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SOME ASPECTS OF THE THEORY OF HIGH TEMPERATURE SUPERCONDUCTORS

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ABSTRACT

A qualitative survey of the theory of high temperature superconductors as strongly correlated electron systems is presented. Various microscopic models and their implications for the theory of the normal state are described. Constraints on the mechanism and many-body theory of the superconducting state are discussed.

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1. INTRODUCTION

The investigation of high-temperature superconductors¹ is at an interesting stage. There is by now a reasonable body of agreed experimental data² which places quite severe constraints on the character of the normal state and the mechanism of superconductivity. At the same time, attention has focussed on a few microscopic models that seem to encompass the essential features of the problem and may well explain what is going on once acceptable means of solving them have been developed. Of course, as with any rapidly moving area of investigation, current views are subject to change as new data or new ideas come along. Nevertheless, the general features of what has to be explained are emerging, and a context for discussing them has been developed. Here an attempt will be made to convey the flavour of the field to those who are not actively working in it. In view of space limitations, band structure, phonon mechanisms, or exotic scenarios such as anyon superconductivity will not be discussed, especially as they are addressed in the articles by W. Pickett³ and D. H. Lee⁴ in this volume. Rather, the focus will be on high-temperature superconductors as strongly-correlated quasi two-dimensional electron systems, and on the implications of experiments for models, the many-body theory, and the mechanism of high-temperature superconductivity. Wherever possible, a comparison with more conventional superconductors and liquid ^3He will be made.

The new materials present a real challenge to solid state theory. It might have been that the basic physics was rather mundane and that progress in the field rested solely on the ingenuity of the materials scientist. Fortunately for theorists, this appears not to be the case, and it seems that genuinely new theoretical insight might be required to unravel the puzzles presented by the experimental data that are already available. Whether some of the more exotic proposals have any basis in fact remains to be seen.

2. PHASE DIAGRAM AND MICROSCOPIC MODELS

The general features of the phase diagram⁵ of high temperature superconductors are illustrated by $La_{2-x}Sr_xCuO_{4+y}$. The undoped stoichiometric material ($x = 0 = y$) is an antiferromagnetic insulator. Adding strontium or oxygen induces a concentration $\delta = x + 2y$ holes into the copper oxide planes, as may be seen by using simple valence counting (assuming La^{3+} , Sr^{2+} , Cu^{2+} , and O^{2-}) to evaluate the charge imbalance. For small δ , the holes are localized by their donors, and qualitatively the properties of the material do not change too much. But when $\delta > 0.02$, the holes delocalize, causing long range antiferromagnetic order to give way to a "frozen spin" phase⁶ which persists up to $\delta \approx 0.15$. Superconductivity appears for $\delta \gtrsim 0.05$: the transition temperature T_c first increases to a maximum of about 38K then flattens out, finally going to zero again when $\delta \gtrsim 0.25$. Similar behavior is seen in all families of high temperature superconductors containing copper oxide planes, although usually it is not possible to go to sufficiently high doping to drive T_c down to zero.

The frozen spin phase is but one indication of the rather "noisy" character of the oxide superconductors. Even in the best samples, with the highest values of T_c in their class, there are background spin⁵ and charge⁷ fluctuations that persist even in the superconducting state. In view of the prevalence of oxygen defects and random elements such as strontium dopants, it is tempting to attribute these fluctuations to the imperfect nature of the materials themselves. But there are indications that the fluctuations are an intrinsic property of strongly-correlated electron systems and are intimately related to the occurrence of high-temperature superconductivity. This possibility will be discussed further in Section 6.

Early band structure calculations³ showed that the relevant states are holes in the $3d_{x^2-y^2}$ orbitals of copper and the $2p_x$ or $2p_y$ orbitals of oxygen. It was then quite natural to base a two-dimensional tight-binding model⁸ solely on these orbitals, introducing a $Cu - O$ hybridization parameter t_{pd} , site energies ϵ_d and ϵ_p , Coulomb energies U_d and

U_p between two holes in the same state and interaction V between holes on neighbouring copper and oxygen sites. Given such a model, the first consideration is whether it is better to work in the strong or weak coupling limit – that is whether some or all of the interactions U_d , U_p , and V are large or small compared to the single-particle terms t , and $\epsilon_p - \epsilon_d$. This is a common problem in solid state physics, especially when the parameters are determined by fitting experiments. Often the two limits work equally well. The first estimates⁸ placed t_{pd} , $(\epsilon_p - \epsilon_d)$, and V in the range $1 - 2\text{eV}$ and gave $U_p \simeq 3\text{eV}$, $U_d = 8 - 10\text{eV}$. These values are not too different from those obtained by more recent cluster calculations and fits to high-energy spectroscopy,^{9,10} which also find a substantial direct oxygen-oxygen hybridization t_{pp} . Clearly, at least U_d is in the intermediate to strong coupling range.

This conclusion is supported by neutron scattering experiments⁵ on undoped La_2CuO_4 which agree well with a Heisenberg model for localized spins having a d -character in excess of 90%. Such a picture is readily understood via strong-coupling perturbation theory. When $t_{pd} = t_{pp} = 0$ and $\epsilon_p > \epsilon_d$, all of the relevant holes are on copper sites in La_2CuO_4 . Switching on t gives the superexchange (see Section 5) interaction J of the Heisenberg model and mixes some fraction of oxygen $2p$ states into the wave functions of the holes. The latter effect reduces the d -character to about 80% for the assumed values of the parameters, a little smaller than suggested by the neutron scattering form factor value, cited above.

Another feature of the superconducting materials that fits quite naturally into the strong-coupling picture is that only the doped holes are mobile.² When $t_{pd} = t_{pp} = 0$, doped holes go onto oxygen sites because $U_d > (\epsilon_p - \epsilon_d + 2V)$ makes it energetically unfavourable to put them on copper. Hybridization ($t_{pd}, t_{pp} \neq 0$) reduces the p -character of the doped holes by admixture of copper d -states and also enables them to move throughout the relatively empty oxygen lattice. However the pre-existing holes (with mainly copper character) remain localized.

Thus strong coupling is a very reasonable starting point for a theory of high-temperature

superconductors and it has been adopted quite widely. More will be said about in the next section. But the weak coupling point of view has not been neglected. Indeed, some experiments may be understood more easily in that way. For example, angle-resolved photoelectron spectroscopy increasingly shows¹¹ that the Fermi surface coincides with the expectations of band structure calculations. Since the parameters of the Hamiltonian were constrained to agree with band structure in mean field theory^{8,10} and the latter is the starting point for weak-coupling calculations, it is clear that the Fermi surface experiments present no problem. However, since the Fermi surface contains all holes, those added by doping as well as the ones that were present before, there appears to be a conflict with the strong-coupling picture of the carrier concentration. In fact, the situation is well-understood in one dimension,^{12,13} and it appears to be simply a technical matter to resolve it for the two-dimensional Hamiltonian adopted here. The point is that the carrier concentration is a property of the *charge degrees of freedom*, but the Fermi surface is a one-electron property involving *both charge and spin*. Provided this distinction is recognized, there is no real conflict. A weak coupling calculation will get the carrier concentration right if it takes proper account of umklapp scattering¹² in this nearly commensurate half-filled band situation. In strong coupling, spin correlations are essential for calculating one-electron.¹³

However, the fundamental question is whether the qualitative physical behaviour is necessarily the same in the two limits. Again the example of one dimension is useful. In the case of the single-band Hubbard model ($U_p = U_d = U$, $\epsilon_p = \epsilon_d$, $V = 0$, $t_{pp} = 0$), it has been shown¹² that the behaviour depends on the sign of U but not its magnitude. In the language of the renormalization group, scaling proceeds to a strong or weak coupling fixed point, whatever the starting value of $|U|$: only the energy scale changes with U . However the same cannot be said for an extended model, ($V \neq 0$). For example, when $V < 0$, superconducting fluctuations give way to phase separation as $|V|$ is increased.¹⁴ The same may well be true in two dimensions (see Section 6) but the phase boundaries of the general

model have yet to be mapped out with any degree of accuracy.

3. STRONG COUPLING LIMIT

Much of the theoretical work on high-temperature superconductors as correlated electron systems has focussed on the strong-coupling limit and, in particular, on the so-called $t - J$ model. The latter assumes that there is only one spatial state per unit cell and that, in undoped materials such as La_2CuO_4 , each site is singly occupied by a hole. Since the energy U of a doubly-occupied site is assumed to be large, the only relevant degree of freedom at low energies is the spin, which is coupled to that of its neighbors by an exchange integral J . Doping produces doubly occupied sites which have no spin degrees of freedom (because they are singlets) but carry the charge of the doped hole. The latter may hop to a neighbouring site with amplitude t , provided that site is not already doubly occupied. This is the simplest model of doped holes in an antiferromagnet, and it was suggested by Anderson¹⁵ that it is a suitable starting point for a theory of high-temperature superconductors. In this approach, the undoped materials are Mott-Hubbard insulators in that there is a gap for charge excitations that necessarily involve doubly occupied sites.

The model described in Section 2 takes a different point of view. There, the low-lying charge excitations of the undoped materials move holes from copper to oxygen sites since $U_d > (\epsilon_p - \epsilon_d + V)$. For this reason, the parent compounds are in the class of “charge-transfer” insulators.¹⁶ This is closely related to the fact that the charge of a doped hole is largely on oxygen, as described in Section 2. In that case, the spin of the doped hole may play a role in the low-energy physics. However, Zhang and Rice¹⁷ have argued that there is a strong superexchange interaction between the spins of the doped hole and the pre-existing hole in the same cell, which causes them to form a singlet pair whose behaviour is the same as that of a doubly-occupied site in the single-band model. If so, the $t - J$ model would also give a reasonable description of the low-energy physics of a charge-transfer insulator. This

suggestion gave rise to a lively debate¹⁸⁻²¹ about the validity of the $t - J$ model, which still goes on. In my view, the equivalence has not been established on the Hamiltonian level, and it is necessary to compare the *solutions* to find out the similarities and differences. Numerical studies of finite systems have been used to support^{19,20} or contradict²¹ the equivalence. But they may be misleading; the use of a very small cluster with a particular geometry²⁰ will bias the outcome. The case of one dimension is particularly clear.²² In the $U_p, U_d \rightarrow \infty$ limit, the doped holes do not form singlets but retain their full spin degree of freedom. However, they join together with the pre-existing holes to form a single spin system described by a Heisenberg Hamiltonian. The main difference is that doping adds spins in the two-band model but removes them in the single-band case. What effect this may have on other properties depends on the parameters of the Hamiltonian.

One final point in the same vein: a phenomenological analysis²³ of nuclear magnetic resonance experiments²⁴ on the normal state has concluded that the assumption of a single spin fluid is sufficient to account for the data. It is sometimes said that this justifies a *single-band* model. This is not the case: as described above there are interactions between the spins of the doped and pre-existing holes which fall short of forming singlets but give a single spin fluid.

To convey a feeling for the physics of the $t - J$ model, it is simplest to consider the case of a single doped hole, which has been investigated by many people.²⁵ In an ordered antiferromagnet on a square lattice the net spin alternates from site to site. Suppose now that a doped hole is initially at an up spin site and then hops to one of the neighbours. Since the total spin is conserved, there is now a down spin at the original location of the doped hole and therefore three antiferromagnetic bonds are broken. As the doped hole hops further, it creates a string of reversed spins each breaking two bonds (assuming for the moment essentially straight strings). Thus the energy is proportional to the length of the string, and consequently the doped hole will actually oscillate back and forth about its original position. The time scale for this oscillation is governed by the ratio t/J , and it

constitutes the fast motion in the physical limit $t \gg J$. Now exchange processes enable the hole-string complex to move about by flipping (i.e. righting) the two spins at the end of the string, thereby moving the tail (which is the centre of oscillation) by two sites. Thus the slow motion consists of a drift of the hole-string complex at a rate of order J . Evidently this simple picture is complicated by loops, broken strings etc., but the general conclusion holds good. Thus holes in an antiferromagnet seriously perturb spin correlations in their immediate neighbourhood and they become heavy – the bandwidth is $2 - 3 J$ rather than $8 t$ as it would be for a freely moving hole. Since J is known to be 0.13eV from a variety of experiments on magnetic properties^{2,5} the predicted bandwidth is determined rather well. The minimum energy occurs at the four points $(\pm\pi/2, \pm\pi/2)$ in momentum space, which is related to the fact that the hopping is to second neighbour sites.

How does this work out in practice? A variety of experiments, including one which essentially measures properties of isolated holes,²⁶ give an effective mass of about $2m_e$ (where m_e is the mass of an electron). This is about a factor four lighter than the $t - J$ model would suggest. Furthermore Trugman²⁷ has used a model which is very close to the $t - J$ model to calculate various properties of doped La_2CuO_4 (susceptibility, specific heat, plasma frequency, Hall effect, and thermopower). He assumed that the holes do not interact but have the energy spectrum of the modified $t - J$ model. Thus the Fermi surface had hole-pockets about the points $(\pm\pi/2, \pm\pi/2)$. With some exceptions he was able to fit the experiments quite well. But the parameters he used imply an exchange integral of 0.53eV , a factor of four larger than the experimental value. Therefore it seems that the simple $t - J$ model does not account for the properties of high-temperature superconductors. Indeed varying the ratio t/J or allowing for the effects of doping on J makes the discrepancy worse. Moreover it is not clear that the observation of dynamical antiferromagnetic fluctuations in doped materials^{5,28} is compatible with a substantial string region around a doped hole. A possible way out is to allow for hopping t' to second neighbour sites, preserving the number of degrees of freedom of the original model. Small cluster calculations²⁰ suggest

that $t' \approx t/6$. But there is very little work on the motion of a single doped hole in such a model, and it remains to be seen if the addition of t' will rectify the situation. Moreover, it also is necessary to take into account the phenomenon of phase separation, to be discussed in Section 6.

4. VALIDITY OF FERMI LIQUID THEORY

Landau's theory of a Fermi liquid²⁹ was developed for systems such as liquid ^3He . It assumes that at temperatures well below the Fermi degeneracy temperature there are well-defined fermion quasiparticles which dominate the properties of the system. The point is that the phase space for quasiparticle-quasiparticle scattering is limited by the exclusion principle and, as a consequence the lifetime τ of a quasiparticle on the Fermi surface is proportional to T^{-2} at low temperatures. Thus the uncertainty $\hbar/\tau$ in the energy of a quasiparticle is small compared to the mean energy (which is proportional to T) and the quasiparticle is well-defined. However, liquid ^3He is almost ferromagnetic and there are important corrections,³⁰ (caused by the effects of spin fluctuations) which must be added to $\hbar/\tau$. The corrections are of order T^3 rather than T^4 , as might otherwise have been expected. Similarly there are contributions of order T^3 to transport scattering rates for quantities such as the viscosity.

As pointed out by Anderson,³¹ high temperature superconductors do not follow simple Fermi liquid behaviour. The in-plane resistivity² is linear in T over a very wide temperature range extending downwards to the neighbourhood of T_c , even when T_c is only 6 K. Similarly analyses of the optical conductivity³² and quasiparticle lifetimes obtained by angle-resolved photoemission spectroscopy¹¹ suggest that quasiparticle and transport scattering rates are linear in T rather than quadratic, as simple Fermi liquid theory might suggest. Of course it is possible that there is a temperature T^* below which the scattering rates become quadratic in T , but the experimental indications are² that T^* must be very small and

may well be zero. Evidently the quasiparticles do not become better defined as $T \rightarrow 0$, and, indeed, further analysis suggests^{31,33} that the weight of the quasiparticles vanishes logarithmically in this limit. Thus the Fermi liquid description is at best marginal.

There is no current consensus on the origin of this behaviour. It has been suggested³⁴ that the situation is similar to that of the one-dimensional electron gas where the quasiparticle spectral weight is known to vanish as a power law at low temperatures.¹² Other proposals are that interaction with charge-³³ or antiferromagnetic spin³⁵ fluctuations might be responsible, or that there is an exceptional quasiparticle scattering rate because the Fermi surface is almost nested.³⁶ Whatever the cause, it clearly is of central importance for understanding the materials themselves and for the nature of high temperature superconductivity.

5. MECHANISM OF HIGH TEMPERATURE SUPERCONDUCTIVITY

The fundamental problem is, of course, to identify the mechanism of high temperature superconductivity, that is the source of the attractive interaction that gives rise to pairing. There have been many suggestions but, rather than giving a catalogue of them, it seems more useful to point out phenomenological limitations provided by a study of the magnetic penetration depth λ as a function of temperature for different materials. Very early on, it was found that the temperature dependence of λ was consistent with a two-fluid model or a BCS strong coupling theory.³⁷ This is a significant constraint on theories of high temperature superconductivity because it is inconsistent with models giving a superconducting energy gap vanishing at points or along lines on the Fermi surface.³⁷ But the study of the zero temperature value of λ as a function of carrier concentration in given classes of materials may prove to be of even greater importance since, it leads to the conclusion that superconductivity is produced by non-retarded processes, or equivalently excitations of a high energy scale.³⁸

According to the BCS theory,^{39,40} superconductivity is a consequence of an instability

of the Fermi sea leading to the formation of (Cooper) pairs of electrons. The pairs form because there is an attractive interaction due to the exchange of phonons. The latter is retarded – the lattice is slow to relax enabling two electrons to feel its influence at different times, thereby avoiding the Coulomb repulsion. By the uncertainty principle, an equivalent way of saying the same thing is that the phonon energy scale ($100K$) is much smaller than the electronic energy scale ($10,000K$). That is why, in BCS theory,^{39,40} T_c is proportional to the Debye temperature θ_D , which is the range of energies over which the electron-phonon interaction is effective. In the other (nonretarded) extreme,⁴¹ where the energy of the excitations that are responsible for pairing is high, T_c is proportional to $p_F^2/2m^*$ which is the Fermi energy for free electrons with Fermi momentum p_F and effective mass m^* . This limit is most easily realized in low-density systems such as nuclear matter or neutron stars. It has been suggested that high temperature superconductors are in the same limit.^{8,42} Now the carrier concentration n_c is equal to A/d , where d is the average spacing between copper oxide planes and A is the areal density which is given by $p_F^2/2\pi\hbar^2$ for a circular Fermi surface. Thus, for fixed d , the relationship between T_c and the Fermi energy may be rewritten $T_c \propto n_c/m^*$. Alternatively, superconductivity produced by Bose condensation of tightly bound pairs of electrons would lead to the same conclusion,³⁸ but that too is most compatible with a non-retarded pairing force.

Measurement of λ is a direct way to test this relationship. In the London limit,⁴⁰ which is appropriate for high temperature superconductors, λ is given by

$$\lambda^{-2} = 4\pi n_c e^2 / m^* c^2 \quad (1)$$

at $T = 0$ where e is the charge of the electron and c the velocity of light. Then, according to the above discussion, non-retardation implies that T_c is proportional to λ^{-2} , provided d is fixed. By now, this broad trend appears to be followed⁴³ for materials with T_c ranging from $30K$ to $125K$. Moreover it has been found that a similar correlation exists between T_c and ω_p^2 where ω_p is the plasma frequency.⁴⁴ Since $\lambda^{-2} = \omega_p^2/c^2$, the two experiments

provide the same information. It should be added that, in any given group of materials, T_c flattens out and eventually decreases as the doping increases. This too is an important aspect of the problem and may be related to a departure from the regime of doped holes in a charge-transfer insulator.²²

Of course it is easier to be in a non-retarded situation if the Fermi temperature T_F is not too high. The penetration depth data show that T_F is about 5000K for the Tl -based materials which have the highest value of T_c . This is indeed quite low, but nevertheless the condition for non-retardation is not satisfied for exchange of phonons or for the simplest versions of exchange of spin fluctuations (for which the energy scale is $2J \approx 3000K$). Only charge fluctuations or near-neighbour superexchange^{8,42} meet the requirements. Consequently the requirement of non-retardation is a severe constraint on the mechanism of superconductivity.

Finally, it is interesting to contrast the ways of thinking about the mechanism in the weak- and strong-coupling limits. In weak coupling, two electrons or holes generate an effective attractive interaction by polarising their surroundings – the lattice or the charge- or spin density of the other electrons. This is an appropriate way of looking at the problem when the interactions are regarded as perturbations. But, in strong coupling, it is better to think in terms of the relief of excessive zero point kinetic energy.

Superexchange provides an example of this idea. It is most easily described for the single-band Hubbard model for which there is one spatial state per unit cell. The parameters of the model are a hybridization or hopping amplitude t and an on-site Coulomb interaction U . In hopping perturbation theory, the unperturbed ground state has exactly one electron per site at half-filling. The mean kinetic energy per site is equal to zero – which is a high zero point energy because it would be negative for noninteracting particles. Motion of the electrons is allowed in second order in t , since neighbouring electrons of opposite spin may hop onto each other's site and back again, giving a net energy $-2t^2/U$ per pair. This process is forbidden by the exclusion principle when the spins are parallel. It is

the physical origin of the antiferromagnetic superexchange interaction in the Heisenberg model.

The lowering of zero point energy is the usual way of generating interactions in hopping perturbation theory, which characterizes the strong-coupling limit. In the copper-oxygen model, a hole added to an oxygen site may, through its interactions, lower the zero point energy of the original copper holes.⁴² Two adjacent oxygen holes may do so more effectively thereby generating an effective attractive interaction.⁴² In such a process, the distinction between charge- and spin fluctuations is much less important than in weak coupling: both are important and it is better simply to talk of an electronic mechanism. This is well-understood for one-dimensional systems.^{12,22} Working out in detail the analogous physics in two-dimensional models is a central problem for understanding the mechanism of high temperature superconductivity.

6. PHASE SEPARATION

This section is concerned with what may be a quite general property of mobile holes in an antiferromagnet, a tendency to separate into two phases, one magnetic, the other superconducting.⁴⁵ The idea of phase separation may be explained most easily in the limit in which the kinetic energy of the mobile holes is small compared to the exchange energy of the copper spins. Suppose for a moment that there is full phase separation into a region with no holes and one with hole concentration δ_0 . Now if one hole were transferred from the second region to the first, it would have a lower zero point kinetic energy because it could move unimpeded by other holes. However, because of its tendency to reverse copper spins, it also would break antiferromagnetic bonds. Therefore there would be a net increase in energy and the system remains phase separated. In the opposite limit in which the kinetic energy of the holes is larger than the exchange energy, phase separation still occurs, but it is a consequence of the frustration of the motion of a hole in an antiferromagnet.⁴⁵

Another way of describing the same situation is to regard the uniform phase as a gas of

holes, spread out over the whole system. Then phase separation is equivalent to the formation of a liquid or a self-bound system of holes filling only part of the container. However, the holes are charged and cannot aggregate in this way unless there is an accompanying clustering of negative charge to compensate. This actually happens⁴⁶ in oxygen-doped La_2CuO_4 , since the excess oxygen ions are mobile at temperatures of the order of 250K where the phase separation first occurs. On the other hand strontium ions are not mobile, consequently bulk phase separation cannot take place in $La_{2-x}Sr_xCuO_4$. Nevertheless, there are inevitable fluctuations in the concentration of strontium ions throughout the sample. A gas of holes that would like to phase separate will be unusually responsive to such fluctuations, and even encourage them. The distance over which variations of strontium concentration take place may not be too large, but it can be significant on the scale of magnetic or superconducting coherence lengths which are no more⁵ than 30Å.

The competition between the short-range interactions, which try to cause condensation of the holes, and the long-range Coulomb interaction which tries to keep the concentration uniform may well be responsible for spin glass ordering⁵ at low hole concentrations and for the low energy charge and spin fluctuations at high hole concentrations.⁴⁷ In that sense, it is an intrinsic property of the oxide superconductors.

But there is another way in which it may be important.⁴⁷ Systems such as liquid 3He and nuclear matter are self-bound: the major effect of the (non-retarded) attractive interactions is the formation of a liquid ground state. There is sufficient residual interaction to produce superconductivity⁴¹ (or fermion superfluidity) but it is not high-temperature superconductivity even when measured on the scale of T_F . (T_c/T_F is an order of magnitude larger in the oxides than in liquid 3He). However, if the system could be expanded in some way so that the carrier concentration were lower, T_c/T_F would increase dramatically. This is clear in the case of nuclear matter⁴¹ where the number density is low enough for controlled calculations to be carried out. At low density an approximate expression for T_c

within BCS theory is⁴¹

$$T_c = \text{const. } T_F \exp \left(- \frac{\pi}{2} \frac{m}{m^*} \cot \delta \right) \quad (2)$$

where δ is the phase shift for two-particle scattering at relative angular momentum ℓ and relative momentum k_F . Now the generic behaviour of δ is that it is positive at small values of k (reflecting the longer range attraction) and then, as k increases, δ passes through a maximum and finally becomes negative (as the short range repulsion dominates). For nuclear matter (with $\ell = 0$) and liquid ^3He (with $\ell = 1$), k_F exceeds the value at which δ is a maximum and, since the rapid variation of the right hand side of Eq.(2) is given by the exponential factor, T_c would increase if k_F were decreased. In other words, if the system were "inflated" to lower density, it would become a high-temperature "superconductor."⁴⁷

The oxides appear to have found a way to bring this about.⁴⁷ Holes in an antiferromagnet would like to condense into a self-bound liquid state. But the Coulomb interactions keep them at a lower density, as prescribed by the charge-compensating background of dopants which donated the holes. In such a situation, the residual attraction is stronger and able to give a higher value of T_c . This seems to be a generic way of obtaining high-temperature superconductivity from non-retarded interactions.

7. CONCLUDING REMARKS

It should be evident that this survey of the theory of high temperature superconductors is highly selective and has omitted the contributions of many people working in the field. I apologise to them for doing so and hope that they acknowledge the impossibility of presenting an exhaustive review in a limited space. Nevertheless it should also be clear that, while progress has been made on the phenomenological and fundamental levels, much work remains to be done before the many-body theory is in good order or there is agreement on the mechanism of high temperature superconductivity. An interesting time lies ahead.

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