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Pion Nucleus Single Charge Exchange Reactions Above the $\Delta(1232)$ Resonance

Sayed H. Rokni*

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ABSTRACT

Pion Nucleus Single Charge Exchange Reactions

Above the $\Delta(1232)$ Resonance

by

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Forward-angle differential cross sections for the (π^+, π^0) reaction leading to the Isobaric Analog State in the residual nuclei at 300, 425, 500 and 550 MeV have been measured. Targets ranged in mass from ${}^7\text{Li}$ to ${}^{208}\text{Pb}$. A description of the experimental setup and the analysis is presented. The 0^0 cross sections are found to rise markedly between 300 and 425 MeV, contrary to the extrapolation from the lower energy data and to the behavior of the free pion-nucleon single charge exchange process. The angular distributions are sharply forward peaked. Systematics of the data indicate increased volume penetration with increasing pion beam energy. The cross sections are compared with the results of Glauber model calculations indicating the significance of higher order processes even at these energies.

(203 pages)

I. INTRODUCTION

Study of the differential cross sections in pion single charge exchange (SCX) reactions to the isobaric analog states (IAS) has been underway since 1979, when the π^0 spectrometer was commissioned at LAMPF.¹ Most of these work have focused on energy regions below and around the $\Delta(1232)$ resonance. At these energy regions the strong coupling of the pions to nucleons, and the rapidly changing nature of the pion-nucleon (πN) interaction have complicated the theoretical calculations. These calculations have generally failed to describe the data at the resonance energy region.

This dissertation describes the results of the first exclusive pion-nucleus experiment at energies well above the $\Delta(1232)$ resonance. At the measured energy region (300 to 550 MeV) the πN coupling is much weaker than at the resonance, furthermore, the nature of the interaction does not change rapidly with energy. These fundamental differences in the nature of the πN interaction with respect to the resonance region are expected to simplify the theoretical treatments of the pion-nucleus reactions at this energy region.

¹Before 1979, measurements using activation technique could only give the angle-integrated cross sections (Chivers *et al.*, 1969); for a detailed review of these work see Alster and Warszawski, (1979).

In chapter 1, we present an introduction to some aspects of the pion physics related to our work. Concepts of the isotopic spin, isobaric analog states, isospin conservation in pion-nucleon and pion-nucleus interactions, a review of the related literature, and finally the motivations for the current work are presented in this chapter. Chapter 2 contains the details of the experimental setup, namely a description of the π^0 spectrometer. The results of this experiment are presented in chapter 3. In chapter 4, we compare these results with theoretical calculation based on the Glauber model. A summary of the results and conclusions is presented in chapter 5.

A. Pion physics

With the advent of the meson facilities such as LAMPF (in Los Alamos) , SIN (in Switzerland), and TRIUMPF (in Canada) intermediate energy (0 - 600 MeV), high flux ($\sim 10^6$ - 10^8) pion beams have now been available for more than a decade. These beams have made possible the study of nearly all the nuclear reaction channels and greatly enhanced the study of nuclear structure.

The pion is a light, isovector ($T=1$) particle, which mediates the long range part of the nucleon-nucleon force. The mass of the $\pi^\pm$ is $139.56 \text{ MeV}/c^2$. These charged pions decay weakly via $\pi^\pm \rightarrow \mu^\pm \nu_\mu$ (100%) with a lifetime of 2.60×10^{-8} seconds. The π^0 has

a mass of $134.96 \text{ MeV}/c^2$ and decays primarily (98.85%) through the $\pi^0 \rightarrow \gamma \gamma$ reaction with a lifetime of 0.83×10^{-16} seconds (Particle Data Group, 1984).

Several properties of the pion make it a unique probe of nuclear structure. It is a pseudo-scalar meson ($J^{\pi}, 0^{-}$), therefore pions can be created or absorbed singly. Since it comes in three charge states, in addition to elastic and inelastic scattering, single- and double- charge exchange reactions can occur and also be studied.

One of the most important features of the pion which makes it a unique probe of the nuclear medium, is its many resonances in pion-nucleon scattering. In contrast, in the nucleon-nucleon interactions there are no known resonances. The most dominant πN resonance is at the total energy of 1232 MeV (the $\Delta(1232)$ resonance), with an approximate width of 110 MeV. It has a spin, isospin of ($J=3/2$, $T=3/2$) and is, therefore, commonly referred to as the (3,3) resonance. One important feature of the (3,3) resonance is that the pion-nucleon cross section in that region is very large, which results in a small mean free path for the pion through the nucleus. Therefore, the nucleus strongly absorbs the pions in this energy region and appears as a "black disk". Some aspects of the pion scattering at resonance can be explained by simple geometrical models.

This spin-isospin dependence of the pion-nucleon interaction makes the intermediate energy pion-nucleus interaction very attractive, in that a range for penetration of the pions into the nuclear medium can be selected by choosing the beam energy.

B. Isospin

By 1936 the analysis of p-p and p-n scattering data showed that after subtracting the Coulomb effects, the p-p and n-n hadronic forces appeared to be equal. This led to the "charge symmetry" principle which states that a system of hadrons is identical to its charge conjugate in the similar spin, parity state with regard to the strong force. It was also observed that if the Coulomb force is neglected (Breit, Condon, and Present, 1936), the n-p and p-p interactions are equal. This led to the stronger "charge independence" principle (Breit, and Feenberg, 1936), which states that apart from the electromagnetic interaction, forces between two nucleons in the same spin, parity state are the same.

Heisenberg (1932) had earlier introduced the isotopic spin notation to express the observed similarity between the proton and the neutron. In analogy with ordinary spin space, an isotopic spin space (charge space) is introduced, and an isotopic spin T is defined. In the absence of the electromagnetic interaction, the proton and the neutron are two degenerate states of the same

particle, namely the nucleon, accordingly the nucleon has $2T+1=2$ possible orientations in charge space, hence $T=1/2$. By definition, the value of the third component of T , namely T_3 , distinguishes between the proton and neutron. Throughout this study we assume $T_3=+1/2$ for the protons, and $T_3=-1/2$ for neutrons.

In terms of the isospin formalism, the charge independence principle states that the strong interaction depends only on T , not T_3 . Therefore it does not distinguish between the proton and neutron in the same spin and parity state. This means that the Hamiltonian for the hadronic interaction (H_h) is invariant under rotation in isospin space and hence only contains scalars.

$$[H_h, T] = 0 \quad (1.1)$$

where $[H_h, T] = H_h T - T H_h$ is the commutator of the H_h and T . In the presence of the strong interaction alone, the $2T+1$ states with different values of T_3 are degenerate, i.e. the proton and neutron would have the same mass.

The charge symmetries observed in the proton-neutron system have also been observed in large groups of particles and nuclear levels, which have different electromagnetic and weak interaction properties but cannot be distinguished in the presence of strong interaction alone. In the next section charge independence would be

generalized to the πN system, and in section D isospin in nuclei will be discussed.

C. Isospin in pion nucleon systems

The pion-nucleon interaction is presumed to be the elementary reaction involved in the pion-nucleus interaction. Charge independence states that the interaction between a pion and a nucleon with definite angular momentum J and given parity depends only on their total isospin T , which can have the values of $T = \frac{3}{2}$ or $T = \frac{1}{2}$. The possible values of T_3 are $\pm \frac{3}{2}$ or $\pm \frac{1}{2}$. For given values of T and T_3 it is possible to write down the following pion nucleon combinations using the Clebsh-Gordan coefficients (Particle Data Group, 1984), where we use the bra-ket ($| \rangle$) notation of Dirac to represent the wave function of T and T_3 , or N and π .

$$|\frac{3}{2}, \frac{3}{2}\rangle = |p, \pi^+\rangle$$

$$|\frac{3}{2}, \frac{1}{2}\rangle = \sqrt{(1/3)} |n, \pi^+\rangle + \sqrt{(2/3)} |p, \pi^0\rangle$$

$$|\frac{3}{2}, -\frac{1}{2}\rangle = \sqrt{(1/3)} |p, \pi^-\rangle + \sqrt{(2/3)} |n, \pi^0\rangle$$

$$|\frac{3}{2}, -\frac{3}{2}\rangle = |n, \pi^-\rangle$$

$$|\frac{1}{2}, \frac{1}{2}\rangle = \sqrt{(2/3)} |n, \pi^+\rangle - \sqrt{(1/3)} |p, \pi^0\rangle$$

$$|\frac{1}{2}, -\frac{1}{2}\rangle = \sqrt{(1/3)}|n, \pi^0\rangle - \sqrt{(2/3)}|p, \pi^0\rangle. \quad (1.2)$$

Conversely, a given pion-nucleon system can be expressed in terms of isotopic spin states as follows:

$$\begin{aligned} |p, \pi^+\rangle &= |\frac{3}{2}, \frac{3}{2}\rangle \\ |p, \pi^0\rangle &= \sqrt{(2/3)}|\frac{3}{2}, \frac{1}{2}\rangle - \sqrt{(1/3)}|\frac{1}{2}, \frac{1}{2}\rangle \\ |p, \pi^-\rangle &= \sqrt{(1/3)}|\frac{3}{2}, -\frac{1}{2}\rangle - \sqrt{(2/3)}|\frac{1}{2}, -\frac{1}{2}\rangle \\ |n, \pi^+\rangle &= \sqrt{(1/3)}|\frac{3}{2}, \frac{1}{2}\rangle + \sqrt{(2/3)}|\frac{1}{2}, \frac{1}{2}\rangle \\ |n, \pi^0\rangle &= \sqrt{(2/3)}|\frac{3}{2}, -\frac{1}{2}\rangle + \sqrt{(1/3)}|\frac{1}{2}, -\frac{1}{2}\rangle \\ |n, \pi^-\rangle &= |\frac{3}{2}, -\frac{3}{2}\rangle. \end{aligned} \quad (1.3)$$

The pion-nucleon interaction cross section is proportional to the square of the scattering amplitudes, which are associated with scattering matrices connecting the initial and final states i.e.

$$\frac{d\sigma}{d\Omega} = \rho |\langle \Psi_f | M | \Psi_i \rangle| \quad (1.4)$$

where ρ is the density of final states. Using relations 1.3, 1.4 the cross sections for elastic scattering

$$\begin{aligned}
 \pi^+ p &\rightarrow \pi^+ p \\
 \pi^- p &\rightarrow \pi^- p
 \end{aligned}
 \tag{1.5}$$

and for the charge exchange reaction,

$$\pi^- p \rightarrow \pi^0 n \tag{1.6}$$

can be expressed in terms of $A(1/2)$ and $A(3/2)$, the scattering amplitudes for $T=1/2$ and $T=3/2$ channels respectively. For the π^+p elastic scattering, equations 1.3,1.4 result in

$$\begin{aligned}
 \sigma_{el}^+ &\propto |\langle \pi^+ p | M | \pi^+ p \rangle|^2 \\
 &\propto |A(3/2)|^2
 \end{aligned}
 \tag{1.7}$$

and for the π^-p elastic scattering equations

$$\begin{aligned}
 \sigma_{el}^- &\propto |\langle \pi^- p | M | \pi^- p \rangle|^2 \\
 &\propto \frac{1}{9} |2A(1/2) + A(3/2)|^2.
 \end{aligned}
 \tag{1.8}$$

For the charge exchange reaction the cross section can be written as

$$\begin{aligned}
 \sigma_{scx} &\propto |\langle \pi^0 n | M | \pi^- p \rangle|^2 \\
 &\propto \frac{2}{9} |A(1/2) - A(3/2)|^2.
 \end{aligned}
 \tag{1.9}$$

The following special cases are of importance:

$$\begin{array}{lll}
A(1/2)=0, A(3/2)\neq 0 & \sigma_{el}^+:\sigma_{el}^-:\sigma_{scx} & 9:1:2 \\
A(1/2)\neq 0, A(3/2)\neq 0 & \sigma_{el}^+:\sigma_{el}^-:\sigma_{scx} & 1:1:0 \\
A(1/2)\neq 0, A(3/2)=0 & \sigma_{el}^+:\sigma_{el}^-:\sigma_{scx} & 0:2:1
\end{array} \quad (1.10)$$

Figure 1.1 shows the cross sections for these processes (Arndt, 1986). The dominant feature in the angle-integrated spectra is around $T_\pi = 180$ MeV (lab) (the $\Delta(1232)$ resonance), where the experimental cross sections are,

$$\sigma_{el}^+/\sigma_{scx}/\sigma_{el}^- = 200\text{mb}/50\text{mb}/25\text{mb}$$

These experimental ratios are quite close to the values 9/2/1 obtained assuming that only the $T=3/2$ amplitude contributes to the scattering in equation 1.10.

At energy region of the present study, 300 to 550 MeV, the $\Delta(1232)$ is no longer dominant; a number of weaker N and Δ resonances have been identified (Particle Data Group, 1984). The $T = 3/2$ channel has only one resonance in this region, the $\Delta(1550)$ or P_{31} resonance which is a one star resonance (evidence weak; could disappear). The $T = 1/2$ channel has one four star resonance (good, clear, and unmistakable) at this region, namely the $N(1440)$ or the P_{11} partial wave. The two other four star resonances are $N(1520)$, or D_{13} , and $N(1535)$, or S_{11} , with widths of 125, and 150 MeV, respectively. From figure 1.1 it is evident that above the $\Delta(1232)$

Fig. 1.1. Total cross sections for π^+p (solid line), π^-p elastic, (dotted line), and single charge exchange (chain-dot) processes (Arndt, 1986).

resonance the total pion-nucleon cross sections are much weaker, and change less drastically than at the resonance region.

The angular distribution. on the other hand, changes drastically over this energy range. The reaction of interest in this study is the pion single charge exchange. Figure 1.2 shows the angular distribution for this process at different energies (Arndt, 1986). At energies below the $\Delta(1232)$ resonance, the interference between s and p wave terms effects the angular distribution shape, and causes a forward dip in the 0° SCX cross section near 50 MeV. In the presence of only s and p waves, the differential cross section can be written as .sp2.eq

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2} (A + B \cos\theta + C \cos^2\theta) \quad (1.11)$$

where A, B and C are given in terms of phase shifts (Kallen, 1964). The $(B\cos\theta)$ term is the s-p wave interference term, and $(C \cos^2\theta)$ is a purely p-wave term. Near 50 MeV, A and B are nearly equal and have opposite signs and both are larger than C. This leads to near cancellation in the zero degree differential cross section. At even lower energies, B and C terms decrease in magnitude and the angular distribution becomes more isotropic. In the $\Delta(1232)$ resonance energy region the angular distribution is nearly symmetric about $\theta_{c.m.} = 90$ degrees, and follows closely the shape

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2} (1 + 3 \cos^2\theta) \quad (1.12)$$

Fig. 1.2. Angular distributions for the $\pi^-p \rightarrow p^0n$ process at 50 MeV (solid line), 180 MeV (dotted line), and 500 MeV (chain-dot line) (Arndt, 1986).

which is expected for a pure p-wave term. At higher energies, higher partial waves must be taken into account. Adding the d-wave, the differential cross section can be written as

$$\frac{d\sigma}{d\Omega} = \frac{1}{k^2} (A + B \cos\theta + C \cos^2\theta + D \cos^3\theta + E \cos^4\theta) \quad (1.13)$$

where A, B, and C are functions of s, p, and d partial waves, D is due to the p and d partial wave interference, and E is a pure d wave term.

D. Isospin in nuclei

One of the consequences of isospin invariance is the existence of isospin multiplets. A hadron has a definite isospin T and 2T+1 possible values of T_3 corresponding to different charge states. These different particles have otherwise the same quantum numbers such as spin, parity and strangeness.

Isospin multiplets also exist in nuclear physics. Nuclear states have definite isospin assignments. Since the nuclear interaction is independent of T_3 , all the 2T+1 states with different values of T_3 but the same values of spin and parity are energy degenerate. The 2T+1 isospin multiplets of an isospin T appear in different isobars (same atomic mass A), and are referred to as isobaric analog states (IAS). For example, the ground state of ^{14}C

($T=1, T_3=-1$) and $^{14}\text{O}(T=1, T_3=1)$ and the 2.3 MeV excited state of $^{14}\text{N}(T=1, T_3=0)$ form an isospin triplet of 0^+ states.

For a nucleus with A nucleons, the isospin T is the vector sum over isospin of all the nucleons (figure 1.3).

$$T = \sum_i T_i \quad (1.14)$$

and

$$T_3 = \sum_i T_{i3} = \frac{1}{2} (Z-N) \quad (1.15)$$

Since isospins add up vectorially, they can add up to a maximum of $A/2$, and since a vector can not be smaller than any of its component, the minimum value of T is $|T_3| = \frac{1}{2} |(Z-N)|$.

There are several methods for assigning an isospin value T to a state. Starting from a nuclear state with a given energy E_0 , the isobars are searched for states with the same quantum numbers. The energy difference due to the electromagnetic interaction can be estimated.

$$\Delta E = E(A, Z+1) - E(A, Z) \sim \Delta E_{\text{Coul}} - (m_n - m_p)c^2 \quad (1.16)$$

where $E(A, Z+1)$ and $E(A, Z)$ represent the energies of the isobaric

Fig. 1.3. Energy levels for $A=14$ isobars, showing the IAS and the DIAS transitions.

analog states in nuclei having the same atomic mass A , with the greater and lesser number of protons respectively. The neutron-proton mass difference is 1.293 MeV. The Coulomb energy difference can be estimated assuming that the charge Ze is distributed uniformly through a sphere of radius R .

$$E_{\text{Coul}} = \frac{3(Ze^2)}{5 \cdot 4\pi R} \quad (1.17)$$

Then the Coulomb energy difference between isobars with charges $Z+1$ and Z is

$$\Delta E_{\text{Coul}} = \frac{6}{5} \frac{(e^2)}{4\pi R} Z \quad (1.18)$$

The Coulomb energy difference varies with the charge and size of the nucleus. For example ΔE is 1.64 MeV for ${}^7\text{Li}$, and increases to 18.68 MeV for ${}^{208}\text{Pb}$. The search for the IAS is limited to the energy range ΔE away from E_0 . The search is continued until all of the $2T+1$ multiplets are found in different nuclei, then T is determined. Other methods based on the use of the isospin conservation in reactions are also widely used.

The electromagnetic interaction violates the charge independence of the strong force and lifts the $2T+1$ -fold degeneracy. It appears in the proton-neutron mass difference, but most importantly it is manifested in the presence of a static Coulomb

force between protons. However, since the Coulomb force is a long ranged force and varies slowly over the dimension of a nucleus, it is approximately the same for the ground state and the low lying excited states of a nucleus. Thus, the relative spacings are not altered. Therefore in light nuclei where $Ze^2 \ll 1$, the isospin invariance holds to a very good approximation.

The electromagnetic interaction also mixes states of given isospin with states having the same spin and parity but different isospins. Since the electromagnetic interaction (H_Y) is much weaker than the nuclear interaction (H_N), perturbation theory may be used, and to first order it gives (Fraunfelder and Henley, 1975).

$$|\psi\rangle = |T_a, T_{3a}\rangle + \sum_{b \neq a} C_b |T_b, T_{3b}\rangle$$

$$C_b = \frac{\langle T_b, T_{3b} | H_Y | T_a, T_{3a} \rangle}{E_a - E_b} \quad (1.19)$$

where the sum is over all the states with the same spin and parity but different isospin. The isospin mixing is defined as $\sum_b |C_b|^2$. H_Y the Coulomb interaction for a nucleus can be written as

$$H_Y = \frac{e^2}{4\pi} \sum_{i < j} \frac{1}{r_{ij}} (T_{3i} + 1/2)(T_{3j} + 1/2) \quad (1.20)$$

where the sum extends over all the nucleons. The Hamiltonian can also be written as

$$\begin{aligned}
H_Y &= T^{(0)} + T_0^{(1)} + T_0^{(2)} \\
T^{(0)} &= \frac{e^2}{4\pi} \sum_{i < j} \frac{1}{r_{ij}} \left(\frac{1}{3} T_i \cdot T_j + 1/4 \right) \\
T_0^{(1)} &= \frac{e^2}{8\pi} \sum_{i < j} \frac{1}{r_{ij}} (T_{3i} + T_{3j}) \\
T_0^{(2)} &= \frac{e^2}{4\pi} \sum_{i < j} \frac{1}{r_{ij}} (T_{3i} + T_{3j} - \frac{1}{3} T_i \cdot T_j) \quad (1.21)
\end{aligned}$$

where $T^{(0)}$ is an isoscalar, $T_0^{(1)}$ is the zero-component of an irreducible tensor of rank 1, and $T_0^{(2)}$ is the zero-component of an irreducible tensor of rank 2. To find out what isospins can be mixed to a state $|T_\alpha, a\rangle$, the matrix elements of H_Y are calculated. $T^{(0)}$ does not contribute to isospin impurity, and the contribution due to $T_0^{(1)}$ is calculated using the Wigner-Eckart theorem (Merzbacher, 1970):

$$\begin{aligned}
\langle T_b T_{3b}, b | T_0^{(1)} | T_a T_{3a}, a \rangle &= \\
1/\sqrt{(2T_b+1)} * \langle T_a T_{3a} \ 1 \ 0 | T_b T_{3b} \rangle \langle T_b, b | T_0^{(1)} | T_a, a \rangle. \quad (1.22)
\end{aligned}$$

The Clebsh-Gordan coefficients are zero unless $T_{3b} = T_{3a}$ and $T_b = T_a, T_a + 1$. Therefore the impurities caused by the isovector part of the electromagnetic interaction are from states with $T = T_a \pm 1$. The isotensor part of the Hamiltonian can come from states with $T = T_a \pm 1, T_a \pm 2$. Therefore only states with the same spin and parity

as the ground state with isospin differing by ± 1 , ± 2 can be admixed to this state. The isospin mixing is weakened by the factor $1/(E_a - E_b)$, as is shown on equation 1.19, where $(E_a - E_b)$ is the energy difference between the state $|T_b, b\rangle$ that contribute to the isospin mixing, and the initial state $|T_a, a\rangle$.

In light nuclei, this difference is large. For example the ground state of ${}^7\text{Li}$ is a $\frac{3^-}{2}$ state, while the next state with the same spin, parity assignment is at an excitation energy of 11.1 MeV. Therefore the mixing of that state would be small. In medium and heavy nuclei, the Coulomb interaction is large and the spacing between the nuclear states is small; therefore the impurity coefficient was expected to be large, and the isospin invariance to be violated severely.

The discovery of the IAS in ${}^{51}\text{V}$, ${}^{51}\text{Cr}$ by Anderson and Wong (1961), proving that isospin can still be a good quantum number in heavy nuclei, was therefore quite surprising. For heavier nuclei the average Coulomb field inside the nucleus is constant. This leads to the lifting of the degeneracy, but it does not contribute to transition between states with different isospins.

This is due to the dilution of the isospin mixing by the excess neutrons. The isospin mixing is due to the protons alone. If $(N-Z)$ is large, the neutrons with no Coulomb interactions dilute this mixing. The contribution of the T_a+1 isospin to the isospin T_a can

be estimated from the relation 1.22, and the Clebsh-Gordon coefficients,

$$\frac{\langle T_a T_a 1 0 | T_{a+1} T_a \rangle}{\langle T_a T_a 1 0 | T_a T_a \rangle} = 1/\sqrt{T_a} = \sqrt{2/(N-Z)}. \quad (1.23)$$

For medium and heavy weight nuclei the (N-Z) factor can be quite considerable, leading to the dilution of the isospin mixing process. Indeed in chapter 3, the data will be presented that will show the IAS of the ground state of ^{208}Pb which has 44 excess neutrons.

E. Pion SCX to the IAS

The SCX transition to the isobaric analog states requires minimum rearrangement in the nuclear wave function. As such, this transition is the simplest inelastic nuclear process. The SCX transitions to the IAS are generally $0^+ \rightarrow 0^+$ which limits their multipolarity to the single value of $\Delta J = \Delta L = 0$; this keeps the angular momentum and spin transfer effects on the angular distributions to a minimum; therefore the SCX to the IAS permits a especially clear study of the reaction mechanisms.

Since the discovery of the IAS in ^{51}V by Anderson and Wong (1961), transitions to these states have been studied in detail using ($^3\text{He}, t$) and (p,n) reactions (Fox and Robson, 1966), and the

pion single charge exchange reaction is complementary to these data. However, the pion has some advantages that makes it a unique probe of these transitions.

The scattering amplitude for the πN reaction can be written as (Sparrow, 1979)

$$T_{\pi N} = f_0 + f_1(T_{\pi} \cdot T_N) + f_2(\vec{\sigma} \cdot \hat{n}) + f_3(T_{\pi} \cdot T_N)(\vec{\sigma} \cdot \hat{n}). \quad (1.24)$$

Two of the contributing amplitudes are of comparable size, namely f_1 , the spin flip amplitude, and f_3 , the non spin flip amplitude. The spin flip term derives from a spin-orbit force; since for every single spin-flip the pion must carry off one unit of angular momentum along the beam axis, this term vanishes at zero degrees. On the other hand, the charge exchange by proton beams involving spin-flip seems to be dominated by the spin-spin tensor forces and is observed to be large at zero degrees. This leads to the strong excitation of Gamow-Teller states at certain energies.

The study of pure Fermi transitions at zero degrees, the possibility of studies of elastic scattering as well as inelastic and double charge exchange, and study of giant isovector resonances (Bowman et al., 1983; Baer et al., 1982), probing the neutral as well as charged nuclear densities, were among the motivations for construction of the π^0 spectrometer at LAMPF.

F. Pion single charge exchange at LAMPF.

The isospin invariance properties of the pion-nucleus interaction have been explicitly used by Johnson (1980) in representing the optical potential in terms of the pion isospin operator T_π and the nuclear isospin T_N .

$$U = U_0 + U_1(T_\pi \cdot T_N) + U_2(T_\pi \cdot T_N)^2 \quad (1.25)$$

where U_0 , U_1 and U_2 are the isoscalar, isovector and the isotensor components of the optical potential. The U_0 , U_1 , and U_2 are functions of nuclear density and the π -nuclear dynamics. Each of these terms are sensitive to one of the scattering processes. The U_0 term is dominated by the elastic scattering, the U_1 term is directly sensitive to the IAS transition, and U_2 is most sensitive to double isobaric analog state (DIAS) transitions. This term arises from the isospin $T=1$ of the pion. Therefore it is not present in the nucleon-nucleus optical potential. The DIAS can also proceed through the sequential or non IAS intermediate process. The U_2 term represents the case where the DIAS proceeds through non IAS intermediate states (Johnson, 1980).

In the lowest order optical potential, U_0 and U_1 depend linearly on the isoscalar density $\rho = \rho_n + \rho_p$ and the isovector density $\Delta\rho = \rho_n - \rho_p$. The lowest order U_2 term contains a $\rho\Delta r$ term and is neglected. This first order optical model accounts for many of the general features of the IAS transitions, such as the trend of the zero degree cross sections with $(N-Z)$ and A , the forward angle distribution of elastic scattering and the SCX to IAS (Sennhauser et al., 1983).

However, the first order calculations underestimate the absolute value of the cross sections. This should not be very surprising, since in the first order calculations effects such as true pion absorption and the propagation of the Δ in the nucleus are neglected. Higher order calculations which have their origins in the Δ -nucleus dynamics have been incorporated in the calculations to obtain agreement with the data at resonance region as well as lower energy regions.

The program at LAMPF for the SCX to the IAS has centered around the measurements of beam energy, mass and angle dependence of the cross sections for the IAS transitions, driven by the U_1 term in the optical potential.

G. Review of previous SCX experiments.

The first results of the pion SCX to the IAS were published in 1980, (Baer, et al., 1980), where the first direct measurements of the forward angle cross sections for (π^+ , π^0) on nuclei throughout the periodic table at 100 MeV were reported. The measured spectra are shown in figure 1.4. The most prominent feature in each spectrum is identified as the IAS for all the target ground states, based on the close correspondence between the expected and observed peak positions for all targets.

The energy dependence of the small angle differential cross sections to the IAS from 20 MeV to 295 MeV has subsequently been studied (Cooper et al., 1982, 1984, Doron et al., 1982a, 1982b, Irom et al., 1983, 1985a, 1985b, Leitch et al., 1986, Ullmann et al., 1986). These cross sections show different trends at different energies. To better understand the different features of the SCX interaction, we have divided the results of the previous experiments into three energy regions. The low-energy region where the pion kinetic energy is between 0 to 80 MeV, the resonance energy region of 80 to 250 MeV, and the high energy region where pions have kinetic energies higher than 250 MeV. This division is based on the distinctively different nature of πN interaction at these regions.

Fig. 1.4. The measured π^0 spectra for the (π^+, π^0) reaction at 98 MeV on several nuclei; the arrow marks the expected position of the IAS states, (Baer et al., 1980).

1. Low energy region
(T_π between 0 and 80 MeV)

As the energy is lowered below 100 MeV, the nucleus becomes more transparent to the pions. This transparency is the result of the decrease in πN cross section at these energies, which leads to higher penetration of pions into the nucleus. Therefore, the interaction samples a larger range of nuclear densities.

At still lower energies the zero degree πN charge exchange cross section is characterized by the interference phenomenon between the repulsive s-wave and attractive p-wave interactions, which causes a deep minimum in the free πN single charge exchange cross sections as mentioned earlier. The corresponding minimum in the pion-nucleus interactions could be modified by medium effects, such as Pauli blocking causing a shift in the energy and the angle of the minimum. Over the past few years extensive data in this energy region have been obtained (Cooper et al., 1984, Irom et al., 1985a, 1985b, Leitch et al., 1986, Ullmann et al., 1986). Figure 1.5 shows the extracted 0° cross sections for the free πN and for nuclei from ${}^7\text{Li}$ to ${}^{120}\text{Sn}$ from Irom et al., 1985b. It is observed that the minimum persists even for the heaviest nucleus, but shifts in energy and broadens for larger A . However, the shift is from 40 to 60 MeV. The analysis of these data shows that the location of the minimum after correcting for the Coulomb effects is nearly constant (within the experimental uncertainty). The width of the minimum also does not seem to increase except for the heaviest

target used, ^{120}Sn . This work shows that cross sections at the minimum position per excess neutron are all statistically consistent with that of free nucleon charge exchange.

This study suggested that the forward angle single charge exchange to the IAS at lower energies is very similar to that of the free nucleon single charge exchange. This result is quite surprising, since it was expected that the penetration of the pions into the the interior of the nucleus at these energies would make it test a large range of nuclear densities, therefore this reaction would be very sensitive to density related effects. The authors concluded that the interplay between various nuclear medium effects, such as the true absorption of pions and the Pauli blocking, seem to cancel. Therefore, the energy dependence of the isovector πN is not strongly modified by the nuclear medium. However, this conclusion does not seem to hold as the angle is increased. The angular distribution of the $^{15}\text{N}(\pi^+, \pi^0)^{15}\text{O}$ reaction (Leitch et al., 1986) shows that the shape of the angular distribution is very energy dependent, reflecting the analogous behavior of the free pion-nucleon SCX process. The measured angular distribution has a fairly flat shape at 32.4 MeV and a deep minimum at zero degrees for 48.2 MeV. The minimum moves to 25 degrees for 55.5 MeV, and finally the angular distribution becomes fairly forward peaked at 63.3 MeV. This energy dependence is caused by a change in the relative strength of s-wave and p-wave amplitudes in the medium. The angular distributions for SCX to IAS in ^7Li , ^{14}C , ^{39}K and ^{48}Ca have also

Fig. 1.5. The measured 0° excitation functions for the free charge exchange process and the IAS transition on different nuclei (Irom et al., 1985b).

been reported, showing the significance of the medium effects at the larger momentum transfer at low energies.

Various optical model calculations at low energies have been used to explain the data. The first order calculation of Johnson (1980) predicts a maximum rather than a minimum for the forward angle cross section in ^{15}N . Second order effects representing nucleon-nucleon correlations and pion absorption have to be added to reproduce the data.

In optical potential calculations of Kaufman and Gibbs (1983), the charge exchange reaction is expressed within a three-body distorted-wave impulse approximation (DWIA) framework, where the incident pion interacts with a nucleon in the nucleus. The strength of this interaction is expressed by a potential. The nuclear binding and the Pauli blocking are explicitly considered in the calculations of the distorted waves. These conditions predict a steep minimum in the 0° excitation function for the π -nucleus interaction around 50 MeV, due to the high transparency of the nucleus to the pions at these energies. The excitation functions predicted by this model for ^7Li and ^{14}C are similar in shape to the 0° excitation function for the free $\pi^-p \rightarrow \pi^0n$ cross sections. This is due to a large reduction of the absorption in the optical potential due to Pauli blocking and is more evident at lower energies. The original Kaufman-Gibbs calculations and the first-order calculations of Johnson (1980) failed to predict

correctly the shape or the location of the minimum in the excitation function for ${}^7\text{Li}$ at low energies (Irom et al., 1985a). This was partially attributed by Kaufman and Gibbs to incorrect phase shift analyses available at these energies. A more refined version of the same model and the second order optical calculation of Johnson (1980) are in agreement with the ${}^{14}\text{C}$ data at 48 MeV (Ullmann et al., 1986).

A simple theoretical approach which can be used as a guide in understanding the relative shift in s-p wave amplitudes at 48 MeV is presented in Ullmann et al., 1986. A plane-wave Born approximation method is used and the cross section for the SCX is represented as

$$\frac{d\sigma}{d\Omega} = (N-Z) \frac{k^2}{k_c^2} |f_s + f_p \cos\theta|^2 * \left| 4\pi \int \frac{\sin(qr)}{qr} \rho(r) r^2 dr \right|^2 (1.26)$$

Here, f_s and f_p are the free s- and p-wave scattering amplitudes, $\rho(r)$ is the valence density of the ground state, where a p-shell harmonic oscillator is used, q and k are the momentum transfer and momentum in the π -nucleus center of mass system, and k_c is the pion momentum in the π -nucleon center of mass. Multiplying the p-wave by 0.9 or the s-wave by 1.1 gives a better agreement with the data, thereby confirming the importance of the relative shifts between p and s waves.

2. Resonance energy region (T_π between 80 and 250 MeV)

In the resonance region, the free πN cross section is dominated by the $\Delta(1232)$ resonance. The mean free path of the pion in the nucleus is very short, and the nucleus seems as a black disk. This feature is evident in π -nucleus elastic scattering. Figure 1.6 shows the angular distributions for π^+ elastic scattering on ^{40}Ca . These data are from Preedom et al., 1981 at 30 MeV; Leitch et al., 1984 at 80 MeV; and Boyer et al., 1984 at 180, and 292 MeV. The data between 80 and 240 MeV clearly show the diffractive nature of the scattering at resonance region.

The strong absorption model of Johnson (1980) predicts the angular distribution of SCX to the IAS at the resonance as

$$\frac{d\sigma}{d\Omega} = g_1 \frac{N-Z}{A^{4/3}} J_0^2(qR). \quad (1.27)$$

The energy and mass dependence of small angle differential cross sections in these energy region has been published (Sennhauser et al., 1983). The mass dependence of the 0° cross sections have consistently followed the form

$$\frac{d\sigma}{d\Omega} = g(E) (N - Z) A^{-\alpha} \quad (1.28)$$

where $g(E)$ and α are the fitted parameters. Sennhauser et al.,

Fig. 1.6. Elastic π^+ scattering on ^{40}Ca at four energies spanning the $\Delta(3,3)$ resonance. The data between 80 and 292 MeV clearly show the diffractive nature of the scattering. These data are from Freedom *et al.*, 1981 at 30 MeV; Leitch *et al.*, 1984 at 80 MeV; and Boyer *et al.*, 1984 at 180, and 292 MeV.

(1983) report values of α at resonance quite consistent with the predictions of the strong absorption model, namely $\alpha = 4/3$. Figure 1.7 shows the results of fits with the above equation to the 0° cross sections from 100 MeV to 290 MeV. The fitted values of α decrease with increasing energies, showing the weakened A-dependence with increasing energy.

The excitation functions for the zero degree cross sections show a peak around 200 MeV, then following the trend of the free SCX process, decrease at higher energies. The fitted values of $g(E)$ seem to follow the trend of the free SCX process. However these values are not accurately predicted by the Johnson model. The angular dependence studies have focused on the forward angles, in expectation that the theoretical treatments would simplify. The angular distributions at these energies are very forward peaked, following a $J_0^2(qR)$ shape closely, thus supporting a picture of diffraction from a black disk. Angular distributions to more backward angles are limited to ^{13}C and ^{15}N at 165 MeV (Doron et al., 1982a, 1982b) and ^3He at 200 MeV (Cooper, et al., 1982). The angular distributions of ^{13}C and ^{15}N show a minimum at 38 ± 1 degree and a second minimum around 52 ± 2 degrees with the same strength. Since ^{13}C and ^{15}N are both P-shell nuclei with an excess of one neutron, and ground states of both nuclei are $\frac{1}{2}^-$, the distribution is expected to be the same, any difference being due to the details of their nuclear structure. Doron et al., (1982a, 1982b) have concluded that the similarity of the angular distributions in both

Fig. 1.7. The A -dependence of the 0° IAS cross sections. The solid lines represents the function $g(N-Z)A^{-\alpha}$ (Sennhauser et al., 1983).

nuclei is a reflection of the same reaction mechanism for both nuclei. In the ${}^3\text{He}(\pi^-, \pi^0){}^3\text{H}$ reaction, the direct term in the charge exchange has a minimum near 55° due to s and p wave interference in the πN amplitude, however the spin-flip term peaks near 40° , resulting in a featureless distribution. The measured angular distribution is indeed featureless, verifying the significance of the spin flip term in light nuclei (Cooper et al., 1984). As the angle increases the contribution of the spin-flip states to the IAS transitions increases thereby making the extraction of the area of the IAS peak quite difficult.

3. High energy region ($T_\pi > 250$ MeV)

The highest kinetic energy of the pion beam used in the previous SCX to IAS studies was 295 MeV. The data (Sennhauser et al., 1983, Irom et al., 1983) following the trends of free $\pi^-p \rightarrow \pi^0 n$ process seem to decrease above the resonance. The first order calculation shown in figure 1.8 (Auerbach et al., 1984) again failed to predict the magnitude or the trend of the 0° cross sections.

In summary, prior to the current study the reported zero degree cross sections were limited in energy to 295 MeV and below. The first-order theoretical treatments are inadequate in describing these data. At low energies, Pauli blocking, Fermi motion and

Fig. 1.8. The zero-degree cross sections for ^{14}C and ^{120}Sn . Solid lines represent the results of the first-order optical model calculations (Auerbach *et al.*, 1984).

binding energy effects, and at the resonance, the strong coupling of pions to many reaction channels complicates these treatments.

H. Motivations for the current study

The current study is based on the LAMPF experiment #827 which was performed in the summer of 1985. In this experiment we measured the forward-angle cross sections for pion SCX reactions to the IAS between 300 to 550 MeV. This is the highest energy available in any meson factory to date.

There are a number of simplifying features in the pion-nucleus interaction which are expected to appear in the measured energy region. The π^+p and π^+n total cross sections are near 30 mb, compared to 200 mb for the π^+p cross section at 180 MeV. Furthermore, their energy dependences are relatively flat, in contrast to the behavior over the resonance region. From these features one expects that the distortion of the incoming and outgoing pion waves would decrease.

Another feature of pion-nucleus scattering which plays an important role in pion-nucleus scattering is pion true absorption. The cross section for this mechanism has a maximum at the resonance region (Ashery et al., 1981). The existing data do not extend to the energies of the present work, but based on the energy dependence

below and above the resonance as suggested from figure 1.9, one might expect a decrease in the strength of this process at higher energies. From these features one might expect that single scattering processes would be more dominant and first-order scattering theories such as DWIA or Glauber model calculations may give a better description of the forward-angle scattering.

At higher incident pion energies, due to the decrease both in π^+p and π^+n total cross sections, greater pion penetration into the nucleus is expected which could lead to rising cross sections at these energies. On the other hand, the previous data (Sennhauser et al., 1983, Irom et al., 1983) showed that the excitation function seem to follow the trend of the free $\pi^-p \rightarrow \pi^0 n$ cross section, decreasing at higher energies. This leads to the interesting question of whether the cross sections would decrease or increase with increasing beam energies.

The resolution of these issues is the major objective of this dissertation.

Fig. 1.9. Total absorption cross sections for positive pions on various nuclei from Ashery et al., 1981.

II. EXPERIMENTAL PROCEDURES

In this chapter the equipment and facilities used at LAMPF for this experiment will be discussed. A general description of the design of the π^0 spectrometer and parameters influencing its performance and its efficiency are described. The data acquisition hardware and software are presented, and the measurements of pion beam fluxes are also discussed.

A. General description of the π^0 spectrometer

The π^0 spectrometer was used in the experiment to detect and measure the direction and energy of the outgoing neutral pions. This spectrometer was designed and constructed at LAMPF by a collaboration of LAMPF, Tel Aviv University, and Case Western Reserve University. Details of its design are given elsewhere (Baer et al., 1981; Gilad et al., 1977; Gilad et al., 1979; Doron, 1981; Erell, 1984). Here a general description of the detection scheme and the spectrometer is given. The π^0 spectrometer measures the π^0 energy and direction by measuring the directions and energies of the two gamma rays resulting from the decay of π^0 . If the laboratory opening angle between the two decay photons is denoted by η , E_1 and

E_2 represent the laboratory photon energies, and Θ_1 and Θ_2 the laboratory opening angles with respect to the beam direction, then the total energy of the π^0 (W_{π^0}) is obtained from the following relation.

$$W_{\pi^0}^2 = \frac{2m_0^2}{(1 - \cosh) (1 - X^2)} \quad (2.1)$$

where

$$X = \frac{E_1 - E_2}{E_1 + E_2} \quad (2.2)$$

is the energy sharing parameter. The π^0 laboratory angle Θ_{π^0} is given by

$$\cos\Theta_{\pi^0} = \frac{E_1 \cos\Theta_1 + E_2 \cos\Theta_2}{\sqrt{(E_1^2 + E_2^2 + 2E_1 E_2 \cosh)}} \quad (2.3)$$

If the two photon energies are nearly equal, X is small and a good measurement of η can yield a good measurement of W_{π^0} . In this way, the W_{π^0} energy resolution depends only weakly on the energy resolution of the photon detectors.

The spectrometer consists of two arms, J and K, each one being a position sensitive high energy photon detector by itself. Figure 2.1 is a schematic diagram showing the detection scheme of the spectrometer. Each photon resulting from the decay of the π^0 is detected by one of the two spectrometer arms. In front of each arm there is a 2.5 cm slab of polyethylene which is used for preventing charged particles from entering the spectrometer, followed by a thin (3 mm) plastic scintillator which serves as a veto for the charged particles which are not stopped in the polyethylene. This is then followed by three independent systems which convert the photons to showers of charged particles, then by a total absorption Cherenkov counter which contains the full shower and measures its energy. Each one of the conversion systems has an active Pb-glass converter, 0.58 radiation length (X_0) thick ($.58 X_0 = 2.2$ cm), which is followed by three planes of multiwire proportional chambers (MWPC). Two planes measure the X-coordinate perpendicular to the scattering plane and the third plane measures the orthogonal Y-coordinate in the scattering plane. Each gamma ray can convert in any one of the three active Pb-glass converters.

Charged particles exiting the back of each converter are detected in one of the three MWPC planes whose information is used to determine the conversion point of the gamma ray. The X-coordinates from the two X-MWPC planes are traced back to a conversion plane in the converter to reconstruct the X-conversion point. The centroid of Y-coordinates in the Y-plane gives the

Fig. 2.1. Schematic diagram of the π^0 spectrometer (Baer, et al., 1981).

Y-conversion point. Since the π^0 energy resolution depends more sensitively on the X-coordinate than the Y-coordinate, two X-planes and one Y-plane in each conversion system are used. In the back of each conversion system is a thin (3 mm) plastic scintillator whose signal tags the system in which conversion takes place. The full shower is absorbed in an array of 15 blocks of Pb-glass located behind the converters. The Cherenkov light output from the Pb-glass converters and the blocks is directly proportional to the incident gamma-ray energy. The line between the conversion point in each gamma-ray detector and the upstream face of the target defines the gamma-ray direction. Thus each gamma-ray detector measures independently the three kinematical variables $(E_\gamma, \theta_\gamma, \phi_\gamma)$ of one photon. From these information the total energy W_π^0 (equation 2.1), and the scattering angle of π^0 , namely θ_π^0 , (equation 2.3), are obtained.

B. Energy resolution

The uncertainty in measurement of the the π^0 energy depends on the uncertainty in the photon energy ΔE_γ and the uncertainty in the opening angle $\Delta\eta$

$$\Delta W_\pi^0 = \sqrt{[(\Delta W_\pi^0)_{\Delta E_\gamma}]^2 + [(\Delta W_\pi^0)_{\Delta \eta}]^2}. \quad (2.4)$$

The contribution to the π^0 energy uncertainty due to the uncertainty in the photon energy for the subset of events for which $X \approx 0$ is

$$(\Delta W_{\pi^0})_{\Delta E_\gamma} = \frac{\sqrt{3}}{2} C^2 \text{ (MeV)} \quad (2.5)$$

where C is approximately 1.10 (Baer et al., 1981). This contribution is independent of the π^0 energy and depends only on the photon detection energy resolution. The contribution to the π^0 energy resolution due to the uncertainty in the opening angle is

$$(\Delta W_{\pi^0})_{\Delta E_\eta} = \frac{m_0}{2} \gamma^2 \sqrt{1 - 1/\gamma^2} \Delta\eta \quad (2.6)$$

At a typical distance of 100 cm, position resolution of 3.7 mm (FWHM) and pion energy of 100 MeV, the contribution due to $\Delta\eta$ alone is 0.85 MeV (FWHM). For this experiment the distance from the target to the first converter was increased to 200 cm which reduced the $\Delta\eta$ to 2.6 mrad. At 500 MeV this uncertainty resulted in $(\Delta W_{\pi^0})_{\Delta E_\eta} = 3.8$ MeV. Thus the π^0 energy resolution as a function of its energy depends on a fixed parameter due to the photon energy resolution and on an energy-dependent parameter due to the uncertainty in the opening angle measurement which increases with energy. These two factors are nearly equal around 135 MeV for $\Delta\eta = 5$ mrad. Figure 2.2 shows the curve combining these two effects.

Fig. 2.2. Energy resolution of the π^0 spectrometer vs. pion kinetic energy obtained from the Monte-Carlo calculations, (Baer, et al., 1981). R is the distance between the target and the first converter.

C. Angular resolution

The measurement of the direction of π^0 , $\Delta\theta_{\pi^0}$, is also subject to the uncertainty in the two measured parameters E_γ and η ,

$$\Delta\theta_{\pi^0} = \sqrt{[(\Delta\theta_{\pi^0})_{\Delta E_\gamma}]^2 + [(\Delta\theta_{\pi^0})_{\Delta E_\eta}]^2}. \quad (2.7)$$

The contribution to the measurement of θ_{π^0} due to the photon energy resolution for the case of $E_1 = E_2$ at $\theta_{\pi^0} = 0$ is

$$(\Delta\theta_{\pi^0})_{\Delta E_\gamma} = \frac{\tan(\frac{\eta}{2})}{\sqrt{2}} \left(\frac{\Delta E_\gamma}{E_\gamma} \right) \quad (2.8)$$

Thus the angular resolution due to energy resolution is directly proportional to the fractional energy resolution of the Pb-glass counters and decreases with increasing energy. The contribution to the uncertainty in measuring η is given by

$$(\Delta\theta_{\pi^0})_{\Delta E_\eta} = \frac{1}{2}\Delta\eta \quad (2.9)$$

which is negligible at θ_{π^0} compared to the contribution from ΔE_γ . Therefore the angular resolution is a decreasing function of the energy of the π^0 . Figure 2.3 describes this dependence.

Fig. 2.3. Angular resolution of the π^0 spectrometer vs. pion kinetic energy obtained, from the Monte Carlo calculations, (Raer, et al., 1981). R is the distance between the target to the first converter.

D. Additional factors influencing the energy resolution

The finite size of the targets used also affects the π^0 energy resolution; these effects include the energy loss of the charged pion in the target, the straggling, the energy spread as a function of the spread in η resulting from π^0 creation at different depths, and the finite size of the beam spot. Additional contributions to the π^0 resolution are beam related. One of the contributing factors is the spread in the beam energy due to the momentum bite. This was added in quadrature to the straggling of the pion beam in the carbon degrader which was inserted in this experiment to reduce the proton contamination. This factor was estimated to be 20% of the beam energy loss in the equivalent thickness of carbon (Bethe and Ashkin, 1953). Table 2.1 shows the contribution of the beam and the spectrometer to the measured energy resolution at different energies.

E. Spectrometer design

Each arm of the spectrometer has 15 Pb-glass blocks arranged in a 3x5 array, each one being 15x15x60 cm³. The back face of 15x15 cm² couples to a 5" photomultiplier tube. The 60 cm length of the blocks allows showers initiated by up to 10 GeV gamma-rays to be detected.

Table 2.1 Measured energy resolution and factors influencing it, in MeV.

T_{π^+}	$\Delta E(\text{Spec})^a$	$\Delta E(\text{Beam})^b$	Measured $\Delta E(\text{Tot})$
300	4.0	3.5	6.0
425	5.5	6.0	8.2
500	6.5	9.5	11.5
550	9.5	11.0	14.5

^a) from the Monte Carlo calculations.

$$^b) \Delta E(\text{Beam}) = \sqrt{(\Delta E(\text{Tot})^2 - \Delta E(\text{Spec})^2}$$

There are three Pb-glass converters in each arm. The face dimensions of each converter are $64 \times 40 \text{ cm}^2$, which corresponds to the face area of the 3×5 array of glass blocks. The conversion efficiency for the converters is defined as the probability that an incident gamma-ray results in at least one detectable charged particle exiting the back face of the converter. The converter thickness is $0.58 X_0$ which results in a conversion probability of 0.28 for a 100 MeV incident photon. Three converters per arm then yield about 63% conversion probability per arm. Each gamma-ray conversion system has three multiwire proportional chamber planes, a total of 18 chambers for the six conversion systems. Each MWPC plane is built as a separate module. There are 192 wires in each

plane, with the wire separation of 2 mm. Each wire in the X-planes is read individually, while every two wires in the Y-planes are connected together and read out digitally with the PICOS II (1976) readout system.

The individual wire readout system is used since the multiparticle events often result in signals on several adjacent wires. The readout system includes chamber mounted electronics to amplify, discriminate, delay and latch the data signals, and four CAMAC modules to encode the data from all 3264 wires. The chamber frames are made of G-10 Fiberglass-hepoxe-laminate, their active area being $64 \times 38 \text{ cm}^2$. The gap is 5.7 mm. The anode wires are Au-plated Tungsten, 20 μm in diameter with 2mm spacing. The X-planes have 192 wires, and the Y-planes have 160 wires. The cathode planes are 2.54 μm mylar with aluminum film evaporated on one of the sides. They are operated at -3.0 KV. A "magic" gas mixture is used, the components of which are 75% Argon, 20% Isobutane, 5% Methylal and 0.5% Freon. In order to obtain the best position resolution, only the information from the most forward prongs in each shower is used. After determining the MWPC coordinates for the most forward prong, these are extrapolated into the converter to an imaginary "conversion plane" located at a depth (from the front face) equivalent to 5/6th of the converter thickness. With this algorithm a position resolution in the X-direction of 6 mm (FWHM) was achieved with a tagged photon beam. Since there is only one Y-plane per system, the conversion

Y-coordinate is determined for each shower by averaging y-coordinates of all the the charged prongs. The resolution achieved for the conversion Y-coordinate is about 12 mm (FWHM) and does not make a significant contribution to the resolution in η . The support structure of each of the two detector arms was designed to permit independent placement of each detector to an accuracy of ± 0.5 mm in all three dimensions over π^0 scattering angles from 0 to 180 degrees.

Over the period of several years the photon energy resolution of the spectrometer obtained from the block signals had deteriorated from 30% at $E_\gamma = 100$ MeV (FWHM) to 50%. The reason for this deterioration was the loss of internal reflection of Cherenkov light at the glass surface due to the presence of a thin layer of optical grease. This grease originated from the phototube-glass interface and had spread over the entire glass surface.

From January to June 1985, the two crates of the spectrometer were fully disassembled. All the Pb-glass elements were de-greased, and the optical grease was replaced by an elastomer optical coupling. The crates were then reassembled and aligned. During a two week period, the glass elements were energy calibrated relative to the Bismuth light pulsers (Bolton et al., 1980) using a 100 MeV electron beam at the LEP channel.

F. Event trigger

The electronic trigger consisted of the coincidence logic pulses from the two arms of the spectrometer (a γ - γ event). Each gamma pulse consisted of the logical condition

$$\text{CONVERTED } \gamma = (S_1 + S_2 + S_3) \cdot \bar{V} \cdot \sum_{ij} (C_i + B_j) \quad (2.10)$$

where S stands for the plastic scintillators behind the converters, V is the veto counter in front of each arm, and C and B are the analog signals from the converters and blocks, respectively. In addition to the " γ - γ " condition, the trigger included hardware requirements that the beam was on, that the computer was not busy, and that a manual switch was on. A logical product of these requirements was made for the π^0 trigger as follows:

$$\pi^0 \text{ EVENT TRIGGER} = (\gamma_j \cdot \gamma_k) \cdot \text{BEAM} \cdot \overline{(\text{COMPUTER BUSY})} \quad (2.11)$$

G. Data acquisition hardware

The data acquisition program "Q" was used. The components of the data acquisition consisted of the computer, the Micro Programmable Branch Driver (MBD), the LAMPF trigger module and the NIM and CAMAC electronic modules. A PDP-11/45 microcomputer under

RSX-11M operating system was used for the data acquisition. The MBD (Biswell and Rajala, 1973) is a small computer which handles the transfer of data from the CAMAC crates to the main computer. The recorded information in each event consisted of up to 256 words which were read out through the CAMAC into the MBD, which buffered the data until they were released to the computer. The CAMAC modules were then reset and the system made active for the next event. Meanwhile the data were transferred from the the MBD to the computer memory and subsequently recorded on the magnetic tape. The information included digitized pulse height of all Cherenkov counters and of all plastic scintillators, digitized times of all the scintillators and Cherenkov counter pulses and of relative timing between the two arms, location and width of all MWPC hits, registers that tagged counters that fired, and scalars that contained information on singles and coincidence in different counters, trigger rates, proton and pion beam toroid counts that were used for beam monitoring, and beam burst counts that were used to monitor the live time of the run. The events are routed to the MBD via the LAMPF trigger module. This is a module with a series of inputs numbered 4 to 11; whenever a NIM signal is received at the trigger module, the corresponding event is triggered.

H. Data acquisition software

Several types of events were defined in the program. They included data taking events such as "singles" events, or coincidence events "gamma-gamma" ,and others such as the "write data on tape" event or "read and write scalars" events. In this experiment events 4,5,7,9,22 and 23 were used. Event 4 is the scalar event, event 5 is used for gain stabilization. Event 7 checks the pedestal shift and event 9 is the main coincidence event. The analyzer subroutine processes a "Gamma-Gamma" event, which supports various levels of processing. The first level consists of retrieving the event data and testing if there are good gamma conversions in both arms. The events that pass these tests are taped. This level also tests the chamber efficiency data. The second level of processing consists of determining the conversion planes, linearizing ADCs, generating converter and block energy sums, calculating total gamma energies and calculating "X" (the energy sharing variable). The conversion coordinates for arm J are determined. In the third level of processing the conversion coordinates for the K-arm are determined. In the fourth level, the kinematics for the event are evaluated, and the final cuts on the data are applied.

Event 22 starts the taking of the pedestal data, and event 23 is a hardware event which clears all the CAMAC modules at the start of the run. The program allowed at any time the histogramming of all the parameters which were necessary to monitor the run

conditions. Hence, in addition to raw parameters (MWPC hits, ADC and TDC spectra), calculated parameters (energy in blocks, energy sums, gamma conversion coordinates) and the π^0 kinematic parameters were histogrammed and monitored during the run to check the stability of the run conditions.

In addition to the accumulated data which were histogrammed, the program allowed for the display of single events that were chosen at random. In this mode, all energies registered in the Cherenkov counters, all MWPC hits, summed energies, calculated gamma conversion coordinates, as well as π^0 kinematic parameters were displayed for a single event. A HP-9000 microcomputer was used to monitor parameters such as current drawn by the MWPC's and temperatures of PICOS electronics.

I. The pion beam

The experiment was performed in the east cave of the pion particle (P^3) channel of the Los Alamos Meson Physics Facility (LAMPF). A brief description of the accelerator and the experimental areas is given in appendix A.

A diagram of the P^3 channel is shown in figure 2.4. The target was located 200 cm from the last quadrupole in the channel. To transport the pions produced in the A-2 production target by the

Fig. 2.4. Layout of the Pion Particle Physics (P^3) channel at LAMPF.

main proton beam to the P^3 experimental area, three rectangular bending magnets (dipoles) are used, BM01-BM03. The bend planes are horizontal. Each bending magnet has two entrance and two exit quadrupole magnets QM01-QM16. This three-bend analyzing system selects and momentum analyzes positive or negative pions. The production angle is 20° . After the first bend which determines the momentum and polarity of the pions, the beam is focused between QM-04 and QM-05 where a set of slits, MS03, defines the channel's momentum acceptance. A degrader may be inserted to provide chromatic separation of π^+ and p which initially have the same momentum. BM02 translates this chromatic separation into spatial separation at the position of the absorber slits. A second focus occurs in the center of the last bending magnet (BM03). This magnet steers the beam to the east or the west cave. Horizontal and vertical slits near the A-2 production target are used to control the pion beam intensity.

The channel was tuned using the result of TRANSPORT (Brown et al., 1977) and TURTLE (Carey et al., 1982) programs. The resulting magnetic field outputs were input to the SHUNT program (Macek and Aldridge, 1978) which produced the the shunt voltages for the beam line magnets.

Fine tuning was accomplished with the aid of a beam profile monitor located at the target position. For the final confirmation of each tune a Polaroid film was located at the target position and

irradiated. The size of the spot on the film and the location of its center were checked for accuracy of the tune. Two steering magnets at the end of the P^3 channel were used to compensate for mis-steering of the beam. The X-steerer was used to steer the beam in the horizontal plane and the Y-steerer, which was usually off during the run, steered the beam vertically. A wall of lead bricks was built around the beam exit to reduce the background into the spectrometer crates.

J. Measurements of the angle of incidence

To find out the angle of incidence of the beam on the target in a test run before the experiment, two beam profile monitors (BPM) were located at the target position and 80 cm downstream of the target. The centers of the two BPM's were optically aligned with beam line. The horizontal and vertical angles of incidence were obtained using the deviation of the beam spot from the center in the two BPMs at different beam energies and degrader thicknesses. The horizontal angle varied from 0.29 degrees to 0.04 degrees and the vertical angle varied from 0.43 degrees to 0.0 degrees. In section III.H we will show that the uncertainties in the zero-degree cross sections due to these measurements are negligible.

K. π flux measurements

For each channel setting, the pion flux was measured by using the scintillator activation technique. In this method plastic scintillator targets are irradiated at each channel setting. The cross section for the $^{12}\text{C}(\pi^+, \pi^+ \text{N})^{11}\text{C}$ reaction have been accurately measured (Dropesky et al., 1979). The irradiation time is between 300 to 600 seconds at steady beam conditions. The targets are optically coupled to a photomultiplier tube which is in contact with a NaI(Tl) detector. The ^{11}C isotope decays by emitting positrons, which are self detected in the scintillator, while the annihilation quanta are detected by the NaI(Tl) which has an electronic window set to include only the 511 keV photopeak. The decay rate is then measured using the β - γ coincidence technique. Since protons in the beam also induce $^{11}\text{C}(\beta)$ activity in the scintillator, a correction had to be applied based on the measured proton contamination. The proton activation cross sections are also known in this energy region (Cumming, 1963). The proton-to-pion ratios were measured using the Sampling-Grid-Scintillator (SGS). The SGS was also used for the absolute rate measurement.

L. The Sampling Grid Scintillator

The use of the Sampling Grid Scintillator (SGS) makes it possible to identify and count the particles in high rate beams. At 6% duty factor the instantaneous rates can be as high as 10^9 particles/second. These rates are too high to count with the usual "counter in the beam" technique, because such high rates would result in a large dead time. For example, for an average rate of $r=2 \times 10^7$ Hz, a pulse width of $t=10^{-8}$ seconds, and duty factor of $d=6\%$, the probability of pile up is $1-e^{-rt/d} = 0.96$, i.e every pulse is piled up. The first SGS tested in the fall of 1984 was a 15×15 array of scintillator cylinders 1.00 mm in diameter mounted in a 1.56 mm thick lucite plate which was connected at each side to a photomultiplier tube. This SGS was similar to a scintillator designed by Holley et al., 1980. The cylinders were spaced on 1 cm centers ($h=1$ cm). This way the SGS detector would only sample a fraction (S) of the particles in the beam,

$$S = \pi r^2 / h^2 = 0.00785. \quad (2.12)$$

The instantaneous rate would reduce to

$$r' = Sr \quad (2.13)$$

which can be detected with a pile up probability of

$$1 - e^{-Srt/d} = 2.5 \times 10^{-2} \quad (2.14)$$

The factors that influence the measurement of the sampling ratio S are the phase space dependence of S and the activation of the SGS. To find out the effects of these factors in the performance of the SGS, a series of measurements were performed in the summer of 1985. The phase space of the beam from the LEP channel is large, and the divergence in the beam can be as large as $\Theta \sim 100$ mr. For normal incidence the sampling fraction S is

$$S = \pi r^2 / h^2 \quad (2.15)$$

while for non-normal incidence the sampling fraction is larger if every particle that passes through a scintillator is counted

$$S = \pi r^2 / h^2 + 2\pi \Theta r t / h^2 + O(\Theta^2). \quad (2.16)$$

In the measured pulse height spectrum of the charged particles passing through the SGS, if only the events that have passed through the top and bottom surfaces are counted, i.e. if the threshold in the discrimination is set high, then

$$S = \frac{\pi r^2}{h^2} - \frac{2\pi \Theta r t}{h^2} \quad (2.17)$$

and if all the events that pass any part of the scintillator buttons are counted, i.e. if the threshold is set very low, then

$$S = \frac{\pi r^2}{h^2} + \frac{2\pi\Theta r t}{h^2}. \quad (2.18)$$

If the discriminator is set at half of the full pulse height, the sampling factor will be independent of the Θ to the first order of approximation.

The second problem was not anticipated. When the SGS was placed in coincidence with a solid counter, the measured value of S agreed with the calculated value. But when the SGS was used alone, the measured value of S was 10% higher. Placing a beam gate requirement on events from the SGS reduced the measured value of S . The excess value of the SGS was attributed to the activation of the SGS.

In order to be used as a flux monitor, the SGS should be able to identify the particle types. During the 1984 run at LEP, the identification was achieved by the time of flight technique relative to a Radio Frequency (RF) signal from the accelerator. The (RF) signal is not available any longer, so a new design was necessary. The 1985 design of the SGS included a second lucite plate (G2) 3.12 mm thick, which also has an array of 15x15 scintillators 2 mm in diameter each (Bowman, 1985). Figure 2.5 shows the layout of the

Fig. 2.5. A schematic diagram of the Sampling Grid Scintillator where the distance between the two grids along the beam axis is exaggerated.

new design of the SGS. The centers of the scintillators in the two arrays are aligned with beam. Since G2 has 4 times as much active area as G1, it was connected to fast phototubes. The improved design aims at particle identification as well as reducing the effects of activation by requiring a coincidence between the two grids. Particles are identified by the amount of energy that they deposit in the G2 plate. The coincidence requirement between G1 and G2 and the fact that G2 has twice the radius of G1 insures that the edge effects are negligible for $\theta < 1.00$ rad. Using the SGS the ratios of π/P were measured at 425 ,500 and 550 MeV.

In the current experiment, the SGS was used to measure the P contamination in the π^+ beam in P^3 channel. Figure 2.6 shows the π -p separation as measured by the SGS at 500 MeV. The results of these measurements are listed in table 2.2.

M. π^0 detection efficiency

The total efficiency of the spectrometer (ϵ) is defined as the ratio of the effective solid angle of the spectrometer ($\Delta\Omega$) divided by the geometrical solid angle ($\Delta\Omega_g$).

$$\epsilon = \frac{\Delta\Omega}{\Delta\Omega_g} \quad (2.19)$$

Fig. 2.6. The pion, proton separation in the 500 MeV π^+ beam.

Table 2.2 Positive pion flux and proton contamination at p^3 for the experiment.

T_{π^+} (MeV)	$(\Delta p/p)^a$ %	Degrader cm of C	π -Flux (pion/sec)	P/π
300	0.2	1.7	1×10^7	-
425	0.2	3.1	5×10^6	0.02
500	0.4	10.0	2.5×10^6	0.29
550	0.5	11.5	8×10^4	0.40

^a)Setting of channel momentum slits. Values of $\Delta p/p$ passed by the channel were actually larger.

The effective solid angle ($\Delta\Omega$) was measured using the CH_2 data and the known $\pi^-p \rightarrow \pi^0 n$ reaction cross sections (Arndt, 1986) at 300, 425, 500, and 550 MeV. The geometrical solid angle ($\Delta\Omega_g$) was calculated by the Monte-Carlo simulations.

A CH_2 target was used for the solid-angle normalization. The contribution from the SCX reaction on the carbon component of the target was measured using a graphite target and subtracted from the CH_2 data. Figure 2.7 shows the CH_2 , carbon and the carbon subtracted CH_2 spectra. The events from the graphite target are at energies lower than the IAS peak. Figures 2.8-2.11 show the π^0 spectra for the $\pi^-p \rightarrow \pi^0 n$ reaction at the measured energies.

Fig. 2.7. The CH_2 , carbon and the carbon subtracted CH_2 spectra for 300 MeV incident π^- beam.

Fig. 2.8. The measured π^0 spectrum for the $\pi^-p \rightarrow \pi^0 n$ reaction at 300 MeV. The solid line represents the result of the Monte Carlo calculations.

Fig. 2.9. The measured π^0 spectrum for the $\pi^-P \rightarrow \pi^0 n$ reaction at 425 MeV. The solid line represents the result of the Monte Carlo calculations.

Fig. 2.10. The measured π^0 spectrum for the $\pi^-p \rightarrow \pi^0 n$ reaction at 500 MeV. The solid line represents the result of the Monte Carlo calculations.

Fig. 2.11. The measured π^0 spectrum for the $\pi^-p \rightarrow \pi^0 n$ reaction at 550 MeV. The solid line represents the result of the Monte-Carlo calculations.

The effective solid angle at each energy and angle setting was obtained from the following equation:

$$\Delta\Omega = \frac{Y e^{\mu \cdot x}}{n_t \cdot \pi \cdot LT \cdot \frac{d\sigma}{d\Omega}} \quad (2.20)$$

where Y is the π^0 yield in each of the angular bin, n_t is the number of target nuclei per cm^2 , and π is the number of pions to hit the target as determined by the activation method.

The $\frac{d\sigma}{d\Omega}$ term is the single charge exchange cross section based on the results of Arndt 1986 phase shift calculations. The cross sections used are given in table 2.3. The $e^{\mu \cdot x}$ term is the photon attenuation in the target.

The term LT is the electronic live time, it represents the fraction of time that pions could generate event triggers. This factor is dependent on the pion beam flux and the amount of the time that computer is busy recording/analyzing the information from the triggered events. During the experiment the live time fraction was around 85%.

The geometrical solid angle ($\Delta\Omega_g$) is the probability for the two photons resulting from a π^0 decay to hit the faces of the two spectrometer arms. This probability is calculated using the

Table 2.3 The angular distribution of the cross section for the $\pi^- p \rightarrow \pi^0 n$ reaction from Arndt (1986).

Angle (C.M.) degrees	$d\sigma/d\Omega$ (LAB) mb/sr			
	T_π (MeV)			
	300	425	500	550
0.00	10.40	9.21	9.56	10.37
0.50	10.39	9.22	9.58	10.39
1.00	10.39	9.17	9.58	10.39
1.50	10.39	9.21	9.57	10.38
2.00	10.38	9.21	9.57	10.37
2.50	10.38	9.21	9.56	10.36
3.00	10.37	9.20	9.55	10.35
3.50	10.36	9.19	9.54	10.34
4.00	10.35	9.18	9.53	10.32
4.50	10.34	9.17	9.51	10.30
5.00	10.33	9.16	9.50	10.28
5.50	10.31	9.15	9.48	10.26
6.00	10.30	9.13	9.46	10.24
6.50	10.28	9.17	9.44	10.21
7.00	10.26	9.10	9.42	10.18
7.50	10.24	9.08	9.39	10.15
8.00	10.22	9.06	9.34	10.12
8.50	10.20	9.04	9.34	10.08
9.00	10.17	9.02	9.31	10.04
9.50	10.15	8.99	9.28	10.00
10.0	10.12	8.97	9.24	9.96

Monte-Carlo code "PIANG" (Baer, 1986; Gilad, 1979). Figure 2.12 shows the solid angles as a function of energy (acceptance) for 300, 425 and 500 MeV measurements respectively. The input to the program includes beam and target properties, geometry, energy sharing parameter, estimated uncertainties in the measurement of gamma-ray conversion points, and energies. Just as in the actual data analysis, the events are divided into four angular bins and the program calculates scattering angles and energy spectra. The mean angles for the angular bins were taken to be the centroids of the calculated angular distributions.

Knowing $\Delta\Omega$ and $\Delta\Omega_g$, the total efficiency of the spectrometer ϵ is then calculated from equation 2.19. In this way, normalization of the cross sections depend on the yields measured in the CH_2 calibration runs and the cross sections for the $\pi^-p \rightarrow \pi^0 n$ process. The efficiencies for each energy and angle bin are tabulated in table 2.4. These data show an increase in slope with the energy and angle. The increase in ϵ with energy for the first angle bin is 3.7% at 425 MeV and 8.0% at 500 MeV, compared to the 300 MeV data. The reason for this increase is the increase in conversion efficiency of the Pb-glass elements with energy. These increases are 2.1% and 8.7%, respectively. The ϵ values resulting from the analysis of the first angular bin are plotted in Figure 2.13. The uncertainties include the fitting and statistical uncertainties, and are listed in table 2.5.

Table 2.4 Efficiency of the spectrometer determined from CH₂ runs (eq. 2.19) as a function of angle and energy.

Angle (deg)	Efficiency (%)	Energy (MeV)
2.9	.35±.017	300
4.8	.37±.019	-
6.9	.42±.024	-
2.5	.34±.016	425 ^a
4.6	.37±.019	-
6.7	.42±.024	-
	.30±.018	425 ^b
3.2	.36±.017	-
5.7	.39±.018	-
2.2	.38±.024	500
3.9	.40±.026	-
6.0	.42±.026	-

^a) Angular bins used for analysis of ⁷Li, ¹⁴C, ²⁷Al and ⁶⁰Ni data.

^b) Angular bins used for analysis of ⁹⁰Zr, ¹²⁰Sn, and ²⁰⁸Pb data.

Table 2.5 Uncertainties involved in solid angle measurements from CH₂ runs.

Energy (MeV)	$\bar{\theta}$ (Deg)	yield	uncertainties in %					total
			statistical	peak fitting	π flux	$\frac{d\sigma_a}{d\Omega}$	target thickness	
300	2.9	894	3.4	0.9	3.4	0	1	4.9
	4.8	785	3.6	0.9	-	-	-	5.0
	6.9	558	4.3	0.9	-	-	-	5.6
425 ^b	2.5	1024	3.1	2.2	3.2	-	-	4.8
	4.6	877	3.4	2.2	-	-	-	5.2
	6.7	567	4.2	2.2	-	-	-	5.8
425 ^c	1.7	334	5.4	3.9	3.2	-	-	6.0
	3.2	938	3.3	0.6	-	-	-	4.6
	5.7	1189	2.9	1.6	-	-	-	4.6
500	2.2	394	5.1	3.1	2.4	-	-	6.4
	3.9	361	5.3	3.0	-	-	-	6.5
	6.0	448	4.7	3.1	-	-	-	6.1

^aNo uncertainty assigned, (Gaille, et al., 1984) report 2% uncertainty for the $\pi^-p \rightarrow \pi^0 n$ cross section at 425 MeV.

^bfor targets ⁷Li to ⁶⁰Ni.

^cfor targets ⁹⁰Zr to ²⁰⁸Pb.

Fig. 2.12. The acceptance of the π^0 spectrometer vs. energy. The solid, chain-dash, and dotted lines represent the acceptance at the first, second, and third angle bins, respectively. The mean scattering angle for each bin is given in table 2.4.

Fig. 2.13. The spectrometer efficiency for the first angle bin as defined by equation 2.18, vs. pion kinetic energy.

The angular dependence of the ϵ was also studied. Figure 2.14 shows that there is a slight increase in ϵ as the angle increases. At 300 MeV, there is a 4.6% increase in ϵ in the second bin over the first bin, and an increase of 14.8% in the third bin over the second bin. At 425 MeV the same quantities are 8.6% and 14.5% respectively, and at 500 MeV, they are 4.5% and 6.3%. The reason for this angular dependence is not well understood.

To better understand the solid-angle normalizations, we have further analyzed the total π^0 detection efficiency of the spectrometer, in terms of the efficiencies of the individual components of the spectrometer. We define the total efficiency ϵ to depend on four factors, the conversion efficiency of the Pb-glass converters $\epsilon_{(\text{conv})}$, photon absorption in the spectrometer $\epsilon_{(\text{abs})}$, chamber efficiency $\epsilon_{(\text{mwpc})}$, and the track finding efficiency $\epsilon_{(\text{track})}$:

$$\epsilon = \epsilon_{(\text{conv})}^{j.k} \cdot \epsilon_{(\text{abs})} \cdot \epsilon_{(\text{mwpc})} \cdot \epsilon_{(\text{track})}. \quad (2.21)$$

This assumes 100% efficient plastic scintillators.

The conversion probability of each converter plane ϵ_c is defined in section II.E as the probability that at least one charged particle exits the back side of the converters. ϵ_c is a slowly varying function of photon energy, and is very well represented by the following empirical relation (Baer, et al., 1981).

Fig. 2.14. The spectrometer efficiency vs. angle for 300 MeV (solid rectangular), 425 MeV (open circle), and (500 MeV) (closed circle) pions.

$$\epsilon_c = 0.28 + (0.1) \log [0.01 E_\gamma (\text{MeV})] \quad (2.22)$$

For three converters in each arm, the total conversion efficiency of the spectrometer for both arms is

$$\epsilon^{j.k} = [1 - (1 - \epsilon_c)^3]^2. \quad (2.23)$$

At each energy, the events which pass all the cuts in each of the three converter planes in each arm of the spectrometer are tabulated in a 3x3 matrix. Each element of the matrix contains all the events in which a given pair of the converter planes passed all the cuts. If the number of conversions in the first converter plane is ϵ_c , the relative number of conversions in the second and third planes are $\epsilon_c(1 - \epsilon_c)$ and $\epsilon_c(1 - \epsilon_c)^2$, respectively. The overall efficiency of each converter plane ϵ_c is then determined by a fit to all 9 parameters of the matrix. In figure 2.15 the results of these measurements are compared with the result of a fit to the values obtained for the same parameter from equation 2.22. The bars reflect only the statistical uncertainties. The scintillator and the chamber efficiencies are not included in these calculations.

The increase in the value of ϵ_c based on the 2.22 equation is 3.2% for 425 MeV, and 4.5% for 500 MeV, over the 300 MeV value. The measured values are 2.1% and 8.7%, respectively. The 425 MeV data point does not follow the trend of the Monte-Carlo calculations very

Fig. 2.15. The deduced conversion efficiency of the glass elements vs. photon energy. The solid line represents the results obtained from equation 2.22.

well, however, it is within one and one half standard deviations of the curve.

The $\epsilon_{(abs)}$ term is the probability for both photons to survive the absorbers in the spectrometer. Each photon passes through one and one half inch of polyethylene that prevents many charged particles from entering into each arm, and a half inch of veto scintillator, for a total of 5.08 cm in front of the first converter.

$$\epsilon_{(abs)} = \exp^{-(2\mu \cdot x)} \quad (2.24)$$

The photon attenuation coefficient $\mu=0.015$ (cm^2/gm) was taken as an average over the energy range used. This resulted in the value of 0.87 for $\epsilon_{(abs)}$.

The $\epsilon_{(mwpc)}$ term for the m th plane is defined as the ratio of the number of times the m th converter and scintillator and MWPC plane fired to the number of times the m th converter and scintillator fired. The $\epsilon_{(track)}$ term is the efficiency of tracing back the track to a conversion point in one of the converters. It determines how efficient the algorithms used for reconstruction of vertices are.

Due to the difficulty of constructing a reliable algorithm for measuring the quantities $\epsilon_{(\text{mwpc})}$ and $\epsilon_{(\text{track})}$, these quantities could not be well determined, therefore the values of ϵ_{mwpc} obtained in the on-line analysis were only used as a relative monitor for the efficiency of the wire chambers. The product $\epsilon_{(\text{mwpc})} \cdot \epsilon_{(\text{track})}$ is, however, quite tightly constrained by the large body of CH_2 data and the accurate $\pi^-p \rightarrow \pi^0 n$ cross sections (Gaille, et al., 1984) existing in the measured energy region (Arndt, 1986). From equation 2.21, one obtains

$$\epsilon_{(\text{track})} \cdot \epsilon_{(\text{mwpc})} = \frac{\epsilon}{\epsilon^{j.k}_e e^{-2\mu \cdot x}} \quad (2.25)$$

Here ϵ is given by equation 2.19. Numerical values are given in table 2.4. $\epsilon^{j.k}_e$ is given by equation 2.23, with ϵ_c given by equation 2.22. The attenuation factor $\exp^{-2\mu \cdot x}$ is calculated as discussed above. Substituting these values into the right side of equation equation 2.25 the values shown in figure 2.16 are obtained. The average value of $\epsilon_{(\text{track})} \cdot \epsilon_{(\text{mwpc})}$ for all the angle bins at 300, 425, and 500 MeV is 0.95. This is consistent with a 99% efficiency for each chamber and nearly 100% track finding efficiency.

It should be emphasized that in the analysis we used the total spectrometer efficiencies that were obtained by normalizing the CH_2 data to the $\pi^-p \rightarrow \pi^0 n$ reaction cross sections (equation 2.19). The

Fig. 2.16. $\epsilon(\text{track}) \cdot \epsilon(\text{mwpc})$ vs. scattering angle for 300 MeV (open rectangular), 425 MeV (closed circle), and 500 MeV (closed rectangular).

decomposition of the ϵ as given by equation 2.21 was used only as a means of understanding of the overall efficiency.

III. DATA ANALYSIS AND RESULTS

In this chapter the data analysis procedure is described and the results of the analysis are presented. The background subtraction and the cross section determination are described next, followed by the zero degree cross section extrapolation procedure. The uncertainties in the cross sections are also discussed. The energy spectra and the angular distributions of the cross sections are presented in this chapter.

A. The "off-line" code

The detailed description of the "off-line" analysis code is described by Doron (1981) and Erell (1984). Here an outline and then an overview of the event analysis procedure are presented. The analyzer finds the conversion plane, calculates the MWPC efficiencies and checks the timing of the signals. It then calculates the energy of each gamma-ray. The coordinates of the conversion points in each arm are then determined and the kinetic energy of the π^0 and its scattering angle are calculated. Here we describe these steps further. Each arm has three conversion systems, and each system is made up of a converter, followed by

three MWPC's. The first and third chambers would give information on the X-direction, and the second one on the Y-direction. A "good" π^0 event has a signal in a converter, and hits in all three chambers following it and no hits in the upstream chambers. Any event that does not satisfy this condition is rejected as a π^0 event.

First, the analyzer finds the conversion planes, where a conversion plane is defined as the first plane that has hits in all of its three chambers. It then calculates the MWPC efficiencies. In obtaining the MWPC efficiencies, the information from the "good" π^0 events as well as the rejected ones is used. As it was mentioned in the last chapter, we used the efficiencies obtained with this method only as a relative monitor of the MWPC's performance. In this analysis the chamber efficiencies were treated as part of the solid angle normalization, obtained from the CH_2 data.

Timing was determined by the scintillators, and the time difference spectrum of the two arms had a FWHM of 2.5 nanoseconds. Gates of three standard deviations were set on the time difference spectrum.

The analyzer then calculates the energy of each gamma ray by summing up the energy deposited in the Pb-glass elements and the scintillators downstream of the conversion plane. The energy sharing parameter X is then calculated.

Next, the analyzer calculates the conversion coordinates. The conversion coordinates of each gamma-ray are obtained using the information from the MWPC's. In the system in which the conversion takes place, the events which cause more hits in the chambers than a certain limit "MXHITS", are rejected. The events with more than "MXHITS" prongs are more likely to have come from multiple scattering, therefore being more deflected from the original track. The hits on the first X and the third X' chambers are traced, and the line that makes the smallest angle with the line connecting the center of the target to the hits on the first chamber determines the X- coordinate of the conversion point. If this angle was larger than a maximum angle "MXSLOP", that event was also rejected. In our experiment "MXHITS" was taken to be 5, and "MXSLOP" was 12° . The Y-coordinate is the average of all the hits in the Y-chamber.

At this point the analyzer transforms these coordinates to the laboratory frame, and using equations 2.1 and 2.3 calculates the kinetic energy of the π^0 and the scattering angle.

B. Additional software cuts

The additional software cuts that were imposed included tests that minimized the shower leakage. A "fiducial area" was defined by requiring that the shower should penetrate at least 8 radiation lengths before exiting from the sides of the block array. This in

turn defines an effective area of the chamber smaller than the physical area. Only events within the fiducial area, were accepted. At the first converter this area was $17.06 \times 28.43 \text{ cm}^2$ and at the second and third converters these areas were $17.98 \times 29.96 \text{ cm}^2$ and $18.87 \times 31.45 \text{ cm}^2$, respectively. A further condition was applied to the events that reached the block elements. Gates were set around the center of shower propagation in the block $\vec{P}(x,y)$, where this center was defined as

$$\vec{P}(x,y) = \frac{\sum E(i)(X_i, Y_i)}{E(i)}. \quad (3.1)$$

From the events that reached the block elements, only events within this window were further analyzed. The width of these windows were ± 50 MWPC wire spaces in each direction. For the same events the distance between $\vec{P}(x,y)$ and the centroid of the electron shower, determined from the chamber information were histogrammed. Events which were outside three standard deviations from channel zero of each histogram were rejected.

The energy sharing parameter X (equation 2.2) was limited to 0.1. Both the Monte-Carlo calculations and the data analysis showed that the values of X greater than ± 0.1 would result in deterioration of the energy resolution. The smaller values of X reduce the number of events linearly. So the value of 0.1 was chosen as a compromise between sufficient statistics and good energy resolution.

The total energy spectrum had a low energy tail, so gates of three standard deviations were set on each side of the central energy value.

Random coincidences were obtained by setting out of coincidence cuts on the time difference spectrum, Then those events were subtracted from each of the π^0 spectra.

In the veto scintillators a long exponentially decaying shoulder was observed; this was attributed to slight light leakage in these scintillators, so appropriate cuts were set.

The spectrometer was set for the central angle of zero degrees and had an angular acceptance of about 15 degrees for the set-ups used in this work. The spectrometer parameters for different energies are listed in table 3.1.

Table 3.1 Geometry of the spectrometer.

Nominal T_π (MeV)	Target-Converter distance (cm)	Spectrometer opening angle (deg)	Acceptance peak (MeV)	Full Solid angle (msr)
300	202.0	36.15	299.99	0.67
425	202.0	28.15	420.05	0.82
500	202.0	24.56	499.64	0.93
550	202.0	24.56	499.64	0.93

The target-to-converter distance was fixed at 202 cm. This facilitated changes of configuration at different set-ups by requiring only a change in opening angle for changes in energies. The same set-up was used for the 500 and 550 MeV.

The events were stored in a two dimensional array, so that the data could be summed up in different number of bins and in bins of different sizes. The number of angle bins were increased so that one would have a better understanding of the angular distribution of forward angle cross sections. This requirement was balanced against the requirement to keep the statistical uncertainty in each bin to 10% or better.

Since the zero-degree cross section extrapolation method that we used depended more on the cross section of the first angle bin, higher statistical accuracy was required for that bin. Based on the above criteria, three angle bins were used in the analysis. The size of these bins were different for different beam energies. At 425 MeV, for targets heavier than ^{60}Ni , a smaller bin size for the first angle bin resulted in a better ratio of peak to valley for the IAS, subsequently the data from those targets were replayed with the new bin sizes. Table 3.2 gives the size of each angle bin.

C. Targets

Data were taken at 300, 425, 500, and 550 MeV on targets across the periodic table. Table 3.3 lists the targets that were used at each energy. The criteria for choosing these targets were: 1) data should exist on the target at lower energies; 2) the case under study should exhibit a strong IAS peak. These requirements insure

Table 3.2 The angular bin sizes chosen for the analysis.

Bin	E MeV	$\Delta\theta$ deg	$\bar{\theta}$ ^a deg	$\langle \bar{\theta} \rangle^{1/2}$ ^a deg
1	300	0.0-3.5	2.9	1.52
	425	0.0-3.5	2.5	1.13
	425	0.0-2.0	1.7	0.92
	500	0.0-3.0	2.2	0.99
2	300	3.5-5.5	4.8	1.55
	425	3.5-5.5	4.6	1.04
	425	2.0-4.0	3.2	1.05
	500	3.0-4.5	3.9	0.94
3	300	5.5-12.5	6.9	1.46
	425	5.5-12.5	6.7	1.05
	425	4.0-12.5	5.6	1.77
	500	4.5-12.5	6.0	1.24
4	300	0.0-12.5	4.5	2.17
	425	0.0-12.5	4.1	1.95
	425	0.0-12.5	4.1	1.95
	500	0.0-12.5	4.0	1.92

^a) From the Monte-Carlo calculations.

that the systematics of data on the same targets can be studied at different energies.

Since the proton contamination in the beam increased with increasing beam energy, degrader had to be inserted to decrease their number, which in turn reduced the pion flux. Therefore, at 550 MeV data were taken only on ^7Li , and ^{14}C , and at 500 MeV on ^7Li , ^{14}C , ^{27}Al and ^{60}Ni . At 425 MeV, data were taken on all the targets, and at 300 MeV on ^7Li , ^{27}Al , and ^{60}Ni . The ^7Li data at 300 MeV were taken as consistency check to compare with data taken at 295 MeV taken at the LEP channel (Sennhauser et al., 1983). The target thicknesses and their isotopic enrichment are listed in table 3.4.

Table 3.3 List of targets used at each energy.

Energy (MeV)	Target
300	^7Li , ^{27}Al , ^{60}Ni
425	^7Li , ^{14}C , ^{27}Al , ^{60}Ni , ^{90}Zr , ^{120}Sn , ^{208}Pb
500	^7Li , ^{14}C , ^{27}Al , ^{60}Ni
550	^7Li , ^{14}C

Table 3.4 Target information.

Target	gm/cm ²	nuclei/cm ²	form	enrichment
C	1.03±.001	5.16x10 ²²	solid	natural
CH ₂	1.17±.001	1.01x10 ²²	solid	natural
⁷ Li	1.00±.01	1.76x10 ²²	solid	99%
¹⁴ C	0.29±.03	1.33x10 ²²	powder	83%
²⁷ Al	0.84±.01	1.87x10 ²²	solid	natural
⁶⁰ Ni	2.16±.17	2.17x10 ²²	powder	99.8%
⁹⁰ Zr	1.80±.015	1.27x10 ²¹	solid	97.7%
¹²⁰ Sn	1.49±.08	7.48x10 ²¹	solid	98.7%
²⁰⁸ Pb	2.24±.05	6.48x10 ²¹	solid	99.1%

D. Differential cross sections.

The spectra were corrected for the energy acceptance of the spectrometer. Each energy spectrum was divided by its appropriate acceptance function. Then the differential cross sections were obtained using,

$$\frac{d\sigma}{d\Omega} = \frac{Y e^{\mu z}}{\pi \cdot n_t \cdot \varepsilon \cdot \Delta\Omega_g} \quad (3.2)$$

where

Y = counts in the IAS peak.

$e^{\mu z}$ = correction factor for photon attenuation in the target.

μ = photon absorption coefficient.

z = target thickness.

π = number of live pions (π incident on the target x live time).

n_t = number of target nuclei per cm^2 .

ε = spectrometer efficiency (equation 2.19).

$\Delta\Omega$ = geometrical solid angle (calculated by the Monte Carlo).

Alternatively, the differential cross sections were obtained by dividing the uncorrected energy spectra, channel by channel, by the product

$$\pi \cdot n_t \cdot \varepsilon \cdot e^{\mu z} \cdot \Delta\Omega_g \quad (3.3)$$

The resulting double differential cross section spectra $d^2\sigma/d\Omega dT$ are shown in figures 3.1(a) to 3.3(d). The IAS transition is the identifiable feature in each spectrum. The differential cross sections $d\sigma/d\Omega$ were determined by extracting area under the IAS peaks.

E. Extraction of the IAS peak.

Each spectrum was decomposed into three components, a constant background, the IAS peak, and the charge exchange continuum.

The source of constant background is the charge exchange in air before and after the target; it was extracted from the kinematically forbidden energy range above the IAS peak.

The position of the IAS peak was fixed to the energy values calculated from the kinematics of the target(π^+ , π^0) reactions.

The energy of the IAS peaks were measured relative to the position of the peak observed in the $\pi^-p \rightarrow \pi^0 n$ reaction at 300, 425, 500, and 550 MeV. The measured values agree with the expected values within ± 1.0 MeV. The expected values are calculated from the difference between the Q-values of πN and πA SCX reactions. The absolute π^0 energies were found to be 1 to 3 MeV higher than the the

values expected from the kinematics of $\pi^-p \rightarrow \pi^0 n$ reactions, indicating a small misalignment of the π^0 spectrometer.

The line shape for the IAS peak is taken from the Monte Carlo simulations that reproduce the $\pi^-p \rightarrow \pi^0 n$ line shapes, also obtained from the CH_2 data.

The continuum originates from the excited non-analog nuclear states, and quasi-free scattering. The average effect of all the measured non-analog-states were approximated by polynomial functions considered as background to the IAS peaks. These polynomials varied smoothly with angle. We used the computer code IASFIT (Cooper, et al., 1982), which uses a maximum likelihood method, and incorporates the above features to extract the areas of the IAS peaks. The solid lines in figures 3.1(a) to 3.4(b) represent the fits to the analog states.

As atomic mass (A) of the target increases, the IAS moves down in energy to the continuum and the peak extraction becomes more difficult. Figure 3.1(a) shows that in the case of ^7Li , the peak to background ratio is large enough so that there is not much uncertainty in extracting the peak's area. However, in the case of ^{208}Pb , figure 3.2(g), the IAS has moved to the continuum and the extraction of the peak's area is more ambiguous. Here we used another feature of the IAS transition, namely its being a very forward peaked transition to verify the results obtained by the

Fig. 3.1(a). Double differential cross sections for the ${}^7\text{Li}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 300$ MeV. The angles are given in the c.m. system.

Fig. 3.1(b). Double differential cross sections for the $^{27}\text{Al}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 300$ MeV. The angles are given in the c.m. system.

Fig. 3.1(c). Double differential cross sections for the $^{60}\text{Ni}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 300$ MeV. The angles are given in the c.m. system.

Fig. 3.2(a). Double differential cross sections for the ${}^7\text{Li}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.2(b). Double differential cross sections for the $^{14}\text{C}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.2(c). Double differential cross sections for the $^{27}\text{Al}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.2(d). Double differential cross sections for the $^{60}\text{Ni}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.2(e). Double differential cross sections for the $^{90}\text{Zr}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.2(f). Double differential cross sections for the $^{120}\text{Sn}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.2(g). Double differential cross sections for the $^{208}\text{Pb}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 425$ MeV. The angles are given in the c.m. system.

Fig. 3.3(a). Double differential cross sections for the $7\text{Li}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 500$ MeV. The angles are given in the c.m. system.

Fig. 3.3(b). Double differential cross sections for the $^{14}\text{C}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 500$ MeV. The angles are given in the c.m. system.

Fig. 3.3(c). Double differential cross sections for the $^{27}\text{Al}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 500$ MeV. The angles are given in the c.m. system.

Fig. 3.3(d). Double differential cross sections for the $^{60}\text{Ni}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 500$ MeV. The angles are given in the c.m. system.

Fig. 3.4(a). Spectra for the ${}^7\text{Li}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 550$. The angles are given in the c.m. system.

Fig. 3.4(b). Spectra for the $^{14}\text{C}(\pi^+, \pi^0)$ reaction at $T_{\pi^+} = 550$ MeV. The angles are given in the c.m. system.

previous methods. Since the IAS transition is sharply forward peaked, its strength decreases with increasing scattering angle. However, the shape of continuum for heavy targets seems to remain relatively constant. Therefore, subtraction of the third angle bin spectrum from the first angle bin (1-3 method) would result in a spectrum which mostly contains the IAS transition. Figure 3.5 shows the results of the overlay of the first and third angle bins for ^{120}Sn , and ^{208}Pb . The mean scattering angle for these spectra is 1.3, and 5.8 degrees, respectively. Figure 3.6 shows the result of the 1-3 subtraction for these two targets. The IAS transition at the expected energy is clearly visible. The results of this method (1-3 method) were in agreement with the methods described earlier.

Another problem in the extraction of the area under the IAS peaks is the limited energy resolution of the π^0 spectrometer. The contribution from the non IAS transitions, mainly from the $\Delta J=1, 2$ transitions, might not be resolved from the IAS transitions. However, $\Delta J=1$ transitions follow an $l=1$ angular distribution which is not as forward peaked as a $l=0$ transition. The $\Delta J=2$ transitions follow an $l=2$ angular distribution which is very suppressed relative to a $l=0$ transition. The spin flip components of these transitions have a $\sin(\theta)$ dependence and are very inhibited at forward angles. Therefore, the IAS is the dominant transition contributing to the area under each peak.

Fig. 3.5. Superposition of the third angle bin (5.8°) on the first angle bin (1.3°) for ^{120}Sn (top), and ^{208}Pb (bottom) at 425 MeV.

Fig. 3.6. The results of subtraction of the third angle bin (5.8°) from the first angle bin (1.3°) for ^{120}Sn (top), and ^{208}Pb (bottom) at 425 MeV.

In summary, the IAS peaks were identified and their areas extracted based on the following criteria:

- 1.-The close correspondence between the measured and expected position of the IAS peak.
- 2.-The observed sharply forward peaked angular distribution shapes.
3. The observed near constancy of the non-analog background over the measured angular range.
- 4.-The well constrained line shapes obtained from the CH₂ data.

F. The angular distributions and the effective scattering angle.

The measured π^0 yields represent a weighted average of the differential cross sections over angular acceptance. Thus, a mean cross section is determined for each angular bin according to the following relation,

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle_i = \int_{\theta_1}^{\theta_2} \frac{d\sigma}{d\Omega} A_i(\theta) d\theta \quad (3.4)$$

where θ_1 , and θ_2 are the lower and upper limits of each bin; i is the bin number, and $A_i(\theta)$ is the angular acceptance of that bin which is calculated by the Monte-Carlo program. The " effective angle " is defined as an angle for which,

$$\left\langle \frac{d\sigma}{d\Omega} \right\rangle_i = \frac{d\sigma}{d\Omega}(\theta_{\text{eff}}). \quad (3.5)$$

Assuming that the cross section for the IAS transition follows a Bessel function of the form $J_0^2(\chi)$ (Johnson, 1980), we have,

$$\frac{d\sigma}{d\Omega} = N J_0^2(\chi) \quad (3.6)$$

where $\chi = qR$, $q = 2 k \sin(\theta/2)$ is the momentum transfer, k is the momentum of the incoming pion and R is the effective radius of interaction. The numerical values of R were obtained from an analysis of the existing π^+ -nucleus elastic data (Appendix B). Substituting 3.5, and 3.6 into 3.4, we have

$$\langle \frac{d\sigma}{d\Omega} \rangle_i = N \eta_i \quad (3.7)$$

where

$$\eta_i = \int_{\theta_1}^{\theta_2} J_0^2(\chi) A_i(\theta) d\theta. \quad (3.8)$$

From definitions 3.5, 3.7, and 3.8, θ_{eff} is an angle which satisfies the following:

$$J_0^2(\chi_{\theta_{\text{eff}}}) = \eta_i. \quad (3.9)$$

This equation was then solved for each angle bin and θ_{eff} for that bin was obtained. Tables 3.5 through 3.8 give the cross sections in the laboratory system for the energies and the targets used. The errors represent the total errors and will be discussed in the next

section. The angular distribution of the IAS cross sections versus θ_{eff} in the laboratory system are plotted in figures 3.7 to 3.9(b). The solid curves represent Bessel functions of $J_0^2(X)$ form normalized to the cross sections of the first angle bins. In figure 3.8(c), the angular distributions for ${}^7\text{Li}$ and ${}^{208}\text{Pb}$ at 425 MeV are compared; the ${}^7\text{Li}$ and ${}^{60}\text{Ni}$ angular distributions at 500 MeV are compared in figure 3.9(b). These targets are the lightest and heaviest targets used at each energy, respectively. The angular distributions show that the scattering minima moves in with increasing mass of the target (A). This clearly demonstrates the diffractive nature of the scattering. These results will be discussed in detail in chapter IV.

Table 3.5 The measured cross sections at 300 MeV in the laboratory system.

Target	θ_{eff} (deg)	$\Delta\theta$ (deg)	Cross-Section (mb/sr)
${}^7\text{Li}$	2.31	0.0-3.5	$2.71 \pm .24$
	4.561	3.5-5.5	$2.39 \pm .22$
	7.32	5.5-12.5	$2.03 \pm .39$
${}^{27}\text{Al}$	2.30	0.0-3.5	$0.56 \pm .07$
	4.55	3.5-5.5	$0.42 \pm .06$
	7.27	5.5-12.5	$0.30 \pm .05$
${}^{60}\text{Ni}$	2.29	0.0-3.5	$1.03 \pm .15$
	4.55	3.5-5.5	$0.90 \pm .12$
	7.20	5.5-12.5	$0.61 \pm .10$

Table 3.6 The measured cross sections at 425 MeV in the laboratory system.

Target	θ_{eff} (deg)	$\Delta\theta$ (deg)	Cross-Section (mb/sr)
${}^7\text{Li}$	2.29	0.0-3.5	$3.57 \pm .32$
	4.53	3.5-5.5	$2.57 \pm .28$
	6.92	5.5-12.5	$2.26 \pm .34$
${}^{14}\text{C}$	2.29	0.0-3.5	$3.60 \pm .58$
	4.52	3.5-5.5	$3.01 \pm .49$
	6.91	5.5-12.5	$1.66 \pm .31$
${}^{27}\text{Al}$	2.28	0.0-3.5	$0.85 \pm .11$
	4.53	3.5-5.5	$0.59 \pm .09$
	6.89	5.5-12.5	$0.39 \pm .08$
${}^{60}\text{Ni}$	2.27	0.0-3.5	$1.83 \pm .29$
	4.51	3.5-5.5	$1.27 \pm .22$
	6.84	5.5-12.5	$0.63 \pm .15$
${}^{90}\text{Zr}$	1.29	0.0-2.0	$2.69 \pm .38$
	3.06	2.0-4.0	$1.65 \pm .30$
	5.79	4.0-12.5	$1.12 \pm .21$
${}^{120}\text{Sn}$	1.29	0.0-2.0	$4.24 \pm .84$
	3.06	2.0-4.0	$2.63 \pm .62$
	5.74	4.0-12.5	$1.34 \pm .40$
${}^{208}\text{Pb}$	1.29	0.0-2.0	4.9 ± 1.1
	3.05	2.0-4.0	2.0 ± 1.0

Table 3.7 The measured cross sections at 500 MeV in the laboratory system.

Target	θ_{eff} (deg)	$\Delta\theta$ (deg)	Cross-Section (mb/sr)
${}^7\text{Li}$	1.97	0.0-3.0	$3.76 \pm .48$
	3.78	3.0-4.5	$3.40 \pm .47$
	6.21	4.5-12.5	$2.73 \pm .41$
${}^{14}\text{C}$	1.97	0.0-3.0	$4.19 \pm .71$
	3.78	3.5-4.5	$3.38 \pm .63$
	6.19	4.5-12.5	$2.74 \pm .55$
${}^{27}\text{Al}$	1.97	0.0-3.0	$1.01 \pm .16$
	3.78	3.5-4.5	$0.86 \pm .15$
	6.16	4.5-12.5	$0.54 \pm .11$
${}^{60}\text{Ni}$	1.96	0.0-3.0	$2.08 \pm .36$
	3.77	3.0-4.5	$1.59 \pm .32$
	6.08	4.5-12.5	$0.80 \pm .19$

Table 3.8 The measured cross sections at 550 MeV in the laboratory system.

Target	θ_{eff} (deg)	$\Delta\theta$ (deg)	Cross Section(mb/Sr) (mb/sr)
${}^7\text{Li}$	1.98	0.0-3.0	3.63 ± 0.75
	3.78	3.0-4.5	3.15 ± 0.54
	6.18	4.5-12.5	2.31 ± 0.40
${}^{14}\text{C}$	1.97	0.0-3.0	5.90 ± 1.46

Fig. 3.7. Angular distribution of the IAS cross sections in the laboratory system at 300 MeV. Solid lines represent $J_0^2(qR)$ functions normalized to the first angle bin cross sections.

Fig. 3.8(a). Angular distribution of the IAS cross sections in the laboratory system at 425 MeV. Solid lines represent $J_0^2(qR)$ functions normalized to the first angle bin cross sections.

Fig. 3.8(b). Angular distribution of the IAS cross sections in the laboratory system at 425 MeV. Solid lines represent $J_0^2(qR)$ functions normalized to the first angle bin cross sections.

Fig. 3.8(c). Angular distribution of the IAS cross sections in the laboratory system for ${}^7\text{Li}$ and ${}^{208}\text{Pb}$ at 425 MeV, the lightest and heaviest targets used at this energy.

Fig. 3.9(a). Angular distribution of the IAS cross sections in the laboratory system at 500 MeV. Solid lines represent $J_0^2(qR)$ functions normalized to the first angle bin cross sections.

Fig. 3.9(b). Angular distribution of the IAS cross sections in the laboratory system for ${}^7\text{Li}$ and ${}^{60}\text{Ni}$ at 500 MeV, the lightest and heaviest targets used at this energy.

G. 0° cross section extraction

Since $J_0^2(\chi)$ is one at zero degrees, N in equation 3.7 is the zero degree cross section and is obtained from the following relation.

$$\frac{d\sigma}{d\Omega}(0^\circ) = N = \frac{\langle \frac{d\sigma}{d\Omega} \rangle_i}{\int_{\theta_1}^{\theta_2} J_0^2(\chi) A_i(\theta) d\theta} \quad (3.10)$$

Since the first angle bin had the best statistics and was closest to zero degree, only the measured first angle bin cross sections were used for the zero degree cross sections extrapolation. The correction factor to the first angle bins, η_1 varied from 2% for ${}^7\text{Li}$ to 9% for ${}^{60}\text{Ni}$ at 300 MeV. At 425 MeV it varied from 3% for ${}^7\text{Li}$ to 14% for ${}^{60}\text{Ni}$. It was 5%, 6% and 10% for ${}^{90}\text{Zr}$, ${}^{120}\text{Sn}$ and ${}^{208}\text{Pb}$, respectively. The correction at 500 MeV was 3%, 4%, 6% and 12% for ${}^7\text{Li}$, ${}^{14}\text{C}$, ${}^{27}\text{Al}$ and ${}^{60}\text{Ni}$ respectively. Table 3.9 gives the extrapolated 0° IAS cross sections.

H. Error analysis

The uncertainties involved in the IAS cross section measurements are listed in detail in tables 3.10 to 3.14. The largest contribution to the total errors comes from the

uncertainties in fitting the IAS peak, with this uncertainty increasing for the second and third angle bins. It varied from 5% for ${}^7\text{Li}$ at 300 MeV to 20% for ${}^{208}\text{Pb}$ at 425 MeV for the first angle bin.

Statistical errors ranged from 3% to 7%. Uncertainty in the target thickness measurement varied from less than 1% to 10%. Thicknesses of ${}^7\text{Li}$, ${}^{27}\text{Al}$ and ${}^{90}\text{Zr}$ were determined to better than 1%, however we used powder ${}^{14}\text{C}$ and ${}^{60}\text{Ni}$ targets which were measured to 10% accuracy. The ${}^{120}\text{Sn}$ and ${}^{208}\text{Pb}$ targets were measured to 5% and 2% respectively. The beam normalization error is related to the uncertainty in ${}^{12}\text{C}(\pi^+, \pi^0 \text{N}) {}^{11}\text{C}$ cross sections and the error in the measurement of proton to pion ratio. This uncertainty was 4% at 300 MeV and 425 MeV and 8% at 500 MeV. The average uncertainty in the effective solid angle measurement was 5%.

The uncertainty in the zero degree cross section extrapolation due to the uncertainty in the measurement of the effective radius of interaction and effective angle were also studied. The change in the zero degree cross section for a 10% change in R was less than 3%, and a 15% uncertainty in the measurement of θ_{eff} resulted in less than 4% change. These errors have been added to the results in table 3.9.

Table 3.9 Parameters involved in extraction of the 0^0 c.m. cross sections.

Energy MeV	Target	first-bin lab cross section	η	$\frac{d\Omega_{\text{c.m.}}}{d\Omega_{\text{lab}}}$	0^0 c.m. cross section
300	^7Li	$2.71 \pm .24$	.982	.88	$2.43 \pm .23$
	^{27}Al	$0.56 \pm .07$	.955	.97	$0.57 \pm .07$
	^{60}Ni	$1.03 \pm .15$	.925	.98	$1.10 \pm .16$
425	^7Li	$3.57 \pm .32$	.975	.85	$3.112 \pm .28$
	^{14}C	$3.60 \pm .58$	.961	.92	$3.45 \pm .56$
	^{27}Al	$.85 \pm .11$	.941	.96	$0.86 \pm .11$
	^{60}Ni	$1.83 \pm .29$	.902	.98	$1.99 \pm .32$
	^{90}Zr	$2.69 \pm .38$	.957	.99	$2.77 \pm .40$
	^{120}Sn	$4.24 \pm .84$	.949	.99	$4.42 \pm .89$
	^{208}Pb	4.9 ± 1.1	.927	.99	5.25 ± 1.21
500	^7Li	$3.76 \pm .48$	.976	.83	$3.20 \pm .41$
	^{14}C	$4.19 \pm .71$	.962	.91	$3.97 \pm .68$
	^{27}Al	$1.01 \pm .16$	.942	.95	$1.02 \pm .17$
	^{60}Ni	$2.08 \pm .36$	.903	.98	$2.25 \pm .40$
550	^7Li	$3.63 \pm .57$	.976	.83	$3.07 \pm .63$
	^{14}C	$5.90 \pm .9$	.962	.90	5.55 ± 1.37

Table 3.10 Uncertainties involved in the IAS cross section measurements at 300 MeV.

Target	θ_{eff}	yield	uncertainties in %					total
			statistical	peak fitting	π flux	solid angle	target thickness	
^7Li	2.31	682	4.3	5	3.8	4.9	1	9.0
	4.56	543	4.8	6	-	5.0	-	9.9
	7.32	296	6.8	7	-	5.6	-	11.9
^{27}Al	2.30	351	7.4	7	3.9	4.9	1	11.9
	4.55	241	9.1	7	-	5.0	-	13.1
	7.27	124	12.9	7	-	5.6	-	16.2
^{60}Ni	2.29	373	6.7	8	3.9	4.9	8	14.5
	4.55	290	7.6	10	-	5.0	-	16.2
	7.20	114	12	10	-	5.6	-	18.8

Table 3.11 Uncertainties involved in the IAS cross section measurements at 425 MeV for targets ${}^7\text{Li}$ to ${}^{60}\text{Ni}$.

Target	θ_{eff}	yield	uncertainties in %					total
			statistical	peak fitting	π flux	solid angle	target thickness	
${}^7\text{Li}$	2.29	940	3.6	6	3.0	4.8	1	9.5
	4.53	641	4.5	8	-	5.2	-	11.0
	6.92	326	6.4	12	-	5.6	-	15.0
${}^{14}\text{C}$	2.29	663	5.1	10	2.7	4.8	10	16.0
	4.52	497	5.2	10	-	4.8	10	16.0
	6.91	178	9.0	12	-	5.6	-	18.9
${}^{27}\text{Al}$	2.28	551	7.4	8	3.9	4.8	1	12.6
	4.53	226	8.9	10	-	5.2	-	14.8
	6.89	98	15.3	12	-	5.6	-	20.6
${}^{60}\text{Ni}$	2.27	556	6.7	10	3.9	4.8	8	15.7
	4.51	369	6.8	12	-	5.5	-	17.2
	6.81	123	15.5	15	-	5.6	-	24.0

Table 3.12 Uncertainties involved in the IAS cross section measurements at 425 MeV for targets ^{90}Zr to ^{208}Pb .

Target	θ_{eff}	yield	uncertainties in %					total
			statistical	peak fitting	π flux	solid angle	target thickness	
^{90}Zr	1.29	197	9.1	8	3.6	6.0	1	14.0
	3.06	324	12.4	12	-	4.0	-	18.2
	5.79	315	7.9	16	-	4.0	-	18.8
^{120}Sn	1.29	434	9.7	15	3.6	6.0	5	19.9
	3.06	810	9.3	20	-	4.0	-	23.4
	5.74	6.05	14.2	25	-	4.0	-	30.0
^{208}Pb	1.29	463	7.1	20	3.7	6.0	2	22.4
	3.05	189	10.0	30	-	4.0	-	

Table 3.13 Uncertainties involved in the IAS cross section measurements at 500 MeV.

Target	θ_{eff}	yield	uncertainties in %					total
			statistical	peak fitting	π flux	solid angle	target thickness	
^7Li	1.97	717	4.2	7	7.7	6.4	1	12.9
	3.78	586	4.6	8	-	6.5	-	14.8
	6.21	582	4.8	10	-	6.1	-	14.8
^{14}C	1.97	571	4.9	8	7.7	6.4	10	17.0
	3.78	398	6.8	10	-	6.5	-	18.6
	6.19	385	7.5	12	-	6.1	-	19.9
^{27}Al	1.97	296	8	10	7.7	6.4	1	16.2
	3.78	216	8.7	12	-	6.5	-	17.9
	6.16	190	10.5	15	-	6.1	-	20.8
^{60}Ni	1.96	511	6.3	10	7.7	6.4	8	17.4
	3.77	380	10.3	12	-	6.5	-	20.4
	6.08	238	11.3	16	-	6.1	-	23.3

Table 3.14 Uncertainties involved in the IAS cross section measurements at 550 MeV.

Target	θ_{eff}	yield	uncertainties in %					total
			statistical	peak fitting	π flux	solid angle	target thickness	
^7Li	1.98	149	8.2	15	9.5	6.4	1	20.6
^{14}C	1.97	110	9.5	17	9.5	6.4	10	24.7

IV. INTERPRETATION OF THE DATA

In this chapter the observed systematics of the data will be discussed. The energy, mass, and the angular dependence of the 0^0 IAS cross sections will be presented and compared with the results of Glauber model calculations.

A. Systematics of the data

Figures 4.1(a), to 4.1(d) show the 0^0 IAS cross sections for seven targets. The excitation functions for all the target nuclei show a sharp rise with increasing energy between 300 and 425 MeV. These results are in contrast with the previous measurements at lower energies which show that the 0^0 IAS cross sections either remain constant or follow the energy dependence of the free π -nucleon single charge exchange cross sections and typically decrease as the pion energy is raised above the $\Delta(1232)$, (Sennhauser et al., 1983; Irom et al., 1983). The observed rise in the 0^0 excitation functions is also in contrast with the SCX cross sections for the free πN process at zero degrees. The cross sections for this process are also shown in the same figures.

Fig. 4.1(a). Excitation functions for the 0° IAS cross sections in the c.m. system for ${}^7\text{Li}$, and ${}^{27}\text{Al}$. The solid line represents the cross section for the free SCX reaction in the c.m. system. The data below 300 MeV are taken from Sennhauser et al., 1983.

Fig. 4.1(b). Excitation function for the 0° IAS cross sections in the c.m. system for ^{14}C . The solid line represents the cross section for the free SCX reaction in the c.m. system. The data below 300 MeV are taken from Sennhauser *et al.*, 1983.

Fig. 4.1(c). Excitation function for the 0° IAS cross sections in the c.m. system for ^{60}Ni , and ^{208}Pb . The solid line represents the cross section for the free SCX reaction in the c.m. system. The data below 300 MeV are taken from Sennhauser *et al.*, 1983.

Fig. 4.1(d). Excitation function for the 0^0 IAS cross sections in the c.m. system for ^{90}Zr , and ^{120}Sn . The solid line represents the cross section for the free SCX reaction in the c.m. system. The data below 300 MeV are taken from Sennhauser *et al.*, 1983.

To quantify the mass-dependence of the 0^0 cross sections, following Baer et al., (1982) and Sennhauser et al., (1983), the function $g(N-Z)A^{-\alpha}$ was fit to the data at each energy, this function has given a good fit to the data at lower energies. MINUIT, a package of CERN computer programs, was used to fit the above function to the data using a χ^2 minimization routine. The values of $[g \pm \Delta g, \alpha \pm \Delta \alpha]$ which resulted in the best fit to the data are listed in table 4.1. The values of the same parameters at 165 MeV from the work of Sennhauser, et al., (1983) are also listed for comparison. Figure 4.2 shows the A-dependence of the experimental 0^0 cross sections. The solid lines represent fits to the data with the above parameters. The value of α at 165 MeV is 1.36 ± 0.04 which is in agreement with the predictions of scattering of pions from a black disk, $\alpha = 1.33$. In the plane wave limit (scattering of pions from a transparent sphere) α is zero. The value of α at 500 MeV (0.82 ± 0.01) is smaller than the resonance value but is still very different from the limit of complete transparency.

The fitted values of g are shown in figure 4.3, where the solid curve represents the free SCX cross sections normalized to the data at 300 MeV. The open circles represent the values of g that would have been expected if the π -nucleus interaction at higher energies followed the free πN SCX cross sections (Alster et al., 1983). The solid rectangular are the values of g obtained in the current experiment. The solid circles represent the previously reported values of g . The values of g determined from the result of the

Table 4.1 The values of g and α obtained from the fits of the expression $g(N - Z)A^{-\alpha}$ to the 0° cross sections obtained in the experiment and the Glauber model.

T_π (MeV)	g Experiment	α	g Glauber Model	α
165 ^a	49.9±9.0	1.36±.04	4.0±.6	1.29±.04
300 ^b	18.0±1.7	1.09±.03	11.4±1.5	0.93±.03
425	19.6±3.0	0.94±.04	13.1±1.6	0.84±.03
500	16.0±4.5	0.82±.10	13.6±1.6	0.83±.03

current experiment do not agree with the extrapolation from the 300 MeV based on the free πN shape.

The measured angular distributions are shown in figures 3.21, and 3.23. One sees that the angular distributions become more forward peaked as the kinetic energy of the incoming pion is increased. These figures also show that at a given beam energy, the angular distributions become more forward peaked with increasing mass of the target.

In the black disk limit, the pion-nucleus scattering is diffractive in nature, and the angular distributions follow the $J_0^2(qR)$ functional form (Johnson, 1980). The increase in the energy

Fig. 4.2. A dependence of the 0° IAS cross sections divided by $(N - Z)$. The solid lines show the $g A^{-\alpha}$ fit to the data. The published cross sections from previous works at 165, and 300 MeV are also included.

Fig. 4.3. Energy dependence of the parameter g from the $g(N - Z)A^{-\alpha}$ fit to the 0^0 IAS cross sections. Solid circles represent the previously published work. Open circles are the expected values of g , and the solid rectangles are determined from the current work. The solid line represent the free SCX cross sections normalized to the data at 300 MeV.

of the incoming pion beam and the size of the target would cause the $J_0^2(qR)$ to be more forward peaked. This behavior is consistent with the observed trends of the measured angular distributions, thereby justifying the use of a $J_0^2(qR)$ function for the extraction of the zero degree cross section.

B. Glauber model

To gain some understanding of the trends of the data, we carried out an analysis within the framework of the Glauber model, (Glauber, 1970, 1986; Bleszynski, 1986). These calculations are described in detail in appendix C. The cross section for the SCX to the IAS states in this model is,

$$\frac{d\sigma}{d\Omega} \pi A = \frac{d\sigma}{d\Omega} \pi N * (N - Z) * \left| \int e^{i\vec{q} \cdot \vec{b}} S(b) T'(b) d^2b \right|^2 \quad (4.1)$$

where

$$S(b) = \exp\left[i\frac{2\pi}{k} \bar{f}(0) A T(b) - \frac{c}{2} A^2 T_2\right] \quad (4.2)$$

and the cross section for the total reaction cross section is,

$$\sigma_R = 2\pi \int (1 - \exp[-\bar{\sigma} A T(b) - c A^2 T_2]) b db. \quad (4.3)$$

The quantity $q = 2k \sin(\theta/2)$ is the momentum transfer in the

laboratory system, where k is the wave number of the incoming pions; θ is the scattering angle, and b is the impact parameter. $\bar{f}(0)$ is the forward angle pion-nucleon scattering amplitude, and is taken from the 1986 Arndt phase shift analysis.

$$\bar{f}(0) = \frac{Z}{A} f_{\pi^+p} + \frac{N}{A} f_{\pi^-p}. \quad (4.4)$$

$T(b) = \int \bar{\rho}(b,z) dz$ is the thickness function and represents the amount of nuclear matter that the pions traverse through the nucleus. $\bar{\rho}(b,z)$ is the isospin averaged nuclear matter density

$$\bar{\rho} = \frac{Z}{A} \rho_p + \frac{N}{A} \rho_n. \quad (4.5)$$

ρ_p and ρ_n are the proton and neutron densities normalized such that $\int \rho dV = 1$. $T'(b) = \int \rho'(b,z) dz$ is the IAS transition density where ρ' is the valence neutron density normalized also such that $\int \rho' dV = 1$. To incorporate the effects of the higher order processes, the term $\frac{c}{2} A^2 T_2$ was added to phenomenologically simulate the effects of higher order processes such as pion true absorption. The free parameter c is determined from the data. We define $T_2 = \int \rho^2(b,z) dz$, since pion absorption is commonly regarded as a two-nucleon process. The term $\bar{\sigma}$ represents the isospin averaged free πN cross section.

$$\bar{\sigma} = \frac{Z}{A} \sigma_{\pi^+p} + \frac{N}{A} \sigma_{\pi^-p}. \quad (4.6)$$

The optical theorem, $\bar{\sigma} = \frac{4\pi}{k} \text{Im} \bar{f}(0)$, can be used to put the $S(b)$ term (equation 4.2) in the following form

$$S(b) = \exp\left[\left(-\frac{\bar{\sigma}}{2} + i \frac{2\pi}{k} \text{Re} \bar{f}(0)\right) A T(b) - \frac{c}{2} A^2 T_2(b)\right]. \quad (4.7)$$

Using the relation $\vec{q} \cdot \vec{b} = qb \cos \phi$, where ϕ is the angle between the impact parameter $\vec{b}$ and the momentum transfer $\vec{q}$, and the relation,

$$\int e^{iqb \cos \phi} d\phi = 2\pi J_0(qb) \quad (4.8)$$

the cross section for the SCX to IAS can be written as

$$\frac{d\sigma}{d\Omega} \text{SCX} = \frac{d\sigma}{d\Omega} \pi N * (N - Z) * \left| \int J_0(qb) S(b) T'(b) 2\pi b db \right|^2. \quad (4.9)$$

It is this form of the model that we used for the computation of the cross sections.

C. Models used for nuclear matter and transition densities.

The nuclear matter densities $\rho(b, z)$ were taken from Hartree-Fock calculations using the Skyrme III force (Negele and Vautherin, 1972; Beiner et al., 1975; Siciliano, 1986). Figures

4.4(a) to 4.4(g) show the proton, neutron and valence neutron densities for all the target nuclei.

The transition densities $\rho'(b,z)$ were assumed to be proportional to the valence neutron densities and were also taken from the (H-F) calculations.

We also performed calculations using densities taken from electron-nucleus scattering data (de Jager et al., 1974). Functional forms used for the densities were,

$$\rho(r) = \rho_0 (1 + \alpha(r/a)^2) \exp(-(r/a)^2) \quad (4.10)$$

$$\rho(r) = \rho_0 / (1 + \exp(r-c)/z) \quad (4.11)$$

$$\rho(r) = \rho_0 (1 + wr^2/c^2) / (1 + \exp((r^2 - c^2)/z^2)). \quad (4.12)$$

These functions represent the Harmonic-Oscillator model (H0), the two parameter Fermi model (2pF), and the three-parameter Gaussian model (3pG). Table 4.2 shows the parameters used for the assumed densities.

Calculations using (H-F) densities result in much larger cross sections than calculations using the densities obtained from electron scattering data. However, at 165 MeV, where we assumed

Fig. 4.4(a). Neutron (dashed line), proton (chaindash line), and the valence neutron (solid) densities for ${}^7\text{Li}$ obtained from the H-F calculations.

Fig. 4.4(b). Neutron (dashed line), proton (chaindash line), and the valence neutron (solid) densities for ^{14}C obtained from the H-F calculations.

Fig. 4.4(c). Neutron (dashed line), proton (chaindash line), and the valance neutron (solid) densities for ^{27}Al obtained from the H-F calculations.

Fig. 4.4(d). Neutron (dashed line), proton (chaindash line), and the valance neutron (solid) densities for ^{60}Ni obtained from the H-F calculations.

Fig. 4.4(e). Neutron (dashed line), proton (chaindash line), and the valance neutron (solid) densities for ^{90}Zr obtained from the H-F calculations.

Fig. 4.4(f). Neutron (dashed line), proton (chaindash line), and the valance neutron (solid) densities for ^{120}Sn obtained from the H-F calculations.

Fig. 4.4(g). Neutron (dashed line), proton (chaindash line), and the valance neutron (solid) densities for ^{208}Pb obtained from the H-F calculations.

Table 4.2 Parameters for the transition densities.

Target	Density	c or a fm	z or α fm	w fm
${}^7\text{Li}$	H0	1.77	0.327	
${}^{14}\text{C}$	H0	1.73	1.38	
${}^{27}\text{Al}$	2pF	3.07	0.519	
${}^{60}\text{Ni}$	2pF	4.15	0.578	
${}^{90}\text{Zr}$	3pG	4.434	2.528	0.350
${}^{120}\text{Sn}$	2pF	5.315	0.576	
${}^{208}\text{Pb}$	2pF	6.624	0.549	

$T' = T(b,z)$, the two types of densities resulted in cross sections which differed by less than 6.5% for ${}^{14}\text{C}$ and by less than 4% for ${}^{120}\text{Sn}$.

D. Model results and comparison with the data.

At zero degrees the cross section for the IAS transition can be written as

$$\frac{d\sigma}{d\Omega} \text{SCX}(0) = \frac{d\sigma}{d\Omega} n_N(0) * (N - Z) * \left| \int S(b) T'(b) 2\pi b db \right|^2 (4.13)$$

This equation was numerically evaluated using densities from the H-F calculations.

The value of c in the above equation was determined by fitting the results of the above equation to the experimental zero-degree SCX cross sections at 425 MeV. These cross sections constituted the most complete set of data. The parameters c which resulted in the best fits for all the target nuclei are plotted in figure 4.5. Since the reported nuclear matter and transition densities for ${}^7\text{Li}$ and ${}^{14}\text{C}$ are not very reliable, only targets with $A \geq 27$ were used in this fit. The solid line represents an A -independent fit to the values of these parameters. This fit resulted in the value of 29.02 for the parameter c .

As a further check on this value, we calculated the total reaction cross sections using equation (4.3), with $c=29.02$, and compared them to the experimental values at 315 MeV from Ashery *et al.*, (1981), which are the highest energy data available for these cross sections. Figure 4.6 shows the experimental data, and the solid line represents a fit to the calculated values. The experimental data and the results of the theoretical calculations are well represented by $\sigma_R = a A^b$, with $a = 68.8 \pm 4$ mb and $b = 0.612 \pm 0.035$ for the data, and $a = 49.4$ mb and $b = 0.677$ for the model.

Fig. 4.5. Parameter c resulting from the Glauber model fits to the 0° IAS cross sections at 425 MeV. The solid line represent an A -independent fit to the data.

Fig. 4.6. Total reaction cross sections (solid line) based on Glauber model calculations using $c=29.02$. Data are from Ashery et al., 1981.

The results of the calculations using $c = 29.02$, and the 0^0 SCX cross sections extracted from the experimental data are shown in figures 4.7(a), to 4.7(g). The calculation performed with $c = 0$ overestimate the 425- and 500- MeV 0^0 cross sections by about a factor of three, as shown for ^{27}Al and ^{60}Ni . This fact suggests that the higher order processes are still significant even at these higher energies.

The calculated cross sections for the target nuclei were obtained from equation 4.1 with α decreasing with increasing energy. They were then fitted to the function $g (N-Z) A^{-\alpha}$ assuming 10% uncertainty for each one of the theoretical data points. The values of g and α obtained from these fits are listed in table 4.1. The model gives a reasonably good description of the energy dependence of α .

At resonance energy region, following Johnson (1930), we used the nuclear matter density $T(b)$ for the transition density T' . If we use the valence neutron transition density T' instead, the theoretical cross sections would scatter around $\alpha = 1.28$ line with a reduced χ^2 of 5.8. Since at the resonance energies pions interact at the very outer surface of the nucleus, the theoretical calculations are very sensitive to the differences in the nuclear matter densities (T), and the valence densities (T'). These differences are not well determined within our model calculations at 165 MeV.

Fig. 4.7(a). Excitation function for the 0° IAS cross sections in the c.m. system for ${}^7\text{Li}$. The solid line represents the results of the Glauber model calculations with $c = 29.02$.

Fig. 4.7(b). Excitation function for the 0^+ IAS cross sections in the c.m. system for ^{14}C . The solid line represents the results of the Glauber model calculations with $c = 29.02$.

Fig. 4.7(c). Excitation function for the 0° IAS cross sections in the c.m. system for ^{27}Al . The solid line represents the results of the Glauber model calculations with $c = 29.02$. The dotted line represent the same calculations with $c = 0$.

Fig. 4.7(d). Excitation function for the 0° IAS cross sections in the c.m. system for ^{60}Ni . The solid line represents the results of the Glauber model calculations with $c = 29.02$. The dotted line represent the same calculations with $c = 0$.

Fig. 4.7(e). Excitation function for the 0° IAS cross sections in the c.m. system for ^{90}Zr . The solid line represents the results of the Glauber model calculations with $c = 29.02$.

Fig. 4.7(f). Excitation function for the 0° IAS cross sections in the c.m. system for ^{120}Sn . The solid line represents the results of the Glauber model calculations with $c = 29.02$.

Fig. 4.7(g). Excitation function for the 0° IAS cross sections in the c.m. system for ^{208}Pb . The solid line represents the results of the Glauber model calculations with $c = 29.02$.

The angular dependences of the IAS cross sections were obtained using equation 4.1. Figure 4.8 shows some representative calculated angular distributions.

E. Discussion

Since the strength of the free SCX process decreases at the higher energies, this might lead to the expectation that the π -nucleus SCX cross sections at these energies would also decrease. On the other hand, due to the decrease in total πN cross sections at this energy range, greater pion penetration into the nuclear volume is expected which could lead to rising cross sections with increasing pion energy. The observed increase in the 0° cross sections suggests that the latter effect dominates the π -nucleus interaction at this energy range.

The Glauber model calculations describe the observed trend of the data (figures 4.4(a), to 4.4(g)). We therefore examine the cause of the rise in the excitation functions in terms of the parameters in the equation 4.1.

The zero-degree cross sections for the free SCX process $\frac{d\sigma}{d\Omega} \pi N (0)$ decrease above the $\Delta(1232)$ resonance. The SCX cross section at resonance is 7.6 mb, while at 425 MeV the value of this parameter has decreased to 4.2 mb. The decrease in this value is compensated for by a sharp rise in the value of N_{eff} , where

Fig. 4.8 The angular distribution of the IAS cross sections at 425 MeV for ${}^7\text{Li}$, ${}^{60}\text{Ni}$, and ${}^{120}\text{Sn}$. The solid lines represent the result of the Glauber model calculations normalized to the 0° cross sections.

$$N_{\text{eff}} = \left| \int S(b) T'(b) 2\pi b db \right|^2 \quad (4.14)$$

is the effective number of contributing valence nucleons. The parameter $S(b)$ (equation 4.2) is a complex variable with a magnitude of

$$|S(b)| = \exp\left[-\frac{\bar{\sigma}}{2} A T(b) - \frac{c}{2} A^2 T_2\right]. \quad (4.15)$$

At resonance, $\bar{\sigma}$ is large (~130 mb), while at 425 MeV the value of this parameter drops to ~40 mb. This fact causes the sharp rise in N_{eff} . Since $S(b)$ represents the attenuation of the incoming and outgoing waves, the reduction in the value of $\bar{\sigma}$ with increasing energy results in lesser attenuation at higher energies. Consequently, the pions penetrate further into the nuclear volume and the rise in the 0° cross sections can be described as a consequence of the increased volume scattering of pions at these energies.

Figure 4.9(a) shows the quantities $T'(b)$, and $|S(b)|$ for ^{14}C at 165 and 425 MeV. One sees that for a fixed impact parameter b , the pions are less attenuated at 425 MeV than at 165 MeV (larger value of $|S(b)|$ at 425 MeV), therefore, the pions penetrate deeper into the nuclear interior. The same point is illustrated in figure 4.9(b) where the product $|S(b)| * T'(b)$ is shown for ^{14}C . At 165 MeV, the interaction is seen to be surface peaked, whereas at 425

Fig. 4.9(a). $T'(b)$, $|S(b)|$ and their product $|S(b)| * T'(b)$ at 165 MeV for ^{14}C .

Fig. 4.9(b). $T'(b)$, $|S(b)|$ and their product $|S(b)| * T'(b)$ at 425 MeV for ^{14}C .

MeV pions sample a larger fraction of the interior of the nuclear volume. This results in a sharp rise in the IAS cross sections. The same quantities are plotted for ^{120}Sn in figures 4.10 and 4.11.

The increased volume scattering at higher energies is also reflected in the systematics of α . At resonance α is 1.36 representing the scattering of pions from a black disk. In the plane wave limit α is zero, representing scattering of pions from a transparent disk. The reduction in the value of α with increasing energy of the incoming pions suggests that the nucleus has become more transparent. However, even at 500 MeV α is 0.80, still quite different from the plane wave limit. This fact indicates that at the measured energy range pions scatter from a fuzzy black disk, (Baer, 1986).

In order to reproduce the experimentally determined IAS cross sections we included a term simulating pion true absorption; this term may represent a variety of higher order processes as well. This suggests that within the framework of the Glauber model the single step processes are not sufficient to describe the IAS cross sections and higher order processes still play a significant role.

Existing optical model calculations (Johnson and Siciliano, 1983a, 1983b; Auerbach et al., 1984; Siciliano et al., 1986) include only s- and p- partial waves and can not be used to compare with the results of the current experiment. However theoretical efforts

Fig. 4.10(a). $T'(b)$, $|S(b)|$ and their product $|S(b)| * T'(b)$ at 165 MeV for ^{120}Sn .

Fig. 4.10(b). $T'(b)$, $|S(b)|$ and their product $|S(b)| * T'(b)$ at 425 MeV for ^{120}Sn .

Fig. 4.11. $T'(b)$, $|S(b)|$ at 165 MeV (dashed line) and 425 MeV for ^{120}Sn .

which include d- and f- partial waves (Ernst, 1986), and more complete Glauber model calculations which explicitly include second and third order terms (Strottman, 1986) are underway to try to explain the current results.

V. SUMMARY AND CONCLUSIONS

Prior to the current study, the reported 0^0 cross sections for the pion SCX reaction leading to the isobaric-analog-states of the target nuclei were limited in energy to the region well below and around the $\Delta(1232)$ resonance. These work had suggested that the 0^0 IAS cross sections follow the trend of the free πN SCX process. Since the πN SCX cross sections decrease with energy above the $(3,3)$ resonance, it was expected that the cross sections for the pion-nucleus SCX reactions would also decrease with energy. However the πN total cross section also decreases at this energy region, resulting in greater pion penetration into the interior of the nucleus which could result in larger cross sections. The empirical resolution of the interplay between these two competing effects was one goal of this dissertation. It was also expected that between 300 and 500 MeV, higher order terms such as nuclear correlation terms (ρ^2) would play a lesser role, therefore the first order theoretical approaches would be more reliable. Another goal of this dissertation was to investigate if single step processes would give a good description of the the data.

We used the π^0 spectrometer at P^3 channel at LAMPF to measure the forward angle cross sections for the (π^+, π^0) reaction at 300, 425 and 500 MeV on variety of targets. The 0^0 cross sections were

extracted from these data. The reported cross sections are the first exclusive pion-nucleus data at these energies.

The 0^0 excitation functions for all the target nuclei show an increase with energy between 300 to 425 MeV. The A-dependence of these cross sections at each energy is well represented by $g(N - Z)A^{-\alpha}$ function, where α , following the pattern observed at lower energies, decreases with increasing energy. The forward-angle IAS cross sections follow the $J_0^2(qR)$ function and decrease rapidly in magnitude with increasing angle.

We studied the above systematics in terms of a Glauber model calculation. This model attributes the rise in the 0^0 IAS cross sections to the increased pion penetration in the interior of the nucleus. This model also described the energy dependence of the parameter α , again indicating increased volume scattering at higher energies. In order to reproduce the absolute value of the 0^0 cross sections, the model must include a term representing higher order processes such as true absorption. This term is consistent with the measured total reaction cross sections. Calculations which do not include this term overestimate the high energy-single charge exchange cross sections by about a factor of three. This indicates that even at the measured energy region the higher order processes can not be neglected. A complete optical-model calculation which includes the higher partial waves may be necessary to study in detail and quantify the effects of all the higher order processes.

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APPENDICES

Appendix A: The linear accelerator at LAMPF.

LAMPF is a half mile long, high intensity, linear proton accelerator. By using high-purity hydrogen gas in special ion sources, it can produce three different beams: H^+ (Proton), H^- (Hydrogen atom with an extra electron), and P^- (Polarized H^- beam). The facility utilizes a three stage linear accelerator (linac) that can simultaneously accelerate the H^+ beam and either the H^- or P^- beam. The first stage is an injector which accelerates the beams from their respective sources to 750 keV. At this stage the ion source is pulsed at a 120 Hz rate, while each beam pulse is 570 microsecond long, giving a duty factor of 6.84% for the accelerator (i.e beam is on only for 570 μ s every 8.33 ms). This is the beam macrostructure. Higher duty factors are possible by lengthening the macropulse.

In the second stage, an Alvarez-type drift tube accelerator operating at 201.25 MHz accepts ions from two of the three injectors, accelerates the ions to 100 MeV, and sends them to the main accelerator. This gives a microstructure to the beam; each of the 570 μ s long macro pulses consists of "micro buckets", every $1/201.25 \text{ MHz} = 4.97 \text{ ns}$ or approximately every 5 ns. This is the 201 MHz pulsed time structure of the beam. In the third and final section of the accelerator, the particles are accelerated up to 800 MeV, using the side-coupled cavity waveguide linac developed at Los

Alamos. The final energy of the proton beam is stepwise variable from 100 MeV to 800 MeV.

The H^+ and the H^- (or P^-) beams enter the beam switchyard area shown in figure 2.4, where they are magnetically separated. In the switchyard the beams are directed to different beam lines. Line X uses either H^- or P^- for the High Resolution Spectrometer (HRS) in area C and the Nucleon Physics Laboratory (NPL) in area B. Line D uses part of the H^+ beam to produce a high intensity short pulse source of neutrons for the newly developed Los Alamos Neutron Scattering Center (LANSCE). Line A gets most of the H^+ beam. There are three production targets in line A, a thin target used for nuclear chemistry studies, the A-1 and A-2 targets. The A-1 target consists of 3 cm of carbon for producing pion beams for the Low Energy Pion channel (LEP), Energetic Pion Channel (EPICS), and the test channel. The A-2 target, a 4 cm carbon target provides the pion beam for the Pion Particle Physics (P^3) channel and a Muon beam for the Stopped Muon Channel (SMC). A fairly detailed description of LAMPF is given in the LAMPF User Handbook (1980).

Appendix B: The effective radius of interaction:

The values of r_0 were calculated from the existing π^+ -nucleus elastic data (Geesaman et al., 1981; Boyer et al., 1984; Marlow et al., 1984). The angular distribution for the pion-nucleus elastic scattering interaction is represented by,

$$\frac{d\sigma}{d\Omega} = \left| iR \frac{J_1(qR)}{qR} \right|^2 \quad (B1)$$

where $q = 2 k \sin(\theta/2)$ is the momentum transfer; k is the momentum and θ is the scattering angle in the center of mass system. The first minimum of J_1 occurs at 3.83, therefore,

$$qR = 2 k R \sin(\theta_{\min}/2) = 3.83. \quad (B2)$$

From the above equation, and the relation, $R = r_0 A^{1/3}$, r_0 was calculated. Table B.1 shows the tabulated values of r_0 and the related quantities. The values of r_0 used in our analysis were obtained from the elastic π^+ scattering on ^{40}Ca at 292 MeV and 672 MeV, and scaled with energy to the 300 MeV, 425 MeV and 500 MeV.

At 300 MeV the calculated value of r_0 was 1.25 fm, at 425 MeV, 1.17 fm, and at 500 MeV, 1.12 fm, respectively.

Table B.1 Summary of elastic scattering data.

T_{π^+}	Target	$\Theta_{\min}^{\pi^+}$	k	R	r_0
(MeV)		(deg)	MeV/c	fm	fm
162	^{12}C	53.0	260.6	3.24	1.42
-	^{28}Si	40.0	264.5	4.17	1.37
	^{58}Ni	32.0	266.1	5.14	1.33
-	^{208}Pb	22.0	267.2	7.40	1.25
178	^{89}Y	26.2	284.7	5.85	1.31
180	^{92}Mo	25.8	286.7	5.89	1.31
-	^{40}Ca	33.0	285.3	4.66	1.36
291	^9Be	48.0	388.1	2.39	1.15
-	^{28}Si	30.0	401.1	3.63	1.20
-	^{58}Ni	23.0	404.4	4.68	1.21
-	^{208}Pb	15.0	406.7	7.11	1.20
292	^{40}Ca	25.0	404.0	4.31	1.26
-	^{54}Fe	24.3	405.2	4.42	1.17
672	^{40}Ca	15.1	782.9	3.57	1.04

Appendix C: The Glauber model calculations

In this appendix the theoretical calculations based on Glauber's theory of multiple scattering will be discussed. These calculations are within the framework of eikonal approximation, which will be discussed first.

The Schrodinger equation can be written as an integral equation, (the Lippman-Schwinger equation)

$$\psi^{\pm}(r) = \phi(r) + \int G^{\pm}(r, r') U(r') \psi^{\pm}(r') d^3r' \quad (C1)$$

where

$$G^{\pm} = \frac{\exp\{\pm i k |\bar{r} - \bar{r}'|\}}{4\pi |\bar{r} - \bar{r}'|} = \int \frac{d^3\bar{k}''}{(2\pi)^3} \frac{\exp[i\bar{k}'' \cdot (\bar{r} - \bar{r}')] }{\bar{k}_0^2 - \bar{k}''^2 \pm i\eta} \quad (C2)$$

In eikonal approximation the trajectory is assumed not to be disturbed by the interaction, $\bar{k} \approx \bar{k}'$, so

$$k^2 - k''^2 = 2\bar{k} \cdot (\bar{k} - \bar{k}''). \quad (C3)$$

This approximation results in

$$G^{\pm}(r, r') = \frac{-i}{2k} \delta^2(b' - b) e^{ik_0(z' - z)} \Theta(z' - z) \text{ for } \pm i\eta \quad (C4)$$

$$= \frac{i}{2k} \delta^2(b' - b) e^{ik_0(z' - z)} \theta(z - z') \text{ for } -i\eta \quad (C5)$$

Substituting the above equation into the equation C1 results in,

$$\psi^\pm = e^{ik_0 r} + \frac{i}{2k_0} \int e^{ik_0(z - z')} \theta(\pm(z - z')) U(b, z - z') \psi^\pm(b, z - z') dz' \quad (C6)$$

continuing the calculations for ψ^+ results in,

$$\psi^+ = e^{+ik_0 \cdot r} - \frac{i}{2k} \int e^{ik_0(z - z')} U(b, z - z') \psi^+(b, z - z') dz' \quad (C7)$$

differentiating with respect to z

$$\begin{aligned} \frac{d}{dz} \psi^+(b, z) &= ik_0 e^{ik_0 \cdot \bar{r}} - \frac{i}{2k_0} U(b, z) \psi^+(b, z) \\ &+ \frac{1}{2} \int e^{ik_0(z - z')} U(b, z') \psi^+(b, z') dz' \end{aligned} \quad (C8)$$

which can be put in the following form,

$$\frac{d}{dz} \psi^+ = ik_0 - \frac{i}{2k_0} U(b, z) \psi^+ \quad (C9)$$

Trying a solution of the form,

$$\psi^+(b, z) = F(b) e^{\frac{-i}{2k_0} \int U(b, z') dz'} e^{ik_0 z} \quad (C10)$$

and substituting it into the equation C9 results in the eikonalized wave function of the form

$$\psi^\pm = e^{i\bar{k}_0 \cdot \bar{r}} e^{\frac{-i}{2k_0} \int U(b, z') dz'} \quad (C11)$$

Now, the eikonalized scattering amplitude,

$$f_{el} = -\frac{1}{4\pi} \int d^3r e^{-i\bar{k}_f \cdot \bar{r}'} U(r') \psi_{k_i}(r') dr', \quad (C12)$$

can be calculated to be

$$f_{el} = -\frac{1}{4\pi} \int d^3r e^{-i(\bar{k}_f - \bar{k}_i) \cdot \bar{r}} U(r') e^{\frac{-i}{2k} \int U(b, z') dz'}. \quad (C13)$$

Assuming cylindrical coordinates such that $\bar{r} = \bar{b} + z\bar{k}$ and defining $\bar{q} = \bar{k}_f - \bar{k}_i$, we can make the following approximation

$$\bar{q} \cdot \bar{r} = \bar{q} \cdot (\bar{b} + z\bar{k}) = \bar{q} \cdot \bar{b} + kz(1 - \cos(\theta)) \approx \bar{q} \cdot \bar{b} \quad (C14)$$

where the terms of order $\theta^2 kz \leq \theta^2 ka$ have been neglected.

Substituting the above approximation in equation C13 results in

$$f_{el} = \frac{k}{2\pi i} \int d^2b \, e^{i\vec{q} \cdot \vec{b}} \left\{ e^{\frac{-i}{2k_0} \int dz' U(b, z')} - 1 \right\} \quad (C15)$$

or

$$f_{el} = \frac{k}{2\pi i} \int d^2b \, e^{-i\vec{q} \cdot \vec{b}} [e^{i\chi(k, b)} - 1] \quad (C16)$$

where the quantity $\chi(k, b) = \frac{1}{2k} \int U(b, z) dz$ is referred to as the eikonal phase shift function. The quantity $\Gamma(b)$ the "profile function" is defined as

$$\Gamma(b) = 1 - e^{i\chi(k, b)} \quad (C17)$$

The equation for the scattering amplitude can therefore be written as

$$f_{el} = \frac{ik}{2\pi} \int d^2b \, e^{i\vec{q} \cdot \vec{b}} \Gamma(b) \quad (C18)$$

i.e the scattering amplitude is the Fourier transform of the $\Gamma(b)$.

Glauber has applied the eikonal approximation to the many body problems, and in particular to the high energy hadron-nucleus scattering. The amplitude for a direct collision making a direct transition from an initial state $|i\rangle$ to a final state $|f\rangle$ is

$$f_m = \frac{k_1}{2\pi i} \int d^2b \, e^{i\vec{q} \cdot \vec{b}} \langle f^* | [\exp\{i\chi_{\text{tot}}(b_1, \dots, b_A)\} - 1] | i \rangle \quad (\text{C19})$$

where

$$\chi_{\text{tot}}(b_1, \dots, b_A) = \sum_j \chi_j(b - b_j) \quad (\text{C20})$$

defining

$$\Gamma_{\text{tot}}(b_1, \dots, b_A) = 1 - \exp[i\chi_{\text{tot}}(b_1, \dots, b_A)] \quad (\text{C21})$$

The multiple scattering amplitude can be written as

$$f_m = \frac{ik_1}{2\pi} \int d^2b \, e^{i\vec{q} \cdot \vec{b}} \langle f^* | \Gamma(b_1, \dots, b_A) | i \rangle \quad (\text{C22})$$

The Γ_{tot} can be written as

$$\Gamma_{\text{tot}} = 1 - \prod_j [1 - \Gamma_j(b - b_j)] \quad (\text{C23})$$

$$\Gamma_{\text{tot}} = \sum_j \Gamma_j - \sum_{j \neq 1} \Gamma_j \Gamma_1 + \dots (-1)^{A-1} \prod_j \Gamma_j \quad (\text{C24})$$

The quantity Γ_j can be obtained in terms of the two body scattering amplitude, since

$$\Gamma_j = \frac{1}{2\pi i k} \int d^2q \, e^{-i\vec{q} \cdot \vec{b}} f_j(q) \quad (\text{C25})$$

Therefore the terms linear in Γ_j describe the single scattering

terms, and the next term describes the double scattering terms, and so on. Assuming that

$$|i\rangle = \psi_1(r_1) \psi_2(r_2) \dots \psi_A(r_A) \quad (C26)$$

is the antisymmetrized wave function of the nucleus in its initial state, and that in the final state, only one of the nucleons is in a different configuration, $\psi_1(r_1)$ changes to $\psi_1^*(r_1)$, (for example in the SCX reactions) the final wave function can be written as eq

$$|f^*\rangle = \psi_1^*(r_1) \psi_2(r_2) \dots \psi_A(r_A) \quad (C27)$$

Assuming that the SCX can take place in any of the $(N - Z)$ excess neutrons, the multiple scattering relation C22 can be expanded in terms of Γ_j 's. The first term results in

$$\langle f^* | \sum_j \Gamma_j | i \rangle = (N - Z) \langle \psi_1' | \Gamma_1 | \psi_1 \rangle \quad (C28)$$

$$\langle f^* | \sum_j \Gamma_j | i \rangle = \sqrt{(N - Z)} \langle \Gamma_1 \rangle^* \quad (C29)$$

where $\langle \Gamma_1 \rangle^*$ represents the inelastic scattering term. The second term results in

$$\langle f^* | \sum_{j \neq 1} \Gamma_j \Gamma_1 | i \rangle = (N - Z) (A - 1) \langle \psi_1^* | \Gamma_1 | \psi_1 \rangle \langle \psi_2 | \Gamma_2 | \psi_2 \rangle \quad (C30)$$

$$\langle f^* | \sum_j \Gamma_j \Gamma_1 | i \rangle = \sqrt{(N - Z)} (A - 1) \langle \Gamma_1 \rangle^* \langle \Gamma_2 \rangle \quad (C31)$$

The $\langle \Gamma_1 \rangle^*$ term represent the elastic scattering term. Summing up all the terms,

$$\langle f^* | \sum_j \Gamma_j | i \rangle = \sqrt{(N - Z)} \langle \Gamma_1 \rangle^* \quad (C32)$$

$$\langle f^* | \sum_{j \neq 1} \Gamma_j \Gamma_1 | i \rangle = \sqrt{(N - Z)} (A-1) \langle \Gamma_1 \rangle^* \langle \Gamma_2 \rangle \quad (C33)$$

$$\langle f^* | \sum_{j \neq 1 \neq m} \Gamma_j \Gamma_1 \Gamma_m | 1 \rangle = \sqrt{(N - Z)} \frac{(A-1)(A-2)}{2!} \langle \Gamma_1 \rangle^* \langle \Gamma_2 \rangle^2 \quad (C34)$$

which results in

$$\begin{aligned} \langle f^* | 1 - \prod_j (1 - \Gamma_j) | i \rangle &= \sqrt{(N - Z)} \langle \Gamma \rangle^* - \sqrt{(N - Z)} (A-1) \langle \Gamma \rangle^* \langle \Gamma \rangle \\ &+ [\sqrt{(N - Z)} \frac{(A-1)(A-2)}{2!}] \langle \Gamma \rangle^* \langle \Gamma \rangle^2 - \dots \end{aligned} \quad (C35)$$

$$\begin{aligned} \langle f^* | 1 - \prod_j (1 - \Gamma_j) | i \rangle &= \sqrt{(N - Z)} \langle \Gamma \rangle^{*-1} (1 - \langle \Gamma \rangle)^{(A-1)} \\ &= \sqrt{(N - Z)} \langle \Gamma \rangle^* e^{-(A-1)\langle \Gamma \rangle} \end{aligned} \quad (C36)$$

For $A \gg 1$ this results in

$$\langle f^* | 1 - \prod_j (1 - \Gamma_j) | i \rangle \approx \sqrt{(N - Z)} \langle \Gamma \rangle^* e^{-A/\langle \Gamma \rangle} \quad (C37)$$

At this point the further assumption that

$$f(q) \approx f(0) = \frac{k\sigma}{4\pi} (i + \alpha) \quad (C38)$$

is made which represents the excellent approximation that the size of nucleons is much less than the size of the nucleus. The Γ can now be calculated

$$\Gamma = \frac{1}{2\pi i k} \int d^2 q \, e^{-i\vec{q} \cdot \vec{b}} f_{el}(q) \quad (C39)$$

$$\Gamma = \frac{\sigma}{2} (1 - i\alpha) \delta^2(b - s) \quad (C40)$$

and the term which represents the average of the elastic scattering amplitude is then

$$\langle \Gamma \rangle = \int d^3 r \, \rho(r) \Gamma(b-s) \quad (C41)$$

$$\langle \Gamma \rangle = \frac{\sigma}{2} (1 - i\alpha) T(b) \quad (C42)$$

where

$$T(b) = \int dz \, \rho(b, z) \quad (C43)$$

To calculate the $\langle \Gamma \rangle^*$ term a similar approximation is made

$$f^{SCX}(q) = f^{SCX}(0) \quad (C44)$$

which results in

$$\Gamma^{SCX}(b) = \frac{1}{2\pi i k} f^{SCX}(0) (2\pi)^2 \delta^2(b) \quad (C45)$$

The multiple scattering term due to the inelastic interaction then becomes

$$\langle \Gamma \rangle^* = \int d^3r \rho_{tr}(r) \Gamma^{SCX} \quad (C46)$$

$$\langle \Gamma \rangle^* = \frac{2\pi}{ik} f^{SCX}(0) T'(b) \quad (C47)$$

where

$$T'(b) = \int dz \rho_{tr}(r) \quad (C48)$$

Substituting the above equations in C24 we obtain,

$$f^{SCX} \approx \sqrt{(N-Z)} f^{SCX}(0) \int d^2b e^{i\vec{q} \cdot \vec{b}} T'(b) e^{-A\sigma(1-i\alpha)T(b)/2} \quad (C49)$$

From this amplitude, the cross section for the SCX is calculated

$$\frac{d\sigma_{SCX}}{d\Omega} = \frac{d\sigma_{\pi N}}{d\Omega}(0) (N-Z) \left| \int e^{(i\vec{q} \cdot \vec{b} + \frac{2\pi i}{k} f(0)AT(b))} T'(b) d^2b \right|^2 \quad (C50)$$

The quantity in the squared bracket is referred to as the effective number of nucleons N_{eff} . The term $\exp[(2\pi i/k)f(0)T(b)]$ represents the distortion of the incoming and outgoing pion waves traversing the nuclear medium. It represents to what extent the pion flux has disappeared from the SCX channel.

In order to account for the second order processes a term $\frac{c}{2} A^2 T_2$ was added to the imaginary part of the optical potential. This term simulates the removal of pion flux from the incoming pion beam due to processes such as pion true absorption. Equation C50 can be written as

$$\frac{d\sigma_{SCX}}{d\Omega} = \frac{d\sigma_{\pi N}}{d\Omega}(0) (N - Z)^* \left| \int e^{i\vec{q} \cdot \vec{b}} + \frac{2\pi i}{k} \bar{f}(0) AT(b) - \frac{c}{2} A^2 T_2 T'(b) d^2b \right|^2. \quad (C51)$$

The parameter c and the term T_2 are described in section IV.2.

The eikonal phase shift χ is,

$$\chi = i \left(\frac{2\pi}{k} \bar{f}(0) T(b) \right) - \frac{c}{2} A^2 T \quad (C52)$$

The cross section for the total reaction cross section is obtained from the following relation

$$\sigma_R = \sigma_{tot} - \sigma_{el} \quad (C53)$$

Using the optical theorem, and the relation C18 the following relation is obtained

$$\sigma_{tot} = \frac{4\pi}{k} \text{Im } f_{el}(0) = 2 \int d^2b \text{Re} \Gamma(b) \quad (C54)$$

σ_{el} is obtained from equation C18,

$$\sigma_{el} = \int d\Omega |f_{el}(q)|^2 \quad (C55)$$

Using relations $d\Omega = d\phi \sin(\theta) d\theta$, and $q = 2k\sin(\theta/2)$, equation C53 can be put in the following form,

$$\sigma_{el} = \int d^2b |\Gamma(b)|. \quad (C56)$$

Substituting equations C55, and C55 in equation C53 results in the following relation for σ_R

$$\sigma_R = \int d^2b |\Gamma(b)|^2. \quad (C57)$$

which can be written as

$$\sigma_R = 2\pi \int b db [1 - e^{-\bar{\sigma}AT(b) - cA^2T_2}]. \quad (C58)$$

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