SCALE LENGTH STUDY IN TFTR

By

S. Hiroe et al.

DECEMBER 1988

PLASMA PHYSICS LABORATORY

PRINCETON UNIVERSITY

PRINCETON, NEW JERSEY

PREPARED FOR THE U.S. DEPARTMENT OF ENERGY,
UNDER CONTRACT DE-AC02-76-CHO-3073.
SCALE LENGTH STUDY IN TFTR*


Princeton Plasma Physics Laboratory, Princeton University, Princeton, New Jersey, United States of America

ABSTRACT. The scale lengths of the electron density ($L_{n_e}$), temperature ($L_T$), and pressure ($L_P$) gradients were investigated during the 1985 operating period of the Tokamak Fusion Test Reactor (TFTR) for gas-fueled plasmas with neutral beam injection and a movable limiter. Although the global energy confinement time degrades as the heating power increases or the plasma current decreases, the radial profiles of the scale lengths ($L_T$ and $L_P$) remain unchanged. Especially, the electron pressure profile is constrained not to change. This trend appears to hold over a fairly wide range of TFTR operational regimes. The radial profiles of $L_{n_e}$ and $\eta_e (= L_{n_e}/L_T)$ also appear to remain unchanged, although the uncertainties of the experimental data for these quantities are greater than those for $L_T$ and $L_P$. The experimental parameters are used to evaluate theoretical predictions of the electron thermal diffusivity, and the results are compared with the empirical thermal diffusivity.

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.


**Permanent address: Oak Ridge National Laboratory, Oak Ridge, Tennessee 37831, U.S.A.

***Permanent address: Japan Atomic Energy Research Institute, Ibaraki-ken, Japan.
1. INTRODUCTION

Energetic neutral particle injection and RF power in the ion cyclotron range of frequencies have been used successfully to heat tokamak plasmas to tens of kilovolts [1,2]. However, the confinement time degrades as the heating power increases. A great deal of effort has been expended in understanding this degradation. Some candidates for the anomalous transport associated with the fluctuations have been discussed. The study of the profiles is important in understanding the enhanced transport due to instabilities.

The electron temperature profile has been reported to remain unchanged over a fairly wide range of TFTR operational regimes [3]. The density and temperature profiles are obtained from the particle and power balances, as described, for example, by Guzdar et al. [4]. The electron anomalous thermal transport associated with the electron temperature gradient instability has been calculated. The electron thermal diffusivity is

$$\chi_e = 0.1 \left( \frac{c^2 s}{\omega_{pe}^2} \right) \frac{v_e}{R} q \eta_e (\eta_e + 1)$$

where $c$ is the light speed, $s$ is the shear [$= (r/q)(dq/dr)$], $\omega_{pe}$ is the electron plasma frequency, $v_e$ is the electron thermal speed, $R$ is the major radius, $q$ is the safety factor, and $\eta_e = L_{ne}/L_{Te}$; $L_{ne}$ and $L_{Te}$ are the scale lengths of the electron density and temperature gradients, respectively. By solving the energy balance equation based on this thermal conduction model, with a Gaussian density profile, Guzdar et al. [4] found that the electron temperature profiles of the resulting parabolic form is similar to the Ohmic case for almost any auxiliary heating profile in low-$q$ discharges. Other electron anomalous transport theories were summarized by Ross et al. [5].

As shown by Eq. (1), the thermal diffusivity is strongly dependent on the scale lengths, which implies that they may play an important role in the plasma performance of tokamaks. In this paper we study the correlation of the scale lengths of the density gradient $L_{ne}$, the temperature gradient $L_{Te}$, and the pressure gradient $L_{pe}$ with the plasma parameters in a specific set of discharges in TFTR. (Recent data for supershots [6] and pellet injection [7] are not included in this analysis.) The data analyzed here were taken in the L-mode regime (gas fueling, co-injection, and steady state) [8]. Because the local scale lengths are not constant from shot to shot, they are treated statistically, as discussed in Section 2.1.

In Section 2.2, we study the radial profiles of the scale lengths for the data of a scan of heating power with toroidal field $B_T$, plasma current $I_p$, and line-averaged density $\bar{n}_e$ constant. In Section 2.3, the studies are expanded into a wider range...
with the data of a current scan and a density scan. In Section 2.4, confinement times and scale lengths are studied to find whether scale lengths correlate with the tokamak plasma performance. In Section 3, predictions of the anomalous electron heat transport from various theories are compared with the observations. In Section 4, the conclusions of this work are summarized.

2. EXPERIMENTAL OBSERVATIONS

2.1. Statistical analysis of scale length

In TFTR, the electron density \( n_e \) and temperature \( T_e \) are measured by several diagnostics \([9]\). Here the 76-point Thomson scattering data are used to analyse the electron density and temperature profiles. The data versus major radius are smoothed with a triangular weighting function whose full width is set to 0.1a, where \( a \) is the minor radius, and then mapped to shifted-circles magnetic equilibria with the SNAP data analysis code. Thus, the real-space data—for example, \( T_e(R) \)—are mapped into the flux coordinate \( T_e(r/a) \) \([10]\). The central ion temperature is estimated by using Ti(XXI) \( K_\alpha \) Doppler broadening measurements \([11]\). The scale lengths of the density, the temperature, and the electron pressure gradient are

\[
L_{n_e}^{-1} = -\frac{d(\ln n_e)}{dr},
L_{T_e}^{-1} = -\frac{d(\ln T_e)}{dr},
L_{P_e}^{-1} = L_{n_e}^{-1} + L_{T_e}^{-1}.
\]

First, the arithmetic mean and the fractional standard deviation for the scale lengths \( L_{n_e} \), \( L_{T_e} \), and \( L_{P_e} \) and the ratio \( \eta_e \) at three radial positions \( r/a = 0.43, 0.57, \) and \( 0.67 \) are examined for the following operational conditions. The magnetic field \( B = 3.9 \) and \( 4.7 \) T; the plasma current \( I_p = 0.9, 1.0, 1.4, 1.8, \) and \( 2.2 \) MA; the line-averaged density is \( (1.4-6.0) \times 10^{19} \) m\(^{-3}\); the heating power is \(<7 \) MW; and the global energy confinement time ranges from 0.10 to 0.45 s. The major radius is \( R = 2.6 \) m, and the plasma radius is \( a = 0.82 \) m. The total number of shots treated here is 106. The safety factor for 80% of the data is in the range \( 2.6 < q_{cy} < 4.3 \). where \( q_{cy} \) is the cylindrical \( q \)-value at the last closed flux surface (limiter). In this range of safety factors, the three radial positions examined lie between the \( q = 1 \) and \( q = 2 \) surfaces. These conditions cover a fairly wide range of the TFTR L-mode operational regimes.

As long as the scale lengths are calculated by taking the derivative of the data, they are not constant, as discussed below. Statistical treatment is one method
of studying the scattered data set. The acceptability of the statistical treatment was evaluated by plotting the probability distribution of \( L_p \) for these 106 shots bounded with \( \Delta L_p = 0.02 \) m. Figure 1 shows that the distribution is normal, with a mean value of 0.22 m and a standard deviation of 0.023 m. Because we are focusing on analysis of the electron component, the symbol \((e)\) for electrons is generally neglected in what follows.

Table I lists the mean values and the fractional standard deviations of \( \nu_n \), \( \nu_T \), \( \nu_p \), and \( \eta_e \) at \( r/a = 0.57 \) for six operational regimes and for all 106 shots (which include other operational regimes). The mean values do not change for these operational conditions. In addition, there are no differences between plasmas with Ohmic heating (OH) and those with neutral beam injection (NBI). For the case of the current scan \((0.8 \leq I_p \leq 1.8 \) MA\) with \( B = 3.9 \) T and NBI, \( \nu_n = 0.56 \) m with \( \sigma/\nu_n = 0.21 \), \( \nu_T = 0.33 \) m with \( \sigma/\nu_T = 0.15 \), \( \nu_p = 0.21 \) m with \( \sigma/\nu_p = 0.11 \), and \( \eta_e = 1.72 \) with \( \sigma/\eta_e = 0.25 \) at \( r/a = 0.57 \). (This data set is discussed in Section 2.3.) These scale lengths are quite similar to the values in Table I, and similar scale lengths are found for all 106 shots. These observations indicate that \( \nu_n, \nu_T, \nu_p \), and \( \eta_e \) remain unchanged over a wide range of operational regimes.

As clearly shown in Table I, the fractional standard deviation \( \sigma/L \) for \( \nu_T \) and \( \nu_p \) is small, implying that both \( \nu_T \) and \( \nu_p \) are constrained. However, \( \sigma/\nu_n \) and \( \sigma/\eta_e \) are twice as large as \( \sigma/\nu_T \) and \( \sigma/\nu_p \). These trends are also observed for \( r/a = 0.43 \) and 0.67. Hereafter a quantity is defined as constrained when the fractional standard deviation is less than 0.2.

The scale lengths for the three radial positions \((r/a = 0.43, 0.57, \) and 0.67\) at constant toroidal magnetic field \((B = 3.9 \) T\) and plasma current \((I_p = 1.8 \) MA\) are plotted at the top of Fig. 2; the fractional standard deviation is plotted at the bottom. No meaningful differences between OH and NBI plasmas are found. It is interesting that the scale lengths decrease as the radius increases, implying that the profiles become steeper as the radial position is increased. The value of \( \eta_e \) also decreases, but it stays above 1.0 near the plasma edge, as discussed later.

The fractional standard deviation also decreases as the radial position increases, indicating that the profile becomes rigid at the outer radii. This is clearly observed for the scale length of the density gradient (see Fig. 2), the standard deviation of which is over 20% at \( r/a < 0.5 \). This feature may result from the sawtooth activity at \( q = 1 \), because the \( q = 1 \) surface is at \( r/a = 0.35 \), as determined by the soft X-ray array [12]. However, this hypothesis has not yet been confirmed. As discussed in
Section 2.2, close to $q = 1$, the scale lengths start to increase sharply, implying more or less flat profiles.

The statistical treatment indicates that the average values of $L_n$, $L_T$, $L_P$, and $\eta_e$ remain unchanged over a fairly wide range of operational regimes. The radial profiles of the scale lengths also appear to remain unchanged regardless of the operational regime. $L_T$ and $L_P$ are well constrained at $r/a > 0.4$. $L_n$ and thus $\eta_e$ change more than the others, but they are still constrained at $r/a > 0.6$. The scatter in the data increases as the radial position approaches the $q = 1$ surface.

2.2. Radial profile of the scale length

When the experimental data are compared with anomalous transport theories, the details of the radial profile of the scale lengths are of importance. The data for one series of TFTR experiments [13], with $B = 4.7$ T, $I_p = 2.2$ MA, $n_e = (4.5 \pm 0.3) \times 10^{19}$ m$^{-3}$, $q_{cy} = 2.8$, and heating power varied from 2 MW (Ohmic) to 6.4 MW (NBI), are summarized in Table II. The global energy confinement time $\tau_E(a)$ degrades as the heating power increases; thus, this series is in the L-mode regime [8].

In Figs. 3 and 4, the radial profiles of the scale lengths of the electron density and temperature gradients are shown for various levels of heating power. In the figures, the arithmetic mean values for operation at $B = 4.7$ T and $I_p = 2.2$ MA (29 shots, see Table I) are shown by the open circles; here the regions within 70% probability ($\pm \sigma$) are indicated by the bars.

The numbers in Figs. 3 and 4 correspond to the case numbers in Table II. (For simplicity, data from cases 3, 5, and 7 are displayed only at $r/a = 0.57$.) The global kinetic energy confinement time $\tau_E(a)$ is 0.41 s for case 1, 0.32 s for case 2, 0.25 s for case 4, and 0.20 s for case 6, indicating no explicit dependence of the scale length on the global energy confinement time or vice versa. (Detailed information on determining the energy confinement time is given in Ref. [13].)

The global shapes of $L_T$ and $L_n$ for the different levels of heating power are more or less identical, as shown in Figs. 3 and 4. For $r/a > 0.7$, $L_n$ and $L_T$ are independent of the heating power. For $0.4 < r/a < 0.7$, both $L_n$ and $L_T$ are scattered.

In Fig. 5, the radial profiles of the scale length of the electron pressure gradient $L_p$ are plotted. At $r/a > 0.4$, $L_p$ for 2.8 MW, 4.2 MW, and 6.1 MW NBI heating is not significantly different from $L_p$ for 2.0 MW Ohmic heating, suggesting that
the radial profile of $L_p$ remains unchanged regardless of the heating power. $L_p$ also increases sharply when the radial position approaches the $q = 1$ surface, indicating that the electron pressure profile is approximately flat inside the $q = 1$ surface. It is clear that there is no systematic relation between $L_p$ and the heating power.

The power dependencies of the scale length $L_p$ at several fixed radial positions are plotted in Fig. 6. For $r/a > 0.4$, it is clear that $L_p$ does not depend on the heating power. Although $L_T$ and $L_n$ are scattered at $r/a \sim 0.57$ (see Figs. 3 and 4), $L_p$ is very well constrained. At $r/a \sim 0.57$ in Figs. 3 and 4 (points 6 and 7), $L_n$ and $L_T$ are anti-correlated. One also finds $\sigma/L_T > \sigma/L_p$ for most of the cases in Table I when $L_p$ is calculated by using $L_n$ and $L_T$. It appears from these data that the electron pressure profile is better constrained than the electron temperature profile. To determine whether $L_p$ or $L_T$ is constrained would require an independent measurement of the pressure profile and/or guidance from the physics model. However, it is worth noting that data from pellet injection experiments on TFTR and elsewhere suggest that $L_n$ is probably more weakly constrained than $L_T$.

We must look at $\eta_e$ to determine whether the profile is constrained by certain classes of microinstabilities. In Fig. 7, the radial profile of $\eta_e$ is plotted for different levels of heating power. As $r/a$ increases, $\eta_e$ decreases from 2.2 at $r/a = 0.43$ to 1.3 at $r/a = 0.9$. Again, $\eta_e$ does not depend systematically on the heating power. The scatter of $\eta_e$ is slightly over the constraint criterion for the fractional standard deviation.

From these observations, we conclude that the scale lengths of the electron density, temperature, and pressure gradients decrease as the radial position increases. The profiles appear to be unaffected by the heating power, as do the local values for $L_T$ and $L_p$ at $r/a > 0.5$. Especially, the shapes and values of $L_p$ for $r > r_1$ (where $r_1$ is the value of $r$ at the $q = 1$ surface) are similar even when the heating power changes. The ratio $\eta_e = L_n/L_T$ decreases as the radius increases, but it is still larger than 1.0 at $r/a = 0.9$. The value of $\omega_n$ (and thus $\eta_e$) is also constrained at $r/a > 0.6$, but the standard deviations are somewhat greater than those of $L_T$ and $L_p$. The scale lengths do not appear to depend on the operational regimes, as pointed out earlier. This issue is discussed further in Section 2.3.
2.3. Scale length for different operational regimes

As shown by Fig. 6, $L_p$ is not a function of the heating power. Thus, one cannot find a difference in $L_p$ for OH and NBI plasmas. In the L-mode, the energy confinement time increases with the plasma current [8]. Radial profiles of $L_p$ are plotted in Fig. 8 for different $q$ values. The radial profile of $L_p$ at high $q$ (open circles in Fig. 8(b)) is slightly different from that at low $q$, but the general tendency is similar, regardless of $q$. Inside $q = 1$, $L_p$ increases sharply. In the range $r_1/a < r/a < 0.6$, $L_p$ appears to be constant. It decreases linearly at $r/a \geq 0.8$ as $r/a$ increases. To clarify the dependence of the scale lengths on the plasma current, $L_n$, $L_T$, $L_p$, and $\eta_e$ at $r/a = 0.57$ are plotted versus plasma current for $B_T = 4.7$ T and 3.9 T in Fig. 9. Both OH and NBI data are included. The cylindrical $q$-value at 1.8 MA and 3.9 T is similar to that at 2.2 MA and 4.7 T and is about 2.8. While $L_n$ increases slightly (~20%) as the plasma current increases (~3 times), $L_T$ and $L_p$ remain unchanged (less than 10%). The data for $\eta_e$ are slightly scattered, but its value is probably constant versus $I_p$, and thus $q$. This is in contrast to results reported for Alcator C [14], which imply $L_T \propto q^{-1}$. These Alcator results have been extensively used in "profile consistency" microinstability models [15].

In Fig. 10, the scale lengths of the electron density, temperature, and pressure gradients at $r/a = 0.67$ are plotted versus the line-averaged density with $B_T = 4.7$ T and $I_p = 2.2$ MA; the heating power is kept constant around 4.37 MW. Under these conditions, at $r/a = 0.67$, $L_n$ is 0.47 m with $\sigma/L_n = 0.26$, $L_T$ is 0.35 m with $\sigma/L_T \sim 0.22$, and $L_p = 0.19$ m with $\sigma/L_p \sim 0.17$, again indicating $\sigma/L_p < \sigma/L_T$. The scatter in $L_n$ and $L_T$ in Fig. 10 is relatively large. However, we can conclude that, while the plasma density changes by a factor of 4, the scale lengths remain nearly unchanged; no strong positive dependence of the scale lengths on $\bar{n}_e$ can be found for this data set.

From these observations, it is reasonable to conclude that the scale lengths $L_T$, $L_n$, and $L_p$ remain unchanged regardless of the operational regime, even though the constraint on $L_n$ is somewhat less rigid. Also, the pressure profile appears to be more constrained than the temperature profile.

2.4. Correlation

The constraining of the scale lengths could result from a marginal stability constraint. If this were the case, one could expect to find some correlation of $\tau_E$
with gradient scale lengths. To test this possibility, the correlations between the scale lengths and $T_g(a)$ were studied. The correlation coefficient \cite{16} between the parameters $x$ and $y$ is defined by

$$C_{xy} = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\left[ \sum (x_i - \bar{x})^2 \times \sum (y_i - \bar{y})^2 \right]^{1/2}},$$

where $\bar{x}$ and $\bar{y}$ are the arithmetic means of $x$ and $y$, respectively. Practically, the logarithm of the parameters is taken into account in the case of the correlation, as long as the functional relation among the parameters is unknown [this in Eq. (3) are expressed in terms of the logarithm of the parameters]. This means that the functional relation is assumed to be a linear product of powers of the variables.

No systematic correlation was found between $T_g(a)$ and any of the scale lengths. For plasmas with $B = 3.9$ T and $I_p = 1.8$ MA, the correlation coefficient between $T_g(a)$ and $L_T$ was 0.53 for an OH plasma and 0.55 for an NBI plasma; the correlation coefficient between $T_g(a)$ and $L_n$ was +0.17 for an OH plasma and -0.1 for an NBI plasma. No systematic differences were found between OH plasma and NBI plasma. The correlations of the scale lengths and $\eta_e$ with the electron energy confinement time at the half-radius $T_{ge}(a/2)$ were also calculated. Again, no correlations were observed. These results are consistent with the experimental observations discussed in the previous sections.

In the well-known neo-Alcator scaling, the confinement time saturates when $g = \bar{n}_e q$ exceeds a critical value. In TFTR \cite{13}, this is about $1.1 \times 10^{20}$. The scale lengths at $g > 1.1 \times 10^{20}$ are $L_n = 0.57$ m with $\sigma = 0.12$, $L_T = 0.33$ m with $\sigma = 0.040$, and $L_p = 0.21$ m with $\sigma = 0.023$. The values at $g < 1.1 \times 10^{20}$ are $L_n = 0.66$ m with $\sigma = 0.20$, $L_T = 0.36$ m with $\sigma = 0.054$, and $L_p = 0.23$ m with $\sigma = 0.034$. Although the scale lengths in the lower-density regime are systematically longer by 10% than those in the higher-density regime, the difference is not significant. Thus, for this data set, the degradation of confinement time in the high-density regime with neo-Alcator scaling cannot be shown to result from changes in the electron scale lengths ($L_T$, $L_n$, $L_p$, or $\eta_e$). This observation may indicate that the saturation of $T_E$ in the high-density regime results from enhanced energy loss through the ion channel rather than the electron channel. An issue related to this possibility is discussed in Section 3.
3. COMPARISON WITH THEORY

In Section 2.4, correlations between local quantities such as the scale lengths and \( \tau_{Ee} \) were studied. No clear correlations were found. However, some local quantities are very much scattered because we have taken the derivatives of plasma quantities. Also, it is difficult to evaluate unknown combinations among the quantities with Eq. (3). Thus, it is important to study the global features of the thermal conductivity, which we do by comparing an empirically obtained \( \chi_e(\tau) \) with a theoretically predicted \( \chi_e(\tau) \).

In this section, several theoretical predictions of \( \chi_e \) summarized in Ref. [5] are evaluated from the experimental data and then compared with the empirically determined \( \chi_e \). Since the ion temperature profile is not measured, the theoretical thermal diffusivities that are very sensitive to this profile, such as the diffusivities associated with the ion temperature gradient mode [17], are not evaluated here. For these calculations, the ion thermal diffusivity has been adjusted to provide agreement with the measurement of the central ion temperature; consequently, the adjustment factor ranges within several times the Chang-Hinton neoclassical diffusivity [18]. This factor, in general, is higher for plasmas with poorer confinement. Recent measurements on TFTR [19] indicate that \( \chi_i \) in high-density Ohmic plasma and in NBI plasmas has a profile shape similar to that of \( \chi_e \) over the region considered here \( (q > 1) \). The calculations shown here, however, essentially assume that all heat flow is in the electron channel. This could amount to an overestimate of a factor of 2 for the empirical \( \chi_e \).

Theoretical diffusivities were calculated with data from shot 14726 (an OH plasma) and shot 14729 (an NBI plasma), both with 4.7 T and 2.2 MA, and compared with the experimental \( \chi_e \). Detailed comparisons were carried out for three OH plasmas and three NBI plasmas with 4.7 T and 2.2 MA and for seven other shots with different \( q \) values. The general trends found from these 13 shots are similar to the theoretical results, except at high \( q \).

It is important to know whether these TFTR plasmas are collisional or collisionless. Since no detailed measurements of instabilities associated with anomalous diffusivity were done, we must assume the wavelengths. When \( 3p_e k \sim 1 \) is employed [5], the effective collision frequency \( \nu_{e\text{ff}} \) is higher than the electron diamagnetic frequency \( \omega_e^* \) for the region \( \tau > \tau_1 \); that is, \( \nu_{e\text{ff}}/\omega_e^* > 1 \), as shown in Fig. 11. Here \( \rho_e \) is an ion Larmor radius with the electron temperature and is about 0.15 cm at \( \tau = 0.55 \) m for shot 14729. It is significant that \( \nu_{e\text{ff}}/\omega_e^* \) decreases as the radius increases...
because the decrease in \( n_e L_n \) is comparable to the decrease in \( T_e^2 \) for both the OH and the NBI plasmas considered here. This tendency holds for all 13 shots analyzed here. The plasma is collisionless from the neoclassical point of view because the collision frequency is less than the bounce frequency except near the plasma edge.

Instabilities associated with the anomalous transport models have not been identified experimentally. Thus, the radii where the dissipative or collisionless theories should be applied are not clear, because the distinction between 'dissipative' and 'collisionless' is dependent on the wavelength for the present TFTR plasma parameters. For example, when the wavelength is longer than 2 cm, TFTR plasmas are dissipative, as seen in Fig. 11. While fluctuations have not been measured in detail in TFTR, there are some preliminary observations [20] at \( r/ a > 0.7 \) (mainly at the scrape-off layer). \( D_\alpha \) light is characterized by a frequency range of <50 kHz and wavelengths in the range of 5-10 cm. When these characteristics are applicable in the region considered here, the TFTR plasma is collisional at \( \tau_i/ a < r/ a \). It is also worth noticing that when the collision frequency \( \nu_{ei} \) instead of the effective collision frequency is compared with \( \omega_e^* \), the ratio \( \nu_{ei}/ \omega_e^* \) is almost flat and is around 1 for \( k_\perp < 1/3 \).

In Figs. 12-18, we compare the experimental values of \( \chi_e \) for shots 14726 and 14729 with values obtained from the theories of Ohkawa and of Mereshkin and Mukhovatov, some related theories, and the drift and trapped electron mode theories. Figure 12 shows \( \chi_e \) derived by Kadomtsev and Pogutse [21],

\[
\chi_e = e^2 \left( \frac{c}{\omega_{pe}} \right)^2 \frac{\omega_e}{(qR)} ,
\]

where \( e \) is \( r/R \). Figure 13 shows \( \chi_e \) derived by Guzdar et al. [4],

\[
\chi_e = 0.1 \left( \frac{c^2 s}{\omega_{pe}} \right) \left( \frac{v_e}{R_q} \right) \eta_e (\eta_e + 1) .
\]

Figure 14 shows \( \chi_e \) derived by Perkins for the dissipative trapped electron (DTE) mode [22],

\[
\chi_e = c^2 (\rho_s c_s/L_n)^2 / \nu_{ei} ,
\]

where \( \nu_{ei} \) is the electron ion collision frequency. Figure 15 shows the electron diffusivity \( \chi_e \) derived by Diamond for the DTE mode [23],

\[
\chi_e = \chi_e [\text{Eq. (5)}] \times \left( c_s e / (L_n \nu_{ei}) \right)^{1/3} / s^2 .
\]

Figure 16 shows \( \chi_e \) derived by Ross et al. [5],

\[
\chi_e = \chi_e [\text{Eq. (5)}] \times \exp(-0.56 \nu_{ei}^{0.62}) ,
\]
where $\nu^*_e$ is the electron collisionality, which is less than 1. Figure 17 shows $\chi_e$ derived by Diamond for the collisionless drift wave (DW) mode [23],

$$\chi_e = \left(\frac{\rho_e c_s}{L_n}\right)^2 \left(\frac{q_R}{\nu_e}\right) \left(\frac{c_s q R}{L_n v_e}\right)^{1/3}. \quad (8)$$

Figure 18 shows $\chi_e$ derived by Perkins for the collisionless trapped electron (CTE) mode [22],

$$\chi_e = 3c^{1/2} \frac{\rho_e^2 c_s}{L_n}. \quad (9)$$

### 3.1. Ohkawa or Merezhkin-Mukhovatov theory and relatives

As shown in Fig. 12, the Kadomtsev-Pogutse formula is in relatively good agreement with observations for the case with Ohmic heating. Even when the ion temperature effect is considered, the agreement is still within the experimental uncertainty. This formula appears to be insensitive to the TFTR plasma parameters. The theoretically predicted $\chi_e$ is constant for $r/a > 0.3$, while the measured $\chi_e$ increases for the case with NBI as the radius increases. Ohkawa's formula [24] is similar to the Kadomtsev-Pogutse formula. It is larger by $2/e$ than the Kadomtsev-Pogutse formula; thus, $\chi_e$ predicted by Ohkawa decreases at the plasma edge because of the effect of $r/R$.

The observations discussed in Section 2 are frequently referred to as profile consistency [14, 15]. As described in Ref. [4], the profile is constrained by the onset of the electron temperature gradient mode. The electron temperature profile was calculated from the electron energy balance equation with the $\chi_e$ formula given by Eq. (1) and under the assumption of a Gaussian-type density profile. When the density profile $n(r)/n(0) = \exp[-\alpha(r/a)^2]$ is assumed, then the energy balance equation readily yields

$$T(r)/T(0) = \left[1 - \left(\frac{r/a}{\beta}\right)^2 \left(\frac{\beta + 1}{2}\right) + 1 - \left(\frac{r_1/a}{\beta}\right)^2 \left(\frac{\beta + 1}{2}\right)\right]^2. \quad (10)$$

In the case of Guzdar et al. [4], $\beta = 2.0$.

When the electron temperature profiles measured in TFTR are compared with this calculation, there is reasonable agreement in the range $r_1/a < r/a < 0.5$. The electron temperature profile is also similar to the Ohmic case for almost any auxiliary heating source in the low-$q$ discharges, when the product of the power deposition profile and the $q$ profile is assumed to be radially uniform [25]. The constraint of the electron temperature profile appears to be consistent with the observations described in Section 2.2.
However, there are several discrepancies between theory and experiments. Figure 7 shows $\eta_e = L_n/L_T$ for operation at $B = 4.7 \text{ T}$ and $I_p = 2.2 \text{ MA}$ calculated from Figs. 3 and 4; $\eta_e$ for operation with $B = 3.9 \text{ T}$ and $I_p = 1.8 \text{ MA}$ is plotted in Fig. 2. The value of $\eta_e$ decreases as the minor radius increases. However, the theoretical value of $\eta_e$ calculated by Guzdar et al. [4] increases as $r/a$ increases (it is infinity at the edge). This is a serious discrepancy because the profile of $\chi_e$ in theory depends strongly on $\eta_e$, scaling as $\eta_e(\eta_e + 1)$. In other words, the increase in $\chi_e$ at the plasma edge may result from the increase in the assumed $\eta_e$. When the theoretical $\chi_e$ is evaluated from the experimental parameters, it decreases or is flat at best as the plasma edge is approached, as shown in Fig. 13. This discrepancy may result from the theoretical choice of the density profile, because the denominator ($L_T$) in $\eta_e = L_n/L_T$ becomes zero [see Eq. (10)] while the numerator ($L_n$) is finite as a result of the Gaussian-type profile when the radius increases.

Recently, Zhang and Mahajan [26] proposed a new formula for turbulent transport. Following the present understanding, the electromagnetic modes associated with this anomalous transport are stable. When this formula is evaluated with experimental parameters as an empirical relation, it agrees reasonably well with the experimental $\chi_e$. This formula also does not agree well with NBI data in the outer region.

### 3.2. Drift wave turbulence theories

Drift wave turbulence theories are discussed in the summary of Ross et al. [5]. The theoretical prediction of $\chi_e$ from the Perkins DTE mode formula (Fig. 14) is about one-third of the empirical value for OH plasmas. For NBI plasmas, the difference becomes larger at $r > 0.5 \text{ m}$ [27]. The Diamond DTE mode formula (Fig. 15) and that of Ross et al. [5] (Fig. 16) are essentially similar to the Perkins DTE mode formula. As noted previously, the discrepancy between theory and experiment can be reduced by the decrease in the empirical $\chi_e$ (by a factor of as much as two) that results from the ion temperature profile effect. If this is the case, these dissipative theories are indeed good candidates for explaining the anomalous electron transport in TFTR [28]. It is worth noting that $\chi_e$ for the dissipative mode is not sensitive to $k_\perp$ because $\chi_e$ is proportional to $(\omega_e^*/k_\perp)^2$.

The theoretically predicted $\chi_e$ for both OH and NBI does not decrease even when the radius increases. This observation holds for all of the 13 shots analyzed here. The anomalous conductivity from drift wave theory has a strong temperature
dependence, scaling as $T_e^{7/2}$. Consequently, $\chi_e$ is often predicted to decrease as a function of the radius, contrary to the experimentally inferred $\chi_e$ profiles. However, $\chi_e$, for example in Eq. (5), is proportional to $T_e^{7/2}/n_e L_n^2$. As seen in Fig. 4, $L_n$ decreases sharply as the radius increases. Thus, the radial profile of $\chi_e$ does not decrease sharply but sometimes increases, in contrast to simple predictions. This is why the dissipative DW models stay within a reasonable value in comparison to the experiments.

In Fig. 17, which shows $\chi_e$ for the Diamond collisionless DW mode, the qualitative tendency near the plasma edge agrees well with observations, because $\chi_e$ in Eq. (8) is strongly dependent on $L_n$ rather than $T_e$, scaling as $\chi_e \propto T_e^{2/3}/L_n^{1/3}$. The theoretical prediction is one to two orders of magnitude smaller than the experimental observation, suggesting that this theory should be applied to really collisionless plasmas. The Diamond collisional DW formula and the Terry-Diamond formula [29] are also one to two orders of magnitude smaller than the observations.

The Perkins CTE mode formula shown in Fig. 18 seems to be in good agreement with the observations, especially for the OH plasma, but the OH plasma is relatively more collisional, as seen in Fig. 11. The radial tendency of this theoretical prediction also differs slightly from the observation at the plasma edge for the TBI plasma, although the values are quite good. To apply this theory, the unstable mode should have $\rho_s k_\perp \gtrsim 1/3$ (the wavelength should be shorter than 1 cm). When the ion temperature effect is considered, the discrepancy becomes greater but the solution is still satisfactory.

Until now in this discussion, it has been assumed that the theoretical predictions alone can explain the diffusivity over a whole plasma region. It is clear that this assumption is incorrect, because the diffusivity inside the $q = 1$ surface, at least, cannot be explained by these predictions. It is possible to explain the behavior of $\chi_e$ by combining several theoretical predictions [30]. This means that each theory may be applicable at a limited location in the plasma. From these studies, in the region between $q = 1$ and $q = 2$ the Perkins DTE theory and its relatives are in good agreement with the observations if $\lambda_\perp > 2$ cm. However, another model may be required to explain the diffusivity at the outer region, especially for NBI plasmas.

There are several unsolved problems. First, the mode numbers of the unstable fluctuations associated with the anomalous transport are unknown; this makes it difficult to determine whether the collisionless mode or the dissipative mode causes the anomalous transport. As pointed out in the experimental observations, the confinement time does not correlate with the scale lengths at all. Moreover, the
experiments indicate that $L_p$ and $L_T$ are strongly constrained regardless of the operational regime. It is not clear how to deduce the electron pressure and temperature profiles from these transport theories and whether the resultant profiles will be consistent with the observed ones.

Furthermore, we have not directly addressed here the question of whether one should be 'surprised' by the apparent constraints on the profile shapes [31]. As a comment on this topic, however, we pointed out the very different profile shapes of $\chi_e$ in the OH and NBI phases, which are apparently required to keep the temperature and pressure profiles unchanged. Whether these changes in $\chi_e(r)$, which are observed consistently in high-density NBI experiments [32], should be considered 'surprising' is perhaps best judged in the light of the theoretical models studied here. None of them show a definitive tendency towards increased peaking to the outside with neutral injection. One may then be left with the hypothesis that the ion transport provides the physical mechanism to maintain the observed constraint on $T_e(r)$ [33] or is enhanced by convection due to the asymmetric potential [34].

4. SUMMARY

The experimental observations are summarized as follows:

1. The scale lengths decrease as the radius increases, indicating sharp profiles in the outer region. The fractional standard deviations also decrease as the radius increases, providing the relatively rigid profiles in the outer region.

2. The radial shapes of the scale lengths of the electron temperature and pressure gradients at $r/a > 0.4$ remain unchanged regardless of heating power and plasma current. $L_p$ appears to be more constrained than $L_T$.

3. The radial shape of the scale length of the electron density gradient for $r/a > 0.5$ appears to be constrained from changing in this data set (gas-puff fueled). In general, however, the fractional standard deviation of $L_n$ is larger than those of $L_T$ and $L_p$.

4. With increasing plasma radius, $\eta_e$ decreases from a value of $\sim 2.2$ at $r/a = 0.43$ to a value of 1.3 at $r/a = 0.9$, which is still larger than 1. The standard deviation of $\eta_e$ is relatively larger than that of the other quantities.

5. These observations appear to hold for a fairly wide range of TFTR operational regimes (OH and L-mode). This indicates that theories which explain the constraint of the profiles should be applicable to both OH and L-mode plasmas.
6. No direct correlations of the scale lengths and $\eta_e$ with the confinement time are found over a wide range of plasma operations. The saturation in energy confinement in the high-density OH regimes for neo-Alcator scaling does not result from changes in the electron scale lengths for this data set.

We used these observations to examine the theoretical predictions of the anomalous electron heat transport, with the following results.

7. When $k_{\perp} \rho_e \leq 1/3$ is assumed, TFTR plasmas are in a dissipative regime. The thermal diffusivity associated with the DTE mode predicted by the theory of Perkins and its relatives appears to be in fairly good agreement with the observations (OH plasmas), because the sharp drop in $L_n$ in the relation $T_e^{7/2}/L_n^2 \eta_e$ compensates for the effect of $T_{e}^{7/2}$.

8. Since the wavelengths have not been measured, it is difficult to determine definitively whether the dissipative models or the collisionless models should be used to establish the electron thermal diffusivities in TFTR. When $k_{\perp} \rho_e \gg 1/3$ is assumed, the thermal diffusivity associated with the Perkins CTE mode is acceptable. Experimental observations of fluctuations will resolve the uncertainty about whether the collisionless or the collisional theories are applicable to TFTR plasmas.

9. The thermal diffusivity predicted by Kadomtsev and Pogutse is also acceptable to explain TFTR OH plasmas, but it has the same problems as the other theories for NBI plasmas.

10. It is not clear whether these turbulent theories can explain the constraint of the profile shape over a wide range of operational regimes from OH to NBI plasmas.

11. The theoretical predictions do not show the tendency of $\chi_e$ to peak towards the outside with increasing NBI power, as observed in the experimental data. This indicates that energy loss near the plasma edge through the ion channel may play an important role in NBI plasmas.
ACKNOWLEDGEMENTS

The ORNL authors (S.H., M.M., C.E.B.) thank Dr. D.M. Meade at PPPL and Dr. J.L. Dunlap at ORNL for their support. One author (S.H.) expresses his appreciation to Prof. D. Sigmar of MIT and Dr. J. LeBoeuf of ORNL for helpful discussions.

This work was sponsored by the Office of Fusion Energy, U.S. Department of Energy, under contract DE-AC02-76-CHO-3073 with Princeton University and contract DE-AC05-84OR21400 with Martin Marietta Energy Systems, Inc.
REFERENCES


<table>
<thead>
<tr>
<th>Plasma conditions</th>
<th>Number of shots</th>
<th>$L_n$ Mean value</th>
<th>$L_T$ Mean value</th>
<th>$L_p$ Mean value</th>
<th>$\eta_e$ Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\sigma$</td>
<td>$\sigma$</td>
<td>$\sigma$</td>
<td>$\sigma$</td>
</tr>
<tr>
<td>$B = 4.7$ T,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_p = 2.2$ MA</td>
<td>14</td>
<td>0.63</td>
<td>0.29</td>
<td>0.34</td>
<td>0.14</td>
</tr>
<tr>
<td>OH</td>
<td>15</td>
<td>0.67</td>
<td>0.33</td>
<td>0.38</td>
<td>0.12</td>
</tr>
<tr>
<td>NBI</td>
<td>16</td>
<td>0.49</td>
<td>0.16</td>
<td>0.34</td>
<td>0.16</td>
</tr>
<tr>
<td>$B = 3.9$ T,</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$I_p = 1.8$ MA</td>
<td>7</td>
<td>0.49</td>
<td>0.16</td>
<td>0.34</td>
<td>0.16</td>
</tr>
<tr>
<td>OH</td>
<td>9</td>
<td>0.61</td>
<td>0.17</td>
<td>0.37</td>
<td>0.09</td>
</tr>
<tr>
<td>NBI</td>
<td>16</td>
<td>0.66</td>
<td>0.19</td>
<td>0.32</td>
<td>0.12</td>
</tr>
</tbody>
</table>

All shots        106  0.63  0.26  0.35  0.14  0.22  0.12  1.8  0.28
TABLE II. DATA FOR ONE SERIES OF TFTR EXPERIMENTS WITH
$B_T = 4.7$ T, $I_p = 2.2$ MA, $q = 2.8$. (DATA BASED ON SNAP ANALYSIS;
SEE REF. [13] FOR ADDITIONAL INFORMATION.)

<table>
<thead>
<tr>
<th>Case</th>
<th>$P_{\text{heat}}$ (MW)</th>
<th>$\tilde{n}_e$ ($10^{19}$ m$^{-3}$)</th>
<th>$n_e(0)$ ($10^{19}$ m$^{-3}$)</th>
<th>$T_e(0)$ (keV)</th>
<th>$T_i(0)$ (keV)</th>
<th>$\tau_E(a)$ (s)</th>
<th>$\tau_{Ee}(a)$ (s)</th>
<th>Shot number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.01</td>
<td>4.41</td>
<td>5.69</td>
<td>2.62</td>
<td>2.60</td>
<td>0.413</td>
<td>0.303</td>
<td>14726</td>
</tr>
<tr>
<td>2</td>
<td>2.83</td>
<td>4.40</td>
<td>5.76</td>
<td>2.89</td>
<td>2.96</td>
<td>0.324</td>
<td>0.222</td>
<td>14732</td>
</tr>
<tr>
<td>3</td>
<td>3.54</td>
<td>4.27</td>
<td>5.70</td>
<td>3.39</td>
<td>3.36</td>
<td>0.277</td>
<td>0.226</td>
<td>14730</td>
</tr>
<tr>
<td>4</td>
<td>4.21</td>
<td>4.45</td>
<td>5.94</td>
<td>3.61</td>
<td>3.92</td>
<td>0.253</td>
<td>0.182</td>
<td>14731</td>
</tr>
<tr>
<td>5</td>
<td>5.15</td>
<td>4.58</td>
<td>6.04</td>
<td>3.85</td>
<td>4.07</td>
<td>0.216</td>
<td>0.156</td>
<td>14729</td>
</tr>
<tr>
<td>6</td>
<td>6.14</td>
<td>4.79</td>
<td>6.30</td>
<td>3.70</td>
<td>4.21</td>
<td>0.201</td>
<td>0.132</td>
<td>14727</td>
</tr>
<tr>
<td>7</td>
<td>6.38</td>
<td>4.73</td>
<td>6.28</td>
<td>4.76</td>
<td>4.80</td>
<td>0.207</td>
<td>0.159</td>
<td>14734</td>
</tr>
</tbody>
</table>
FIGURE CAPTIONS

FIG. 1. Normal distribution of $L_p$ values with $L_p = 0.22$ m and $\sigma = 0.023$ m versus the percentage probability in the interval $\Delta L_p = 0.02$ m for a 106-shot data set. $L_p$ is evaluated at $r/a = 0.57$. The statistical analysis of this group is acceptable.

FIG. 2. Top: scale lengths and $\eta_e$ for a $B = 3.9$ T, $I_p = 1.8$ MA plasma. The bar on $\eta_e$ represents $\pm \sigma$. Bottom: fractional standard deviations of $L_n$ and $L_p$. Open data points: OH plasmas. Closed data points: NBI plasmas.

FIG. 3. Radial profile of the scale length of the electron temperature profile. The data point numbers correspond to the case numbers in Table II. Data for cases 3, 5, and 7 are plotted only at $r/a = 0.57$. $B = 4.7$ T, $I_p = 2.2$ MA.

FIG. 4. Radial profile of the scale length of the electron density profile. The data point numbers correspond to the case numbers in Table II. Data for cases 3, 5, and 7 are plotted only at $r/a = 0.57$. $B = 4.7$ T, $I_p = 2.2$ MA.

FIG. 5. Radial profile of the scale length of the electron pressure profile. The experimental conditions are the same as those in Figs. 3 and 4.

FIG. 6. $L_p$ versus heating power at several radial positions. The plasma conditions are those of Table II. Open points: OH plasmas. Solid points: NBI plasmas.

FIG. 7. Radial profile of $\eta_e$ for different levels of heating power. The plasma conditions are those of Table II.

FIG. 8. Radial profiles of the scale length of the electron pressure profile for operation at different $q$-values. At $r < r_1$, $L_p$ increases sharply. At $r_1 < r < 0.6$, $L_p$ is almost constant. When $r > 0.6$, $L_p$ decreases linearly. These trends are observed over a fairly wide range of operational regimes except at very high $q$ (see the case of $B = 4.7$ T and $I_p = 0.8$ MA).

FIG. 9. $L_n$, $L_T$, $L_p$, and $\eta_e$ versus $I_p$ for $B = 3.9$ T (solid circles) and 4.7 T (open circles). The data are from both OH and NBI plasmas.

FIG. 10. $L_n$, $L_T$, and $L_p$ versus line-averaged density for 4.7 T, 2.2 MA NBI operation. Data are taken at $r/a = 0.57$.

FIG. 11. $\nu_{eff}/\omega_e$ as a function of the radius, where $3\rho_s k_\perp \sim 1$ is assumed.

FIG. 12. $\chi_e$ proposed by Kadomtsev and Pogutse (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas.
FIG. 13. $\chi_e$ derived by Guzdar et al. (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas.

FIG. 14. $\chi_e$ derived by Perkins for the DTE mode (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas.

FIG. 15. $\chi_e$ proposed by Diamond for the DTE mode (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas.

FIG. 16. $\chi_e$ derived by Ross (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas.

FIG. 17. $\chi_e$ proposed by Diamond for the collisionless DW mode (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas. The calculation displayed here is multiplied by 10.

FIG. 18. $\chi_e$ proposed by Perkins for the CTE mode (open circles) and the experimental value (solid line) with $B = 4.7$ T, $I_p = 2.2$ MA (a) for OH plasmas and (b) for NBI plasmas.
Fig. 24

ORNL-DWG 87-2660R

TFTR

\[ \exp \left[ -\frac{(L_{p_e} - \overline{L}_{p_e})^2}{2\sigma^2} \right] \]

\( \overline{L}_{p_e} = 0.22 \)

\( \sigma = 0.023 \)

PROBABILITY

0.15 0.20 0.25 0.30

Fig. 24
Fig. 2
TFTR
B = 4.7 T
I_p = 2.2 MA

Fig. 3
Fig. 4

TFTR
B=4.7 T
I_p=2.2 MA

L_e (m) vs r/a

27
Fig. 5
TFTR
4.7 T, 2.2 MA

Fig. 6
ORNL-DWG-87-2674R2 FED

TFTR
B = 4.7 T
I_p = 2.2 MA

\[ \frac{\eta_e}{I_e} = \frac{L_{n_e}}{L_{T_e}} \]

Fig. 7
Fig. 8a
Fig. 8b
Fig. 9
Fig. 10
Fig. 11
Fig. 12
Fig 13

OH NO. 14726

NBI NO. 14729
Fig. 14
Fig. 15
Fig. 16
Fig. 17
Fig. 18