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EDGE PLASMA

BY

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ELECTROSTATIC ANALYSIS OF THE TOKAMAK

EDGE PLASMA

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ABSTRACT

The inclusion of an equipotential poloidal limiter into the edge plasma of a circular tokamak discharge distorts the axisymmetry in two ways. First, it 'artificially' shorts out the top-to-bottom Pfirsch-Schlüter loop potentials, and second, it creates zones of back current flow into the limiter. The resulting boundary mismatch between the outer layers and the inner axisymmetric Pfirsch-Schlüter layer provides free energy to drive the edge plasma instabilities. Operational limiters are proposed to symmetrize the edge plasma and thereby reduce the electrical and MHD activity of the boundary layer.

I. INTRODUCTION

A tokamak discharge, if "dissected" in the minor cross section (Fig. 1), consists of a number of overlying nested surfaces [1] like the layers of an onion. From the innermost collisionless "banana" core to the outer collisional Pfirsch-Schlüter layer [2] the equilibrium states are axisymmetric, i.e., the equilibrium currents and driving potentials depend only on the poloidal angle θ and not on the toroidal angle ϕ .

Beyond the Pfirsch-Schlüter regime there exists one more layer -- the ring or scrape-off plasma outside the limiting radius at $r = a$. This region has received little attention in the literature; it is usually lumped into the inner Pfirsch-Schlüter layer. This layer is indeed collisional, but it is not identical to the classical Pfirsch-Schlüter regime. The scrapeoff layers are in good electrical contact with the front and back surfaces of the limiter, so that the potentials and currents which are to prevent charge separation in this region are strongly modified from the corresponding quantities in the interior.

In this paper we give an electrostatic analysis of the (nonrotating) plasma behind a circular, equipotential, poloidal limiter, deriving the currents, the potentials, and the poloidal electric fields in the region $r > a$. We find that the axisymmetric state of the inner layer is strongly modified by the intrusion of the limiter. In the limiting case of few collisions the outer layer exhibits helical symmetry. In fact, the potential profile is similar to that which would arise if a double helix, one positive, the other negative, were to arise on opposing sides of the minor cross section. In the more realistic heavily collisional case the helical symmetry in turn becomes modified.

Because of the distortion of the symmetry, it is in general impossible to match the inner and outer states stably across the $r = a$ boundary. For an equipotential limiter there does exist, however, a condition of least mismatch in a plane midway between the front and back surfaces of the limiter, provided the collision frequency is properly adjusted at the plasma edge.

To avoid this mismatch and to minimize the free energy to drive instabilities at the boundary, we propose two types of novel limiters -- one, an active, two-faced, poloidal sandwich limiter on which one places prescribed potentials, and the second, biased toroidal rail limiters.

11. PLASMA STATE $r < a$

In this section we outline an elementary derivation of the equilibrium currents and potentials in the axisymmetric Pfirsch-Schlüter region of a symmetric torus just inside the limiting radius at $r = a$. A rotational transform $q \approx 2\pi/q$ is created by axial current flow in the hot inner regions of the torus. The plasma is collisional and in general may be characterized by different electron and ion temperatures T_e and T_i . The product of the toroidal magnetic field and the major radius BR is constant. We assume that the discharge has been centered in the torus by proper choice of the equilibrium field. Ions and electrons drift vertically (Fig. 2) in opposite directions at a speed $\pm 2ckT_{i,e}/eBR$ as a result of field curvature and toroidal inhomogeneity [3]. If the plasma is radially inhomogeneous, these motions give rise to a cross field charge accumulation in each cm^3 of surface volume of

$$\frac{\partial Q}{\partial t} = nev(r_L/\Lambda) (1/R_0) \sin \theta \quad (1)$$

where $v = (2kT/m_e)^{1/2}$, $r_L = c (2kTm_e)^{1/2}/eB$, $T = T_i + T_e$, $\Lambda = |nkT/d|$ (nkT/dr), R_0 is the major radius, and we restrict our attention to $r = a$. This charge accumulation must drain off by current flow along the helical flux surfaces encircling the torus. Integrating the charge along a line of force from the top of the torus, we obtain the magnitude of the neutralizing equilibrium currents [4-6]:

$$j_{||} = \int \partial\rho/\partial t \, dL = q R_0 \int_{-\pi/2}^{\pi/2} \partial\rho/\partial t \, d\theta = -a \cos \theta, \quad (2)$$

where $dL = q \, d\theta$ is a unit length along a helical flux tube and $a = nev(r_L/\Lambda)q$. The sign of $j_{||}$ in Eq. (2) differs from the "normal" sign because the loop current and the magnetic field are opposed. The field lines then form a left-handed helix.

In a plasma characterized by a resistivity $\eta = m_p c^2 / 4\pi e^2 \nu$ (ν = collision frequency) there arises an electric field $E_{||} = \eta j_{||}$ to drive the current along the field lines and a corresponding potential distribution $\phi = - \int_0^L E_{||} \, dL + \phi_0$ such that

$$eV/kT_e = \beta \sin \theta + eV_0/kT_e \quad (3)$$

where $\beta = 2q^2 (R_0/\Lambda) (1/T_e) (v/\omega_c)$ and V_0 is the plasma potential on the midplane. The current flow patterns and potential maximum predicted by Eq. (2) and (3) are summarized in Fig. 3 for a "linearized" tokamak,

i.e., one in which the torus has been severed through the minor cross section and stretched along a line. Current flows from the top to the bottom of the torus along helical field lines intersecting both the inside and outside of the torus (toward and away from the observer).

There is also a poloidal electric field $E_\theta = - dV/ad\theta$ and a cross field drift velocity $v_r = - cE_\theta/B$

$$v_r = (qr_L)^2 \frac{R_0}{a} \frac{v}{\Lambda} \cos \theta . \quad (4)$$

The plasma flows as if a uniform vertical electric field, $- 2q^2 (R_0/\Lambda) (v/\omega_c) (kT/ea)$, were applied to the torus. Averaging the drift motion around the major and minor circumference, $\bar{v}_r = (q r_L)^2 v/\Lambda$, which is equivalent to the Pfirsch-Schlüter result [2], if one neglects diamagnetism.

Equations (3) and (4) must be corrected for plasma rotation [4]. If the rotation speed is comparable to the electron diamagnetic velocity, the corrections should be minor [5,7,8].

1.1. PLASMA STATE BEYOND LIMITER

Currents must also flow in the plasma outside the limiter at $r = a$ to neutralize the charge accumulation of Eq. (1). In this region direct paths between the top and bottom of the torus are intercepted by the limiter. This departure from axisymmetry modifies both the potential and the current contours from the equilibrium solutions of Eqs. (2) and (3). The current flow now depends on which sector of the torus is

encircled by the helical flux tubes — note Fig. 4. There are two sectors of purely external current flow — those that encircle the top and bottom of the torus. In these zones the current pattern is very similar to that of the interior.

Quite a different pattern emerges in the loops that encircle the inner and outer midplane. Here current can flow either internally, connecting the electron and ion areas, or externally, into the limiter. The potential maximum in these latter zones will occur not at $\theta = \pi/2$, but at an angle intermediate between 0 and $\pi/2$. The current flow patterns and potential maxima and minima are illustrated in Fig. 4.

To consider the problem quantitatively we introduce the boundary conditions on the back surface of the limiter in the form of the classical Langmuir relation between the current j_1 and the potential V_1 immediately outside the thin sheath separating the plasma from the limiter [9,10],

$$j_1 = nev(m_e/2m_i)^{1/2} [C \exp(-e[V_1 - V_L]/kT_e) - 1], \quad (5)$$

where $C = (m_i T_e/m_e T)^{1/2}$, j_1 and V_1 are functions of θ and V_L is the limiter potential. Equation (5) expresses the condition of quasi-charge neutrality for the bounding plasma in contact with the limiter. The sum of these currents and the currents flowing into the front of the limiter has been detected and measured with a six-element segmented limiter in the T-5 tokamak by Mukhovatov [11].

It is instructive to examine the implications of Eq. (5). Rearranging the variables

$$eV_1/kT_e = eV_L/kT_e + \ln C - \ln \left[1 + j_1 / nev (m_e/2m_i)^{1/2} \right] \quad (5a)$$

A current-free plasma in contact with conductors assumes a positive potential $\ln C(kT_e/e) \sim 3 kT_e/e$ to trap electrons and equalize the escape rate of the two plasma species. The tokamak plasma is not current free; a current j_1 must flow into the limiter to relieve the space charge accumulation. The last term of Eq. (5a) shows that the current gives rise to an additional increment of potential. As j_1 approaches $-nev (m_e/2m_i)^{1/2}$ (electron flow away from the conductor, in our convention), the plasma potential goes to infinity. If j_1 exceeds this limit, there is no steady-state solution to the equations. One would expect this state to be highly unstable.

A. Outer State in Limit of Infrequent Collisions

In general one must seek solutions that consistently take account of the charge accumulation, the current distribution needed to remove the charge, the (parallel) electric field to drive the current, and the boundary conditions on the two faces of the limiter. Before considering the general problem, it is instructive to consider the low density, high temperature limit in which the ohmic potential drop $\int \eta j_{||} dL$ is small compared with the plasma potential V_1 . In this limit the plasma along a helical flux tube linking the two faces of the limiter is an equipotential; the currents at each end

of the flux tube extending from a poloidal angle θ_1 at the back of the limiter (Fig. 2) to $\theta_2 = \theta_1 + 2\pi/q$ at the front are equal and opposite:

$$j_1 = - (qR_0/2) \int_{\theta_1}^{\theta_2} \frac{\partial p}{\partial t} d\theta = - (\alpha/2) (\cos \theta_1 - \cos \theta_2) . \quad (6)$$

The effect of the equipotential limiter (in this limit) is to shift the angle of maximum current flow from $\theta = 0, \pi$ to $\theta = \pm \pi/2 - \pi/q$ at the limiter. From Eqs. (5) and (6)

$$e(V_1 - V_L)/kT_e = \ln C - \ln[1 - \gamma(\cos \theta_1 - \cos \theta_2)/2] , \quad (7)$$

where $\gamma = q(r_L/\Lambda) (2m_i/m_e)^{1/2}$ is the ratio of the electron drift speed to the sound speed. If $\gamma \ll 1$, then

$$e(V_1 - V_L)/kT_e \approx \ln C + \gamma(\cos \theta_1 - \cos \theta_2)/2 . \quad (8)$$

The potential at any toroidal angle ϕ may be obtained by the transformation $\theta = \theta_1 + \phi/q$.

The potential distribution of Eq. (8) is in phase with Eq. (3) only at $\phi = 0$. As one moves toward the limiter the pattern rotates with the pitch of the helical field lines. The equivalent electric field, $-(\gamma kT_e/e\alpha) \sin \pi/q$, rotates from the vertical at $\phi = \pi$ to $\pm \pi/q$ off the vertical at the limiter, i.e., the field pattern exhibits helical, rather than axial symmetry, as shown in Fig. 5. A comparable symmetry also shows up in the ambipolar flow patterns.

The effect is equivalent to the formation of two charged helices, one positive, the other negative, on opposite sides of the minor cross section.

Under no conditions can the interior and exterior states be matched everywhere around the torus, just as a helix cannot be matched to a straight line. Equation (8) and the symmetry of the torus suggest that the two states can be matched at $\phi = \pi$ — halfway between the two faces of the limiter — if the collisional parameter β is properly adjusted.

B. $\int \eta j_{\parallel} dL \neq 0$

To include resistivity in the analysis we must take account of local current flow as well as external flow into the limiters. The solutions will be similar to those of Eqs. (6) and (8) near the zones of symmetric current flow, i.e., $\theta_1^* \sim \pi/2 - \pi/q, 3\pi/2 - \pi/q$. In regions of antisymmetric flow ($\theta_1 \sim \pi/q, \pi - \pi/q$) Eqs. (6) and (8) are incorrect.

The current and potential near the back of the limiter, j_1 , and V_1 , are related by the Langmuir relation, Eq. (5). To compute these quantities at all positions around the periphery of the torus we integrate $\partial\phi/\partial t$ and the electric field $E_{\parallel} = \eta j_{\parallel}$ along the helical flux tubes from the back of the limiter

$$j_{\parallel} = j_1 + qR_0 \int_{\theta_1}^{\theta} \partial\phi/\partial t d\theta = j_1 + \alpha (\cos \theta_1 - \cos \theta) , \quad (9)$$

and

$$\begin{aligned} eV/kT_e &= eV_1/kT_e - q R_0 n (e/kT_e) \int_{\theta_1}^{\theta} j_{\perp} d\theta \\ &= eV_1/kT_e - \beta [(j_1/\alpha + \cos \theta_1) (\theta - \theta_1) + \sin \theta_1 - \sin \theta] \end{aligned} \quad (10)$$

where $\theta = \theta_1 + \pi/q$. A similar set of relations holds for the front of the limiter (side 2).

The quantities $j_{1,2}$ and $V_{1,2}$ are related by four boundary conditions on the front and back faces of the limiter; the first two are derived from the Langmuir relation; the third, from the total charge to be drained; and the fourth, from the condition that the total potential difference is $-\eta \int_0^L j_{\perp} dL$:

$$e(V_{1,2} - V_L)/kT_e = \ln C - \ln (1 \pm \gamma j_{1,2}/\alpha) \approx \ln C \mp \gamma j_{1,2}/\alpha, \quad (11)$$

$$j_2 = j_1 + q R_0 \int_{\theta_1}^{\theta_2} \partial p / \partial t d\theta = j_1 + \alpha (\cos \theta_1 - \cos \theta_2), \quad (12)$$

$$\begin{aligned} e(V_2 - V_1)/kT_e &= -q R_0 (e/kT_e) \int_{\theta_1}^{\theta_2} E_{\perp} d\theta = -\beta [(j_1/\alpha + \cos \theta_1) 2\pi/q \\ &\quad + \sin \theta_1 - \sin \theta_2]. \end{aligned} \quad (13)$$

The four boundary conditions can now be solved to obtain the current flow into the back face of the limiter, j_1 :

$$j_1/\alpha = \frac{1}{2} \frac{\cos \theta_2 - \cos \theta_1 - (\beta/\gamma) (\sin \theta_1 - \sin \theta_2 + (2\pi/q) \cos \theta_1)}{1 + \pi \beta/q\gamma} \quad (14)$$

and

$$\theta_2 = \theta_1 + 2\pi/q.$$

The quantities j_1 , and V are now completely defined. The solutions derived for a $q = 3$ tokamak discharge are displayed in Fig. (6) to (9). In this general case there is a further degradation of the symmetry. The ohmic potential drop ηj_1 is now evident. The potential contours for $r > a$ and $r < a$ are similar only near the two zones $\theta_1^* = \pi/2 - \pi/q, 3\pi/2 - \pi/q$. In the two zones rotated by $\pm \pi/2$ from these there are now potential maxima and minima as a result of back current flow into the limiter. The poloidal potential contours are similar to those of the interior only at $\phi = \pi$ (Fig. 7); near the limiter the drop in potential due to the (partial) short circuiting is evident in Fig. 8.

In Fig. (9) it is clear that the current contours of the two states are identical only along loops beginning near θ_1^* . In zones rotated from these by $\pm \pi/2$ the current flow is much smaller. In fact near the limiter the current is reversed from the normal ($-\cos \theta$) equilibrium flow. High density ($\beta/\gamma > 1$) enhances the back current

flow; in this limiting case the limiter acts as a true short circuit. The effects of this reversed current flow on the balance of magnetic and pressure forces in the scrape off layer need further investigation to determine whether any real "equilibrium" of forces actually does exist. In our analysis we have explicitly assumed that such an equilibrium does exist.

As in section A it is impossible to match the inner and outer states across the $r = 1$ boundary. Real discontinuities in the potential $\phi = e k T_{e0} / e$ at low density and $\phi = k T_{e0} / e$ (at high density) appear in the surface layer. If this potential develops over a distance of order $1/r_0$, then the surface layers may be unstable to Kelvin-Helmholtz (12) shearing instabilities, a condition now being met in years old machines (13). Strong Kelvin-Helmholtz excitation is to be expected in high density tokamaks, such as the Alcator series. In Alcator A, for example, $\beta \approx 1$, so that the ratio of electric field at the boundary will be $\approx 10^4$ V/cm. Since the threshold condition for shearing instabilities is $E \times 10^4 \text{ V/cm} > 1$ V/cm, Kelvin-Helmholtz waves should be strongly excited during low density operation. Indeed, the scattering experiments of Sisson and Surko (14) have shown strong electrostatic surface wave excitation in Alcator A if the plasma density exceeded 10^{14} cm^{-3} .

Although the inner and outer states may never be matched everywhere around the periphery of the torus, it is possible to minimize this mismatch by adjusting the surface collision frequency. By change of variable $\rho' = \rho + \rho_0$ one can transform Eq. (10) to a plane halfway

around the torus from the limiter. It can then be readily seen that if

$$g/\gamma = \sqrt{2} q (R_0/r_L) (T/T_e) (v/v_e) (m_e/m_i)^{1/2}$$

$$= 1/\tan(\pi/2 - \pi/q) \quad (15)$$

$$1.73, 1, 0.57 \text{ for } q = 3, 4, 6.$$

then the interior and exterior states match at the midplane $\theta = \pi$, as demonstrated in Fig. 7. No matching is possible for $1 < q < 2$. As one increases the toroidal current in a tokamak plasma (decreasing q) it will be necessary to increase the collisionality in order to maintain partial matching across the $r = a$ boundary. This observation is consistent with the practice on Alcator A [15], in which high current operation is achieved by raising the plasma density gradually by gas puffing. The surface variables making up the constants β and γ have not been measured in most tokamaks. In PDX $\beta/\gamma \approx 1$ for $q = 3$ circular discharges in deuterium [16].

The characteristics of the instabilities excited by the boundary mismatch, the onset criteria, and the mode structure can be obtained only from a detailed stability analysis. To guide experimental work, however, it may be useful to point out explicitly those conditions leading to strong boundary mismatch: (1) $g/\gamma \gg 1/\tan(\pi/2 - \pi/q)$ and (2) $\sqrt{\beta} \gg 1$. In case (1), important for high density

tokamaks, the boundary discontinuity should be especially prominent near the limiter, where the conductor short circuits the interior potential $\beta < T_e/e$. In case (2) (surface overheating) the limiter sheaths cannot support the current needed to remove the charge accumulation. The potential in the zone near $\theta = \theta_1 + \pi/4$ rises to infinity (in our linear analysis). In this latter case one would expect the instabilities to be strongest in loops girdling the top of the torus.

IV. ACTIVE LIMITERS

A. Poloidal Limiters

The mismatch between the inner and outer tokamak states has its origin (at low density) in the formation of a current-carrying plasma sheath and at high density in the short circuiting of the equilibrium potentials by the poloidal limiter. Both of these perturbations can in principle be removed by replacing the equipotential limiter by a driven variable potential limiter constructed with two faces insulated from each other (Fig. 10). The potential profile on each side of the limiter must be chosen such that the interior plasma looks into a matched load (both V and $j_{\parallel}$ must match all around the periphery of the torus). On the back side of the limiter (Fig. 2 the potential should be (in the small current approximation)

$$eV_{L1}/kT_e = \beta \sin \theta - \alpha \cos \theta, \quad (16)$$

The first term of Eq. (16) matches the inner and outer plasma potentials; the second term corrects for the potential perturbation induced by the current flow through the plasma-limiter sheath. Insertion of Eq. (16) into Eq. (11), together with the equilibrium requirement that $j_1 = j_0$, yields

$$eV_1/kT_e = \beta \sin\theta + 2nC, \quad (17)$$

a relation identical (within a constant) to Eq. (3). The potential and current just outside the limiter sheath therefore matches the equilibrium potential. Provided the charge accumulation is also the same (identical pressure gradient for $r < a$ and $r > a$), then the current flow and potential at all points around the periphery of the torus are identical to that in the interior.

The analysis just given holds only for that side of the limiter facing the loop current (side 1). On the opposing face the current is reversed with respect to the collecting electrodes, so that the sheath potential [2nd term of Eq. (16)] must also be reversed, i.e.,

$$eV_{L2}/kT_e = \beta \sin\theta + \gamma \cos\theta, \quad (18)$$

this relation follows from Eq. (11) and the requirement that $j_2 = j_0$. The conclusion to be drawn from Eq. (17) and (18) is that the limiter must be a sandwich, with an insulating layer to separate the two faces. The general symmetry relation between the two sides is

$$V_1(u) = -V_2(-u) .$$

(19)

It should be noted that the second term of Eq. (16) is an approximation that breaks down if the electron drift speed approaches or exceeds the sound speed ($\gamma = 1$). As $\gamma \rightarrow 1$ the number of electrons in the limiter sheath drops sharply, setting an upper limit to the current. At the same time the plasma potential can rise well above $3kT_e/e$. If $\gamma > 1$ the limiter sheath cannot support the current; there is no steady-state solution to the equations in the scrapeoff layer. The plasma potential will rise to a level determined by nonlinear processes not considered in the theory. Boundary matching by active limiters becomes impossible. This highly unstable condition may be retarded by choosing a limiter material with a large secondary emission coefficient.

The magnitude of γ will also be affected by the loop current j_ℓ that flows outside $r = a$. Since $j_\ell \sim V_\ell/2\pi R_0 n$, the condition that $|j_\ell| < |j_{\text{lim}}|$ reduces to

$$V_\ell < 2\pi R_0 n a , \quad (20)$$

where the loop voltage V_ℓ depends on the total integrated resistance of the plasma column. Discharges in PLT/PDX typically meet this criterion, since $V_\ell \sim 1$ volt, about 1/3 the upper limit set by Eq. (20) if $q = 3$. In general, however, the loop current will introduce further toroidal asymmetry in the potential contours, especially if $\gamma \sim 1$.

Equation (16) may break down for other reasons: (1) if the surface plasma rotates at a speed much in excess of the electron diamagnetic velocity; (2) if the plasma pressure is not negligible compared with the magnetic field pressure; (3) if other non-axisymmetric perturbations become prominent (e.g., asymmetric surface heating). It may be wise to use Eq. (16) only as a rough guide and to derive the potentials to drive the limiter elements from the surface plasma itself by means of either voltage or current sensing elements.

Two types of active poloidal limiters are illustrated in Fig. 10: (a) current control and (b) voltage control. In (b) potentials are applied to the individual segments of a multi-element limiter. This type is easily regulated; one adjusts the voltage to force a $\cos \theta$ dependence on the current. The number and spacing of the elements depend on how closely one wishes to tailor the current and potential profiles. In the 16-element limiter shown, the potential matching is worst near the midplane, where the interior potential along one element varies by $\beta kT_e/e \sin 22.5^\circ = 0.38 \beta kT_e/e$. A more efficient design would suggest elongated elements near $\theta = \pi/2, 3\pi/2$ and foreshortened elements near $0, \pi$ (if $\beta > \gamma$).

Biasing of the limiter elements can be accomplished in a variety of ways. If $\beta > \gamma$, for example, one can generate the potentials $\beta kT_e/e$ with one main power supply and a variable resistor chain. Auxiliary center-tapped power supplies connected between the left

and right hand elements of one face can supply the surface potentials. The second face of the limiter can be driven with another power supply. Alternately, one can use the same power supply and merely interchange the left and right hand connections. If the bias is properly adjusted, there should be no current drawn on the main power supply.

The segmented limiter permits only a stepwise adjustment of current and potential. Its most serious deficiency is that the poloidal component of the magnetic field, E_θ , is not continuous, but is non zero only near the segment boundaries. In this respect the current drive limiter of Fig. 10a, in which the resistor chain is incorporated continuously in the limiter itself, is far superior. A minimum of four electrical contacts is required; two vertical contacts to approximate (linearly) the $\sin \theta$ term and two horizontal contacts for the $\cos \theta$ term. Front and back surfaces of the limiter should be connected (vertically) in parallel and (horizontally) in anti-parallel. This type of limiter cannot be regulated as conveniently as its segmented mate. Possibly one can monitor the space potential or the noise of the surface plasma.

In principle the inner and outer states must be matched with a sandwich limiter. In practice this complication can be avoided in an important class of torus - the high density tokamak such as Alcator A or C. In these devices the voltage required to drive the equilibrium current is much greater than the voltage drop across the limiter sheath ($\beta \gg \gamma$). Equation (16) reduces to a simple $\sin \theta$

dependence. Potentials at the back and front can be identical, so that the limiter reduces to a more conventional single-faced configuration. This is a fortunate circumstance, since the boundary mismatch is especially acute at high densities (β can exceed 1).

B. Toroidal Limiters

It is questionable whether the poloidal limiters discussed in Section A will be useful in limiting hot reactor-grade plasmas. The presence of insulators inside the vacuum vessel close to the hot plasma will pose severe shielding problems. These problems may be rendered more tractable by converting to toroidal limiters, as proposed for TFTR by J. Cecchi [17]. Toroidal rail limiters are inherently more attractive, because their area is larger than that of poloidal limiters by the factor $R_0/a \sim 3-4$. The power and current-handling capacities are correspondingly greater. Furthermore, $\theta = \text{constant}$ for toroidal limiters, so that one can satisfy Eq. (16) merely by biasing each complete toroidal loop with one power supply. In principle one should also split each rail and insulate the top and bottom portions to satisfy the γ factor of Eq. (16). In practice this term will be reduced by the factor a/R due to the increased area of current interception, so that it may in a practical case be negligible compared with β .

We illustrate toroidal limiters designed for two kinds of tokamaks in Fig. 11. For a tokamak of circular cross section there is a four rail limiter. Two of these can be grounded to the vessel. The upper (or bottom) pair must be insulated and connected to a power supply. The insulators can be recessed into the shielding to

protect them from the hot toroidal plasma. If the insulation should become a serious problem, then it may be feasible to convert to a two-rail grounded limiter on the inner and outer midplane. This is certainly not an ideal solution, since this location would not be accessible for diagnostics and heating beams, either neutral beam or radiofrequency beams.

A possible way to circumvent this problem is shown in Fig. 11b. Here we show a vertically-elongated torus, a design also favored for other reasons, principally stability. In this configuration the orbit drifts near the midplane are parallel to the poloidal contours, so that no equilibrium currents need to flow around the midplane. Instead, the currents from the top and bottom of the torus are intercepted by grounded, split toroidal limiters.

Sandwich limiters such as those discussed in this paper have never been tested in tokamak discharges. Mukhovatov [11], however, reported experiments in the T-5 tokamak with a 6-element, single-layer limiter. He noted that a current of ~ 8 A flowed from the top to the bottom of the torus when the individual elements were shorted together. This observation showed clearly that the currents in the scrapeoff layer did not follow the classic $\cos \theta$ behavior of the Pfirsch-Schlüter regime. Biasing did not appear to affect the overall confinement in the torus, but no measurements of the scrapeoff layers were undertaken. More recent experiments by Strait on the Wisconsin octupole [18] showed that radial electric fields could be set up by biasing segments of the limiting aperture.

V. SUMMARY AND DISCUSSION

The outer, collisional layers of a tokamak plasma act in some respects as a battery. A top-to-bottom plasma potential must be generated to complete the current loops initiated by the orbit drifts in the inhomogeneous magnetic field. If the plasma is terminated by an equipotential poloidal limiter, the Pfirsch-Schlüter driving potentials will be shorted out in the scrapeoff layers behind the limiter, so that the potential and current profiles become strongly altered from the normal equilibrium profiles. The resulting mismatch across the bounding radius provides free energy to drive the edge plasma unstable. We propose that this mismatch be avoided, either by applying potentials to segments of a poloidal sandwich limiter, or by converting to toroidal rail limiters. Prescriptions for matching potentials to a nonrotating plasma in the small current approximation are given.

The analysis applies in detail only to a tokamak plasma with a circular, nonemitting limiter. Extension of the analysis to include secondary emission [19] or thermionic emission is straight forward. Interfacing problems such as those described must also be present in other types of tori: tokamaks confined by horizontal and vertical bars, tokamaks with divertors, stellarators, and the bumpy torus. These and other variants will require more complicated interfacing formulae.

In this article we have directed attention only to the electrostatic aspects of the tokamak boundary. Experimental evidence shows that the boundary also exhibits a variety of poorly understood MHD activity [20], so that a fruitful area for future studies might well be the interaction of the (modified) equilibrium currents with the vertical magnetic field.

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Fig. 1 The minor cross section of a circular tokamak discharge, showing the three inner axisymmetric layers and the outer scrapeoff layer in the shadow of the limiter.

81X0308

81X0306

Fig. 3 Current patterns (arrows) for $r < a$ in a "linearized" Tokamak plasma. The toroidal field and loop current are opposed. $q = 3$.

81 X 0307

Fig. 4 Current pattern in a cylindrical tube with a magnetic field.

Fig. 5 Regions of maximum (++) and minimum (--) potential in the limiter shadow in the limit of low collisionality. $q = 3$.

Fig. 6 Solid lines show the potential variation along the helical field lines, $r > a$; θ_1 is the angle of interception of the flux tube at the back of the limiter; $V_L = 0$; $q = 3$; $\beta = 1$, $\gamma = 0.2$. The dashed line shows the potential variation for $r < a$, $\theta_1 = 30^\circ$.

81X0405

Fig. 7 Potential variation in the poloidal direction in the plane $\phi = \pi$;

$q = 3$; $v_L = 0$.

81X0492

FIG. 8. Potential profiles for $\phi = 0^\circ$ and $\beta/\gamma = 0.5$.

Fig. 9 Equilibrium currents along the field lines; $q = 3$.

81X0335

Fig. 10 Active poloidal sandwich limiters: (a) current-carrying carbon loops, and (b) a 16-element segmented voltage-control limiter.

Fig. 11 Toroidal rail limiters: (a) a 4-rail limiter for a tokamak of circular cross section; (b) split midplane limiters for a vertically elongated tokamak.