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NUMERICAL SIMULATION OF BOOTSTRAP CURRENT

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Numerical Simulation of Bootstrap Current

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Abstract

The neoclassical theory of Bootstrap Current in toroidal systems is calculated in magnetic flux coordinates and confirmed by numerical simulation. The effects of magnetic ripple, loop voltage, and magnetic and electrostatic perturbations on bootstrap current for the cases of zero and finite plasma pressure are studied. The numerical results are in reasonable agreement with analytical estimates.

1 Introduction

In a tokamak, the magnetic confinement configurations rely on a rotational transform traditionally described through the safety factor $q(r) = rB_\phi / RB_\theta$, created by the toroidal current, which prevents particles drifting vertically to the wall. Here θ and ϕ are poloidal and toroidal angles, r the minor radius, R the major radius, and $\vec{B}$ the magnetic field. The usual way to produce such a current is through an inductive electric field (i.e. through $E_{||} = -\frac{\partial}{\partial t} \iint \vec{B} \cdot d\vec{A}$). Since any Ohmic transformer can only contain a finite amount of volt-seconds, it is impossible to produce a steady state tokamak in this way. Some other mechanisms have been proposed for driving current, including neutral beam injection [1], fast wave and lower hybrid wave current drive [2] and neoclassical bootstrap current. The first two have associated problems concerning accessibility and efficiency. Bootstrap current, however, originally proposed by Bickerton *et al.* [3], has lead to an attractive

idea of operating a tokamak in steady state with $E_{\parallel} = 0$. Boozer [4] [5] has shown that 3D resistive MHD could, through a magnetic reconnection process similar to an inverse sawtooth oscillation, also provide a current density at the magnetic axis, leading to a parallel current necessary for producing the poloidal field driven by the pressure gradient only. Thus a 100% bootstrap current tokamak with no additional current drive could result. Even without this process, control of density and temperature profiles can at high β produce bootstrap current profiles which meet most of the current density requirements. Present designs for high β operation of next generation tokamaks rely upon a significant bootstrap contribution to the total plasma current [6] [7] [8] [9].

Many authors have investigated the bootstrap current, including analytical studies of the dependence on the aspect ratio [10] [11] [12] and the collision frequency [13]. A clear confirmation of the bootstrap current has also been found in several different experiments such as those performed on TFTR [14] and JET [15]. In this paper, numerical simulation as well as analytic estimates are used to investigate the neoclassical bootstrap current theory. The effects of toroidal field ripple, and magnetic and electrostatic perturbations in low and high β equilibria are investigated.

Consider an axisymmetric equilibrium of arbitrary aspect ratio, shape, and plasma pressure. The magnetic field can be written in contravariant and covariant form as

$$\vec{B} = \nabla\psi \times \nabla\theta - \nabla\psi_p \times \nabla\xi \quad (1)$$

$$\vec{B} = g(\psi_p) \nabla\xi + I(\psi_p) \nabla\theta + \delta(\psi_p, \theta) \nabla\psi_p \quad (2)$$

where $\psi, \psi_p, \xi, \theta$ are respectively toroidal and poloidal flux and toroidal and poloidal angles. The functions g and I are related to the poloidal and toroidal current and δ describes the degree of nonorthogonality of the system. The toroidal and poloidal flux are related to the safety factor through $q(\psi_p) = d\psi/d\psi_p$. In these variables the guiding center equations are [16]:

$$\begin{aligned} \dot{\psi}_p = & -\frac{g}{D}(\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \theta} + \frac{\rho_{\parallel} B^2 g}{D} \frac{\partial \alpha}{\partial \theta} - \frac{g}{D} \frac{\partial \Phi}{\partial \theta} + \frac{I}{D} \frac{\partial \Phi}{\partial \xi} \\ & - \frac{\rho_{\parallel} B^2 I}{D} \frac{\partial \alpha}{\partial \xi} + \frac{I(\mu + \rho_{\parallel}^2 B)}{D} \frac{\partial B}{\partial \xi} \end{aligned} \quad (3)$$

$$\dot{\xi} = \frac{\rho_{\parallel} B^2 (q + \rho_c I')}{D} - (\mu + \rho_{\parallel}^2 B) \frac{\partial B}{\partial \psi_p} \frac{I}{D} + \rho_{\parallel} B^2 \frac{\partial \alpha}{\partial \psi_p} \frac{I}{D} - \frac{\partial \Phi}{\partial \psi_p} \frac{I}{D} \quad (4)$$

$$\dot{\theta} = \frac{\rho_{\parallel} B^2 (1 - \rho_c g')}{D} + (\mu + \rho_{\parallel}^2 B) \frac{g}{D} \frac{\partial B}{\partial \psi_p} + \frac{g}{D} \frac{\partial \Phi}{\partial \psi_p} - \frac{\rho_{\parallel} B^2 g}{D} \frac{\partial \alpha}{\partial \psi_p} \quad (5)$$

$$\begin{aligned} \dot{\rho}_{\parallel} = & \frac{\mu + \rho_{\parallel}^2 B}{D} \left[\left(g \frac{\partial \alpha}{\partial \psi_p} - 1 + \rho_c g' \right) \frac{\partial B}{\partial \theta} - \frac{\partial B}{\partial \psi_p} \left(g \frac{\partial \alpha}{\partial \theta} - I \frac{\partial \alpha}{\partial \xi} \right) \right] - \frac{\partial \alpha}{\partial t} \\ & - \frac{1 - \rho_c g'}{D} \frac{\partial \Phi}{\partial \theta} - \frac{\mu + \rho_{\parallel}^2 B}{D} \frac{\partial B}{\partial \xi} (q + \rho_c I') - \frac{q + \rho_c I'}{D} \frac{\partial \Phi}{\partial \xi} \end{aligned} \quad (6)$$

with

$$\rho_{\parallel} = v_{\parallel} / B \quad (7)$$

$$\rho_c = \rho_{\parallel} + \alpha \quad (8)$$

and

$$D = \rho_c (g I' - I g') + I + q g \quad (9)$$

where we have introduced a perturbing magnetic field $\delta \vec{B} = \nabla \times \alpha \vec{B}$ and a perturbing electrostatic potential Φ . All primes refer to differentiation with respect to ψ_p . Second order quantities in α and third order quantities in the gyroradius ρ are neglected in this formalism, but the equations of motion exactly conserve the Hamiltonian.

Numerical simulations are carried out using a Hamiltonian guiding center code which has been previously described [17]. A Monte Carlo pitch-angle collision operator is used. Thus we directly simulate only the electron contribution to the bootstrap current caused by collisions with cold background ions. The effects of electron-electron collisions are ignored and the electron temperature T_e is taken as constant for simplicity. A direct comparison with experiment requires the inclusion of the ion contribution. Nevertheless, the study of the electron contribution alone permits examination of dependence on aspect ratio, plasma shape, and collisionality and is thus of considerable interest. Dimensionless units are used above and in the following. The unit of length is the radius of the magnetic axis R_0 ; and the unit of time is given by the reciprocal of the gyrofrequency in the toroidal field $\Omega = e B_T / m$. In these units the basic unit of energy is $m \Omega^2 R_0^2$. For a given field a particle orbit is uniquely determined by ρ , the ratio of gyro radius to system size, and the initial conditions.

In section 2 we briefly introduce analytical estimates for the bootstrap current and its modification by magnetic and electrostatic perturbations. The results of the numerical simulations are discussed in section 3. In section 4 are some concluding remarks.

2 Analytical Estimates

The bootstrap current is due to the collisional trapping and detrapping of particles in the low collision frequency regime. Toroidal curvature and the existence of banana orbits produces a small perturbation of the particle distribution from that of uniform pitch, leading to a net toroidal current. Consider a flux surface ψ , and two associated electron banana orbits with opposite velocities at $\theta = 0$. The inside banana is counter-moving and the outside one is co-moving with respect to the Ohmic current. Because of the density gradient the number of counter-moving particles will be greater than the number of co-moving ones by an amount $-\sqrt{r} W_b dn/dr$, with $W_b \simeq 2 q \rho / \sqrt{r}$ the banana width. A priori there is no such relation between co- and counter-moving passing particles, in the absence of collisions these populations are independent of one another. The bootstrap current is due to an induced asymmetry in the co- and counter-moving passing particle distributions necessary to maintain steady state in the presence of trapping and detrapping through small pitch angle scattering.

With the presence of a magnetic or electrostatic field perturbation, the banana orbit does not close, and the gap can be calculated directly from the equations of motion. If the perturbation is sufficiently large so that the gap is comparable to the banana width, the co- and counter-moving banana orbits are no longer correlated with a density differential given by a banana width due to the random phases, and the bootstrap current is destroyed. We discuss primarily the effect of static perturbations. Fluctuating perturbations will have even larger effects if frequencies exceed the bounce frequency (see Fig. 10 in Sec. 3).

2.1 Neoclassical theory

The *Collisional-Drift-Kinetic* equation in flux coordinates is

$$v_{\parallel} \frac{\vec{B} \cdot \nabla \theta}{B} \left\{ \frac{\partial f_1}{\partial \theta} + g(\psi_p) \frac{\partial}{\partial \theta} \left(\frac{v_{\parallel}}{\Omega} \right) \frac{\partial F_M}{\partial \psi_p} \right\} = C(f_1) \quad (10)$$

where

$$C(f) = \frac{\nu}{B} v_{\parallel} \frac{\partial}{\partial \mu} v_{\parallel} \mu \frac{\partial f}{\partial \mu} \quad (11)$$

and $B \simeq g(\psi_p)/R$, $\mu = v_{\perp}^2/2B$, F_M a Maxwellian.

Expand the first order function f_1 in the low collision frequency limit, i.e. let $f_1 = f_1^0 + \nu f_1^1 + \dots$, we have

$$\frac{\partial}{\partial \theta} \left(f_1^0 + g \frac{v_{\parallel}}{\Omega} \frac{\partial F_M}{\partial \psi_p} \right) = 0 \quad (12)$$

and

$$\frac{\partial f_1^1}{\partial \theta} = \frac{B}{\vec{B} \cdot \nabla \theta} \frac{C(f_1^0)}{v_{\parallel}} \quad (13)$$

or

$$f_1^0 = h - g \frac{v_{\parallel}}{\Omega} \frac{\partial F_M}{\partial \psi_p} \quad (14)$$

and

$$\oint \frac{B}{\vec{B} \cdot \nabla \theta} \frac{C(f_1^0)}{v_{\parallel}} d\theta = 0 \quad (15)$$

here h is an arbitrary function in μ , v , and ψ_p but independent of θ .

Substituting Eq. (12) into Eq. (13), we find h and thus the distribution function f to first order

$$f = F_M - \frac{g}{\Omega} \frac{\partial F_M}{\partial \psi_p} \left[v_{\parallel} - \begin{cases} 0 & \text{if } \mu > \mu_0 \\ \int_{\mu}^{\mu_0} d\mu \frac{B}{<v_{\parallel}>} & \text{if } \mu \leq \mu_0 \end{cases} \right] \quad (16)$$

here

$$\langle A(\psi, \theta) \rangle = \frac{\oint A(\psi, \theta) j d\theta}{\oint j d\theta} \quad (17)$$

where $\mu_0 = v^2/2B_{max}$ defines the boundary between trapped and passing particles and $j = (\vec{B} \cdot \nabla \theta)^{-1}$ is the flux coordinate Jacobian.

Although the distribution function f appears to be independent of the collision frequency, the collision operator modifies f through the solubility condition, Eq. (13).

The corresponding local bootstrap current density is then:

$$j_b = -\langle \int f e v_{\parallel} d^3 \vec{v} \rangle \quad (18)$$

which gives

$$j_b = -f_t R_0 T_e \frac{dn(\psi_p)}{d\psi_p} \quad (19)$$

where f_t , the trapped particle fraction, is

$$f_t = \frac{\langle R \rangle}{R_0} - \frac{3}{4} \frac{\langle B^2 R \rangle}{B_{max}^2 R_0} \int_0^1 \frac{u du}{\langle \sqrt{1 - \frac{B}{B_{max}}} u \rangle} \quad (20)$$

The analytic calculation of bootstrap current j_b thus requires the calculation of the trapped particle fraction f_t . In the ideal (i.e. low β , circular profile) case, Rutherford [18] found $f_t \simeq 1.6\sqrt{r}$ as $r \rightarrow 0$. Since we know that $f_t = 1$ when $r \rightarrow 1$ from Eq. (14) ($\mu_0 = 0$ in this case), if we assume that the trapped particle fraction has the form $f_t = c_1\sqrt{r} + c_2 r$, we find $f_t = (1.6\sqrt{r} - .6r)$. This expression agrees very well with the numerical calculation of Eq. (18). With this approximation, the explicit expression for bootstrap current in the ideal case is:

$$j_b = -(1.6\sqrt{r} - .6r) \frac{T_e}{B_\theta} \frac{dn(r)}{dr} \quad (21)$$

Although we use a simplified collision operator, the expression of f_t obtained is very close to those given by Hazeltine [10] and Hirshman [12]. The comparison is depicted in Fig. 1.

In the presence of significant plasma pressure or elongation of the equilibrium, the flux surfaces are more complex, and the calculation of f_t is not tractable analytically (An analytical calculation of f_t has not been given by either Hazeltine et al. or Hirshman for high β non circular equilibria). In these cases, we numerically calculate f_t from the given numerical equilibrium.

Define an effective inverse aspect ratio through the cylindrical coordinate $R(\psi_p, \theta)$

$$r_e(\psi_p) = \frac{R(\psi_p, 0) - R_0}{R_0} \quad (22)$$

(Note $R_0 = R(0,0)$ is the radius of the magnetic axis, not the geometric plasma center, and in high β equilibria they are very different). With this definition we find

$$f_t = 1.6 \sqrt{r_e} - .6 r_e \quad (23)$$

is still a very good approximation to the trapped particle fraction for a large aspect ratio tokamak.

To find the collision frequency dependence it is necessary to solve the *Collisional-Drift-Kinetic* equation to second order. We do not attempt to do that here, but we will numerically examine the dependence on collisionality directly.

2.2 Magnetic Perturbation

Consider an axisymmetric equilibrium magnetic field $B = 1 - r \cos \theta$ and a perturbing field $\delta \vec{B} = \nabla \times \alpha \vec{B}$ with $\alpha = \sum \alpha_{mn} \sin(n\phi - m\theta + \delta_{mn})$ which produces islands at rational surfaces $q = m/n$ with width $W_{mn} = 4q \sqrt{\alpha_{mn}/(rq')}$ where $q' = dq/dr$, and δ_{mn} are random phases. With this simple large aspect ratio equilibrium the generalized toroidal coordinate ξ is simply the cylindrical coordinate ϕ . Our simulations are carried out locally, in the vicinity of a given flux surface. Thus for simplicity, we choose a $q(r)$ profile linear in r , and a spectrum of modes with a fixed value of n , with a range of m values, giving a distribution of islands uniformly spaced in r . Amplitudes α_{mn} are chosen to give a uniform value of stochasticity parameter S over the domain of interest. The stochasticity parameter S is defined as island width divided by island spacing, so $S = 1$ corresponds to the Chirikov overlap criterion. However, as is well known, stochastic threshold occurs for significantly smaller values of S , and the exact value of S giving stochastic threshold depends on the equilibrium and on the spectrum used. In particular we found by direct examination of the Poincare map that for constant perturbation strength higher β equilibria have a lower stochastic threshold value.

Now consider the effect of the field perturbations on the particle orbits. After a straightforward calculation, we find from Eq. (3) that a banana orbit fails to close in one bounce time by an amount given by

$$\Delta(r) = \sum_{m,n} \frac{4mq\theta_b}{r} \alpha_{mn} \frac{\sin[(m-nq)\theta_b]}{(m-nq)\theta_b} \cos \delta_{mn} \quad (24)$$

here θ_b is the poloidal angle of the bounce point, $\langle \theta_b \rangle \simeq \pi/2$. Obviously, the only modes which are relevant are those near resonance, i.e. we have $q(r) \simeq m/n$. Thus

$$\frac{\sin[(m-nq)\theta_b]}{(m-nq)\theta_b} \simeq 1 \quad (25)$$

The *cosine* factor is random, so we find for the magnitude of the drift

$$\Delta_m \simeq \frac{4mq\theta_b}{r} \alpha_{mn} \quad (26)$$

Using $b_{mn} \simeq m\alpha_{mn}/r$, where b_{mn} is the (m, n) Fourier component of $\delta B_r/B_0$, and requiring that the orbit perturbation exceed the banana width, $\Delta_m \geq W_b$, we have

$$\alpha_{mn} \geq \frac{\sqrt{r}}{2m\theta_b} \rho \quad (27)$$

or

$$b_{mn} \geq \frac{\rho}{2\theta_b\sqrt{r}} \quad (28)$$

which gives

$$\langle (\frac{\delta B_r}{B_0})^2 \rangle^{1/2} \geq \sqrt{\frac{M}{2}} b_{mn} = \sqrt{\frac{M}{2r}} \frac{\rho}{\pi} \quad (29)$$

Here M is the total number of the Fourier components. The value of $\delta B_r/B_0$ for typical tokamak parameters is about 10^{-4} .

The above result is valid for only those mode spectrum with the perturbation amplitude constant everywhere.

In the case of a linear $q(r)$ profile, since

$$S = 4m \sqrt{\frac{q' \alpha_{mn}}{r}} \quad (30)$$

we find the condition for the bootstrap current to be destroyed to be:

$$S \geq 4 \sqrt{\frac{mq' \rho}{\pi \sqrt{r}}} \quad (31)$$

where ρ is the gyroradius of the electrons in units of R_0 .

Choosing $q = 3$, $q' = 80$, $r_0 = .1$, $T_e = 1KeV$ (or $\rho = 8.9 \times 10^{-6}$), $n = 7$, $17 \leq m \leq 25$, gives $S \sim 0.47$.

2.3 Electrostatic Perturbation

With an electrostatic perturbation $\Phi = \sum \Phi_{mn} \sin(n\phi - m\theta + \delta_{mn})$, the banana tip shift in one bounce time is:

$$\Delta(r) = - \sum_{m,n} \frac{4 m q \Phi_{mn}}{\rho r^{3/2}} \int_0^{\theta_b} d\theta \frac{\cos[(m - nq)\theta + \delta_{mn}]}{\sqrt{\cos \theta - \cos \theta_b}} \quad (32)$$

Average $\Delta(r)$ over the mode spacing $\tau = (nq')^{-1}$, i.e. calculate

$$\langle \Delta(r) \rangle = \frac{1}{2\tau} \int_{r_s - \tau}^{r_s + \tau} \Delta(r) dr \quad (33)$$

here r_s is the nearest rational surface given by $q(r_s) = m/n$, noting that only resonant terms contribute, we find

$$\langle \Delta(r) \rangle = \sum_{m,n} \frac{4 m q \Phi_{mn}}{\rho r^{3/2}} \int_0^{\theta_b} d\theta \frac{\sin \theta}{\theta \sqrt{\cos \theta - \cos \theta_b}} \cos \delta_{mn} \quad (34)$$

Using

$$\int_0^{\theta_b} d\theta \frac{\sin \theta}{\theta \sqrt{\cos \theta - \cos \theta_b}} \simeq \frac{2}{\theta_b} \quad (35)$$

we find the magnitude of the shift

$$\Delta_e \simeq \frac{16 m q \Phi_{mn}}{\pi \rho r^{3/2}} \quad (36)$$

The condition for destruction of the bootstrap current then becomes

$$\frac{\Phi_{mn}}{T} \geq \frac{\pi r}{4 m} \quad (37)$$

or

$$\frac{\delta \Phi}{T} \geq \frac{\pi r}{4 \sqrt{2 M}} \quad (38)$$

Again, M is the total number of the Fourier components. In a tokamak the right hand side is about a few percent.

For a particle with pitch angle equal to 1, the electrostatic perturbation is equivalent to a magnetic perturbation with an island width $W_{mn} = 4 q \sqrt{\Phi_{mn}/(r q' \rho)}$, this gives the minimum equivalent stochastic parameter

$$S \geq \sqrt{2 \pi m q' \rho} \quad (39)$$

Again, if we use the mode spectrum given above, we find the bootstrap current to be destroyed at $S \sim 0.3$.

2.4 Magnetic Ripple

The effect of toroidal field ripple, due to the discrete toroidal field coils in a tokamak can be examined using the model magnetic field

$$B = 1 - r \cos \theta + \delta \sin(N\phi) \quad (40)$$

where δ is the ripple magnitude and N is the number of toroidal field coils.

To calculate the shift across magnetic surfaces of the bounce point caused by ripple, we find the difference between the cross surface drift with and without ripple. Consider a deeply trapped particle, with energy $E = v_{\parallel}^2/2 + \mu B$ and $v_{\parallel}^2 \ll E$ throughout the orbit. In a low β , cylindrical equilibrium, for an average bounce point $\theta_b = \pi/2$, this shift is

$$\Delta r \simeq \left(\frac{q}{r}\right)^{3/2} \sqrt{2\pi N} \rho \delta \quad (41)$$

Goldston *et al* [19] have discussed the confinement of high energy (MeV) trapped particles in tokamaks. They found that if the ripple exceeds a certain limit,

$$\delta \geq \left(\frac{r}{2\pi N q}\right)^{3/2} / (\rho q'), \quad (42)$$

the radial drifts dominate, the phase space of banana tip trajectories become stochastic, and there is a resultant loss of all trapped particles. Of course one can not expect bootstrap current to persist in this case.

For low energy (KeV) particles the above condition is difficult to satisfy. However for the destruction of the bootstrap current it is sufficient to require $\Delta \geq W_b$, i.e.

$$\delta \geq \sqrt{\frac{2}{N\pi q}} r \quad (43)$$

For $N = 16, q = 3, r_0 = .2$, we find $\delta \sim 2\%$.

3 Numerical Simulation

The bootstrap current is measured locally. An initial Monte Carlo particle distribution is produced with a triangular density profile, extending from ψ_1 to ψ_2 , centered at $\psi = \psi_0$ with $\psi_1 < \psi_0 < \psi_2$, uniformly distributed in pitch λ , and in θ , and ξ . Each particle is advanced in time using the guiding center

equations. Once a particle exits from the domain, it is reinserted at ψ_0 with λ, θ, ξ random. The particles are advanced in time to produce a steady state density distribution. The velocity distribution is Maxwellian generated using Monte-Carlo techniques. The run time, and the time for a particle to diffuse through the domain of interest, are chosen to be at least a few collision times so that there is enough time for equilibration between trapped and passing populations. Typically 10^5 particles are used and the domain ψ_1, ψ_2 extends over 20 banana widths. After collisional equilibration steady state is achieved with approximately constant density gradient, positive for $\psi < \psi_0$ and negative for $\psi > \psi_0$. Associated bootstrap currents are produced in each domain opposite in sign, allowing the independent determination of both bootstrap current and Ohmic current in the case of an applied toroidal electric field.

First consider experiments carried out with no magnetic or electrostatic perturbations of the equilibrium field. Figures 1 - 3 show the numerical simulation results for a low β , circular analytic equilibrium. The comparison of f_i found in this paper to two previously derived expressions [10] [12] is shown in Fig. 1. The expression given in reference [10] gives $f_i > 1$ for $r > .60$, which is clearly nonphysical, so this result is not valid for large r . The expression given in reference [12] is the one most commonly used in the literature. Our result agrees with it within 20% in spite of the simplified collision operator used. Fig. 2 depicts the dependence of j_b on the local inverse aspect ratio. Even though Eq. (19) is a simple interpolation of the two limiting results, the numerical data agrees with the theoretical value within 40 % over the whole domain. Fig. 3 gives the dependence on the collision frequency. From Hinton and Rosenbluth [11], the frequency ν dependence is $(1 + \sqrt{\nu^*} + .54\nu^*)^{-1}$, where $\nu^* = \nu_{eff}/\omega_b$, $\nu_{eff} = \nu/r$, $\omega_b = v_{th}\sqrt{r}/qR_0$ in the large aspect ratio limit. But our numerical experiments show a much weaker frequency dependency. Even for $\nu^* = 1$ there still exists about 60% of the low collision frequency value of the bootstrap current.

For equilibria with high β or noncircular cross section analytical estimates are more difficult. However, it is very easy to calculate f_i numerically and thus calculate from Eq. (17) the theoretical value for the bootstrap current. Fig. 4 gives the trapped fraction at three quite different β values. Shown is the normalized trapped particle fraction as a function of effective inverse aspect ratio r_e . Note that Eq. (21) is a reasonable estimate of the trapped fraction even at high β for a large aspect ratio tokamak. It should be pointed

out that it is not correct to use the similar formula for high β but medium or small aspect ratio tokamaks because the magnetic field and the flux surfaces deviate considerably from the low β case. We must numerically measure the trapped particle fraction in this case. Fig. 5 shows the ratios of the numerical simulation values of the bootstrap current to the theoretical calculations as a function of the poloidal flux surface ψ_p normalized to the outermost flux surface. It is clear that the analytic calculation is reasonable for both low and high β cases. This is contrary to the analytic result recently found by Cowley [20] who showed that in the high β limit the bootstrap current is multiplied by a factor $\sqrt{\varepsilon/(\beta q^2)}$ because of a reduction of the trapped particle fraction. In our case, this factor is about $1/q$ or about $1/3$, but no such reduction is observed.

Now consider the effects of field perturbations. The effects of magnetic and electrostatic perturbations on bootstrap current in a low β circular equilibrium are given by Fig. 6 and Fig. 7. For these experiments, there were 9 modes with $n = 7, 17 \leq m \leq 25$. Three of these modes had rational surfaces within the domain $.098 \leq r \leq .102$ which was the domain used for the bootstrap determination. The q profile is given by $q(r) = 80r - 5$. The perturbation thresholds given by our analytic estimates fit the experiments within a factor of 2. From Fig. 6 and Fig. 7 it is also clear that the loop voltage does not significantly affect the result. Fig. 8 shows that the bootstrap current is only affected by the perturbations of those rational surfaces located in the relevant domain, i.e. the addition of other nonresonant perturbations has no influence on the magnitudes of the bootstrap current. Modes (5 - 15) are not resonant within the domain $\psi_1 \leq \psi \leq \psi_2$. Fig. 9 depicts the influence of the magnitude of the perturbation on the bootstrap current using high β numerical equilibria. It is clear that the bootstrap current in higher β equilibria is more sensitive to perturbations than in lower β equilibria due to the lower stochastic threshold for the fields. The effect of magnetic perturbations with time-dependent amplitude is shown in Fig. 10. The modes were of the form $\sum \sin(n\phi - m\theta + \delta_{mn} - \omega t)$ and the amplitudes were chosen with $S = .1$, so that at low frequency they produce negligible effect. It is clear that the perturbation has no significant influence on the bootstrap current until the frequency reaches the bounce frequency, at which point the particle response is no longer adiabatic and significant orbit modification results. The destruction of bootstrap current due to magnetic ripple is shown in Fig. 11.

Again, the numerical results agree with our analytic calculation very well. Note that the ripple has negligible influence on the bootstrap current until it reaches the critical point. In all the plots from Fig. 6 to Fig. 11 the bootstrap currents are normalized to the values obtained in the absence of field perturbations.

4 Conclusion

In this paper, the neoclassical bootstrap current is calculated in magnetic flux coordinates by using a simple pitch-angle scattering collision operator and confirmed directly by numerical simulations. Modification of bootstrap current due to magnetic ripple, magnetic and electrostatic perturbations and finite β have been studied. Analytical estimates are given for each of the perturbations considered. The mechanism for the destruction of the bootstrap current is easily understood. Once the banana tip shift due to the perturbation is comparable to the banana width, the bootstrap current will be destroyed. It is apparent from the numerical experiments that the bootstrap current is sensitive neither to the loop voltage nor to a finite β . But the bootstrap current is more easily destroyed by field perturbations in a high β equilibrium than in a low β equilibrium. It is also found that the current only responds to the local perturbations, that is, the local bootstrap current is only affected by perturbations with rational surfaces located within one banana width.

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Figure 1: Comparison of f_t by Hazeltine et al (Dashed), by Hirshman (Solid) and by Wu and White (Dotted).

Figure 2: Bootstrap current j_b vs local inverse aspect ratio r .

Figure 3: Normalized bootstrap current j_b versus ν^* . The dashed curve is the theoretical prediction given by Hinton and Rosenbluth.

Figure 4: The trapped particle fraction f_t for three different β values, using numerical equilibria. (f_t^* is normalized value to the f_t defined in Eq. (21))

Figure 5: Bootstrap current from theory and numerical experiment for low and high β equilibria.

Figure 6: Destruction of bootstrap current j_b by magnetic perturbation in a low β circular equilibrium.

Figure 7: Destruction of bootstrap current j_b by electrostatic perturbation.

Figure 8: Perturbation mode number dependence at $S = .2$.

Figure 9: The effects of magnetic perturbation on low and high β numerical equilibrium profiles.

Figure 10: The perturbation frequency dependence of j_b at $S = .1$.

Figure 11: Destruction of bootstrap current j_b by magnetic ripple.

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