Development and Utilization of New Diagnostics for Dense-Phase Pneumatic Transport

Quarterly Technical Progress Report (July 1, 1991 to September 30, 1991)

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Summary of accomplishments

Dense-phase pneumatic transport is an attractive means of conveying solids. Unfortunately, because of the high solid concentrations, this transport method is a difficult regime in which to carry out detailed measurements. Hence most details of the flow are unknown.

In this context, the main objective of this work is to develop probes for local measurements of solid velocity and holdup in dense gas-solid flows. In particular, we have designed capacitance probes to measure local, time-dependent particle concentrations, and a new optical fiber probe based on laser-induced-phosphorescence to measure particle velocities. We have described the principle of the capacitance diagnostic in [Particulate Science & Tech., 7 51:59 (1989)] and its calibration in [Powder Tech. 62, 85-94 (1990)]. In addition, we have recently published data on the optical anemometer in [Applied Optics 30, 1976-81 (1991)].

Because we anticipate that recent theories of rapid granular flows will bring insight to the dense pneumatic transport of particles, we have sought to substantiate these theories through computer simulations. There we have verified the theory of Hanes, Jenkins & Richman (1988) for the rapid, steady shear flow of identical, smooth, nearly elastic disks driven by identical, parallel, bumpy boundaries [Phys. Fluids A 2 (6), 1042-44 (1990)]. Also, in examining the theory of Jenkins & Richman (1988) for the rapid flows of uniform smooth inelastic disks under simple shear, we have revealed the formation of microstructures there [Phys. Fluids A 3 (1), 47-57 (1991)]. In addition, because granular flows depend strongly on the nature of their interaction with a boundary, we have verified the boundary conditions calculated by Jenkins (1991) for spheres interacting with a flat,

In the previous reporting period, we have concluded a series of experiments with plugs of flour in the dense-phase pneumatic setup. We have interpreted these experiments in the present reporting period.

**Progress Report**

1) **A model for gas flow in cohesive plug transport**

In the previous quarterly report, we have presented data gathered at Cornell in the dense-phase transport of cohesive flour plugs. In these experiments, we have shown that (1) the velocity difference between the gas and the solid plug is small; (2) that the pressure across the plug rises linearly with gas velocity; and (3) that the extrapolated pressure drop at zero velocity is proportional to the plug length (Fig. 1). In this reporting period, we have developed a simple model of the gas flow in the porous plug that explain these effects, at least qualitatively. We begin with a description of this model.

![Fig. 1](image-url)

**Fig. 1.** Pressure drop across plugs vs. gas velocity. The open circles, closed squares, closed circles and open squares represent plugs of length 4, 4.5, 8 and 11 cm, respectively. The solid lines are best fits through the data.
We consider the flow of a porous plug of constant velocity \( v \) and constant voidage \( \varepsilon \) in a vertical tube of radius \( R \). The acceleration of gravity is \( g \). We assume that the gas flow in the plug is fully-developed, axisymmetric, steady, laminar, and in the viscous regime everywhere.

The particle and gas momentum balances are linked by an interaction term. In determining the form of the interaction, we follow the variational analysis of Jenkins [J. Appl. Mech. 47, 493 (1980)] for the pressure part and extend this to a general state of stress. In general, one part of this interaction contributes \((-p\delta_{ij} + \tau_{ij}) \partial(1-\varepsilon)/\partial x_j\) to the gas phase momentum balance, where \( \tau_{ij} \) is the shear stress tensor and \( p \) is the pressure. Then in the gas phase momentum balance \( S \) appears outside the derivative of all stress terms:

\[
(-p\delta_{ij} + \tau_{ij}) \partial(1-\varepsilon)/\partial x_j + \partial(-\varepsilon p\delta_{ij} + \varepsilon \tau_{ij})/\partial x_j = \varepsilon \left[ \partial(-p\delta_{ij} + \tau_{ij})/\partial x_j \right].
\]

Because the flow is fully-developed, the radial velocity component of the gas is zero, and the gas momentum Eq. becomes

\[
0 = -\varepsilon \frac{\partial p}{\partial z} + \varepsilon \frac{1}{r} \frac{d}{dr}(r \mu \frac{du}{dr}) - C_d (u-v), \tag{1}
\]

where \( u \) is the vertical gas velocity component, \( C_d (u-v) \) is the drag force exerted on a unit volume of the plug, \( \mu \) is the molecular viscosity of the gas, \( r \) is the radial coordinate, and \( \partial p/\partial z \) is the gradient of pressure in the gas.

The drag force is determined by analogy with D'Arcy's semi-empirical relation for a stationary porous medium in the viscous regime:

\[
-\frac{\partial p}{\partial z} = 150 \frac{(1-\varepsilon)^2}{\varepsilon^3} \frac{\mu u_g}{(\phi d)^2}, \tag{2}
\]

where \( u_g \equiv u \) is the superficial gas velocity, \( d \) is the mean Sauter diameter of the particles, and \( \phi \) is their sphericity. For a stationary plug, the combination of Eqs. (1) and (2) yields

\[
C_d = 150 \frac{(1-\varepsilon)^2}{\varepsilon} \frac{\mu}{(\phi d)^2}. \tag{3}
\]

The boundary conditions for the gas velocity are that it vanishes at the wall, and that its derivative with respect to the radial coordinate vanishes at the centerline by symmetry. For convenience we define the local slip velocity \( U \equiv (u-v) \) between the gas and the plug, and the constants

\[
A \equiv - \frac{1}{\mu} \frac{\partial p}{\partial z}; \quad B \equiv 150 \frac{(1-\varepsilon)^2}{\varepsilon} \frac{\mu}{(\phi d)^2} \tag{4}
\]
Thus the slip velocity satisfies the ODE

$$0 = A + \frac{1}{r} \frac{d}{dr} \left( r \frac{dU}{dr} \right) - BU$$

(5)

with the boundary conditions

$$\frac{dU}{dr} = 0 \text{ at } r=0 ,$$

(6a)

and

$$U = -\nu \text{ at } r=R .$$

(6b)

The solution is

$$U = \frac{A}{B} - (v + \frac{A}{B}) \frac{I_0(r\sqrt{B})}{I_0(R\sqrt{B})} ,$$

(7)

where $I_0$ is the modified Bessel function of the second kind [Handbook of Math. Funct., p.377]. At the wall, the shear stress is

$$\tau_w = \mu \frac{dU}{dr} = - (v + \frac{A}{B}) \frac{\mu \sqrt{B} I_1(R\sqrt{B})}{I_0(R\sqrt{B})} .$$

(8)

For typical conditions, $R\sqrt{B} >> 1$ and $\nu >> A/B$. Because as $z$ becomes large,

$$I_0(z) \sim I_1(z) \sim \frac{ez}{\sqrt{2\pi z}} ,$$

(9)

the wall shear stress is nearly equal to

$$\tau_w \sim - \mu \nu \sqrt{B} ,$$

(10)

and the slip velocity $U$ is small except in the near vicinity of the wall. At the centerline,

$$U = A/B \sim 0;$$

(11a)

and near the wall,

$$U = - \nu \frac{I_0(r\sqrt{B})}{I_0(R\sqrt{B})} \sim - \nu \frac{e\sqrt{B}}{eR\sqrt{B}} \left( \frac{R}{r} \right)^{1/2} .$$

(11b)

Thus under typical conditions, the gas velocity is within 1% of that of the plug at a distance $l$ from the wall of order the particle mean diameter:

$$l = \ln(100)/\sqrt{B} \sim 0.4 \phi d \varepsilon/(1-\varepsilon)$$

(12)

Assuming that the solid shear stress is constant along the plug, a global balance of forces on the suspension at constant elevation yields
\[ 0 = \left( -e \frac{\partial p}{\partial z} + \frac{2}{R} \varepsilon \tau_w \right) + \left( -\frac{d\sigma_{zz}}{dz} + (1-e) \frac{\partial p}{\partial z} - \rho_s (1-e) g + \frac{2}{R} \tau_s \right), \tag{13} \]

where \( \rho_s \) is the material density of the particles, \( \tau_s \) is the shear stress in the particle phase at the wall (in general, \( \tau_s < 0 \)), and \( \sigma_{zz} \) is the normal stress on horizontal surfaces of the particle phase. In (13), the first two terms arise from the gas phase, and the other four are, respectively, the gradient of normal stress \( \sigma_{zz} \), the buoyancy force exerted by the gas on the particle phase, the gravitational force, and the solid shear at the wall. Using a Mohr circle construction, Konrad, et al. (1980) derived a relation between \( \sigma_{zz} \) and \( \tau_s \). From that relation and the assumption of a constant \( \tau_s \), it follows that \( d\sigma_{zz}/dz = 0 \). For the entire plug of length \( L \), the absolute pressure drop in the gas is therefore

\[ \frac{\Delta p}{L} = \rho_s (1-e) g + \frac{2 \varepsilon \mu \sqrt{B}}{R} + \frac{2 |\tau_s|}{R}. \tag{14} \]

Figure (2) illustrates the predictions of the Eq. (14) for the gas pressure across plugs of various lengths. These plugs are suspended in air with bulk density \( \rho_s (1-e) = 420 \text{kg/m}^3 \), mean modified Sauter diameter \( \phi_d = 5 \mu \text{m} \), \( (1-e) = 65\% \) and \( |\tau_s|/R = 390 \text{ Pa/m} \) in a tube of 1 cm ID. These numerical values are reasonable estimates for the flour used in the present experiments, and the agreement with experimental data is reasonable.

![Figure 2](image_url)

**Fig. 2.** The solid lines are predictions of Eq. (14) for the parameters mentioned in the text. The symbols have the same meaning as in Fig. 1.

2) Optical fiber measurements of particle concentration in dense suspensions

In suspensions with relatively large density \( (1-e \geq 10\%) \), it is convenient to employ optical fiber sensors to measure the local volume fraction of solids. This technique was
discussed in our 8-th quarterly report. Its main difficulty is to obtain a quantitative
calibration of the sensor output with local particle volume fraction. We have attached to the
present report a paper submitted to *Applied Optics* on calibration and modeling techniques
that we have developed for this sensor.

3) **Next research and conclusions**

In this reporting period, we have primarily developed a simple model for gas flow
in cohesive plugs. This quarterly report concludes the work carried out under the present
contract. During the no-cost extension of this work approved by the DOE, we will prepare
the final report.

**References**


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M.Y. Louge & J.T. Jenkins: "Development and Utilization of New Diagnostics for Dense-Phase Pneumatic
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M. Louge, J. Jenkins & M. Hopkins: "The relaxation of the second moments in rapid shear flows of
Clarkson University, August 1991.

**Appendix**

Preprint of the paper by D.J. Lischer and M.Y. Louge: "Optical fiber measurements of

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