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BEAM-BEAM INTERACTIONS IN P-P COLLIDING ACCELERATORS

George Parzen

January 1983

ACCELERATOR DEPARTMENT

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Contents

1. Empirical Model
2. ν -Shifts and Stop-Band Widths Summary
 - 2.1 The Perfect Machine
 - 2.2 Closed Orbit Errors and Random β -Function Errors
 - 2.3 Comparison of CBA and ISR Stop Bands
3. Effects of a Spectrometer on the Beam-Beam Interaction
4. Effect of β_x , β_y and α on the Beam-Beam Interaction
5. Numerical Computation of Stop-Bands

Abstract

Non-linear stop-bands, due to the beam-beam interaction in a P-P colliding accelerator, are computed and used to estimate the strength of the beam-beam interaction. Results for the stop-band widths due to various causes are summarized. These results are used to compare the effects of beam-beam non-linear resonances in the ISR with those in CBA (Colliding Beam Accelerator at BNL), to estimate the effect of a spectrometer magnet placed at one of the beam crossing points, and to compare the beam-beam interactions for colliding beam accelerators with different crossing angles and different β -functions at the crossing points.

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I. Empirical Model

One model for beam growth due to the beam-beam interaction in P-P colliding accelerators is that it is due to the presence of non-linear forces generated by the fields produced by the beam plus some randomizing effect like noise, or a tune modulation. According to this model, to limit beam-beam effects, one should try to limit the size of the non-linear forces and the sources of noise or tune modulation.

This model can also be used to compare the severity of beam-beam effects in two situations by comparing the size of the non-linear forces. In this paper, this approach will be used to study three problems:

1. to compare the effects of beam-beam non-linear resonances in the ISR at CERN with those in the CBA (Colliding Beam Accelerator at BNL).
2. to estimate the strength of a spectrometer magnet that may be placed at one of the beam crossing points, without appreciably increasing the beam-beam effects.
3. to compare the beam-beam interaction for colliding beam accelerators with different crossing-angles and different β_x and β_y at the crossing points.

To make these comparisons, one needs a measure of the non-linear forces. The measure used in this paper is the stop-band widths, $\Delta\nu_{mn}$, for the resonance $m\nu_x + n\nu_y = \text{integer}$, using formulae of the type

$$\Delta v_{mn} = C(n,m,\epsilon_x,\epsilon_y) \int ds \beta_x^{m/2} \beta_y^{n/2} \frac{\partial^{m+n} V}{\partial x^m \partial y^n}, \quad (1.1)$$

where $C(n,m,\epsilon_x,\epsilon_y)$ is some function of n,m,ϵ_x and ϵ_y ; ϵ_x, ϵ_y are the beam emittances, and V is the potential function for the fields due to one of the beams.

It is possible to suggest other measures of the non-linear forces than that given by Eq. (1.1). One can suggest including the contributions of higher order terms to the stop-band width of a resonance, or one can suggest a different measure based on a more detailed model of the beam-beam interaction. Considering the inaccuracies present in the model, it is not clear that these other measures of the non-linear forces are better, and the measure given by Eq. (1.1) was used in this paper.

Sometimes the beam-beam V -shift is used as a measure of the beam-beam interaction. It has been found that in some cases the linear V -shift can be misleading, and the stop-band widths of the higher order resonances need to be examined.

II. V-Shifts and Stop-Band Widths

This section will review some of the properties of the V-shifts and the non-linear stop bands introduced by the beam-beam interaction.

For two proton beams crossing at a small angle, the force on one beam due to the other is given by

$$F = (1 + \beta_1 \beta_2) E$$

where F is the perpendicular component of the force on beam 1, the test beam, due to the other beam. β_1 and β_2 are the velocities of the two beams. E is the electric field due to beam 2 at the location of beam 1.

The V-shift of beam 1 is given by

$$\Delta v_x = - \frac{1}{4\pi} \int ds \frac{\beta_x}{B_0} \frac{\partial}{\partial x} (F_x / \beta_e) ,$$

and a similar expression for Δv_y

Since the crossing angle is assumed small, E_x or E_y , in the coordinate system of the strong beam, beam 2, may be regarded as being perpendicular to the weak beam, beam 1.

The results given in this paper are for beams with a Gaussian transverse distribution with the charge density

$$\rho(x, y) = \frac{\lambda}{2\pi\sigma_x\sigma_y} \exp(-x^2/2\sigma_x^2 - y^2/2\sigma_y^2) .$$

The electric field due to this charge distribution may be derived from the potential function,

$$V = \frac{-\lambda}{4\pi\epsilon_0} \int_0^\infty 2udu \frac{1 - \exp(-x^2(2\sigma_x^2 + u^2) - y^2(2\sigma_y^2 + u^2))}{(2\sigma_x^2 + u^2)^{1/2} (2\sigma_y^2 + u^2)^{1/2}} . \quad (2.1)$$

The expression used in this paper for computing the stop-band half-widths,¹ Δv_{nm} , is

$$\Delta v_{nm} = \frac{1}{2^{n-2}} \frac{1}{n!m!} (n^2 + m^2)^{1/2} \epsilon^{(N-2)/2} \frac{1+\beta^2}{\beta} \\ \frac{1}{4\pi B\rho} \int ds \exp(i(n\psi_x + m\psi_y)) \beta_x^{n/2} \beta_y^{m/2} D_x^n D_y^m v ,$$

where ϵ is the emittance divided by π , assumed to be the same horizontally and vertically, $N = n+m$ where n and m are assumed to be positive, and $D_x = \partial/\partial x$, $D_y = \partial/\partial y$.

2.1 The Perfect Machine

The accelerator is assumed in this section to have no closed orbit errors, or other orbit errors, and the proton beams are assumed to cross at a small but non-zero angle. Then, under certain assumptions, described below, the only resonances driven are the even order vertical resonances. Horizontal and coupling resonances are not excited. The important assumption made is that β_x, β_y are constant, independent of the path lengths.

The above result can be seen as follows: The v_x shift is given by

$$\Delta v_x \sim \int ds \frac{\partial E_x}{\partial x} ,$$

where the integral is over the beam path, $x = \alpha s$, $y = 0$. Noting that

$$\frac{d}{ds} E_x = \frac{\partial}{\partial x} E_x \frac{dx}{ds} + \frac{\partial}{\partial y} E_x \frac{dy}{ds} , \\ = \alpha \frac{\partial}{\partial x} E_x ,$$

one finds

$$\Delta v_x \sim \int ds \frac{1}{\alpha} \frac{d}{ds} E_x = 0 .$$

In a similar manner one can show that $\Delta v_{mn} \propto \int ds \partial^{m+n} E_x / \partial x^m \partial y^n$ and that $\Delta v_{mn} = 0$ if $m \neq 0$.

On the other hand, Δv_y is not zero and if β_y can be regarded as constant, then v_y can be found from the charge density of the beam $\rho(x, y)$.

$$\Delta v_y \propto \int ds \frac{\partial}{\partial y} E_y ,$$

and

$$\frac{\partial}{\partial y} E_y + \frac{\partial}{\partial x} E_x = \frac{1}{\epsilon_0} \rho .$$

Thus,

$$\Delta v_y \propto \frac{1}{\epsilon_0} \int ds \rho(x, y) ,$$

which gives the result

$$\Delta v_y = \frac{1 + \beta^2}{\beta} \frac{\beta_y}{B_0} \frac{I}{\beta} \frac{1}{(2\pi)^{1/2} \sigma_y \alpha} \times 10^{-6} , \quad (2.1-1)$$

where I is in amperes and B_0 is in kG-m.

In a similar manner one can evaluate the even order vertical stop bands Δv_{on} ,

$$\Delta v_{on} \propto \int ds \frac{\partial^{n-1} E_y}{\partial y^{n-1}} ,$$

$$\Delta v_{on} \propto \frac{1}{\epsilon_0} \int ds \frac{\partial^{n-2}}{\partial y^{n-2}} \rho(x, y) ,$$

where $x = \alpha s$ and $y = 0$. This gives the result for even n

$$\Delta v_{on} = \Delta v \frac{1}{2^{(n-2)/2}} \frac{1}{[(n-2)/2]!} \frac{1}{(n-1)} ,$$

where Δv is the linear v -shift, Eq. (2.1-1).

The above results for the perfect machine assumed that β_x, β_y may be considered constant over the region where the beam-beam interaction is strong.

Another factor that enters is the beam-beam interaction cut-off which occurs when the beams have separated enough that they can be shielded from each other. It has been pointed out² that in the case of the linear v-shift there may be an appreciable long-range v-shift because β_y increases like s^2 , since $\beta_y = \beta_y^* + s^2/\beta_y^*$, while the $\partial E_y/\partial y$ decrease like $1/s^2$.

For small β_y at the crossing point, β_y will vary more rapidly with s , and for small crossing angles, the beam will interact over a longer path length and the long range effects become more important. The v-shifts and the stop bands can become considerably larger than the results given above. Also, one can get an appreciable horizontal v-shift, and also appreciable horizontal and coupling resonances.³

For one design of the CBA colliding proton beams accelerator, where $\alpha = 11$ mr, $\beta_y = 7.5$, $\beta_x = 40$ at the crossing points and $\sigma_y = .75$ mm, $\sigma_x = 1.8$ mm at 30 GeV, numerical computations give the result

$$\Delta v_y = .979 \Delta v$$

$$\Delta v_x = -.001 \Delta v$$

On the other hand, for another CBA design where $\alpha = 2$ mr, $\beta_y = \beta_x = 1$ m, and the same beam emittance $\epsilon = .5\pi \times 10^{-6}$ rm, one finds

$$\Delta v_y = 2.89 \Delta v$$

$$\Delta v_x = -2.43 \Delta v$$

A comparison of the non-linear beam-beam stop bands for these two designs are given in section IV.

2.2 Closed Orbit Errors and Random β Function Errors

The previous section treated the non-linear stop-bands of the perfect machine. The stop-bands of resonances produced by machine errors, like the closed orbit errors, are often more important as these resonances lie close to the operating ν_x, ν_y .

Closed Orbit Errors

Assuming that β_x, β_y near the crossing points may be considered as constant, independent of the length s , and that the closed orbit errors $\Delta x, \Delta y$ near the crossing points may also be considered as constant, then one finds that again only vertical resonances are excited. No horizontal or coupling resonances are excited. The derivation of this result given for the perfect machine in the previous section will also apply for closed orbit errors.

For the vertical resonances, one finds

$$\Delta \nu_{on} \sim \int ds \frac{\partial^{n-1} E_y}{\partial y^{n-1}},$$

$$\Delta \nu_{on} \sim \frac{1}{\epsilon_0} \int ds \frac{\partial^{n-2}}{\partial y^{n-2}} \rho(x, y),$$

where $x = \alpha s + \Delta x$, $y = \Delta y$.

Putting in for $\rho(x, y)$ the Gaussian charge distribution, and expanding in powers of the vertical closed orbit error Δy , one finds that for odd n , $\Delta \nu_{on} \sim \Delta y$, and for even n , $\Delta \nu_{on} \sim \Delta y^2$. The results for $\Delta \nu_{on}$ are^{1,4}

n odd

$$\Delta \nu_{on, rms} = \Delta \nu \frac{\sqrt{N_{cp}}}{2^{(n-1)/2}} \frac{1}{((n-1)/2)!} \frac{y_{rms}}{\sigma_y}, \quad (2.2-1)$$

n even

$$\Delta \nu_{on, rms} = \Delta \nu \frac{\sqrt{N_{cp}}}{2^{(n-2)/2}} \frac{1.41}{((n-2)/2)!} \frac{y_{rms}^2}{2 \sigma_y^2}, \quad (2.2-2)$$

where Δv is the linear vertical v-shift given for Eq. (2.1-1), y_{rms} is the rms vertical closed orbit error, and N_{cp} is the number of crossing points.

For even n , when n is a multiple of N_{cp} , the statistical distribution of Δv_{on} has the above rms value around the average value of Δv_{on} which for this case is not zero, and is given by

$$\overline{\Delta v_{on}} = -\Delta v \frac{N_{cp}}{2^{(n-2)/2}} \frac{1}{((n-2)/2)!} \frac{y_{rms}^2}{2\sigma_y^2}.$$

The closed orbit errors produce a vertical v-shift. The distribution of this Δv_y has the rms value

$$\Delta v_{y,rms} = 1.41 \sqrt{N_{cp}} \frac{y_{rms}^2}{2\sigma_y^2} \Delta v,$$

and the distribution has the average value,

$$\overline{\Delta v_y} = -N_{cp} \frac{y_{rms}^2}{2\sigma_y^2} \Delta v.$$

The above results assume that β_x and β_y are constant. Table I below compares the analytic results for the stop bands with the results of a numerical computation where β_x , and β_y are allowed to vary according to $\beta = \beta^* + s^2/\beta^*$. The results are for an CBA design for which $\beta_y = 7.5$, $\beta_x = 40$, $\sigma_x = 1.8$ mm, $\sigma_y = .75$ mm, $\alpha = .011$, $y_{rms} = .05$ mm. Note that only Δv_{mn} for m even are allowed because $x = \alpha s$ is an odd function of s .

Random β -Function Errors

Random errors in β_y at the crossing points will also drive the even order vertical resonance. Since the linear vertical v-shift, Δv , varies like $\beta_y/\sigma_y \sim \beta_y^{1/2}$, then a random error in β_y causes a relative error in Δv of $\Delta\beta_y/2\beta_y$. This leads to the even order stop bands, Δv_{on} , n even, given by⁴

$$\Delta v_{on,rms} = \Delta v \frac{N_{cp}^{1/2}}{2^{(n-2)/2}} \frac{1}{((n-2)/2)!} \frac{1}{(n-1)} \frac{1}{2} \frac{\Delta \beta_{y,rms}}{\beta_y}$$

Table 1

Resonance	$\Delta v_{nm}/\Delta v$	
	Computed Result β_x, β_y . Not Constant	Analytic Result β_x, β_y . Constant
Δv_{02}	.0075	.008
Δv_{20}	.00093	0
Δv_{03}	.16	.13
Δv_{21}	.0002	0
Δv_{04}	.0095	.006
Δv_{22}	.0003	0
Δv_{40}	.0000	0
Δv_{05}	.04	.02
Δv_{23}	.0001	0
Δv_{41}	.0000	0

For CBA, $\Delta \beta_{y,rms}/\beta_y = .05$, and for the eighth order resonance which may be of interest, the above gives $\Delta v_{08} = 9.5 \times 10^{-7}$. This resonance is also excited by orbit errors, and for the vertical orbit error $y_{rms} = .05$ mm, one obtains $\Delta v_{08} = 9.2 \times 10^{-7}$. This even order resonance, Δv_{08} , appears to be excited about equally by closed orbit errors and by random $\Delta \beta_y/\beta_y$ errors.

2.3 Comparison of CBA and ISR Stop Bands

Using the above results for Δv_{on} due to orbit errors, and random β -function errors at the crossing-point, one can compare⁵ the strength of the eighth order beam-beam resonances in CBA and the ISR storage accelerators. The important parameters of the two accelerators are

$$\begin{aligned} \text{CBA, } \Delta v &= 5.8 \times 10^{-3}, \sigma_y = .75 \text{ mm, } I = 8\text{A, } \beta_y = 7.5 \text{ m} \\ \text{ISR, } \Delta v &= 1.4 \times 10^{-3}, \sigma_y = 1 \text{ mm, } I = 30\text{A, } \beta_y = 14 \text{ m} \end{aligned}$$

The eighth order resonance is particularly interesting as it is likely to be close to the initial operating point of CBA, $v_x \approx v_y \approx 22.62$. This resonance is driven by orbit errors, and the stop band width is proportional to y_{rms}^2 . It is also driven by random errors in β_y .

First, let us consider the stop-band width due to closed orbits errors, which according to Eq. (2.2) varies like

$$\Delta v_{\text{on,rms}} \sim \Delta v y_{\text{rms}}^2 / \sigma_y^2$$

y_{rms} , the orbit error at the crossing point, is assumed to be equal for both accelerators. Using the above parameters for Δv and σ_y , one finds that $\Delta v_{0,8}$ is 7.4 times stronger for CBA than for the ISR.

There is also a contribution to $\Delta v_{0,8}$ from random $\Delta\beta_y/\beta_y$ errors. For CBA, $\Delta v_{0,8}$ due to orbit errors and $\Delta\beta_y/\beta_y$ errors are about equal. For the ISR, it is assumed that there is little contribution to $\Delta v_{0,8}$ from $\Delta\beta_y/\beta_y$ errors, as the random field errors which generate the $\Delta\beta_y/\beta_y$ errors are considerably smaller in the ISR. Adding the contribution to $\Delta v_{0,8}$ from $\Delta\beta_y/\beta_y$ errors, one finds that $\Delta v_{0,8}$ is about 10 times stronger for CBA than for the ISR at 30 GeV.

III. Effect of a Spectrometer Magnet on the Beam-Beam Interaction

A possible spectrometer magnet has been proposed for one of the CBA crossing points. This magnet might be about 6 m in diameter and have a uniform magnetic field of up to 15 kG. This distorts the particle orbits near the crossing points, and thus makes the beam-beam interaction different at one crossing point, and generates non-linear resonances close to the generating point. This subject is studied in detail in reference 6.

To study the effect of the spectrometer, the beam-beam stop-band introduced by the spectrometer were computed numerically, and these stop bands were compared to those produced by closed orbit errors. If the spectrometer stop-bands are smaller than the closed orbit stop bands, then one might conclude that the presence of the spectrometer is not increasing the harmful beam-beam effects already present.

Figure 1 shows the widths of the spectrometer stop bands as a function of the order of the stop-band. For each order, the width of the strongest stop-band is shown. The large crosses are the stop-band widths due to random closed-orbit errors of $y_{rms} = .05 \text{ mm}$. Results are given for two kinds of spectrometers, one producing a horizontal bend and one producing a vertical bend. The spectrometer has a 6 m diameter, and results are given for 3 field levels 5, 10 and 15 kG. One would conclude from Fig. 1, that up to 15 kG, the spectrometer will not seriously increase the beam-beam interaction effects.

Spectrometer Figure

IV. Effect of β_x , β_y and α on the Beam-Beam Interaction

In a colliding beam accelerator, the luminosity depends on the choice of β_y at the crossing point, and the crossing angle α ; smaller β_y or a smaller α leads to larger luminosities. However, the choice of β_x , β_y and α also effects the magnitude of the beam-beam interaction. The analytic results given in section II for the beam-beam ν -shift and stop bands are valid only if β_x , β_y may be considered as constant, independent of the path length s . As β_x , β_y and α become smaller, this assumption becomes less valid; the long range beam-beam interaction may appreciably increase the beam-beam ν -shifts, and one may find appreciable horizontal and coupling resonances and an appreciable horizontal ν -shift.

Table II compares the stop bands for two CBA designs at 30 GeV. One design, labeled CBA, has $\alpha = .011$, $\beta_y = 7.5$ m, $\beta_x = 40$, while the second design, labeled ISAP, has a smaller crossing angle, $\alpha = .002$, and smaller β functions, $\beta_x = \beta_y = 1$ m. In Table II, the first 2 columns list the stop-bands and ν -shifts per crossing point for the perfect machine without closed orbit or other errors. The results in Table II are given in terms of the linear vertical ν -shift, $\Delta\nu$. One sees that ISAP has considerably larger ν shifts and considerable $\Delta\nu_x$, and appreciable horizontal and coupling resonances. The third and fifth order resonances are also excited in ISAP by the asymmetry in the β -functions around the crossing point.

The stop bands due to closed orbit errors are also larger for ISAP by about a factor of 3. The stop bands due to vertical closed orbit errors of $y_{rms} = .05$ mm are shown in the last two columns of Table II.

Table II

	<u>Perfect Machine</u>		<u>$y_{rms} = .05 \text{ mm}$</u>	
	CBA	ISAP	CBA	ISAP
Δv_y	.9902	2.89	.007	.058
Δv_x	-.0010	-2.43	0	.0022
Δv_{03}	0	0	.151	.450
Δv_{12}	0	.1617	0	.0001
Δv_{21}	0	0	.0017	.0817
Δv_{30}	0	.336	0	0
Δv_{04}	-.4134	-.5220	.078	.075
Δv_{13}	0	0	0	0
Δv_{22}	.0016	.464	0	.0078
Δv_{31}	0	0	0	0
Δv_{40}	.0001	-.160	0	.0002
Δv_{05}	0	0	.0355	.098
Δv_{14}	0	-.0166	0	0
Δv_{23}	0	0	.0001	.027
Δv_{32}	0	-.0110	0	0
Δv_{41}	0	0	0	.0044
Δv_{50}	0	-.0140	0	0

V. Numerical Computational of Δv_{mn}

In order to compute the derivatives of $V(x,y)$, $\partial^{n+m} V / \partial x^n \partial y^m$, V was computed on a mesh, $V(I,J)$. The mesh intervals, Δx , Δy , have to be small compared to the beam size. For the CBA case $\Delta x = \Delta y = .1 \text{ mm}$ to $.05 \text{ mm}$ was used. The extent of the mesh in the x and y directions has to cover the region where the beams interact. Along the s -direction, the beams will continue to interact until each beam starts moving in its own vacuum chamber, which shields them from each other. The beams will interact over a distance of a number of meters, about 8 to 20 m for CBA, which together with the small mesh interval size of about $.1 \text{ mm}$ could lead to a large number of mesh points at which $V(x,y)$ has to be computed.

One way to reduce the number of mesh points at which the potential function is to be computed, is to compute $V(I,J)$ only in an interval around the path of the particle. For each value of x , the horizontal separation of the beams, the test beam will be at $y(x)$ given by the vertical path of the beam. $V(I,J)$ was computed at just 4 mesh points above, and 4 mesh points below $y(x)$. This is sufficient to compute up to the fifth order resonance for reasonable test beam paths.

A numerical problem arises in computing the long range contribution to the v -shifts. At large distances from the strong beam, the potential function varies slowly but there still may be an appreciable contribution to the v -shift as the β -function is getting large. It becomes difficult to compute $V(I,J)$ from the integral in Eq. (2-1) with sufficient accuracy to be able to compute derivatives of the potential. A procedure to avoid this difficulty is to compute $V(I,J)$ from its asymptotic form $V = (1/2\pi\epsilon_0) \log (r/r_0)$, beyond a certain cut-off point in x . This cut-off point is chosen empirically and was

found to occur at about $15 \sigma_x$. This procedure also has the effect of reducing the number of mesh points that needs to be computed.

Another numerical problem arises in computing $V(x,y)$ from the integral formulation given by Eq. (2.1) with sufficient accuracy. The integrand has a long tail as $\mu \rightarrow \infty$. This tail can be compressed by a transformation to z where

$$z = 2\sigma_y^2 / (2\sigma_y^2 + u^2),$$

$$u = (2\sigma_y^2(1/z-1))^{1/2},$$

$$dz/z = -(zu/\sigma_y^2) du,$$

$$V = \frac{-\lambda}{4\pi\epsilon_0} \int_0^1 \frac{dz}{z^2} \frac{2\sigma_y^2 \{1 - \exp(-x^2/(2\sigma_x^2 + u^2) - y^2/(2\sigma_y^2 + u^2))\}}{(2\sigma_x^2 + u^2)^{1/2} (2\sigma_y^2 + u^2)^{1/2}}$$

In the integration, intervals in z of $\Delta z = .1$ to $.01$ were used, which correspond to u -intervals, near the beam, of $\Delta u = .1 \sigma_y$ to $.01 \sigma_y$.

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