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STOCHASTICITY AND SUPERADIABATICITY
IN RADIOFREQUENCY PLASMA HEATING

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ON STABILITY AND SPATIAL LOCALIZATION IN PLASMA BEAMS
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ABSTRACT

An ensemble of non-interacting particles, in the presence of a resonant particle - component just a member of the total velocity distribution - which are mutually affected. Under near-fusion conditions, the interaction by Coulomb collisions is slow, and non-linear stochasticity can play an important role in determining $f(v)$. It is found that the common set of interactions, including Landau, cyclotron and transit-time damping, can be fitted in a unified manner for a single two-step one-parameter (ϵ) mapping which can display collision-free stochastic or adiabatic (also called superadiabatic) behavior, depending on the choice of ϵ . The effect on the evolution of the space-averaged $f(x,v,t)$ is reasonably well described by a "pseudo-stochastic diffusion function", $D_{ps}(v,\epsilon)$ which is the quasilinear diffusion coefficient but with appropriate widening of the delta-function spikes.

Coulomb collisions, leading to $D_{\text{Coul}}(v)$ which may be added and directly compared to $D_{\text{ps}}(v, \epsilon)$, are introduced by Langevin terms in the mapping equations.

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INTRODUCTION

In the range of densities and temperature appropriate for controlled fusion, Coulomb mean free paths are extremely long compared to device sizes and anisotropies in the particle velocity distribution are relaxed by collisions only very slowly. It is then reasonable to ask on what time scale can an energy deposition scheme which only perturbs the particle velocity distribution over a relatively small portion of its range be accurately viewed as a plasma "heating" scheme. Absorption of radiofrequency energy by a plasma is just such a deposition scheme -- the arguments of quasilinear theory show that the tendency of rf interaction is to produce local plateaus in the space-averaged velocity distribution. The aim of this paper is to examine a number of such rf heating processes and to display their common behavior -- the occurrence of trapped adiabatic orbits in rf waves (superadiabaticity) which can be destroyed by collisionfree stochasticity (island overlap) as effectively as by Coulomb collisions. It is shown that Landau, cyclotron and transit-time damping can be modelled by a simple two-step one-parameter (ϵ) mapping which can display superadiabatic or stochastic behavior, depending on the choice of ϵ . Coulomb collisions may be introduced into the mapping by the addition of Langevin terms.

I. SYSTEMS WITH SYMMETRY

The simplest application of radiofrequency fields to a plasma occurs when there is underlying symmetry to the combined configuration. An example would be the case of a running wave in a homogeneous plasma. In the reference frame moving with the wave phase velocity, the electric and magnetic field are static and the Hamiltonian for the motion of a single charged particle is time-independent. Familiar illustrations of this situation are the nonlinear electrostatic waves described by Bernstein, Greene and Kruskal [1]. Individual particles in the wave are either trapped or untrapped; the wave fields are, of course, self-consistent with the motions of the plasma particles, but the plasma is considered collisionless and the energy of each particle (kinetic plus electrostatic) is rigorously conserved. Electromagnetic modes - say Alfvén waves or electron cyclotron waves - may, of course, also display such nonlinear undamped forms which are stationary in the wave-frame.

Another familiar feature of these waves is the picture, in phase space, of the particle trajectories, sketched in Fig. 1. The untrapped particles lie on open surfaces, while the trapped particles lie on closed surfaces. A separatrix between the two types of surfaces defines a chain of islands in phase space. Now it is

often possible to destroy the exact symmetry of such a configuration without modifying the topology of orbit surfaces in phase space. Thus the superposition of "slow" variations may lead only to slow adiabatic changes in the particle motions. Individual particle energies will no longer be conserved, but the area of orbits in phase space (the "action") can be, under these circumstances, an adiabatic invariant. The question as to whether a given perturbation leaves invariant surfaces for the particle trajectories or, on the other hand, causes stochastic destruction of these surfaces is one which we discuss in the next section. But in the adiabatic regime - when particles remain on their invariant surfaces - stochasticity is introduced into the situation only by actual collisions.

Up to now, we have discussed only a steady-state wave. It is also relevant to plasma heating to consider the evolution of the distribution function for particles in a plasma as a heating wave is turned on. Prior to "turn-on", the contours in a phase space picture corresponding to Fig. 1 would be a family of horizontal lines, and each contour would be loaded with particles according to the equilibrium $f(v)$. Turning on the wave distorts these contours and it is useful to estimate the rate at which the new velocity distribution, self-consistent with the wave fields, is established. Following through the

formalism of the quasilinear calculation but for a monochromatic wave $E \cos(kx - \omega t)$ leads to a velocity-space "diffusion" coefficient

$$D_{QL}(v) = \frac{1}{2} \frac{\langle (\Delta v)^2 \rangle}{\Delta t} = \frac{\pi}{2|k|} \left(\frac{eE}{m} \right)^2 \delta(v - \omega/k) \quad (1)$$

In ordinary quasilinear theory, which pertains to the evolution of unstable modes, the Dirac delta function in (1) is replaced by a resonance function, with finite width due to a finite growth rate for the wave. But for application to rf heating of a finite-sized plasma, ω is real and the k -spectrum is discrete. On the other hand, the size of the island of trapped particles is finite, and an approximate description of the perturbation of $f(v)$ toward island formation can be given by a coefficient for "pseudo-stochastic diffusion", $D_{PS}(v)$, of rectangular shape,

$$\begin{aligned} D_{PS}(v) &= \pi |k| \left(\frac{w}{8} \right)^3 \quad \text{for} \quad \frac{\omega}{k} - \frac{w}{2} \leq v \leq \frac{\omega}{k} + \frac{w}{2} \\ &= 0 \text{ elsewhere} \end{aligned} \quad (2)$$

spread out [2] over the full island width, w ,

$$w = 4(eE/mk)^{1/2} \quad (3)$$

Alternatively, a crude estimate of $D(v)$ can be made noting that a typical velocity deviation might be $w/4$, i.e., half the distance from the island center to the separatrix, while a typical time associated with this movement might be $1/2\pi$ times the bounce period, $\omega_b^{-1} = (m/ekE)^{1/2}$, so that $D(v) = (1/2) \langle (\Delta v)^2 \rangle / \Delta t \approx w^2 \omega_b / 32 = |k/2| (w/4)^3$, in rough agreement with $D_{PS}(v)$ above.

Quasilinear-type diffusion looks only at the $k = 0$ (space-averaged) component of $f(x, v, t)$ and is oblivious to detailed phase information such as the existence of invariant surfaces for particle trajectories in phase space. On the other hand, the attainment of a steady-state traveling wave will be dutifully observed in this formalism by the cessation of all further diffusion as $\partial f / \partial v$ reaches zero in the range $\omega/k - w/2 \leq v \leq \omega/k + w/2$. For the purpose of plasma heating calculations, one may study the evolution of the space-averaged $f(v, t)$ by the one-dimensional model Fokker-Planck equation,

$$\frac{\partial f}{\partial t} = \frac{\partial}{\partial v} \left[v \left(v f + \frac{kT}{m} \frac{\partial f}{\partial v} \right) \right] + \frac{\partial}{\partial v} D_{PS} \frac{\partial f}{\partial v} \quad (4)$$

in which, for a rough estimate, one may take for $v(v)$ the 90° -collision frequency. The pseudo-stochastic diffusion term will attempt to flatten $f(v, t)$ within the range $\pm w/2$ around ω/k , while the Coulomb term will attempt to restore a Maxwellian shape to $f(v)$. Equation (4)

admits of an immediate steady-state solution,

$$f = \text{constant} \cdot \exp \left[- \int \left(\frac{\kappa T}{m} + \frac{D_{PS}}{v} \right)^{-1} v dv \right] \quad (5)$$

in which both D_{PS} and v may be functions of velocity, v .
[An independent second steady-state solution to (4) exists, but exhibits a constant velocity-space flow and pertains to a situation with sources and sinks.] By (2), D_{PS} in (5) is different from zero only in the range $\omega/k - w/2 \leq v \leq \omega/k + w/2$. When $w \ll v_{\text{thermal}}$, it is a good approximation to take $v = \text{constant}$ in (5), in which case $f(v)$ is a continuous function piecewise of Maxwellian form, with effective temperature κT where $D_{PS} = 0$ and effective temperature

$$\kappa T_H = \kappa T \left[1 + \left(\frac{w}{w_c} \right)^3 \right] \quad , \quad (6)$$

$$w_c \equiv 8 \left| \frac{1}{\pi} \frac{v}{k} \frac{\kappa T}{m} \right|^{1/3}$$

in the range $\omega/k - w/2 \leq v \leq \omega/k + w/2$.

Pertinent to plasma heating, Eqs. (4) and (5) also yield a rapid estimate of the steady-state power input, P , from the wave field to the plasma. With a simple integration by parts, then using (5),

$$P = \frac{1}{2} \int m v^2 \frac{\partial}{\partial v} D_{PS} \frac{\partial f}{\partial v} dv = \int \left(1 + \frac{v k T}{m D_{PS}}\right)^{-1} v m v^2 f(v) dv. \quad (7)$$

When w is small compared to v_{thermal} , an approximate evaluation of Eq. (7), based on (2), is

$$P \approx v m \left(\frac{\omega}{k}\right)^2 f\left(\frac{\omega}{k}\right) \frac{w^4}{w^3 + w_c^3} \quad (8)$$

where v is the collision frequency appropriate to $v_n = \omega/k$. The limit $w \ll w_c$ leads to the familiar linear-theory collisionfree result, while results with $w \gtrsim w_c$ include the effect of plateau formation and a reduced rf absorption rate.

We recall from its definition, Eq. (3), that $w \sim E^{\frac{1}{2}} \sim P_D^{\frac{1}{4}}$, where P_D is the rf driving power. Eq. (8) thus shows that the absorbed power, P , varies as P_D for small P_D , but only as $P_D^{\frac{1}{2}}$ for large P_D due to plateau formation. Splitting the available source power among several rf generators operating at different frequencies can help recover the more favorable scaling.

Finally, Eq. (5) may be used to evaluate the steady-state macroscopic velocity induced by wave-plasma interaction. Again invoking $v=\text{constant}$ where $D_{PS} \neq 0$, one may evaluate $n\langle v \rangle = \int f v dv$ to find

$$n\langle v \rangle = 2\pi \left(\frac{w}{8}\right)^3 \frac{|k|}{v} \sinh\left(-\frac{m\omega w}{2k\kappa_{TH}}\right) \exp\left(-\frac{m\omega^2}{8\kappa_{TH}}\right) f\left(\frac{\omega}{k}\right) \quad (9)$$

For w small compared to v_{thermal} , the exponential may be replaced by unity and the $\sinh$ by its argument, leaving

$$n\langle v \rangle \approx \frac{\omega}{k} f\left(\frac{\omega}{k}\right) \frac{w^4}{w^3 + w_c^3} \quad (10)$$

Eqs. (8) and (10) may then be combined to reveal a simple relation between rf power and the induced macroscopic particle flow,

$$P = vnm\langle v \rangle \frac{\omega}{k} \quad (11)$$

which provides a rough basis for understanding the efficiency for rf-driven plasma currents. [3,4]

II. ASYMMETRIC SYSTEMS

The physical problem treated in the previous section was the response of a homogeneous plasma to a single plane wave. The symmetry of the problem can be destroyed in a number of ways; one such way is to superimpose a second plane wave which moves with phase velocity different from the first. We may consider, for example, the motion of an electron according to

$$m_e \frac{dv}{dt} = -eE \sin(kx) - eE_1 \sin(k_1 x - \omega_1 t) \quad (12)$$

For small values of E_1 , the trajectories, examined at time intervals $\Delta t = 2\pi/\omega_1$, still fall on invariant surfaces in plasma space. For larger values of E_1 — i.e., for values of E_1 such that the islands in phase space would overlap — the trajectories wander about in a stochastic manner filling up much of the phase space volume formerly defined by the two sets of islands. This problem has been examined by a number of authors [5-7] and the following main points emerge from the analysis:

- a. Resonance between the perturbation field (E_1) and particle motions along a primary orbit leads to the formation of "secondary islands" chained together along what had been a primary orbit.

b. A denumerable infinity of such secondary islands appears on each side of a primary separatrix, and overlap [8,9] of these secondary islands produces a stochastic layer of finite thickness in the separatrix region.

c. Considering E_1 as a perturbation, a fraction of phase space occupied by the primary island which goes stochastic varies roughly as $E_1 \exp(-2\pi\omega_1/kw)$, where $w = 4(eE/mk)^{1/2}$ as before. The dependence on the degree of overlap, i.e., on the ratio of island width (w) to inter-island spacing (ω_1/k_1), is therefore very strong and modest changes in this ratio can effect dramatic differences in the amount of stochasticity.

With stochastic wandering taking place between two (or more) chains of overlapping islands, the pseudo-stochastic diffusion coefficient can be applied to the entire band of velocities spanned by the overlapping islands. The formal quasilinear result, due to the superposed fields $E = \sum_j E_j \cos(k_j x - \omega_j t)$, would be

$$D_{QL} = \frac{\pi}{2} \sum_j \frac{1}{|k_j|} \left(\frac{eE_j}{m} \right)^2 \delta \left(v - \frac{\omega_j}{k_j} \right) \quad (13)$$

Replacing the delta functions by rectangular spikes of width w_j leads, as in Eq. (2), to the pseudo-stochastic coefficient for diffusion,

$$D_{PS}(v) = \pi |k| \left(\frac{\omega_j}{8} \right)^3 \left(\frac{\omega_j}{u_j} \right) \quad \text{for} \quad \frac{\omega_j}{k_j} - \frac{u_j}{2} \leq v \leq \frac{\omega_j}{k_j} + \frac{u_j}{2} \quad (14)$$

$$= 0 \text{ elsewhere}$$

where

$$\omega_j = 4(eE_j/mk_j)^{1/2},$$

$u_j = w_j$ in the non-overlapping regions,

$$u_j = \left| \frac{\omega_j}{k_j} - \frac{\omega_{j+1}}{k_{j+1}} \right| \quad \text{in the regions of island overlap.}$$

Fig. 2 sketches the appearance of D_{PS} in a representative case. In the non-overlap regions, the intra-island regions of rapid diffusion are isolated, one from another, by Coulomb transport. A smooth transition is made to the overlap condition, where stochastic diffusion may swamp Coulomb transport. For even spacing of the successive phase velocities (ω_j/k_j) the envelope of the rectangular spikes in D_{PS} is determined by the local value of the power spectrum for E , namely E_j , in accordance with Eq. (14).

With respect to Fig. 2, it should be kept in mind that the representation by diffusion according to rectangular blocks constitutes only an approximation to the actual transport situation. The pseudo-stochastic representation makes no attempt to distinguish between the oscillation in phase space associated with motion along

invariant surfaces, and true stochastic diffusion such as occurs under overlap conditions. Moreover, computer calculations (as in Figs. 3-5) show that stochastic diffusion between adjacent island chains builds up rapidly but not infinitely rapidly as the overlap condition is approached. Nevertheless, this representation can be useful in understanding the evolution of the space-averaged $f(v)$ and in the calculation of plasma heating.

III. COMPUTER MODELLING: SUPERADIABATICITY AND STOCHASTICITY

The problem of stochastic heating lends itself well to computer calculation and a number of results appear in the literature [10-12]. One very simple model [13-15] is based on the generalization of Eq. (12) to the superposition of many plane waves, evenly spaced in phase velocity. In dimensionless variables, we write

$$\frac{dx}{dt} = v \quad (15)$$

$$\frac{dv}{dt} = \epsilon^2 \sum_{-N}^N \cos(x - nt)$$

In the circumstances that $k = k_1$ and $E = E_1$ in Eq. (12), this equation is equivalent to set (15) with $N = \frac{1}{2}$ and $\epsilon = (ekE/m\omega_1^2)^{1/2} = |kw/4\omega_1|$. For small values of ϵ , islands of full width 4ϵ appear, centered around the lines $v = n$, n integral. It is important to note that the rapid falloff mentioned in paragraph IIc above implies that the dominant effect on the motion of particles trapped near any $n = n_0$ will come from the nearest-neighbor fields, $n = n_0 \pm 1$, and the effect of the more asynchronous fields will be exponentially small by comparison.

For $2N$ integral, the summation in (15) can be easily performed, leading to

$$\frac{dx}{dt} = v \quad (16)$$

$$\frac{dv}{dt} = \epsilon^2 \cos x \frac{\sin[(2N+1)t/2]}{\sin(t/2)} + 2\pi\epsilon^2 \cos x \sum_{n=-\infty}^{\infty} \delta(t-2\pi n)$$

The last expression on the right corresponds to the limit $N \rightarrow \infty$. In this form, integrating until just after the $(m+1)^{\text{th}}$ jump and changing variables according to $x = (4u-1)\pi/2$, set (16) changes into the simple two-step mapping,

$$u_{m+1} = u_m + v_m \quad (17)$$

$$v_{m+1} = v_m - 2\pi\epsilon^2 \sin(2\pi u_{m+1})$$

The mapping (17) is area-preserving, symmetric with respect to changes in u by integral amounts, and also symmetric with respect to $u \rightarrow -u$, $v \rightarrow -v$. By interleaving two such mappings, the method may be applied with high accuracy to the two-wave case, Eq. (15), and Figs. 3-5 show some typical results from the computer, for various values of the overlap parameter ϵ .

For small values of ϵ , the computer mappings indeed show adiabatic behavior, with the appearance of islands of full width 4ϵ centered on the lines $v = 0$ and $v = 1$. This small- ϵ behavior proceeds exactly as anticipated from Eq. (15), with each component of the electric field spectrum producing its own small phase-space island moving at the phase velocity of that component. As noted in Section IIc, the effect of even the nearest-neighbor fields is exponentially small.

Viewing the same effect from the equivalent mapping, Eqs. (17), however, this adiabatic behavior is somewhat remarkable. There is no long-term coherence that is obvious in the mapping equations. Therefore one might expect that the argument of the sine function will, after a number of iterations, become completely decorrelated from its early values and, as a result, that the trajectories will diffuse throughout velocity space over the full range of v values. That the trajectories are, for small ϵ values, confined to phase-space islands is the same effect which, in another context (see Section IVh below), Rosenbluth has termed "super-adiabaticity".

For larger values of ϵ , the islands grow and approach the overlap condition. As noted in Sections IIa,b, and c above, secondary islands also start to appear and undergo overlap. The superadiabatic behavior breaks down and

stochastic wandering between evenly-spaced islands [15] begins around $\epsilon = 0.18$, and is already quite strong at $\epsilon = 0.25$, which is the calculated point where the two sets of islands should just begin to touch ($w = w_1/k_1$, or $4\epsilon = 1$).

Clearly seen in the figures is the sharp cessation of stochasticity outside the outer boundaries of the original two islands. This point may be understood from a careful theoretical analysis [14].

A final point in the discussion of the mapping given in (17) is the predicted rate of diffusion. For ϵ large enough so that stochasticity is reasonably well established, the phase $2\pi u_{m+1}$ in the argument of the sine function may be considered a random variable. The mean change in v^2 at each iteration ($\Delta t = 2\pi$) is therefore $(1/2) (2\pi\epsilon^2)^2$ and the corresponding diffusion coefficient in the stochastic regions is

$$D_{\text{stoch}}(v, \epsilon) = \frac{1}{2} \frac{\langle (\Delta v)^2 \rangle}{\Delta t} = \frac{\pi \epsilon^4}{2} \quad \text{in overlap regions.} \quad (18)$$

Then, by analogy with (14), we can write down a pseudo-stochastic form for the regions of non-overlap,

$$D_{\text{PS}}(v, \epsilon) = \pi \epsilon^3 / 8, \quad n_0 - 2\epsilon \leq v \leq n_0 + 2\epsilon$$

(19)

$$= 0 \quad \text{otherwise.}$$

Restoring dimensions,

$$D_{PS}(v, \epsilon) = \frac{\pi^2 \epsilon^4 u_0^2}{|\Delta t|} \left(\frac{u_0}{u} \right) \quad \text{where } v_p - \frac{u}{2} \leq v \leq v_p + \frac{u}{2} \quad (20)$$

$$= 0 \text{ elsewhere,}$$

where v_p is the particle velocity for the p^{th} resonance,
 Δt is the time interval between successive iterations,
and u_0 and u are defined

$$u_0 \equiv |v_p - v_{p+1}|$$

$u = u_0$ in the regions of island overlap

$= 4\epsilon u_0$ in the non-overlapping regions.

If, in Eq. (14), $k_j = k$ and $E_j = E$ for all j , $\omega_j = j\omega$,
 j integral, then $\epsilon = (ekE/m\omega^2)^{1/2} = kw/4\omega, \Delta t = 2\pi/\omega$,
 $v_j = j\omega/k$, $u_{ci} = \omega/k$, and Eq. (20) may be seen to be
identical to Eq. (14).

IV. SOME APPLICATIONS

Although it was derived from the specific physical problem of the linear acceleration of a charged particle by a superposed group of plane waves, the mapping presented in (17) can be applied to a host of situations spanning most of the possibilities for radiofrequency plasma heating.

a. Lower Hybrid Heating in a Torus

For lower-hybrid electron-Landau-damping heating of a tokamak plasma, we consider waves launched from an array which is small in its dimensions when compared to the major radius of the torus [16]. The spatial boundaries of the wave packet follow along resonance cones as they propagate radially into the plasma. Thus the extent of the wave packet, d , in the toroidal direction is substantially independent of radius. Consider now that an electron with velocity v_n hits the beginning of the wave packet at time t_n . The equations of motion are

$$z = \int_{t_n}^t v \, dt \quad (21)$$

$$m_e \frac{dv_n}{dt} = -eE \cos(kz - \omega t) \quad .$$

Integration from $z = 0$ to $z = d$ with the free-streaming trajectory $v_n = \text{constant}$ leads to the velocity at exit, v_{n+1} . The electron now travels one or more times around the torus (depending on the rotational transform, etc.) and re-enters the wave-packet at the time $t_{n+1} = t_n + L/v_{n+1}$, having travelled a total distance L . (To simplify the problem, we overlook the fact that the distance d is traversed at a velocity not exactly equal to v_{n+1}). The integrated equation of motion may now be presented as a mapping,

$$v_{n+1} = v_n - \frac{eE}{m_e} \cdot \frac{d}{v_n} \cdot \frac{\sin \psi_n}{\psi_n} \cos (\tau_n - \psi_n) \quad (22)$$

$$\tau_{n+1} = \tau_n + \frac{\omega L}{v_{n+1}}$$

$$\text{where } \tau_n \equiv \omega t_n \text{ and } \psi_n \equiv \frac{(kv_n - \omega)d}{2v_n}$$

Resonances occur in the vicinity of velocities which are integral sub-multiples of $\omega L/2\pi$, and the mapping can be linearized around any one such velocity, v . The changes in ψ_n due to changes in v_n are smaller by the factor d/L than the changes in τ_n and can be neglected. The linearized form of (22) is then equivalent to set (17) with

$$\epsilon^2 = \left| \frac{1}{2\pi} \frac{(\Delta v)_{\max}}{u_0} \right| = \frac{m_s}{2\pi} \frac{(\Delta v)_{\max}}{v} \quad (23)$$

where

$$\begin{aligned}
 (\Delta v)_{\max} &= \frac{eE}{m_e} \frac{d}{v} \frac{\sin \psi}{\psi} \\
 u_0 &= 2\pi v^2 / \omega L \\
 m_s &= \omega L / 2\pi v = v / u_0 \\
 \Delta t &= L / v
 \end{aligned}
 \tag{24}$$

$(\Delta v)_{\max}$ is the maximum velocity change which can occur on one pass through the wave packet, and m_s is the sub-harmonic number of the particle resonance. u_0 is (approximately) the spacing between successive resonant values of the velocity, v , the island full width is $w = 4\epsilon u_0$, and the nominal overlap condition is $4\epsilon = 1$. Substitution of (23) and (24) into (20) provides the pseudo-stochastic diffusion function for this case.

For application to plasma heating calculations, it is useful to write the heating rate in terms of the time, Δt , between successive momentum impulses

$$N \equiv \frac{\text{Characteristic Local Heating Time}}{\text{Time Between Iterations}} = \frac{v^2}{\langle (\Delta v)^2 \rangle}
 \tag{25}$$

where $\langle (\Delta v)^2 \rangle$ is the mean square Δv per transit of the heating wave-packet. Then, using (23) and (25), the stochasticity parameter ϵ can be written as a ratio of two dimensionless numbers

$$\epsilon^2 = \frac{m_s}{\pi} \left(\frac{1}{2N} \right)^{1/2}
 \tag{26}$$

A typical value of the overall heating time in current experiments might be 100 milliseconds. Assuming that the lower-hybrid heating process is active over only 10% of the cross-section, the necessary local heating time would be 10 milliseconds. The circumnavigation time for an electron with $v_{\perp} = (kT_e/m_e)^{1/2}$ and $kT_e = 3000$ eV is about 0.4 microseconds ($R = 140$ cm), so $N = 25,000$ (taking the effect of rotational transform into account, N would be $\sim 25,000/q$), at 800 MHz and for electrons of this same value of $v_{\perp}$, $m_s = 300$. Thus $\epsilon \approx 0.65$ which is well above the stochasticity threshold ($\epsilon \sim 0.18$). The separate harmonic resonances are connected by overlap and the envelope of the resultant diffusion coefficient will be governed by the single-pass resonance shape, $(\sin \psi/\psi)^2$, Eqs. (20), (22), and (23).

b. Cyclotron Harmonic Heating

Single-particle motion in cyclotron harmonic acceleration has been studied by several authors [17-20], all of whom note that stochastic effects can appear in the presence of a single monochromatic accelerating wave. In the theoretical analysis, the underlying physics is somewhat obscured by the need to manipulate a host of Bessel functions, but for our present purposes, it is sufficient to approximate the circular cyclotron orbit by one of rectangular shape, in which case Eqs. (22) with minor modifications, can be directly applied. Orienting the orbit so

that $\vec{k}$ is directed along the top leg in the direction of particle motion, the first equation may be taken over without change. Assuming $\vec{E}$ is electrostatic (parallel to $\vec{k}$), no acceleration occurs along the two side legs of the square, and along the bottom leg the particle is moving in the direction opposite to the running wave and will experience negligible net acceleration. We need only put in the time change between successive passes through the top leg, but this step introduces a subtlety of the problem: to an excellent approximation, the cyclotron period is independent of the particle velocity. The determination of the phase-space trajectories requires, however, knowledge of the velocity dependence of the orbiting frequency, however weak this dependence is. Such dependence can be due to an inhomogeneous magnetic field or to relativistic mass change, or to the finite amplitude of the perturbing electric field. Having found such an effect, we can rewrite set (22) as

$$\begin{aligned} v_{n+1} &= v_n + \frac{qE_1}{m_i} \frac{d}{v_n} \frac{\sin \psi_n}{\psi_n} \cos (\tau_n - \psi_n) \\ \tau_{n+1} &= \tau_n + \frac{2\pi\omega}{\omega_c} + \frac{\omega L}{v_{n+1}} - \frac{\omega L}{v_r} \end{aligned} \quad (27)$$

where v_r is some fixed reference velocity in the mid-range of the v_n . In the formal theory, L now turns out to be very much smaller than the circumference of the (cyclotron)

orbit. Moreover, "stop bands" at $\sin\psi = 0$ show up as the zeros of the Bessel function $J_m(k_\perp v_\perp / \omega_c)$. By proper choice of $v_\perp$, the second and fourth terms on the right hand side in the equation for r_{n+1} cancel, and (27) reduces exactly to the form of (22). The interested reader may wish to compare Eqs. (27) with the mapping, based on circular orbits, derived by Karney [21]. The overlap condition for this case, calculated by Fukuyama et al., [18] is

$$\epsilon^2 = \frac{2}{\pi} \left| \frac{pqE_\perp}{m_i} \right|^2 \frac{1}{\omega_c k_\perp v_\perp^3} \quad (28)$$

where q is the ion charge, m_i its mass, and p the cyclotron harmonic number. Noting that $k_\perp v_\perp \approx p\omega_c$, we can approximate (28),

$$\epsilon^2 \approx p \left(\frac{a_{\text{electric}}}{a_{\text{magnetic}}} \right)^2$$

i.e., p times the square of the ratio of electric to magnetic particle acceleration. For tokamaks and mirror machines, powerful rf electric fields are necessary to achieve stochastic diffusion, $\epsilon > 0.18$.

c. Slow-Wave Alfvén Heating

Several authors have, in recent years, studied Alfvén-wave plasma heating at frequencies well below the ion cyclotron frequency [22-24]. Two quite different situations can arise, and they should be carefully distinguished. In a homogeneous cold plasma, there are two Alfvén modes, the compressional mode and the shear mode. Propagating across $\vec{B}$ into a plasma of increasing density, the radial wave number of the shear mode becomes very large (infinite in mhd theory) and the wave undergoes mode conversion into a quasi-acoustic mode. From the outside of the plasma, this process appears as absorption and the spectrum of a plasma-filled waveguide is a continuous one for this mode. The variation of perpendicular wavenumber with density is sketched in Fig. 6 and the critical parameter for the validity of the behavior as shown in Fig. 6 on the left (hot case) is that the parameter A be positive where

$$A = \frac{1}{K_{zz}} + \frac{3\omega^2 k T_i}{4\omega_{ci}^2 m_i c^2} \quad (29)$$

in which K_{zz} is the real part of the zz element of the plasma dielectric tensor, the term in T_i is the finite-ion-Larmor-radius correction to the mode conversion wave equation, and A is to be evaluated at the critical layer (i.e., where the parallel phase velocity of the wave is

equal to the local Alfvén speed). Roughly speaking, $A > 0$ if, at the critical layer, either $\beta^{(e)}$ or $\beta^{(i)}$ is greater than $2m_e/m_i$. Since T_e is generally greater than T_i in a tokamak, the $A > 0$ condition is most likely to imply that $\beta^{(e)} > 2m_e/m_i$ and a corollary is that the electron thermal velocity at the critical layer is greater than the parallel wave phase velocity there. Moreover, as T_e increases toward the magnetic axis, one may conclude that the entire region inside the critical layer is subject to quite strong electron Landau damping, and that the small-amplitude absorption lengths for the mode-converted kinetic Alfvén wave are of the order of $(1/k_{\parallel})$ in the toroidal direction and of the order of a few ion Larmor radii in the radial direction.

On the other hand, such strong damping implies that the wave packet dies out close to the antenna structure which launches it, and the calculations of Section IVa above, Eqs. (22) - (26) for electrons in a torus heated by a wave packet of limited spatial extent, may apply. However, it is interesting to consider the situation for slow-wave Alfvén heating at low frequency, $\omega = k_{\parallel} v_{\text{Alfvén}} = (n/R) v_{\text{Alfvén}}$ in a torus with, say, the choice $n = 1$ or 2 . Engineering requirements could well dictate that the toroidal extent of the antenna (and of the wave packet) be small compared to the major radius, R , of the torus. Then, in contrast with the case for lower-hybrid heating, ψ_n , except for very small v_n , is

always small compared to π . Moreover from

$(\Delta v)_{\max} \approx (eE/m_e) (d/v)$ and from (20), (23) and (24), it is clear that $D_{PS} \sim v^{-1} \sim m_s$ in the overlapping range and that $D_{PS} \sim v^{1/2} \sim m_s^{-1/2}$ in the non-overlap region. The qualitative behavior of the diffusion coefficient is sketched in Fig. 7; nominal overlap begins at that value of v for which $4\epsilon = 1$, from Eq. (23) with $|\sin\psi/\psi| = 1$.

Divergence of D_{PS} as $v \rightarrow 0$ does not occur because $(\sin\psi/\psi)^2$ becomes small for small v . It is interesting to note in Fig. 7 that the effect of the very broad shape-factor, $|\sin\psi/\psi|$, is to cause diffusion of electrons in the velocity spectrum in regions very distant from the fundamental Landau-Alfvén resonance, $v_{\parallel} = \omega/k_{\parallel} = v_{\text{Alfvén}}$.

From another point of view, if the wave packet launched by the antenna is subjected to Fourier analysis, the harmonic number v in the expansion of the wave field according to

$$E(r, z, t) = \sum_v E_v(r) \exp(2\pi i v z/L - i \omega t)$$

corresponds exactly to our particle-resonance sub-harmonic number m_s . Moreover, it is Landau damping by the high- $k_{\parallel}$ components of the wave packet which is responsible for the low- v high- m_s diffusion, and the pseudo-stochastic

diffusion function portrayed in Fig. 7 may be regarded as the quasilinear diffusion coefficient for the power spectrum defined by the wave packet. In this vein, we may note that the harmonic components $\nu > 1$ have correspondingly slower toroidal phase velocities and would only "see" Alfvén resonances at densities ν^2 times the $\nu = 1$ critical-layer density. If no such resonance occurs in the plasma, a description of the plasma heating by non-resonant electron transit-time magnetic pumping (paragraph e below) may be more appropriate.

d. Fast-Wave Alfvén Heating

The object of fast-wave Alfvén heating [24] is the excitation of high-Q toroidal eigenmodes with good coupling efficiency from a simple high-radiation-resonance antenna structure. This situation is favored if A, Eq. (29), is negative at the critical layer, in which case the variation of k_x^2 with density appears as in Fig. 6, on the right (cold case). The resonance region around the critical layer is very thin and transmission, via tunneling, into the propagating compressional Alfvén wave inside the critical layer should be close to unity. (Tunneling into this mode will also occur even when the parameters are nominally appropriate for slow-wave Alfvén heating, Fig. 6, hot case, and the balance between absorption from each of the two branches remains to be calculated). As a high-Q mode (also awaiting calculation

is the effect on the Q of this mode due to tunneling and mode conversion to the surface wave outside the critical layer, Fig. 6, cold case) the parallel wavenumber will be sharply defined and the heating will be described by the single-mode analysis, Eqs. (2) - (11).

e. Magnetic Pumping

Plasma heating by classical magnetic pumping involves no coupling to propagating waves in the plasma, and the wave packet from the antenna will evanesce rapidly with distance. The analysis of Section (c), for slow-wave Alfvén heating, is therefore applicable to this situation, with either electrons or ions considered as the heated particles, depending on the choice of frequency and antenna dimensions.

In the low-collision limit, trapping in the moving components of the applied electromagnetic field becomes a dominant process; a provocative study of heating by magnetic transit-time pumping as an example of neo-classical diffusion has been made by Canobbio [25].

f. Phase Stability; Fermi-Ulam Acceleration

Previous occurrences of the concepts used here deserve mention, and one of the first applied to the "phase stability" of particles accelerated in a synchrotron [26,27]. The rising magnetic field together with the electric field across the accelerating gap are synchronized with the orbiting of a selected ion,

and the fields are shaped so that other ions which are nearby in phase-space will execute small "phase-stable" oscillations around the exactly synchronous ion.

Another mapping which, in its linearized form, can lead to set (17) is the description of the one-dimensional motion of a particle bouncing between a fixed wall and an oscillating piston. S. Ulam [28] suggested this physical situation as an elementary model of the Fermi cosmic-ray acceleration mechanism [29] and it has since been studied by a number of other authors [30-32].

g. Electron Cyclotron Heating in a Mirror Machine

Electron cyclotron heating in a mirror machine gives rise to another example of the concepts surrounding the mapping set (17). Consider an rf field in a mirror machine with frequency close to the fundamental or to an integer harmonic of the electron cyclotron frequency. The electron enters the field with a certain relative gyration phase, experiences resonant acceleration and undergoes a change in its magnetic moment, μ . It then streams along $\vec{B}$ with constant μ , is reflected by the magnetic mirror, and re-enters the accelerating rf field with a new phase, the phase difference being a calculable function of μ and the known $\vec{B}(\vec{r})$. A new adiabatic invariant (for small acceleration values - corresponding to the areas interior to the island orbits) was discovered in

1964 by M. Seidl [33] (a phenomenon later termed "super-adiabaticity" - see paragraph h below) and the transition between adiabaticity and stochasticity as well as the effect of Coulomb collisions has been extensively explored by Lichtenberg and co-workers [34].

h. Ion Cyclotron Heating in Tori and Mirrors:

Superadiabaticity

As the final set of examples to which the mapping in (17) can apply, we examine ion cyclotron heating in magnetically confined plasmas. Under cyclotron heating, ions following the $\vec{B}$ lines in a torus with rotational transform pass through relatively short regions of cyclotron resonance - just where their orbits intersect the $\omega = \omega_{ci}(r)$ resonance cylinder - and receive a kick in $v_{\perp}$. (A small kick in $v_{\parallel}$ accompanies the $v_{\perp}$ kick because ion energy is conserved in the wave-frame.) Circumnavigating the torus, the change in the relative gyration phase of the ion and the rf field is a calculable function of $\vec{B}(\vec{r})$ and of the starting values for $v_{\perp}$ and $v_{\parallel}$. For example, using $B(\vec{r}) = B_0 [1 + (r/R) \cos \theta]^{-1}$ and, working at the p^{th} cyclotron harmonic, assuming $\omega = p\omega_{ci}$ at $\theta = \pi/2$, the total phase change in half of the circumnavigation in the poloidal direction is ϕ

$$\begin{aligned} \phi &\equiv \frac{1}{2\pi} \int [\omega - p\omega_{ci}(\vec{r})] dt \\ &= \frac{\omega q r^2}{4v_{\parallel} R} [1 + v_{\perp}^2/2v_{\parallel}^2] + \dots \end{aligned} \tag{30}$$

to lowest order in r/R and where $q = rB_t/RB_p$.

The integral values of ϕ are, for this case, the effective sub-harmonic numbers of the particle resonance, m_s [compare Eqs. (23) and (24)]. [Values of $v_{\perp}^2/v_n^2 > R/r$ correspond to trapped ions and should not be considered in this calculation but rather by Eqs. (33-35) below.] A two-step mapping can be formed from the one-pass acceleration equation [35,36] giving the change in $v_{\perp}$, and from the linearized form of (30) for the change in ϕ ,

$$\begin{aligned} v_{\perp}^{(n+1)} &= v_{\perp}^{(n)} + (\Delta v_{\perp})_{\max} \cos(2\pi\phi_n) \\ \phi_{n+1} &= \phi_n + \left[\frac{\omega q r^2 v_{\perp}}{4 R v_n^3} \right] v_{\perp}^{(n+1)} \end{aligned} \quad (31)$$

which leads to

$$\epsilon^2 = \frac{1}{8\pi} \frac{\omega q r^2 v_{\perp} (\Delta v_{\perp})_{\max}}{R v_n^3} = \frac{p q}{8\pi} \cdot \frac{r^2}{R \rho_{Li}} \cdot \left(\frac{v_{\perp}}{v_n} \right)^3 \frac{(\Delta v_{\perp})_{\max}}{v_{\perp}} \quad (32)$$

$$u_0 = 4 R v_n^3 / (\omega q r^2 v_{\perp})$$

$$\Delta t = \pi q R / v_n$$

and which may be used in (20) to evaluate $D_{ps}(v_{\perp}, \epsilon)$. Use of the heating rate argument posed at the end of Section IVa, with parameters representative of current tokamaks implies that phase-space islands in this instance will occur both with and without overlap depending rather sensitively on the values of r , $v_{\perp}$ and v_n for the particular orbits. [37].

The two-step mapping for the case of ion cyclotron heating in mirror geometry has been worked out by Rosenbluth [38] and is similar to work cited for electron cyclotron heating [34]. The context of the calculation is the question of enhanced mirror loss by electrostatic fluctuations and the regime of non-overlap and of low Coulomb scattering is there labelled "superadiabatic" [39]. The mirror equations can also be applied to the group of trapped ions in a toroidal device. Following an ion in a mirror field, $B = B_0 [1 + (z/L)^2]$, from $z = 0$ through its reflection in the mirror and back to $z = 0$, one finds

$$\phi = \frac{1}{2\pi} \int (\omega - p\omega_{ci}) dt = \frac{\omega L v_n^2}{4v_\perp^3} \quad (33)$$

$$\Delta t = \oint \frac{ds}{v_n} = \frac{\pi L}{v_\perp}$$

for the case of $\omega = p\omega_{ci}(z)$ at $z = 0$. Integral values of ϕ again give the subharmonic mode number, m_s , and Eqs. (33) may be used to derive a mapping similar to (31) and to compute ϵ^2 and u_0 ,

$$\epsilon^2 = \frac{3p}{8\pi} \frac{L}{\rho_{Li}} \left(\frac{v_n}{v_\perp} \right)^2 \frac{(\Delta v_\perp)_{\max}}{v_\perp} \quad (34)$$

$$w_0 = \frac{4v_\perp}{3p} \frac{\rho_{Li}}{L} \left(\frac{v_\perp}{v_n} \right)^2$$

Values from (33) and (34) may be used, as before, to compute $D_{PS}(v_\perp, \epsilon)$ by (20).

V. COULOMB COLLISIONS

The last topic to be considered in this paper is the question of Coulomb collisions and the relative effect of their contribution to stochasticity compared, for instance, to $D_{PS}(v)$. As indicated in Section I, Fokker-Planck forms each for quasilinear (or pseudo-stochastic) and for Coulomb diffusion can be used on the right-hand side of the Boltzmann equation. A more transparent presentation of the effect is to add Langevin terms to the mapping of set (17) which then becomes

$$\begin{aligned} u_{m+1} &= u_m + v_m \\ v_{m+1} &= v_m - 2\pi e^2 \sin(2\pi u_{m+1}) + \alpha v_m + \beta R(m) \end{aligned} \tag{35}$$

where α introduces viscous drag (dynamic friction), $R(m)$ is a sequence of random numbers with average value zero and rms value unity, and β is the coefficient of velocity dispersion. The corresponding diffusion coefficient is $D_{Coul} = (1/2) \langle (\Delta v)^2 \rangle / \Delta t = \beta^2 / 4\pi$, which may be compared to the dimensionless $D_{PS} = \pi \epsilon^4 / 2$ (overlap) or $D_{PS} = \pi \epsilon^3 / 8$ (non-overlap). Dimensions can be eliminated from the customary format for the Coulomb coefficients,

$$\alpha = \left| \frac{\Delta t}{v} \right| \langle \Delta v \rangle_S$$

$$\beta^2 = \left| \frac{\Delta t}{u_o} \right|^2 \langle (\Delta v)^2 \rangle_S \quad (36)$$

where the sub-script S on the right indicates the notation of Spitzer [40]. As in Eq. (20), Δt is the time between successive iterations, and u_o is the velocity spacing between successive resonant velocities. In each application, the proper interpretation of (35) and (36) requires some thought to express Δv appropriately in terms of the actual Spitzer-Chandrasekhar scattering increments parallel and perpendicular to $\vec{v}$, but for purposes of rough estimation, it can suffice to approximate

$$\alpha \approx v \Delta t$$

$$\beta^2 \approx v \Delta t \left(\frac{2kT}{m u_o^2} \right) \quad (37)$$

where v is the 90° -collision rate.

In another context, the effect of collisions on (non-overlapping) island-trapped particles has seen considerable attention. This is just the realm of neoclassical theory, and neoclassical diffusion in velocity space has been examined by several authors [25, 41].

A crude examination of the competition between D_{Coul} and D_{PS} shows that collisions play a decreasing role at lower

frequencies (ICRF, Alfvén, Magnetic Pumping) and at higher temperatures (say above 3 keV). Under these conditions superadiabaticity and stochastic diffusion merit careful examination. But even under conditions of moderate collisionality, the formalism of the pseudo-stochastic diffusion function and of the Langevin-form mapping equations, set (35), can provide highly useful tools for studying the evolution and shape of the space-averaged $f(v,t)$.

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FIGURE CAPTIONS

FIG. 1. Islands in phase space.

FIG. 2. Sketch illustrating a representative velocity dependence of the pseudo-stochastic diffusion function, D_{PS} .

FIG. 3. Surfaces of section for set (10) with $N = 1/2$; computer was programmed to interleave two mappings of the type in set (12). $\epsilon = 0.125$; 19 starts $(x_0 = 0, y_0 = -0.4771, \Delta y_0 = 0.1086)$, each with 500 points.

FIG. 4. $\epsilon = 0.175$; 17 starts $(x_0 = 0, y_0 = -0.4943, \Delta y_0 = 0.1243)$, each with 1000 points.

FIG. 5. $\epsilon = 0.250$; 29 starts $(x_0 = 0, y_0 = -0.9, \Delta y_0 = 0.1)$, each with 500 points.

FIG. 6. Dispersion relations $[k_\perp^2$ versus density] for Alfvén waves; hot electrons (left), cold electrons (right).

FIG. 7. Sketch showing qualitatively the pseudo-stochastic diffusion function for slow-wave Alfvén heating.

ISLANDS IN PHASE SPACE

Fig. 1. 793329

Fig. 2. 793328

DLT1= 0.5000 DLT2= 0.5000 NST= 19 NCT= 500 NN= 1 MM= 8 KK= 1
 X = 0.0000 Y = -0.4771 DY = 0.1086 BNDRY = 0.500 PHASE= 0.0000 LL= 8

Fig. 3. 793326

DLT1= 0.7000 DLT2= 0.7000 NST= 17 NCT= 1000 NH= 1 MH= 8 KK= 1
 X = 0.0000 Y = -0.4943 DY = 0.1243 BNDRY = 0.500 PHASE= 0.0000 LL= 8

Fig. 4. 793324

DLT1= 1.0000 DLT2= 1.0000 NST= 29 NCT= 500 NM= 1 MM= 8 KK= 1
X = 0.0000 Y = -0.9000 DY = 0.1000 BNDY = 0.500 PHASE= 0.0000 LL= 8

Fig. 5. 793325

ALFVEN WAVES

Fig. 6. 793327

D_{PS} FOR SLOW ALFVEN WAVE

Fig. 7. 793330

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