FREE ELECTRON LASERS FOR THE XUV SPECTRAL REGION*

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1) The interest sparked by the operation of the first Free Electron Laser (FEL) by John Madey and his collaborators in 1975 has lead to some exciting results.¹⁻⁶ In the last two years remarkable progress has been made toward the realization of the FEL as a source of high power, tunable, coherent radiation. FEL oscillators and amplifiers have been operated at wavelengths varying from the centimeter to the near ultraviolet and at peak power levels up to a hundred megawatts. This wealth of experimental results is due to researchers at numerous laboratories: MSNW, TRW-Stanford, LASL, LBL-LLNL, MIT, NRL, Columbia-NRL, UC-Santa Barbara, Orsay, and Frascati.

The theory of FELs has at the same time reached a high level of completeness and is in good agreement with the experimental results. As a result of the experimental and theoretical progress we now have a good understanding of the physics and technology of FELs, which can be used to design systems operating in new wavelength regions, like the XUV spectral region.

The possibility of building a FEL operating at wavelengths shorter than 1000 Å is a result of the progress made in producing high density relativistic electron beams using electron storage rings. Storage rings specially

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designed for FEL applications and capable of accommodating undulator magnets 5 to 15 meters long, should offer the possibility of producing coherent radiation down to a few hundred Angstrom with average powers of the order of watts and peak powers up to hundreds of megawatts. One such ring is being built at Stanford University,⁷ while similar rings are also being studied at other laboratories^{8,9}. In this paper we want to briefly review the different operation modes of a FEL in the XUV region (Sections 2,3,4), we will then discuss in more detail, the Self Amplified Spontaneous Emission mode (Sections 5,6,7,8,9).

2) In a FEL a relativistic electron beam and an electromagnetic wave traverse an undulator. The coupling of the wave and the transverse electron beam current, induced by the undulator, can produce an energy transfer between the beam kinetic energy and the radiation field energy if a synchronism condition is satisfied. This condition relates the radiation field wavelength λ , the undulator period λ_0 , the undulator field B_0 , and strength parameter $K = e\lambda_0 B_0 / 2\pi mc^2$, and the beam energy γ measured in rest energy units¹⁰:

$$\lambda = \frac{\lambda_0}{2\gamma^2} (1 + K^2) \quad (1)$$

Notice that this wavelength is also the wavelength at which the spontaneous radiation from an electron traversing an undulator is emitted.¹¹

An important property of the FEL is that the energy transfer between the beam and the radiation can be enhanced by a collective instability producing an exponential growth of the radiation¹². When this instability

becomes important the FEL is said to operate in the High Gain Regime. The existence of this regime is very important for FEL operation in the XUV region where we do not have optical components with sufficiently high reflectivity and small absorption.¹³

Three modes of operation of a FEL can be considered. In the first mode, Self Amplified Spontaneous Emission, (SASE) the initial spontaneous radiation emitted by the electrons is amplified; this system does not require any optical components but needs a high density electron beam and a rather long undulator.^{8,12}

The second mode is the FEL oscillator; an optical cavity is used to reflect back and forth the radiation for further amplification by another electron bunch; this system can get by with a smaller electron beam density and a shorter undulator but needs mirrors for the cavity; at wavelengths smaller than 1000 Å these mirrors have yet to be developed and their reflectivity is expected to be on the order of 50%.

In the third mode, or Transverse Optical Klystron,¹⁴ an external laser beam at the spontaneous radiation wavelength is used to modulate the beam energy and longitudinal density distribution leading to the emission of coherent radiation at the higher harmonics of the input laser. Of the three modes this is the one requiring the least stringent electron beam parameters. In addition it does not need optical elements but on the other hand it requires an undulator with rather strict magnetic field tolerances and produces the smallest coherent radiation power.

3) In all three modes the FEL can be approximately characterized by one parameter, the FEL e-folding length, $4\pi\rho$, measured in number of undulator periods N_u ¹²

$$4\pi\rho = 4\pi \left(\frac{\kappa \lambda_o \Omega_p}{8\pi c} \right)^{2/3} \quad (2)$$

where Ω_p is the electron beam plasma frequency, defined in terms of the electron density n_o , and energy γ , by

$$\Omega_p = (4\pi r_e c^2 n_o / \gamma^3)^{1/2} \quad (3)$$

r_e being the classical electron radius.

For an oscillator to operate at short wavelength, where the optical cavity losses can be on the order of 100% per round trip, one needs a value of $4\pi\rho N_u$ on the order of one, i.e. a number of undulator periods

$$N_u \sim \frac{1}{4\pi\rho}.$$

In the case of SASE¹² the value of $4\pi\rho N_u$ must be on the order of 10.

In the case of SASE & oscillator modes the energy transfer from the beam to the radiation field is on the order of ρ , while in the TOK case the transfer from the input laser to the harmonics is rather small.

The expression (2) for the FEL growth rate applies only if two other conditions on the electron beam are satisfied. One is a condition on the beam energy spread, which must be less than ρ ; the second is a condition on the beam emittance, which must be smaller than the radiation wavelength. If these conditions are not satisfied the radiation growth rate decreases and the output laser power is reduced.⁸

For wavelengths in the millimeter region and electron energy of a few MeV the value of ρ can be on the order of 1. In the VUV region with electron energies of several hundred MeV, ρ is on the order of 10^{-3} and one can expect an energy transfer from the beam to the radiation on the order of a few part in a thousand.

4) In the wavelength region below 1000 Å, the best accelerators available to produce the high density electron beams required to operate in the SASE mode are electron storage rings. Existing storage rings, such as the VUV ring of the National Synchrotron Light Source at Brookhaven, can provide an average emittance on the order of 10^{-8} mrad, an energy spread of about 10^{-3} and a peak current of 60 A at an energy of 750 MeV.¹⁵ A ring like this, with straight sections capable of accommodating undulators 5 to 6 meters long, would allow us to produce coherent radiation in the 1000 Å region.

We believe that it is now possible to design a storage ring with an energy of 700 to 1000 MeV, the same energy spread and an emittance smaller by an order of magnitude than that of the VUV ring, and peak currents in the range of 100 to 200 A. Such a ring would enable us to produce radiation in the wavelength range of 100 to 500 Å using undulators about 10 meters long.^{7,8,9}

Using this ring, the peak radiation power that one can obtain in the SASE mode is on the order of 10^{-3} times the beam peak power, or 100 MW. This pulse would have a duration of about 100 ps and a repetition rate of 10 Hz, for an average radiation power of 0.1 W.

With the same system operating in the oscillator mode, one can obtain an average output power of the order of 1 W, pulse duration of about 100 ps and a repetition rate determined by the ring revolution time to be on the order of a few MHz, and a peak power of about 10 KW. For this oscillator it is also possible, by modulating the system gain, to reduce the repetition rate and increase the peak power.

For the TOK mode one can expect conversion efficiencies on the order of 10^{-6} around the tenth harmonic, so that starting with a 100 MW peak power laser at 2000 Å one should be able to produce about 100 W at around 200 Å.

In all of these cases the angular distribution of the radiation is determined by the electron beam radius, a , and the radiation wavelength; the characteristic angle is on the order of λ/a , i.e. of a few tenth of milliradians. The line width is on the order of the wavelength divided by the electron bunch length, i.e. 10^{-6} - 10^{-5} , for the oscillator and the TOK mode. For the SASE mode it depends on the details of the system and is intermediate between the oscillator limit and the inverse of the number of periods in the undulator, i.e. between 10^{-3} and 10^{-5} .

5) FEL Equations

In the remainder of this paper we will discuss the High Gain Regime and the SASE mode of operation of a FEL. Following the work of other authors^{16,17} we write the FEL equations using the phase and energy as electron variables and use the slowly varying amplitude and phase approximation for the radiation field. These equations can be written in a

very general form including the effects of space-charge fields and higher harmonics of the radiation field.¹⁸ To simplify our discussion we neglect these terms and use the results of reference (18) to evaluate their effects. Our notations are the following: z is the electron beam and electromagnetic wave direction of propagation; x and y are the transverse coordinates; B_0 is the undulator magnetic field (we use a helical undulator for simplicity) and λ_0 , N_w are its period and length in number of periods respectively; λ is the wavelength of the radiation field; γ is the electron energy in units of $m_0 c^2$; $\beta_z = 1$ is the longitudinal electron velocity and $\beta_{\perp} = K/\gamma$ the amplitude of the transverse velocity; the electron phase relative to the electromagnetic wave, ϕ , is related to z and t by $\phi = 2\pi z/\lambda_0 + 2\pi(z - ct)/\lambda$; the resonant energy γ_R is related to λ_0 , λ and K by $\gamma_R^2 = \lambda_0(1 + K^2)/2\lambda$; the undulator frequency ω_0 is $\omega_0 = 2\pi c\beta_z/\lambda_0$; the amplitude, E_0 , and phase, θ_0 , of the radiation field are combined to yield a complex amplitude $\alpha = iE_0 e^{i\theta_0}$.

To write the FEL equations it is convenient to use a set of normalized variables and introduce some quantities to characterize the beam properties.^{12,18} We will use the relativistic beam plasma frequency already introduced in Sec. 3, which for a beam with energy dispersion is given by

$$\Omega_p = \left(\frac{4\pi r_e n_0 c^2}{\langle \gamma_0 \rangle^3} \right)^{1/2} \quad (4)$$

where $\langle \gamma_0 \rangle$ is the average value of the initial electron energy; we introduce also the quantities

$$\rho = \left[\frac{K}{4} \left(\frac{\langle \gamma_0 \rangle}{\gamma_R} \right)^2 \frac{\Omega_p}{\omega_0} \right]^{2/3} \quad (5)$$

and

$$\dot{\phi}_0 = \omega_0 \left(1 - \frac{\gamma_R^2}{\langle \gamma_0 \rangle^2} \right) \quad (6)$$

and a normalized time

$$\tau = 2\omega_0 \rho \left(\frac{\gamma_R}{\langle \gamma_0 \rangle} \right)^2 t \quad (7)$$

Using these definitions we can construct a set of dimensionless variables

$$\psi = \phi - \dot{\phi}_0 t \quad (8)$$

$$\Gamma = \frac{1}{\rho} \frac{\gamma}{\langle \gamma_0 \rangle} \quad (9)$$

$$A = \frac{ae^{i\phi_0 t}}{[4\pi mc^2 \langle \gamma_0 \rangle n_0 \rho]^{1/2}} \quad (10)$$

and write the FEL equations as¹²

$$\dot{\psi}_1 = \frac{1}{2\rho} \left(1 - \frac{1}{\rho^2 \Gamma_1^2} \right) \quad i = 1, 2, \dots, N \quad (11a)$$

$$\dot{\Gamma}_1 = -\frac{1}{\rho} \left(\frac{Ae^{i\psi_1}}{\Gamma_1} + \text{c.c.} \right) \quad (11b)$$

$$\dot{A} = i\delta A + \frac{1}{\rho} \left\langle \frac{e^{-i\psi_1}}{\Gamma_1} \right\rangle \quad (11c)$$

with

$$\delta = \frac{\Delta}{\rho} \quad (11d)$$

and Δ the detuning parameter

$$\Delta = \frac{\gamma_0^2 - \gamma_R^2}{2\gamma_R^2} \quad (11e)$$

The dot indicates differentiation with respect to τ . The angular brackets indicate an average over the particle initial phases, i.e. $\langle \rangle = \frac{1}{N} \sum_j$ where N is the number of particles.

From these equations we can show that the quantity:

$$H = \langle \Gamma \rangle + |A|^2 \quad (12)$$

is an invariant. In terms of laboratory variables this can be written as

$$\bar{H} = mc^2 n_0 \langle \gamma \rangle + \frac{E_0^2}{4\pi} = \text{constant} \quad (13)$$

which is seen to be the conservation of energy relation for the electron beam-radiation field system. It is also convenient, using (9), to rewrite (12) as

$$\left\langle \frac{\gamma - \langle \gamma_0 \rangle}{\langle \gamma_0 \rangle} \right\rangle = \rho (|A|^2 - |A_0|^2) \quad (14)$$

which relates directly the change in the field amplitude A to the average change in electron energy. One can see from (14) that, assuming $|A_0| \ll |A|$, the quantity $\rho |A|^2$ measures the efficiency of energy transfer from the electron beam to the radiation field.

In integrating the FEL equations the maximum time is defined by the undulator length $t_{\max} = N_w \lambda_0 / c$. In terms of the scaled time τ this becomes

$$\tau_{\max} = 4\pi\rho \left(\frac{\gamma_R}{\langle \gamma_0 \rangle} \right)^2 N_w. \quad (15)$$

6) The FEL collective instability and coherent radiation generation

The system of Eqs. (11a)-(11c) has been discussed in ref. 12 where it has been shown that for $\delta < \delta_{th} \approx 1.9$ the system is unstable and the field amplitude A grows exponentially. Both the radiation field and the beam

bunching grow exponentially. We can characterize the bunching by the parameter $b = |\langle e^{-i\psi} \rangle|$. The nonlinear regime and saturation that follow the initial exponential growth have also been studied in these papers.

In this paper we want to discuss a collectively unstable system using the parameters that apply to an electron beam obtained from a storage ring.

We assume that the initial field amplitude is zero and we introduce an initial noise in the electron phase distribution so that the initial value of $b = |\langle e^{-i\psi} \rangle|$ is $1/\sqrt{N_\lambda}$, N_λ being the number of electrons in one radiation wavelength. In figure 1 we show the evolution of the field amplitude $|A|$ versus τ , for different values of the initial electron beam r.m.s. energy spread, σ_{e0} , and for $\delta = 0$. One can see that for $\sigma_{e0} \ll \rho$ the field amplitude $|A|$ reaches a value of the order of unity, so that, from (14), we have an energy transfer efficiency of the order of ρ , i.e., at the peak of $|A|$ we have transferred a fraction ρ of the beam energy to the radiation field.

Figure 2 shows the evolution of the r.m.s. beam energy spread for the same values of σ_{e0} as in Figure 1. One can see that when the field peaks the energy spread becomes on the order of ρ provided $\sigma_{e0} \leq \rho$.

The time needed to reach the peak can be seen from Figures 1 and 2 to be $\tau \approx 10$. Assuming $\gamma_0 = \gamma_R$ we can see from (15) that to reach the peak we need an undulator with a number of periods $N_u \approx 1/\rho$.

Let us summarize the results of this section:

- 1) the electron beam, undulator magnet, radiation field system is unstable, if $\delta < \delta_{th}$, and both the field amplitude, $|A|$, and the beam bunching, b , will grow exponentially up to a saturation level where $|A| \sim 1$ and $b \sim 1$;

2) if the system initial conditions are $|A_0| = 0$, b_0 determined by noise, $\sigma_{e0} < \rho$, the electron beam will transfer a fraction ρ of its energy in a number of undulator periods of the order of $1/\rho$.

3) after traversing the undulator we have $|A| \approx 1$, $b \approx 1$ and $\sigma_e = \rho$.

7) The electron beam-undulator system

As we wish to discuss the operation of an FEL over a large wavelength range (30Å-2000Å) we will consider operating the storage ring at energies ranging from 300-500 MeV. In addition we will consider 3 undulator designs, a 5 mm period undulator for λ in the range 30-100 Å, a 1 cm period undulator for λ in the range 100-250 Å and one with $\lambda_0 = 2.5$ cm for λ in the range (500 - 2000Å).

To calculate the undulator properties we assume it to be of the hybrid (permanent magnet and iron) type and calculate the magnetic field from¹⁹

$$B_0 = 3.33 \exp \left(- 5.47 \frac{g}{\lambda_0} + 1.8 \frac{g^2}{\lambda_0^2} \right) \quad (\text{Tesla}) \quad (16)$$

where λ_0 and g are the period and the gap respectively. A complete listing of the undulator data can be found in Table 1. The output radiation wavelengths for the 3 undulators are signified by D's in Figures 5a,b,c.

The electron beam described in Table 2 can be obtained in a storage ring, as we will discuss in Section 5. However, if we tried to install the undulators described in Table 1 directly in the ring such that the electron beam would pass through the small aperture of the undulator on each revolution, it would become impossible to operate the ring. The small aperture (gap) of the undulator would result in vanishing small beam

lifetimes due to collisions with undulator walls. The minimum allowable gap depends on both the ring and undulator parameters and must be determined experimentally.

For this reason we propose to install the undulator in a ring bypass, as shown in Fig. 3. The electron beam would normally circulate in the ring, where the effect of synchrotron radiation damping would produce the beam properties of Table 2. About once per damping time, of the order of 50 ms for the storage ring illustrated in Table 3, the beam is taken into the bypass and focused in the undulator by a special quadrupole triplet. In going through the undulator the electron beam produces the radiation, its energy is decreased by ρE_T and its energy spread increases from its initial value to about ρ . The beam is then taken back into the storage ring and left there for a time long enough for synchrotron radiation damping to bring its characteristics back to their starting value. A more detailed discussion of the storage ring and bypass system is given in the next section.

As the electron beam circulates in the ring it performs both vertical and horizontal oscillations, the so-called betatron oscillations. The betatron functions, β_H and β_V , which characterize the amplitude and period of the betatron oscillations, are a measure of the focussing properties of the magnetic lattice. Together with the emittance, ϵ , which is the area in the position-angular deviation phase space in which the beam is contained, the beta functions determine the beam size, i.e., the r.m.s. beam height, $\sigma_H = \sqrt{\epsilon_H \beta_H}$. The choice of the beta functions in the bypass is determined by the requirement of keeping ρ as large as possible, which

requires small β_H, β_v , without violating the energy spread condition $\sigma_e < \rho$. In effect for a beam with nonzero transverse emittances ϵ_H, ϵ_v , it is necessary to add to the real energy spread an effective spread given by²⁰

$$\sigma_{e,eff} = \frac{1}{2} \left(\frac{2\pi}{\lambda_0} \right)^2 \frac{K^2}{1+K^2} \epsilon_{v,v} + \frac{\gamma^2}{1+K^2} \left(\frac{\epsilon_H}{\beta_H} + \frac{\epsilon_v}{\beta_v} \right) \quad (17)$$

In what follows we will make sure that the condition $\sigma_{e,eff} < \sigma_e$ is always satisfied.

In the cases that we will consider in the next section the undulator length varies between two and three meters and is determined by the condition $N_w = 1/\rho$. This length is also consistent with our assumption on the beta-functions.

8) The electron storage ring and bypass section

The storage ring that we consider is similar to those used as synchrotron radiation sources, for instance the National Synchrotron Light Source VUV ring.¹⁵ Its main characteristics are given in Table 3.

Since we want to maximize the electron density to obtain a large value of ρ in the undulator, we have chosen a ring design which minimizes the beam emittance and the bunch length. When the beam enters the bypass section it undergoes additional focussing to increase ρ , as shown in Table 2.

The ring has two 10 m long straight sections, one used for the radiofrequency system and one for the bypass switching magnets. The arcs joining the two long straights each have three equal periods. Each period has 2 dipole magnets with a focussing quadrupole between them and two quadrupole

doublets on the external sides. The ring energy dispersion is controlled by the central quadrupole and is non zero only in the dipoles and in the region between them.

The momentum compaction, $\alpha = (dE/E)/(dl/l)$, relates the change in orbit length to the relative energy deviation from the design energy E_0 of the ring. For a ring with this magnetic structure the momentum compaction α , and the horizontal emittance, ϵ_H , are approximately given by²¹

$$\alpha = \frac{1}{6} \left(\frac{\pi}{M} \right)^2 \frac{R_B}{R_{av}} \quad (18)$$

$$\epsilon_H = 7.7 \times 10^{-13} \frac{\gamma^2}{M^3} \quad (\text{m-rad}) \quad (19)$$

where R_B , R_{AV} are the bending and average ring radii and M is the number of achromatic bends. An achromatic bend, typically consists of two dipole magnets with a horizontally focussing quadrupole in between, and is designed to focus all the entering electrons, regardless of energy, to the same point on exiting the bend. The vertical emittance is determined by the coupling between horizontal and vertical oscillation due to magnet misalignment, $\epsilon_V = \chi \epsilon_H$.

At zero or small current the r.m.s. energy spread and the bunch length are determined by synchrotron radiation and are given by²²

$$\sigma_{\epsilon_0} = 4.38 \times 10^{-7} \frac{\gamma}{R_B^{1/2}} \quad (20)$$

$$\sigma_{p_0} = \frac{\alpha R_{AV}}{v_s} \sigma_{\epsilon_0} \quad (21)$$

where $\nu_s = \omega_s/\omega_0$ is the ring synchrotron oscillation tune. At large currents the microwave instability,²³ caused by the beam interaction with the broad-band high frequency storage ring impedance can increase the energy spread, σ_E , and the bunch length, σ_p . An increase of σ_p reduces the value of ρ while at the same time σ_E increases and the condition $\sigma_E < \rho$ can be violated.

To evaluate this effect we use the approximate condition²³

$$eI_p \left| \frac{Z(n)}{n} \right| \leq 2\pi E_0 \alpha \sigma_E^2 \quad \text{for } n \geq \frac{R_{AV}}{\sigma_p} \quad (22)$$

where I_p is the peak current, related to the average bunch current, I_0 , by

$$I_p = (2\pi)^{1/2} \frac{R_{AV}}{\sigma_p} I_0 \quad (23)$$

and $\left| \frac{Z(n)}{n} \right|$ is the effective longitudinal coupling impedance of the ring.

From eqs. 21-23 an expression for the microwave instability limited bunch length and energy spread can be obtained

$$\sigma_p = R_{AV} \left(\frac{\alpha I_0 e}{\sqrt{2\pi} E \nu_s^2} \left| \frac{Z_n}{n} \right| \right)^{1/3} \quad (24)$$

$$\sigma_E = \frac{\nu_s}{\alpha R_{AV}} \sigma_p \quad (25)$$

The storage ring coupling impedance is determined by the vacuum chamber geometry and by the bending radius in the curved section^{23,24} and is a quantity difficult to calculate "a priori..." However in modern storage rings values of the order of 1 Ω have been obtained. Since this quantity is very important in determining the performance of our system we have chosen to use

in our calculations three values of $\left|\frac{Z(n)}{n}\right|$, i.e. 0.1, 1 and 10 Ω . Let us notice that a 10 Ω coupling impedance is large, and is a pessimistic assumption, while a 1 Ω value is realistic and has been already obtained. On the other hand, a 0.1 Ω value would require a breakthrough in storage ring design.

The microwave instability limited bunch lengths and peak currents, which depend on the value of the coupling impedance but not the energy, are given in Table 4. The bunch lengths are typically a few centimeters and the peak currents are in the 100-400 amp range.

To test the beam for stability against transverse coherent oscillations we have used the conditions that the coherent betatron tune shift be smaller than the synchrotron tune²³

$$\delta\nu_{\beta} = \frac{eI_p}{\pi \nu E} Z_{T,eff} < \nu_s \quad (26)$$

with the transverse coupling impedance $Z_{T,eff}$ evaluated from the longitudinal impedance as²³

$$Z_{T,eff} = 2\left(\frac{R}{b}\right)^2 \left|\frac{Z(n)}{n}\right| \quad (27)$$

The ring described in Table 3 will be free from transverse instability problems provided the effective impedance can be kept on the order 1 Ω or less.

As a final measure of the ring's feasibility we compute the Touschek lifetime,²⁵ τ_t . The Touschek lifetime is the time in which losses due to coulomb collisions between electrons in the same bunch have reduced the beam current to half of its initial value. For the range of ring parameters given in Table 3 $\tau_t > 1$ hour.

9) Results

In Figures 4a, 4b and 4c we plot the FEL parameter, ρ , and the microwave instability limited energy spread, σ_E , versus the beam energy for the 5 mm, 1 cm and 2.5 cm undulators respectively. Recalling that the limit on the allowable energy spread is $\sigma_E \leq \rho$ it can be seen from Fig. 4a that if the ring impedance can be kept at or below $.1\Omega$ one can expect to obtain high intensity coherent soft-X-Rays in the range 30-85Å. From Figs. 4b and 4c it can be seen that the energy spread in the ring will not pose any problems for the generation of intense radiation in the range 85-2000 Å.

Figures 5a,b and c are plots of the peak power versus energy for the three undulator designs. The peak power is calculated assuming that the radiation pulse length is equal to the electron bunch length.

10. Conclusions

Using the system described, an electron storage ring with an undulator in a special bypass section, we can obtain high intensity coherent radiation by sending the beam through the undulator and using the FEL collective instability to produce radiation. Compared to other systems, such as an FEL oscillator or a transverse optical klystron, this system has the advantage that it does not require mirrors to form an optical cavity or an input high power laser to bunch the electron beam. On the other hand, by its very nature, this system can only produce high intensity, short radiation pulses with a repetition rate of the order of 10 Hz.

The storage ring needed to operate the system is characterized by a small transverse emittance. The other important ring parameter is the longitudinal coupling impedance. For a value of the order of 1Ω one can obtain peak

powers on the order of 500 MW down to wavelength of about 500 Å and 50 MW to 80 Å; the power decreases sharply at lower wavelength. If it should become possible to reduce $\left| \frac{Z(n)}{n} \right|$ to 0.1 Ω one could get peak powers on the order of 20 MW down to 30 Å.

One should also remember that in this paper we have concentrated our attention on the first harmonic production only; however, from the results of ref. 18, we know that the system will also produce higher harmonics and this can shift down the lower limit for soft X-ray production.

We want to emphasize that the results presented here are preliminary, and that one might improve the system performance by optimizing other ring parameters such as the momentum compaction or the radiofrequency voltage and frequency. To obtain a more complete understanding of the system one should investigate diffraction effects on the radiation due to the finite beam radius and consider a three dimensional calculation taking into account the electron density variation in both the transverse and longitudinal direction.

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FIGURE CAPTIONS

1. Plot of $|A|$ versus τ for $b = 9.1 \times 10^{-3}$, $\rho = 3 \times 10^{-3}$ and several values of the initial rms energy spread, $\sigma_{E_0} = .1\rho$, $.75\rho$, and ρ , labeled a, b & c respectively.
2. Plot of σ_E versus τ for $b = 9.1 \times 10^{-3}$, $\rho = 3 \times 10^{-3}$ and several values of the initial rms energy spread, $\sigma_{E_0} = .1\rho$, $.75\rho$, and ρ , labeled a, b & c respectively.
3. Sketch of storage ring and bypass section.
4. Plots of the FEL parameter, ρ , and the microwave instability limited rms energy spread, σ_E , versus energy for a) $\lambda_0 = 5$ mm, b) $\lambda_0 = 1$ cm and c) $\lambda_0 = 2.5$ cm. The ρ values are signified by a $\square$ and the σ_E values are given by a $\circ$. Each figure displays the ρ & σ_E values for 3 values of $Z_{eff} = 0.1, 1.0$ and 10 ohms. A solid line (—) corresponds to $Z = .1 \Omega$, a dashed line (---) to $Z = 1 \Omega$ and (-.-.) to $Z = 10 \Omega$. The lines are not fitted to the points, they are drawn simply to indicate trends.
5. Plot of the output wavelength λ and the estimated peak power output versus energy for a) $\lambda_0 = 5$ mm, b) $\lambda_0 = 1$ cm and c) $\lambda_0 = 2.5$ cm. The λ values are signified by a $\square$ and the power values are given by a $\circ$. For the power curves, a solid line (—) corresponds to $Z = .1\Omega$, a dashed line (---) to $Z = 1\Omega$ and (-.-.) to $Z = 10\Omega$.

Table 1: Undulator Magnets

Period, λ_o (cm)	0.5	1.0	2.5
Gap, g (cm)	0.1	0.2	0.5
Pump strength, B_o (Tesla)	1.2	1.2	1.2
Undulator parameter, K	0.56	1.12	2.80

Table 2: Electron beam parameters in the bypass section

Energy E(MeV)	500.00	400.00	300.00
Beta horizontal, β_x (m)	3.00	3.00	3.00
Beta vertical, β_y (m)	1.00	1.00	1.00
Coupling,X	0.01	0.01	0.01
rms horizontal beam radius, σ_x (m)	1.01E-04	8.10E-05	6.07E-05
rms horizontal angular spread, σ'_x (rad)	3.37E-05	2.70E-05	2.02E-05
rms vertical angular spread, σ'_y (rad)	5.81E-06	4.65E-06	3.49E-06
rms vertical beam radius, σ_y (m)	5.81E-06	4.65E-06	3.49E-06

Table 3: Storage Ring Parameters

Energy, E , (MeV)	500	400	300
Gamma, γ	978	782	587
Bending radius, R_b (m)	4	4	4
Average radius, R_{av} (m)	15	15	15
Number of achromatic bends, M	6	6	6
RF voltage, V (MV)	1	1	1
Harmonic number, h	100	100	100
Number of bunches	1	1	1
Average current, I_0 (amp)	0.10	0.10	0.10
Electron number, N	1.97E+11	1.97E+11	1.97E+11
Synchronous phase, ϕ_s	1.38E-03	5.67E-04	1.79E-04
Momentum compaction, α	1.22E-02	1.22E-02	1.22E-02
Horizontal emittance, ϵ_x (m-rad)	3.41E-09	2.18E-09	1.23E-09
Vertical emittance, ϵ_y (m-rad)	3.38E-11	2.16E-11	1.22E-11
Zero current bunch length, σ_{p0} (m)	1.99E-03	1.42E-03	9.24E-04
Zero current energy spread, σ_{E0}	2.14E-04	1.71E-04	1.29E-04
Synchrotron tune, ν_s	1.97E-02	2.20E-02	2.54E-02
Synchrotron radiation loss, U_0 (MeV)	1.38E-03	5.67E-04	1.79E-04
RF acceptance, ϵ_{RF}	3.23E-02	3.61E-02	4.17E-02

Table 4: Bunch Length & Peak Current

Effective coupling impedance (ohm)	0.1	1.0	10
Bunch length, σ_p [u-wave limit] (cm)	0.95	2.0	4.4
Peak current, I_p (amp)	397	184	86

Fig. 1.

Fig. 2.

Fig. 3.

Fig. 4.

Fig. 5.