LA-UR--84-3496

DE65 003779

TITLE: Correlation of Uniaxial and Biaxial Fracture
Strengths of Ceramics

AUTHOR(S): Dinesh K. Shetty, J. J. Petrovic, A. R. Rosenfield and
W. H. Duckworth

SUBMITTED TO: Sixth International Conference on Fracture
4-10 December 1984, New Delhi, India

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.

By acceptance of this article, the publisher recognizes that the U.S. Government retains a nonexclusive, royalty-free license to publish or reproduce the published form of this contribution, or to allow others to do so, for U.S. Government purposes.

The Los Alamos National Laboratory requests that the publisher identify this article as work performed under the auspices of the U.S. Department of Energy.

MASTER

Los Alamos Los Alamos National Laboratory
Los Alamos, New Mexico 87545

CORRELATION OF UNIAXIAL AND BIAXIAL FRACTURE STRENGTHS OF CERAMICS

D. K. Shetty,* J. J. Petrovic,** A. R. Rosenfield*
and W. H. Duckworth*

*Battelle, Columbus Laboratories, 505 King Avenue,
Columbus, Ohio 43201

**Los Alamos National Laboratory, Group MST-5,
MS G730, Los Alamos, New Mexico 87545

ABSTRACT

The increased utilization of brittle materials in structural applications has necessitated a need for better understanding of factors governing fracture strengths in multiaxial stress states. The research on which this paper is based examines the problem of relating fracture strengths of ceramics in biaxial stress states to strengths in uniaxial stress states. The problem is divided into two parts: in the first part fracture criteria for failure of brittle materials from single flaws are evaluated through controlled-flaw experiments in different stress states and fracture mechanics theory. The derived fracture criteria are then employed in a statistical fracture theory framework to predict strengths of ceramics failing from surface flaws.

KEYWORDS

Ceramics; surface flaws; mixed-mode fracture; weakest link theory; biaxial strengths.

INTRODUCTION

The increasing application of ceramics as structural materials has fostered research on the stress-state effects on fracture strength (Duckworth and Rosenfield, 1980). The objective of this research is to develop a sound theoretical basis to account for the two fundamental characteristics of brittle fracture strength: (1) the dispersions in fracture strength resulting from the variability of the critical flaw sizes and the resulting dependence of strength on size, and (2) the interaction of principal stresses in defining a fracture criterion for a single flaw (Shetty, Rosenfield and Duckworth, 1982). The statistical theory of strength of Weibull (1939), incorporating the concept of weakest links or extreme-value statistics, is commonly used in the probabilistic design of ceramic components (Paluszny and Nicholls, 1978). The Weibull theory is often satisfactory to explain the size dependence of strengths of ceramics (Bansal, Duckworth and Niesz, 1976; Katayama and Hattori, 1982). But recent studies have shown that it is not adequate to

account for the stress-state effects on strengths of ceramics. Thus, for example, the present authors in biaxial flexure tests of a glass-ceramic (Shetty and coworkers, 1981) and an alumina ceramic (Shetty and co-workers, 1983) found the biaxial fracture strengths measured in an inert environment to be significantly greater than the Weibull theory predictions based on the uniaxial flexure data. Similar observations have been made in biaxial tension and tension-compression fracture studies by Giovan and Sines (1979) and Petrovic and Stout (1981), respectively.

In a number of papers Batdorf and Heinisch (1978a, 1978b) have discussed the limitations of Weibull theory. They attributed the discrepancies between the Weibull theory predictions and the biaxial strengths measured by Giovan and Sines to the limitations of the normal stress fracture criterion that is implicitly assumed in the Weibull theory. In essence, this explanation says that the normal stress criterion in a weakest-link statistical approach predicts conservatively low strengths in biaxial tension. On the other hand the shear-sensitive or the mixed-mode fracture criteria predict higher biaxial strengths. Batdorf was able to explain the biaxial strength data of Giovan and Sines by invoking a mixed-mode strain-energy-release rate criterion.

In the present paper we examine the above explanation to account for the discrepancies we noted in the Weibull analyses of the strength data for the alumina ceramic. Data reported in the literature for mixed-mode fracture from indenter flaws are reviewed to assess the degree of shear-sensitivity of fracture of ceramics. Weakest-link fracture statistics are, then, formulated for both physically-based and empirically-fitted shear-sensitive fracture criteria. Results of these calculations do indicate the trends suggested by Batdorf. But these refined statistical calculations show only a moderate improvement in terms of agreement between theory and experiments. These results support the view that the measured higher strength in biaxial tension has its origin in a nonstatistical stress-state effect.

INFLUENCE OF SHEAR ON FRACTURE FROM SURFACE FLAWS

Surface flaws produced by microhardness indentation have been employed extensively in recent years in ceramic fracture studies (for a review of this subject see Lawn, 1983). The semicircular surface cracks produced by indentation are generally viewed as model surface flaws with characteristics similar to those of a broad class of surface flaws generated in contact damage processes like particle impact, or surface finishing operations like grinding. Surface flaws produced with a Knoop microhardness indenter have been employed to study mixed-mode fracture in ceramics. The influence of shear stress on fracture inferred from these controlled surface flaw tests was, therefore, considered relevant to formulating mixed-mode fracture criteria for the grinding flaws that controlled strength in the alumina ceramic.

Petrovic and Mendiratta (1976, 1977), Freiman et al. (1979) and Petrovic (1983) have studied mixed-mode fracture from Knoop indenter flaws. The results of the mixed-mode fracture tests are presented in Fig. 1. The tensile stress normal to the crack plane at fracture, σ_N , is plotted versus the shear stress parallel to the crack at fracture, τ . The fracture stresses for the three different materials were normalized with respect to σ_c , the tensile fracture stress when the shear is zero, i.e. when the crack plane is perpendicular to maximum tension. The experimental data in Fig. 1 do suggest a trend of decreasing tensile stress with increasing shear. But there is also significant discrepancy between the results of the tension-torsion tests (filled points) and the results of the bending tests (open points), particularly at high shear

stresses. Reasons for this discrepancy are not clear.

Fig. 1. Shear sensitivity of fracture stresses for ceramics failing from Knoop indenter flaws.

A number of criteria have been put forward for treating brittle fracture from crack-like flaws under conditions of mixed-mode loading. All of them are based on linear elastic fracture mechanics theory. The most rigorous and realistic are those that predict both the critical conditions of loading as well as the direction of crack extension. These criteria have been critically reviewed by Swedlow (1976) and they include the following:

- (a) Maximum hoop stress criterion (Erdogan and Sih, 1963)
- (b) Strain energy release rate criteria (Husain, Pu and Underwood, 1974; Palaniswamy and Knauss, 1978)
- (c) Minimum strain energy density criterion (Sih, 1974).

For the purpose of the present discussion it is only necessary to recognize two significant aspects of the criteria listed above. First, although the different criteria have distinctly different physical bases their resulting predictions of the fracture stresses and the direction of crack growth are very similar. Experimental data, particularly those obtained on ceramics, are not adequate to discriminate among these criteria. Second, as suggested by Palaniswamy and Knauss (1978), for two dimensional crack problems the following empirical relation is a good approximation to the exact relations derived on fracture mechanics grounds:

$$\frac{\sigma_N}{\sigma_c} + \frac{3}{2} \left(\frac{\tau}{\sigma_c} \right)^2 = 1 \quad (1)$$

Figure 1 shows a plot of this fracture criterion. Comparison with the experimental data suggests that it is a poor fit. Several factors make this comparison somewhat qualitative and not precise. The experimental data were obtained with surface cracks, a three-dimensional problem in fracture mechanics, while the fracture criteria discussed above were based on a two-dimensional crack problem. Surface crack fronts are subject to Mode III loading in addition to the Modes I and II discussed in the theories. A more serious problem arises from the nature of the indenter flaws. It has been demonstrated that indenter cracks undergo precursor stable growth prior to unstable extension as a result of the residual plastic wedging force (Lawn, 1983). Under mixed-mode conditions the stable growth extends the cracks out of their original planes and this adds an additional complication to the comparison of the experiments and the theories (Marshall, 1984).

Since the existing mixed-mode fracture theories are not completely adequate to describe the test data for the indenter flaws, an empirical data fit was thought to be a useful approach for the purpose of the present study. With the exception of the result obtained in the pure shear test the remaining data in Fig. 1 can be fitted reasonably well with the following empirical fracture criterion:

$$\left(\frac{\sigma_N}{\sigma_c}\right)^3 + \left(\frac{\tau}{0.6\sigma_c}\right)^3 = 1 \quad . \quad (2)$$

Finally, Fig. 1 also shows the normal stress fracture criterion:

$$\frac{\sigma_N}{\sigma_c} = 1 \quad . \quad (3)$$

The experimental data clearly show that the normal stress criterion is non-conservative, i.e., it predicts unrealistically high tensile stress at fracture and neglects the influence of shear. In the following section we will compare the relative fracture stresses in equibiaxial and uniaxial tension predicted from a weakest-link statistical formulation employing each of the above three fracture criteria.

RELATIVE FRACTURE STRESSES IN UNIAXIAL AND BIAxIAL STRESS STATES

The fracture statistics approach used in this study to calculate fracture probabilities for ceramics subject to uniaxial and biaxial tension is the one outlined by Batdorf and Heinisch (1978a). A summary of their approach is given in Appendix A. In essence, it combines the principles of fracture mechanics and the concept of weakest-flaw fracture for a population of flaws subject to stress.

It is instructive to examine the statistical fracture predictions in two stages. In this section we consider only the stress-state effects by comparing the relative fracture stresses in uniaxial and biaxial tension at equal fracture probabilities. In the following section this is combined with the test specimen size, shape and stress gradient effects to make a quantitative comparison with the data for the alumina ceramic.

Figure 2 shows plots of the ratio of the fracture stresses in uniform equibiaxial tension and uniaxial tension over unit surface areas as a function of the flaw-size distribution parameter more commonly referred to as the

Fig. 2. Ratio of fracture stresses in equibiaxial tension and uniaxial tension for equal fracture probabilities as a function of Weibull modulus.

Weibull modulus, m (see Appendix). The three lines plotted in the figure represent the biaxial stress-state effects on fracture predicted by the three fracture criteria discussed in the previous section. All the three criteria predict a lower fracture stress in equibiaxial tension than in uniaxial tension. The degree of this "weakening" in biaxial tension is dependent on the specific fracture criterion employed and the value of the Weibull modulus. The normal stress criterion (curve C in Fig. 2) predicts the highest degree of weakening particularly for low values of the Weibull modulus, m . This result is due to the fact that the influence of shear stress on fracture is ignored in uniaxial tension and an artificially high fracture stress is predicted for a given probability of fracture. With increasing influence of shear taken into account (curves B and A), the fracture probability in uniaxial tension increases or fracture stress at a given probability decreases and the ratio, σ_b/σ_u , approaches a value closer to unity. But the effect of biaxial tension is always greater than the combined effect of uniaxial tension and shear, and the statistical theory always predicts a net weakening.

It is also noted in Fig. 2 that the Weibull modulus has a strong influence on the fracture predictions. This can be qualitatively understood as follows. When the Weibull modulus is large only the flaws oriented nearly normal to the principal stress contribute significantly to the fracture in uniaxial tension. This causes both a decrease in the relative influence of shear as well as in the overall degree of weakening predicted by the statistical theory.

The qualitative trends shown in Fig. 2 are very similar to those reported by Batdorf and Heinisch (1978a), although the specific fracture criteria investigated here are quite different.

UNIAXIAL AND BIAXIAL FRACTURE STRESSES OF AN ALUMINA CERAMIC

The fracture mechanics and statistical analyses discussed in this paper were primarily aimed at critically examining the relations between the fracture stresses of an alumina ceramic tested in uniaxial (four-point and three-point) and biaxial (uniform-pressure loaded disks) flexure tests. The test material, specimen preparation, the strength tests and the application of the Weibull fracture statistics have been described in two earlier papers (Shetty and co-workers, 1983; Shetty, Rosenfield and Duckworth, 1982). Here we will only consider the final strength data in relation to the refined fracture mechanical and statistical analyses discussed in this paper.

Fig. 3. Weibull plots of fracture stresses for sintered alumina tested in inert environment (dry N_2)

Figure 3 shows the Weibull plots of the fracture stresses for the alumina ceramic tested in an inert environment. The fracture stresses measured in four-point bending were used as the experimental base line for the statistical analyses. The Weibull fit to these data (square points and heavy solid line in Fig. 3) gave $m = 23.8$. The three light solid lines identified by the letters A, B and C are the biaxial fracture stress predictions based on the respective fracture criteria shown in Fig. 2. The unmarked light solid line is the three-point bend fracture prediction from the four-point bend data. Note that the statistical prediction of the three-point bend stresses is independent of the fracture criterion. This is because the effects of fracture criteria are the same for the two uniaxial bend tests, while their differences only arise from the size effect, i.e. differences in surface areas of the specimens subject to tension.

Examination of Fig. 3 reveals that the size effect in uniaxial tension is

accurately predicted (note that the solid and the dotted lines overlap for the three-point tests). This result was of course anticipated and redundant in light of our earlier Weibull analyses. The three biaxial fracture predictions show significant discrepancy relative to the measured biaxial stresses. The discrepancy is largest with the prediction based on the normal stress fracture criterion (Weibull statistics) and smallest with the prediction based on the noncoplanar crack extension criterion (Eq. 1). The improvement in the agreement between the theory and the experiment by employing the mixed-mode fracture criterion is at best modest. Also, the empirical fracture criterion (Eq. 2) does not produce any better result than the physically-based mixed-mode fracture criterion. The major reason for the small differences among the predictions of the three fracture criteria is the relatively high Weibull modulus for the ceramic. As noted earlier in relation to Fig. 2 significant differences in predictions arise only when the Weibull modulus is about 10 or smaller. The results of Figure 3 confirm our earlier conclusion that was based on the Weibull analysis. The existing statistical fracture theories are adequate to treat the size effect on strengths of ceramics, but they are not as satisfactory in treating the stress-state effect on strength in biaxial tension.

SUMMARY DISCUSSION AND CONCLUSIONS

The experimental results and the analyses presented in this paper serve to clarify several issues involved in relating the strengths of ceramics in uniaxial and biaxial tension. The first issue is the limitation of the Weibull theory. As pointed out initially by Batdorf and Heinisch (1978a) and confirmed in this study the limitation of the Weibull theory resides in its adoption of the normal stress fracture criterion. It neglects the influence of shear in uniaxial tension fracture and as a result predicts conservatively low biaxial fracture stresses. Statistical fracture formulations that use the mixed-mode fracture criteria do show the correct qualitative trends, but the improvements achieved in terms of the agreement with the experimental data is marginal for ceramics with a flaw population characterized by a high value of the Weibull modulus. As a result a significant discrepancy persists between the measured biaxial fracture stresses and those calculated from the uniaxial data.

Reasons for the failure of the statistical fracture theory to adequately account for the biaxial fracture stresses of the alumina ceramic are not exactly clear. But the recent fractographic observations of Rice, Mecholsky and Becher (1981) provide some insight to the problem. Surface flaws generated in grinding of polycrystalline ceramics are not randomly oriented semicircular surface cracks assumed in the statistical fracture theory. Surface flaws in grinding tend to be oriented in two preferred directions; the direction of grinding and the direction normal to it. Secondly, the two sets of orthogonal surface flaws are likely to interact and the resulting strength-controlling flaw complex is likely to behave differently than an idealized single surface crack. Preliminary results from research currently in progress indicate that a statistical fracture theory suitably modified to account for the above characteristics of grinding damage flaws can better account for the biaxial fracture stresses of ceramics.

ACKNOWLEDGEMENTS

This paper was based on research supported by the Division of Materials Sciences, Office of Basic Energy Sciences, U. S. Department of Energy under

Contract W-7504-Eng-92, Task 127. The authors are grateful to Mr. P. R. Held for his assistance in experimental work.

REFERENCES

- Bansal, G. K., W. H. Duckworth and D. E. Niesz (1976). J. Am. Ceram. Soc., 59, 472-478.
- Batdorf, S. B. and H. L. Heinisch, Jr. (1978a). J. Am. Ceram. Soc., 61, 355-358.
- Batdorf, S. B. and H. L. Heinisch, Jr. (1978b). Eng. Frac. Mech., 10, 831-841.
- Duckworth, W. H. and A. R. Rosenfield (1980). In F. Vincenzini (Ed.), Energy and Ceramics. Elsevier Scientific Pub. Co., Amsterdam. pp. 645-660.
- Erdogan, F. and G. C. Sih (1963). J. Basic Eng., Trans. ASME, 85D, 519-527.
- Freiman, S., A. C. Gonzalez and J. J. Mecholsky (1979). J. Am. Ceram. Soc., 62, 206-208.
- Giovan, M. and G. Sines (1979). J. Am. Ceram. Soc., 62, 510-515.
- Hussain, S. L. Pu and J. Underwood (1974). In Fracture Analysis, ASTM STP 560, American Society for Testing and Materials. pp. 2-28.
- Katayama, Y. and Y. Hattori (1982). J. Amer. Ceram. Soc., 65, C 164-C165.
- Lawn, B. R. (1983). In R. C. Bradt, A. G. Evans, D.P.H. Hasselman and F. F. Lange (Eds.), Fracture Mechanics of Ceramics, Vol. 5, Plenum Press, New York. pp. 1-25.
- Marshall, D. B. (1984). J. Am. Ceram. Soc., 67, 110-116.
- Palaniswamy, K. and W. G. Knauss (1978). In S. Nemat-Nasser (Ed.), Mechanics Today, Vol. 4, Pergamon Press, Oxford. pp. 87-148.
- Paluszny, A. and P. F. Nicholls (1978). In J. J. Burke, E. N. Lenoe and R. N. Katz (Eds.), Ceramics For High Performance Applications, Vol. 2, Brook Hill Pub. Co., Chestnut Hill, Mass. pp. 95-112.
- Petrovic, J. J. and M. G. Mendiratta (1976). J. Am. Ceram. Soc., 59, 163-167.
- Petrovic, J. J. and M. G. Mendiratta (1977). J. Am. Ceram. Soc., 60, 463.
- Petrovic, J. J. and M. G. Stout (1981). J. Am. Ceram. Soc., 64, 656-660.
- Petrovic, J. J. (1983). In R. C. Bradt, A. G. Evans, D.P.H. Hasselman and F. F. Lange (Eds.), Fracture Mechanics of Ceramics, Vol. 6, Plenum Press, New York. pp. 63-76.
- Rice, R. W., J. J. Mecholsky and P. F. Becher (1981). J. Mater. Sci., 16, 853-862.
- Shetty, D. K., A. R. Rosenfield, G. K. Bansal and W. H. Duckworth (1981). J. Am. Ceram. Soc., 64, 1-4.
- Shetty, D. K., A. R. Rosenfield and W. H. Duckworth (1982). In S. W. Freiman (Ed.), Structural Reliability of Brittle Materials, American Society for Testing and Materials (In Press).
- Shetty, D. K., A. R. Rosenfield, W. H. Duckworth and P. R. Held (1983). J. Am. Ceram. Soc., 66, 36-42.
- Sih, G. C. (1974). Int. J. Frac., 10, 305-320.
- Swedlow, J. L. (1976). In Cracks and Fracture, ASTM STP 501, American Society for Testing and Materials. pp. 506-521.
- Weibull, W. (1939). Ing. Vetenskaps Akad. Handl., 151, 5-45.

APPENDIX A. FRACTURE STATISTICS FOR SURFACE FLAWS

For brittle materials that fail from surface cracks the cumulative probability of fracture, F , can be described as (Batdorf and Heinisch 1978a)

$$F = 1 - \exp \left[- \rho \int_0^{\sigma} \int_A f(\sigma, \sigma_c) \frac{dN(\sigma_c)}{d\sigma_c} d\sigma_c dA \right] \quad (A1)$$

where

ρ = surface-crack density, i.e. number of cracks per unit area

A = surface area stressed in tension

σ = remote applied stress, i.e. maximum principal stress.

$N(\sigma_c)$ = a cumulative flaw-size distribution function expressed as the fraction of the cracks that fracture when tension normal to the crack plane is $< \sigma_c$

$f(\sigma, \sigma_c)$ = fraction of the cracks that can cause fracture in the size range σ_c to $\sigma_c + d\sigma_c$.

The flaw-size function, $N(\sigma_c)$, is generally an intrinsic characteristic of the material, but in the case of surface cracks introduced in surface-finishing operations such as grinding or polishing it is also a characteristic of the surface-finishing process. The fraction, f , on the other hand is a function of the orientation distribution of the cracks, the fracture criterion governing the extension of the cracks, and the applied stress state. Thus, f serves as a link between the flaw-size function and the fracture stress distribution.

In the absence of an experimental technique for directly assessing the flaw-size function, $N(\sigma_c)$, one is normally forced to assume a convenient and flexible distribution function, for example, the Weibull distribution function.

$$N(\sigma_c) = 1 - \exp \left[- \left(\frac{\sigma_c}{\sigma_{c0}} \right)^m \right] \quad (A2)$$

where m and σ_{c0} are the well known shape (Weibull modulus) and the scale parameters, respectively. For large values of m ($m \gg 1$) and large numbers of cracks sampled (i.e., $\sigma_c \ll \sigma_{c0}$) Equation A2 can be simplified to

$$N(\sigma_c) = \left(\frac{\sigma_c}{\sigma_{c0}} \right)^m \quad (A3)$$

Before one can formulate f it is necessary to know the orientation distribution of the strength-controlling flaws. If experiments do not show any evidence of preferred flaw orientation one can assume a random orientation distribution (i.e., equal probability for all orientations). Further, in the case of surface cracks produced in external machining operations like grinding it is reasonable to expect the crack planes to be perpendicular to the ground surface. Thus, the orientation distribution would be limited to the plane of the surface. In equibiaxial tension ($\sigma_1 = \sigma_2 = \sigma$), the flaws are subject to pure remote tension, σ , independent of their orientations. For this special case, therefore,

$$f = 1 \quad (A4)$$

For uniaxial tension the fraction of cracks that can cause fracture at any applied stress, f , can be formulated as

$$f = \frac{2\theta_c}{\pi} \quad (A5)$$

where θ_c is the critical maximum angle between the normal to a crack plane and the maximum principal stress ($\sigma_1 = \sigma$) direction. For a given flaw-size range (specified by σ_c) the critical angle θ_c is defined by solving for θ in the fracture criteria equations (Eqs. 1, 2, and 3) by making the following substitutions

$$\sigma_N = \sigma \cos^2 \theta \quad (A6)$$

and

$$\tau = \sigma \sin \theta \cos \theta \quad (A7)$$

As an example, it can be easily shown that the normal stress fracture criterion (Eq. 3) leads to

$$\theta_c = \cos^{-1} \left[\left(\frac{\sigma_c}{\sigma} \right)^{1/2} \right] \quad (A8)$$

For the mixed-mode fracture criteria, Eq. 1 and 2, θ_c can be easily solved by a numerical procedure such as the secant iteration method.

The fracture probabilities can now be calculated for both the uniaxial and the biaxial stress states by substituting the appropriate formulations of f and the flaw-size distribution function, $N(\sigma_c)$, in Eq. A1. It should be noted that both the specimen size and stress-gradient effects on fracture probabilities are taken into account in the integration of Eq. A1 over the specimen areas subject to tensile stresses. Fracture probabilities for specimen surfaces subject to compressive stresses are assumed to be zero. This assumption is generally valid when the compressive stresses are of a magnitude comparable to or smaller than the tensile stresses.