```

NE=RNK*EZTD/ZNZ

```

```

TAUS=DSQRT(TAU)

```

```

Q1=(1.0-J)*2.0*NE/TAUS

```

```

ALQ1=DLOG(Q1)

```

```

XE=-TAU*ALQ1

```

```

Q2=DSQRT(1.0+PI*XE)

```

```

NEI=1.0-NE/Q2

```

```

Q3=Q2**3

```

```

R=(2.0*NEI-NE*PI*TAU/Q3)/(PI*TAU)

```

```

FLAG=1.0

```

```

C

```

```

C

```

```

C

```

```

12

```

```

8

```

```

TITX=0.D0

```

```

YIO=0.D0

```

```

CSFO=CDRO

```

```

YI(1)=-H1*AB*CPO*(1.0-CPO*CPO)-0.5*H1*H1*AB*(1.0-3.0*CPO*CPO)*
*CDRO

```

```

YI(2)=H1*(-AB*CPO*(1.0-CPO*CPO)-4.0*AB*CP(1)*(1.0-CP(1)*CP(1))-AB
**CP(2)*(1.0-CP(2)*CP(2)))/3.0

```

```

KM1=KI-1

```

```

DO 4 L=2,KM1

```

```

YI(L+1)=H1*(-AB*CP(L-1)*(1.0-CP(L-1)*CP(L-1))-4.0*AB*CP(L)*(1.0-
*CP(L)*CP(L))-AB*CP(L+1)*(1.0-CP(L+1)*CP(L+1)))/3.0+YI(L-1)

```

4 CONTINUE

C

ALM=DSQRT(TER)*SLM
FO=1.0/DSQRT(DABS(R))-ALM*NE*J
PA=DSQRT(PI/2.0)/3.0
KO=(1.0+TAU)*PA/FO
ALMS=3.44D-5*TA*COIA/P
PRC=DSQRT(TCE)*NE*J*ALM
DEN2=TAJ*TCE*PI/2.0
DEN1=DSQRT(DEN2)
IF(CP(KI).LT.1.4D-3) GO TO 15
GO TO 6

15 DEN=DEN1+PRO

GO TO 20

6 DEN=DEN1+PRO*CPO/CP(KI)

20 K1=PA*(ICE+TAU)/DEN

C

CIO=0.D0
CI(1)=-0.5*H1*H1*AB*CPO*(1.0-CPO*CPO)
CI(2)=H1*(4.0*YI(1)+YI(2))/3.0
DO 35 L=2,KM1
CI(L+1)=H1*(YI(L-1)+4.0*YI(L)+YI(L+1))/3.0+CI(L-1)
CONTINUE

35

C

CO=ALMS*KO*CDRO/EC
C(KI)=-ALMS*K1*CDR(KI)/EC
YTO=C(KI)-CO-CI(KI)
CDRO=YTO
IF(CP(KM2).GT.0.9) GO TO 69
IF(CP(KM2).GT.0.75) GO TO 70
DO 16 L=1,KI
CDR(L)=YTO+YI(L)
BJ=L
C(L)=CO+YTO*H1*BJ+CI(L)
CONTINUE
GO TO 72
DO 71 L=1,KI
CDR(L)=(YTO+YI(L)+CDR(L))/2.0
BJ=L
C(L)=(CO+YTO*H1*BJ+CI(L)+C(L))/2.0

16

CONTINUE

GO TO 72

70

DO 71 L=1,KI

CDR(L)=(YTO+YI(L)+CDR(L))/2.0

BJ=L

C(L)=(CO+YTO*H1*BJ+CI(L)+C(L))/2.0

71

CONTINUE

CO=(CPO+CO)/2.0

CDRO=(CDRO+CSRO)/2.0

72

IF(LABS(C(40)).GT.1.0) GO TO 82

SCP=0.D0

SCPO=(CPO-CO)*(CPO-CO)

DO 7 L=1,KI

SCP1=(CP(L)-C(L))*(CP(L)-C(L))

SCP=SCP+SCP1

7

CONTINUE

SCP=SCP+SCPO

ERROR=SCP/(C(KM2)*C(KM2))

FRINT 123,ERROR

123

FORMAT(' ERROR=',1P1E10.3)

IF(ERROR.GT.E53) GO TO 9

GO TO 11

IV G LEVEL 21

MAIN

DATE = 78236

20/18/5

```

9      CPO=CO
      DO 10 L=1,KI
      CP(L)=C(L)
10     CONTINUE
      TITX=TITX+1.0
      IF (TITX.GT.100.0) GO TO 82
      GO TO 8

C
C
69     ACOO=1.0
      AKD=(ALMS*KO/DC)**2
      FCD=1.0+1.0/(AB*AKD)
      DFCD=DSQRT(FCD*FCD-1.0)
      SQCO=FCD-DFCD
      CO=DSQRT(SQCO)
      CDRO=DSQRT(AB*(0.5*CO**4-CO*CO+0.5))
      CPO=CO
      KN=KI
      KN1=KN/2
      I=0
      CO1=CO
      H=H1*A1
31     CQ=0.5*CO1**4-CO1*CO1+0.5
      K10=H*DSQRT(CQ)
      K20=-H*CO1*(1.0-CO1*CO1)
      CB=CO1+0.5*K10
      CQ1=0.5*CB**4-CB*CB+0.5
      K11=H*DSQRT(CQ1)
      K21=-H*CB*(1.0-CB*CB)
      CB1=CO1+0.5*K11
      CQ2=0.5*CB1**4-CB1*CB1+0.5
      K12=H*DSQRT(CQ2)
      K22=-H*CB1*(1.0-CB1*CB1)
      CB2=CO1+K12
      CQ3=0.5*CB2**4-CB2*CB2+0.5
      K13=H*DSQRT(CQ3)
      K23=-H*CB2*(1.0-CB2*CB2)
      SK1=(K10+2.0*K11+2.0*K12+K13)/6.0
      SK2=(K20+2.0*K21+2.0*K22+K23)/6.0
      I=I+1
      C(I)=CO1+SK1
      IF (C(I).GT.1.0) C(I)=1.0
      CO1=C(I)
      IF (I.EQ.KN1) GO TO 33
      GO TO 31
33     CDRO2=-CDR(KI)
      AX=PA*(TCE+TAU)
      EX=DSQRT(TAU*TCE*PI/2.0)
      EX=ALM*NE*J*DSQRT(TCE)
52     CDROT=CDRO2
      ADB=ALMS*AX*CDRO2/(DC*BX)
      ECB=EX*CO/BX
      FXY=CDRO2*CDRO2+AB*(ADB-ECB)**2-AB*(ADB-ECB)**4/2.0-AB/2.0
      DFX=2.0*CDRO2+2.0*AB*(ADB-ECB)*ADB/CDRO2-2.0*AB*(ADB-ECB)**3*
      *ADB/CDRO2
      IF (DFXY.EQ.0.0) GO TO 82

```

```

CDRO2=CDRO2-FXY/DFXY
IF (DABS (CDROT-CDRO2) .GT.ES) GO TO 52
CDR (KI) =-CDRO2
C (KN) =ADE-ECB
CP (KN) =C (KN)
KN3=KN1+1
KN2=KN
CO1=C (KN2)
32  CQ=0.5*CO1**4-CO1*CO1+0.5
    K10=H*DSQRT (CQ)
    K20=-H*CO1*(1.0-CO1*CO1)
    CB=CO1+0.5*K10
    CQ1=0.5*CB**4-CB*CB+0.5
    K11=H*DSQRT (CQ1)
    K21=-H*CB*(1.0-CB*CB)
    CB1=CO1+0.5*K11
    CQ2=0.5*CB1**4-CB1*CB1+0.5
    K12=H*DSQRT (CQ2)
    K22=-H*CB1*(1.0-CB1*CB1)
    CB2=CO1+K12
    CQ3=0.5*CB2**4-CB2*CB2+0.5
    K13=H*DSQRT (CQ3)
    K23=-H*CB2*(1.0-CB2*CB2)
    SK1=(K10+2.0*K11+2.0*K12+K13)/6.0
    SK2=(K20+2.0*K21+2.0*K22+K23)/6.0
    KN2=KN2-1
    C (KN2) =CO1+SK1
    IF (C (KN2) .GT.1.0) C (KN2) =1.0
    CO1=C (KN2)
    IF (KN2.EQ.KN3) GO TO 11
    GO TO 32

C
C
C
11  RM=4.0*SLM*NE*KO*DSQRT (TAU) *CDRO/PI
    VCC=1.0/CO
    DO 125 L=1,KI
    VC (L) =1.0/C (L)
125  CONTINUE
    SH=0.D0
    DO 135 L=2,KM2
    NS=2*L-2
    SH1=VC (NS)
    SH=SH+SH1
135  CCNTINUE
    TH=0.D0
    DO 136 L=1,KM2
    NSS=2*L-1
    TH1=VC (NSS)
    TH=TH+TH1
136  CONTINUE
    RS=H1*(VCO+VC (KI) +2.0*SH+4.0*TH)/3.0
    RO=KM*RS
    ELI=0.D0
    ALO=DLOG ((1.0-J)/(J*NEI))
    RC=TAU*ALO/J+2.0*(TAU-1.0+ELI)/(J*J)

```

```
IF (DABS (RC-RO) .GT. ES3) GO TO 90
GO TO 91
90  DIRC=RC-RO
    ZNZ=CO*ZNZ1
    BANE=NE*ZNZ
    RNER=BANE/RNR
    RST=RO/SLM
    PRINT 121, DIRC, NE, RNER, KO, K1, RO, RST
121  FORMAT (/ ' DR=', 1P1D10.3, 2X, ' NE=', 1P1D10.3, 2X, ' SCHO=', 1P1D10.3,
* 2X, ' KO=', 1P1D10.3, 2X, ' K1=', 1P1D10.3, 2X, ' RO=', 1P1D10.3, 2X, ' RST=',
* 1P1D10.3)
    PRINT 222, NEI, R, XE, ELI, J
222  FORMAT ( ' NEI=', 1P1D10.3, 2X, ' R=', 1P1D10.3, 2X, ' XE=', 1P1D10.3, 2X,
* ' ELI=', 1P1D10.3, 2X, ' J=', 1P1D10.3 /)
    IF (M.EQ.1) GO TO 93
    RC2=RC
    R2=RO
    J2=J
    J12=J1-J2
    R12=R1-R2
    RC12=RC1-RC2
    J= ((J12*R2-R12*J2) - (J12*RC2-RC12*J2)) / (RC12-R12)
    RC1=RC2
    R1=R2
    J1=J2
    GO TO 92
93  RC1=RC
    R1=PO
    J1=J
    J=J+0.05
    M=M+1
    GO TO 92
91  VDE= (1.0-TAU-ELI) *2.0/J
    TS= (TE/3000.0) **1.5
    TS1=-15.0*(3000.0-TE)/TE
    TS2=DEXP (TS1)
    TBA=DSQRT (TS2*TS)
    CNT=CO*NE*ZNZ1
    DLCR=DLOG (CNT/RNR)
    TMT=-DLCR
    IF (FLAG.EQ.1.0) GO TO 81
    IF (TMT) 80,81,81
80  FLAG=1.0
    GO TO 92
81  BGJ=J*NZ*CO*TBA*DSQRT (TEI) *PTA*1.654D3
    SMV=VDE+TMT-FI
    BGV=SMV*TEI*8.62D-5
    SMVV=VDE+TMT-FI-PIW
    EGVV=SMVV*TEI*8.62D-5
    DO 25 L=1,10
    SJ (L)=0.1*L
25  SVC (L)=2.0*(1.0-TAU-ELI)/SJ (L)
    PRINT 190, J, BGJ, VDE, EGV, TMT, BGVV, ELI
190  FORMAT (/ ' J=', 1P1D10.3, 2X, ' BGJ=', 1P1D10.3, 2X, ' VD=', 1P1D10.3, 2X,
* ' BV=', 1P1D10.3, 2X, ' DXE=', 1P1D10.3, 2X, ' BVV=', 1P1D10.3, 2X, ' ELI=',
* 1P1D10.3 /)
```

```

      SX1(II)=BGV
      SY1(II)=EGJ
      SY2(II)=R
      SY3(II)=NE
      SY4(II)=NEI
      SY5(II)=KO
      SY6(II)=K1
      SY7(II)=RO
      SY8(II)=XE
      SY9(II)=FNER
      SX3(II)=VDE
      SY10(II)=J
      PRINT 182
182   FORMAT(14X,'NUMBER DENSITY DISTRIBUTION'/)
94    PRINT 160,CO,(C(L),L=1,KI)
160   FORMAT (3X,1P7D15.3)
      SY(1)=CO
      DO 23 I=1,50
23    SY(I+1)=C(I)
      CALL SETSY(5.)
      IF(II.EQ.1) CALL DFIPS1(SX,SY,-51,51,5.,0.,0.2,0.,0.2,0,0,0)
      IF(II.GT.1) CALL DFIPS2(SX,SY,51,51)
      C1J=C(KI)*(1.0-J)/(J*CO)
      C1JL=TAU*DLOG(C1J)
      XC=XE+C1JL
      XP=XC-XE-VDE
      PRINT 122,XC,XP
122   FORMAT(/' XC=' ,1P1D10.3,2X,'XP=' ,1P1D10.3)
      SY11(II)=XC
      SY12(II)=XP
      IF(ACOO.EQ.1.0) GO TO 225
      IF(FLAG.EQ.0.00) GO TO 223
      TAU=TAU+0.01
      GO TO 224
223   TAU=TAU+0.01
      GO TO 224
225   TAU=TAU+0.05
224   TE=TAU*TEI
      IF(BGV.LT.1.5) GO TO 24
      ACO=1.0
      GO TO 49
27    II=II-1
24    SXM=SX1(II)
      PSIED=35.0
      NP=PSIED+1.0
      DO 26 L=1,NP
      SX4(L)=2.15+(L-1.0)/100.0
26    SY13(L)=120.0*TZI*TEI*DEXP(-1.16D4*SX4(L)/TEI)
      CALL SETSY(6.)
      CALL DFIPS1(SX1,SY1,-II,51,5.,1.5,0.18,0.,10.,0,0,0)
      CALL DFIPS2(SX4,SY13,NP,51)
      SY2D=1.0/5.0
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY2,-II,51,5.,1.6,0.08,0.,SY2D,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY3,-II,51,5.,1.6,0.08,0.,SY2D,0,0,0)

```

```
CALL SETSY(5.)
CALL DFIPS1(SX2,SY4,-II,51,5.,1.6,0.08,0.,SY2D,0,0,0)
CALL SETSY(5.)
CALL DFIPS1(SX2,SY5,-II,51,5.,1.6,0.08,0.,SY2D,0,0,0)
CALL SETSY(5.)
CALL DFIPS1(SX2,SY6,-II,51,5.,1.6,0.08,0.,SY2D,0,0,0)
SY7D=4.0/5.0
CALL SETSY(5.)
CALL DFIPS1(SX2,SY7,-II,51,5.,1.6,0.08,0.,SY7D,0,0,0)
CALL SETSY(5.)
CALL DFIPS1(SX2,SY8,-II,51,5.,1.6,0.08,0.,0.6,0,0,0)
SY9D=SY3(11)/5.0
CALL SETSY(5.)
CALL DFIPS1(SX2,SY9,-II,51,5.,1.6,0.08,0.,SY9D,0,0,0)
CALL SETSY(5.)
CALL DFIPS1(SX2,SY11,-II,51,5.,1.6,0.08,0.,0.6,0,0,0)
SY12D=0.6
CALL SETSY(5.)
CALL DFIPS1(SX2,SY12,-II,51,5.,1.6,0.08,0.,SY12D,0,0,0)
CALL SETSY(5.)
CALL DFIPS1(SX3,SY10,-II,51,5.,-3.4,0.28,0.,0.2,0,0,0)
STCP
END
```

C
C

```
SUBROUTINE MATINV(A,NX,N,JX)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(NX),L(10),M(10)
DOUBLE PRECISION DABS
JX=1
E=1.0
KK=-N
DO 80 K=1,N
  NK=NK+N
  L(K)=K
  M(K)=K
  KK=NK+K
  BIGA=A(KK)
  DO 20 J=K,N
    IZ=N*(J-1)
    DO 20 I=K,N
      IJ=IZ+I
10    IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20
15    BIGA=A(IJ)
    L(K)=I
    M(K)=J
20    CONTINUE
    J=L(K)
    IF(J-K) 35,35,25
25    KI=K-N
    DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
30    A(JI)=HOLD
35    I=M(K)
    IF(I-K) 45,45,38
38    JP=N*(I-1)
    DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
40    A(JI)=HOLD
45    IF(BIGA) 48,46,48
46    D=0.0
    IF(D.EQ.0.) JX=-1
    RETURN
48    DO 55 I=1,N
      IF(I-K) 50,55,50
50    IK=NK+I
      A(IK)=A(IK)/(-BIGA)
55    CONTINUE
    DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
```



```
      IJ=IJ+N
      IF (I-K) 60,65,60
60     IF (J-K) 62,65,62
62     KJ=IJ-I+K
      A(IJ)=HOLD*A(KJ)+A(IJ)
65     CONTINUE
      KJ=K-N
      DO 75 J=1,N
      KJ=KJ+N
      IF (J-K) 70,75,70
70     A(KJ)=A(KJ)/BIGA
75     CONTINUE
      D=D*BIGA
      A(KK)=1.0/BIGA
80     CONTINUE
      K=N
100    K=K-1
      IF (K) 150,150,105
105    I=L(K)
      IF (I-K) 120,120,108
108    JQ=N*(K-1)
      JR=N*(I-1)
      DO 110 J=1,N
      JK=JQ+J
      HOLD=A(JK)
      JI=JR+J
      A(JK)=-A(JI)
110    A(JI)=HOLD
120    J=M(K)
      IF (J-K) 100,100,125
125    KI=K-N
      DO 130 I=1,N
      KI=KI+N
      HOLD=A(KI)
      JI=KI-K+J
      A(KI)=-A(JI)
130    A(JI)=HOLD
      GO TO 100
150    RETURN
      END
```

2. Non-isothermal Case

The main control parameters are as follows:

- MM = iteration counter for calculating the normalized plasma resistance. When $MM > 1$, new J is obtained by interpolation
- FLAG = 0, motive peak exists
 = 1, no motive peak (modified as needed during iteration)
- II = counter of solutions along C-V characteristics
- B = 0, flag to change FACTO
 = 1, holds FACTO constant
- W = flag to change from the isothermal case to the non-isothermal case

The main variables used in this computer program are given as follows:

FACTO = correction factor for the electron thermal conductivity

$$(k_e = \frac{2}{FACTO} \times D_e n_p k)$$

DTAU0 = dimensionless plasma electron temperature at the emitter sheath edge

DTAU = dimensionless plasma electron temperature across the converter (a subscripted variable)

TE00 = plasma electron temperature at the emitter sheath edge (°K)

- DTE = plasma electron temperature across the converter (a subscripted variable)
- CNDO = coefficient of the term for the plasma electron temperature gradient at the emitter sheath edge (see Eq. (3.15))
- CND = coefficient of the term for the plasma electron temperature gradient across the converter (a subscripted variable)
- DTA = cesium atom temperature across the converter (a subscripted variable)
- DAKPO = dimensionless motive always taken to be zero at the emitter sheath-plasma interface
- DAKP = dimensionless motive in the plasma region calculated from the electron momentum consideration (a subscripted variable)
- CO = normalized plasma number density at the emitter sheath edge
- C = normalized plasma number density across the converter (a subscripted variable)
- CDRO = derivative of the normalized plasma number density at the emitter sheath edge
- CDR = derivative of the normalized plasma number density across the converter (a subscripted variable)
- ABO = the parameter A evaluated at the emitter sheath edge
- DAB = the parameter A across the converter (a subscripted variable)

ZNZE = Saha number density at the emitter sheath edge (cc^{-1})
 DZNZ1 = Saha number density across the converter (a subscripted variable)
 KO = K_0
 K1KI = K_1
 RNER = ratio of $\hat{n}_E$ to the Richardson number density
 ES3 = convergence criterion for calculating the normalized plasma number density (normally 5×10^{-4})
 VDE = converter voltage arc-drop/ kT_E
 DAP = dimensionless plasma motive drop calculated from the electron continuity considerations
 DDAP = convergence criterion for calculating the dimensionless plasma motive drop (normally 4×10^{-2})
 RZNZ1 = normalized Saha number density across the converter (a subscripted variable)
 BGJ = converter current density (amps/cm^2)
 BGV = sum of output voltage and work function of the collector (volts)
 BGVV = output voltage (volts)

```

IMPLICIT REAL*8 (A-H,O-Z)
REAL*8 NE,NEI,J,NET,NEIT,K10,K20,K11,K21,K12,K22,K13,K23,KO,K1,
*J1,J2,J12,K1KI,NTD,JC
DIMENSION P(4),A(4,4),CP(50),CDR(50),YI(50),CI(50),C(50),VC(50),
*DTAU(50),DST(50),TCC(50),DTE(50),DT(50),EDTAU(50),FITC(50)
*,SITC(50),DAKP(50),DTA(50),CND(50),DDTAU(50),T4C(50),DTE3(50),
*DTEE(50),DTER(50),DRTER(50),DPO(50),DDE(50),DAB(50),DA1(50),
*DPTA(50),DTE31(50),DTE4(50),DTEX(50),DSTEX(50),DZNZ1(50),DZNZQ(50)
*,RZNZ(50),ZI(50),DDD(50),CDDC(50),RAMB(50),DDDD(50),DIAMB(50),
*ZZ(50),CNU(50),CMD(50),DELTA(50),PSDEL(50),DELTS(50),PSDES(50),
*RZNZ1(50),SNDD(51,6),TDIS(51,6),PDIS(51,6),SAHA(51,6)
REAL*4 SX(100),SY(100),SX1(100),SX2(100),SX3(100)
*,SY1(100),SY2(100),SY3(100),SY4(100),SY5(100),SY6(100),
*SY7(100),SY8(100),SY9(100),SY10(100),SY11(100)
DC=1.0
SLM=2.0/3.0
PSIE=1.15
COWO=1.1
ES=1.D-4
P=1.0
TC=1500.0
TEI=1500.0
TA=(TC+TEI)/2.0
DBR=3.45
QIA=3.D-14
ES3=5.0D-4
E=1.0
KI=50
KM2=KI/2
H1=0.02
FLAG=0.D0
RNER=0.DC
MM=1
W=1.0
II=0
FACTO=0.32/1.5
DO 21 I=1,51
21  SX(I)=(I-1)*0.02
    PI=4.D0*DATAN(1.D0)
    READ (5,151) DCTO,(DTAU(L),L=1,50)
151  FORMAT (1P8D10.3)
    READ (5,152) CO,(C(L),L=1,50)
152  FORMAT (1P8D10.3)
    READ (5,153) DAKPO,(DAKP(L),L=1,50)
153  FORMAT (1P8D10.3)
    READ (5,154) NE,XE,J,R,NEI,CDRO
154  FORMAT (1P6D10.3)
    READ (5,155) CDRO,(CDR(L),L=1,50)
155  FORMAT (1P8D10.3)
    PD=P/10.0
    PD1=PD*PD
    CDBR=3.45/DBR
    CQIA=3.D-14/QIA
    TCE=TC/TEI
    FI=-1.602D4*PSIE/(1.381*TEI)
    FIW=1.602D4*COWO/(1.381*TEI)

```

I IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

IPI=DEXP (PI)
RJR=120.0*TEI*TEI*DFI
BNR=0.62D12*RJR*DSQRT (1000.0/TEI)
PA=DSQRT (PI/2.0)/3.0
ETAO=DSQRT (P/TEI)
DTAUO=DDTO
TAU=DDTO
TE=TAU*TEI
TEOO=DDTO*TEI
DSTO=DSQRT (DTAUO)
DO 315 L=1,KI
DTA (L)=TEI+(TC-TEI)*H1*I
315 CONTINUE
IF (B.EQ.1.0) GO TO 314
II=II+1

C
C
342 CNDO=1.5*FACTO*NE*DC/(5.16D-5*TEI*CQIA/P)
DO 304 L=1,KI
CND (L)=1.5*FACTO*CO*NE*DC/(C (L)*5.16D-5*DTA (L)*CQIA/P)
304 CONTINUE
NTD=NE*TEI*DSQRT (R)
CNTD=45.858/NTD
DEX4=0.D0
DEX5=0.D0
CT=0.D0
305 PNC=P*NE*CO*DC*TEI**1.5
335 DO 18 L=1,KI
DTE (L)=DTAU (L)*TEI
18 CONTINUE
CT=CT+1.0
IF (CT.GT.100.0) GO TO 82
KIM1=KI-1
KIM2=KI-2
KIM3=KI-3
341 DDD (KI)=C.5*J*(2.0*DTAU (KI)-2.0*DTAUO-XE+DAKP (KI))
CDDC (KI)=6.58D-4*CDR (KI)*(DTA (KI)+DTE (KI))*CQIA*DSQRT (DTA (KI))/
*FNC
T4C (KI)=CND (KI)*(DTAUO/DSQRT (DTAU (KI))-(1.0-DDD (KI)))/
*DSQRT (DTAU (KI))-(CDDC (KI)-CNTD)/DSQRT (DTAU (KI))
ETAU (KI-1)=DTAU (KI)-H1*T4C (KI)
TKO=-2.0*H1*T4C (KI)
THK=DTAU (KI)-0.5*TKO
TK1=-2.0*H1*CND (KI-1)*(DTAUO/DSQRT (THK)-(1.0-0.5*J*(2.0*THK
*-2.0*DTAUO-XE+DAKP (KI-1)))/DSQRT (THK)-(6.58D-4*CDR (KI-1)*
*(DTA (KI-1)+DTE (KI-1))*CQIA*DSQRT (DTA (KI-1))/PNC-CNTD)/DSQRT (THK))
THK1=DTAU (KI)-0.5*TK1
TK2=-2.0*H1*CND (KI-1)*(DTAUO/DSQRT (THK1)-(1.0-0.5*J*(2.0*THK1
*-2.0*DTAUO-XE+DAKP (KI-1)))/DSQRT (THK1)-(6.58D-4*CDR (KI-1)*
*(DTA (KI-1)+DTE (KI-1))*CQIA*DSQRT (DTA (KI-1))/PNC-CNTD)/DSQRT (THK1))
THK2=DTAU (KI)-TK2
TK3=-2.0*H1*CND (KI-2)*(DTAUO/DSQRT (THK2)-(1.0-0.5*J*(2.0*THK2-2.0*
*DTAUO-XE+DAKP (KI-2)))/DSQRT (THK2)-(6.58D-4*CDR (KI-2)*(DTA (KI-2)+
*DTE (KI-2))*CQIA*DSQRT (DTA (KI-2))/PNC-CNTD)/DSQRT (THK2))
ETAU (KI-2)=DTAU (KI)+1.0/6.0*(TKO+2.0*TK1+2.0*TK2+TK3)
DO 207 L=1,KIM3

```

```

KIM2L=KIM2-L
KIM23=KIM2L+3
KIM21=KIM2L+1
DO 208 M=KIM21,KIM23
  T4C(M)=CND(M)*(DTAUO/DSQRT(DTAU(M))-(1.0-0.5*J*(2.0*DTAU(M)-2.0*
*DTAUO-XE+DAKP(M)))/DSQRT(DTAU(M))-(6.580D-4*CDR(M)*(DTA(M)+
*DTTE(M))*CQIA*DSQRT(DTA(M))/PNC-45.858/NTD)/DSQRT(DTAU(M)))
208  CONTINUE
  ETAU(KIM2L)=DTAU(KIM2L+1)-1.0/12.0*H1*(23.0*T4C(KIM2L+1)-16.0*
*T4C(KIM2L+2)+5.0*T4C(KIM2L+3))
209  DDTAU(KIM2L)=DTAU(KIM2L)
  DDD(KIM2L)=0.5*J*(2.0*DTAU(KIM2L)-2.0*DTAUO-XE+DAKP(KIM2L))
  CDDC(KIM2L)=6.58D-4*CDR(KIM2L)*(DTA(KIM2L)+DTE(KIM2L))*CQIA*
*DSQRT(DTA(KIM2L))/PNC
  T4C(KIM2L)=CND(KIM2L)*(DTAUO/DSQRT(DTAU(KIM2L))-(1.0-DDD(KIM2L))/
*ESQRT(DTAU(KIM2L))-(CDDC(KIM2L)-CNTD)/DSQRT(DTAU(KIM2L)))
  DTAU(KIM2L)=DTAU(KIM2L+1)-1.0/12.0*H1*(5.0*T4C(KIM2L)+8.0*
*T4C(KIM2L+1)-T4C(KIM2L+2))
  IF(DABS(EDTAU(KIM2L)-ETAU(KIM2L)).GT.E53) GO TO 209
207  CONTINUE
  DDTO=DTAU(1)-1.0/12.0*H1*(23.0*T4C(1)-16.0*T4C(2)+5.0*T4C(3))
340  DDTAO=DDTO
  DDDO=0.5*J*(2.0*DDTO-2.0*DTAUO-XE)
  CDDO=6.580D-4*CDRO*(TEI+TEOO)*CQIA/(P*NE*CO*DC*TEI)
  T4CO=CND*(DTAUO/DSQRT(DDTO)-(1.0-DDDO)/DSQRT(DDTO)-(CDDO-CNTD)
*/DSQRT(DDTO))
  DDTO=DTAU(1)-1.0/12.0*H1*(5.0*T4CO+8.0*T4C(1)-T4C(2))
  IF(DABS(DDTAO-DDTO).GT.E53) GO TO 340
  DTAUD=DDTO-DTAUO
  IF(DABS(ETAUD).LT.E53) GO TO 381
  IF(DTAUD.LT.0.E0) GO TO 382
  IF(DEX4.EQ.1.0) GO TO 384
  DSTAU=DTAU(KI)
  DTAU(KI)=DTAU(KI)-0.05
  DEX5=1.0
  GO TO 305
384  ESTAU=DTAU(KI)
  DTAU(KI)=(DTAU(KI)+DUTAU)/2.0
  DEX5=1.0
  GO TO 305
382  DUTAU=DTAU(KI)
  IF(DEX5.EQ.1.0) GO TO 385
  ETAU(KI)=DTAU(KI)+0.05
  DEX4=1.0
  GO TO 305
385  DTAU(KI)=(DSTAU+DTAU(KI))/2.0
  DEX4=1.0
  GO TO 305
381  TEOO=DDTC*TEI
C
C
C
PRINT 339,J
339  FORMAT(3X,1P1D15.3)
PRINT 171,DDTO,DTAU(1),DTAU(49),DTAU(50)
171  FORMAT(3X,1P4D15.3)

```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

395  DTE30=3000.0/TE00
      DTEEO=DTE30**5.5
      CTERO=TECO/TEI
      DRTEO=DTERO**1.5/(1.0+1.0/DTERO)
      DPOO=-15.0*DTE30
      DDEO=DEXP(DPOO)
      ABO=1.96D10*PD1*DTEEO*DRTEO*DDEO*CDBR/CQIA*DC*DC
      TE40=-15.0*(3000.0-TE00)/TE00
      TEXO=DEXP(TE40)
      STEO=DSQRT(TEXO)
      TE30=(TE00/3000.0)**0.75
      ZNZE=3.33D16*PTAO*TE30*STEO
      ZNZQO=ZNZE**2
      DO 330 L=1,KI
      DTE(L)=DTAU(L)*TEI
      DTE3(L)=3000.0/DTE(L)
      DTEE(L)=DTE3(L)**5.5
      DTER(L)=DTE(L)/DTA(L)
      DRTER(L)=DTER(L)**1.5/(1.0+1.0/DTER(L))
      EPO(L)=-15.0*DTE3(L)
      DDE(L)=DEXP(DPO(L))
      DAB(L)=1.96D10*PD1*DTEE(L)*DRTER(L)*DDE(L)*CDBR/CQIA*DC*DC
      DA1(L)=DSQRT(DAB(L))
      DPTA(L)=DSQRT(P/DTA(L))
      DTE31(L)=(DTE(L)/3000.0)**0.75
      ETE4(L)=-15.0*(3000.0-DTE(L))/DTE(L)
      DTEX(L)=DEXP(ETE4(L))
      ESTEX(L)=DSQRT(DTEX(L))
      DZNZ1(L)=3.33D16*DPTA(L)*DTE31(L)*DSTEX(L)
      RZNZ1(L)=DZNZ1(L)/ZNZE
      DZNZQ(L)=DZNZ1(L)**2
      RZNZ(L)=ZNZQO/DZNZQ(L)
      RAMB(L)=DSQRT(TEI)*((TEI+TE00)/(DSQRT(DTA(L))*(DTA(L)+DTE(L))))
      DDDD(L)=1.0/(DSQRT(DTA(L))*(DTA(L)+DTE(L)))
330  CONTINUE
      RNER=NE*CO*ZNZE/RNR
      PRINT 320,A30,DAB(KI),RNER
320  FORMAT(3X,1P3D15.3)
C
C
92  CAI=PI*TAU
2   NET=NE
    NEIT=NEI
    XET=XE
    ET=R
1   CAB=DSQRT(1.0+PI*DABS(XE))
    CA=CAB**3
    CAC=PI*R
    CAD=DSQRT(1.0+DABS(CAC*XE))
    CAE=-XE/TAU
    CAF=DEXP(CAE)
    IF((RNER-1.0).GT.0.D0) GO TO 40
    FLAG=0.D0
    F(1)=NE+(NEI-1.0)*CAE
    F(2)=(2.0*NEI-CAI*R)*CA-NE*CAI
    F(3)=2.0*(CAD-1.0)+2.0*R*NE*(1.0-CAB)+CAC*NEI*TAU*(CAF-1.0)

```


IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

5      F(4)=2.0*NE*(J-1.0)+DSQRT(TAU)*CAF*NEI
      A(1,1)=1.00
      A(1,2)=CAB
      A(1,3)=0.00
      A(1,4)=PI*(NEI-1.0)/(2.0*CAB)
      A(2,1)=-CAI
      A(2,2)=2.0*CA
      A(2,3)=-CAI*CA
      A(2,4)=1.5*PI*(2.0*NEI-CAI*R)*CAB
      A(3,1)=2.0*R*(1.0-CAB)
      A(3,2)=CAI*R*(CAF-1.0)
      A(3,3)=PI*XE/CAD+2.0*NE*(1.0-CAB)+CAI*NEI*(CAF-1.0)
      A(3,4)=CAC/CAD-CAC*NE/CAB-CAC*NEI*CAF
      A(4,1)=2.0*(J-1.0)
      A(4,2)=0.00
      A(4,3)=0.00
      A(4,4)=-CAF/DSQRT(TAU)
      CALL MATINV(A, 16,4,JX)
      DO 3 K=1,4
      NE=NE-A(1,K)*F(K)
      NEI=NEI-A(2,K)*F(K)
      R=R-A(3,K)*F(K)
      XE=XE-A(4,K)*F(K)
3      CONTINUE
      IF(DABS(NE-NET).GT.E5) GO TO 2
      IF(DABS(NEI-NEIT).GT.E5) GO TO 2
      IF(DABS(R-RT).GT.E5) GO TO 2
      IF(DABS(XE-XET).GT.E5) GO TO 2
      IF(W.EQ.1.0) GO TO 364
40     XK=2.0*(2.0*(CAD-1.0)/CAC-NEI*TAU*(1.0-CAF)-2.0*NE*(CAB-1.0)/PI)
      DXK=DSQRT(DABS(XK))
      DDXK=DSQRT(DXK)
      IF(W.EQ.1.0) GO TO 306
      ZNZ=CO*ZNZ1
      GO TO 307
306     ZNZ=CO*ZNZE
307     ZNZO=(ZNZ/1.0D14)**0.25
      SZTD=0.276*ZNZO*TEO*DDXK
      EZTD=DEXP(SZTD)
      NE=RNREZTD/ZNZ
      TAUS=DSQRT(TAU)
      Q1=(1.0-J)*2.0*NE/TAUS
      ALQ1=DLOG(Q1)
      XE=-TAU*ALQ1
      Q2=DSQRT(1.0+PI*XE)
      NEI=1.0-NE/Q2
      Q3=Q2**3
      R=(2.0*NEI-NE*PI*TAU/Q3)/(PI*TAU)
      FLAG=1.0

C
C
364     DEX1=0.00
      DEX2=0.00
      DEX3=0.00
      TI=0.00
      DN2KI=DTAU(KI)*TCE*PI/2.0

```

```

DN1KI=DSQRT(DN2KI)
TERKI=DTE(KI)/DTA(KI)
ALMKI=DSQRT(TERKI)*SLM
PROKI=DSQRT(TCE)*NE*J*ALMKI
343 ALMO=DSQRT(DTAUO)*SLM
CLM=0.D0
ALMSO=3.44D-5*TEI*CQIA/P
FOO=1.0/DSQRT(DABS(R))-ALMO*NE*J
KO=(1.0+ETAUO)*PA/FOO
AMBO=DSQRT(TEI)*(TEI+TECO)
DDDDO=1.0/AMBO
CO=ALMSO*KO*CDRO/DC
FRINT 365,CDRO,CO
365 FORMAT(3X,1P2D15.3)
IF(CO.GT.ES3) GO TO 504
CO=0.D0
504 AMBCI=(1.0/DDDD(1)-AMFO)/H1
K10=2.0*H1*CDRO
K20=2.0*H1*(-ABO*CO*(1.0-CO*CO))
CB=CDRO+0.5*K20
K11=2.0*H1*AMBO*DDDD(1)*CB
CBB=CO+0.5*K10
K21=-2.0*H1*DAB(1)*CBE*(1.0-CBB*CBB*RZNZ(1))
CB1=CDRO+0.5*K21
CBB1=CO+0.5*K11
K12=2.0*H1*AMBO*DDDD(1)*CB1
K22=-2.0*H1*DAB(1)*CBE1*(1.0-CBB1*CBB1*RZNZ(1))
CB2=CDRO+K22
CBB2=CO+K12
K13=2.0*H1*AMBO*DDDD(2)*CB2
K23=-2.0*H1*DAB(2)*CBE2*(1.0-CBB2*CBB2*RZNZ(2))
C(2)=CO+1.0/6.0*(K10+2.0*K11+2.0*K12+K13)
ZZ(2)=CDRO+1.0/6.0*(K20+2.0*K21+2.0*K22+K23)
CDR(2)=AMBO*ZZ(2)*DDDD(2)
DELTO=1.0-CO
DELTA(2)=1.0-C(2)/DZNZ1(2)*ZNZE
PDELT=DELTA(2)
PSDEL(2)=PDELT**2+DELTO**2
PPSDE=PSDEL(2)
KI21=KI/2-1
DO 344 L=1,KI21
L2=2*L
L2M1=L2+1
K10=2.0*H1*CDR(L2)
K20=-2.0*H1*DAB(L2)*C(L2)*(1.0-C(L2)*C(L2)*RZNZ(L2))
CB=ZZ(L2)+0.5*K20
CBB=C(L2)+0.5*K10
K11=2.0*H1*AMBO*DDDD(L2M1)*CB
K21=-2.0*H1*DAB(L2M1)*CBE*(1.0-CBB*CBB*RZNZ(L2M1))
CB1=ZZ(L2)+0.5*K21
CBB1=C(L2)+0.5*K11
K12=2.0*H1*AMBO*DDDD(L2M1)*CB1
K22=-2.0*H1*DAB(L2M1)*CBE1*(1.0-CBB1*CBB1*RZNZ(L2M1))
CB2=ZZ(L2)+K22
CBB2=C(L2)+K12
K13=2.0*H1*AMBO*DDDD(L2+2)*CB2

```

```
K23=-2.0*H1*DAB(L2+2)*CEB2*(1.0-CBB2*CBB2*RZNZ(L2+2))
C(L2+2)=C(L2)+1.0/6.0*(K10+2.0*K11+2.0*K12+K13)
IF(C(L2+2).GT.1.0) C(L2+2)=1.0
ZZ(L2+2)=ZZ(L2)+1.0/6.0*(K20+2.0*K21+2.0*K22+K23)
CDR(L2+2)=AMBO*ZZ(L2+2)*DDDD(L2+2)
CL=L2+2
IF(C(L2+2).GT.0.90) CIM=C(L2+2)
DELTA(L2+2)=1.0-C(L2+2)/DZNZ1(L2+2)*ZNZE
SDELT=DEITA(L2+2)
PSEL(L2+2)=PDELT**2+SDELT**2
SPSDE=PSEL(L2+2)
SSSDE=SPSDE
PDELT=DEITA(L2+2)
IF(PPSDE.GT.SPSDE) GO TC 500
FPSDE=PPSDE
GO TO 509
500 PPSDE=SPSDE
509 IF(C(L2+2).LT.0.D0) GO TO 345
CNU(L2+2)=C(L2+2)*ZNZE
CMD(L2+2)=(DZNZ1(L2+2)-CNU(L2+2))/DZNZ1(L2+2)
IF(CMD(L2+2).LT.0.D0) GO TO 346
GO TO 373
347 PRINT 400,SSSDE
400 FORMAT(3X,1P1D15.3)
SZNZ1=(DZNZ1(L2+2)-DZNZ1(L2+1))/H1
KW1=L2+4
KW=(KI-KW1)/2
IF((L2+2).EQ.KI) GO TC 508
L2P2=L2+3
DENU=SZNZ1/ZNZE
DMS=DENU-CDR(L2+2)
PRINT 368,SZNZ1,DENU,DMS,CDR(L2+2)
368 FORMAT(3X,1P4D15.3)
GO TO 348
373 IF((L2+2).EQ.KI) GO TC 508
344 CONTINUE
508 TI=TI+1.0
DENKI=DN1KI+PROKI*CO/C(KI)
K1KI=PA*(TCE+DTAU(KI))/DENKI
SMALL=C(KI)+ALMSO*K1KI*CDR(KI)/DC
PRINT 502,SMALL,PPSDE,K1KI
502 FORMAT(3X,1P3D15.3)
IF(DABS(SMALL).LT.E3) GO TO 348
IF(TI.EQ.1.0) GO TO 501
SCDR=CDRO
IF(TI.GT.100.0) GO TC 82
CDRO=CDRC-SMALL*((CDRO-PCDR)/(SMALL-SMAL1))
PRINT 503,CDRO
503 FORMAT(3X,1P1D15.3)
FCDR=SCDR
SMAL1=SMALL
PPSD1=PPSDE
GO TO 343
501 SMAL1=SMALL
PPSD1=PPSDE
FCDR=CDRC
```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

      CDRO=CDRC+0.002
      GO TO 343
345  PRINT 505,C(L2+2),CDR(L2+2),CL
505  FORMAT(3X,1P3D15.3)
      IF(DEX1.EQ.1.0) GO TO 349
      DEX3=DEX3+1.0
      CURO=CDRO
      CDRO=CDRO+0.5
      DEX2=1.0
      IF(DEX3.GT.50.0) GO TO 82
      GO TO 343
349  CURO=CDRC
      DEX3=DEX3+1.0
      CDRO=(CSDRO+CDRO)/2.0
      DEX2=1.0
      IF(DEX3.GT.50.0) GO TO 82
      GO TO 343
346  PRINT 367,CMD(L2+2),CNU(L2+2),DZNZ1(L2+2),C(L2+2),CL,CDR(L2+2)
367  FORMAT(3X,1P6D15.3)
      CSDRO=CDRO
      DEX3=DEX3+1.0
      IF(DEX2.EQ.1.0) GO TO 350
      CDRO=CDRO-0.5
      DEX1=1.0
      GO TO 343
350  CDRO=(CSDRO+CURO)/2.0
      DEX3=DEX3+1.0
      DEX1=1.0
      GO TO 343
348  LRP2=L2/2
      C(1)=(C0+C(2))/2.0
      CDR(1)=(CDRO+CDR(2))/2.0
      DO 351 L=1,LRP2
      I3=2*L+1
      I3M=L3-1
      I3P=L3+1
      C(I3)=(C(L3M)+C(L3P))/2.0
      CDR(I3)=(CDR(L3M)+CDR(L3P))/2.0
      IF(L3.EQ.49) GO TO 510
351  CONTINUE
      GO TO 512
510  PRINT 511,C0,(C(L),L=1,KI)
511  FORMAT(3X,1P7D15.3)
      GO TO 83
C
C
C
512  XED1=0.00
      XED2=0.00
358  C(KI)=-ALMSO*PA*(TCE+ETAU(KI))*CDR(KI)/(DC*DN1KI)-PROKI*C0/DN1KI
      ZZ(KI)=CDR(KI)*DDDDO/DDDD(KI)
      PRINT 550,C(KI),CDR(KI)
550  FORMAT(3X,1P2D15.3)
      PDELS=1.0-C(KI)/DZNZ1(KI)*ZNZE
      DO 452 L=1,KW
      M2=KI-2*L

```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

M22=M2+2
M21=M2+1
K10=-2.0*H1*CDR(M22)
K20=2.0*H1*DAB(M22)*C(M22)*(1.0-C(M22)*C(M22)*RZNZ(M22))
CB=ZZ(M22)-0.5*K20
CBB=C(M22)-0.5*K10
K11=-2.0*H1*AMBO*DDDD(M21)*CB
K21=2.0*H1*DAB(M21)*CEB*(1.0-CBB*CBB*RZNZ(M21))
CB1=ZZ(M22)-0.5*K21
CBB1=C(M22)-0.5*K11
K12=-2.0*H1*AMBO*DDDD(M21)*CB1
K22=2.0*H1*DAB(M21)*CEE1*(1.0-CBB1*CBB1*RZNZ(M21))
CB2=ZZ(M22)-K22
CBB2=C(M22)-K12
K13=-2.0*H1*AMBO*DDDD(M2)*CB2
K23=2.0*H1*DAB(M2)*CEB2*(1.0-CBB2*CBB2*RZNZ(M2))
C(M2)=C(M22)+1.0/6.0*(K10+2.0*K11+2.0*K12+K13)
ZZ(M2)=ZZ(M22)+1.0/6.0*(K20+2.0*K21+2.0*K22+K23)
CDR(M2)=AMBO*ZZ(M2)*DEED(M2)
CM2=M2
DELTS(M2)=1.0-C(M2)/DZNZ1(M2)*ZNZE
SDELS=DELTS(M2)
PSDES(M2)=SDELS**2+PDELS**2
FDELS=DELTS(M2)
SSSES=PSDES(M2)
IF(SSSES.LT.1.0D-5) GO TO 355
CNU(M2)=C(M2)*ZNZE
CMD(M2)=(DZNZ1(M2)-CNU(M2))/DZNZ1(M2)
IF(C(M2).LT.0.D0) GO TO 353
IF(CMD(M2).LT.0.D0) GO TO 354
IF(DABS(CMD(M2)).LT.1.0D-10) GO TO 355
452 CONTINUE
GO TO 353
355 PRINT 600,SSSES
600 FORMAT(3X,1P1D15.3)
SZNZ1=(DZNZ1(M2)-DZNZ1(M21))/H1
L3P3=M2-1
DENU=SZNZ1/ZNZE
DMS=DENU-CDR(M2)
PRINT 514,SZNZ1,DENU,DMS,CDR(M2)
514 FORMAT(3X,1P4D15.3)
GO TO 356
353 PRINT 551,C(M2),CDR(M2),CM2
551 FORMAT(3X,1P3D15.3)
IF(XED1.EQ.1.0) GO TO 357
CURO=CDR(KI)
CDR(KI)=CDR(KI)-0.5
XED2=1.0
GO TO 358
357 CURO=CDR(KI)
CDR(KI)=(CSDRO+CDR(KI))/2.0
XED2=1.0
GO TO 358
354 PRINT 513,CMD(M2),CNU(M2),DZNZ1(M2),C(M2),CE2
513 FORMAT(3X,1P5D15.3)
CSDRO=CDR(KI)

```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

      IF (XED2.EQ.1.0) GO TO 359
      CDR(KI)=CDR(KI)+0.5
      XED1=1.0
      GO TO 358
359   CDR(KI)=(CSDRO+CURO)/2.0
      XED1=1.0
      GO TO 358
356   LRP3=(KI-M2)/2
      DO 360 L=1,LRP3
      LR4=L*2-1
      N2=KI-LR4
      N2P=N2+1
      N2M=N2-1
      C(N2)=(C(N2P)+C(N2M))/2.0
      CDR(N2)=(CDR(N2P)+CDR(N2M))/2.0
360   CONTINUE
      DO 361 L=L2P2,L3P3
      C(L)=DZNZ1(L)/ZNZE
361   CONTINUE
      DO 363 L=L2P2,L3P3
      CDR(L)=(C(L+1)-C(L-1))/(2.0*H1)
363   CONTINUE
      DENKI=DN1KI+PROCKI*CO/C(KI)
      K1KI=PA*(TCE+DTAU(KI))/DENKI
      PRINT 362,CO,(C(L),L=1,KI)
362   FORMAT(3X,1P7D15.3)
      C
      C
83    TCCO=DTAUO*CDRO/CO
      PRINT 405,CLM
405   FORMAT(3X,1P1D15.3)
      DTC=DSTC/CO
      DO 301 L=1,KI
      DST(L)=DSQRT(DTAU(L))
      TCC(L)=DTAU(L)*CDR(L)/C(L)
      DTE(L)=DTAU(L)*TEI
      DT(L)=DST(L)/C(L)
301   CONTINUE
      COEF=4.0*SLM*NE*KO*CDRO/J/PI
      FITC(2)=E1*(TCC(2)+4.0*ICC(1)+TCCO)/3.0
      SITC(2)=COEF*H1*(DT(2)+4.0*DT(1)+DTC)/3.0
      LAKP(2)=DTAUO-DTAU(2)-FITC(2)-SITC(2)
      DAKP(1)=LAKP(2)/2.0
      DO 302 L=2,KM2
      KKM=L*2
      FITC(KKM)=FITC(KKM-2)+H1*(TCC(KKM)+4.0*TCC(KKM-1)+TCC(KKM-2))
      * /3.0
      SITC(KKM)=SITC(KKM-2)+H1*COEF*(DT(KKM)+4.0*DT(KKM-1)+DT(KKM-2))
      * /3.0
      DAKP(KKM)=DTAUO-DTAU(KKM)-FITC(KKM)-SITC(KKM)
      LLM=KKM-1
      DAKP(LLM)=(DAKP(KKM)+LAKP(KKM-2))/2.0
302   CONTINUE
303   VDE=2.0/J*(1.0-DTAUO+J*(DTAUO-DTAU(KI))-45.858/(NE*TEI)*(1.0/
      *LSQRT(R)+C(KI)/CO*DSQRT(DTAU(KI))*1.25))
      TAUO1=DTAU(KI)/DTAUO

```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

      DAP=-DTAU(KI)*DLOG((1.0-J)/J*DSQRT(TAU01)*C(KI)/CO)+VDE-(TAU01-
*1.0)*XE
      IF(DABS(DAKP(KI)-DAP).GT.4.5D-2) GO TO 371
      GO TO 392
371    DDAP=DAKP(KI)-DAP
      PRINT 372,DDAP
372    FORMAT(3X,1P1D15.3)
      IF(MM.EQ.1) GO TO 312
      DAP2=DAP
      DAKP2=DAKP(KI)
      J2=J
      J12=J1-J2
      DAKP3=DAKP1-DAKP2
      DAP12=DAP1-DAP2
      PRINT 375,J1,J2,DAKP1,DAKP2,DAP1,DAP2
375    FORMAT(3X,1P6D15.3)
      J=((J12*DAKP2-DAKP3*J2)-(J12*DAP2-DAP12*J2))/(DAP12-DAKP3)
      IF(J.LT.0.00) GO TO 376
      IF(J.GT.1.0) GO TO 376
      DAP1=DAP2
      DAKP1=DAKP2
      J1=J2
      GO TO 342
312    DAP1=DAP
      DAKP1=DAKP(KI)
      J1=J
      J=J+0.001
      MM=MM+1
      GO TO 342
376    PRINT 377,J
377    FORMAT(3X,1P1D15.3)
      GO TO 82
C
C
392    TS=(TE/3000.0)**1.5
      TS1=-15.0*(3000.0-TE)/TE
      TS2=DEXP(TS1)
      TBA=DSQRT(TS2*TS)
      IF(W.EQ.1.0) GO TO 390
      CNT=CO*NE*ZNZ1
      GO TO 391
390    CNT=CO*NE*ZNZE
391    DLCR=DLOG(CNT/RNR)
      TMT=-DLCR
      IF(FLAG.EQ.1.0) GO TO 81
      IF(TMT) 80,81,81
80    FLAG=1.0
      GO TO 92
81    EGJ=J*NE*CO*TBA*DSQRT(TEI)*52.26
      SMV=VDE+TMT-FI
      BGV=SMV*TEI*8.62D-5
      SMVV=VDE+TMT-FI-FIW
      BGVV=SMVV*TEI*8.62D-5
      PRINT 187,NE,XE,J,F,NEI,CDRO
187    FORMAT(/'NE=',1P1D10.3,2X,'XE=',1P1D10.3,2X,'J=',1P1D10.3,2X,
*'R=',1P1D10.3,2X,'NEI=',1P1D10.3,2X,'SLOPE=',1P1D10.3)

```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```

200 PRINT 200,KO,K1KI
    FORMAT(/'KO=',1P1D10.3,2X,'K1=',1P1D10.3)
    PRINT 190,J,BGJ,VDE,EGV,TMT,BGVV
190  FORMAT(/' J=',1P1D10.3,2X,'BJ=',1P1D10.3,2X,'VD=',1P1D10.3,2X,
    *'BV=',1P1D10.3,2X,'DYE=',1P1D10.3,2X,'BVV=',1P1D10.3/)
    SX1(II)=EGV
    SY4(II)=BGJ
    SY5(II)=R
    SY6(II)=NE
    SY7(II)=NEI
    SY8(II)=KO
    SY9(II)=K1KI
    SY10(II)=XE
    SX3(II)=VDE
    SY11(II)=J
    PRINT 182
182  FORMAT(14X,'NUMBER DENSITY DISTRIBUTION'/)
94  PRINT 160,CO,(C(L),L=1,KI)
160  FORMAT(3X,1P7D15.3)
    PRINT 185
185  FORMAT(/,14X,'TEMPERATURE DISTRIBUTION'/)
    PRINT 331,DTAUO,(DTAU(L),L=1,KI)
331  FORMAT(3X,1P7D15.3)
    PRINT 186
186  FORMAT(/,14X,'POTENTIAL DISTRIBUTION'/)
    PRINT 333,DAKPO,(DAKP(L),L=1,KI)
333  FORMAT(3X,1P7D15.3)
    PRINT 188
188  FORMAT(/,14X,'SLOPE DISTRIBUTION'/)
    PRINT 189,CDRO,(CDR(L),L=1,50)
189  FORMAT(3X,1P7D15.3)
    PRINT 310
310  FORMAT(/,14X,'SAHA NUMBER DENSITY DISTRIBUTION'/)
    PRINT 311,(RZNZ1(L),L=1,50)
311  FORMAT(3X,1P7D15.3)
    IF(II.EQ.1 .OR. II.EQ.5) GO TO 71
    IF(II.EQ.10 .OR. II.EQ.15) GO TO 71
    IF(II.EQ.20 .OR. II.EQ.25) GO TO 71
    GO TO 72
71  IF(II.EQ.1) NI=1
    IF(II.GT.1) NI=II/5+1
    SNDD(1,NI)=CO
    TDIS(1,NI)=DTAUO
    PDIS(1,NI)=DAKPO
    SAHA(1,NI)=1.0
    DO 79 I=1,50
    IL=I+1
    SNDD(IL,NI)=C(I)
    TDIS(IL,NI)=DTAU(I)
    PDIS(IL,NI)=DAKP(I)
    SAHA(IL,NI)=RZNZ1(I)
79  CONTINUE
72  IF(BGV.LT.1.5) GO TO 27
314  DTAUO=DTAUO+0.005
    TAU=TAU+0.005
    DO 394 L=1,KI

```



```

394   DTAU(L)=CTAU(L)+0.005
      CONTINUE
      MM=1
      TE=DTAUO*TEI
      TEOO=DTAUO*TEI
      ISTO=DSQRT(DTAUO)
      II=II+1
      SX2(II)=CTAUO
      GO TO 395
27    CALL SETSY(6.)
      CALL DFIPS1(SX1,SY4,-II,51,5.,1.5,0.18,0.,10.,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY5,-II,51,5.,1.6,0.08,0.,0.2,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY6,-II,51,5.,1.6,0.08,0.,0.2,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY7,-II,51,5.,1.6,0.08,0.,0.2,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY8,-II,51,5.,1.6,0.08,0.,0.2,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY9,-II,51,5.,1.6,0.08,0.,0.2,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX2,SY10,-II,51,5.,1.6,0.08,0.,0.6,0,0,0)
      CALL SETSY(5.)
      CALL DFIPS1(SX3,SY11,-II,51,5.,-3.4,0.28,0.,0.2,0,0,0)
      PUNCH 161,DTAUO,(DTAU(L),L=1,KI)
161   FORMAT (1P8D10.3)
      PUNCH 162,CO,(C(L),L=1,KI)
162   FORMAT (1P8D10.3)
      PUNCH 163,DAKPO,(DAKP(L),L=1,KI)
163   FORMAT (1P8D10.3)
      PUNCH 164,NE,XE,J,R,NEI,CDRO
164   FORMAT (1P8D10.3)
      PUNCH 165,CDRO,(CDR(L),L=1,KI)
165   FORMAT (1P8D10.3)
      DO 73 NI=1,6
      SY(1)=SNED(1,NI)
      DO 23 I=1,50
      IL=I+1
23    SY(IL)=SNDD(IL,NI)
      CALL SETSY(5.)
      IF(NI.EQ.1) CALL DFIPS1(SX,SY,-51,51,5.,0.,0.2,0.,0.2,0,0,0)
      IF(NI.GT.1) CALL DFIPS2(SX,SY,51,51)
73    CONTINUE
      DO 74 NI=1,6
      SY2(1)=TDIS(1,NI)
      DO 25 I=1,50
      IL=I+1
25    SY2(IL)=TDIS(IL,NI)
      CALL SETSY(5.)
      IF(NI.EQ.1) CALL DFIPS1(SX,SY2,-51,51,5.,0.,0.2,1.5,0.1,0,0,0)
      IF(NI.GT.1) CALL DFIPS2(SX,SY2,51,51)
74    CONTINUE
      DO 75 NI=1,6
      SY3(1)=PDIS(1,NI)
      DO 26 I=1,50

```

IV G LEVEL 21

MAIN

DATE = 78248

13/51/24

```
      IL=I+1
26     SY3(IL)=PDIS(IL,NI)
      CALL SETSY(5.)
      IF(NI.EQ.1) CALL DFIPS1(SX,SY3,-51,01,5.,0.,0.2,-5.,1.0,0,0,0)
      IF(NI.GT.1) CALL DFIPS2(SX,SY3,51,01)
75     CONTINUE
      EO 76 NI=1,6
      SY1(1)=1.0
      DO 24 I=1,50
      IL=I+1
24     SY1(IL)=SAHA(IL,NI)
      CALL SETSY(5.)
      IF(NI.EQ.1) CALL DFIPS1(SX,SY1,-51,01,5.,0.,0.2,0.,0.2,0,0,0)
      IF(NI.GT.1) CALL DFIPS2(SX,SY1,51,01)
76     CONTINUE
82     STOP
      END
```

C
C

```
SUBROUTINE MATINV(A,NX,N,JX)
IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(NX),L(10),M(10)
DOUBLE PRECISION DABS
JX=1
D=1.0
NK=-N
DO 80 K=1,N
  NK=NK+N
  L(K)=K
  M(K)=K
  KK=NK+K
  BIGA=A(KK)
  DO 20 J=K,N
    IZ=N*(J-1)
    DO 20 I=K,N
      IJ=IZ+I
10    IF(DABS(BIGA)-DABS(A(IJ))) 15,20,20
15    BIGA=A(IJ)
    L(K)=I
    M(K)=J
20    CONTINUE
    J=L(K)
    IF(J-K) 35,35,25
25    KI=K-N
    DO 30 I=1,N
      KI=KI+N
      HOLD=-A(KI)
      JI=KI-K+J
      A(KI)=A(JI)
30    A(JI)=HOLD
35    I=M(K)
    IF(I-K) 45,45,38
38    JP=N*(I-1)
    DO 40 J=1,N
      JK=NK+J
      JI=JP+J
      HOLD=-A(JK)
      A(JK)=A(JI)
40    A(JI)=HOLD
45    IF(BIGA) 48,46,48
46    D=0.0
    IF(D.EQ.0.) JX=-1
    RETURN
48    DO 55 I=1,N
      IF(I-K) 50,55,50
50    IK=NK+I
      A(IK)=A(IK)/(-BIGA)
55    CONTINUE
    DO 65 I=1,N
      IK=NK+I
      HOLD=A(IK)
      IJ=I-N
      DO 65 J=1,N
```

```
        IJ=IJ+N
        IF (I-K) 60,65,60
60      IF (J-K) 62,65,62
62      KJ=IJ-I+K
        A(IJ)=HOLD*A(KJ)+A(IJ)
65      CONTINUE
        KJ=K-N
        DO 75 J=1,N
        KJ=KJ+N
        IF (J-K) 70,75,70
70      A(KJ)=A(KJ)/BIGA
75      CONTINUE
        D=D*BIGA
        A(KK)=1.0/BIGA
80      CONTINUE
        K=N
100     K=K-1
        IF (K) 150,150,105
105     I=L(K)
        IF (I-K) 120,120,108
108     JQ=N*(K-1)
        JR=N*(I-1)
        DO 110 J=1,N
        JK=JQ+J
        HOLD=A(JK)
        JI=JR+J
        A(JK)=-A(JI)
110     A(JI)=HOLD
120     J=M(K)
        IF (J-K) 100,100,125
125     KI=K-N
        DO 130 I=1,N
        KI=KI+N
        HOLD=A(KI)
        JI=KI-K+J
        A(KI)=-A(JI)
130     A(JI)=HOLD
        GO TO 100
150     RETURN
        END
```

3. Procedures to Run Additional Cases For Different Electron Thermal Conductivities

A. Run program with data deck and with new parameters, for example, changing the input data or the control parameter FACTO

B. The output of the program will be

- (1) new data cards for the dimensionless plasma electron temperature distribution, normalized plasma number density distribution, dimensionless motive distribution in the plasma region, the derivative distribution of the normalized plasma number density, the four parameters in the sheath region analysis and the normalized converter current density at the point for $V + \phi_c < 1.5$ volts on the C-V curve. These data cards can be used directly as inputs to additional calculations.
- (2) numerical solutions printed out on the computer sheets
- (3) plots listed on the computer chart sheets in the following order

(a) J .vs. $V + \phi_c$	(g) χ_E .vs. τ_0
(b) $\hat{\alpha}$.vs. τ_0	(h) j .vs. V_d
(c) N_E .vs. τ_0	(i) C .vs. ξ
(d) N_e .vs. τ_0	(j) τ .vs. ξ
(e) K_0 .vs. τ_0	(k) $\Delta\chi$.vs. ξ
(f) K_1 .vs. τ_0	(l) $\frac{n_s}{\bar{n}_s}$.vs. ξ