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MASTER

RESEARCH ON THE
"HYLIFE" LIQUID-FIRST-WALL CONCEPT
FOR FUTURE LASER-FUSION REACTORS

LIQUID SHEET JET EXPERIMENTS:
COMPARISON WITH A NONLINEAR THEORY

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DISCLAIMER

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ABSTRACT

Experiments were performed on planar sheet jets of water flowing vertically downward. The nozzles were vibrated transversely to excite the sinuous mode of instability, and the sheet jets were photographed from the edge view to obtain data on the wave growth parameters. The data are compared to a second-order nonlinear theory. Qualitative agreement is obtained with regard to the stabilizing nature of the second-order terms, and reasonably quantitative agreement is obtained on the fall distance for the maximum wave amplitude.

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NOMENCLATURE

A_e	=	Wave envelope amplitude
A_o	=	Amplitude of the nozzle excitation
B_2	=	Coefficient of the second-order term
b	=	Sheet jet thickness (2h)
C_r	=	Wave speed
f	=	Forcing frequency
h	=	Sheet jet half-thickness (b/2)
k	=	Wave number, $2\pi/\lambda$
LT	=	Linear theory (constant U_o)
LTX	=	Approximate linear theory with variable velocity, $U(x)$
NT	=	Nonlinear theory (constant U_o)
NTX	=	Approximate nonlinear theory with variable velocity, $U(x)$
p	=	Pressure
ρ_d	=	Ratio of ambient density to liquid density, ρ_a/ρ
t	=	Time
U	=	Vertical component of mean jet velocity
w	=	Sheet jet width
x	=	Vertical coordinate
δ	=	Spatial wave number of the wave envelope oscillation
η	=	Wave surface amplitude
λ	=	Wave length
ρ	=	Density
σ	=	Surface tension coefficient
ω	=	Complex frequency

Subscripts

a	=	ambient (or aerodynamic)
c	=	critical
co	=	cross-over point
i	=	imaginary part
m	=	maximum point
o	=	at the nozzle exit (initial value)

LIQUID SHEET JET EXPERIMENTS: COMPARISON WITH A NONLINEAR THEORY

H. R. Asare and M. A. Hoffman

1. INTRODUCTION

A series of experiments on planar liquid sheet jets flowing vertically downward from slit nozzles has been performed by Takahashi [1] and Asare [2]. The nozzles were vibrated transversely by an electromechanical shaker, and the stability of the jets was studied and compared to the linear temporal stability theory of Squire [3] and Hagerty and Shea [4] in a previous paper [5]. It was shown that the linear theory could explain only the initial portion of the wave envelope, as expected. A second-order nonlinear theory was then developed by Asare [2] based on perturbation methods such as described by Nayfeh [6]. Nayfeh and Saric [7] have solved the somewhat similar problem of the Kelvin-Helmholtz instability on the free surface of a semi-infinite liquid sheet in the presence of a gas flow over the surface, but in their case the gravity was perpendicular to the free surface. Asare has solved the case of the inviscid, incompressible planar sheet jet with two free surfaces in the absence of gravity forces. Specifically he solved for the sinuous mode of instability which results from the relatively large-amplitude, transverse forced vibration imposed on the nozzle.

In Section 2, we will review the key results of the nonlinear theory. Then in Section 3, the experimental facility and procedures will be described along with the uncertainty estimates. Finally in Section 4, the experimental results for two of the nozzles used will be presented and compared to the nonlinear theory.

2. RESULTS OF THE NONLINEAR THEORY

The method of derivative expansion using multiple time and length scales was used by Asare [2] to solve for the second-order terms in the sheet jet wave growth. Only the key results are given here; the interested reader is referred to Refs. [2] and [8] for details.

As in the case of the linear stability theory [3, 4, 5], the liquid jet and surrounding air are assumed to be incompressible, inviscid and initially irrotational. The continuity and transverse momentum equations are solved for each of the three regions (Fig. 1) subject to the usual kinematic and dynamic boundary conditions at the two interfaces. The linear theory reveals that the sinuous mode induced by the transverse forced nozzle vibration will be unstable for forcing frequencies, f , below a critical frequency given approximately by:

$$f_c = \frac{\rho_a U_0^3}{2\pi\sigma} \quad (1)$$

for $r_d \coth(kh)_0 \ll 1$ which is valid for all the experiments to be described. ρ_a is the air density, U_0 is the initial jet velocity which is assumed to remain constant (and the air velocity is assumed to be zero), σ is the surface tension coefficient, $r_d = \rho_a/\rho$ is the ratio of the air to liquid density, $k_0 = 2\pi/\lambda_0$ is the initial wave number, λ_0 is the initial wave length and h_0 is the initial half-thickness of the jet, all assumed to remain constant for this constant-jet-velocity solution.

The sinuous mode (sketched in Fig. 1) is shown in Ref. [4] to have a higher growth rate than the dilational mode where the two surfaces are 180° out of phase. In addition, it was shown in Ref. [5] that the

growth rates predicted by the linear temporal theory, ω_i , are essentially the same as those predicted by the linear spatial theory using the transformation:

$$k_i = \frac{\omega_i}{C_g} = \frac{\omega_i}{C_r} \approx \frac{\omega_i}{U_0} \quad (2)$$

where k_i is the spatial growth rate, C_g and C_r are the group velocity and wave velocity, respectively, and ω_i is the temporal growth rate. The same transformation has been shown by Asare to be valid for the nonlinear theory [2]. The spatial growth rates predicted by the linear theory are shown in Figure 2 for the two nozzle flows to be discussed later in this paper.

The solution for the dimensionless wave surface amplitude, $\tilde{\eta}(x,t) \equiv \eta(x,t)/\lambda_0$ was assumed to take the following form in terms of a small dimensionless parameter, ϵ :

$$\tilde{\eta}(x,t) = \epsilon \tilde{\eta}_1(x,t) + \epsilon^2 \tilde{\eta}_2(x,t) + O(\epsilon^3) \quad (3)$$

$$\text{where} \quad \tilde{\eta}_1 = \text{Re} \{ C_1 e^{i(\omega t - kx)} + C_2 e^{-i(\omega t - kx)} \} \quad (4)$$

$$\text{and} \quad \tilde{\eta}_2 = \text{Re} \{ D_1 e^{2i(\omega t - kx)} + D_2 e^{-2i(\omega t - kx)} \} \quad (5)$$

where Re means the "real part" of the term in the brackets. The velocity potential is assumed to have an analogous form. After substitution into the equations of motion for each of the three regions of Figure 1 and into the boundary conditions, and removing secular terms from the second and third order solutions, ϵ and the coefficients C_1 , C_2 , D_1 , and D_2 were found to be as follows

$$\epsilon \equiv \frac{\pi f A_0}{\omega_i \lambda_0} \quad (6)$$

$$C_2 = -C_1 = 1 \quad (7)$$

$$D_2 = +D_1 = \frac{B_2 \lambda_0}{4} \quad (8)$$

The parameter B_2 was assumed to be a weak function of x and t :*

$$B_2 = B_2(x_1, x_2, t_1, t_2) + O(\epsilon^3) \quad (9)$$

$$= B_2(\epsilon x, \epsilon^2 x, \epsilon t, \epsilon^2 t) + O(\epsilon^3) \quad (10)$$

B_2 was then evaluated numerically by solving an approximate second-order differential equation obtained as a consequence of suppressing the secular terms in η_2 and η_3 [2].

Results for this nonlinear theory AD2 of Ref. 2 (referred to as NT hereafter) are compared to the linear theory (referred to as LT hereafter) for one typical set of run conditions for Nozzle A on Figure 3. The constant-velocity linear theory (LT, shown by the solid curve) predicts that the sheet jet is unstable at these conditions, and that the mean wave envelope amplitude, A_e , will grow essentially exponentially with the growth rate shown on Figure 2 [5]. It can be seen from Figure 3 that the nonlinear theory (NT, shown by the dash-dot curve) begins to deviate significantly (> 5%) from the linear theory at values of A_e/A_0 of around 10 for this relatively small initial forcing amplitude, A_0 , of only 0.2 mm.

The second-order nonlinear term can be seen to be stabilizing; i.e., it causes the wave envelope amplitude to reach a maximum and then decrease toward zero very rapidly. The stabilizing effect of the nonlinear terms implies that the stabilizing surface tension force increases faster than the destabilizing aerodynamic force as the wave form begins to depart from

* In Ref. [2], $B_2 \equiv B^{(2)} = A^{(2)} A_1^2$ for the nonlinear theory AD2. (B_2 has the units of 1/length.)

a near-sinusoidal shape. It should be noted that the second-order nonlinear theory is no longer valid as the wave envelope amplitude approaches zero, because this implies that the magnitude of the second-order term is approaching that of the first order term.

The nonlinear theory (NT) predicts that the maximum wave amplitude will be reached at a fall distance given by:

$$x_m \approx \frac{U_0}{\omega_i} \ln \left[-\frac{2}{\epsilon \lambda_0 B_2} \right] \quad (11)$$

The zero cross-over point is predicted to be at:

$$x_{co} \approx \frac{U_0}{\omega_i} \ln \left[-\frac{4}{\epsilon \lambda_0 B_2} \right] \quad (12)$$

These predictions will be compared to the actual behavior of the sheet jets in the following sections.

Since the sheet jets were directed vertically downward in our experiments, the jets accelerated and the jet velocity, U , and thickness, b , did not remain constant as assumed above. An attempt was made to include these effects in both the linear theory (LTX) and the nonlinear theory (NTX) using the following approximate method [2]. The equations were integrated for a small Δx starting at the nozzle exit. Then the conditions at the end of the integration step were used as new initial conditions for evaluating all the parameters (such as growth rate, B_2 , etc) for the next integration step. In this way a few approximate variable velocity predictions were obtained at the end of our research program. The variable-velocity linear theory is shown on Figure 3. The difference between the variable-velocity (LTX) and constant-velocity (LT) linear theory for this high initial jet velocity of 5.0 m/s can be seen to

be small. Difficulties were experienced in calculating the variable-velocity nonlinear theory (NTX) for most of the unstable cases as well as for the cases where the jets were initially stable but accelerated into the unstable regime. Time and funding limitations did not allow us to resolve this problem.

3. EXPERIMENTAL APPARATUS

The apparatus has been described in detail elsewhere [1, 2, 5]. It consisted of a cylindrical, cast acrylic vacuum tank 120 cm high with the nozzles mounted in the top. An electromechanical shaker was used to vibrate the nozzles approximately transversely. The nozzles were formed out of plexiglas tubes which were heated at one end and then carefully pressed into the slit nozzle shape. The water flowing to the nozzles came from the industrial water supply lines and the velocity was measured by a standard ASME orifice plate with an estimated uncertainty of about $\pm 3\%$ for the 1 atm runs and about $\pm 5\%$ for the subatmospheric runs due to the presence of some bubbles in the flow. (All uncertainties quoted are limited sample values with approximately 20:1 odds.)

The nozzle forcing amplitudes, A_o , were measured to $\pm 4\%$ using calibrated accelerometers. The mean wave envelope amplitudes, A_e , were read off flash photographs of the edge view of the sheet jets using the following procedure. The mean wave envelope was first drawn on the photographs through the mid-points of the wave edges at the wave crests. This method eliminated most of the errors which might have been introduced by the increasing sheet edge thickness. The edges of the sheet tend to accumulate fluid due to surface tension forces at the edges which are trying to reduce the surface area and convert the sheet jets into cylindrical jets. As a result of these forces, the cross-section of the sheet jet takes on a dumbbell-like shape. Using the above reading method, the estimated uncertainties in the mean wave envelope amplitudes, A_e , were about ± 0.75 mm for Nozzle A and ± 1.0 mm for Nozzle B. However, these were more like bias errors, since the eye tended to smooth the envelope curve in the process of drawing it on the photograph.

For the subatmosphere runs, a water-jet ejector pump was able to reduce the vacuum tank pressure down to about 0.2 atm. These subatmospheric pressures were measured with a mercury manometer with an uncertainty of about $\pm 10\%$, due primarily to pressure fluctuations.

4. RESULTS AND DISCUSSION

Extensive experimental results using three different nozzles were obtained in the course of this research [1, 2]. Due to space limitations, only a small but representative sample of these experimental results will be compared to the nonlinear theory in the following sections.

4.1 Results for Nozzle A

Nozzle A produced the thinnest sheet jets with initial thicknesses, $b_0 = 2h_0 = 1.59$ mm and an initial sheet width, $w_0 = 78$ mm. Photographs of the edge views of the jets for four different forcing amplitudes, A_0 , at a forcing frequency of 70 Hz are shown on Figure 4. The linear theory predicts that all these jets will be unstable at 70 Hz as can be seen from Figure 2 where the critical frequency for Nozzle A is about 332 Hz. It can be seen that the sheet jets all tend to grow rapidly initially, but then appear to reach a "saturation" amplitude. Closer examination of the photographs on a light table revealed that the jets actually started to break up when they first reached the "saturation" amplitudes and then the assembly of fluid ligaments and drops just continued to fall, giving the appearance of a nearly constant or saturated amplitude at large fall distances.

The nondimensional data for the mean wave envelope amplitudes, A_e/A_0 , are plotted in Figure 5 along with the linear theory. The constant-velocity linear theory (LT) can be seen to agree with the data for short fall distances only, as expected. The approximate linear theory with variable velocity (LTX) only improves the agreement slightly for these jets with high initial velocities of 5 m/s.

It is somewhat surprising at first to note that the data show that the higher the forcing amplitude, A_0 , the smaller the maximum wave amplitude ratio, and that this maximum or saturated amplitude occurs at smaller fall distances for larger A_0 . The constant-velocity nonlinear theory curves, NT, can be seen to agree with these trends in data. The nonlinear theory tends to predict the fall distance at which the jets first reach the maximum amplitude quite well. However, the experimental data all show a higher amplitude than predicted by the constant-velocity nonlinear theory (NT).

Beyond the maximum point the constant-velocity nonlinear theory predicts that the wave envelope should decrease rapidly toward zero. It is hypothesized that the sheet jets cannot actually follow these rapid changes imposed by the nonlinear effects and begin to breakup as a result.

The fall distances for breakup read from many photographs for Nozzle A [2] and also for earlier runs using Nozzle C [1] (with $b_0 = 2.5$ mm and $w_0 = 76$ mm) are compared to x_m (Eq. 11) and x_{co} (Eq. 12) in Figure 6 in terms of the appropriate dimensionless parameters from the nonlinear theory. The agreement can be seen to be reasonably good with jet breakup tending to occur between x_m and x_{co} .

Some photographs for three runs at three different ambient pressures are shown on Figure 7 and the data are compared to the theories in Figure 8. For all three cases, the constant-velocity linear theory (LT) predicts that the sheet jets will be unstable, although at $p_a = 0.17$ atm the jet is only very slightly unstable (i.e., the growth rate predicted by the linear theory is quite small [5]). The nonlinear theory (NT) can be seen to predict the observed fall distance to the maximum points reasonably well, but it does not predict the observed maximum amplitudes at breakup very well. As in Figure 5, the experimental data show a higher amplitude than predicted by the nonlinear theory.

4.2 Results for Nozzle B

Nozzle B produced thicker jets than Nozzle A. The initial thickness, b_0 , of Nozzle B jets was about 7.9 mm and the initial width, w_0 was 75 mm. For this large a flow area, the maximum velocity at which we could obtain reliable data was about 2 m/s. This had two potentially important consequences. First, the jets accelerated sufficiently to almost double the jet velocity in the 60 cm fall distance of our apparatus. Thus variable velocity effects may have been much more important for Nozzle B than for Nozzle A (where U increased only about 20% in 60 cm). Secondly, for forcing frequencies above the initial critical frequency of about 23 Hz, the linear theory predicts that the sheet jets will be stable at $U_0 = 2$ m/s. In the stable regime, the wave envelope oscillates with a spatial wave number, δ , as indicated on Figure 9.* However as the sheet jet accelerates, the critical frequency increases (Eq. 1) and the jet soon enters the unstable regime where the spatial growth rate $k_i \approx \omega_i(x)/U(x)$ is positive. These changes in the envelope oscillation and growth rates with fall distance are plotted for Nozzle B in Figure 9 for three typical forcing frequencies. It can be seen that the jet acceleration results in quite complex jet stability characteristics for the Nozzle B runs.

Two typical edge-view photographs for Nozzle B are shown in Figure 10, one for 20 Hz forcing frequency where the jet is always unstable and one for 50 Hz where the jet is initially stable. This second jet is predicted

* The envelope spatial wave number, $\delta \approx 2\pi/\Lambda$, where Λ is the wave length of the envelope oscillation.

by linear theory to become unstable at a transition distance, x_L , of about 14.6 cm, as can be seen from Figure 9 and as indicated by the horizontal line drawn on the right-hand photograph of Figure 10. The dramatic difference in the wave shapes for these two cases can be seen clearly in Figure 10. This is due in part to the fact that we are able to see so many wavelengths of the jet at 50 Hz compared to only about four wavelengths at 20 Hz in the full length available in our apparatus. Thus we are able to see the nonlinear wave form develop to the extreme zig-zag shape for the 50 Hz run. This shape is also somewhat evident for the Nozzle A runs in Figure 4, but is probably more fully developed for Nozzle B runs at 50 Hz because the forcing amplitudes were larger for the latter cases.

The data for the 20 Hz run where $A_0 = 1.6$ mm are compared to the theories in Figure 11. This was the only case where the nonlinear theory with variable velocity (NTX) appeared to give reasonable results. The NTX theory can be seen to agree somewhat better with the data, than the linear theories or the constant-velocity nonlinear theory (NT).

As a final set of data, we present the photographs for four different forcing amplitudes at 50 Hz forcing frequency in Figure 12. The photographs in all cases show strongly non-sinusoidal, zig-zag type waveforms. The appearance of fluid "globules" or "blobs" at the two edges of the sheet jets becomes more pronounced as A_0 is increased. By viewing the front of the sheet jets with a stroboscope during the experiments, these fluid globules were clearly seen to be at the two side edges of the sheet jets near the wave crests; this is not obvious from the edge view photographs, however.

The mean wave envelope amplitude, A_e , is defined excluding these fluid globules (as implied on Fig. 1), and the data presented in all figures including Figure 13 have been read off the photographs accordingly. Only the data for the smallest and the largest forcing amplitudes are compared to the theories in Figure 13 for clarity.

The constant-velocity linear theory, LT, predicts that the wave envelope will be sinusoidal. This is due to the fact that the wave amplitude grows initially due to the initial transverse momentum imparted to the jet by the transverse forced vibration, not due to any instability. After a certain time the surface tension forces (which dominate the aerodynamic forces for the stable case) succeed in slowing and reversing this transverse motion of the fluid particles. Since the theories are inviscid, the wave envelope is predicted to continue to grow and decay sinusoidally in the stable regime; this oscillation of the wave envelope would continue indefinitely in the absence of damping according to linear theory. As mentioned above, the wave number for the envelope oscillation is denoted δ on Figure 9.

The constant-velocity nonlinear theory (NT) introduces second-order effects which tend to increase the effectiveness of the surface tension forces and flatten the wave envelope. The agreement with the data on Figure 13 can be seen to be reasonably good for short fall distances. However, the constant-velocity nonlinear theory is still for a sheet jet which remains in the stable regime. Since these jets enter the unstable regime at a fall distance, x_L , of about 14.6 cm, the variable velocity effects should be important for large fall distances.

The approximate effect of variable velocity on the linear theory is shown by the dashed curve (LTX) on Figure 13. It can be seen that when the jet goes unstable at $x_t \approx 14.6$ cm, this theory shows the wave envelope growing rapidly, contrary to the trends exhibited by the data.

Attempts were made to estimate the variable velocity effects on the nonlinear theory (NTX) using the approximate method described in Section 2, but the computer code always experienced difficulties near the transition point, x_t . As mentioned previously, there was not sufficient time or funds to solve this problem, unfortunately.

It should be noted that we did not see breakup of the Nozzle B jets in the observable fall distance of 60 cm. The rather flat maximum of the wave envelope and gradual variation of the wave envelope amplitude with fall distance may result in effectively delaying the breakup, but this is only a speculation.

5. CONCLUSIONS

The nonlinear theory developed by Asare [2,8] agrees only in part with the experimental results. In particular for the thinnest sheet jets from Nozzle A:

- a) The nonlinear theory correctly predicts that the second-order term is stabilizing.
- b) The nonlinear theory also predicts the fall distance at which the wave envelope amplitude reaches its maximum value quite well.
- c) The breakup of the jet tends to begin at this fall distance where the nonlinear theory predicts the wave amplitude will be a maximum. The breakup may be induced partly by the very rapid decrease in the wave envelope amplitude predicted by the theory after the maximum point.
- d) The actual wave amplitudes at breakup tend to be significantly larger than predicted by the constant-velocity nonlinear theory.

For the thickest sheet jets from Nozzle B (which experience large velocity increases with fall distance):

- a) In the unstable regime at low forcing frequencies below the critical frequency, the sheet jet behavior is best described by the approximate version of the nonlinear theory which accounts for the variable jet velocity with fall distance.
- b) In the higher forcing frequency regime where the jets are initially stable, the agreement between the nonlinear theory for constant jet velocity and the data is reasonably good for short fall distances. The jets quickly accelerated into the unstable regime, and hence the constant-velocity nonlinear theory

would not be expected to give good agreement. Also, the effective initial forcing amplitudes were very large, causing the waveform to become highly non-sinusoidal in shape. The resulting zig-zag shape would be expected to cause the aerodynamic and surface tension forces and their distribution on the sheet jet surfaces to be strongly affected, so that even a second-order nonlinear theory might be inadequate. Additional theoretical studies are required to fully explain the nonlinear behavior of these rapidly accelerating liquid jets.

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FIGURE TITLES

- Figure 1 Edge view of a liquid sheet jet showing the sinuous mode of instability.
- Figure 2 Spatial growth rates predicted by the linear theory for constant jet velocity for flows from two nozzles used in the experiments.
- Figure 3 Comparison of two versions of the linear theory with the constant-velocity nonlinear theory for one typical set of experimental conditions for Nozzle A.
- Figure 4 Edge views of the sheet jets from Nozzle A ($h_0 = 0.79$ mm, $U_0 = 5.0$ m/s, $f = 70$ Hz, $p_a = 1$ atm) for four nozzle forcing amplitudes, A_0 : 0.08, 0.1, 0.2, and 0.3 mm (going from left to right).
- Figure 5 Comparison of the linear and nonlinear theories with the experimental data for Nozzle A from Figure 4.
- Figure 6 Comparison of the predictions of the nonlinear theory (NT) for the fall distance to the maximum wave envelope amplitude, x_{m1} , and to the zero cross-over point, x_{c0} , with the data points where the sheet jets begin to break up.
- Figure 7 Edge views of the sheet jets from Nozzle A ($h_0 = 0.79$ mm, $U_0 = 5.0$ m/s, $A_0 = 0.2$ mm, $f = 50$ Hz) for three ambient pressures: 0.17 atm, 0.68 atm, and 1.0 atm (going from left to right).
- Figure 8 Comparison of the linear and nonlinear theories with the experimental data for Nozzle A of Figure 7.
- Figure 9 Approximate local variations of the stable oscillation rate and the unstable growth rate with fall distance due to the acceleration of gravity for sheet jets from Nozzle B.
- Figure 10 Edge views of the sheet jets from Nozzle B ($h_0 = 3.94$ mm, $U_0 = 2.05$ m/s, $p_a = 1$ atm) for two conditions: $f = 20$ Hz ($< f_c$) and $A_0 = 1.6$ mm on the left, and $f = 50$ Hz ($> f_c$) and $A_0 = 0.8$ mm on the right.
- Figure 11 Comparison of the linear and nonlinear theories with the experimental data for Nozzle B at 20 Hz.
- Figure 12 Edge views of the sheet jets from Nozzle B ($h_0 = 3.94$ mm, $U_0 = 2.05$ m/s, $f = 50$ Hz, $p_a = 1$ atm) for four nozzle forcing amplitudes, A_0 : 0.6, 0.8, 1.0, and 1.4 mm (going from left to right).
- Figure 13 Comparison of the linear and nonlinear theories with experimental data for Nozzle B for two of the nozzle forcing amplitudes of Figure 12.

Figure 1 Edge view of a liquid sheet jet showing the sinuous mode of instability.

Figure 2 Spatial growth rates predicted by the linear theory for constant jet velocity for flows from two nozzles used in the experiments.

Figure 3 Comparison of two versions of the linear theory with the constant-velocity nonlinear theory for one typical set of experimental conditions for Nozzle A.

Figure 4 Edge views of the sheet jets from Nozzle A ($h_0 = 0.79$ mm, $U_0 = 5.0$ m/s, $f \approx 70$ Hz, $p_a = 1$ atm) for four nozzle forcing amplitudes, A_0 : 0.08, 0.1, 0.2, and 0.3 mm (going from left to right).

Figure 5 Comparison of the linear and nonlinear theories with the experimental data for Nozzle A from Figure 4.

Figure 6 Comparison of the predictions of the nonlinear theory (NT) for the fall distance to the maximum wave envelope amplitude, x_m , and to the zero cross-over point, x_{co} , with the data points where the sheet jets begin to break up.

Figure 7 Edge views of the sheet jets from Nozzle A ($h_0 = 0.79$ mm, $U_0 = 5.0$ m/s, $A_0 = 0.2$ mm, $f = 50$ Hz) for three ambient pressures: 0.17 atm, 0.68 atm, and 1.0 atm (going from left to right).

Figure 8 Comparison of the linear and nonlinear theories with the experimental data for Nozzle A of Figure 7.

Figure 9 Approximate local variations of the stable oscillation rate and the unstable growth rate with fall distance due to the acceleration of gravity for sheet jets from Nozzle B.

Figure 10 Edge views of the sheet jets from Nozzle B ($h_0 = 3.94$ mm, $U_0 = 2.05$ m/s, $p_a = 1$ atm) for two conditions: $f = 20$ Hz ($< f_c$) and $A_0 = 1.6$ mm on the left, and $f = 50$ Hz ($> f_c$) and $A_0 = 0.8$ mm on the right.

Figure 11 Comparison of the linear and nonlinear theories with the experimental data for Nozzle B at 20 Hz.

Figure 12 Edge views of the sheet jets from Nozzle B ($h_0 = 3.94$ mm, $U_0 = 2.05$ m/s, $f = 50$ Hz, $p_a = 1$ atm) for four nozzle forcing amplitudes, A_0 : 0.6, 0.8, 1.0, and 1.4 mm (going from left to right).

Figure 13 Comparison of the linear and nonlinear theories with experimental data for Nozzle B for two of the nozzle forcing amplitudes of Figure 12.