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**Comparison of Austin Research Associates
and Los Alamos Scientific Laboratory
Loop-Drive Boundary Conditions**

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COMPARISON OF AUSTIN RESEARCH ASSOCIATES AND
LOS ALAMOS SCIENTIFIC LABORATORY LOOP-DRIVE BOUNDARY CONDITIONS

by

William R. Shanahan

ABSTRACT

A comparison of two different models for boundary conditions appropriate to the resonant-loop-drive slow-wave structure is presented. The first model is based on a low frequency approximation in which a capacitive field-current relationship is invoked. The second model is essentially a rigorous field theory approach in which no such low-frequency assumption is made. Significantly lower growth rates are obtained from the latter approach than from the former. Reasons for this difference are sought in an examination of the radial eigenfunctions of the azimuthal electric field.

Interaction of a relativistic electron beam with the resonant-loop-drive slow-wave structure has been proposed by Austin Research Associates (ARA) as a method of achieving slow cyclotron wave growth alternative to that provided by use of a helical structure.¹ At the time that this proposal was made, an accompanying analytic study seemed to indicate that growth rates and lengths comparable to those achievable by the use of a helix were to be expected. During the past few months, we at LASL have conducted an independent study of this proposed method of wave growth.² Early in the course of this study, it became apparent that the validity of several of the assumptions in the ARA analysis was not altogether self-evident. Since these assumptions, particularly those pertaining to boundary conditions, were made in the interests of obtaining an analytically tractable model of the loop-drive, we embarked upon a purely numerical investigation with what were deemed to be a more realistic set of boundary conditions. Growth rates lower than an order of magnitude of those predicted by ARA have

been obtained. The purpose of this report is to delineate the differences between ARA and Los Alamos Scientific Laboratory (LASL) boundary conditions and to present numerical evidence for the importance of these differences.

The resonant-loop-drive slow-wave structure, proposed by ARA, consists of a z-slotted waveguide about which are placed conducting straps, which are interrupted periodically, both axially and azimuthally, by capacitive gaps. The periodicity introduced into configuration space by the presence of the gaps is mirrored, via Floquet's theorem, by the w-k space structure of the positive-energy vacuum mode, thereby intersecting the less affected negative-energy cyclotron mode. A resonant transfer of energy from the latter to the former is thus effected, giving rise to growth of the desired cyclotron mode. Physically, the radial modulations requisite to cyclotron mode growth may be viewed as arising from interaction between the first-order azimuthal velocity perturbation and the zero-order axial magnetic field. The first-order velocity perturbations are driven by azimuthal electric fields generated inductively by the time-dependent first-order axial magnetic field which is produced by the azimuthal currents flowing in the conducting straps. These currents are driven by the azimuthal electric fields, thus completing the feedback loop necessary for any instability.

The ARA analysis of the growth mechanism described above proceeded on the basis of a number of assumptions.¹ Firstly, although the azimuthal capacitive gaps are in reality situated at discrete points about the circumference, it was assumed that these are sufficiently numerous that the capacitance may be assumed to be uniformly distributed. In the spirit of low-frequency lumped-element circuit theory, it was asserted that from this it followed that the current in the inductive strap and the azimuthal electric field were related by the capacitive relation:

$$I_{\theta} = C \frac{1}{d} \int_0^d \dot{E}_{\theta}(z) dz \quad , \quad (1)$$

where an average of the electric field has been effected over the axial length of the strap. This equation was then combined with well-known j' conditions on the tangential magnetic field to give

$$\Delta B_z(z) = \frac{4\pi C}{d} \sum_{n=-\infty}^{\infty} \text{Str}(z) \int_{nL}^{d+nL} \dot{E}_\theta(z') dz' , \quad (2)$$

where $\text{Str}(z)$ is that combination of step functions specifying the position of the conducting straps. Using Eq. (2), supplemented by the corresponding conditions for the tangential electric field, matching was effected between interior and exterior solutions to yield the desired dispersion relation. Analysis of this dispersion relation yielded the significant growth rates referred to above.

Our principal reservations concerning this analysis center about an apparent conflict between the assumptions which underlie the capacitive relation, Eq. (1), and the basic jump condition leading to Eq. (2). Essentially, Eq. (1) implies an identification of the conduction with the displacement current, which is equivalent to assuming $\text{curl } B = 0$, whereas the jump condition Eq. (2) requires that this quantity be finite. Further, the preceding analysis assumes that the azimuthal electric field is finite at every point about the circumference of the loop, where in reality the electric field will vanish, to within a very good approximation, on the conducting straps. Since the azimuthal electric fields play a key role in the growth mechanism, it is not altogether clear that this assumption is not without significant consequences. Of course, it was also recognized that the assumption of continuously distributed capacitance precluded the possibility of examining the coupling among azimuthal modes induced by the azimuthal gaps.

Despite these objections, it might be thought that the ARA analysis would nevertheless yield qualitatively correct results and that the points just alluded to would yield only refinements of these results. Such basically low frequency arguments are frequently used in microwave engineering and often lead to quite satisfactory results. To resolve these issues, we have conducted a numerical investigation of the resonant-loop-drive employing a more realistic set of boundary conditions. These boundary conditions are simply that the azimuthal electric fields vanish everywhere on the conducting straps and possess continuous radial derivatives in the gaps. The z-slotted waveguide assures that $E_z = 0$ at the loop radius. These conditions may be summarized as

$$\left(\frac{\partial E_{\theta}^{(1)}(r)}{\partial r} - \frac{\partial E_{\theta}^{(2)}(r)}{\partial r} + \left\{ E_{\theta}^{(1)}(r) - \left[\frac{\partial E_{\theta}^{(1)}(r)}{\partial r} - \frac{\partial E_{\theta}^{(2)}(r)}{\partial r} \right] \right\} \times \text{Str}(z, \theta) \right)_{r=a} = 0, \quad (3)$$

where a designates the radius of the slow-wave structure and $\text{Str}(z, \theta)$ is that combination of step functions specifying both the axial and azimuthal positions of the conducting straps. The superscripts (1) and (2) refer respectively to the regions interior and exterior to the loop drive. Further, the entire beam slow-wave structure system was assumed to be surrounded by an outer conducting wall at which the usual metallic boundary conditions apply.

These boundary conditions have been incorporated into a numerical code, GRADR, designed previously by B. Godfrey³ to investigate nonperiodic geometries. GRADR performs normal mode analyses of linear perturbations made about self-consistently generated equilibria. GRADR produces not only the proper eigenfrequency, but also plots of the corresponding radial eigenfunction. For purposes of comparison, two different versions of ARA boundary conditions were also implemented in the code. One version was identical to that used by ARA, whereas the other dispensed with the z -averaging present in Eq. (1). Because GRADR deals with normal modes of given axial wave number and azimuthal quantum number, the above equations must be subjected to several manipulations before they are programmed. Details appear elsewhere.² Here we summarize the results.

Our results are presented in three sets of two graphs each. The first graph in each set exhibits the growth rate plotted as a function of axial wave number. The second exhibits the nature of the azimuthal electric field as a function of the radial coordinate for an eigenfunction corresponding to a point near the maximum of the first graph. These graphs have been selected from a more complete set of radial eigenfunction graphs generated by GRADR. As will be noted further below, the azimuthal electric field plot selected for display here is the most revealing of these graphs.

The parameters used to generate these graphs were chosen to model the proof-of-principle experiment. These were radius of beam = $2.65 c/\omega_b$, loop radius = $3.8 c/\omega_b$, and $\omega_c/\omega_b = 2$, where ω_c is the zero-order cyclotron frequency and ω_b is the beam plasma frequency. In addition, the outer wall has a radius

fifteen times that of the loop which, for these purposes, is essentially infinite. Further, while the tuning of the slow-wave structure may be simply achieved in the case of ARA boundary conditions by the adjustment of a free parameter, for the LASL boundary conditions this must be accomplished by appropriate selection of the various geometric factors involved. For the cases presented here, we have chosen d/L and $\theta_s/\theta_0 = 0.5$, where L is the axial periodicity length, d is the axial length of the strap, and θ_0 and θ_s are the corresponding azimuthal quantities. Further, $2\pi/L = 1$ and two azimuthal gaps have been employed. It was found that adjustment of these parameters alone would not yield the desired low resonant frequency. Further reduction was obtained by filling the space between the slow-wave structure and the outer conducting wall with a material of constant dielectric constant. Here we have chosen this constant to be 30. The unit of length is here c/ω_b , which, for projected ARA parameters, is approximately equal to a centimeter.

Figure 1 corresponds to runs made with LASL boundary conditions, Fig. 2 to those made with ARA boundary conditions, while Fig. 3 is identical to Fig. 2 except that the z -averaging of the azimuthal electric field has been removed. The most striking aspect of these graphs is, of course, that the LASL boundary conditions yield much reduced growth rates compared to those obtained with ARA boundary conditions. Consistently, the range of unstable axial wave numbers is also rather narrower. Removing the z -averaging slightly increases the growth rates obtained with ARA boundary conditions and broadens the range of instability. The group velocity observed in the region of instability for all three cases was approximately 0.5. Thus, these temporal growth rates are relevant to the question of growth length.

It is difficult to come to a definitive conclusion regarding the qualitative source of the quantitative differences in the growth rate exhibited here. Examination of the radial eigenfunctions of three cases considered generally fails to reveal any striking differences. However, a comparison of the azimuthal electric field plots reveals an exception to this statement. For all three cases the real parts of this component are approximately equal, whereas for ARA boundary conditions, both averaged and unaveraged, significantly larger values of the imaginary components at the position of the loop are observed. The absolute values of the azimuthal electric field at the loop for the ARA boundary conditions are approximately three times greater than those obtained with LASL boundary conditions. This observation leads us to speculate that the

conducting nature of the inductive straps, which is contained implicitly in the LASL boundary conditions but is absent from the ARA formulation, is a principal source of the differences in growth rate.

Originally, we had intended an extensive study of the resonant-loop-drive in a variety of parameter regimes, including the study of the variation of growth rate with beam energy and magnetic field strength. The observation of such small growth rates, however, seems to render such studies of purely academic interest.

REFERENCES

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Fig. 1. LASL boundary conditions.

Fig. 2. ARA boundary conditions with z-averaging.

Fig. 3. ARA boundary conditions without z-averaging.