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Neutral Technicolor Pseudo Goldstone Bosons Production and QCD
Background at the SSC

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Neutral technicolor pseudo Goldstone bosons
production and QCD background at the SSC

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The production of the neutral technicolor pseudo Goldstone bosons, $P^{0'}$ and $P_8^{0'}$, at large transverse momentum in pp collisions, $pp \rightarrow g(q)P^{0'}(P_8^{0'})X$ has been investigated in reactions at a high energy collider such as the SSC. The major two-body and three-body decay modes in tree diagrams are investigated in detail. The $t\bar{t}$ decay channel would dominate both the decays of $P^{0'}$ and $P_8^{0'}$ if it is allowed. Otherwise, gg and $3g$ will be the dominant decay modes unless the mass of the $P^{0'}$ and $P_8^{0'}$ are below 40 GeV, where $b\bar{b}$ becomes dominant. According to the QCD backgrounds, which we have also investigated in detail in this work, the signal for $t\bar{t}$ is much larger than the background and will be the ideal signal for detecting these bosons. However, in the absence of the $t\bar{t}$ channel, the $\tau\bar{\tau}$ mode can be used to identify $P^{0'}$ up to $m_P = 300$ GeV in the transverse momentum range $P_\perp \lesssim 100$ GeV. Similarly, the $b\bar{b}$ decay mode can serve us a signal to identify $P_8^{0'}$ up to $m_P = 300$ GeV for $P_\perp$ between 500 and 700 GeV. Our results show that these high transverse momentum production processes are useful for the searching for the $P^{0'}$ and $P_8^{0'}$ at the SSC.

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1. INTRODUCTION

During the last two decades, particle physics has made significant progress in uncovering the fundamental constituents of matter and their interactions. Particles colliding at high energy gives us precious information of their behaviors. The next generation collider, which will reach TeV colliding energy, may reveal an entirely new world. The current view is that the microscopic world is composed of leptons and quarks building blocks (Table 1.1) and their interactions are described by gauge theory.

The standard model, which combines the Glashow-Weinberg-Salam [1, 2] electroweak model $SU(2)_L \times U(1)_Y$ and strong interactions $SU(3)_c$ gauge groups, have succeeded in every test of experiments thus far. Despite the success, however, it has serious drawbacks, e.g., seventeen undetermined parameters and the hierarchy problem. These drawbacks have inspired work on the extensions of the standard model, including supersymmetry, composite models, technicolor, etc. The technicolor theory, whose phenomenology is the main topic of this thesis, is one of the extensions without the hierarchy problem. The following paragraphs outline the current facts and basics in particle physics.

Before we embark on the task of describing the technicolor theory of some of the phenomenology, we summarize the current status of the study of fundamental

Table 1.1. Some quantum number of fermions

leptons	SU(2)	SU(3)	U(1)	Q
$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_{L_1}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_{L_1}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_{L_1}$	doublet	singlet	$-\frac{1}{2}$	$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$
e_R, μ_R, τ_R	singlet	singlet	-1	-1
quarks	SU(2)	SU(3)	U(1)	Q
$\begin{pmatrix} u \\ d \end{pmatrix}_{L_1}, \begin{pmatrix} c \\ s \end{pmatrix}_{L_1}, \begin{pmatrix} t \\ b \end{pmatrix}_{L_1}$	doublet	triplet	$\frac{1}{6}$	$\begin{pmatrix} \frac{2}{3} \\ \frac{1}{3} \\ -\frac{1}{3} \end{pmatrix}$
u_R, c_R, t_R	singlet	triplet	$\frac{2}{3}$	$\frac{2}{3}$
d_R, s_R, b_R	singlet	triplet	$-\frac{1}{3}$	$-\frac{1}{3}$

particles. The current fundamental particle spectrum duplicates the pattern into three families (see Table 1.1). Each family behaves just like the other families except for their mass differences. However, the top quark has not been found to complete the third family. The current lower bound on top quark mass is 78 GeV from the CDF group at Fermi National Laboratory [3].

Are there more families? According to the recent data on Z particles from the CERN LEP experiments [4], the measured width of Z, at the 95 % confidence level, allows only three families, if one assumes the neutrino of the fourth family to be light. Would the quarks and leptons be composite [5] as a recurrence of the past history? To find the answer, a very short distance probe has to be made, probably $\leq 10^{-17}$ cm. No current accelerator in the world is able to break the shell of compositeness if it exists. The projected Superconducting Super Collider (SSC), 40 TeV proton-proton collider, may just barely cross the line. Consequently compositeness is still an open question awaiting a more powerful collider to resolve it.

What are the interactions among the fundamental constituents? Four fundamental forces have been known for some time. They are gravitation, electromagnetism,

(a)

(b)

Figure 1.1. Two concepts of gravitation

(a) curved space-time gives gravitation (b) quantum gravity

weak, and strong interactions. Gravitation, the force which guides the motion of universe, is perceived as a consequence of a curved space-time as suggested by Einstein, Figure 1.1 (a). Due to severe singularity, the quantized theory of gravitation (Figure 1.1 (b)) was unable to handle the divergence to make the theory renormalizable. Only recently, the divergence could be canceled for the first time in superstring theory [6]. However, we still are not sure if the superstring is the only solution to this problem. Electromagnetism, the force between charged objects, is the best known one among the four forces. The quantum version of electromagnetism (Figure 1.2 (a)), quantum electrodynamics (QED), not only can be made finite (renormalizable), but also describes the interaction of charged particles with amazing accuracy.

The weak interactions mediate the decay of nuclei, e.g., neutron decaying to proton. The weak force exists also among leptons, for example, a muon decays to an electron, an antineutrino and a muon neutrino. In the early days, the weak interaction

Figure 1.2. The quantum interactions: (a) a QED process, (b) a QCD process

was described by Fermi's four-fermion interactions [7], see Figure 1.3. The Fermi coupling constant, G_F , is measurable from β decay, muon decay, or kaon decay, and the current experimental value is $1.16637 \times 10^{-5} \text{GeV}^{-2}$ [8]. However, the four-fermion interaction scheme is unrenormalizable and violates unitarity at high energy. Later on the four-fermion interaction scheme was viewed as a low energy effective interaction mediated by heavy particles W^+ and W^- . This intermediate-vector-boson (IVB) theory faces same difficulties, for example, the high energy behavior of the process, $\nu_e + \bar{\nu}_e \rightarrow W^+ + W^-$. The amplitude violates unitarity at high energy and the S-matrix is unrenormalizable in high order calculations. These problems can be resolved by adding a neutral particle W^0 and requiring certain relations between the couplings of neutral W and charged W to fermions, see Figure 1.4.

Glashow proposed an interesting model [1] based on the $SU(2) \times U(1)$ gauge group. Not only did the model give the right relationships for couplings, but it also unified electromagnetism and weak interactions. Unfortunately, gauge theory

Figure 1.3. Four-fermions and IVB interactions
 (a) Fermi's four-fermions interactions (b) IVB interactions

as known did not allow massive gauge particles such as W^+ , W^- , and W^0 . Later, Salam and Weinberg [2] discovered independently that the Higgs mechanism [9] can break the gauge symmetry (spontaneously symmetry breaking) and then give masses to the gauge bosons. Furthermore, t'Hooft and Veltman [10] proved that the Higgs mechanism does not spoil the renormalizability. The neutral current predicted in the Glashow-Weinberg-Salam model was discovered in the 1973 [11] and the particles mediating weak interactions, $W^\pm$ and Z (a combination of W^0 field of $SU(2)$ and B field of $U(1)$), in the model were discovered in 1983 [12].

Nucleons are held together in nuclei by a force we call strong interactions. This force was long known to be responsible for hadron resonances. Further analysis of systematics of such resonances led to their description in terms of three quarks, (u,d,s), substructure [13]. Now, the quantum chromodynamics (QCD) with six quark flavors and eight gluons is used to describe the strong interactions. QCD is based on the

Figure 1.4. $\nu_e \bar{\nu}_e \rightarrow W^+ W^-$

(a) unitarity violation at high energy

(b) cancels bad behavior of (a) to hold the unitarity

$SU(3)$ gauge group [14, 15]. The strong force of QCD is mediated by gluons (Figure 1.2 (b)) and is asymptotically free at short distances. Because the strong interactions become strong and nonperturbative at low energy, and nonperturbative calculation is still under development, the only method to deal with the nonperturbative region currently is the lattice gauge theory. Although all relevant experimental results seem to agree with QCD, more work on theory and experiment are needed to test QCDs further.

A common ground for all interactions is the local gauge symmetry first discovered by H. Weyl [16] for electromagnetism and then later applied to the non-Abelian gauge by Yang and Mills [17] for nucleon interactions. We have already seen the gauge theories for electroweak and strong interactions. Gauge theory may not be the correct or final theory, but probably the principles learned from gauge theory for the fundamental interactions will be lasting.

Figure 1.5. Different running coupling constants vs. energy

Nature may have more symmetry than we see in everyday life. What we usually see is a world with broken symmetry. We do not see electroweak gauge symmetry macroscopically because that symmetry only appears in short distances ($\leq 10^{-17}$ cm); see Figure 1.5. The symmetry breaking in the standard model is done by Higgs potential. The ground state (vacuum) is not trivial and the vacuum expectation value of the Higgs field fails to vanish, $\langle \phi \rangle \neq 0$. Hence the solution of equations lose some or all symmetries the equation had before. This broken symmetry scheme is well known in a ferromagnet. The equations of motion of spins in a ferromagnet are invariant under rotation. However, all spins of ground state align in one direction when the temperature is low. Thus, the solutions no longer have rotational symmetry.

In spontaneously symmetry breaking theory, a fundamental scalar field must be

introduced. One $SU(2)$ doublet complex scalar field is needed in the minimum standard model. If Nature has more than one symmetry breaking step at different energy scales, more Higgs fields are needed. Higgs fields from the lower scale tend to merge to a higher scale through radiative corrections and spoil the large gaps among the multi-breaking scales unless some other mechanism enters to prevent it. One mechanism is supersymmetry [18] which connects boson and fermion sectors. Supersymmetry introduces new particles which are partners of ordinary bosons and fermions with different spins such that the contributions of super particles cancel out the contributions of ordinary particles in radiative corrections. Another mechanism assumes the Higgs fields to be composite particles and the symmetry breaking is due to new interactions, dynamical symmetry breaking. A theory based on new interactions that has drawn much attention is technicolor theory (TC) [19]. There are various models in TC. Generally, they contain new chiral fermions (techni-fermions) engaging in new interactions (TC force), Figure 1.6. Techni-fermions have global chiral symmetry if we neglect other interactions at high energy. The TC force gets stronger as energy becomes smaller (or distance becomes larger). When energy reaches a critical value, the TC force becomes so strong that techni-fermions begin to form bound states which give rise to a nontrivial vacuum and break the global chiral symmetry to a global vector symmetry. According to Goldstone [20], breaking a global symmetry introduces massless Goldstone bosons. Since the global vector symmetry remains valid, all Goldstone bosons are pseudo-scalar. The pseudo Goldstone bosons, which have the same quantum numbers as $W^\pm$ and W^0 , can couple and give masses to $W^\pm$ and W^0 . The rest of the pseudo Goldstone bosons are not massless because they still can acquire masses from QCD and electroweak interactions. The existence of pseudo

Figure 1.6. Techni-fermions exchange TC gauge bosons

Goldstone bosons is a strong signal for a new interactions such as TC force.

Besides the problem in Higgs sector, the standard model contains too many uncalculable parameters (see Chapter 2) and makes itself unlikely to be an ultimate theory. The proposed machines of LHC (large hadronic collider) at CERN and SSC in the United States may be able to investigate the electroweak interaction above the symmetry breaking scale. Their main goals are to discover Higgs particles and further test electroweak theory. Future experiments performed at these accelerators will undoubtedly shed light on the mystery of Higgs sector or the compositeness of fermions.

In this thesis we will focus on the phenomenology of TC theory at the SSC. The standard model is briefly reviewed in Chapter 2. The TC model we will use is the Farhi-Susskind model [21] which has four $SU(2)_L$ doublets of fermions with the $SU(4)$ TC gauge group. The details will be reviewed in Chapter 3. Eichten et al. (EHLQ) [22] examined the productions of neutral technipions, $P^0 (\sim \bar{Q}\gamma_5 Q - 3\bar{L}\gamma_5 L)$ and

$P_8^{0'} (\sim \bar{Q} \gamma_5 \lambda_a Q, \lambda_a \text{ are Gell-Mann SU(3) matrices}),$ at the SSC through the process

$$p + p \longrightarrow P^{0'}(P_8^{0'}) + X. \quad (1.1)$$

EHLQ also calculated the branching ratios of various two-body tree decay channels of $P^{0'}$ and $P_8^{0'}$ but neglected three-body decay channels. McKay et al. show that three-body tree decay channels are significant [23]. We will examine a production process in Chapter 4 for $P^{0'}$ and $P_8^{0'}$ at the SSC [23, 24],

$$p + p \longrightarrow P^{0'}(P_8^{0'}) + g(q) + X. \quad (1.2)$$

The total widths and branching ratios of the decay channels, in tree graphs and up to three-body for $P^{0'}$ and $P_8^{0'}$, have been recalculated and their analytical forms are presented in Chapter 5. Our calculation is not exact, since we did not include one-loop effects of the two-body decay modes. The one-loop effect together with the corresponding tree diagrams of three-body decay channel should eliminate the infrared divergence. Instead, we use a cutoff to circumvent the infrared divergence. The backgrounds of different signals from two-body decay channels are also investigated in Chapter 6. The use of (1.2) as process to search for $P^{0'}$ and $P_8^{0'}$ is discussed in Chapter 7 where we also present our conclusions.

2. THE STANDARD MODEL

The standard model is based on the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group. So far no experiments disagree with the standard model. However, in the theoretical point of view the standard model is incomplete and allows for several possible extensions. We will review the gauge principle in Section 1. In Section 2, we discuss the Glashow-Weinberg-Salam electroweak theory. The theory of the strong interaction is discussed in Section 3. Section 4 discusses briefly the unanswered questions of the standard model.

2.1. The Gauge Principle

The idea of gauging a field was invented by H. Weyl [16]. Yang and Mills [17] applied the idea to the isospin group of protons and neutrons. Their work inspired a lot of interest in the gauge field, but nothing was taken seriously until the unification of electromagnetism and weak force proposed by Glashow, Weinberg, and Salam [1, 2]. The gauge principle has become a guiding principle for all fundamental interactions: gravitation, weak, electromagnetic, and strong.

Consider the Lagrangian of free massless fermion,

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi. \quad (2.1)$$

Make a U(1) Abelian gauge transformation,

$$\psi \rightarrow \psi' = U(\alpha)\psi = e^{-iq\alpha}\psi. \quad (2.2)$$

The Lagrangian (2.1) is invariant under the global transformation (2.2) where the transformation parameter, α , is a constant. The conserved current of this global transformation is

$$\partial^\mu J_\mu = 0, \quad (2.3)$$

where

$$J_\mu = q\bar{\psi}\gamma_\mu\psi. \quad (2.4)$$

The charge operator Q which is defined as

$$Q \equiv \int J_0 d^3x \quad (2.5)$$

is conserved. Q is a physical quantity not related to the dynamics. If we require that the Lagrangian is also invariant under a local U(1) gauge transformation where $\alpha(x)$ is an arbitrary function of spacetime coordinates, we must change (2.1). Under the local U(1) gauge transformation

$$\begin{aligned} \partial_\mu\psi \rightarrow (\partial_\mu\psi)' &= U(\alpha)(\partial_\mu - iq\partial_\mu\alpha)\psi \\ &\neq U(\alpha)(\partial_\mu\psi). \end{aligned} \quad (2.6)$$

A necessary change is to introduce a vector field A_μ and replace ∂_μ by a covariant derivative D_μ defined as

$$D_\mu \equiv \partial_\mu + iqA_\mu. \quad (2.7)$$

If one requires the vector field A_μ transformation as

$$A_\mu \rightarrow A'_\mu = A_\mu + \partial_\mu\alpha, \quad (2.8)$$

the transformation of $D_\mu\psi$ becomes

$$D_\mu\psi \rightarrow (D_\mu\psi)' = U(\alpha)(D_\mu\psi). \quad (2.9)$$

A_μ is called a gauge field. Using the covariant derivative D_μ , we rewrite the Lagrangian (2.1)

$$\begin{aligned} \mathcal{L} &= i\bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - q\bar{\psi}\gamma^\mu\psi A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu} \\ &= i\bar{\psi}\gamma^\mu\partial_\mu\psi - J^\mu A_\mu - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \end{aligned} \quad (2.10)$$

where

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu. \quad (2.11)$$

The last term in (2.10) is the kinetic energy part of A_μ and is invariant under the transformation (2.8). Hence, the Lagrangian (2.10) is invariant under the local U(1) gauge transformation. From the local gauge invariance, the conserved current, J^μ , appears in the Lagrangian to interact with the field A_μ . If we identify q as the charge of the fermion and A_μ as the electromagnetic potential, the Lagrangian (2.10) is just the Lagrangian for a fermion field interacting with the electromagnetic field. The process described in Eqs. (2.7)–(2.11) to introduce an interaction is called gauging a field. Note that a vector mass term $m^2 A^\mu A_\mu$ is not invariant under the transformation (2.8). So, the gauge field A_μ must be massless.

In the non-Abelian case, we consider the fermion fields which form an SU(2) doublet,

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}. \quad (2.12)$$

Under an SU(2) transformation, we have

$$\psi \rightarrow \psi' = U(\theta)\psi = e^{-iT \cdot \theta} \psi, \quad (2.13)$$

where $T = (T^1, T^2, T^3)$ and $\theta = (\theta^1, \theta^2, \theta^3)$. T^i are the SU(2) generators and satisfy the following commutation relation,

$$[T^i, T^j] = i f^{ijk} T^k \quad i, j, k = 1, 2, 3. \quad (2.14)$$

$f^{ijk} = \epsilon^{ijk}$ are the structure constants. θ^i are the transformation parameters. The free Lagrangian (2.1) is invariant under global SU(2) transformation (θ^i are constants), but is not invariant under local SU(2) transformation. Similarly to the U(1) Abelian case, replacing ∂_μ by the covariant derivative D_μ

$$D_\mu \equiv (\partial_\mu - igT \cdot B_\mu), \quad (2.15)$$

where g is the coupling constant in SU(2), requires $D_\mu \psi$ to transform as

$$D_\mu \psi \rightarrow (D_\mu \psi)' = U(\theta)(D_\mu \psi). \quad (2.16)$$

Then B_μ field should be transformed as

$$T \cdot B_\mu \rightarrow T \cdot B_\mu' = U(\theta)(T \cdot B_\mu)U^{-1}(\theta) - \frac{i}{g}[\partial_\mu U(\theta)]U^{-1}(\theta). \quad (2.17)$$

For an infinitesimal transformation $\theta^i(x) \ll 1$,

$$U(\theta) \cong 1 - iT \cdot \theta, \quad (2.18)$$

Eq. (2.17) becomes

$$B_\mu^i \rightarrow B_\mu'^i = B_\mu^i + f^{ijk}\theta^j B_\mu^k - \frac{1}{g}\partial_\mu \theta^i. \quad (2.19)$$

The field strength tensor $F_{\mu\nu}$ for non-Abelian gauge field is defined as

$$F_{\mu\nu}^i = \partial_\mu B_\nu^i - \partial_\nu B_\mu^i + g f^{ijk} B_\mu^j B_\nu^k. \quad (2.20)$$

$\text{Tr}(F^{\mu\nu} F_{\mu\nu})$ is gauge invariant. The gauge invariant Lagrangian of this example can be written as

$$\mathcal{L} = i\bar{\psi}\gamma^\mu D_\mu\psi - \frac{1}{4}\text{Tr}(F^{\mu\nu} F_{\mu\nu}). \quad (2.21)$$

The Lagrangian (2.21) describes the interaction among the gauge fields B_μ^i and SU(2) doublet fermions.

Different gauge group requires different gauge invariant Lagrangian and different interaction. If the gauge principle is indeed the principle by which interactions are introduced, finding the right gauge group becomes an ultimate goal in particle physics research.

2.2. Glashow, Weinberg, and Salam Model

Weak interactions are responsible for numerous types of particle decay such as the nuclear β decay, muon decay, kaon decay, and the like. Fermi proposed the four-fermion interaction for the β decay of the neutron n into proton p , electron e , and antineutrino $\bar{\nu}$, with the Lagrangian density $\mathcal{L}_F$,

$$n \rightarrow p + e + \bar{\nu}$$

$$\mathcal{L}_F = -\frac{G_F}{\sqrt{2}} [\bar{p}\gamma_\lambda n][\bar{e}\gamma^\lambda \nu] + h.c., \quad (2.22)$$

where G_F is the Fermi coupling constant [7]. Later on, extensive investigations on various decays unveiled a number of violations of the conservation law hold in the strong interactions. The non-conserved quantum numbers are space parity P, charge

conjugation parity C, combined parity CP, strangeness, charm, and some others. All known quarks and leptons can engage in weak interactions. Fermi's four-fermion interaction can be viewed as an effective low energy theory describing the interactions between fermion and the weak interaction force fields (see Figure 1.3). The interaction part of the Lagrangian can be written as

$$\mathcal{L}_I = g(J^\mu W_\mu + h.c.). \quad (2.23)$$

Lagrangian (2.23) is identical to the gauge interaction discussed earlier. We can treat the intermediate-vector-bosons as gauge fields. Since the Gauge bosons have to be massive, we can simply add its gauge non-invariant mass term in the Lagrangian by hand. However, such a theory is not renormalizable and violates unitarity at high energy. Glashow, Weinberg, and Salam (GWS) [1, 2] proposed a model that not only unifies the weak interactions and the electromagnetic force, but also proves to be renormalizable according to 't Hooft and Veltman [10].

The GWS model [25] is based on the $SU(2)_L \times U(1)_Y$ gauge group. Quarks and leptons are arranged into families (or generations) to form $SU(2)_L$ left-handed doublets and right-handed singlets:

quarks

$$q_L : \begin{pmatrix} u^i \\ d'^i \end{pmatrix}_L, \begin{pmatrix} c^i \\ s'^i \end{pmatrix}_L, \begin{pmatrix} t^i \\ b'^i \end{pmatrix}_L, \quad q_R : u_R^i, d_R^i, s_R^i, c_R^i, b_R^i, t_R^i, \quad i = r, g, y$$

and leptons

$$l_L : \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L, \quad l_R : e_R, \mu_R, \tau_R.$$

The superscript i is the color index. The d' , s' , and b' are used to distinguish them from the mass eigenstates, d , s , and b . The $L(R)$ denotes left(right)-handed states. There are no right-handed neutrinos, since they have not been seen. From the gauge principle developed in Section 2.1, the $SU(2)_L \times U(1)_Y$ gauge invariant Lagrangian takes the form

$$\begin{aligned} \mathcal{L}_1 = & -\frac{1}{4} \text{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \\ & \sum_{q_L^i} i \bar{q}_L^i \gamma^\mu (\partial_\mu + i g \frac{\sigma}{2} \cdot W_\mu + i \frac{g'}{2} Y B_\mu) q_L^i + \\ & \sum_{l_L} i \bar{l}_L \gamma^\mu (\partial_\mu + i g \frac{\sigma}{2} \cdot W_\mu + i \frac{g'}{2} Y B_\mu) l_L + \\ & \sum_{q_R^i} i \bar{q}_R^i \gamma^\mu (\partial_\mu + i \frac{g'}{2} Y B_\mu) q_R^i + \sum_{l_R} i \bar{l}_R \gamma^\mu (\partial_\mu + i \frac{g'}{2} Y B_\mu) l_R, \end{aligned} \quad (2.24)$$

where the W s are $SU(2)_L$ gauge fields, B is $U(1)_Y$ gauge field, and σ^i are the Pauli matrices. Y is the hypercharge of $U(1)_Y$; g and g' are the coupling constants for each group. In order to give masses to the W fields, the spontaneously symmetry breaking (SSB), i.e., the Higgs mechanism [9], is used. One introduces a complex doublet of scalar fields,

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \quad (2.25)$$

which transforms as an $SU(2)_L$ doublet. The Lagrangian of the Higgs fields is

$$\mathcal{L}_H = (D^\mu \phi)^\dagger (D_\mu \phi) - V(\phi^\dagger \phi), \quad (2.26)$$

where

$$V(\phi^\dagger \phi) = \mu^2 (\phi^\dagger \phi) + \lambda (\phi^\dagger \phi)^2; \quad \lambda > 0. \quad (2.27)$$

$\mathcal{L}_H$ is gauge invariant. For $\mu^2 < 0$, the potential $V(\phi^\dagger\phi)$ has minima at

$$\langle\phi^\dagger\phi\rangle_0 = \frac{v^2}{2} \quad \text{with } v = \sqrt{\frac{-\mu^2}{\lambda}}. \quad (2.28)$$

We choose the physical vacuum as the non-vanishing vacuum expectation value of the Higgs field,

$$\langle\phi\rangle_0 = \begin{pmatrix} 0 \\ \frac{v}{\sqrt{2}} \end{pmatrix}. \quad (2.29)$$

The vacuum remains invariant under a generator $\mathcal{G}$, if

$$e^{i\theta\mathcal{G}}\langle\phi\rangle_0 = \langle\phi\rangle_0. \quad (2.30)$$

For an infinitesimal transformation, the equation becomes

$$\mathcal{G}\langle\phi\rangle_0 = 0. \quad (2.31)$$

For the generators of $SU(2)_L \times U(1)_Y$, we have

$$\sigma_j\langle\phi\rangle_0 \neq 0 \quad j = 1, 2, 3 \quad (2.32)$$

$$Y\langle\phi\rangle_0 = \langle\phi\rangle_0 \neq 0, \quad (2.33)$$

and

$$Q\langle\phi\rangle_0 = \frac{1}{2}(\sigma_3 + Y)\langle\phi\rangle_0 = 0. \quad (2.34)$$

Although $SU(2)_L \times U(1)_Y$ symmetry is spontaneously broken, the symmetry corresponding to charge Q remains unbroken. The latter is the generator of the electromagnetic $U(1)_{\text{em}}$ group. We obtained the Gell-Mann-Nishijima relation for the charge Q

$$Q = I_3 + \frac{Y}{2}, \quad (2.35)$$

where $I_3 = \sigma_3/2$ is the third component of the weak isospin. According to the Goldstone theorem [20], there is one massless spin 0 particle (Goldstone boson) created for each broken generator of a global symmetry. However, due to Higgs theorem, the Goldstone bosons become the longitudinal part of the gauge fields associated with the broken symmetry. These gauge fields become massive. The scalar doublet can be expressed as

$$\phi = U(\theta) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} = \exp \left(\frac{-i\theta \cdot \sigma}{2v} \right) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix}. \quad (2.36)$$

Transforming to the unitary gauge,

$$\phi \rightarrow \phi' = U^{-1}(\theta)\phi = \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} \quad (2.37)$$

$$\sigma \cdot W_\mu \rightarrow \sigma \cdot W'_\mu \quad (2.38)$$

$$B_\mu \rightarrow B'_\mu, \quad (2.39)$$

we have the mass terms

$$(\mathcal{L}_H)_{\text{mass}} = -\mu^2 H^2 \quad (2.40)$$

$$(\mathcal{L}_{GB})_{\text{mass}} = \frac{g^2 v^2}{8} (|W_\mu^1 - iW_\mu^2|)^2 + \frac{v^2}{8} (g' B_\mu - gW_\mu^3)^2. \quad (2.41)$$

We call H the Higgs boson whose mass is

$$M_H = \sqrt{-2\mu^2} > 0. \quad (2.42)$$

Define the charged gauge fields

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}. \quad (2.43)$$

The first term of Eq. (2.41) now reads

$$\frac{g^2 v^2}{8} (|W_\mu^+|^2 + |W_\mu^-|^2). \quad (2.44)$$

Thus, the masses of charged intermediate-vector-bosons are

$$M_{W^+} = M_{W^-} = \frac{gv}{2}. \quad (2.45)$$

Similarly defining

$$Z_\mu = \frac{-g' B_\mu + g W_\mu^3}{\sqrt{g^2 + g'^2}} \quad (2.46)$$

and

$$A_\mu = \frac{g B_\mu + g' W_\mu^3}{\sqrt{g^2 + g'^2}}, \quad (2.47)$$

we have the second term of Eq. (2.41)

$$\frac{v^2 \sqrt{g^2 + g'^2}}{8} Z^\mu Z_\mu \quad (2.48)$$

The neutral intermediate-vector-boson has gained a mass

$$M_Z = \sqrt{g^2 + g'^2} \frac{v}{2} \quad (2.49)$$

$$= M_W \sqrt{1 + \frac{g'^2}{g^2}}. \quad (2.50)$$

Using the fact

$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}, \quad (2.51)$$

hence, the value of v is

$$v = (G_F \sqrt{2})^{\frac{1}{2}} \simeq 250 \text{ GeV}. \quad (2.52)$$

The field A_μ remains a massless gauge boson corresponding to the $U(1)_{\text{em}}$ symmetry and is identified as the photon field. We have just shown the Higgs mechanism generating masses to gauge fields in Eqs. (2.36)-(2.50).

How to generate fermion mass? Because ψ_L is a doublet and ψ_R is a singlet under $SU(2)_L$, the mass term $\bar{\psi}_L \psi_R$ is not gauge invariant. However, $\bar{\psi}_L \psi_R$ can couple with a Higgs doublet from Yukawa coupling,

$$-G_Y \left[\bar{\psi}_R (\phi^\dagger \psi_L) + (\bar{\psi}_L \phi) \psi_R \right]. \quad (2.53)$$

Then, fermions can acquire masses from the SSB. Take the electron as an example. The Yukawa coupling in the unitary gauge reads

$$\begin{aligned} \mathcal{L}_{Y_e} &= -G_e \left[\bar{e}_R \left(0, \frac{v+H}{\sqrt{2}} \right) \begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix} + (\bar{e}_L, \bar{\nu}_L) \begin{pmatrix} 0 \\ \frac{v+H}{\sqrt{2}} \end{pmatrix} e_R \right] \\ &= \frac{-G_e v}{\sqrt{2}} \bar{e}e - \frac{G_e}{\sqrt{2}} H \bar{e}e. \end{aligned}$$

Accordingly, the electron has a mass

$$m_e = \frac{G_e v}{\sqrt{2}} \quad (2.54)$$

The Yukawa coupling (2.53) can give masses to the isospin down type fermions. However, unlike massless neutrinos, the weak isospin up type quarks, (u, c, t) , are massive. We need a charge conjugate scalar doublet

$$\phi_c = i\sigma^2 \phi^* \quad (2.55)$$

to form another Yukawa coupling that generates masses for quarks (u, c, t) . Unlike leptons, the mass matrix for quarks is not diagonal. Hence the weak isospin states are not the same as the mass eigenstates. Conventionally, in the weak interactions we keep the mass eigenstates of up-type quarks as weak isospin states and define proper weak isospin states of down-type quarks, (d', s', b') , in terms of their mass eigenstates,

$(d, s, b),$

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = U_{KM} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad (2.56)$$

We call U_{KM} the Kobayashi-Maskawa (KM) matrix [26]. U_{KM} is a unitary matrix that contains three angles and one CP violation phase, usually expressed as

$$U_{KM} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \quad (2.57)$$

Here, $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$. With the frame work of the standard model, the four parameters in the KM matrix are determined by experiments not by theory.

Through the SSB of Higgs mechanism

$$SU(2) \times U(1)_Y \xrightarrow{SSB} U(1)_{em},$$

we get three massive gauge bosons corresponding to the broken symmetries, one photon field to the surviving $U(1)_{em}$ group, and a massive Higgs particle. The Yukawa couplings give masses to all fermions except neutrinos.

The outcome of the GWS model is very impressive. Three massive gauge bosons, W^+ , W^- , and Z , have been discovered. All known results of experiments on weak interactions agree with the GWS model. The t quark has not been discovered, but most of us expect to see it in the near future. Despite the enormous phenomenological success of the GWS model, there are still many unanswered questions in the model. More details of the unanswered questions will be presented in Section 2.4. A crucial part of the GWS model is the Higgs sector for the SSB. However, the Higgs bosons

have yet to be discovered. One of the main purposes of the planned Superconducting Super Collider (SSC) is to find the Higgs bosons. On the theoretical side, the GWS model has been extended in order to provide answers to some of the problems not answerable in the standard model. Most of the extension can be tested at the SSC. Technicolor is one of the extensions worthwhile investigating at the SSC. In Chapter 4, we will address the production of the neutral technicolor pseudo Goldstone bosons and discuss the possibilities for finding them in SSC experiments.

2.3. Strong Interactions

Strong interactions are the interactions among hadrons, for example, the interaction which governs the scattering between protons, pion and proton, etc. The classification studies of hadrons, hadron mass spectra, and hadronic interactions in the sixties strongly suggested that hadrons have common building blocks [13] which are called quarks. The deep inelastic electron-proton scattering experiments to probe the structure of the proton at the SLAC confirmed the prediction by Bjorken of scaling behavior [27] in the structure function of the proton. To explain Bjorken scaling, Feynman [28] proposed the parton model where the electrons interact with almost free partons inside the proton in the deep inelastic scattering. The partons were later identified with quarks.

The interaction between quarks must be weaker at shorter distance in the parton model. Because we have not observed free quarks, the strong interaction must be sufficiently strong at larger distances so that quarks are permanently confined. Quantum chromodynamics (QCD) based on $SU(3)_c$ non-Abelian gauge theory satisfies all the requirements of this strong interaction among quarks. In this theory,

each quark flavor forms a $SU(3)_c$ triplet and carries a quantum number called color. The interaction is mediated by the gluons (QCD gauge bosons). 't Hooft [29], Gross and Wilczek [14], and Polizer [15] proved independently that QCD is asymptotically free. So QCD satisfies the condition of free partons inside the proton when probed by high energy photons. Furthermore, by its severe infrared properties, QCD also confines quarks. Only colorless states can exist as isolated objects. This explains why we have not observed free quarks. In the following, we list some of the evidence for color quantum number [30]:

(i) Fermi statistics. Consider the Δ^{++} resonance in the $J_z = \frac{3}{2}$ state

$$|\Delta^{++}, J_z = \frac{3}{2}\rangle = |u \uparrow, u \uparrow, u \uparrow\rangle. \quad (2.58)$$

The ground state of Δ^{++} is symmetric in flavor, spin, and space. This clearly violates the Fermi statistics. Fermi statistics can be restored if an extra quantum number 'color' is introduced. Three up quarks in the state (2.58) are different in their color. Each quark flavor forms a color triplet. The state (2.58) is modified to

$$\frac{1}{\sqrt{6}} \epsilon^{rby} |u_r \uparrow, u_b \uparrow, u_y \uparrow\rangle, \quad (2.59)$$

which is anti-symmetric under the exchange of any two quarks.

(ii) Cancellation of anomalies. In the GSW model, the triangular anomalies (see Figure 2.1) are proportional to

$$\sum_{\text{fermions}} \text{Tr}(Y T^a T^a) \propto \sum_{\text{doublets}} Y, \quad (2.60)$$

where T^a are the generators of $SU(2)_L$ and Y is the hypercharge. Each lepton doublet with $Y=-1$ comes with a quark doublet with $Y=\frac{1}{3}$. If each flavor has three colors, then the triangular anomalies are cancelled.

Figure 2.1. The triangular anomalies

(iii) $\pi^0 \rightarrow 2\gamma$. The decay of neutral pion into two photons is given by the triangle diagram (see Figure 2.2). The width is

$$\Gamma(\pi^0 \rightarrow 2\gamma) = N_c^2 (Q_u^2 - Q_d^2)^2 \alpha^2 m_\pi^3 (64\pi^3 F_\pi^2)^{-1} \quad (2.61)$$

where N_c is the number of colors and Q_u and Q_d are the charges for u and d , respectively. The experimental data [31] are

$$\Gamma_{\text{exp}} = 7.48 \pm 0.33 \text{ eV}.$$

If we take $N_c = 3$, we obtain

$$\Gamma(\pi^0 \rightarrow 2\gamma) = 7.6 \text{ eV},$$

which agrees with the experimental data.

(iv) $e^+ - e^-$ annihilations. Hadrons are formed from the fragmentations of quark-antiquark pairs which are produced by $e^+ - e^-$ annihilations (see Figure 2.3).

The ratio R_{had} is defined by

Figure 2.2. $\pi^0 \rightarrow 2\gamma$ through fermion loop

Figure 2.3. Hadron production from $e^+ - e^-$ annihilations

Figure 2.4. The R_{had} vs. energy; QPM: quantum parton model, EW: electroweak

$$\begin{aligned}
 R_{\text{had}} &\equiv \frac{\sigma(e^+e^- \rightarrow \text{hadrons})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} \\
 &= N_c \sum_{\text{flavors}} Q_i^2
 \end{aligned}
 \tag{2.62}$$

where N_c is the number of color and Q_i is the charge of the i th quark. Equation (2.62) sums over the flavors which are allowed by the energy. For $N_c = 3$ and five flavors, we have

$$R_{\text{had}} = \frac{11}{3},$$

which agrees with data [32] (see Figure 2.4).

We write the Lagrangian of QCD as

$$\mathcal{L}_{QCD} = \sum_{\text{flavors}} \bar{\psi}_f (i\gamma^\mu D_\mu - m_f) \psi_f - \frac{1}{4} \text{Tr} F^{\mu\nu} F_{\mu\nu} + \mathcal{L}_{GF} + \mathcal{L}_{FP}, \quad (2.63)$$

where $\mathcal{L}_{GF}$ is a gauge fixing term necessary for covariantly quantizing the gauge fields and $\mathcal{L}_{FP}$ removes contributions of all non-physical states. In the standard model, the masses of quarks are from the Yukawa couplings.

QCD brings new features of the fundamental forces, i.e., asymptotic freedom and confinement. Since the QCD interaction becomes strong at lower energy, we enter the non-perturbative regime. This remains a big challenge in high energy physics. The test of QCD are continuously undertaken. Experiments from hadron colliders as well as e^+e^- accelerators continually provide information on strong interactions. From these, we will learn more about the strong interactions.

2.4. Unanswered Questions

More than two decades ago, we had many questions about elementary particles. Today, we have a theory, the standard model, which is capable of accounting for all experimental data in particle physics. The standard model does not unify all the fundamental forces. However, its success firmly establishes the gauge principle as a guiding principle in describing fundamental interactions. From the gauge principle many attempts to unify all forces have been made. Gravity, which was a classical theory for a long time, has recently been formulated as a quantum theory in the superstring theory. Although no attempts can claim to be totally successful and the road to the true theory of everything is far from clear, these attempts do open new windows for us to gaze into nature.

The standard model contains many unexplained facts. Except for the gauge principle and Higgs mechanism, the other facets of the standard model are input from experimental observations and do not have satisfactory theoretical explanations. Even the Higgs mechanism has unsatisfactory properties when the strong interactions and electroweak interactions are unified in a simple symmetry, implying the existence of an energy scale much higher than the electroweak energy scale; this difference in scales is known as the hierarchy problem. There is a strong indication that the standard model may be incomplete and is only an effective theory which is valid at low energies. We list some of the unanswered questions in the standard model [33]:

- (i) What determines the mass hierarchies of quarks and leptons?
- (ii) What determines the Kobayashi-Maskawa matrix?
- (iii) How many generations are there?
- (iv) What is the origin of CP violation?
- (v) Why are there so many elementary particles? Are they composite?

Many extended theories of the standard model have been developed in order to give answers to some of these questions, such as the $SU(5)$ grand unified theory; these include $SU(5) \times U(1)$ flipped grand unified theory, technicolor, supersymmetry, the superstring inspired E_6 model, strong CP, compositeness. Experiments at the next generation accelerators in the TeV energy region may give answers to some of these questions. This is the reason why the Superconducting Super Collider is eagerly awaited by all high energy physicists.

3. THE TECHNICOLOR THEORY

Using the Higgs mechanism to break the symmetry has the hierarchy problem and therefore requires fine tuning. The mechanism of dynamical symmetry breaking due to a new strong force, the technicolor force, was invented to eliminate the hierarchy problem [19]. The hierarchy problem of Higgs fields is addressed in Section 1. Then, the mechanism of dynamical symmetry breaking is introduced in Section 2. Section 3 briefly reviews the technicolor theory [34].

3.1. The Hierarchy Problem

The scalar Higgs fields in the standard model possess the hierarchy problem, when the model is grand unified to include a very high energy scale. This is a general feature of a theory which contains spin zero fields. The hierarchy problem stems from the loop corrections to the potential of the Higgs fields. Take the $SU(5)$ grand unified theory [35] as an example. The $SU(5)$ has two scales corresponding to the symmetry breaking on the grand unified symmetry scale and the weak symmetry scale

$$SU(5) \xrightarrow{\langle \Phi \rangle} SU(3)_c \times SU(2)_L \times U(1)_Y \xrightarrow{\langle \phi \rangle} SU(3)_c \times U(1)_{em} \quad (3.1)$$

with

$$\langle \Phi \rangle = 10^{13} \langle \phi \rangle. \quad (3.2)$$

Figure 3.1. A sample of the radiative corrections to the Higgs potential

The Higgs potential reads

$$V(\Phi, \phi) = \lambda_1(\Phi^2 - \langle \Phi \rangle^2)^2 + \lambda_2(\phi^2 - \langle \phi \rangle^2)^2. \quad (3.3)$$

The quantum radiative corrections would introduce couplings between Φ and ϕ , e.g., through their coupling to gauge bosons as illustrated in Figure 3.1. The correction to the $V(\Phi, \phi)$ is roughly

$$\sim (8\pi^2)^{-1} g^4 \Phi^2 \phi^2. \quad (3.4)$$

The new minima of the potential are

$$\langle \Phi \rangle_{NEW} \sim \langle \Phi \rangle \quad (3.5)$$

and

$$\langle \phi \rangle_{NEW} \sim \frac{g^2}{4\pi\sqrt{\lambda_2}} \langle \Phi \rangle. \quad (3.6)$$

The ratio of two new vacuum expectation values becomes

$$\frac{\langle \phi \rangle_{NEW}}{\langle \Phi \rangle_{NEW}} \sim \frac{g^2}{4\pi\sqrt{\lambda_2}}, \quad (3.7)$$

which is of the order $O(1)$. The loop corrections are so large that the Higgs field of the low scale moves to the high scale and is therefore 10^{13} time too large for the electroweak symmetry breakdown. In order to separate the symmetry breaking between the grand unified and the weak scales, we have to fine tune the physical parameters to incredible accuracy and to redo it in every order of perturbation. In t' Hooft's criterion of naturalness [36], the physical parameters should not depend on a fixed value, precise to 10^{-26} , in order to make the system stable under perturbation. There are two ways to cure the unnaturalness of Higgs fields. One is to introduce an extra symmetry, the supersymmetry [18], such that the leading quadratic divergences of the radiative correction are canceled by adding loops of supersymmetric particles (see Figure 3.2). Another way is to make the Higgs fields as composite particles which are bound states of new strongly interacting particles [19]. In the former, supersymmetry introduces a new group of supersymmetric particles which are partners of the ordinary particles. None of these particles have been discovered yet. In the latter, the theory is called the technicolor theory and brings in new particles which also have not been observed. One major task of the experiments of the next generation, e.g., SSC, will be devoted to testing the Higgs sector in the standard model to find out the true nature of the electroweak symmetry breaking.

3.2. The Dynamical Symmetry Breaking

In this section, we will discuss dynamical symmetry breaking, an alternative to the spontaneous symmetry breaking scheme. We take QCD as an example. At high energy, the isospin doublet (u,d) quarks, whose masses can be neglected, have a global $SU(2)_L \times SU(2)_R$ chiral symmetry, where L (R) stands for left (right). The strong

Figure 3.2. A diagram of radiative correction from supersymmetry particals, gauginos

interaction coupling strength increases as the energy decreases. When the coupling strength reaches a critical value, the u and d quarks form condensates. This feature is very similar to vapor condensation to water when the temperature hits the transition point. The quark-antiquark condensates develop non-zero vacuum expectation value

$$\langle \bar{u}u \rangle_0 = \langle \bar{d}d \rangle_0 \neq 0. \quad (3.8)$$

The condensates, $\bar{u}u$ and $\bar{d}d$, still have $SU(2)_V$ ($SU(2)_{L+R}$) vector symmetry. The global chiral symmetry is broken to the smaller $SU(2)_V$ symmetry by the strong QCD force at energy scale $\Lambda_{QCD} \sim 200$ MeV

$$SU(2)_L \times SU(2)_R \xrightarrow{\Lambda_{QCD}} SU(2)_V. \quad (3.9)$$

According to the Goldstone theorem, we have three massless Goldstone bosons corresponding to the three degrees of freedom of the broken $SU(2)_A$ ($SU(2)_{L-R}$) symmetry, where A stands for the axial vector. They are

$$\pi^+ \equiv \bar{d}i\gamma_5 u \quad (3.10)$$

$$\pi^0 \equiv \frac{1}{\sqrt{2}}(\bar{u}i\gamma_5 u - \bar{d}i\gamma_5 d) \quad (3.11)$$

$$\pi^- \equiv \bar{u}i\gamma_5 d. \quad (3.12)$$

The three Goldstone bosons are coupled to the three axial isospin currents

$$\langle 0 | J_{5a}^\mu | \pi_b(q) \rangle = i f_\pi q^\mu \delta_{ab} \quad (3.13)$$

where J_{5a}^μ are defined as

$$J_{5+}^\mu = \bar{\psi} \gamma^\mu \gamma_5 \frac{\sigma^1 - i\sigma^2}{2} \psi \quad (3.14)$$

$$J_{5-}^\mu = \bar{\psi} \gamma^\mu \gamma_5 \frac{\sigma^1 + i\sigma^2}{2} \psi \quad (3.15)$$

$$J_{50}^\mu = \bar{\psi} \gamma^\mu \gamma_5 \frac{\sigma^3}{2} \psi \quad (3.16)$$

with $\psi = (u, d)$ and the σ^i are the Pauli matrices. If we include the electroweak interactions, $SU(2)_L \times U(1)_Y$, the hadronic current J_{5a}^μ could couple to the electroweak gauge fields. The W 's couple to J_{5a}^μ with strength $g/2$ where g is the $SU(2)_L$ gauge coupling constant. The B_μ field of $U(1)_Y$ couples to J_{50}^μ with strength $g'/2$ where g' is the $U(1)_Y$ gauge coupling constant. Consider the propagators of $W^\pm$ modified by the hadronic current

$$\frac{g^{\mu\nu} - q^\mu q^\nu / q^2}{q^2} \rightarrow \frac{g^{\mu\nu} - q^\mu q^\nu / q^2}{q^2(1 + \Pi(q^2))}, \quad (3.17)$$

where $\Pi(q^2)$ is the hadronic contribution to the vacuum polarization (see Figure 3.3).

$\Pi(q^2)$ develops a pole at $q^2 = 0$ due to massless pions

$$\Pi(q^2) \xrightarrow{q^2 \rightarrow 0} \frac{g^2 f_\pi^2}{4q^2}. \quad (3.18)$$

From the modified propagators, $W^\pm$ has a mass

$$M_W = \frac{g f_\pi}{2}. \quad (3.19)$$

Figure 3.3. Vacuum polarization due to pions

Figure 3.4. Mixing diagrams between W^0 and B gauge bosons

For the neutral gauge bosons, we have a mass matrix because of the mixing diagrams (see Figure 3.4)

$$M^2 = \frac{f_\pi^2}{4} \begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix}. \quad (3.20)$$

The eigenvalues of the matrix are

$$M_\gamma^2 = 0 \quad (3.21)$$

$$M_Z^2 = \frac{1}{4}(g^2 + g'^2)f_\pi^2. \quad (3.22)$$

The eigenstates A_μ and Z_μ are the same as those in the standard model and are identified as photon field and neutral Z boson, respectively. Three of the gauge bosons acquire mass. In this scheme, the pions replace the Higgs fields and the strong interactions replace the Higgs potential. Hence, we have an alternative to break the symmetry and to make gauge bosons massive. But since f_π is about 93 MeV and the desired value is about 250 GeV, the massive gauge bosons cannot get enough mass from QCD condensation. To overcome this problem, Weinberg and Susskind [19] postulated in 1979 a new strong interaction (technicolor) among new fermions (technifermions) that is responsible for the electroweak symmetry breaking. The point is to assume the decay constant of the new pions (technipions) to be about 2,000 times larger than that of the ordinary pions. With this much larger decay constant, the technipions could generate the correct masses for the $W^\pm$ and Z.

3.3. The Technicolor Theory

We assume that TC is a $SU(N_{tc})$ QCD-like non-Abelian gauge theory and is asymptotically free. To increase the f_π value by a thousand times, we naively have a relation

$$\frac{F_\Pi}{f_\pi} \sim \frac{\Lambda_{TC}}{\Lambda_{QCD}} \sim O(10^3), \quad (3.23)$$

where Λ_{TC} (Λ_{QCD}) is the technicolor (QCD) condensate scale. F_Π is the decay constant of technicolor pions (technipions). If $\Lambda_{QCD} \sim 200$ MeV, then Λ_{TC} is about 200 GeV. This give us a rough idea of where the TC scale is.

We will focus on the popular one-family TC model, i.e., the Farhi-Susskind model

[21]. The technifermions form $SU(2)_L$ doublets

$$\begin{pmatrix} U^i \\ D^i \end{pmatrix}_L, \quad U_R^i, \quad D_R^i \quad (3.24)$$

$$\begin{pmatrix} N \\ E \end{pmatrix}_L, \quad N_R, \quad E_R, \quad (3.25)$$

where i is the QCD color index. The technicolor indices are suppressed. We use capital letters to represent the technifermions. The technifermions have weak interactions, but only U and D have QCD interactions. All of the technifermions carry technicolor so that they strongly interact with each other. In the high energy region of order of 1 TeV, the TC force becomes very strong and the standard model forces become very weak, and therefore we can ignore the QCD and electroweak coupling. In the Farhi-Susskind model the technifermions have a global $SU(8)_L \times SU(8)_R$ chiral symmetry for the 8 fermions, $U^r, U^b, U^y, D^r, D^b, D^y, N, E$. Just as in QCD, when the TC force becomes strong at $\Lambda_{TC} \sim O(\text{TeV})$, the technifermions form condensates. These condensates give rise to non-zero vacuum expectation value

$$\langle \bar{U}U \rangle_0 = \langle \bar{D}D \rangle_0 = \dots \neq 0. \quad (3.26)$$

Then the chiral symmetry is broken into a vector symmetry

$$SU(8)_L \times SU(8)_R \xrightarrow{\Lambda_{TC}} SU(8)_V \quad (3.27)$$

Hence, $8^2 - 1 = 63$ degrees of freedom are lost, resulting in 63 massless pseudoscalar Goldstone bosons (PGBs). These massless PGBs are technipions. We list the 63 PGBs in Table 3.1.

Table 3.1. The 63 PGBs in one-family model. Q' is a colored quark doublet and L is the lepton doublet. λ_a are the Gell-Mann SU(3) matrices and τ^a are the Pauli matrices. The first number in the parentheses is the representation in $SU(3)_c$ and the second one is the representation in $SU(2)_L$.

$$(8, 3) \quad \Theta_a^\alpha \equiv \bar{Q} \gamma_5 \lambda_a \tau^\alpha Q$$

$$(8, 1) \quad P_8^{\alpha\prime} \equiv \bar{Q} \gamma_5 \lambda_a Q$$

$$(\bar{3}, 3) \quad \bar{T}_i^\alpha \equiv \bar{Q}' \gamma_5 \tau^\alpha L$$

$$(\bar{3}, 1) \quad \bar{T}_i \equiv \bar{Q}' \gamma_5 L$$

$$(3, 3) \quad T_i^\alpha \equiv \bar{L} \gamma_5 \tau^\alpha Q'$$

$$(3, 1) \quad T_i \equiv \bar{L} \gamma_5 Q'$$

$$(1, 3) \quad \Pi^\alpha \equiv \bar{Q} \gamma_5 \tau^\alpha Q + \bar{L} \gamma_5 \tau^\alpha L$$

$$(1, 3) \quad \begin{aligned} P^\pm &\equiv \bar{Q} \gamma_5 \tau^\pm Q - 3 \bar{L} \gamma_5 \tau^\pm L \\ P^3 &\equiv \bar{Q} \gamma_5 \tau^3 Q - 3 \bar{L} \gamma_5 \tau^3 L \end{aligned}$$

$$(1, 1) \quad P^{0\prime} \equiv \bar{Q} \gamma_5 Q - 3 \bar{L} \gamma_5 L$$

Figure 3.5. Gauge bosons coupled to Π

The PGBs can couple to the axial technihadronic currents

$$\langle 0 | J_{5a}^\mu | \Pi_b \rangle = F_\Pi q^\mu \delta_{ab} \quad (3.28)$$

where Π denotes technipions. The F_Π are of the same value for all PGBs due to the residue $SU(8)_V$ symmetry. There are appropriate J_5^μ which are coupled to the electroweak gauge fields. Similar to the QCD case, the modified propagators of $W^\pm$, Z , and B develop poles as $q^2 \rightarrow 0$ (see Figure 3.5). The results are

$$M_W = \frac{g}{2} \sqrt{r} F_\Pi \quad (3.29)$$

$$M_Z = \frac{\sqrt{g^2 + g'^2}}{2} \sqrt{r} F_\Pi \quad (3.30)$$

$$M_\gamma = 0, \quad (3.31)$$

where r is the number of doublets; $r = 4$ in one-family model. So F_Π becomes

$$F_\Pi = \frac{2M_W}{\sqrt{r}g} = \frac{250 \text{ GeV}}{\sqrt{r}}. \quad (3.32)$$

This would lower the technicolor scale by $\frac{1}{\sqrt{r}}$. Three PGBs will be identified as the longitudinal parts of W^+ , W^- , and Z . Assume the TC gauge group to be a

SU(N). To estimate the value of Λ_{TC} , we must know the N dependence of the F_{Π} . From the $\frac{1}{N}$ expansion [37], F_{Π} goes $\sqrt{N}$ times the fundamental scale as N gets large. Therefore we change Eq. (3.23) to

$$\frac{\sqrt{N}\Lambda_{TC}}{\sqrt{3}\Lambda_{QCD}} \sim \frac{F_{\Pi}}{f_{\pi}}. \quad (3.33)$$

Hence, we see that the TC scale Λ_{TC} gets smaller as N grows. For one-family in the SU(4) technicolor group, the TC scale would be $\Lambda_{TC} \sim 233$ GeV.

The TC model does not have the kind of Yukawa couplings that in the standard model generate masses for fermions. However, the masses could come from the four-fermion coupling (see Figure 3.6)

$$g_4 \bar{f} f \bar{F} F \quad (3.34)$$

after the $\bar{F}F$ condense. Since the coupling constant g_4 has mass dimension -2, the theory would not be renormalizable. Similar to the Fermi's four-fermion interaction, the coupling (3.34) can be viewed as the low energy effective Lagrangian from the exchange of a massive vector boson (see Figure 3.6). The effective coupling g_4 would be

$$g_4 = g_{ETC}^2 / M_{ETC}^2, \quad (3.35)$$

where g_{ETC} is the coupling constant between ordinary and technifermions. The M_{ETC} are the masses of new gauge bosons. The ETC represents extended technicolor which was introduced to connect the ordinary fermions and technifermions such that fermions acquire masses after TC condensation [38, 39]. The fermions masses have the form

$$m_f = (g_{ETC}^2 / M_{ETC}^2) (\bar{F}F)_0 \sim O(\Lambda_{TC}^3) (g_{ETC}^2 / M_{ETC}^2). \quad (3.36)$$

Since the ordinary fermions have different masses, there are different mass values of the extended gauge bosons

$$M_{ETC_f}^2 = O(\Lambda_{TC}^3)/m_f. \quad (3.37)$$

Because more than one M_{ETC_f} can contribute to the mass of one given fermion, Eq. (3.37) is not exact and we would have mixing among quarks similar to the Kobayashi-Maskawa matrix. Equation (3.37) is only used to give some idea about M_{ETC_f} . The approximate masses of ETC bosons are shown as follows [40]:

$$\begin{aligned} m_u = 6\text{MeV} & \quad M_{ETC_u}^2 \approx 2200\text{TeV}^2 \\ m_d = 10\text{MeV} & \quad M_{ETC_d}^2 \approx 1200\text{TeV}^2 \\ m_s = 200\text{MeV} & \quad M_{ETC_s}^2 \approx 60\text{TeV}^2 \\ m_c = 1.5\text{GeV} & \quad M_{ETC_c}^2 \approx 8\text{TeV}^2 \\ m_b = 5\text{GeV} & \quad M_{ETC_b}^2 \approx 2.4\text{TeV}^2 \\ m_t = 80\text{GeV} & \quad M_{ETC_t}^2 \approx 0.16\text{TeV}^2. \end{aligned}$$

To simplify the ETC, we assume that the ETC group commutes with the electroweak's $SU(2)_L \times U(1)_Y$ and QCD's $SU(3)_c$. The breaking pattern of the ETC is in a sequence of the masses of ETC bosons

$$G_{ETC} \xrightarrow{10^3\text{TeV}^2} G_{E'TC} \xrightarrow{30\text{TeV}^2} G_{E''TC} \xrightarrow{1\text{TeV}^2} G_{TC}. \quad (3.38)$$

We are not worried about what causes the breaking pattern here. To make the ETC group closed, we generally would have ETC interactions among the ordinary fermions. This can be seen as follows. Through ETC interaction an ordinary fermion can transform into a TC fermion and similarly a TC fermion can transform into

Figure 3.6. Two Feynman diagrams
 (a) Four fermion coupling (b) Intermediating a gauge boson

an ordinary one. Combining these two operations results in a transformation of an ordinary fermion into another ordinary fermion

$$[\bar{f}_i F, \bar{F} f_j] = \bar{f}_i f_j. \quad (3.39)$$

These interactions are called "horizontal ETC" (HETC).

The HETC can contribute to the flavor changing neutral current (FCNC). A way to suppress the FCNC from HETC is to assume there are many generations of technifermions. Each TC generation is only coupled to one fermion generation such that

$$[\bar{f}_i F_a, \bar{F}_b f_j] = 0 \quad \text{for } i \neq j, a \neq b, \quad (3.40)$$

where i, j , and a, b are the family indices. This totally eliminates the FCNC through HETC channels but this scheme has problems [41] concerned with light spin-zero bosons [42, 43] and breaking the degeneracy between the different generations. The diagram in Figure 3.7 shows that $K^0 - \bar{K}^0$ mixing arises at the tree level from

Figure 3.7. $K^0 - \bar{K}^0$ mixing from exchanging HETC gauge boson

exchanging HETC bosons [40]. The effective Lagrangian of the diagram reads

$$\mathcal{L}_{\bar{s}d}^{eff} = \{(\bar{s}_L \gamma_\mu d_L)^2, (\bar{s}_L \gamma_\mu d_L)(\bar{s}_R \gamma_\mu d_R), (\bar{s}_R \gamma_\mu d_R)^2\} \left(\frac{\theta^2}{M_{Ed}^2}, \frac{\theta^2}{M_{Es}^2} \right), \quad (3.41)$$

where the θ are Cabibbo-like angle factors of HETC. The subscript E is the short hand for ETC . The present data on $K^0 - \bar{K}^0$ mixing give the lower bound on M_{Ed}^2 and M_{Es}^2

$$M_{Ed}^2, M_{Es}^2 \gtrsim 5 \times 10^4 \text{ TeV}^2, \quad (3.42)$$

which is much larger than the value needed to generate masses for the u and d quarks. A similar situation also occurs when we examine the $D^0 - \bar{D}^0$ mixing which gives the lower bound

$$M_{Eu}^2, M_{Ec}^2 \gtrsim 800 \text{ TeV}^2. \quad (3.43)$$

The 800 TeV^2 is two orders of magnitude of the desired value 8 TeV^2 estimated before. This is a serious problem and we will come back to this topic later.

In standard model, Higgs boson couples to fermions with coupling,

$$g_{H\bar{f}f} = \frac{m_f}{v}. \quad (3.44)$$

Similar to Higgs boson, the naive coupling of neutral PGBs to fermions is

$$g_{P\bar{f}f} \sim \frac{m_f}{\Lambda_{TC}} \sim \frac{m_f}{F_\Pi} \sim \frac{m_f}{gM_W} \quad (3.45)$$

So the neutral PGBs can contribute to the FCNC at the tree level. The data on the $K^0 - \bar{K}^0$ mixing give the lower bound

$$m_P \gtrsim 6000 \theta \text{ GeV}, \quad (3.46)$$

where the θ is Cabibbo-like angle. Equation (3.46) is the most troublesome for the light neutral PGBs. However, there is a way out. In the standard model, the Higgs bosons do not contribute to the FCNC because the same charge fermions, e.g., (u, c, t), are coupled to one Higgs doublet. When we diagonalize the mass matrix, we simultaneously diagonalize the couplings between the Higgs bosons and fermions. It was proposed by Ellis et al. [44] (monophagic model) that fermions of the same charge get their masses from a single technifermion condensate. This makes the FCNC vanish at the tree level of a single PGB exchange diagram.

The neutral PGBs are also coupled to fermions from the effective four-fermion interactions

$$(\bar{f}_L \gamma_\mu f_L) \left(\frac{g_{ETC}^2}{M_{ETC}^2} \right) (\bar{F}_L \gamma_\mu f_L). \quad (3.47)$$

Making the Fierz transformation on (3.47) we have

$$(\bar{f}_L \gamma_\mu f_L) \left(\frac{g_{ETC}^2}{M_{ETC}^2} \right) (\bar{F}_L \gamma_\mu f_L) + (L \leftrightarrow R). \quad (3.48)$$

The neutral PGBs are coupled to $\bar{F}_L \gamma_\mu f_L$ with magnitude $ip_\mu F_\Pi$. Equation (3.48) becomes

$$i(\bar{f}_L \gamma_\mu f_L) \left(\frac{g_{ETC}^2}{M_{ETC}^2} \right) p_\mu F_\Pi P. \quad (3.49)$$

The coupling reads

$$g'_{Pff} = m_f \left(\frac{g_{ETC}^2}{M_{ETC}^2} \right) F_{\Pi} = \frac{m_f}{g M_W} \left(\frac{g F_{\Pi} \sqrt{r}}{2 M_W} \right) \left(\frac{2 g_{ETC}^2 M_W^2}{\sqrt{r} M_{ETC}^2} \right). \quad (3.50)$$

Comparing with (3.45), this new coupling is suppressed by a factor M_W^2/M_{ETC}^2 . Recently, the slowly running coupling TC [45] and the fixed point TC [46] have been proposed to enhance the condensates, $\langle \bar{F}F \rangle_0$. Since

$$m_f = \left(\frac{g_{ETC}^2}{M_{ETC}^2} \right) \langle \bar{F}F \rangle_0, \quad (3.51)$$

for fixed m_f , the larger $\langle \bar{F}F \rangle_0$ can suppress the value of g_{ETC}^2/M_{ETC}^2 . This, in turn, suppresses the FCNC from ETC and g'_{Pff} .

The 63 massless PGBs can acquire masses from QCD, electroweak, and ETC interactions which explicitly break some of the spontaneously broken symmetries. The degeneracy of the vacuum states is broken after we include all interactions in the model besides the TC [43]. There is less degeneracy of the vacuum states and some PGBs obtain masses. In the spontaneously broken symmetry, we have

$$e^{-iQ_a^A \Lambda_a} |\Omega_0\rangle = |\Omega(\Lambda)\rangle \neq 0, \quad (3.52)$$

where Q_a^A are the broken axial charges [33]. By adding a small perturbation, δH_I , we define an energy $E(\Lambda_a)$ by

$$E(\Lambda_a) \equiv \langle \Omega_0 | e^{-iQ_a^A \Lambda_a} \delta H_I e^{iQ_a^A \Lambda_a} | \Omega_0 \rangle. \quad (3.53)$$

If the subset Q_{UB}^A of Q^A are unbroken by δH_I , the new physical vacuum states will be

$$|\Omega_{phy}(\Lambda)\rangle = e^{iQ_{UB}^A \Lambda_a} |\Omega_0\rangle. \quad (3.54)$$

Rewrite E in terms of the physical vacuum states

$$E(\Lambda') \equiv \langle \Omega_{phy} | e^{-iQ_{Ba}^A \Lambda'_a \delta H_I} e^{iQ_{Ba}^A \Lambda'_a} | \Omega_{phy} \rangle, \quad (3.55)$$

where Q_B^A are the explicitly broken charges. We then have equations

$$\left. \frac{\partial E}{\partial \Lambda'_a} \right|_{\Lambda'_a=0} = 0 \Rightarrow \langle \Omega_{phy} | [Q_{Ba}, \delta H_I] | \Omega_{phy} \rangle = 0 \quad (3.56)$$

and

$$\left. \frac{\partial^2 E}{\partial \Lambda'_a \partial \Lambda'_b} \right|_{\Lambda'_{a,b}=0} = M_{ab}^2 \Rightarrow \langle \Omega_{phy} | [Q_{Ba}, [\delta H_I, Q_{Bb}]] | \Omega_{phy} \rangle = M_{ab}^2. \quad (3.57)$$

From Dashen's formula [47], the mass squared matrix m_{ab}^2 of PGBs is related to M_{ab}^2

$$m_{ab}^2 = M_{ab}^2 / F_{\Pi}^2. \quad (3.58)$$

The masses of PGBs have been calculated [43] and are listed in Table 3.2. Note that the neutral color singlets, $P^{0'}$ and P^3 , are still massless since their associated symmetries are not explicitly broken by QCD and electromagnetism.

It has been shown that the Pati-Salam $SU(4)$ symmetry [48] on multiplets,

$$(U^r, U^b, U^y, N)^T \quad \text{and} \quad (D^r, D^b, D^y, E)^T, \quad (3.59)$$

can generate about 10 to 40 GeV to the masses of $P^{0',3}$ [39]. The four-fermion interactions which give masses to the $P^{0',3}$ and $P^\pm$ are

$$\frac{g_{ETC}^2}{M_{ETC}^2} [\bar{Q} Q \bar{L} L - \bar{Q} \gamma_5 \tau^a Q \bar{L} \gamma_5 \tau^a L], \quad (3.60)$$

which is $SU(3)_C \times SU(2)_L \times U(1)_Y \times TC$ invariant. Here Q is $(U^i, D^i)^T$ and L is $(N, E)^T$. These four-fermion interactions violate separate techniquark and technilepton chiral symmetries, and generate masses for $P^{0',3}$ and $P^\pm$

$$m_P = a \cdot g_{ETC} \langle \bar{F} F \rangle_0 / (F_{\Pi} M_{ETC})$$

Table 3.2. The masses of PGBs

$PGBs$	$Mass$
$\Theta_a^\alpha, P_8^{(1)}$	$\approx 245 \text{ GeV}$
$T_i^\alpha, T_i, \bar{T}_i^\alpha, \bar{T}_i$	$\approx 160 \text{ GeV}$
$P^\pm$	$O(10 \text{ GeV})$
$P^{(1)}, P^3$	0

$$= \frac{a \cdot M_{ETC}/g_{ETC}}{F_\Pi} m_f \quad (3.61)$$

where a is a coefficient of order unity. The slowly running coupling TC and the fixed point TC can enhance the vev of condensate. In the scheme of slowly running coupling TC, Appelquist et al. [45] estimated the masses of PGBs from the four fermion interactions,

$$m_P = a \cdot (50 - 70 \text{ GeV}), \quad (3.62)$$

where a is $O(1)$. This can eliminate the embarrassing light PGBs and suppress the FCNC from exchanging PGBs (see Eq. (3.46)). Appelquist, Lane, and Mahanta [49] has shown that in the next leading order of the ladder approximation to the Schwinger-Dyson gap equation, which is used to calculate the vev of condensate, there is a 1%-20% correction to the first order depending on the fermion representation. They assert that the ladder approximation may be a good method to handle the gap equation. In other words, the scenarios of the slowly running coupling TC or the fixed point TC may be realistic.

It seems that the large FCNC in TC can be tamed by the slowly running coupling TC or the fixed point TC. A direct evidence of TC is the discovery of PGBs with masses on the order of hundreds of GeV. In the next chapter, We will focus on the production of neutral PGBs (or technipions), $P^{0'}$ and $P_8^{0'}$, at the SSC.

4. THE PRODUCTION OF NEUTRAL TECHNIPIONS, $P^{0'}$ AND $P_8^{0'}$

The fundamental scalar fields in the standard model cause the hierarchy problem and also have not yet been discovered. Searching for extensions of the standard model at the least leads us two alternatives, the supersymmetry and technicolor (TC), to solve the hierarchy problem. In this chapter, we will discuss the signals of TC from neutral technipion production through

$$p + p \rightarrow g(q) + P^{0'}(P_8^{0'}) + X \quad (4.1)$$

at the SSC.

4.1. The Motivations

A crucial of the signal of TC below $\Lambda_{TC} \sim O(\text{TeV})$ is the existence of hadron-like heavy pseudoscalar particles, i.e., the PGBs. The masses of PGBs are model-dependent, but most model estimates give mass ranging from the order of 10 GeV to a few hundred GeV [43, 39, 45]. If TC is indeed responsible for the spontaneous symmetry breaking of the electroweak interactions, many PGBs should be produced in the energy range of hundreds of GeV. The direct production of $P^{0'}$ and $P_8^{0'}$ at the SSC have been investigated in Eichten et al. (EHLQ) [22]. The signals that EHLQ

looked into are the two-body final states

$$p + p \rightarrow P^{0'}(P_8^{0'}) + X \quad (4.2)$$

and

$$P^{0'}(P_8^{0'}) \rightarrow gg, Q\bar{Q} \quad (4.3)$$

$$P^{0'} \rightarrow \tau\bar{\tau}, \quad (4.4)$$

where Q is a heavy quark such as the b and t . The decay channel of two gluon jets for $P^{0'}$ is hard to detect due to a very large QCD background of continuous mass spectrum of two jets. The background of $P^{0'}$ from $b\bar{b}$ pairs is 2 orders of magnitude larger than the corresponding signal. The signal and the background of $\tau\bar{\tau}$ for $P^{0'}$ are approximately of the same order. EHLQ further pointed out that the efficiency on measuring the invariant mass of the $\tau\bar{\tau}$ pair is poor so that it is hard to use this signal. The background of gg for $P_8^{0'}$ is much larger than the signal. However the $t\bar{t}$ signal for $P_8^{0'}$ is comparable to the background. Hence, EHLQ concluded that the useful signals are $\tau\bar{\tau}$ for $P^{0'}$ and $t\bar{t}$ for $P_8^{0'}$, but we need high efficiency in identifying particles and high precision in measuring invariant mass of pairs. It has been shown by D. McKay et al. [23] that three-body decay modes are significant for heavy $P^{0'}$. Thus, we expect $P_8^{0'}$ would be similar to $P^{0'}$. The three-gluon ($3g$) and two-gluon modes dominate the decay of $P^{0'}$ when $M_P^{0'}$ is greater than 50 GeV but less than the $t\bar{t}$ threshold. Once the $P^{0'}$ mass is large enough to make $t\bar{t}$ decay, $t\bar{t}$ channel dominates the decay. The $q\bar{q}g$ channel also has a significant contribution. So for large $P^{0'}$ mass, the signal to look for is $t\bar{t}$ if it is allowed. More details on the decays of $P^{0'}$ and $P_8^{0'}$ are given in the next chapter.

Going beyond what has been considered in EHLQ, we will investigate other production processes of $P^{0'}$ and $P_8^{0'}$, i.e.,

$$p + p \rightarrow g(q) + P^{0'}(P_8^{0'}) + X \quad (4.5)$$

so that $P^{0'}$ and $P_8^{0'}$ can be produced in high transverse momentum ($P_\perp$). In EHLQ, $P^{0'}$ and $P_8^{0'}$ are produced at zero $P_\perp$. For high $P_\perp$, the recoiled parton jet can serve as a tagging jet to facilitate the identification of the PGBs. In the next section, we will address a technique used to simplify the calculations involving multi-gluons.

4.2. Multi-Gluon Processes and the Slavnov-Taylor Identity

The calculations of gluon processes are generally very complicated, even for simple tree diagrams. The complication is caused by two factors due to the trilinear and quartic couplings: (1) there are many diagrams, and (2) the amplitude satisfies the Slavnov-Taylor identity (STI) [50] rather than the simpler Ward-Takahashi identity (WTI) [51] of the Abelian case.

For a massless vector particle, the polarization projection is given by

$$\sum_{\lambda=1}^2 \epsilon_\mu^*(\lambda, k) \epsilon_\nu(\lambda, k) = -g_{\mu\nu} + (k_\mu n_\nu + k_\nu n_\mu)/k \cdot n + n^2 k_\mu k_\nu / (k \cdot n)^2, \quad (4.6)$$

where n is an arbitrary vector orthogonal to ϵ , $\epsilon \cdot n = 0$. The use of (4.6), in general, gives rise to a large number of terms in the intermediate steps of a calculation. In the Abelian case, e.g., QED, since the WTI is satisfied, we can drop the k_μ dependent terms in (4.6). The polarization projection is reduced to

$$\sum_{\lambda=1}^2 \epsilon_\mu^*(\lambda, k) \epsilon_\nu(\lambda, k) = -g_{\mu\nu} \quad (4.7)$$

which simplifies the calculation enormously.

In QCD, the use of (4.6) can be avoided and replaced by (4.7). However, this requires the addition of ghost diagrams in order to compensate the scalar and longitudinal gluon contributions, where the ghost diagrams are obtained from the original amplitude by replacing, in turn, each pair of external gluon lines by a pair of ghost lines. Some internal gluon lines can also be replaced by ghost lines if they are not connected to lines other than gluon lines.

It was observed in Ref. [52] in the calculation of the cross section of $g + g \rightarrow q + \bar{q}$ that dropping of the terms of the amplitude which vanish due to the transversality condition $k \cdot \epsilon = 0$ allows the use of (4.7) without the compensating ghost diagrams.

We will give a general proof that the transversality condition when incorporated in the Feynman amplitude of a multi-gluon process makes the reduced amplitude satisfy the WTI and, therefore, (4.7) can be used for the cross section calculation without the compensating ghost diagrams. This technique applies to QCD processes as well to processes containing non-QCD couplings.

Consider the Feynman amplitude of a process with n gluons

$$T_{\mu_1 \mu_2 \dots \mu_n}(k_1, k_2, \dots, k_n; q_1, q_2, \dots, q_f), \quad (4.8)$$

where $k_1, \dots, k_n$ with the Lorentz indices $\mu_1, \dots, \mu_n$, respectively, represent the external gluon momenta; $q_1, \dots, q_f$ are the momenta of the external particles other than the gluons. The amplitude satisfies the STI

$$\begin{aligned} k_1^{\mu_1} T_{\mu_1 \mu_2 \dots \mu_n} &= k_2^{\mu_2} S_{\mu_3 \dots \mu_n}^{(12)} + k_3^{\mu_3} S_{\mu_2 \mu_4 \dots \mu_n}^{(13)} + \\ &= \dots + k_n^{\mu_n} S_{\mu_2 \dots \mu_{n-1}}^{(1n)}, \end{aligned} \quad (4.9)$$

where $S_{\mu_3 \dots \mu_n}^{(12)}$, etc., are the ghost terms obtained from (4.8) by replacing the gluon

lines of momenta k_1 and k_2 , etc., with ghost lines of the corresponding momenta and color indices. In the calculation of the cross section with (4.8), one uses either (4.6), or (4.7) together with the ghost diagrams.

Note the structure of (4.9): Each ghost term is associated with a definite momentum factor which is the momentum of the replaced gluon other than the contracting momentum. This suggests a way to obtain a modified amplitude which satisfies the WTI by eliminating the ghost terms: drop all the terms in (4.8) which are proportional to $k_{i\mu_i}$, $i = 1, 2, \dots, n$, by the transversality condition $k \cdot \epsilon = 0$. Denote the modified amplitude by

$$\tilde{T}_{\mu_1 \mu_2 \dots \mu_n}. \quad (4.10)$$

Hence, the amplitudes (4.8) and (4.10) give the same cross section. We have two cases to consider: with fermions ($f \neq 0$) and without fermions ($f = 0$).

First consider the case $f \neq 0$. We can choose $k_1, k_2, \dots, k_n, q_1, \dots, q_{f-1}$ to be the independent momentum set. Consider the contraction of (4.10) with $k_1^{\mu_1}$. The only possibility that the $k_{2\mu_2}, \dots, k_{n\mu_n}$ terms may be regenerated is the presence of terms in (4.10) which are proportional to $g_{\mu_1 \mu_i}$, $i = 2, \dots, n$. We write

$$\begin{aligned} \tilde{T}_{\mu_1 \mu_2 \dots \mu_n} = & g_{\mu_1 \mu_2} f_{\mu_3 \dots \mu_n}^{(12)} + g_{\mu_1 \mu_3} f_{\mu_2 \mu_4 \dots \mu_n}^{(13)} + \\ & \dots + g_{\mu_1 \mu_n} f_{\mu_2 \dots \mu_{n-1}}^{(1n)} + \text{other terms}. \end{aligned} \quad (4.11)$$

Then

$$\begin{aligned} k_1^{\mu_1} \tilde{T}_{\mu_1 \mu_2 \dots \mu_n} = & k_{1\mu_2} f_{\mu_3 \dots \mu_n}^{(12)} + k_{1\mu_3} f_{\mu_2 \mu_4 \dots \mu_n}^{(13)} + \\ & \dots + k_{1\mu_n} f_{\mu_2 \dots \mu_{n-1}}^{(1n)} + \text{other terms}. \end{aligned} \quad (4.12)$$

Since k_1 is independent of $k_2, \dots$ and k_n , no terms proportional to $k_{2\mu_2}, \dots, k_{n\mu_n}$ can appear in (4.12). A similar argument holds for the contraction of $\tilde{T}_{\mu_1 \dots \mu_n}$ with

$k_2^{\mu_2}, \dots, k_n^{\mu_n}$. Hence, we have

$$(k_1^{\mu_1}, k_2^{\mu_2}, \dots, k_n^{\mu_n}) \tilde{T}_{\mu_1 \mu_2 \dots \mu_n} = 0. \quad (4.13)$$

Thus we can use (4.7) for all the gluons when the modified amplitude (4.10) is used in the cross section calculation.

The $f = 0$ case, i.e., all the external lines are gluon lines, is different since only $n-1$ of the gluon momenta can be linearly independent. Take the first $n-1$ momenta to be linearly independent. Again, the terms proportional to $k_1^{\mu_1}, k_2^{\mu_2}, \dots, k_{n-1}^{\mu_{n-1}}$ do not appear in an expression similar to (4.12). But a term which is proportional to $(k_1 + k_2 + \dots + k_{n-1})^{\mu_n} = -k_n^{\mu_n}$ can appear. Therefore, the ghost term associated with the k_n gluon line cannot be eliminated. In order to eliminate this ghost term, we define another amplitude

$$\bar{T}_{\mu_1 \mu_2 \dots \mu_{n-1}} = \tilde{T}_{\mu_1 \mu_2 \dots \mu_n} \epsilon_n^{\mu_n}, \quad (4.14)$$

where ϵ_n is the polarization vector of n th external gluon. Equation (4.14) satisfies the WTI with respect to the first $n-1$ gluon momenta

$$(k_1^{\mu_1}, k_2^{\mu_2}, \dots, k_{n-1}^{\mu_{n-1}}) \bar{T}_{\mu_1 \mu_2 \dots \mu_{n-1}} = 0. \quad (4.15)$$

In the cross section calculation with (4.14), we can use (4.7) for the first $n-1$ gluons and (4.6) for the last gluon.

Equations (4.13) and (4.15) hold to all orders in the perturbation expansion as long as the STI in the form of (4.9) holds. A technical point to be noted is that, to obtain (4.10), it is necessary to expand the trilinear vertices in order to eliminate all the terms proportional to $k_i^{\mu_i}$, if there are two or more adjacent trilinear vertices.

Two other powerful methods have been proposed in the literature for the calculation of multi-gluon processes involving massless quarks in tree diagrams:

(a) The use of helicity amplitudes instead of tensor amplitudes is proposed in Ref. [53] for the calculation of multi-photon processes. Generalization to non-Abelian cases is given in Ref. [54]. This method modifies the polarization vector of the gauge boson. It is suitable for processes involving massless fermions which interact with gauge bosons in definite helicity states and where the axial vector current conservation holds.

(b) An extension of helicity amplitudes which imbeds QCD in a $N = 2$ supersymmetry has been proposed in Ref. [55]. In this approach, some of the external gluon lines are replaced by scalar lines and therefore, this greatly simplifies cross section calculations. This technique also relies on the chiral separation of the interaction of gauge bosons with massless fermions.

These methods are very useful only in a process with massless fermions. In particular, the technique (b) relies on the supersymmetrization of the theory under consideration. It is not clear how to apply (b) straightforwardly to an effective theory such as the technicolor's low energy effective Lagrangian. However, the present method [56] depends only on the gauge invariance of the theory, i.e., the WT identity. It applies to processes involving massive fermions and in principle to high order processes.

4.3. The Production of $P^{0'}$

The production of $P^{0'}$ from the processes

$$g(p_1) + g(p_2) \rightarrow P^{0'}(p_3) + g(p_4) \quad (4.16)$$

$$g(p_1) + q(p_2) \rightarrow P^{0'}(p_3) + q(p_4) \quad (4.17)$$

has been calculated by D. McKay, et al. [23], for light $P^{0'}$ with a mass of the order of 10 GeV. Because the light $P^{0'}$ suffers from the large FCNC, the enhancement of TC condensates in the slowly running coupling TC or in the fixed point TC can raise the mass of $P^{0'}$ to an order of hundreds of GeV, so as to suppress the FCNC (see Chapter 3, Section 3). Thus, we recalculate the production rate for $m_P = 100$ to 400 GeV. We ignore the production from $q\bar{q}$ fusion

$$q(p_1) + \bar{q}(p_2) \rightarrow P^{0'}(p_3) + g(p_4) \quad (4.18)$$

because the contribution from this process is negligible compared to the processes (4.16) and (4.17) in high energy p-p colliders such as the SSC.

The vertices of $P^{0'}$ coupling to gluons are shown in Appendix. The amplitude squares of (4.16) and (4.17) after averaging over the spins and colors of the initial state and summing the spins and colors of the final state are given by

$$\begin{aligned} \overline{\mathcal{M}}(gg \rightarrow P^{0'} g) &= \frac{\alpha_s^3(Q^2)}{\pi F_{\Pi}^2 \hat{s}} \\ &\left\{ m_P^2 \left[1 + m_P^2 \left(\frac{1}{\hat{s}} + \frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right) + \frac{\hat{s}^2}{\hat{u}\hat{t}} + \frac{\hat{u}^2}{\hat{s}\hat{t}} + \frac{\hat{t}^2}{\hat{u}\hat{s}} \right] + \right. \\ &\quad \left. \frac{\hat{u}\hat{t}}{\hat{s}} + \frac{\hat{t}\hat{s}}{\hat{u}} + \frac{\hat{u}\hat{s}}{\hat{t}} \right\} \end{aligned} \quad (4.19)$$

and

$$\begin{aligned} \overline{\mathcal{M}}(gq \rightarrow P^{0'} q) &= \frac{2\alpha_s^3(Q^2)}{9\pi F_{\Pi}^2 \hat{s}\hat{t}^2} \\ &\left\{ 2m_P^2 \hat{s}\hat{u} + (\hat{s} + \hat{u}) \left[\hat{s}^2 + \hat{u}^2 - m_P^2 (\hat{s} + \hat{u}) \right] \right\}, \end{aligned} \quad (4.20)$$

Figure 4.1. Feynman diagrams for $gg \rightarrow P^{0'}g$

where

$$\hat{s} = (p_1 + p_2)^2$$

$$\hat{t} = (p_1 - p_3)^2$$

$$\hat{u} = (p_1 - p_4)^2.$$

α_s is the strong coupling constant and is a function of interaction energy Q . We neglect all fermions- $P^{0'}(P_8^{0'})$ vertices since they are proportional to $\frac{m_f}{F_\Pi}$ and are small for light fermions. The Feynman diagrams for (4.16) and (4.17) are shown in Figures 4.1 and 4.2. The numerical results will be presented in Section 4.5.

4.4. The Production of $P_8^{0'}$

Similarly to $P^{0'}$, we focus on two modes,

$$g(p_1) + g(p_2) \rightarrow P_8^{0'}(p_3) + g(p_4) \quad (4.21)$$

$$g(p_1) + q(p_2) \rightarrow P_8^{0'}(p_3) + q(p_4) \quad (4.22)$$

Figure 4.2. Feynman diagrams for $gq \rightarrow P''q$

The $q\bar{q}$ mode is again ignored because it is small as compared to (4.21) and (4.22). The mode (4.22) has three diagrams, shown in Figure 4.3. See Appendix for the vertices of $P_8^{0'}$ coupling to gluons. We only calculate diagram (c) since we neglect the PGB-fermion vertex. The averaged amplitude square of (4.22) is

$$\overline{\mathcal{M}}(gq \rightarrow P_8^{0'} q) = \frac{20\alpha_s^3(Q^2)}{9\pi F_\Pi^2 \hat{t}^2} \times \left\{ (\hat{s} + \hat{u}) \left[(\hat{s}^2 + \hat{u}^2 - m_P^2(\hat{s} + \hat{u})) \right] + 2m_P^2 \hat{s} \hat{u} \right\} \quad (4.23)$$

The differences between the gq production of $P^{0'}$ and $P_8^{0'}$ are the color factor and PGB-gluon vertex (see Appendix). The $P_8^{0'}$ is enhanced by a factor 10 ($\frac{10}{6} \times 6 = 10$).

The $P_8^{0'}$ production, through gluon fusion (4.21), has 7 Feynman diagrams which are shown in Figure 4.4. To avoid the complicated polarization projection of gluon in (4.6) or the compensating ghost diagrams in using (4.7), we use the modified amplitude discussed in the preceding section. The final result is

$$\overline{\mathcal{M}}(gg \rightarrow P_8^{0'} g) = \frac{320\alpha_s^3(Q^2)}{\pi F_\Pi^2} \frac{1}{\hat{s}^2} \times$$

$$\begin{aligned}
& \left\{ -2m_P^4 \left(\frac{\hat{s}}{\hat{s} - m_P^2} + \frac{\hat{t}}{\hat{t} - m_P^2} + \frac{\hat{u}}{\hat{u} - m_P^2} \right) - \right. \\
& 2m_P^2 \frac{\hat{s}\hat{t}\hat{u}}{(\hat{s} - m_P^2)(\hat{t} - m_P^2)(\hat{u} - m_P^2)} - \\
& 3m_P^2 + \frac{2}{\hat{s}\hat{t}\hat{u}} [(\hat{s}\hat{t})^2 + (\hat{u}\hat{s})^2 + (\hat{u}\hat{t})^2 + \\
& \left. m_P^2(\hat{s}^3 + \hat{t}^3 + \hat{u}^3) + m_P^4(\hat{s}\hat{t} + \hat{t}\hat{u} + \hat{s}\hat{u}) \right] \Big\}. \quad (4.24)
\end{aligned}$$

The pole terms in Eq. (4.24) do not cause any trouble in the kinematic region of our concern, since none of the variables, $\hat{s}$, $\hat{t}$, and $\hat{u}$, will be near the resonance $P_8^{0'}$ and zero. The next section presents some numerical results of the differential cross section.

4.5. The Numerical Results

The measurable cross section is a convolution with the parton distribution functions and is given by

$$\left. \frac{d\sigma}{dP_\perp dy} \right|_{y=0} = 2P_\perp \sum_{ij} \int_{x_{\min}}^1 dx_a \frac{f_i^a(x_a, Q^2) f_j^b(x_b, Q^2) \hat{\sigma}_{ij}(\hat{s}, \hat{t}, \hat{u})}{x_a s + u - m_P^2}, \quad (4.25)$$

for fixed transverse momentum and rapidity of $P^{0'}$ and $P_8^{0'}$, where

$$\hat{\sigma} \equiv \hat{s} \frac{d\sigma}{d\hat{t}} = \frac{1}{16\pi\hat{s}} \overline{\mathcal{M}} \quad (4.26)$$

$$\hat{t} = -\sqrt{s}m_\perp e^{-y} + m_P^2 \quad (4.27)$$

$$\hat{u} = -\sqrt{s}m_\perp e^y + m_P^2 \quad (4.28)$$

$$\hat{s} = x_a x_b s \quad (4.29)$$

$$\hat{t} = -\sqrt{s}x_a m_\perp e^{-y} + m_P^2 \quad (4.30)$$

$$\hat{u} = -\sqrt{s}x_b m_\perp e^y + m_P^2 \quad (4.31)$$

Figure 4.3. Feynman diagrams for $gg \rightarrow P_R^{(1)} q$

Figure 4.4. Feynman diagrams for $gg \rightarrow P_N^{II'} g$

with

$$m_{\perp}^2 = P_{\perp}^2 + m_P^2 \quad (4.32)$$

$$x_b = \frac{-x_a t - (1 - x_a)m_P^2}{x_a s + u - m_P^2} \quad (4.33)$$

$$x_{min} = \frac{-u}{s + t - m_P^2} \quad (4.34)$$

$$y = \frac{1}{2} \ln \left(\frac{E_3 + p_{3l}}{E_3 - p_{3l}} \right). \quad (4.35)$$

The function f is the parton distribution function and i, j denote the initial partons. We will use the EHLQ's set 2 parton distribution functions (PDF) [22] in our calculation.

The running coupling constant of the strong interactions $\alpha_s(Q^2)$ in the matrix elements (4.19), (4.20), (4.23), and (4.24), is usually expressed as

$$\alpha_s(Q^2) \equiv \left(\frac{33 - 2N_f}{12\pi} f \ln(Q^2/\Lambda^2) \right)^{-1} \quad (4.36)$$

where Λ is QCD scale and N_f is the number of quark flavors. Because the values of Λ have been determined for $N_f = 4$, it is more suitable to use the following expression [22]

$$\alpha_s \equiv \left(\frac{25}{12\pi} \ln(Q^2/\Lambda^2) - \frac{1}{6\pi} \sum_{i=b,t,\dots} \theta(Q^2 - 16m_i^2) \ln(Q^2/16m_i^2) \right)^{-1}, \quad (4.37)$$

EHLQ gives two sets of parametrizations with different values for Λ , one set has $\Lambda = 200$ MeV, the other $\Lambda = 290$ MeV. In expression (4.37), the contributions of all quarks become equal when Q^2 approaches infinity. Since the decay branching ratios of $P^{0'}$ and of $P_8^{0'}$ depend on $\alpha_s(Q^2)$, the ratio of the signal for the production of $P^{0'}$ or $P_8^{0'}$ to the background also depends on $\alpha_s(Q^2)$. Therefore, the choices

of different expressions of $\alpha_s(Q^2)$ will affect this latter ratio. However, the changes are not significant if we consider all uncertainties in the theory. We will adopt the expression (4.37); Q^2 is set to $P_\perp^2$ to characterize the interaction energy. The range of α_s used in this section is from 0.135 at $Q = 100$ GeV to 0.093 at $Q = 1500$ GeV for $m_t = 80$ GeV and $m_b = 5$ GeV.

Figure 4.5 shows that the gg fusion dominates the production. The gq fusion is gradually close to the gg fusion in higher $P_\perp$ because the parton distribution functions favor the quark when the value of x is not so small. We present the differential cross section of $P^{0'}$ in Figure 4.6 for various values of m_P , $m_P = 50, 100, 200, 300$ and 400 GeV. The production rates decrease from $\sim 10^{-3}$ nb/GeV to $\sim 10^{-6}$ nb/GeV as $P_\perp$ increases from 100 GeV to 1500 GeV. There are not much changes caused by the variation of m_P . Similar plot for $P_8^{0'}$ is shown in Figure 4.7. The rate of $P_8^{0'}$ is roughly an order of magnitude larger than the rate of $P^{0'}$. The main reason for the larger $P_8^{0'}$ production is due to QCD color enhancement. Apart from the production rate, the features of the production cross section of $P_8^{0'}$ are as same as those of $P^{0'}$.

The projected luminosity of the SSC is $\sim 3 \times 10^7$ nb $^{-1}$ year $^{-1}$ [57]. With a mass of a few hundred GeV, 10^6 to 10^3 $P_8^{0'}$ bosons can be produced at the SSC per year per one GeV bin of $P_\perp$ for $y = 0$ and $P_\perp = 100$ to 700 GeV. The production of $P^{0'}$ is about one order of magnitude less. Tables 4.1 and 4.2 show expecting numbers of $P^{0'}$ and $P_8^{0'}$ produced at the SSC per GeV bin of $P_\perp$ per year at $y = 0$. The numbers are quite large. Are we able to see them at the SSC if they do exist? It depends how much the background would be. We will address that in the next two chapters.

Table 4.1. The number of $P^{0'}$ bosons produced at the SSC per GeV bin of $P_{\perp}$ per year at $y = 0$

$P_{\perp}$ GeV	$m_P =$	50	100	200	300	400	GeV
100		1.4×10^5	1.1×10^5	7.2×10^4	5.4×10^4	4.2×10^4	
300		6.6×10^3	6.3×10^3	5.4×10^3	4.5×10^3	3.9×10^3	
500		1.3×10^3	1.3×10^3	1.2×10^3	1.1×10^3	9.9×10^2	
700		4.2×10^2	4.2×10^2	3.9×10^2	3.9×10^2	3.6×10^2	

Table 4.2. The number of $P_g^{0'}$ bosons produced at the SSC per GeV bin of $P_{\perp}$ per year at $y = 0$

$P_{\perp}$ GeV	$m_P =$	100	200	300	400	GeV
100		1.0×10^6	6.6×10^5	4.8×10^5	3.9×10^5	
300		6.0×10^4	5.1×10^4	4.2×10^4	3.6×10^4	
500		1.3×10^4	1.2×10^4	1.1×10^4	9.3×10^3	
700		4.2×10^3	3.9×10^3	3.6×10^3	3.3×10^3

Figure 4.5. Rate for $P^{0'}$ of $m_{P^{0'}} = 300$ GeV. Total rate (solid line), gg fusion (dash line), and gq fusion (dotted line)

Figure 4.6. Rates for P'^0 of different masses. From high to low, each line represents $m_{P'} = 50, 100, 200, 300$, and 400 GeV, respectively

Figure 4.7. Rates for $P_{\perp}'$ of different masses. From high to low, each line represents $m_{P'} = 100, 200, 300$, and 400 GeV, respectively

5. THE DECAY OF NEUTRAL TECHNIPIONS, $P^{0'}$ and $P_8^{0'}$

We have discussed the production rates of $P^{0'}$ and $P_8^{0'}$ in the previous chapter. The signals of their existence are from the observation of their decay particles. So here we will discuss the major tree level two-body and three-body decay channels of $P_8^{0'}$. D. McKay et al. have done the same thing for $P^{0'}$ [23]. However, their cutoffs on three-body decay channels to avoid the infrared divergences are done separately on each Feynman diagram. Here we use a different approach. We first sum up all the calculated diagrams and then impose a cutoff on the invariant masses of two-gluon ($2g$) pairs and quark-gluon pairs in three-body decay modes. The numerical results for different masses of $P^{0'}$ and $P_8^{0'}$ and for two values of top quark mass, $m_t = 80$ and 120 GeV, will be presented.

5.1. The Decay of $P^{0'}$

We examine the major decay modes of $P^{0'}$ at the tree level. Our result will not be exact because we do not include loop diagrams. The loop diagrams can modify the width and will cancel the infrared divergence present in some of the three-body modes. Therefore, our results will be approximate results. However, the approximation is reliable in making an order of magnitude estimation. We eliminate the infrared divergences by using an invariant mass cutoff on two-gluon and quark-gluon pairs.

$P^{0'}$ can decay into a pair of gauge bosons, B_1 and B_2 . The amplitude is [42, 58, 44]

$$\mathcal{A}_{PB_1B_2} = \frac{S_{PB_1B_2}}{8\pi^2\sqrt{2}F_\Pi} \varepsilon_{\mu\nu\lambda\rho} \varepsilon_1^\mu \varepsilon_2^\nu p_1^\lambda p_2^\rho, \quad (5.1)$$

where $S_{PB_1B_2}$ is determined from a triangle (anomaly) graph similar to the graph for the decay $\pi^0 \rightarrow \gamma\gamma$ and can be found in Table 5.1 [22]. The decay rates are

$$\Gamma_{B_1B_2}(P^{0'}) = (1 + \delta_{B_1B_2}) \frac{m_P^3}{32\pi} \left[\frac{S_{PB_1B_2}}{8\pi^2\sqrt{2}F_\Pi} \right]^2, \quad (5.2)$$

when the product masses are negligible. From Eq. (5.2), we have

$$\frac{\Gamma_{\gamma\gamma}}{\Gamma_{gg}} = \left(\frac{1}{36} \right) \left(\frac{\alpha_{em}}{\alpha_s} \right)^2 \approx 1.5 \times 10^{-4} \quad (5.3)$$

$$\frac{\Gamma_{Z\gamma}}{\Gamma_{gg}} = \left(\frac{x_w}{1-x_w} \right) \left(\frac{1}{72} \right) \left(\frac{\alpha_{em}}{\alpha_s} \right)^2 \approx 2.2 \times 10^{-5}, \quad (5.4)$$

and

$$\frac{\Gamma_{ZZ}}{\Gamma_{gg}} = \left(\frac{x_w}{1-x_w} \right)^2 \left(\frac{1}{36} \right) \left(\frac{\alpha_{em}}{\alpha_s} \right)^2 \approx 1.3 \times 10^{-5}. \quad (5.5)$$

These show that the $P^{0'}$ decaying to a pair of gauge bosons is dominated by the $2g$ mode. The other two gauge boson modes will be neglected.

The major decay channels of $P^{0'}$ and their partial widths are listed in the following:

$$(1) P^{0'} \rightarrow g + g.$$

The decay width is

$$\Gamma_{gg}(P^{0'}) = \frac{\alpha_s^2 m_P^3}{6\pi^3 F_\Pi^2} \left(\frac{N_{tc}}{4} \right)^2, \quad (5.6)$$

where N_{tc} is the rank of the technicolor gauge group, $SU(N_{tc})$; $N_{tc} = 4$, in Farhi-Susskind model.

Table 5.1. Anomaly factors $S_{P^{0'} B_1 B_2}$ for $P^{0'}$ in the Farhi-Susskind model;
 $x_w = \sin^2 \theta_w$

Vertex $P^{0'} B_1 B_2$	$S_{P^{0'} B_1 B_2}$
$P^{0'} g_a g_b$	$g_s^2 (N_{tc}/\sqrt{6}) \delta_{ab}$
$P^{0'} \gamma \gamma$	$-e^2 (4N_{tc}/3\sqrt{6})$
$P^{0'} Z \gamma$	$e^2 (4N_{tc}/3\sqrt{6}) [x_w/(1-x_w)]^{1/2}$
$P^{0'} Z Z$	$-e^2 (4N_{tc}/3\sqrt{6}) [x_w/(1-x_w)]$
$P^{0'} W^+ W^-$	0

$$(2) P^{0'} \rightarrow f + \bar{f}.$$

The coupling between PGB and fermions arising from the extended technicolor theory is model-dependent. The coupling of PGB-fermion is proportional to m_f/F_Π (see Appendix), where m_f is the mass of fermion and $F_\Pi = 125$ GeV in Farhi-Susskind model. The width of $P^{0'}$ decaying into a fermion pair is

$$\Gamma_{f\bar{f}}(P^{0'}) = \delta_c \frac{m_P}{8\pi} G_0^2 \frac{m_f^2}{F_\Pi^2} \sqrt{1 - 4m_f^2/m_P^2} \quad (5.7)$$

where $\delta_c = 1$ for a lepton pair and $\delta_c = 3$ for a quark pair. The factor G_0 is of order unity and model dependent (see Appendix). Since the width (5.7) is proportional to the square of fermion mass, the decay channels to heavy fermion pairs are dominant.

$$(3) P^{0'} \rightarrow g(k_1) + g(k_2) + g(k_3).$$

The amplitude square can be obtained by redefining $\hat{s}$, $\hat{t}$, and $\hat{u}$, and multiplying

$8 \times 8 \times 2 \times 2 = 256$ in (4.19) since we don't need to average the colors and spins here.

The matrix element is

$$\begin{aligned} \overline{\mathcal{M}}(P^{0'} \rightarrow 3g) = & \frac{16N_c^2\alpha_s^3}{\pi F_\Pi^2} \left\{ \frac{\hat{u}\hat{t}}{\hat{s}} + \frac{\hat{t}\hat{s}}{\hat{u}} + \frac{\hat{u}\hat{s}}{\hat{t}} + \right. \\ & \left. m_P^2 \left[1 + m_P^2 \left(\frac{1}{\hat{s}} + \frac{1}{\hat{t}} + \frac{1}{\hat{u}} \right) + \frac{\hat{s}^2}{\hat{u}\hat{t}} + \frac{\hat{u}^2}{\hat{s}\hat{t}} + \frac{\hat{t}^2}{\hat{u}\hat{s}} \right] \right\}, \quad (5.8) \end{aligned}$$

where

$$\begin{aligned} \hat{s} &= (k_1 + k_2)^2 \\ \hat{t} &= (k_2 + k_3)^2 \\ \hat{u} &= (k_1 + k_3)^2 \end{aligned} \quad (5.9)$$

We obtain the partial width after the integration of phase space

$$\Gamma_{3g}(P^{0'}) = \frac{m_P}{2} \left(\frac{1}{2\pi} \right)^5 \left(\frac{\pi}{2} \right)^2 \int dx dy \overline{\mathcal{M}}(P^{0'} \rightarrow 3g) \quad (5.10)$$

where $x = m_{23}^2/m_P^2$, $y = m_{13}^2/m_P^2$, $m_{23}^2 = (k_2 + k_3)^2$, and $m_{13}^2 = (k_1 + k_3)^2$.

To avoid divergence we do cutoff on m_{23} and m_{13} . The result, after most of the integration, is

$$\begin{aligned} \Gamma_{3g}(P^{0'}) = & \frac{\alpha_s m_P^3}{6\pi^4 F_\Pi^2} \times \\ & \left\{ 3 \int_a^{1-2a} dx \frac{\ln(1-a-x)}{x} + \frac{91}{12} - 29a + \frac{57}{4}a^2 + \frac{27}{2}a^3 + \right. \\ & \left. \ln\left(\frac{1-2a}{a}\right) \left[2a^3 - 3a^2 + 12a - \frac{11}{2} - 3\ln(a) \right] \right\}, \quad (5.11) \end{aligned}$$

where $a = m_0^2/m_P^2$; m_0 is the cutoff mass on gluon pair.

$$(4) P^{0'} \rightarrow q(k_1) + \bar{q}(k_2) + g(k_3).$$

Three decay diagrams are similar to the diagrams in Figure 4.2. However, we cannot

use the previous amplitude square because we neglected the PGB-fermion vertex for light quark in the production and, here, the heavy quark dominates the decay. The amplitude square is

$$\begin{aligned}
\overline{\mathcal{M}}(P^{0'} \rightarrow q\bar{q}g) = & 64\pi\alpha_s \times \\
& \left\{ G_0^2 \left(\frac{1}{(\hat{u} - m_q^2)^2} + \frac{1}{(\hat{t} - m_q^2)^2} \right) [(\hat{u} - m_q^2)(\hat{t} - m_q^2) - 2m_q^2 m_P^2] + \right. \\
& \frac{2G_0^2}{(\hat{u} - m_q^2)(\hat{t} - m_q^2)} [(m_P^2 - m_q^2 - \hat{t})(m_P^2 - m_q^2 - \hat{u}) + 2m_q^2(\hat{s} - 2m_q^2)] + \\
& \frac{V_0^2}{2\hat{s}^2} [(\hat{t} - m_q^2)(m_P^2 - \hat{s})(m_P^2 + m_q^2 - \hat{t}) + 2m_q^2(m_P^2 - \hat{s})^2] + \\
& (\hat{u} - m_q^2)(m_P^2 - \hat{s})(m_P^2 + m_q^2 - \hat{u}) - 2m_P^2(\hat{u} - m_q^2)(\hat{t} - m_q^2)] + \\
& \left. G_0 V_0 \left[\frac{m_q(m_P^2 - \hat{s})^2}{\hat{s}(\hat{u} - m_q^2)} + \frac{m_q(m_P^2 - \hat{s})^2}{\hat{s}(\hat{t} - m_q^2)} \right] \right\}, \quad (5.12)
\end{aligned}$$

where $V_0 = -N_{tc}\alpha_s/(2\sqrt{3}\pi F_\Pi)$. The decay width can be expressed as

$$\Gamma_{q\bar{q}g}(P^{0'}) = \frac{m_P}{2} \left(\frac{1}{2\pi} \right)^5 \left(\frac{\pi}{2} \right)^2 \int dx \frac{(x-r)\lambda(1,r,x)}{x(1-x-r)} \int dy \overline{\mathcal{M}}(P^{0'} \rightarrow q\bar{q}g), \quad (5.13)$$

where

$$r \equiv \frac{m_q^2}{m_P^2} \quad (5.14)$$

$$\lambda(c,d,e) \equiv \sqrt{c^2 + d^2 + e^2 - 2cd - 2ce - 2de}. \quad (5.15)$$

The divergences are avoided by cutoffs on m_{23} and m_{13} . The decay width is expressed as

$$\begin{aligned}
\Gamma_{q\bar{q}g}(P^{0'}) = & \int_b^{(1-\sqrt{b})^2} dx \frac{\alpha_s m_P (x-r)\lambda(1,r,x)}{2\pi^2 x(1-x-r)} \times \\
& \left\{ G_0^2 \left[\left(x-r + \frac{1-2r}{x-r} \right) \ln \left(\frac{y_2-r}{y_1-r} \right) + 2r \left(\frac{1}{y_2-r} - \frac{1}{y_1-r} \right) + \right. \right.
\end{aligned}$$

$$\begin{aligned}
& \frac{x-r-2}{x-r}(y_2-y_1) \Big] + \\
& \frac{V_0^2 m_P^2}{4} \left[((x-r)(1+r-x) - x + 3r) \ln \left(\frac{z_2-r}{z_1-r} \right) + \right. \\
& \left. r \left(\frac{1}{z_2-r} - \frac{1}{z_1-r} + z_1 - z_2 \right) \right] - \\
& \left. \frac{\sqrt{r} G_0 V_0 m_P}{x-r} \left[\ln \left(\frac{z_2}{z_1} \right) - 2(z_2 - z_1) + \frac{z_2^2 - z_1^2}{2} \right] \right\}, \quad (5.16)
\end{aligned}$$

where

$$b \equiv \frac{\Delta m^2}{m_P^2}, \quad m_{13}, m_{23} \geq m_q + \Delta m \quad (5.17)$$

$$y_1 = \frac{x-r_0}{2}(1-x-r_0-\lambda(1, r_0, x)) + r_0, \quad r_0 = \frac{(m_q + \Delta m)^2}{m_P^2} \quad (5.18)$$

$$y_2 = \frac{x-r_0}{2}(1-x-r_0+\lambda(1, r_0, x)) + r_0 \quad (5.19)$$

$$z_1 = 1+r-x-y_1, \quad z_2 = 1+r-x-y_2. \quad (5.20)$$

These complete the decay formula for $P^{0'}$. The decay width and the branching ratios are presented later in Section 5.3.

5.2. The Decay of $P_8^{0'}$

$P_8^{0'}$ can also decay to a pair of gauge bosons which are in the color octet state. The coupling of $P_8^{0'}$ to two gauge bosons also arises from a triangle graph. The amplitude with the $P_8^{0'} B_1 B_2$ coupling has the same form as Eq. (5.1). The factors of $S_{PB_1 B_2}$ for $P_8^{0'}$ are listed in Table 5.2 [22]. Using Eq. (5.2) for the decay rates if the masses of the final state particles are negligible, we obtain

$$\frac{\Gamma_{\gamma g}}{\Gamma_{gg}} = \left(\frac{1}{30} \right) \left(\frac{\alpha_{em}}{\alpha_s} \right) \approx 2.4 \times 10^{-3} \quad (5.21)$$

Table 5.2. Anomaly factors $S_{P_{H_1} B_2}$ for $P_8^{0'}$ in the Farhi-Susskind model;
 $x_w = \sin^2 \theta_w$.

Vertex $P_8^{0'} B_1 B_2$	$S_{P_{H_1} B_2}$
$P_{8a}^{0'} g_b g_c$	$g_s^2 N_{tc} d_{abc}$
$P_{8a}^{0'} \gamma g_b$	$eg_s (N_{tc}/3) \delta_{ab}$
$P_{8a}^{0'} Z g_b$	$-eg_s (N_{tc}/3) \delta_{ab} [x_w / (1 - x_w)]^{1/2}$

and

$$\frac{\Gamma_{Zg}}{\Gamma_{gg}} = \left(\frac{1}{30}\right) \left(\frac{x_w}{1 - x_w}\right) \left(\frac{\alpha_{em}}{\alpha_s}\right) \approx 7.3 \times 10^{-4}. \quad (5.22)$$

Definitely, the Z mass is large. The decay rate involving Z would be smaller than our simple calculation. We will only consider the gg decay mode for $P_8^{0'}$ decaying to a pair of gauge bosons. The major decay channels of $P_8^{0'}$ are as same as $P^{0'}$, including the two-body and the three-body final states. We have completed all major decay formula and list them as follows.

$$(1) P_8^{0'} \rightarrow g + g.$$

The decay width is from Eq. (5.2)

$$\Gamma_{gg}(P_8^{0'}) = \frac{5\alpha_s^2 N_{tc}^2 m_P^3}{384\pi^3 F_\Pi^2}, \quad (5.23)$$

where N_{tc} is the rank of the technicolor gauge group, $SU(N_{tc})$, which equals 4 in the minimum Farhi-Susskind model.

$$(2) P_8^{0'} \rightarrow f + \bar{f}.$$

The coupling between $P_8^{0'}$ -fermion is from the extended technicolor sector and is model-dependent. These couplings are proportional to $\frac{m_f}{F_\Pi}$. The width of $f\bar{f}$ mode is

$$\Gamma_{f\bar{f}}(P_8^{0'}) = \frac{1}{16\pi} G_8^2 \frac{m_f^2}{F_\Pi^2} m_P \sqrt{1 - \frac{4m_f^2}{m_P^2}}, \quad (5.24)$$

where G_8 is model-dependent and defined in the $P_8^{0'}-f\bar{f}$ vertex (see Appendix). Comparing (5.24) to $\Gamma_{f\bar{f}}(P^{0'})$, we have

$$\frac{\Gamma_{f\bar{f}}(P_8^{0'})}{\Gamma_{f\bar{f}}(P^{0'})} = \frac{G_8^2}{2G_0^2}. \quad (5.25)$$

In the Ellis et al. monophagic model [44], the value of this ratio is 12.

$$(3) P_8^{0'} \rightarrow g(k_1) + g(k_2) + g(k_3).$$

The seven diagrams are the same as the diagrams of production in Figure (4.4), except for the incoming and outgoing arrows. We obtain the amplitude square from the production of $P_8^{0'}$ via gluon fusion (see Eq. (4.23)).

$$\begin{aligned} \overline{\mathcal{M}}(P_8^{0'} \rightarrow 3g) = & \frac{10\alpha_s^3(Q^2)}{\pi F_\Pi^2} \frac{1}{\hat{s}^2} \left\{ -2m_P^4 \left(\frac{\hat{s}}{\hat{s} - m_P^2} + \frac{\hat{t}}{\hat{t} - m_P^2} + \frac{\hat{u}}{\hat{u} - m_P^2} \right) - \right. \\ & 2m_P^2 \frac{\hat{s}\hat{t}\hat{u}}{(\hat{s} - m_P^2)(\hat{t} - m_P^2)(\hat{u} - m_P^2)} - 3m_P^2 + \frac{2}{\hat{s}\hat{t}\hat{u}} \times \\ & [(\hat{s}\hat{t})^2 + (\hat{u}\hat{s})^2 + (\hat{u}\hat{t})^2 + m_P^2(\hat{s}^3 + \hat{t}^3 + \hat{u}^3) + \\ & \left. m_P^4(\hat{s}\hat{t} + \hat{t}\hat{u} + \hat{s}\hat{u}) \right] \Big\}. \quad (5.26) \end{aligned}$$

where $\hat{s}$, $\hat{t}$, and $\hat{u}$ are defined in (5.9). Use (5.10) to calculate the width of the 3g mode. We again make cut on the mass of gluonic pairs in the phase space integral to avoid the infrared divergence. After we carry out most of the integration, the width

is

$$\begin{aligned}
\Gamma_{3g}(P_8^{0'}) &= \frac{5\alpha_s^3 N_c^2 m_P^3}{768\pi^4 F_{II}^2} \times \\
&\left\{ 12 \int_a^{1-2a} \frac{\ln(1-a-x)}{1-x} dx + \right. \\
&(4a^3 - 6a^2 + 24a - 11) \ln\left(\frac{1-2a}{a}\right) + \\
&(12\ln(a) + 2a^3 + 12a + 6) \ln\left(\frac{2a}{1-a}\right) + \\
&\left. 12a \left(\frac{1}{2a} - \frac{1}{1-a}\right) - \frac{27}{8}a^4 - 18a^3 + \frac{81}{4}a^2 - 58a + \frac{141}{8} \right\}, \quad (5.27)
\end{aligned}$$

where a is defined as $a \equiv \frac{m_0^2}{m_P^2}$; m_0 is the cutoff mass on the gluon pair.

$$(4) P_8^{0'} \rightarrow q(k_1) + \bar{q}(k_2) + g(k_3).$$

There are four diagrams for $P_8^{0'}$ decaying into $q\bar{q}$ pair and a gluon (see Figure 4.3).

The amplitude square is

$$\begin{aligned}
\overline{\mathcal{M}}(P_8^{0'} \rightarrow q\bar{q}g) &= 64\pi\alpha_s \times \\
&\left\{ \frac{G_8^2}{6} \left[((\hat{u} - m_q^2)(\hat{t} - m_q^2) - 2m_P^2 m_q^2) \left(\frac{1}{(\hat{u} - m_q^2)^2} + \frac{1}{(\hat{t} - m_q^2)^2} \right) - \right. \right. \\
&\frac{1}{4(\hat{u} - m_q^2)(\hat{t} - m_q^2)} ((m_P^2 - m_q^2 - \hat{t})(m_P^2 - m_q^2 - \hat{u}) + 2m_q^2(\hat{s} - m_q^2)) \left. \right] + \\
&\frac{5V_8^2}{96\hat{s}} [(\hat{t} - m_q^2)(m_P^2 - \hat{s})(m_P^2 + m_q^2 - \hat{t}) + \\
&(\hat{u} - m_q^2)(m_P^2 - \hat{s})(m_P^2 + m_q^2 - \hat{u}) - 2m_P^2(\hat{u} - m_q^2)(\hat{t} - m_q^2) + \\
&2m_q^2(m_P^2 - \hat{s})] + \\
&\frac{5G_8 V_8 m_q}{48\hat{s}} (m_P^2 - \hat{s})^2 \left(\frac{1}{\hat{t} - m_q^2} + \frac{1}{\hat{u} - m_q^2} \right) + \\
&\frac{3G_8^2}{32(\hat{s} - m_P^2)} \left[\frac{1}{\hat{u} - m_q^2} [4m_P^2 \hat{s} + 2(\hat{u} - m_q^2)^2 - 2m_P^2(\hat{u} - m_q^2)] + \right.
\end{aligned}$$

$$\left. \frac{1}{\hat{t} - m_q^2} \left(4m_P^2 \hat{s} + 2(\hat{t} - m_q^2)^2 - 2m_P^2(\hat{t} - m_q^2) \right) \right] - \frac{-3G_8^2 \hat{s}(\hat{s} + m_P^2)}{8(\hat{s} - m_P^2)^2} \Bigg\}, \quad (5.28)$$

where $V_8 = -N_{tc}\alpha_s/(\sqrt{2}\pi F_{II})$. There are singularities in the expression and all come from the soft gluon region. We take mass cuts on the gq pair to avoid these infrared divergences. From Eq. (5.13), we obtain the following expression for the decay width

$$\begin{aligned} \Gamma_{q\bar{q}g}(F_8^{J'}) &= \int_b^{(1-\sqrt{b})^2} dx \frac{\alpha_s m_P \lambda(1, r, x)(x-r)}{4\pi^2 x(1-r-x)} \times \\ &\left\{ \frac{G_8^2}{3} \left[\left(x-r - \frac{1-2r}{8(x-r)} \right) \ln \left(\frac{y_2-r}{y_1-r} \right) + 2r \left(\frac{1}{y_2-r} - \frac{1}{y_1-r} \right) + \right. \right. \\ &\quad \left. \frac{2+r-x}{8(x-r)} (y_2-y_1) \right] + \\ &\quad \frac{5}{48} V_8^2 m_P^2 \left[((x-r)(1+r-x) - x + 3r) \ln \left(\frac{z_2-r}{z_1-r} \right) + \right. \\ &\quad \left. r \left(\frac{1}{z_2-r} - \frac{1}{z_1-r} \right) - r(z_2-z_1) \right] - \\ &\quad \frac{3}{8} G_8^2 \left[\left(1 + \frac{2}{(2r-x-y_1)(2r-x-y_2)} + \frac{2}{x-r} \right) (y_2-y_1) - \right. \\ &\quad \left. \left(3 + \frac{(x-r)^2 - x + r + 2}{x-r} \right) \ln \left(\frac{2r-x-y_2}{2r-x-y_1} \right) \right] - \\ &\quad \left. \frac{5\sqrt{r}G_8V_8m_P}{24(x-r)} \left[\ln \left(\frac{z_2}{z_1} \right) - 2(z_2-z_1) + \frac{z_2^2-z_1^2}{2} \right] \right\}, \quad (5.29) \end{aligned}$$

where $b, r, \lambda, y_1, y_2, z_1$, and z_2 are defined in preceding section.

5.3. The Numerical Results on the Decay of $P^{0'}$ and $P_8^{0'}$

Here we present some results of the branching ratios of dominant decay channels and the approximate total width for $P^{0'}$ and $P_8^{0'}$. Since the top quark has not been discovered, we show the results with and without opening the $t\bar{t}$ channel. The present lower bound of t mass so far is 78 GeV given by CDF group at Fermilab [3]. We take two values of t mass, $m_t = 80$ and 120 GeV, in the numerical results. Because the couplings of PGB-fermions are model dependent, we use the couplings given in Ellis et al.'s monophagic model [44] in which the FCNC vanishes from exchanging one neutral PGB at tree level. In monophagic model, the coupling of neutral PGBs to fermion pair are

$$\frac{\phi^0}{F_\Pi} \frac{1}{2} \left[\sqrt{\frac{1}{3}} (\bar{p} m_p \gamma_5 p + \bar{n} m_n \gamma_5 n) - \sqrt{3} (\bar{l} m_l \gamma_5 l) \right], \quad (5.30)$$

$$\frac{\phi_a^8}{F_\Pi} \sqrt{2} \left[\bar{p} m_p \gamma_5 \frac{\lambda_a}{2} p + \bar{n} m_n \gamma_5 \frac{\lambda_a}{2} n \right] \quad (5.31)$$

where ϕ^0 and ϕ_a^8 are $P^{0'}$ and $P_8^{0'}$ fields respectively, $p = (u, c, t)^T$, $n = (d, s, b)^T$, $l = (e, \mu, \tau)^T$, and $m_{p,n,l}$ are the diagonalized mass matrices. According to (9.5) and (9.6), $G_8 = \sqrt{2}$ and $G_0 = 1/(2\sqrt{3})$ for quark, and $G_0 = \sqrt{3}/2$ for lepton.

To avoid the infrared divergence, a cutoff is made. In $3g$ decay mode, an invariant mass cutoff on the gluon pair is imposed to make the energy of each gluon greater than 1.5 GeV for m_P between 20 and 120 GeV and greater than 2 GeV between 120 and 450 GeV. The relationship between the mass cutoff, m_0 , and the gluon minimum energy, $E_{\min}$, is

$$m_0 = \sqrt{2E_{\min} m_P}. \quad (5.32)$$

In $q\bar{q}g$ mode, we imposed an invariant mass cut to make the angle between quark and

gluon greater than 10 degrees. This cut is better than the cut based on the minimum energy of the gluon in considering the separation quark and gluon jets, if the quark mass is large. To get a relationship between the minimum angle and the mass cut, we start with two identities

$$\begin{aligned} (k_1 + k_2)^2 &= M^2 = (m_q + m_0)^2 \\ &= E_1 E_2 - E_2 \sqrt{E_1^2 - m_q^2} \cos \theta, \end{aligned} \quad (5.33)$$

$$E_t = E_1 + E_2 = \frac{m_P^2 + M^2 - m_q^2}{2m_P} \quad (5.34)$$

where E_1 is the quark energy and E_2 is the gluon energy. Combine Eqs. (5.33) and (5.34) and we obtain

$$E_t = \frac{m_P^2/2 + E_1 \sqrt{E_1^2 - m_q^2} \cos \theta - E_1^2}{m_P - E_1 + \sqrt{E_1^2 - m_q^2} \cos \theta} \quad (5.35)$$

For a given $\theta_{\min}$, there is a maximum of E_t in Eq. (5.35). Once we get the maximum of E_t , the invariant mass cut can be obtained from Eq. (5.34). The following are the results of the decay formulae in previous sections; the major decay channels for $P^{0'}$ and $P_8^{0'}$ are summarized in Table 5.3.

(1) $t\bar{t}$ channel not open

$P^{0'}$ case: The branching ratios (BR) are shown in Figures 5.1 and 5.2. The $3g$ and $2g$ modes dominate except for $m_P < 40$ GeV where the $b\bar{b}$ channel becomes dominant. The $\tau\bar{\tau}$ channel gives 20% to 5% of the total rate if $20 \text{ GeV} < m_P < 100 \text{ GeV}$. The $q\bar{q}g$ channel, which includes all the light quarks (u,d,c,s), has 4% to 8% BR for $20 \text{ GeV} < m_P < 300 \text{ GeV}$. There is about a 2% contribution from $b\bar{b}g$ when $m_P > 40 \text{ GeV}$. The total width is very narrow from 10^{-3} to $3 \times 10^{-1} \text{ GeV}$ in the mass region 20 to 300 GeV (see Figures 5.4 and 5.5).

Table 5.3. The major decay channels for $P^{0'}$ and $P_8^{0'}$

PGBs	No $t\bar{t}$	$t\bar{t}$
$P^{0'}$	$2g, 3g, b\bar{b}, \tau\bar{\tau}, q\bar{q}g$	$t\bar{t}, 2g, 3g$
$P_8^{0'}$	$2g, 3g, b\bar{b}, q\bar{q}g$	$t\bar{t}, 2g, 3g$

$P_8^{0'}$ case: We take the $P_8^{0'}$ mass to lie in the region between 100 and 300 GeV. The BRs are shown in Figure 5.3. The $2g$ and $3g$ modes dominate here as well, except when m_P is close to 100 GeV where $b\bar{b}$ contributes about 20% of the total decay rate. The $b\bar{b}$ mode still holds 6% BR at $m_P = 300$ GeV. The BR for $2g$ is about 40% throughout the region. The BR of $3g$ increases from about 20% for $m_P = 100$ GeV to about 40% for $m_P = 300$ GeV. The BR of the $q\bar{q}g$ channel is around 7%. The $b\bar{b}g$ channel has about a 3% BR. In Figure 5.6, we can find the total width ranging from 4×10^{-2} to 0.34 GeV. It, too, is very narrow.

(2) $m_t = 80 \text{ GeV}$

$P^{0'}$ case: Figure 5.7 shows the BRs of major decay channels. The $t\bar{t}$ mode is the dominant decay channel. Its BR is about 84% for $m_P = 200$ GeV and only decreases to 68% for $m_P = 400$ GeV. The BR of $3g$ and of $2g$ channels are about 12% and 10%, respectively, and both rise slowly with increasing m_P . The $q\bar{q}g$ channel has about a 1% BR. The $b\bar{b}g$ and the $t\bar{t}g$ channels are negligible. The total width is presented in Figure 5.11 and has a value increasing from 0.6 GeV

Figure 5.1. BRs of $P^{0'}$ without the $t\bar{t}$ channel: $b\bar{b}$ (upper solid), $\tau\bar{\tau}$ (dotdash), $3g$ (dots), gg (upper dashes), $b\bar{b}g$ (lower solid), and $q\bar{q}g$ (lower dashes)

Figure 5.2. BRs of P'' without the $t\bar{t}$ channel: $b\bar{b}$ (upper solid), $\tau\bar{\tau}$ (dotdash), $3g$ (dots), gg (upper dashes), $b\bar{b}g$ (lower solid), and $q\bar{q}g$ (lower dashes)

Figure 5.3. BRs of $P_8^{0'}$ without the $t\bar{t}$ channel: $b\bar{b}$ (solid), $3g$ (dots), gg (upper dashes), $b\bar{b}g$ (lower solid), and $q\bar{q}g$ (lower dashes)

Figure 5.4. The total width of P'' without the $t\bar{t}$ channel

Figure 5.5. The total width of P'' without the $t\bar{t}$ channel

Figure 5.6. The total width of $P_8^{0'}$ without the $t\bar{t}$ channel

at $m_P = 200$ GeV to about 2 GeV at $m_P = 400$ GeV.

$P_8^{0'}$ case: As with $P^{0'}$, $t\bar{t}$ dominates the decay, from 95% down to 88% for $m_P = 200$ GeV to 400 GeV (see Figure 5.8). The BR of $2g$ is between 2% and 5%. The BR of $3g$ is about the same size as $2g$'s. The other channels are all negligible. The total width is around 2 GeV to 7 GeV for $m_P = 200$ GeV to 400 GeV and is shown in Figure 5.12.

(3) $m_t = 120$ GeV.

The general properties of decay for $P^{0'}$ and $P_8^{0'}$ at $m_t = 120$ GeV are the same as those at $m_t = 80$ GeV. Since the $t\bar{t}$ channel dominates and the PGB-fermions couplings are proportional to $\frac{m_f}{F_\Pi}$, the higher the m_t , the more $t\bar{t}$ dominates and the larger the total width.

$P^{0'}$ case: The BRs are shown in Figure 5.9. In the mass region of 250 GeV to 450

Figure 5.7. BRs of $P^{0'}$ with $m_t = 80$ GeV: $t\bar{t}$ (solid), $3g$ (dots), gg (dashes), and $q\bar{q}g$ (lower dashes)

Figure 5.8. BRs of $P_8^{0''}$ with $m_t = 80$ GeV: $t\bar{t}$ (solid), $3g$ (dots), gg (dashes), and $q\bar{q}g$ (lower dashes)

GeV, the $t\bar{t}$ mode contributes about 80% of the total decay rate. The other sizable channels are $3g$ and $2g$; each contributes about 10%. The BR of $q\bar{q}g$ is around 1%. We can neglect the other channels. The total width runs from 0.7 GeV to 3.6 GeV for $m_P = 250$ to 400 GeV and is presented in Figure 5.11.

$P_8^{0'}$ case: Figure 5.10 shows the BRs of major decay channels. The $t\bar{t}$ channel totally dominates the decay and its BR is about 93% for m_P between 250 GeV and 400 GeV. The $2g$ and $3g$ channels have about the same size BR, around 3% each. Again, the other channels are negligible. The total width increases from 2.5 GeV for $m_P = 250$ GeV to 12.5 GeV for $m_P = 400$ GeV and is shown in Figure 5.12.

Figure 5.9. BRs of P'' with $m_t = 120$ GeV: $t\bar{t}$ (solid), $3g$ (dots), gg (dashes), and $q\bar{q}g$ (lower dashes)

Figure 5.10. BRs of $P_8^{0'}$ with $m_t = 120$ GeV: $t\bar{t}$ (solid), $3g$ (dots), gg (dashes), and $q\bar{q}g$ (lower dashes)

Figure 5.11. The total width of P'' with $m_t = 80$ GeV (dashes) and $m_t = 120$ GeV (solid)

Figure 5.12. The total width of P''_8 with $m_t = 80$ GeV (dashes) and $m_t = 120$ GeV (solid)

6. THE BACKGROUNDS AT THE SSC

In Chapters 4 and 5, we showed the production rates and the decay widths of the neutral technipions, $P^{0'}$ and $P_8^{0'}$. However, the discovery of these particles at the SSC depends on the backgrounds. In this chapter, we will examine the backgrounds in the two-body decay channels

$$p + p \rightarrow P^{0'}(P_8^{0'}) + g(q) + X$$

$\downarrow$
 $t\bar{t}, g\bar{g}, b\bar{b}, \tau\bar{\tau}$

(6.1)

(the $\tau\bar{\tau}$ mode is only for $P^{0'}$ decays.). The light quark channel, $q\bar{q}$, is ignored because its branching ratio (BR) is much less than those of the heavy quark pair gg channels (see Chapter 5). The backgrounds are from the following QCD processes

$$p + p \longrightarrow t + \bar{t} + g(q) + X \quad (6.2)$$

$$p + p \longrightarrow b + \bar{b} + g(q) + X \quad (6.3)$$

$$p + p \longrightarrow g(q) + g(q) + g(q) + X \quad (6.4)$$

$$p + p \longrightarrow \tau + \bar{\tau} + g(q) + X \quad (6.5)$$

We will evaluate and discuss each of these in this chapter.

6.1. The Background of the $t\bar{t}$ and $b\bar{b}$ Channels

There are three subprocesses for the processes (6.2) and (6.3)

$$g(p_1) + g(p_2) \longrightarrow Q(p_3) + \bar{Q}(p_4) + g(p_5) \quad (6.6)$$

$$g(p_1) + q(p_2) \longrightarrow Q(p_3) + \bar{Q}(p_4) + q(p_5) \quad (6.7)$$

$$q(p_1) + \bar{q}(p_2) \longrightarrow Q(p_3) + \bar{Q}(p_4) + g(p_5) \quad (6.8)$$

Ellis and Sexton [59] have calculated the squared amplitudes for these subprocess.

They are

$$\overline{\mathcal{M}}(gg \rightarrow Q\bar{Q}g) = \frac{g_s^6}{4(N^2 - 1)^2} B(p_4, p_3, p_5, -p_1, -p_2) \quad (6.9)$$

$$\overline{\mathcal{M}}(gq \rightarrow Q\bar{Q}q) = -\frac{g_s^6}{4N(N^2 - 1)} A(p_4, p_3, p_5, -p_2, -p_1) \quad (6.10)$$

$$\overline{\mathcal{M}}(q\bar{q} \rightarrow Q\bar{Q}g) = \frac{g_s^6}{4N^2} A(p_4, p_3, -p_2, -p_1, p_5) \quad (6.11)$$

where

$$A(p_1, p_2, p_3, p_4, p_5) =$$

$$\left[\frac{(13)^2 + (23)^2 + (14)^2 + (24)^2 + m_Q^2((12) + (34) + m_Q^2)}{2\hat{s}(34)} \right] \times$$

$$\left[\frac{4}{N}(N^2 - 1)^2 \left(\frac{(13)}{(15)(35)} + \frac{(24)}{(25)(45)} \right) + \right.$$

$$\left. \frac{4}{N}(N^2 - 1) \left(\frac{2(14)}{(15)(45)} + \frac{2(23)}{(25)(45)} - \frac{(13)}{(15)(35)} \right) \right]$$

$$\begin{aligned}
& \left. \frac{(24)}{(25)(45)} - \frac{(12)}{(15)(25)} - \frac{(34)}{(35)(45)} \right) \Big] - \\
& \frac{(N^2 - 4)(N^2 - 1)}{N} \frac{2m_Q^2}{\hat{s}(34)} \left[\frac{(13) - (14)}{(25)} - \frac{(23) - (24)}{(15)} \right] + \\
& \frac{4}{N} (N^2 - 1) m_Q^2 \left[\frac{1}{\hat{s}^2} \left(\frac{(35)^2 + (45)^2}{(35)(45)} \right) - \right. \\
& \left. \frac{1}{2\hat{s}} \left(\frac{1}{(15)} + \frac{1}{(25)} + \frac{1}{(35)} + \frac{1}{(45)} \right) - \right. \\
& \left. \frac{1}{4(34)} \left(\frac{1}{(15)} + \frac{1}{(25)} + \frac{m_Q^2}{(15)^2} + \frac{m_Q^2}{(25)^2} + \frac{4}{\hat{s}} \right) - \right. \\
& \left. \frac{\delta_1^2 + \delta_2^2 + \delta_3^2 + \delta_4^2}{4(34)^2} \right] - \\
& \frac{2(N^2 - 1)}{N} \frac{m_Q^2}{\hat{s}(34)} \left[1 + \frac{2(34)}{\hat{s}} + \frac{m_Q^2}{(15)} + \frac{m_Q^2}{(25)} + \frac{(35)^2 + (45)^2}{(15)(25)} + \right. \\
& \left. ((13) - (14)) \frac{\delta_1 - \delta_2}{(34)} + ((23) - (24)) \frac{\delta_3 - \delta_4}{(34)} \right], \tag{6.12}
\end{aligned}$$

$$B(p_1, p_2, p_3, p_4, p_5) = \sum_{\text{perm.}} F(p_1, p_2, p_3, p_4, p_5), \tag{6.13}$$

with

$$F(p_1, p_2, p_3, p_4, p_5) =$$

$$\frac{(N^2 - 1)(N^2 + 1)}{4N^2} \frac{\left((12) \left((15)^2 + (25)^2 + 2m_Q^2((35) + (45)) \right) \right)}{(13)(14)(23)(24)} -$$

$$\begin{aligned}
& \frac{(N^2 - 1)(N^2 + 1)m_Q^2 \left(\hat{s}^2 + 2(13)(14) + 2(23)(24) + 4m_Q^2(12) \right)}{N^2 \cdot 8(13)(14)(23)(24)} - \\
& (N^2 - 1) \left[\frac{(13) \left((13)^2 + (23)^2 + 2m_Q^2((34) + (35)) \right)}{(34)(14)(15)(25)} + \right. \\
& \left. \frac{\left((15)^2 + (25)^2 + 2m_Q^2((35) + (45)) \right)}{2(34)(14)(23)} \right] + \\
& (N^2 - 1)m_Q^2 \left[\frac{(12) - 3m_Q^2}{(34)(15)(25)} + \frac{3(13)(23) + 5(13)(25) - 2(13)^2}{(13)(14)(23)(24)} \right] + \\
& (N^2 - 1)m_Q^4 \left[\frac{(13)^2 + (23)^2 + (15)^2 + (15)(25) - \hat{s}(13)}{(34)(13)(24)(15)(25)} + \right. \\
& \left. \frac{(15) + (24)}{(13)(23)(14)(25)} \right] + \\
& 2N^2(N^2 - 1) \left[\frac{(23) \left((13)^2 + (23)^2 + 2m_Q^2((34) + (35)) \right)}{\hat{s}(34)(45)(25)} \right] + \\
& N^2(N^2 - 1) \left[\frac{(14)(24) \left((14)^2 + (24)^2 + 2m_Q^2((34) + (45)) \right)}{\hat{s}(34)(45)(13)(25)} \right] + \\
& 2N^2(N^2 - 1)m_Q^2 \left[\frac{4(15)(25) + (13)(23) + (14)(24) - \frac{1}{8}\hat{s}(\hat{s} + 2(34))}{\hat{s}(34)(15)(25)} \right] + \\
& N^2(N^2 - 1)m_Q^2 \left[\frac{(34) - 2m_Q^2}{\hat{s}(13)(24)} - \frac{\hat{s}}{4(34)(15)(25)} \right] -
\end{aligned}$$

$$\begin{aligned}
& 2N^2(N^2 - 1)m_Q^4 \frac{\frac{1}{4}\hat{s}^2 + (34)^2 + (45)^2}{\hat{s}(34)(45)(13)(25)} + \\
& 4N^2(N^2 - 1)m_Q^2 \left[\frac{(34)^2 + (35)^2 + (45)^2}{\hat{s}^2(34)^2} + \frac{2(23)^2}{\hat{s}(34)^2(15)} \right] + \\
& (N^2 - 1)m_Q^2 \left[\frac{1}{(34)^2} \left(\frac{(13)}{(25)} - \frac{(23)}{(15)} \right)^2 + \frac{2m_Q^2}{(34)(15)^2} + \frac{m_Q^2((34) + (45))}{(34)(45)(13)(25)} \right] + \\
& \frac{(N^2 - 1)^2 m_Q^2}{4N^2} \left[\frac{(13)^2 + (23)^2 - (45)((13) + (23))}{(13)(23)(14)(25)} + \right. \\
& \left. \frac{2\hat{s}m_Q^2}{(13)(23)(14)(25)} - \frac{4m_Q^4}{(13)(14)(25)^2} \right] + \\
& \frac{(N^2 - 1)^3 m_Q^2}{2N^2} \left[\frac{m_Q^4 + m_Q^2(24) - 2(24)(25)}{(13)^2(24)^2} + \right. \\
& \left. \frac{\hat{s}(34) + 2(15)(25) - (14)(24)}{(13)(23)(14)(24)} \right] + \\
& \frac{(N^2 - 1)m_Q^2}{N^2} \left[\frac{m_Q^4}{(13)(23)(14)(25)} - \frac{5\hat{s} + 8(23) + 6(25) - 4(13)}{4(14)(23)(24)} \right], \tag{6.14}
\end{aligned}$$

and where

$$\begin{aligned}
\hat{s} &= (p_1 + p_2)^2, \quad (ij) = p_i \cdot p_j, \quad p_1^2 = p_2^2 = m_Q^2, \\
\delta_1 &= \frac{(13)}{(25)} - \frac{2(35)}{\hat{s}}, \quad \delta_2 = \frac{(14)}{(25)} - \frac{2(45)}{\hat{s}}, \\
\delta_1 &= \frac{(23)}{(15)} - \frac{2(35)}{\hat{s}}, \quad \delta_1 = \frac{(24)}{(15)} - \frac{2(45)}{\hat{s}}. \tag{6.15}
\end{aligned}$$

We have taken the strong interaction gauge group to be $SU(N)$. (Of course, $N = 3$ for QCD.) The differential cross section which we are interested in can be written in general as

$$\frac{d\sigma}{dM dP_{\perp} dy} = \left(\frac{1}{4\pi}\right)^4 P_{\perp} M \times \sum_{i,j} \int_{x_{min}}^1 dx_a \frac{f_i^a(x_a, Q^2) f_j^b(x_b, Q^2)}{s(x_a s + u - M^2)} \sqrt{1 - \frac{4m_Q^2}{M^2}} \int d\Omega \mathcal{M} \quad (6.16)$$

where all functions and parameters, excepting the solid angle Ω , were defined in Chapter 4, Section 2. The angles of θ and ϕ in Ω are defined as

$$\cos(\theta) = \frac{\vec{p}_3 \cdot \vec{p}_5}{|\vec{p}_3| |\vec{p}_5|} \Big|_{\vec{p}_3 + \vec{p}_4 = 0} \quad (6.17)$$

$$\cos(\phi) = \frac{(\vec{p}_3 \times \vec{p}_5) \cdot (\vec{p}_1 \times \vec{p}_5)}{|\vec{p}_3 \times \vec{p}_5| |\vec{p}_1 \times \vec{p}_5|} \Big|_{\vec{p}_3 + \vec{p}_4 = 0} \quad (6.18)$$

M , y , and $P_{\perp}$ are the invariant mass, rapidity, and transverse momentum of the heavy quark pair, respectively. We first integrate out the angular variables in the rest frame of the quark pair, and then transform all terms into Lorentz invariant quantities. Finally, the last integration is done in the Lab frame.

In the production of $P^{0'}$ and $P_8^{0'}$, we look for events with momentum transverse to the beam direction. So we require $y = 0$ for the heavy quark pair, which has a total invariant mass in the range from $(m_P - \Gamma/2)$ to $(m_P + \Gamma/2)$, where Γ is the total decay width of either $P^{0'}$ or $P_8^{0'}$. Figure 6.1 shows that gg fusion dominates the background at SSC energies.

In Figure 6.2, there are three curves labeled with the $P^{0'}$ mass $M=200, 300$, and 400 GeV, for $m_t = 80$ GeV. The differential cross section does not change significantly for different values of M . However there is strong dependence on the transverse

momentum. The differential cross section decreases from about 10^{-4} nb/GeV² to about 10^{-9} nb/GeV² as $P_{\perp}$ increases from 100 GeV to 1500 GeV. The Figure 6.3 shows the differential cross section for $m_t = 120$ GeV. The cross sections for $m_t = 80$ GeV are slightly larger than the corresponding cross section for $m_t = 120$ GeV. The quantity of signal, S , is defined as

$$S \equiv (\text{Production Rate of } P^{0'} \text{ or } P_8^{0'}) \times (\text{BR}) \quad (6.19)$$

The quantity of background, B , is approximately defined as

$$B \approx (d(\text{Background Cross Section})/dM) \times (\text{Width}) \quad (6.20)$$

Due to the detector resolution of 2-jet invariant masses [60], we may not be able to use the real width in (6.20). If the width is smaller than the mass resolution, we replace the width by the mass resolution which we take to be

$$\Delta m \approx 5 \text{ GeV} \sqrt{\frac{M}{M_w}}, \quad M_w = 81 \text{ GeV}. \quad (6.21)$$

Otherwise the real width is used.

It is likely that the t quark mass will be found to be higher than 80 GeV; the current published low bound is 78 GeV [3]. We take the case of $m_t = 80$ GeV here to demonstrate the trend of results when the t quark mass varies from the lower $m_t = 80$ GeV to the higher $m_t = 120$ GeV. Tables 6.1 and 6.2 give the signal to background ratios S/B for $P^{0'}$ decaying to $t\bar{t}$ at different values of $P_{\perp}$ and m_P . For fixed m_P and m_t , S/B increases as $P_{\perp}$ increases. However there is a trade-off as the larger $P_{\perp}$ gives lower cross sections. So we only show S/B for $P_{\perp}$ up to 700 GeV. The signals we are looking for are larger than the backgrounds in most cases. The numbers of $t\bar{t}$ events per GeV bin of $P_{\perp}$ per year at the SSC are also shown in these Tables. Since

the number doesn't include the efficiency in identifying a $t\bar{t}$ pair in a real experiment, the actual number of events will be smaller. Hence, we conclude that if the efficiency of identifying a $t\bar{t}$ pair is not too low, $P^{0'}$, if it exists and decay into a $t\bar{t}$ pair, could be seen at the SSC for $P_{\perp} \leq 500$ GeV.

Similar calculations for $P_8^{0'}$ are presented in tables 6.3 and 6.4. The S/B for $P_8^{0'}$ are larger than the S/B for $P^{0'}$ and is of order 10. Since more $P_8^{0'}$ than $P^{0'}$ can be produced at the SSC, it may not be hard to discover in the $t\bar{t}$ channel for $P_{\perp}$ up to 700 GeV.

The S/B ratios also increase as m_t increases for the given values of m_P and $P_{\perp}$, because higher m_t means a higher BR for $t\bar{t}$ channel, and a lower QCD background. So, as long as the number of events of $P^{0'}$ or $P_8^{0'}$ decaying to a $t\bar{t}$ pair is not too small, the SSC can investigate the $P^{0'}$ and the $P_8^{0'}$ beyond $m_P = 400$ GeV.

If these pseudo-goldstone bosons don't decay into $t\bar{t}$, we have to consider other signals, e.g., $b\bar{b}$, $2g$, and $\tau\bar{\tau}$. Four sets of $b\bar{b}g(q)$ background are examined for four different pseudo-Goldstone boson mass values $M = 50, 100, 200$, and 300 GeV. The results are shown in Figure 6.4. The cross sections range from about 10^{-3} to about 10^{-9} nb/GeV² for $P_{\perp}$ varying from 100 GeV to 1500 GeV.

Table 6.5 shows the S/B and the numbers of events of $P^{0'}$ decaying to $b\bar{b}$ per GeV bin of $P_{\perp}$ per year at the SSC. In most cases, the background is 2 orders of magnitude larger than the signal. The signal is comparable to the background only at $P_{\perp} = 500$ and 700 GeV when $m_P = 50$ GeV. However, the number of events in these cases is only a few hundred per GeV bin of $P_{\perp}$ per year. So, it would be difficult to see the $b\bar{b}$ signal of $P^{0'}$ at the SSC.

Since the production rate and the $b\bar{b}$ BR of $P_8^{0'}$ are larger than those for $P^{0'}$, the

Table 6.1. S/B values for $P^{0'}$ decaying to $t\bar{t}$, $m_t = 80$ GeV. The numbers in the parentheses are the number of events of signal per GeV bin of $P_{\perp}$ per year at the SSC

$P_{\perp}$ GeV	$m_P =$	200	300	400	GeV
100		0.54(6×10^1)	0.6(4.2×10^1)	0.73(2.9×10^1)	
300		1.2(4.5×10^3)	1.5(3.5×10^3)	1.5(2.7×10^3)	
500		2.9(10^3)	2.9(860)	2.9(670)	
700		4.8(330)	5.0(300)	5.1(240)	

signal is improved. The S/B and the number of events per GeV bin of $P_{\perp}$ per year are shown in Table 6.6. The signal is comparable to the background except at low $P_{\perp} = 100$ GeV. The signal is around thousands events per GeV bin of $P_{\perp}$. Hence, through the $b\bar{b}$ channel, if $t\bar{t}$ is not allowed, the SSC may be able to probe $P_8^{0'}$ in the mass region of 100 GeV to 300 GeV.

6.2. The Background of the gg Channel

The signals of $P^{0'}$ and $P_8^{0'}$ from the $2g$ decay channel are 3-jet events. The background includes all 3-jet events but heavy quark jets such as b and t , which we treated in the preceding section as the backgrounds of $b\bar{b}$ and $t\bar{t}$ channels, are not included. There are 11 QCD processes which can produce three-parton final states, e.g., gg fusion and scattering, $q\bar{q}$ fusion and scattering, qq scattering, etc. We list all of them in Table 6.7. Essentially, we only need to know the squared amplitudes

Figure 6.1. Differential cross section of $pp \rightarrow g t \bar{t} X$ for $m_t = 80 \text{ GeV}$ and $M_{tt} = 300 \text{ GeV}$: total (solid), gg (dashes), gq (dots), and $q\bar{q}$ (dotdash)

Figure 6.2. Differential cross section of $pp \rightarrow \gamma t \bar{t} X$ for $m_t = 80 \text{ GeV}$ and $M_H = 200, 300$ and 400 GeV (lower mass has higher rate)

Figure 6.3. Differential cross section of $pp \rightarrow g t \bar{t} X$ for $m_t = 120 \text{ GeV}$:
 $M_H = 300 \text{ GeV}$ (upper line) and $M_H = 400 \text{ GeV}$ (lower line)

Figure 6.4. Differential cross section of $pp \rightarrow gbbX$ for $M_b = 50, 100, 200,$ and 300 GeV (lower mass has higher rate)

Table 6.2. S/B values for $P^{0'}$ decaying to $t\bar{t}$, $m_t = 120$ GeV. The numbers in the parentheses are the number of events of signal per GeV bin of $P_\perp$ per year at the SSC

$P_\perp$ GeV	$m_{P^{0'}} =$	300	400	GeV
100		1.4(4.6×10^1)	1.3(3.4×10^1)	
300		2.5(3.8×10^3)	2.5(3.2×10^3)	
500		4.3(940)	4.5(800)	
700		7.2(330)	7.4(290)	

Table 6.3. S/B values for $P_8^{0'}$ decaying to $t\bar{t}$, $m_t = 80$ GeV. The numbers in the parentheses are the number of events of signal per GeV bin of $P_\perp$ per year at the SSC

$P_\perp$ GeV	$m_{P_8^{0'}} =$	200	300	400	GeV
100		5.8(6.3×10^5)	6.5(4.4×10^5)	8.5(3.4×10^5)	
300		13(4.8×10^1)	15(3.9×10^1)	18(3.2×10^1)	
500		32(1.1×10^1)	34(1.0×10^1)	34(8.2×10^3)	
700		53(3.7×10^3)	51(3.3×10^3)	61(2.9×10^3)	

Table 6.4. S/B values for $P_8^{0'}$ decaying to $t\bar{t}$, $m_t = 120$ GeV. The numbers in the parentheses are the number of events of signal per GeV bin of $P_\perp$ per year at the SSC

$P_\perp$ GeV	$m_P =$	300	400	GeV
100		14(4.6×10^5)	13(3.7×10^5)	
300		24(4×10^4)	23(3.4×10^4)	
500		46(1.1×10^4)	43(8.7×10^3)	
700		71(3.4×10^3)	67(3.1×10^3)	

Table 6.5. S/B values for $P^{0'}$ decaying to $b\bar{b}$; no $t\bar{t}$ allowed. The numbers in the parentheses are the number of events of signal per GeV bin of $P_\perp$ per year at the SSC

$P_\perp$ GeV	$m_P =$	50	100	200	300	GeV
100		0.01 (3.8×10^1)	5.2×10^{-3} (1.2×10^1)	3.0×10^{-3} (2.5×10^3)	2.8×10^{-3} (920)	
300		0.1 (1.8×10^3)	0.05 (660)	0.02 (190)	0.01 (77)	
500		0.3 (350)	0.15 (140)	0.06 (42)	0.03 (19)	
700		0.56 (110)	0.31 (44)	0.12 (14)	0.06 (7)	

Table 6.6. S/B values for $P_8^{0'}$ decaying to $b\bar{b}$; no $t\bar{t}$ allowed. The numbers in the parentheses are the number of events of signal per GeV bin of $P_\perp$ per year at the SSC

$P_\perp$ GeV	$m_P =$	100	200	300	GeV
100		0.14(2.9×10^5)	0.09(7.3×10^4)	0.08(2.2×10^4)	
300		1.3(1.7×10^4)	0.56(5.6×10^3)	0.29(2.0×10^3)	
500		3.9(3.8×10^3)	1.8(1.3×10^3)	1.0(520)	
700		8.4(1.2×10^3)	3.7(430)	2.0(190)	

of four processes which have been calculated by Berends et al. [53]. Others can be obtained by crossing symmetry (interchanging momenta) from these four. We list the four squared amplitudes in the following;

$$(1) \quad g(p_1) + g(p_2) \longrightarrow g(p_3) + g(p_4) + g(p_5)$$

It has very compact form

$$\overline{\mathcal{M}} = G(p_1, p_2, p_3, p_4, p_5) \quad (6.22)$$

where

$$G(p_1, p_2, p_3, p_4, p_5) = \frac{27}{160} \frac{\sum_{i < j} (ij)^4}{\prod_{i < j} (ij)} \sum_{\text{perm.}} [12345], \quad (6.23)$$

with

$$[12345] \equiv (12)(23)(34)(45)(51). \quad (6.24)$$

$$(2) \quad q_m(p_1) + \bar{q}_m(p_2) \longrightarrow g(p_3) + g(p_4) + g(p_5)$$

The subscript m denotes the quark flavor. The amplitude squared, averaged over the

spins and the colors of the initial partons, is

$$\overline{\mathcal{M}} = H(p_1, p_2, p_3, p_4, p_5), \quad (6.25)$$

where

$$\begin{aligned} H(p_1, p_2, p_3, p_4, p_5) = & \frac{2}{81} \sum_{l=1}^3 \frac{(1l)(2l)((1l)^2 + (2l)^2)}{(13)(14)(15)(23)(24)(25)} \left\{ 5\hat{s} - 9 \left[\frac{(12; 34)}{(34)} + \frac{(12; 13)}{(35)} + \frac{(12; 45)}{(45)} \right] + \right. \\ & \left. \frac{81}{\hat{s}} \left[\frac{(12; 55)(12; 34)}{(35)(45)} + \frac{(12; 33)(12; 45)}{(34)(35)} + \frac{(12; 44)(12; 35)}{(34)(45)} \right] \right\}, \end{aligned} \quad (6.26)$$

with

$$(kl; ij) \equiv (ki)(lj) + (kj)(li). \quad (6.27)$$

$$(3) \quad qm(p_1) + qn(p_2) \longrightarrow qm(p_3) + qn(p_4) + g(p_5), \quad m \neq n.$$

The averaged amplitude squared is

$$\overline{\mathcal{M}} = F(p_1, p_2, p_3, p_4, p_5), \quad (6.28)$$

where

$$\begin{aligned} F(p_1, p_2, p_3, p_4, p_5) = & \frac{\tilde{s}^2 + s'^2 + \tilde{u}^2 + u'^2}{8\tilde{t}t'(15)(25)(35)(45)} \times \\ & \left\{ \frac{16}{27} [(\tilde{u} + u')(\tilde{s}s' + \tilde{t}t' - \tilde{u}u') + \tilde{u}(\tilde{s}\tilde{t} + s't') + u'(\tilde{s}t' + s'\tilde{t})] - \right. \\ & \left. \frac{2}{27} [(\tilde{s} + s')(\tilde{s}s' - \tilde{t}t' - \tilde{u}u') + 2\tilde{t}t'(\tilde{u} + u') + 2\tilde{u}u'(\tilde{t} + t')] \right\}, \end{aligned} \quad (6.29)$$

with

$$\begin{aligned}\tilde{s} &= (p_1 + p_2)^2, & s' &= (p_3 + p_4)^2, \\ \tilde{t} &= (p_1 - p_3)^2, & t' &= (p_2 - p_4)^2, \\ \tilde{u} &= (p_1 - p_4)^2, & u' &= (p_2 - p_3)^2.\end{aligned}\tag{6.30}$$

$$(4) \quad qm(p_1) + qm(p_2) \longrightarrow qm(p_3) + qm(p_4) + g(p_5)$$

The averaged amplitude squared is

$$\overline{\mathcal{M}} = F'(p_1, p_2, p_3, p_4, p_5),\tag{6.31}$$

where

$$\begin{aligned}F'(p_1, p_2, p_3, p_4, p_5) = & \\ & F(p_1, p_2, p_3, p_4, p_5) + F(p_1, p_2, p_4, p_5, p_5) + \\ & \frac{(\tilde{s}^2 + s'^2)(\tilde{s}s' - \tilde{t}t' - \tilde{u}u')}{8\tilde{t}t'\tilde{u}u'(15)(25)(35)(45)} \times \\ & \left\{ \frac{2}{9}(\tilde{s} + s')(\tilde{s}s' - \tilde{t}t' - \tilde{u}u') + \frac{4}{81} [\tilde{t}t'(\tilde{u} + u') + \tilde{u}u'(\tilde{t} + t')] - \right. \\ & \left. 2\tilde{s}(\tilde{t}\tilde{u} + t'u') - 2s'(\tilde{t}u' + t'\tilde{u}) \right\}.\end{aligned}\tag{6.32}$$

The quantities $\tilde{s}$, $\tilde{t}$, $\tilde{u}$, s' , t' , and u' are defined in Eq. (6.30).

The differential cross section, which we are interested in, is shown in Eq. (6.16).

The light quarks are taken to be massless. Since we cannot distinguish quark jets

from gluon jets, all the final particles are identical. In order to achieve that, we symmetrize the squared amplitudes by adding new terms in which momentum labels are exchanged between non-identical final partons in the original amplitudes. Then, the differential cross section in Eq. (6.16) must be multiplied by $(3/3!)=1/2$; the factor $1/3!$ is for 3 assumed identical final partons and the factor 3 is for 3 ways to form a pair among three partons. We also require $|y_i| < 1.5$ for every jet so that they are not close to the beam line.

The integration is done by Monte Carlo method. Numerical results are shown in Figure 6.5. A comparison of the production rates of $P^{0'}$ and $P_8^{0'}$ is given in Figures 4.6 and 4.7 shows that this 3-jet background production rate is larger than the signals. When the BR and width of $P^{0'}$ and $P_8^{0'}$ are taken into account, the background is at least two order of magnitude larger than the signals. So, the signals of $P^{0'}$ and $P_8^{0'}$ decaying to $2g$ cannot be observed at the SSC.

6.3. The Background of the $\tau\bar{\tau}$ Channel

Although the $\tau\bar{\tau}$ mode of $P^{0'}$ has a small branching ratio, varying from 20% to 5% for $m_P = 20$ to 100 GeV, the signal is comparable to the background for $15 \text{ GeV} < m_P < 60 \text{ GeV}$ in EHLQ study [22]. Thus, the $\tau\bar{\tau}$ mode may be useful in our reactions for the discovery of $P^{0'}$ at the SSC. We investigate next the background of the three particle final state, $\tau\bar{\tau}g(q)$. This background comes from the following processes

$$q(p_1) + \bar{q}(p_2) \longrightarrow g(p_3) + \tau(p_4) + \bar{\tau}(p_5) \quad (6.33)$$

$$q(p_1) + g(p_2) \longrightarrow q(p_3) + \tau(p_4) + \bar{\tau}(p_5) \quad (6.34)$$

Table 6.7. Various 3-jet processes and their averaged amplitudes squared

processes	$\overline{\mathcal{M}}$
$gg \longrightarrow ggg$	$G(p_1, p_2, p_3, p_4, p_5)$
$gg \longrightarrow q_m \bar{q}_m g$	$\frac{9}{64} H(-p_1, -p_3, -p_2, -p_4, p_5)$
$q_m g \longrightarrow q_m gg$	$\frac{-3}{8} H(p_1, -p_3, -p_2, p_4, p_5)$
$q_m g \longrightarrow q_m q_m \bar{q}_m$	$\frac{-3}{8} F'(p_1, -p_5, p_3, p_4, -p_2)$
$q_m g \longrightarrow q_m q_n \bar{q}_n$	$\frac{-3}{8} F(p_1, -p_5, p_3, p_4, -p_2)$
$q_m q_n \longrightarrow q_m q_n g$	$F(p_1, p_2, p_3, p_4, p_5)$
$q_m q_m \longrightarrow q_m q_m g$	$F'(p_1, p_2, p_3, p_4, p_5)$
$q_m \bar{q}_n \longrightarrow q_m \bar{q}_n g$	$F(p_1, -p_4, p_3, -p_2, p_5)$
$q_m \bar{q}_m \longrightarrow q_m \bar{q}_m g$	$F'(p_1, -p_4, p_3, -p_2, p_5)$
$q_m \bar{q}_m \longrightarrow q_n \bar{q}_n g$	$F(p_1, -p_3, -p_2, p_4, p_5)$
$q_m \bar{q}_m \longrightarrow ggg$	$H(p_1, p_2, p_3, p_4, p_5)$

Figure 6.5. The differential cross section for $p + p \rightarrow g(q) + g(q) + g(q) + X$. M and y are the invariant mass and rapidity of a pair of partons. $|y_i| < 1.5$: $M = 100 \text{ GeV}$ (upper curve), $M = 300 \text{ GeV}$ (lower curve)

Their Feynman diagrams are shown in Figures 6.6 and 6.7. The matrix elements of the processes, (6.34) and (6.33), were calculated by Aurenche and Lindfors [61]. We list them in the following

$$\begin{aligned} \overline{\mathcal{M}}_i = & 4c_i g_s^2 e^4 K^2 \left\{ \frac{Q_q^2}{K^4} T_i|_{1,0}^{1,0} + \right. \\ & \frac{1}{\sin^4 \theta_w \cos^4 \theta_w} \frac{1}{(K^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2} T_i|_{A_q, B_q}^{A_l, B_l} - \\ & \left. 2Q_q \frac{1}{\sin^2 \theta_w \cos^2 \theta_w} \frac{K^2 - m_Z^2}{K^2 [(K^2 - m_Z^2)^2 + (\Gamma_Z m_Z)^2]} T_i|_{a_q, b_q}^{a_l, b_l} \right\}, \quad (6.35) \end{aligned}$$

where Q_q is the electric charge of the quark. $i = 1$ is for $q\bar{q}$ annihilation and $i = 2$ is for qg scattering,

$$\begin{aligned} T_1|_{a_2, b_2}^{a_1, b_1} = & -a_1 a_2 \left\{ 1 + \frac{\hat{s} - 2t_1 - K^2}{\hat{t}} - \frac{t_1^2 + t_2^2 + \hat{s}(t_1 + t_2 + K^2)}{\hat{t}\hat{u}} + \right. \\ & \left. [\hat{u} \leftrightarrow \hat{t}, t_1 \leftrightarrow t_2] \right\} - \\ & b_1 b_2 \left\{ \frac{\hat{s} + 2t_2 + K^2}{\hat{t}} + \frac{t_1 - t_2}{\hat{t}\hat{u}} - [\hat{u} \leftrightarrow \hat{t}, t_1 \leftrightarrow t_2] \right\}, \quad (6.36) \end{aligned}$$

$$\begin{aligned} T_2|_{a_2, b_2}^{a_1, b_1} = & -a_1 a_2 \left\{ \frac{\hat{t} - 2(t_1 + K^2)}{\hat{s}} + \frac{\hat{s} + 2(t_1 + t_2)}{\hat{t}} + \right. \\ & \left. \frac{2}{\hat{s}\hat{t}} [(t_1 + t_2 + K^2)^2 + t_1^2 - t_2 K^2] \right\} - \\ & b_1 b_2 \left[\frac{2(t_1 + K^2) - \hat{t}}{\hat{s}} + \frac{\hat{s} + 2(t_1 + t_2)}{\hat{t}} - \right. \\ & \left. \frac{2K^2(2t_1 + t_2 + K^2)}{\hat{s}\hat{t}} \right], \quad (6.37) \end{aligned}$$

Figure 6.6. Feynman diagrams for $q + \bar{q} \rightarrow g + \tau + \bar{\tau}$

and

$$\hat{s} = (p_1 + p_2)^2, \quad t_1 = (p_1 - p_4)^2,$$

$$\hat{t} = (p_2 - p_3)^2, \quad t_2 = (p_2 - p_4)^2,$$

$$\hat{u} = (p_1 - p_3)^2, \quad K^2 = (p_4 + p_5)^2,$$

$$c_1 = \frac{4}{9},$$

$$c_2 = \frac{1}{6},$$

$$a_l = -\frac{1}{4} + \sin^2 \theta_w,$$

$$b_l = \frac{1}{4},$$

$$a_u = a_c = \frac{1}{4} - \frac{2}{3} \sin^2 \theta_w, \quad b_u = b_c = -\frac{1}{4},$$

$$a_d = a_s = -\frac{1}{4} + \frac{1}{3} \sin^2 \theta_w, \quad b_d = b_s = \frac{1}{4},$$

$$A_f = a_f^2 + b_f^2,$$

$$B_f = 2a_f b_f.$$

In the above matrix elements, the mass of the τ is set to zero. Using the physical mass of the τ makes almost no difference in the numerical values of the matrix element (6.35). This is expected, because of the high center of mass energy of the processes involved.

Figure 6.7. Feynman diagrams for $q + g \rightarrow q + \tau + \bar{\tau}$

The differential cross sections, evaluated according to (6.16), are shown in Figure 6.8. The QCD backgrounds are very small and can be neglected. As shown in Table 6.8, the number of events where $P^{0'}$ decays into a $\tau\bar{\tau}$ pair is not large. It seems that $P^{0'}$ can only be seen for masses $\lesssim 300$ GeV for $P_{\perp}$ around 100 GeV.

Figure 6.8. The differential cross section for $p + p \rightarrow \tau + \bar{\tau} + g(q) + X$: From top to bottom, $M_{\tau\tau} = 100, 50, 200$, and 300 GeV

Table 6.8. The number of events of $P^{0'}$ decaying to $\tau\bar{\tau}$ per GeV bin on $P_{\perp}$ at the SSC

$P_{\perp}$ GeV	$m_P =$	50	100	200	300	GeV
100		1.5×10^4	4.4×10^3	940	324	
300		700	250	70	27	
500		140	52	10	7	
700		45	17	5	2	

7. CONCLUSIONS

The main purpose of this work is to study the production and decay of the neutral technicolor pseudo Goldstone particles, $P^{0'}$ and $P_8^{0'}$ in the processes

$$p + p \longrightarrow g(q) + P^{0'}(P_8^{0'}) + X \quad (7.1)$$

at large transverse momentum at the SSC energy regime. The background due to QCD processes has been investigated in detail in order to determine the best signals to detect these particles.

For the production, we found that the lowest order differential cross sections $d\sigma/(dP_\perp dy)|_{y=0}$ are about the order of 10^{-3} to 10^{-5} nb/GeV for $P^{0'}$ and of 10^{-2} to 10^{-4} nb/GeV for $P_8^{0'}$ for $m_P = 100$ to 400 GeV and $P_\perp = 100$ to 700 GeV. With the projected luminosity of the SSC, $\mathcal{L} \sim 3 \times 10^7 \text{ nb}^{-1} \text{ year}^{-1}$, The number of events is of the order of 10^5 to 10^2 per GeV bin of $P_\perp$ per year at $y=0$ depending on $P_\perp$. The number of $P_8^{0'}$ produced is about one order of magnitude larger.

For the decays of $P^{0'}$ and $P_8^{0'}$, we have investigated the major decay channels of two-body and three-body final states at tree level. We have presented the analytic form of partial decay widths for all important decay channels. We found that the total widths of $P^{0'}$ and $P_8^{0'}$ are small, from $O(10^{-2})$ GeV without the $t\bar{t}$ channel to $O(1)$ GeV with the $t\bar{t}$ channel for $P^{0'}$ and from $O(10^{-1})$ GeV to $O(1 \rightarrow 10)$ GeV for $P_8^{0'}$. The $t\bar{t}$ mode dominates if it is allowed. The $t\bar{t}g$ channel is negligible up to

$m_P = 400$ GeV. The $2g$ and $3g$ channels dominate if the $t\bar{t}$ channel is not allowed, except at $m_P < 40$ GeV where the $b\bar{b}$ channel becomes dominant. The branching ratios (BR) of the $q\bar{q}g$ and the $b\bar{b}g$ modes are about few percent when the $t\bar{t}$ channel is not present. The $\tau\bar{\tau}$ channel gives more than a 10% contribution for $m_P < 50$ GeV, and decreases to $\sim 1\%$ after m_P reaches 300 GeV, again assuming that the $t\bar{t}$ channel is not available.

Our results show that the background of $2g$ mode is so large that it renders the $2g$ mode useless for the purpose of finding $P^{0'}$ and $P_8^{0'}$. If there is no $t\bar{t}$ decay channel, we may be able to probe $P^{0'}$ through the $\tau\bar{\tau}$ channel up to $m_P = 300$ GeV at $P_\perp = 100$ GeV. However, the actual detection of $P^{0'}$ depends on the efficiency of the detector used to identify the $\tau\bar{\tau}$ pair.

We found that the $b\bar{b}$ signal for $P^{0'}$ is buried under the background and is not useful for identifying the $P^{0'}$. But for $P_8^{0'}$, the $b\bar{b}$ signal is comparable to the QCD background for m_P up to 300 GeV except for $P_\perp < 300$ GeV where the background becomes much larger than the signal. Using the $b\bar{b}$ signal, it is possible to investigate the existence of $P_8^{0'}$ in the mass region up to 300 GeV, in case there is no $t\bar{t}$ channel.

When the $t\bar{t}$ channel is available, the situation becomes much better. For $P^{0'}$, the signal is larger than the background except at $P_\perp = 100$ GeV and $m_t = 80$ GeV, where the signal and the background become comparable. In the case of $P_8^{0'}$, the signals are much larger than the backgrounds so that we can simply neglect the background. The discovery potential of this channel, again, depends on the efficiency in identifying the $t\bar{t}$ pair.

The mass of $P^{0'}$ is estimated less than 40 GeV from extended technicolor theory by Eichten and Lane [39]. However, Appelquist et al. [45] proposed a slow running

coupling scheme for the technicolor interaction which could bring the $P^{0'}$ mass to $a \cdot (50-70)$ GeV; a is the order of unity. From these $P^{0'}$ mass predictions and the fact that the t quark mass is greater than 78 GeV, the $P^{0'}$ is unlikely to decay into a $t\bar{t}$ pair. Thus, the $\tau\bar{\tau}$ channel is the only possibility for identifying the $P^{0'}$. Furthermore, the search for the $\tau\bar{\tau}$ signal has to be carried out at low transverse momentum, around 100 GeV, as the signal deteriorates rapidly as $P_{\perp}$ increases.

The mass of $P_8^{0'}$ mainly comes from QCD interactions and is around 240 GeV in the Farhi-Susskind one family model. If we add the mass contribution due to the mechanism of Appelquist et al., the $P_8^{0'}$ mass would be increased to around 300 GeV. The upper bound of t mass is about 200 GeV from the study of the radiative corrections to the ρ parameter in the standard model [62]. Since $b\bar{b}$ decays can serve as an effective signal for the $P_8^{0'}$ mass for m_P up to 300 GeV, it would be very difficult to detect $P_8^{0'}$ decays in the region $300 \text{ GeV} < m_P < 400 \text{ GeV}$ if $m_t = 200 \text{ GeV}$. In the EHLQ production scheme, the detection of $P_8^{0'}$ in the absence of the $t\bar{t}$ channel is also very difficult for $m_P < 600 \text{ GeV}$. For $m_P > 600 \text{ GeV}$, the $2g$ signal is comparable to the background and can be used as a signal. Despite the small rate of the $b\bar{b}$ signal, it may provide a way to overcome these difficulties in the mass region 300—400 GeV. However, this would require a long running time at the SSC.

There are several possibilities for detecting high transverse momentum decays of the $P^{0'}$ and $P_8^{0'}$ at the SSC. Since the $P^{0'}$ and $P_8^{0'}$ are produced nearly transverse to the beam direction, it should be easier to detect the signals. Because there are two-body decay channels which are larger than or comparable to the corresponding backgrounds, we conclude that (7.1) are promising processes in the investigation of the neutral technicolor particles, $P^{0'}$ and $P_8^{0'}$.

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9. APPENDIX

The couplings of pseudo-Goldstone bosons to gauge bosons can be determined by the anomalous terms in the low energy effective Lagrangian, known as the Witten-Wess-Zumino [63] effective Lagrangian. The couplings involving $P^{0'}$ and $P_8^{0'}$ are the following

$$\frac{-N_{tc}g_s^2}{16\pi^2 F_\Pi} \left\{ \frac{1}{4\sqrt{3}} \phi^0 G_{\mu\nu}^a \tilde{G}^{a\mu\nu} + \frac{1}{2\sqrt{2}} d^{abc} \phi^{8a} G_{\mu\nu}^b \tilde{G}^{c\mu\nu} \right\} \quad (9.1)$$

with

$$G_{\mu\nu}^a = \partial_\mu G_\nu^a - \partial_\nu G_\mu^a - gf^{abc} G_\mu^b G_\nu^c \quad (9.2)$$

where ϕ^0 and ϕ^8 are fields representing $P^{0'}$ and $P_8^{0'}$ respectively.

The $P^{0'}$ and $P_8^{0'}$ couplings to fermions come from the extended technicolor theory, which gives a four-fermion type interaction, and thus generates mass term for fermions when techni-condensates are formed. Similar to the neutral Higgs field coupling to the fermions, which reads

$$g_H f \bar{f} = \frac{m_f}{v}, \quad (9.3)$$

the coupling of neutral PGB and fermions is given by

$$g_{Pf\bar{f}} \sim \frac{g_{ETC}^2}{M_{ETC}^2} \Lambda_{TC}^2 \sim \frac{m_f}{\Lambda_{TC}} \sim \frac{m_f}{F_\Pi}. \quad (9.4)$$

In General, these neutral PGB couplings will induce flavor changing neutral currents (FCNC). In Ellis et al. [40], a monophagic model is proposed. In this model, the up-type quarks couple to only one type of technifermions and the down-type quarks couple to another type. Then, there is no FCNC due to neutral PGBs. Ellis et al. estimated the couplings of $P^{0'}$ and $P_8^{0'}$ to fermions which we give below;

$$\frac{\phi^0}{F_\Pi} \frac{1}{2} \left[\sqrt{\frac{1}{3}} (\bar{p} m_P \gamma_5 p + \bar{n} m_n \gamma_5 n) - \sqrt{3} (\bar{l} m_l \gamma_5 l) \right], \quad (9.5)$$

and

$$\frac{\phi_a^8}{F_\Pi} \sqrt{2} \left[\bar{p} m_P \gamma_5 \frac{\lambda_a}{2} p + \bar{n} m_n \gamma_5 \frac{\lambda_a}{2} n \right] \quad (9.6)$$

where $F_\Pi = 125\text{GeV}$, $p \equiv (u, c, t)^T$, $n \equiv (d, s, b)^T$, $l \equiv (e, \mu, \tau)^T$, and $m_{p,n,l}$ are the diagonalized mass matrices for the up-, down-, and charged lepton-type fermions.

Figures 9.1 and 9.2 show the vertices of $P^{0'}$ and $P_8^{0'}$ coupling to the gluons and to the fermions. The constants, Γ and Γ' , are defined as

$$\Gamma = \frac{-N_{tc} g_s^2}{16\sqrt{3}\pi^2 F_\Pi} \quad (9.7)$$

$$\Gamma' = \frac{-N_{tc} g_s^2}{8\sqrt{2}\pi^2 F_\Pi}. \quad (9.8)$$

In the monophagic model, $G_0 = 1/2\sqrt{3}$ for quarks, $G_0 = \sqrt{3}/2$ for leptons, and $G_8 = \sqrt{2}$ for quarks.

$$2i\Gamma\epsilon^{\mu\nu\alpha\beta}\delta^{ab}k_{1\alpha}k_{2\beta}.$$

$$2g_s\Gamma\epsilon^{\mu\nu\alpha\beta}f^{abc}(k_1+k_2+k_3)_\mu.$$

$$G_0\frac{m_f}{F_\Pi}\gamma_5.$$

Figure 9.1. The vertices of P'' coupling to the gluons and to the fermions

$$2i\Gamma'\epsilon^{\mu\nu\alpha\beta}d^{abc}k_{1\alpha}k_{2\beta}.$$

$$2g_s\Gamma'\epsilon^{\mu\nu\alpha\beta}(d^{abe}f^{cde}k_{1\mu} + d^{ace}f^{dbe}k_{2\mu} + d^{ade}f^{bce}k_{3\mu}).$$

$$2ig_s^2\Gamma'\epsilon^{\mu\nu\alpha\beta}d^{agh}(f^{bcg}f^{deh} + f^{bdg}f^{ech} + f^{beg}f^{cdh}).$$

$$-g_s f^{abc}(k_1 + k_2)_\mu.$$

$$G_8 \frac{m_f}{F_\Pi} T^a \gamma_5.$$

Figure 9.2. The vertices of P''_μ coupling to the gluons and to the fermions

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