**Numerical Analysis of
Temperature and Flow Effects in a
Dry, One-Dimensional Aquifer
Used for
Compressed Air Energy Storage**

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February 1979

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NUMERICAL ANALYSIS OF TEMPERATURE
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FOREWORD

The Compressed Air Energy Storage Technology Program at the Pacific Northwest Laboratory is sponsored by the U.S. Department of Energy, Division of Energy Storage Systems. The program scope includes a group of studies directed at developing a new energy storage technology to improve the cost and efficiency of electrical power utilization and reducing the dependence on premium fuels such as oil and natural gas. The program has two major thrusts --- Reservoir Stability Criteria Studies and Advanced Concepts Studies. These have the following objectives:

Reservoir Stability Criteria

- Develop design and stability criteria for long-term operation of underground reservoirs used for CAES in order to accelerate the commercialization of the concept.

Advanced Concepts Studies

- Develop and assess advanced CAES concepts that require little or no supplementary firing by premium fuels in order to eliminate the dependence of CAES on premium fuels.

The ultimate objective of this program is to reduce the consumption of natural gas and oil, used for peak-power generation plants, by about 100,000,000 barrels per year. This could be accomplished by replacing conventional gas turbine peaking plants currently being used by utilities with CAES plants.

The following documents have been issued by PNL or by subcontractors to PNL reporting the results of the work toward these objectives.

- Air Storage Peaking Power Plants Utilizing Modified Industrial Gas Turbines and Cavities with Nuclear Explosives, B.C. Fryer, BNWL-1748, May 1973.
- Technical and Economic Feasibility Analysis of the No-Fuel Compressed Air Energy Storage Concept, D.K. Kreid, BNWL-2065, May 1976.
- FY-1977 Progress Report - Stability and Design Criteria Studies for Compressed Air Energy Storage Reservoirs, G.C. Smith, J.A.

Stottlemire, L.E. Wiles, W.V. Loscutoff and H.J. Pincus, PNL-2443, March 1978.

- FY-1977 Progress Report Compressed Air Energy Storage Advanced Systems Analysis, D.K. Kreid and M.A. McKinnon, PNL-2464, March 1978.
- Preliminary Stability Criteria for Compressed Air Energy Storage in Porous Media Reservoirs, J.A. Stottlemire, PNL-2685, June 1978.
- Preliminary Long-Term Stability Criteria for Compressed Air Energy Storage Caverns in Salt Domes, R.L. Thoms and J.D. Martinez, August 1978.

This report examines the temperature and flow effects in a dry, one-dimensional aquifer model used for compressed air energy storage. It is a revised and updated version of the material presented in PNL-2443, originally issued in March 1978.

SUMMARY

This report presents a detailed description of the method of analysis and the results obtained for an investigation of the hydrodynamic and thermodynamic response of a model of a dry porous media reservoir used for compressed air energy storage (CAES). Results were obtained from a one-dimensional simulation of the cycling of heated air to and from a radial flow field surrounding a single well in a porous rock. It was assumed that the performance of the bulk of the reservoir could be characterized by the performance of a single well.

The analysis was based on a computer code developed at PNL which uses a finite difference method to solve the coupled, nonlinear, transient equations that describe the transport of mass, momentum and energy in the reservoir. The code was used to evaluate the sensitivity of the reservoir performance to changes in various reservoir parameters. The parameters that could be varied included those related to material properties, geometry, and operating conditions. The two basic responses that were evaluated were the hydrodynamic and thermodynamic behavior of the reservoir.

The hydrodynamic, or flow field, response was obtained primarily from the solution of the equations of continuity and momentum which yielded the pressure distribution in the reservoir as a function of time. Significant results that were obtained from analysis of the parameters which affect the flow field include the following.

- The parameters having the most significant impact on the hydrodynamic behavior were the mass flow rate per unit thickness of the reservoir and the permeability of the porous material.
- Efficient reservoir operation may require permeabilities on the order of 500 md or more.
- The hydrodynamic response becomes less sensitive to permeability as permeability increases.
- For a fixed number of wells and given permeability, the hydrodynamic response is nearly independent of the structure of the storage zone,

i.e., whether the volume of the storage zone is composed of a large region of low porosity or a small region of high porosity.

- Well bottom diameter is important for large flow rates.
- The well deliverability is reduced as injection temperature is increased.
- For some conditions the assumption of Darcy flow may not be adequate to describe the flow field.

The thermodynamic response, or temperature distribution, was obtained from the solution of the energy equation although it is dependent on the convective transport term obtained from the solution of the flow field. Thermal behavior of the reservoir was assumed to be characterized by rapid thermal equilibrium achieved between the air and rock. Significant results obtained from the thermodynamic analysis include the following.

- Due to the relatively large volumetric thermal capacity of the rock, most of the thermal energy injected into the reservoir is deposited within a region very near the well.
- The growth of the thermal pillar is slow so that even after 5 years the region more than 50 ft from the wellbore is relatively unaffected.
- The average thermal energy recovery in the first year of simulated reservoir operation is about 73%. During the tenth year of operation, approximately 88% of the injected thermal energy is recovered. Thus, the porous media reservoir acts as a good thermal energy storage system.
- Because of the deposition of injected thermal energy near the well during the reservoir charging cycle and the extraction of the energy during reservoir discharge, significant thermal cycling occurs in this region. For most conditions, however, the phenomenon of thermal cycling is only observable to a radius of about 10 ft.
- For a broad range of reservoir parameters, the thermal behavior and rate of thermal development of the reservoir were relatively unaffected.

The analysis and results described in this report are the prelude to the development of more sophisticated models designed to predict the

flow dynamics and the thermal and mechanical response of multi-dimensional, water-bearing porous reservoirs.

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TABLE

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NOMENCLATURE

C_p	=	constant pressure specific heat of fluid
C_s	=	specific heat of solid
D	=	reservoir diameter
$\bar{F}$	=	shear and drag force vector
$\bar{g}$	=	gravity vector
h, H	=	enthalpy of fluid
H_v	=	volumetric heat transfer coefficient
K	=	fluid thermal conductivity
K_s	=	solid thermal conductivity
K_e	=	equivalent thermal conductivity
m	=	mass flow rate
M_{max}	=	maximum mass in reservoir during week
P	=	pressure
P_{max}	=	maximum weekly reservoir pressure
P_o	=	initial aquifer pressure
r	=	radial distance from well center
R	=	gas constant
Re	=	Reynolds number
s	=	reservoir thickness
t	=	time
T	=	fluid temperature
T_s	=	solid temperature
v	=	radial superficial flow velocity
$\bar{v}$	=	superficial flow velocity vector
$\underline{v}_a$	=	velocity absolute value component tensor

v_r, v_θ, v_z	=	components of $\underline{v}_a$ for cylindrical coordinates
$\overline{v}_0$	=	open flow velocity vector
β	=	turbulence factor
κ	=	permeability
λ	=	fraction of total air mass cycled
μ	=	viscosity
ϕ	=	porosity
ρ	=	density of fluid
ρ_s	=	density of solid
$\underline{\tau}$	=	fluid stress tensor
Φ	=	energy dissipation function for open flow
Φ_p	=	energy dissipation function for flow through porous media
ψ	=	fraction of energy stored over weekend
C	=	flow conductance
F	=	weighted volumetric flux
H	=	weighted heat flux
K	=	thermal conductance
j	=	spatial index
n	=	temporal index
P_0	=	old time pressure
P_G	=	guessed pressure
P_N	=	new time pressure
T_0	=	old time temperature
T_G	=	guessed temperature
T_N	=	new time temperature

NUMERICAL ANALYSIS OF TEMPERATURE AND FLOW EFFECTS IN A DRY ONE-DIMENSIONAL AQUIFER USED FOR COMPRESSED AIR ENERGY STORAGE

1.0 INTRODUCTION

Compressed air energy storage (CAES) appears to be an economically and technically viable method for energy storage. The surface facilities are conventional in nature and the natural gas storage industry supports over 50 years of subsurface storage experience. However, it was deemed necessary to explore how a reservoir might respond over a 20- to 40-year time frame to the somewhat unique temperature, pressure, and humidity characteristics associated with CAES.

Candidate reservoirs for the storage of compressed air include: 1) porous media such as sandstone aquifers or carbonate reefs, 2) solutioned salt cavities, and 3) hard rock caverns. Due to the relatively high number of unknown phenomena the porous media alternative was addressed first. A one-dimensional, radially symmetric and dry aquifer system is the subject of this report. Subsequent studies will address multi-dimensional water-bearing porous reservoirs.

The objective of the mathematical modeling phase of this study is to establish preliminary stability and design criteria for CAES porous rock reservoirs based on theoretical investigations. Computer codes that incorporate both flow and thermal effects are being used in conjunction with inputs from laboratory testing and literature surveys to define expected reservoir pressure, temperature, and humidity profiles, and to examine the effects of reservoir parameter variation on reservoir performance. This report presents the results of preliminary investigations of flow and thermal effects on the operation of a dry, one-dimensional porous reservoir.

2.0 RESULTS

Preliminary investigations of flow and thermal effects on the operation of a dry, one-dimensional (radial) porous media reservoir were performed. Several reservoir parameters were examined to determine their influence on reservoir operation. Flow field response investigations yielded seven significant results:

- The parameters having the most significant impact on the hydrodynamic behavior were the mass flow rate per unit thickness of the reservoir and the permeability of the porous material.
- Efficient reservoir operation may require permeabilities on the order of 500 md or more.
- The hydrodynamic response becomes less sensitive to permeability as permeability increases.
- For a fixed number of wells and given permeability, the hydrodynamic response is nearly independent of the structure of the storage zone, i.e., independent of whether the volume of the storage zone is composed of a large region of low porosity or a small region of high porosity.
- Well bottom diameter is important only for large flow rates.
- The well deliverability is reduced as injection temperature is increased.
- For some conditions the assumption of Darcy flow may not be adequate to describe the flow field.

Five significant results related to the thermal behavior of the reservoir were found:

- The porous media reservoir may serve as a fairly good thermal energy storage system.
- The growth of a thermal pillar surrounding the wellbore occurs very slowly.
- Significant thermal cycling can occur in the vicinity of the well.
- Thermal cycling is confined to a region within about 10 ft of the well.
- Thermal behavior is not strongly dependent upon reservoir geometry or material properties.

3.0 AQUIFER MODEL EQUATIONS

3.1 GENERAL EQUATIONS

The most general equations governing the flow of compressible gases in a dry porous solid may be written as follows:

- a. conservation of mass

$$\phi \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \bar{V} = 0 \quad (1)$$

- b. conservation of momentum

$$\phi \left[\frac{\partial \bar{V}}{\partial t} + \frac{\bar{V}}{\phi} \cdot \nabla \bar{V} \right] + \phi [\nabla P - \rho \bar{g}] = \bar{F} \quad (2)$$

- c. conservation of energy (fluid phase)

$$\rho \left(\phi \frac{\partial h}{\partial t} + \bar{V} \cdot \nabla h \right) - \left(\phi \frac{\partial P}{\partial t} + \bar{V} \cdot \nabla P \right) - \phi \nabla \cdot K \nabla T - \mu \phi = H_V (T_S - T) \quad (3)$$

- d. conservation of energy (solid phase)

$$(1 - \phi) \left(C_S \rho_S \frac{\partial T_S}{\partial t} - \nabla \cdot K_S \nabla T_S \right) = H_V (T - T_S) \quad (4)$$

For a more detailed discussion of equations and symbols, see Appendix A and Nomenclature.

3.2 SIMPLIFIED EQUATIONS

Several simplifying assumptions were made to obtain more tractable forms of the governing equations. These simplifications are listed below.

1. Viscous dissipation of energy was assumed negligible.
2. Gas kinetic energy was disregarded.
3. Ideal gas behavior was assumed.
4. Darcy or Forchheimer flow was assumed, depending on velocity.
5. Rapid fluid-solid local thermal equilibrium was assumed.

6. Properties of reservoir solid were assumed stepwise constant.
7. Flow was constrained to be one-dimensional radial.

The validity of these assumptions is discussed in some detail in Appendix A.

Incorporation of these simplifying assumptions allows the conservation equations to be rewritten in the following forms:

- a. conservation of mass

$$\phi \frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{P}{T} r v \right) = 0$$

- b. conservation of momentum

$$\frac{\mu}{\kappa} v + \beta \left(\frac{P}{RT} \right) |v| v + \frac{\partial P}{\partial r} = 0$$

- c. conservation of energy (includes fluid and solid)

$$\frac{\partial T}{\partial t} = \frac{1}{(1 - \phi) \rho_s C_s} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(K_e r \frac{\partial T}{\partial r} \right) + \left(1 - \frac{C_p}{R} \right) \phi \frac{\partial P}{\partial t} - \left(\frac{C_p}{R} \right) \frac{1}{r} \frac{\partial}{\partial r} (r P v) \right]$$

This one-dimensional radial flow model was used to analyze the dynamics of flow of air injected or produced by a single well in porous rock.

3.3 BOUNDARY CONDITIONS

The number of wells in an aquifer air storage field may reach several hundred and will probably be arranged in a uniform pattern as shown in Figure 1. Since a single well is likely to be surrounded by other wells being pressurized or depressurized at about the same rate, interference between these wells would have the effect of creating a boundary located halfway between adjacent wells, across which there would be no transport of mass or energy. The result is that the performance of the bulk of the reservoir can be characterized by analyzing the performance of a single well. For this analysis, the single well reservoir was modeled as a cylindrical region as shown in Figure 2.

FIGURE 1. Single Well Reservoir
Within CAES Reservoir

FIGURE 2. One-Dimensional
Radial Reservoir

During the power production phase, the flow to a CAES system turbine requires throttling to an essentially constant inlet pressure to produce constant power. The air mass flow rate required is approximately proportional to the power produced. Therefore, a constant flow boundary condition at the well was chosen for the analytical model during reservoir discharge (power production phase) to approximate constant power production.

Although air inlet conditions to a CAES compressor train are fixed at ambient, the compressor back pressure rises as the reservoir pressure builds during periods of air injection. If the available compression power is fixed, then the mass flow rate will decrease as reservoir pressure rises, assuming that compressor efficiency remains constant. This fact is shown graphically in Figure 3, where the ratio of mass flow rate to compression power is plotted as a function of compressor back pressure. From this figure it can be seen, however, that although the mass flow rate decreases with increasing back pressure (assuming constant power input), the rate of decrease becomes less for increasingly higher back pressures, with the slope of the curve approaching the horizontal. At storage pressures being contemplated for CAES application, relatively large changes in reservoir pressure (or compressor back pressure) cause relatively small corresponding changes in air mass flow rates. In addition, if a given CAES design maintains a large cushion air mass in the reservoir, total reservoir pressure swings will be small so that the constant mass flow rate corresponds even more closely to a constant power input. The constant flow rate wellbore boundary condition was therefore assumed to reasonably represent a CAES plant during the reservoir charging phase, and was incorporated into the model. During the idle periods of CAES operation there is no flow into or out of the reservoir. Therefore, a no-flow boundary was assumed at the well during these periods.

3.4 LIMITATIONS OF THE MODEL

Many of the complexities of actual underground porous rock air storage reservoirs obviously cannot be treated with the simplified reservoir model presented in this report. Heat and mass flow in real reservoirs will occur in three dimensions and will be influenced by the geometry, rock structure,

FIGURE 3. Power of Air Compression

and moisture conditions prevailing at any given site.

This model best represents a single well located in a relatively shallow, horizontal region of an aquifer air storage field, in which wells reach to the bottom rock and where the effects of gravity on flow are minimal. Some aquifer fields may have sizable regions that fit this description (Figure 1) although almost all reservoirs will also have inclined regions to provide closure for trapping the air. Wells on the outer edges of reservoir fields will function differently from those in the center and will not have the symmetrical boundary assumed in the model. If the porous layer is so deep

that aquifer water is not expelled from the region between the caprock and bottom rock, but instead the air bubble is trapped between the domed caprock and a water "floor", the vertical mass flow becomes important, and this one-dimensional model less applicable. Air still enters and leaves the wells in a radial manner, however, and temperature and pressure profiles in the near vicinity of the wells can be reasonably predicted by this simplified model.

Rock structures found in actual reservoirs are not likely to exhibit the homogeneity assumed in this model, but reservoirs will more typically exhibit varying degrees of non-uniformity. There may be regions containing fractured rock (such as joints, faults, fissures, etc.) and/or regions containing inclusions of impermeable materials which perturb or block the flow of fluids. The storage regions may consist of layers of rock of differing porosities and permeabilities, perhaps with impermeable layers sandwiched in between. Reservoir properties, such as permeability, porosity, thermal diffusivity, etc., may vary considerably from region to region throughout the reservoir. These conditions cannot be treated in detail with the symmetrically homogeneous model used here. The model can, however, be viewed as somewhat of an "average" of the many possibilities that exist, and is useful for examining the overall influence and relative importance of parameters influencing reservoir behavior.

Because only one dimension was included in the model, heat transfer through the caprock and bottom of the reservoir are not accounted for. Conduction of heat through these rock formations is very slow, however, and the effects on porous reservoir temperatures will be noticeable only after a relatively long period of time. In addition, this is a boundary effect that is of diminishing importance for thicker porous reservoirs.

The most serious deficiency of this model is its assumption that the reservoir is totally dry. This will never be the case for air storage reservoirs developed within aquifers. Although the bulk of the water is pushed out as the air bubble is developed, as much as 20-25%⁽¹⁾ of the water originally in the rock remains within the air zone trapped by capillary forces and in dead-end pores. As discussed in detail in Section 2, the presence

of this residual water will significantly affect the flow and thermal development of an aquifer storage field.

In view of the limitations mentioned above, the results presented in this report must be recognized as a first-cut analysis of porous rock air reservoir flow dynamics. They serve primarily to predict the relative influence of various reservoir parameters as well as to provide experience and direction for the development of more realistic analytical and experimental models.

4.0 NUMERICAL MODEL

The development of the finite-difference model for the governing mass and energy transport equations is summarized in this section. Additional details of the development of the difference equations are presented in Appendix B.

4.1 FINITE-DIFFERENCE EQUATIONS

The difference equations are developed from Equations 5 and 7 which represent the transport of mass and energy, respectively. The governing equations are written in difference form using an upstream difference technique. The one-dimensional difference equations are fully implicit.

The grid system for the difference equations is presented in Figure 4. Variable node spacing is available. Pressures and temperatures are defined at the node points j . A volumetric flux based on the product of a flow conductance and the pressure difference between adjacent nodes is the basis for evaluating transported quantities. Node NR+1, located beyond the reservoir boundary, is a "mirror" of node NR. The result is that there is no transport of either mass or energy across the outer reservoir boundary. All computations are based on a vertical grid thickness of one unit.

The development of the difference equation for the mass transport begins from Equation 5.

$$\phi \frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{P}{T} r v \right) = 0 \quad (5)$$

FIGURE 4. Finite Difference Grid

A weighted volumetric flux, F , is defined from the product of the radius and velocity, rv . The flux between nodes j and $j+1$ is written as

$$F_j = -C_j (P_{j+1} - P_j) \quad (8)$$

The flow conductance, C_j , is based on the difference cell geometry, the viscosity of air, and the permeability of the porous material. The flow calculation can be based on either the Darcy or Forchheimer momentum model by the appropriate calculation of C_j .

Substitution of the flux term transforms Equation 5 into

$$\frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \frac{1}{\phi r} \frac{\partial}{\partial r} \left[\left(\frac{P}{T} \right) F \right] = 0 \quad (9)$$

The implicit upstream difference form of Equation 9 is

$$P_{j,n+1} + \frac{\Delta t T_{j,n+1}}{\phi r_j \Delta r_j} \left[C_{j-1} \left[\left(\frac{P}{T} \right)_{j-1,F+} + \left(\frac{P}{T} \right)_{j,F-} \right] (P_j - P_{j-1}) \right. \\ \left. + C_j \left[\left(\frac{P}{T} \right)_{j,F+} + \left(\frac{P}{T} \right)_{j+1,F-} \right] (P_j - P_{j+1}) \right]_{n+1} = \frac{T_{j,n+1}}{T_{j,n}} P_{j,n} \quad (10)$$

where Equation 8 has been used for the flux. If the flux at a cell interface is radially outward (i.e., a positive flux), the density terms designated with the subscript F+ are used while the terms identified with the subscript F- are zero. If the flux is negative, the inverse of this convention applies.

The advected property in Equation 10 is the density, P/T . The pressures associated with the flux term represent the driving force for the mass transport. For the solution to be conservative in mass and energy, it was necessary to distinguish the two pressures from one another in the model.

The development of the difference equation for the energy transport begins from Equation 7.

$$\frac{\partial T}{\partial t} = \left[\frac{1}{r} \frac{\partial}{\partial r} \left(K_e r \frac{\partial T}{\partial r} \right) + \left(1 - \frac{C_p}{R} \right) \phi \frac{\partial P}{\partial t} - \left(\frac{C_p}{R} \right) \frac{1}{r} \frac{\partial}{\partial r} (Prv) \right] \frac{1}{(1 - \phi) \rho_s C_s} \quad (7)$$

Using a method similar to that for the development of the volumetric flux, Equation 8, a heat flux, $K_e r \frac{\partial T}{\partial r}$, is defined from the conduction term. The heat flux between nodes j and $j+1$ is written as

$$H_j = K_j (T_{j+1} - T_j) \quad (11)$$

The last term on the right-hand side of Equation 7, the spatial derivative of Prv , represents the advection of energy and is the dominant term for energy transport. The product of radius and velocity results in the same form as Equation 8. In this case, however, the transported quantity is thermal energy rather than mass.

Inclusion of the flux terms transforms Equation 7 into

$$\frac{\partial T}{\partial t} = \left[\frac{1}{r} \frac{\partial H}{\partial r} + \left(1 - \frac{C_p}{R} \right) \phi \frac{\partial P}{\partial t} - \left(\frac{C_p}{R} \right) \frac{1}{r} \frac{\partial}{\partial r} (PF) \right] \frac{1}{(1 - \phi) \rho_s C_s} \quad (12)$$

In finite difference form, Equation 12 becomes

$$\begin{aligned}
 T_{j,n+1} = & T_{j,n} + \frac{\Delta t}{(1-\phi)\rho_s c_{s,r}} \left[\frac{1}{r_j \Delta r_j} \left(K_{j-1} T_{j-1} - (K_{j-1} + K_j) T_j + K_j T_{j+1} \right)_{n+1} \right. \\
 & + \left(1 - \frac{C_p}{R} \right) \phi \left(\frac{P_{j,n+1} - P_{j,n}}{\Delta t} \right) \\
 & \left. - \frac{C_p}{R} \frac{1}{r_j \Delta r_j} \left(P_j F_{j,F+} - P_{j-1} F_{j-1,F+} - P_j F_{j-1,F-} + P_{j+1} F_{j,F-} \right) \right] \quad (13)
 \end{aligned}$$

The subscripts F+ or F- on the last terms have the same implication as described in the discussion following Equation 10.

The method used for solving the coupled Equations 10 and 13 is outlined in Appendix B.

4.2 CODE VERIFICATION

A computer code (described in Appendix B) was written to simultaneously solve the finite difference Equations 10 and 13. To promote confidence in the accuracy of the model, several problems were analyzed and the results compared to known solutions. These test problems were of three types:

1. isothermal airflow: pressure profile solution
2. non-isothermal airflow: temperature and pressure profile solutions
3. heat flow: temperature profile solution.

For isothermal flow through a porous medium (assuming Darcy flow) the describing equations can be reduced to the single equation

$$\frac{\partial P^2}{\partial t} + \frac{\kappa}{\mu \phi} P \nabla^2 P^2 = 0 \quad (14)$$

Except for the non-linearity which appears as the pressure P, this equation has the same form as the conventional heat equation. If the pressure is not changing too rapidly, then the degree of non-linearity will be small and the

equation can be made linear (with minor effect on the results) by making the term $\frac{\kappa P}{\mu \phi}$ constant at some average or representative pressure. In this way the equation becomes analogous to the heat equation for which the solution is well known.⁽²⁾ Figure 5 compares the linearized analytic (heat analogy) solution to the non-linear numerical computer model solution for the problem of a constant pressure applied to the wellbore of a one-dimensional radial reservoir with isothermal flow. As can be seen, the results are very close.

In one phase of this study, Intercomp Resource and Development, Inc. was given a contract to establish temperature and pressure profiles for certain reservoir conditions. Comparisons of solutions obtained with the PNL model to some of Intercomp's results show good agreement for non-isothermal flow behavior.

The thermal model was also verified by comparison with solutions of heat conduction through a cylindrical wall. To do this, an effectively zero mass flux situation was applied with the temperatures at the inner and

FIGURE 5. Comparison of Numerical Solution with Linear Heat Flow Solution

outer boundaries specified. Again, agreement with the known solution was excellent.

5.0 ANALYSIS OF CAES RESERVOIRS

The computer program described in the last section was used to examine the influence of various parameters on CAES reservoir operation. A reference reservoir geometry was specified and analysis was performed by varying reservoir properties and boundary parameters to observe the effects on reservoir operation. Two types of reservoir response were examined: 1) flow response, which relates to reservoir deliverability and can be examined within relatively short time intervals; and 2) thermal response, which is the growth of heated regions around reservoir wells when high temperature air is injected on a weekly CAES cycle over long periods of time. The results of these analyses are presented in the sections that follow.

5.1 FLOW ANALYSIS

The influence on the flow behavior of a CAES porous rock reservoir was examined for several parameters, including permeability, well diameter, reservoir diameter and porosity, and air injection temperature. In addition, results obtained assuming Darcy flow were compared to results obtained using the more exact Forchheimer momentum equation (Appendix A).

5.1.1 Reference Reservoir

A reservoir geometry was established to provide a reference condition from which to observe the effects of changes in reservoir parameters. To approximate flow into a single well within a well field, a cylindrical reservoir 400 ft (122 m) in diameter was chosen (to simulate well spacing of approximately 400 ft (122 m), Figure 1) with one-dimensional radial flow from a 7-in. (18-cm) casing. The reference case porosity was set at 20% and the permeability at 500 md. The reservoir temperature was assumed to be initially uniform at 100°F (38°C), and reservoir pressure initially uniform at 735 psi (5070 kPa, 50 atm). Air charging temperature was set at

450°F (232°C) to approximate the highest allowable temperatures from presently available centrifugal compressors.^(a) To simulate daily CAES charging, air was injected for 10-hour periods at constant values of mass flow rate per foot of reservoir thickness, ranging from 0.05 to 0.5 lbm/sec-ft (0.07 to 0.74 kg/sec-m). Figure 6 shows the typical behavior of the reservoir as it is charged for a 10-hour period. The top curve plots the well pressure as it increases with time from the initial reservoir pressure of 735 psi (5070 kPa) with a constant mass injection rate of 0.5 lbm/sec-ft (0.74 kg/sec-m). The bottom curve plots the pressure at the outer boundary of the single well reservoir (midpoint between wells if the well is within a large reservoir well field). When flow reaches quasi-steady state, the difference between the well pressure and reservoir boundary pressure remains constant as these two pressures increase at the same rate. If air were being extracted instead of injected, the resulting curves would be approximately mirror images about the reservoir initial uniform pressure level.

5.1.2 Permeability

The quasi-steady state difference between the well pressure and the reservoir boundary pressure (as depicted in Figure 6) is the driving force necessary to maintain constant flow into a given reservoir. Permeability is the characteristic of a reservoir which quantifies the inverse of resistance to fluid flow; low permeability indicates high flow resistance. Thus, high well pressures are required to inject air into low permeability reservoirs.

Figure 7 presents the pressure profiles after injection of 0.10 lbm/sec-ft (0.15 kg/sec-m) for a 10-hour period into the reference reservoir for various values of permeability. Figure 8 shows the well pressure as a function of time. Similar curves are presented in Figure 9 for a flow rate of 0.05 lbm/sec-ft (0.07 kg/sec-m). Although not shown in these figures, the reservoir boundary pressure behaves similarly to that of the lower curve of Figure 6, rising at a rate to parallel the well pressure after a short transient.

(a) Axial compressors can withstand higher temperatures on the order of 600°F (315°C), but are generally pressure limited.

FIGURE 6. Typical Reservoir Pressure Behavior

FIGURE 7. Reservoir Pressure Profiles for Various Permeabilities After 10 Hours of Charging

FIGURE 8. Well Pressure During Injection
for Various Permeabilities
[$m = 0.10 \text{ lbm/sec-ft}$
($m = 0.15 \text{ kg/sec-m}$)]

FIGURE 9. Well Pressure During Injection
for Various Permeabilities
[$m = 0.05 \text{ lbm/sec-ft}$
($m = 0.07 \text{ kg/sec-m}$)]

It can be seen from these curves that increases in reservoir permeability eventually bring diminishing returns in terms of lowering the required well pressure for a given injection rate. This is shown more dramatically in Figure 10 where the quasi-steady state difference between the well pressure and reservoir pressure is plotted as a function of reservoir permeability for several values of air mass flow rate per foot of reservoir depth. Beyond about 2000 md the curves become quite flat, indicating little benefit for increases in permeability.

The merits of at least reasonably high permeability are readily evident, however, from these same curves. For example, suppose it were required that 2000 lbm/sec (900 kg/sec) of air be injected into (or produced from) a CAES reservoir, with a maximum pressure drop between well bottom and reservoir of 20 psi (140 kPa). If the available reservoir has a permeability of 500 md, then, according to Figure 10, the flow rates must be less than 0.10 lbm/sec-ft (0.15 kg/sec-m). If the reservoir is 50 ft (15 m) thick, then the required number of wells is

$$2000 \frac{\text{lbm}}{\text{sec}} \times \frac{\text{sec-ft}}{0.10 \text{ lbm}} \times \frac{\text{well}}{50 \text{ ft}} = 400 \text{ wells} . \quad (15)$$

If the reservoir thickness were 100 ft (30 m), the number of required wells would be reduced to 200. If, however, the permeability were only 250 md, then the maximum flow rate would be 0.05 lbm/sec-ft (0.07 kg/sec-m) (from Figure 10), and the number of wells required for a 100-ft (30-m) thick reservoir would increase to 400. Conversely, if the permeability were 1100 md, the flow rate could be as high as 0.20 lbm/sec-ft (0.30 kg/sec-m) and only 100 wells would be required for the 100-ft (30-m) thick reservoir.

Although prevailing economic factors will determine the optimal number and spacing of wells for a specific CAES installation, the need for reasonably high permeability underground structures is evident, particularly if the reservoir is thin. But the diminishing importance of very high permeability (>2000 md) structures should also be noted.

FIGURE 10. Driving Pressure Difference Versus Permeability
for Various Mass Flow Rates

5.1.3 Well Diameter

Figure 11 is a plot of well pressure versus time for injecting 0.1 lbm/sec-ft (0.15 kg/sec-m) of air for 10 hours into the reference reservoir through wells of various diameters. After a relatively short transient, quasi-steady state injection conditions are reached, and pressure rises at a constant rate. Figure 12 shows the final quasi-steady state difference between well pressure and reservoir boundary pressure as a function of well diameter and for several mass flow rates. As can be seen, the well diameter has little effect on the required injection pressures at low flow rates but becomes of increasing importance as flow rates rise. This is another factor to be considered in optimization of a CAES plant design, although the effect of well shaft diameter on frictional losses through the shaft to the surface may be more constraining on design than on flow resistance to the reservoir.

5.1.4 Porosity and Reservoir Diameter

The volume available for air storage in a radial porous rock reservoir is (neglecting well volume):

$$\text{volume} = \phi \pi \frac{D^2}{4} s \quad (16)$$

where ϕ is porosity, D is reservoir diameter, and s is thickness. The storage volume fed by a single well thus consists of an areal extent of pore space within the solid medium, and can be increased by increasing porosity, thickness, or the effective single well reservoir diameter (well spacing). Figure 13 shows the effect of changing porosity (and thus volume) on the pressure required to inject 0.1 lbm/sec-ft (0.15 kg/sec-m) of air into the reference reservoir (assuming all other reference parameters constant). The effect on injection pressure is obvious: as the porosity (volume) is decreased, the pressure rises at an increased rate as a constant mass flow is maintained.

FIGURE 11. Well Pressure During Injection
for Various Well Diameters
[$m = 0.10 \text{ lbm/sec-ft}$
($m = 0.15 \text{ kg/sec-m}$)]

FIGURE 12. Driving Pressure Difference Versus Well Diameter for Various Mass Flow Rates

FIGURE 13. Well Pressure During Injection
for Various Porosities
[$m = 0.1 \text{ lbm/sec-ft}$
($m = 0.15 \text{ kg/sec-m}$)]

Very similar curves are shown in Figure 14, where porosity is held constant at 20% and reservoir diameter is varied to obtain reservoirs having total volumes exactly equal to the 400-ft (122-m) diameter reservoirs with the various porosities represented in Figure 13.

The difference between these two sets of injection curves is very small. This indicates that the driving pressure difference between the well and the reservoir outer boundary necessary to maintain a given flow rate is about the same, whether the total storage volume is spread out in a large diameter reservoir or concentrated within a higher porosity region closer to the well. For example, the quasi-steady state difference between the well pressure and the outer boundary pressure of the 282-ft (36-m) diameter (20% porosity) reservoir is 18.78 psi (129.5 kPa), compared to 19.38 psi (133.6 kPa) for the 400-ft (122-m) diameter (10% porosity) reservoir having the same total volume. Since the permeability is the same for both cases, the additional flow resistance in the larger diameter reservoir is due only to the larger radial distance through which some of the flow must travel. Pressure gradients in this outer region are very small and velocities are very low, so that the effect of increased resistance is almost negligible on the overall operation of the reservoir.

5.1.5 Injection Temperature

Air charged into a reservoir at high temperatures is more viscous than cooler air (see Appendix C) and hence a higher pressure difference between well and reservoir is required to maintain a desired flow rate. It is true that hot air injected into a cooler porous rock reservoir will rapidly cool to very near the rock temperature, and, for the most part, flow will be nearly isothermal. However, as shown in Figure 15, injection of hot air does require higher pressure differences between well and reservoir, even if the reservoir is initially cool, as is the case for the bottom four curves of the figure. If hot air is cycled into the reservoir over long periods of time the rock temperature will gradually rise so that high temperature flow occurs in an increasingly larger region growing away from the well. The top curve of

FIGURE 14. Well Pressure During Injection for Various Reservoir Diameters
 $[m = 0.1 \text{ lbm/sec-ft}]$
 $[m = 0.15 \text{ kg/sec-m}]$

FIGURE 15. Well Pressure During Injection for Various Injection Temperatures
 $[m = 0.1 \text{ lbm/sec-ft}]$
 $[m = 0.15 \text{ kg/sec-m}]$

Figure 15 shows the well pressure required to inject 450°F (232°C) air into the reference reservoir after it has "aged" for a period of several years (see Section 5.2). It is seen that injection of hot air will reduce deliverability as the reservoir is heated. This is due to the higher air temperatures, and thus, higher viscosities that influence larger regions of the flow field. This continuous increase in flow resistance must be accounted for when designing CAES facilities which utilize hot air storage.

5.1.6 Darcy Versus Forchheimer Flow

The Darcy flow assumption stipulates that flow velocity increases proportionally with increased pressure drop. The simplicity of this assumption has led to its widespread use in simulation of porous reservoir flow, and its accuracy in many instances is well documented. It has been found, however, that at high flow rates inertia effects become important and increases in velocity cease to be proportional to changes in pressure drop (see Appendix A). Pressure drop instead becomes a quadratic function of velocity (Forchheimer flow).

The relative importance of the inertia effects is best expressed by the local Reynolds number defined as $Re = (\rho |v|)(\beta \kappa)/\mu$ in which $\beta \kappa$ may be interpreted as a characteristic length of the porous medium. The magnitude of the Reynolds number is a direct indication of deviation from Darcy flow. It arises directly from writing the Forchheimer momentum equation (Equation 6) in the following form:

$$\frac{\partial P}{\partial r} = - \frac{\mu v}{\kappa} \left[1 + \frac{\rho |v| (\beta \kappa)}{\mu} \right] = - \frac{\mu v}{\kappa} [1 + Re] \quad (17)$$

As the Reynolds number becomes small, the flow behaves according to Darcy's Law, $\frac{\partial P}{\partial r} = - \frac{\mu v}{\kappa}$.

The error that may occur by assuming Darcy flow is evidenced in Figure 16, where well pressure is plotted for a 10-hour period of charging into the reference reservoir. It is seen that for low injection rates (below about 0.1 lbm/sec-ft (0.15 kg/sec-m) for the reference reservoir), the Darcy assumption

FIGURE 16. Comparison of Darcy and Forchheimer Flow for Various Mass Flow Rates

produces results closely matching the curves obtained from inclusion of Forchheimer's quadratic term in the momentum equation. For higher flow rates the error of the Darcy assumption may become important.

The Darcy and Forchheimer curves match reasonably well for a flow rate of 0.1 lbm/sec-ft (0.15 kg/sec-m) into the reference reservoir. However, Figure 17 indicates that, for the same flow rate, the Darcy and Forchheimer results will disagree if the value of the reservoir permeability is decreased. Similar trends can be shown for decreases in well diameter, as in Figure 18. Thus, if more accurate results are desired, care must be taken to include the Forchheimer assumption whenever flow conditions warrant.

5.2 THERMAL RESPONSE

To observe the process of heating a CAES porous rock reservoir subjected to hot air injection, a weekly injection-production cycle was applied to the reference reservoir for a period of 10 years. In addition, the effects of changing various reservoir parameters on the thermal response of the reservoir were examined by varying the individual parameters (including permeability, porosity, well spacing, rock thermal conductivity, rock thermal capacity, air storage pressure, and percentage of air mass cycled) one at a time from the reference condition, and applying the weekly air cycle for a period of one year. Thermal response of the reservoir was slow, and in most cases even a year was insufficient time to show significant deviation from the reference solution for reasonable changes in reservoir parameters. The trends, however, were observable. The results are discussed in the following sections.

5.2.1 Weekly CAES Cycle

The system load characteristics of any given electrical utility are unique, and design of a CAES system would be specific to the needs of the system it would serve. A schedule of power generation mix for a typical utility might be as shown in Figure 19, where the peak regions of the curve could represent that power to be supplied by CAES installations. It has been predicted that a CAES plant to serve a typical Midwest utility might

FIGURE 17. Comparison of Darcy and Forchheimer Flow for Various Permeabilities
 $[m = 0.1 \text{ lbm/sec-ft } (m = 1.5 \text{ kg/sec-m})]$

FIGURE 18. Comparison of Darcy and Forchheimer Flow for Various Well Diameters
 $[m = 0.1 \text{ lbm/sec-ft (} m = 0.15 \text{ kg/sec-m)}]$

FIGURE 19. Typical Power System Weekly Load

have approximately a 10- to 12-hour daily power generation period and would derive about 40% of its energy from the power grid over the weekends.⁽³⁾ These criteria were used to specify a hypothetical weekly CAES air cycle to examine the long-term effects changing various parameters might have on reservoir performance and thermal development.

This idealized weekly cycle is shown in Figure 20 for a reservoir with a maximum air mass content of $M_{\max}$. Already specified for the cycle is $\psi = 0.40$, which is the fraction of energy supplied to the CAES plant on the weekend. The other variable, λ , is the fraction of the total air mass which is cycled. This fraction, like any other specified variables, would be determined for a particular site from economic and operational considerations. Bush et al.,⁽³⁾ found that for a CAES plant conceptually designed for a site near Brookville, IL, cycling about 12% of the total stored air mass in the reservoir was optimal. Katz and Lady⁽⁴⁾ designed cycling to be about 9% if a Michigan reef (Belle River Mills) were used. A value of less than 7% was calculated for a site designed using the Galesville Aquifer near Media, IL. Although this is a function of several site-specific factors, 12% cycling ($\lambda = 0.12$) was chosen as a reasonable reference case value for this study. Equal daily charging and discharging times of 10 hours each were specified. In developing this weekly cycle, an assumption was made that the air temperature in the reservoir was uniform so that the air mass in the reservoir was directly proportional to the spatial average of reservoir pressure. The term $M_{\max}$ in Figure 20 could therefore be replaced by $P_{\max}$ (representing the maximum average reservoir pressure) to convert the figure to a representation of the spatial average reservoir pressure throughout the week. Although injection of hot air into the reservoir will affect the accuracy of this uniform temperature assumption, the bulk of the air within the reservoir will remain cool due to the very high thermal inertia of the rock mass, and any deviation from the original average pressure cycle will be small and gradual.

FIGURE 20. Idealized Weekly CAES Cycle

In order to prevent continued growth or shrinkage of a developed aquifer reservoir, it is necessary that

$$\int_{t=0}^{t=1 \text{ week}} [P(t) - P_0] dt \approx 0 \quad (a) \quad (17)$$

where $P(t)$ is spatial average reservoir pressure, P_0 is initial (before bubble growth) aquifer pressure, and t is time. Integrating the weekly pressure cycle curve (Figure 20) and solving for P_0 yields

$$P_0 = (1 - 0.6275 \lambda + 0.0466 \frac{\lambda}{\psi}) P_{\max} \quad (18)$$

For the weekly cycle reference case, P_0 was arbitrarily chosen as 50 atm (5070 kPa), or 735 psi. This results (for $\lambda = 0.12$ and $\psi = 0.40$) in a maximum reservoir pressure on Monday morning of $P_{\max} = 783$ psi (5400 kPa), and a minimum reservoir pressure on Friday evening of 689 psi (4750 kPa).

The reference geometry of the single well reservoir for weekly cycling is the same as that for short term studies, i.e., a cylindrical reservoir 400 ft in diameter with one-dimensional radial flow from a 7-in. (18-cm) well. The reference case porosity is 20%, permeability is 500 millidarcys, and the reservoir temperature is assumed to be initially uniform at 100°F (38°C). The rock thermal conductivity is assumed constant at 1.25 Btu/hr-ft-°F (216 W/m·°C). Rock density and specific heat are 165 lbm/ft³ (2643 kg/m³) and 0.22 Btu/lbm-°F (921 J/kg-°C), respectively (see Appendix C).

In order to follow the designated weekly cycle within the maximum and minimum pressures of 783 psi and 689 psi, the air injection and production rates were calculated to be 0.119 lbm/sec-ft (0.177 kg/sec-m) and 0.158 lbm/sec-ft (0.235 kg/sec-m), respectively. The final weekly reservoir pressure cycle used as the reference case for examining long-term effects of reservoir thermal development is given in Figure 21.

(a) In the natural gas storage industry, where cycles are annual rather than weekly, this criterion is known as the pound-day concept and integration is for one year.⁽⁴⁾

FIGURE 21. Reference Weekly Cycle (Neglecting Heating Effect)

5.2.2 Thermal Development

As the weekly CAES cycle is applied to the reference reservoir, the well pressure and the reservoir pressure at the outer perimeter respond according to Figure 22 during the first week. The maximum well pressure occurs on Sunday at 4:00 p.m. and reaches a value of 816 psi (5630 kPa). The minimum well pressure, 660 psi (4550 kPa), is reached at the end of the final production period on Friday at 6:00 p.m. The reservoir pressure responds according to the cycle described in the last section ($P_{\max} \approx 783$ psi (5400 kPa), $P_{\min} \approx 689$ psi (4750 kPa)).

The reservoir temperature is initially uniform at 100°F (38°C). Air is injected at 450°F (232°C) and extracted at whatever temperature is available after heat exchange with the reservoir rock. Figure 23 shows the first weekly cycle of temperature at the well-reservoir boundary in addition to various reservoir locations within the vicinity of the well. Large temperature swings characterize the well-reservoir boundary. Injection air heats the rock next to the well to 450°F (232°C), but the temperature drops quickly as air is produced. The temperature of the extracted air reaches a minimum of 244°F (118°C) at the end of the last weekly extraction period on Friday evening.

This large amplitude temperature cycling damps out rapidly as one moves away from the well, and at a radius of less than 10 ft (3 m) is almost completely damped, with temperature rising slowly but with little daily cycling.

After one year of cycling, the temperature curves are those shown in Figure 24. Temperature swings at the well have decreased and average reservoir temperatures have risen. The minimum extraction temperature (occurring again on Friday evening) has increased to 336°F (169° C). Ten years of cycling produces the curves of Figure 25, with the minimum extraction temperature having risen to 365°F (185°C).

If the minimum air extraction temperature is plotted as a function of time, the curve in Figure 26 results, showing that the greatest increase occurs within the first year. Further increases in minimum extraction temperature are very gradual.

FIGURE 22. Weekly Pressure Cycle

FIGURE 23. Reservoir Temperature Cycle at Various Locations for First Week

FIGURE 24. Reservoir Weekly Temperature Cycle at Various Locations After One Year (52 Consecutive Weeks)

FIGURE 25. Reservoir Weekly Temperature Cycle at Various Locations After Ten Years (52 Consecutive Weeks)

FIGURE 26. Minimum Air Extraction Temperature
(Initial reservoir temperature =
100°F, air injection temperature =
450°F)

Temperature profiles in the reservoir are shown in Figure 27 after cycling for the time periods shown. Due to the direct contact between hot injected air and the porous rock, and the large thermal capacitance of the rock, the thermal energy carried into the reservoir by the hot air is absorbed in the rock close to the well and transmission into the outer regions is slow. Because of the relatively low thermal conductivity of the rock, temperatures in the regions away from the reservoir rise very slowly, and the higher temperatures remain fairly localized around the wells.

FIGURE 27. Thermal Growth of Reservoir

One more result is observed as the average reservoir temperature rises. Since the reservoir volume of the one-dimensional model is held constant, and the amount of air mass injected is equal to that produced, the rise in average reservoir temperature causes a corresponding increase in reservoir pressure. Figure 28 shows the increase in maximum and minimum well pressure for constant injection and production flow rates over the 10-year period. In actual practice, the average pressure could not be allowed to increase in an aquifer without causing further growth of the reservoir. This could be easily controlled, however, by lowering the amount of air injected into the reservoir during one cycle. In actual operation, a CAES plant would be required to follow the system peaking load with adjustments, such as reservoir pressure level, being continually made.

FIGURE 28. Pressure Rise Due to Thermal Development

5.2.3 Response to Parameter Variation

The sensitivity of the thermal behavior of the reservoir to adjustments in several reservoir parameters was examined by changing parameters one at a time from their reference value and applying the weekly air cycle for a period of one year. The results were then compared with those of the reference reservoir after one year. The temperature and pressure profiles are so similar to those of the reference case discussed in the preceding section that for most cases they are not given here. The minimum extraction temperature (an indication of thermal growth) and the maximum and minimum average reservoir pressures after one year are presented in Table 1 for various parameters. It is seen that the only parameters having any noticeable effect on extraction temperature after one year are mass flow rates and the rock thermal conductivity. As might be expected, increasing the flow increases the rate of thermal growth in the reservoir. Increased thermal conductivity causes energy to dissipate away from the well more rapidly so that temperatures near the well are decreased and extraction temperatures are lower.

Only a small change in minimum extraction temperature occurred when the rock thermal capacity was lowered 25%. Figure 29 shows that the rock temperature throughout most of the reservoir was higher for the lower thermal capacity rock reservoir. The well boundary temperature was slightly less. Temperature gradients were reduced.

The temperature wave did not reach the outer radius of even the smallest diameter reservoir in the one-year period studied. Thus all of the included reservoirs are, in effect, infinite with regard to thermal growth. Extraction temperatures remained unaffected by changes in reservoir diameter.

Changes in porosity and reservoir diameter are direct changes in reservoir volume and, thus, the reservoir pressure. Their effects (as well as those of simply changing the average storage pressure) on the minimum extraction temperature are small. Also, the permeability of a given rock reservoir has no effect on the thermal growth rate.

TABLE 1. Minimum Extraction Temperatures and Reservoir Pressures After One Year of CAES Cycling for Various Parametric Conditions

Parameter with Reference Value	Off-Reference Condition	Minimum Extraction Temperature, °F(°C)	Maximum Reservoir Pressure, psi (kPa)	Minimum Reservoir Pressure, psi (kPa)
Reference Solution	--	337.12 (169.51)	791.5 (5457)	696.3 (4801)
Permeability	200	337.11 (169.51)	791.5 (5457)	696.3 (4801)
500 md	2000	337.13 (169.52)	791.5 (5457)	696.3 (4801)
Porosity	0.10	336.69 (169.27)	839.3 (5787)	649.2 (4476)
0.20	0.30	337.82 (169.90)	776.3 (5352)	712.6 (4913)
Average Storage Pressure	20 (2030)	337.13 (169.52)	345.7 (2384)	250.7 (1729)
50 atm (5070 kPa)	80 (8100)	337.09 (169.49)	1237.2 (8530)	1141.9 (7873)
Reservoir Diameter	282 (86)	338.36 (170.20)	849.4 (5856)	656.5 (4526)
400 ft (122 m)	490 (149)	336.96 (169.42)	772.6 (5327)	709.4 (4891)
Mass Flow Rates	0.050; 0.066 (0.074; 0.098)	330.66 (165.92)	759.9 (5239)	720.4 (4967)
Inject 0.119 lbm/sec-ft; Produce 0.158 lbm/sec-ft (0.177 kg/sec-m; 0.235 kg/sec-m)	0.298; 0.395 (0.443; 0.588)	341.89 (172.16)	863.7 (5955)	621.1 (4282)
Rock Thermal Conductivity	0.313 (0.542)	342.09 (172.27)	790.4 (5450)	695.3 (4794)
1.25 Btu/hr-ft-°F (2.16 W/m-°C)	3.13 (5.42)	328.30 (164.49)	793.3 (5470)	697.9 (4812)
Rock Thermal Capacity ($\rho_s C_s$)	30 (2.01)	336.93 (169.41)	794.0 (5474)	698.4 (4815)
39.1 Btu/ft ³ -°F (2.62 MJ/m ³ -°C)

FIGURE 29. Reservoir Temperature Profiles After One Year for Two Values of Rock Thermal Capacity

Changes in most parameters, within reasonable limits, did not significantly change the minimum extraction temperature within one year, except to indicate the direction of change. The thermal capacitance of the rock is so large in comparison to the air that the effects of changes in most parameters are masked.

6.0 RECOMMENDATIONS FOR CONTINUED STUDY

Due to the limited nature of the one-dimensional dry reservoir model, the results presented in this report are considered to be preliminary to more detailed studies presently underway or planned. Among the problems which will be addressed in the next phases of analytical study are:

1. the dynamics of non-radial heat and mass flow - Extension of the flow model to more than one dimension will permit examination of such things as:
 - natural convection within the reservoir,
 - the interaction of wells in a multiple well field,
 - heat and mass losses through the reservoir bottom and caprock,
 - performance (deliverability, thermal development, etc.) of asymmetric reservoirs, and
 - the effects of directional permeability on reservoir operation.
2. the effects of porous structure heterogeneity on flow dynamics - Non-uniformities in reservoir rock may cause preferential airflows (fingering and/or wafering) affecting well deliverability as well as reservoir heating profiles and thermal and mechanical stress development in the rock structures.
3. bubble development and movement of the water front surrounding the air bubble - The optimal rate and manner of bubble growth will directly affect the economics of compressed air storage. Cyclic movement of the developed air bubble/water front interface could conceivably affect well deliverability (depending on reservoir properties) as well as contribute to possible geochemical reactions and undesirable product generation and transmission.
4. effect of hot air injection on well integrity - The thermo-mechanical response of steel air storage well casings and cement grouting exposed to possible elevated and cyclic air temperatures is important because

it affects the suitability of porous underground media for hot compressed air storage. For the same reason, the problem of heat transfer into the surrounding earth between the reservoir and the surface is also important.

5. heat and mass transfer through the caprock - This will affect not only the thermal development of the heated reservoir, but may also damage caprock impermeability.
6. the presence of water vapor and liquid phases in the reservoir and their effects on flow dynamics and thermodynamics - Rough computer calculations indicate that the presence of moisture left in the air storage zone after bubble development may substantially retard the development of the heated region around the well when hot air is injected. Incorporation of moisture effects in the reservoir flow model will be necessary to more carefully and completely analyze this problem.
7. drying of the air storage zone - Preliminary calculations indicate that drying of the air storage zone (which may be necessary from a geochemical standpoint if air is stored hot, as well as a flow standpoint) may take several years to accomplish, possibly requiring large-scale surface dehydration equipment, such as refrigeration facilities or cooling towers. There is certainly motivation for more detailed study of this issue.
8. stresses and stress fluctuations in the rock matrix due to temperature and pressure gradients and cycling - Mechanical damage to the reservoir structure due to severe stress conditions may result in serious flow obstructions, or to carryover of sand particles to surface machinery with harmful results. This is an important problem which will require thorough analysis. Computer codes are presently available which, when coupled to the flow models, are capable of performing this stress analysis work.

These are not meant to represent all of the problems that may arise in the use of porous media reservoirs for CAES. They do, however, represent current thinking as to those of most importance which will require thorough analysis in order to solve. All of the analytical studies are being integrated, when possible, with the experimental studies of CAES aquifer reservoirs.

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APPENDIX A

DEVELOPMENT OF DIFFERENTIAL EQUATIONS

DEVELOPMENT OF DIFFERENTIAL EQUATIONS

The governing equations of compressible flow through a dry porous rigid structure are developed in Appendix A. Starting from general forms of the conservation laws, simplifications applicable to flow regimes that may be encountered in CAES systems are made to develop the final forms of the equations used.

GENERAL CONSERVATION LAWS

In classical fluid mechanics, the conservation laws of mass, momentum, and energy can be written, respectively, as

$$\frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \bar{v}_0) = 0 \quad (A-1)$$

$$\rho \left[\frac{\partial \bar{v}_0}{\partial t} + \bar{v}_0 \cdot \nabla \bar{v}_0 \right] = -\nabla P - [\nabla \cdot \underline{\tau}] + \rho \bar{g} \quad (A-2)$$

and

$$\rho \left(\frac{\partial h}{\partial t} + \bar{v}_0 \cdot \nabla h \right) = \nabla \cdot K \nabla T + u \Phi + \left(\frac{\partial P}{\partial t} + \bar{v}_0 \cdot \nabla P \right) \quad (A-3)$$

If the flow is within a porous solid, these equations become

$$\phi \frac{\partial \rho}{\partial t} + (\nabla \cdot \rho \bar{v}) = 0 \quad (A-4)$$

$$\rho \left[\frac{\partial \bar{v}}{\partial t} + \frac{\bar{v}}{\phi} \cdot \nabla \bar{v} \right] = \phi \left[-\nabla P - \frac{\bar{F}}{\phi} + \rho \bar{g} \right] \quad (A-5)$$

and

$$\rho \left(\phi \frac{\partial h}{\partial t} + \bar{v} \cdot \nabla h \right) = \phi \nabla \cdot K \nabla T + u \Phi_p + \left(\phi \frac{\partial P}{\partial t} + \bar{v} \cdot \nabla P \right) + H_v (T_s - T) \quad (A-6)$$

In these equations, the term ϕ is known as the porosity (open volume divided by total volume) and reflects the presence of the solid phase within the flow field. The velocity $\bar{v}$ in these equations is the superficial velocity (volume rate of flow through a unit cross-sectional area) averaged over a small region of space--small with respect to macroscopic dimensions of the flow system but large with respect to pore size. All fluid properties are averaged over a region available to flow that is large with respect to pore size.

The fluid phase viscous shear force term $-\nabla \cdot \underline{\tau}$ in the momentum equation has been replaced by the term $\frac{F}{\phi}$ to include not only viscous shear forces, but also form drag forces imposed on the flowing fluid by the solid structure. Similarly, the viscous energy dissipation function ϕ in the energy equation has been replaced by ϕ_p to account for dissimilar viscous energy dissipation due to the effects of the porous medium. An additional term, $H_V(T_S - T)$, is added to register the heat transfer to the fluid from the solid phase.

One additional equation must be written for energy balance in the porous solid. In terms of the characteristic rock temperature T_S , it is

$$(1 - \phi) \left(C_S \rho_S \frac{\partial T_S}{\partial t} - \nabla \cdot K_S \nabla T_S \right) = H_V (T - T_S) \quad (A-7)$$

SIMPLIFYING ASSUMPTIONS

Equations A-4, A-5, A-6 and A-7 represent the general equations of flow in a porous medium. Several simplifications can be made to reduce these to forms more suitable for analysis of CAES porous rock air reservoirs. Simplifying assumptions incorporated into this study are discussed in the sections that follow.

Negligible Viscous Dissipation of Energy

It is well known that for gases (where Prandtl numbers are near 1) the velocity must approach the speed of sound before viscous dissipation terms become significant in comparison to other terms of the energy equation. CAES reservoir flow velocities will be several orders of magnitude below the sonic level so that neglect of the dissipation term $u\phi_p$ in Equation A-6 is perfectly warranted.

Kinetic Energy Disregarded

For the low velocities of CAES reservoir flows, the kinetic energy terms are also negligible in comparison to other forms of energy. In fact, calculations show that disregarding kinetic energy causes errors of only a very small fraction of a percent in the solutions to the flow equations. If kinetic energy is neglected, the term $\bar{v} \cdot \nabla P$ drops from the energy equation.

Ideal Gas Behavior

As can be seen from Figure A-1, air deviates only slightly from ideal gas behavior under conditions contemplated for compressed air energy storage. The ideal gas equation of state

$$\rho = \frac{P}{RT} \quad (A-8)$$

is therefore justified to relate air density, pressure and temperature.

The relationship between gas enthalpy and temperature is established from the thermodynamic relationship for an ideal gas,

$$dH = C_p dT \quad (A-9)$$

Thus, $\frac{\partial h}{\partial T} = C_p$, and $\nabla h = C_p \nabla T$.

Darcy or Forchheimer Flow Assumed

If the pore Reynolds number is small the inertial terms of the momentum equation (Equation A-5) may be neglected to yield

$$\frac{\bar{F}}{\phi} = (-\nabla P + \rho \bar{g}) \quad (A-10)$$

For flow in this velocity regime, the viscous forces have been shown experimentally to be proportional to velocity, i.e.,

$$\frac{\bar{F}}{\phi} = C \bar{v} \quad (A-11)$$

FIGURE A-1. Compressibility Factors for Air

where the constant of proportionality, C , should be expressed as the ratio $\frac{\mu}{\kappa}$. μ is the absolute viscosity of the fluid, and κ is the porous medium permeability. Thus, the momentum equation may be written as

$$\bar{v} = - \frac{\kappa}{\mu} [\nabla P - \rho \bar{g}] \quad (A-12)$$

which is known as Darcy's law for flow through porous media.

Darcy's law says that in the flow of a fluid through a porous medium, the flow rate increases proportionally with increased pressure drop. Data gathered by Fancher, Lewis and Barnes⁽¹⁾ (Figure A-2) show that, similar to the case for flow in pipes, this "viscous" flow occurs only over a range of flows. At some higher flow rate the increased rate no longer matches the increased pressure drop (in pipes this indicates the onset of turbulence)

FIGURE A-2. Flow Through Porous Media⁽³⁾

and the Darcy equation is no longer adequate. Forchheimer⁽²⁾ suggests the addition of a quadratic term to the Darcy momentum equation to more correctly match the nonlinear relationships between pressure drop and velocity. The Forchheimer momentum equation is expressed as

$$\frac{\mu}{\kappa} \bar{v} + \beta \rho [\underline{v}_a \cdot \bar{v}] = -\nabla P + \rho \bar{g} \quad (A-13)$$

where $\underline{v}_a = \begin{pmatrix} |v_r| & 0 & 0 \\ 0 & |v_\theta| & 0 \\ 0 & 0 & |v_z| \end{pmatrix}$

where the term β is traditionally called the turbulence factor, and has been correlated with reservoir permeability and porosity by Katz and others⁽³⁾ (Figure A-3). The Forchheimer equation is sometimes referred to as the modified Darcy equation, and, in fact, reduces to the Darcy equation when velocities are small.

Rapid Thermal Equilibrium Between Solid and Fluid Phases

Due to the intimate contact between fluid and solid, the assumption of local thermal equilibrium between solid and fluid ($T_s = T$) is commonly employed in porous media flow analyses. In reality, the temperatures of the solid matrix and fluid are coupled by the internal volumetric heat transfer coefficient H_v . An exact determination of this coefficient by theoretical analysis is not possible as a result of the complex flow patterns and geometric characteristics of porous media. Although correlation formulae have been developed (using dimensional analysis and experimental measurement) for specialized porous structures with varying degrees of success, experiments have upheld the validity of the local thermal equilibrium assumption for flow conditions similar to those expected in CAES reservoirs. This assumption simplifies the problem by reducing the number of energy equations from two to one.

Solid Properties Assumed Stepwise Constant

Already inherent in the conservation equations is the assumption that the reservoir properties of porosity, permeability, rock specific heat, and rock

FIGURE A-3. Correlation of Turbulence Factor β with Permeability(1)

density are at least stepwise constant. They are assumed independent of temperature and pressure, and to be adequately represented by appropriate stair functions of reservoir geometry.

FINAL EQUATIONS

Incorporation of the assumptions discussed in the preceding section reduces the conservation laws to the following:

Conservation of Mass

$$\phi \frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \nabla \cdot \left(\frac{P}{T} \right) \bar{v} = 0 \quad (A-14)$$

Conservation of Momentum

$$\frac{\mu}{\kappa} \bar{v} + \beta \left(\frac{P}{RT} \right) [\bar{v}_a \cdot \bar{v}] = -\nabla P + \rho \bar{g} \quad (A-15)$$

Conservation of Energy

$$\frac{C_p P}{RT} \left(\phi \frac{\partial T}{\partial t} + \bar{v} \cdot \nabla T \right) - \phi \frac{\partial P}{\partial t} = \phi \nabla \cdot K \nabla T + (1 - \phi) \nabla \cdot K_s \nabla T - (1 - \phi) \rho_s C_s \frac{\partial T}{\partial t} \quad (A-16)$$

Since the porosity ϕ is constant (at least stepwise) the conductivity terms may be written

$$\nabla \cdot \phi K \nabla T + \nabla \cdot (1 - \phi) K_s \nabla T = \nabla \cdot [\phi K + (1 - \phi) K_s] \nabla T$$

Defining an equivalent thermal conductivity

$$K_e = \phi K + (1 - \phi) K_s \quad (A-17)$$

this reduces to $\nabla \cdot K_e \nabla T$.

It should be noted that although the thermal conductivity of air varies with temperature, its contribution to the equivalent matrix conductivity in porous rock is so small (less than 1%) that it can be assumed constant with negligible effect. The uncertainty of determining appropriate values for rock conductivity will have a much greater effect on the analysis of a given reservoir.

As written, the continuity and momentum equations are both in what are called "conservative forms" suitable for finite differencing for numerical solution. By "conservative" it is meant that no variables appear both outside as well as inside a differential. This is not the case for the energy equation as it appears in Equation A-16, but it can be put into a numerically conservative form by adding

$$\frac{C_p T}{R} \left[\phi \frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \nabla \cdot \left(\frac{P}{T} \right) \bar{v} \right] = 0$$

to Equation A-16 and simplifying to

$$\left(\frac{C_p}{R} - 1 \right) \phi \frac{\partial P}{\partial t} + \left(\frac{C_p}{R} \right) \nabla \cdot P \bar{v} = \nabla \cdot K_e \nabla T - (1 - \phi) \rho_s C_s \frac{\partial T}{\partial t}$$

Rearranged, the energy equation, including energy of both fluid and solid, becomes

$$\frac{\partial T}{\partial t} = \frac{1}{(1 - \phi) \rho_s C_s} \left[\nabla \cdot K_e \nabla T + \left(1 - \frac{C_p}{R} \right) \phi \frac{\partial P}{\partial t} - \left(\frac{C_p}{R} \right) \nabla \cdot P \bar{v} \right] \quad (A-18)$$

which is a conservative form.

ONE-DIMENSIONAL RADIAL FLOW

Air entering or leaving a wellbore at the bottom of a well flows radially through the porous rock. Thus, a reasonable yet simple model for flow to and from a single well is the one-dimensional radial model depicted in Figure 3-2. Since there is no component of gravity in the horizontal direction, the term $\rho \bar{g}$ in the momentum equation disappears, and all equations can be rewritten in terms of the single radial coordinate r . The velocity vector $\bar{v}$ reduces to the radial component v .

Conservation of Mass

$$\phi \frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left[r \left(\frac{P}{T} \right) v \right] = 0 \quad (A-19)$$

Conservation of Momentum

$$\frac{\mu}{\kappa} v + \beta \left(\frac{P}{RT} \right) |v| v + \frac{\partial P}{\partial r} = 0 \quad (A-20)$$

Conservation of Energy

$$\frac{\partial T}{\partial t} = \frac{1}{(1 - \phi) \rho_s C_s} \left[\frac{1}{r} \frac{\partial}{\partial r} \left(k_e r \frac{\partial T}{\partial r} \right) + \left(1 - \frac{C_p}{R} \right) \phi \frac{\partial P}{\partial t} - \left(\frac{C_p}{R} \right) \frac{1}{r} \frac{\partial}{\partial r} (r P v) \right] \quad (A-21)$$

These are the final forms of the one-dimensional conservation equations used in developing a numerical computer model for determining pressure and temperature gradients in air-filled porous rock reservoirs.

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APPENDIX B

DEVELOPMENT AND SOLUTION OF THE DIFFERENCE EQUATIONS

DEVELOPMENT AND SOLUTION OF THE DIFFERENCE EQUATIONS

In Appendix B the difference equations are derived from the governing difference equations. Details of the development of the difference Equations 3-10 and 3-13 are presented. The method of solution of the equations is subsequently developed.

MASS TRANSPORT DIFFERENCE EQUATIONS

The difference equation for the mass transport is developed from the continuity equation, Equation 5.

$$\phi \frac{\partial}{\partial t} \left(\frac{P}{T} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left(\frac{Prv}{T} \right) = 0 \quad (5)$$

A weighted volumetric flux, F , is defined from the product of radius and velocity.

$$F = rv \quad (B-1)$$

For the Darcy flow assumption the velocity can be written as

$$v = - \frac{\kappa}{\mu} \frac{\partial P}{\partial r} \quad (B-2)$$

so that

$$F = - \frac{\kappa r}{\mu} \frac{\partial P}{\partial r} \quad (B-3)$$

The pressure and radius can be separated and the result integrated between two radial positions:

$$\int_{r_i}^{r_o} F \frac{\partial r}{r} = - \int_{P_i}^{P_o} \frac{\kappa}{\mu} \partial P \quad (B-4)$$

which yields

$$F = - \frac{\frac{\kappa}{\mu} (P_o - P_i)}{\ln (r_o/r_i)} \quad (B-5)$$

Since the flux from a node to the difference cell boundary is equal to the flux from the boundary to the adjacent node, then the flux across the boundary can be written as

$$F_{j+1/2} = \frac{-(P_{j+1/2} - P_j)}{\left(\frac{\mu}{\kappa}\right)_j \ln \left(\frac{r_{j+1/2}}{r_j}\right)} = \frac{-(P_{j+1} - P_{j+1/2})}{\left(\frac{\mu}{\kappa}\right)_{j+1} \ln \left(\frac{r_{j+1}}{r_{j+1/2}}\right)} \quad (B-6)$$

The boundary pressure, $P_{j+1/2}$, is eliminated to give

$$F_{j+1/2} = \frac{-(P_{j+1} - P_j)}{\left(\frac{\mu}{\kappa}\right)_j \ln \left(\frac{r_{j+1/2}}{r_j}\right) + \left(\frac{\mu}{\kappa}\right)_{j+1} \ln \left(\frac{r_{j+1}}{r_{j+1/2}}\right)} \quad (B-7)$$

A flow conductance between nodes j and $j+1$ is defined as

$$C_j = \frac{1}{\left(\frac{\mu}{\kappa}\right)_j \ln \left(\frac{r_{j+1/2}}{r_j}\right) + \left(\frac{\mu}{\kappa}\right)_{j+1} \ln \left(\frac{r_{j+1}}{r_{j+1/2}}\right)} \quad (B-8)$$

In this way the flux between nodes j and $j+1$ is defined as

$$F_j = - C_j (P_{j+1} - P_j) \quad (B-9)$$

For high velocity flows the Forchheimer equation was used to define the velocity:

$$\beta \frac{P}{RT} v |v| + \frac{\mu}{\kappa} v = \frac{\partial P}{\partial r} \quad (B-10)$$

Define

$$\eta = -1 \text{ for } v < 0, \text{ i.e., } \frac{\partial P}{\partial r} > 0$$

and

$$\eta = +1 \text{ for } v \geq 0, \text{ i.e., } \frac{\partial P}{\partial r} < 0$$

so that

$$\eta \beta \frac{P}{RT} v^2 + \frac{\mu}{\kappa} v = - \frac{\partial P}{\partial r} \quad (B-11)$$

and

$$v = \frac{- \frac{\mu}{\kappa} + \sqrt{\left(\frac{\mu}{\kappa}\right)^2 - 4 \eta \beta \frac{P}{RT} \frac{\partial P}{\partial r}}}{2 \eta \beta \frac{P}{RT}} \quad (B-12)$$

The product $\eta \frac{\partial P}{\partial r}$ is always negative so that the velocity is written as

$$v = \frac{- \frac{\mu}{\kappa} + \sqrt{\left(\frac{\mu}{\kappa}\right)^2 + 4 \beta \frac{P}{RT} \left| \frac{\partial P}{\partial r} \right|}}{2 \eta \beta \frac{P}{RT}} \quad (B-13)$$

From Equation B-10

$$v = \frac{- \frac{\partial P}{\partial r}}{\frac{\mu}{\kappa} + \beta \frac{P}{RT} |v|} \quad (B-14)$$

or

$$v = \frac{-\frac{\partial P}{\partial r}}{\frac{\mu}{\kappa} + \eta \frac{\beta P |v|}{RT}} \quad (B-15)$$

Substituting Equation B-13 into Equation B-15 gives

$$v = \frac{-2 \frac{\partial P}{\partial r}}{\frac{\mu}{\kappa} + \sqrt{\left(\frac{\mu}{\kappa}\right)^2 + 4 \beta \frac{P}{RT} \left|\frac{\partial P}{\partial r}\right|}} \quad (B-16)$$

By treating $\left|\frac{\partial P}{\partial r}\right|$ as a constant, an integration similar to Equation B-4 can be made. Subsequently, the flux can be defined as

$$F_j = -C_j (P_{j+1} - P_j) \quad (B-17)$$

where the high velocity form of the flow conductance is written as

$$C_j = \left[\left\{ \frac{\mu}{\kappa} + \sqrt{\left(\frac{\mu}{\kappa}\right)^2 + 4 \beta \frac{P}{RT} \left|\frac{\partial P}{\partial r}\right|} \right\}_j \ln \left(\frac{r_{j+1/2}}{r_j} \right) + \left\{ \frac{\mu}{\kappa} + \sqrt{\left(\frac{\mu}{\kappa}\right)^2 + 4 \beta \frac{P}{RT} \left|\frac{\partial P}{\partial r}\right|} \right\}_{j+1} \ln \left(\frac{r_{j+1}}{r_{j+1/2}} \right) \right]^{-1} \quad (B-18)$$

With the flux terms properly defined, the donor cell differencing leading to Equation 3-10 is straightforward.

ENERGY TRANSPORT DIFFERENCE EQUATIONS

The difference equation for the energy transport is developed from Equation 7,

$$\frac{\partial T}{\partial t} = \left[\frac{1}{r} \frac{\partial}{\partial r} \left(K_e r \frac{\partial T}{\partial r} \right) + \left(1 - \frac{C_p}{R} \right) \phi \frac{\partial P}{\partial t} - \left(\frac{C_p}{R} \right) \frac{1}{r} \frac{\partial}{\partial r} (Prv) \right] \frac{1}{(1 - \phi) \rho_s C_s} \quad (7)$$

The only term for which a detailed development is considered necessary is the heat conduction term. In this case a heat flux is defined similar to the weighted volumetric flux of Equation B-1.

$$H = K_e r \frac{\partial T}{\partial r} \quad (B-19)$$

The temperature and radius can be separated and the result integrated between two radial positions.

$$\int_{r_i}^{r_o} H \frac{\partial r}{r} = \int_{T_i}^{T_o} K_e \partial T \quad (B-20)$$

yields

$$H = \frac{K_e (T_o - T_i)}{\ln(r_o/r_i)} \quad (B-21)$$

Since the heat flux between two nodes is constant, the flux at the cell boundary can be written as

$$H_{j+1/2} = \frac{K_{e,j} (T_{j+1/2} - T_j)}{\ln\left(\frac{r_{j+1/2}}{r_j}\right)} = \frac{K_{e,j+1} (T_{j+1} - T_{j+1/2})}{\ln\left(\frac{r_{j+1}}{r_{j+1/2}}\right)} \quad (B-22)$$

The temperature at the interface, $T_{j+1/2}$, is eliminated to give

$$H_{j+1/2} = \frac{T_{j+1} - T_j}{\frac{1}{K_{e,j}} \ln\left(\frac{r_{j+1/2}}{r_j}\right) + \frac{1}{K_{e,j+1}} \ln\left(\frac{r_{j+1}}{r_{j+1/2}}\right)} \quad (B-23)$$

A heat conductance between nodes j and $j+1$ is defined as

$$K_j = \frac{1}{\frac{1}{K_{e,j}} \ln\left(\frac{r_{j+1/2}}{r_j}\right) + \frac{1}{K_{e,j+1}} \ln\left(\frac{r_{j+1}}{r_{j+1/2}}\right)}$$

In this way the heat flux between the nodes is defined as

$$H_j = K_j (T_{j+1} - T_j)$$

From this point the donor cell differencing leading to Equation 13 is straightforward.

SOLUTION METHOD

This section of Appendix B provides the details of the solution of Equations 10 and 13. A flow chart of the computer solution is given in Figure B-1.

In the solution there are essentially three levels of time distinguished during each time step. Properties to be evaluated are designated as new time values by the subscript $n+1$ in Equations 10 and 13. The properties from which the solution is achieved are designated as old time values by the subscript n . The third condition is a result of the nonlinearity of the mass transport equation as well as the coupling of the transport equations. To obtain a reasonably conservative solution it is necessary to distinguish the pressure in the advected density from the pressure of the flux term.^(a) This requires guessing the new time values of the density. The guessed values are iteratively improved until they converge with the new time solution.

-
- (a) Previous numerical investigations of mass flow through porous media have typically not distinguished between the pressure associated with the density and with the flux. This results in the continuity equation combining with Darcy's law to form

$$\phi \frac{\partial P^2}{\partial t} + \frac{\kappa}{\mu r} P \frac{\partial}{\partial r} \left(r \frac{\partial P^2}{\partial r} \right) = 0$$

The most significant assumption inherent in deriving this equation is that the temperature is constant. This form of the equation provides a good pressure solution for the isothermal problem but is not conservative when temperatures vary.

FIGURE B-1. Computational Flow Chart

Designating pressure and temperature at old time with P_0 and T_0 , guessed values with P_G and T_G , and new time with P_N and T_N , Equations 10 and 13 are reconstructed to improve the visibility of these different levels of time.

$$P_{Nj} + \frac{\Delta t}{\phi} \frac{T_{Gj}}{r_j \Delta r_j} \left[C_{j-1} \left[\left(\frac{P_G}{T_G} \right)_{j-1, F+} + \left(\frac{P_G}{T_G} \right)_{j, F-} \right] (P_{Nj} - P_{Nj-1}) \right. \\ \left. + C_j \left[\left(\frac{P_G}{T_G} \right)_{j, F+} + \left(\frac{P_G}{T_G} \right)_{j+1, F-} \right] (P_{Nj} - P_{Nj+1}) \right] = \frac{T_{Gj}}{T_{0j}} P_{0j} \quad (B-24)$$

$$T_{Nj} = T_{0j} + \frac{\Delta t}{(1 - \phi) \rho_s C_s} \left[\frac{1}{r_j \Delta r_j} (K_{j-1} T_{Gj-1} - (K_{j-1} + K_j) T_{Gj} + K_j T_{Gj+1}) \right. \\ \left. + \left(1 - \frac{C_p}{R} \right) \phi \left(\frac{P_{Nj} - P_{0j}}{\Delta t} \right) \right. \\ \left. - \left(\frac{C_p}{R} \right) \frac{1}{r_j \Delta r_j} (P_{Nj} F_{j, F+} - P_{Nj-1} F_{j-1, F+} \right. \\ \left. - P_{Nj} F_{j-1, F-} + P_{Nj+1} F_{j, F-}) \right] \quad (B-25)$$

The required sequence of the solution is now more obvious. All the pressures and the flux terms used in Equation B-25 are obtained from the solution of Equation B-24.

In Equation B-24 the pressures associated with the density are iteratively improved. The determination of the sign of the flux term is based on these guessed values of pressure.

In Equation B-25 the conduction terms on the right-hand side could be transposed to the left side and thereby be designated as new time values. This would require a simultaneous solution technique such as a Gaussian elimination. However, since the primary mode of energy transport is by advection which, computationally, depends only on the pressure solution, it is not necessary to solve exactly for the temperatures. Instead, the best prediction of the mass and energy transport is gained by the best possible solution of Equation B-24.

BOUNDARY CONDITIONS

The boundary conditions are applied in a direct manner except for the mass flux specified at the wellbore.

The outer reservoir boundary is insulated to the passage of either mass or energy. During mass injection the temperature of the injected air is specified at the wellbore. During extraction, the wellbore node is given the temperature of the second node at the previous time step. When there is no flow the wellbore node is disconnected from the solution by setting the mass conductance, C_1 (from Equation 8) and the thermal conductance, K_1 (from Equation 11) to zero. In this way there is no source or sink for mass or energy.

The forcing function for the advection of mass and energy is the pressure specified at the wellbore. Therefore, to apply a mass flux boundary condition it is necessary to specify a pressure. This presents difficulties because the new time pressure must be known. Since the pressure cannot be known until the solution is obtained, the application of the mass flux boundary condition is an iterative procedure. The procedure is included in Figure B-1.

APPENDIX C

AIR AND ROCK PROPERTIES

AIR AND ROCK PROPERTIES

PROPERTIES OF AIR

The gas constant for air used in this study is

$$R = \frac{1.987 \text{ Btu/lbm-mole}^\circ\text{R}}{28.97 \text{ lbm/lbm-mole}} = 0.06859 \text{ Btu/lbm-}^\circ\text{R} = 287.2 \text{ J/kg-}^\circ\text{K}.$$

The constant pressure specific heat of air was held constant at

$$C_p = 0.242 \text{ Btu/lbm-}^\circ\text{F} = 1013 \text{ J/kg-}^\circ\text{C}.$$

Air viscosity as a function of temperature and pressure is presented in Figure C-1.

FIGURE C-1. Viscosity of Air

PROPERTIES OF TYPICAL RESERVOIR ROCK

Since the original source of most sand grains, as pointed out by Landes,⁽¹⁾ is granite or granite gneiss, the solid, or sand grain, phase of the porous reservoir can be assumed to have physical properties similar to these substances.

The density of granite or gneiss is in the range of 165 to 175 lbm/ft³ (2640 to 2800 kg/m³).⁽²⁻⁴⁾ The specific heat, C_s , and thermal conductivity, K_s , of high SiO₂ rocks, including granite, are given as functions of temperature in Figures C-2 and C-3, respectively. Values of rock properties used in this study were assumed constant at the following values:

- density, $\rho_s = 165 \text{ lb/ft}^3$ (2643 kg/m³)
- specific heat, $C_s = 0.22 \text{ Btu/lb-}^\circ\text{F}$ (921 J/kg- $^\circ\text{C}$)
- thermal conductivity, $K_s = 1.25 \text{ Btu/hr-ft-}^\circ\text{F}$ (2.16 W/m- $^\circ\text{C}$).

FIGURE C-2. Specific Heat of Some High SiO₂ Rocks⁽⁵⁾

FIGURE C-3. Thermal Conductivity of Some High SiO_2 Rocks⁽⁴⁾

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