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FLUID FLOW EFFECTS ON ELECTROPLATING

J. R. Kirkpatrick

Computing and Telecommunications Division
at the
Oak Ridge Gaseous Diffusion Plant
Post Office Box 2003
Oak Ridge, Tennessee 37831-7039

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NOMENCLATURE*

c_f	skin friction coefficient, nondimensionalization of shear stress
d	diameter of cylinder, disk, or sphere; base diameter of cone (length)
D	mass diffusivity of ions in electrolyte (length ² /time)
Nu	Nusselt number, nondimensionalization of heat transfer coefficient
Pr	$Pr = \nu/\alpha$, Prandtl number (α = thermal diffusivity)
r	local radius (length)
R	$R = d/2$, radius of cylinder, disk, or sphere; base radius of cone (length)
Re_d	$Re_d = d V/\nu$, Reynolds number based on diameter or base diameter
Re_r	$Re_r = r v/\nu$, Reynolds number based on local radius
Re_R	$Re_R = R V/\nu$, Reynolds number based on radius or base radius
Sc	$Sc = \nu/D$, Schmidt number
Sh	Sherwood number, nondimensionalization of mass transfer coefficient
u	a scale velocity for use in the skin friction coefficient (length/time)
v	Ωr , local surface angular velocity (length/time)
V	ΩR , surface angular velocity at perimeter of object (length/time)
x	scale length for use in Nusselt and Sherwood numbers (length)
α	cone half-angle (degrees or radians); also thermal diffusivity (length ² /time)
δ_c	concentration boundary layer thickness (length)
δ_m	momentum boundary layer thickness (length)
δ_T	thermal diffusion boundary layer thickness (length)
θ	angle between axis of sphere and local radius vector (degrees or radians)
ν	kinematic viscosity (length ² /time)
Ω	angular rotation rate of object (radians/time)

*Symbols that are not nondimensional have dimensions indicated.

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J. R. Kirkpatrick

Computing and Telecommunications Division
Oak Ridge Gaseous Diffusion Plant

ABSTRACT

The effects of fluid flow patterns on the electroplating of rotating, cylindrically symmetric objects are examined. Ways are outlined for preventing undesirable spiral patterns on the plated surface. Estimates are given for the diffusion boundary layer thickness for cylinders, disks, spheres, and cones.

INTRODUCTION

When an object that is axisymmetric is to be electroplated, the object is usually rotated about its axis of symmetry. This rotation drives a circulation in the electrolyte, which helps to transport the ions from the anode to the cathode. There are numerous papers in the open literature that deal with the influence of the fluid circulation on the plating process.^{1,2} Usually, the fluid effects are lumped into an effective concentration boundary layer thickness, which is used to construct an additional electrical resistance due to fluid flow.

Some electroplating experiments have shown differences in the plating rate and in the quality of the plate that are caused by changes in the rotation rate of the part being plated. Thus, it is clear that fluid flow effects can be significant on electroplating.

One example of a significant fluid flow effect is the patterns of spiral grooves that have appeared on the surfaces of the plated parts in some electroplating experiments. These grooves correspond to a fluid flow pattern that appears as part of the transition from laminar to turbulent flow in rotating objects. The electroplating process is intended to produce plated layers that are as close as feasible to the specified thickness, so that these grooves are undesirable. Thus, there is a considerable advantage to arranging the electroplating apparatus so that no grooves are formed.

The body of this report contains two major parts. The first one deals with the subject of the spiral grooves, the origin of these grooves, and how to prevent them. The second part discusses the concept of the concentration boundary layer thickness; it contains estimates of this thickness for four specific part shapes (cylinders, disks, spheres, and cones).

SPIRAL PATTERNS: THEIR ORIGIN AND WAYS TO PREVENT THEM

ORIGIN OF THE SPIRALS

I presume that the grooves that sometimes appear on the surfaces of plated parts correspond to spirals in the flow field of the electrolyte. The spiral flow pattern is not a quirk of the electroplating process but rather a well-known flow that has been observed for spinning disks, spheres, cones, and cylinders. The spirals represent an intermediate flow between the fully laminar and the fully turbulent. They are part of the process of laminar to turbulent transition for rotating objects.

SPIRALS ON CYLINDERS

There is a considerable amount of literature on the subject of flow instabilities for concentric rotating cylinders. The cases that are of most interest for this study are those for which the inner cylinder is rotating and the outer one is fixed. It is well known that this is an unstable situation. At some value of the Reynolds number (see the Nomenclature section of this report), the flow changes from an azimuthal rotation with no axial or radial velocities to a Taylor vortex flow. The Taylor vortex is an azimuthally symmetric, closed circulation. In other words, it is a vortex that is closed upon itself and rotates around an axis that is a closed ring. The axis of this ring is coincident with the axis of the rotating cylinder. At some larger value of the Reynolds number, the flow field adds a set of vortices whose axes trace spirals from the ends of the cylinder toward the middle. (If there are flanges on the end of the cylinder, the path of the spirals is usually from the middle toward the ends.) These spirals appear in a small portion of the gap next to the inner, rotating cylinder. I presume that it is these spirals whose paths are shown by the grooves in the electroplated material.

As the Reynolds number continues to increase, the eventual result is turbulence. The progression to turbulence in rotating objects seems to be a smooth process in which the spirals develop additional modes of instability while gaining more energy until they disintegrate into microscopic, random vortices and thus the flow becomes turbulent. This idea of a continuous transition from laminar flow to turbulence has been reinforced by the moment coefficient curve in Theodorsen and Regier,³ which shows a relatively smooth change from laminar Couette flow to full turbulence. There is no clear jump in moment coefficient to mark the transition to turbulence in contrast to the situation with rotating disks (see, for example, Fig. 2 of ref. 3) and, of course, the familiar friction coefficient curve for flow in pipes.

The subject of the onset of Taylor vortices in the gap between cylinders is one that is of interest to large numbers of applied mathematicians. Unfortunately, the next transition, the one to spirals, is a more difficult calculation and has not been studied very much. The only theoretical work I have found is one by DiPrima and Rogers⁴ for the case with a narrow gap. For the electroplating cases in which I am interested, the gap is usually larger than the radius of the spinning part so that the results of DiPrima and Rogers may not be very useful. DiPrima and Rogers have

commented that the Reynolds number for the onset of spirals for cases with a narrow gap is about 3-5% larger than that for the onset of Taylor vortices, but it becomes about 100 times that for the onset of Taylor vortices if the gap is the same width as the radius of the inner cylinder.

One objective of this study of the transition processes for cylinders is to determine the conditions that would prevent spirals. In this context, Snyder⁵ had reported an equation that gives the Reynolds number for the onset of instabilities in Taylor vortices as a function of the ratio of the gap to the radius of the inner cylinder. The minimum value given by this equation is for a gap that is exactly the same as the inner cylinder radius. This minimum value is Re_d about 550. According to DiPrima and Rogers, the spirals occur (for wide gaps) when the Reynolds number is as much as 100 times the value for the onset of Taylor vortices. The onset of Taylor vortices for wide gaps seems to occur for Re_d about 20, which makes the estimate of DiPrima and Rogers more or less consistent with Snyder's result. Kasagi and Hirata⁶ had suggested that the onset of spirals occurs at Re_d about 700. For some lead cylinder electroplating experiments,⁷ spiral grooves were seen on the surface of the cylinder for $Re_d = 2000$. This compares to a value of about 1250 given by Snyder's fit for the onset of Taylor vortices for that experiment. (This value was 1250 rather than 550 because the gap was narrower than the radius of the inner cylinder.) In another of the same series of experiments, no spirals were seen for Re_d about 480. There are some doubts about the utility of Snyder's fit for short gaps. However, the fact that none of the sources cited in this paragraph reported spirals at a Reynolds number less than 550 seems to indicate that the minimum value for Snyder's fit may be usable. Thus, the conclusion I draw is that running at $Re_d < \text{about } 550$ is probably sufficient to prevent spirals from appearing on a cylindrical part.

The other half of the question of how to prevent spirals on a cylinder is what value of the Reynolds number is necessary to reach fully turbulent flow. The data I collected are measurements from Theodorsen and Regier,³ some others quoted in Schlichting⁸ (pp. 500-503), and the paper by Kasagi and Hirata.⁶ The measurements described in Schlichting were made for narrow to very narrow gaps. Those from Theodorsen and Regier were apparently made for a wide gap. Those of Kasagi and Hirata were definitely for a wide gap. For the results from Schlichting, turbulence was seen at Re_d about 1.7×10^5 for the very narrow gap and about 4.2×10^4 for the narrow gap. For the Theodorsen and Regier data, the moment coefficient measurements reached the curve fit through the turbulent data for Re_d about 2×10^3 . (Note that the cylinder Reynolds numbers reported in Theodorsen and Regier are based on cylinder radius, Re_R , and are thus half as large as those based on diameter, Re_d .) Kasagi and Hirata claimed that the flow was "substantially" turbulent for a cylinder with a very wide gap in air for Re_d as small as 1×10^3 . However, it is not clear whether the flow patterns for a Reynolds number that small are fully turbulent or are a mixture of small spirals with true turbulence. For all but the very narrow gaps, it is probable that the cylinder should be fully turbulent for $Re_d > \text{about } 2 \times 10^3$.

In summary, my conclusions for the cylinder are as follows:

1. Running at $Re_d < \text{about } 550$ is probably sufficient to prevent the formation of spirals on a cylindrical part.

2. For all but very narrow gaps, it is probable that the cylinder should be fully turbulent for $Re_d > \text{about } 2 \times 10^3$.

SPIRALS ON DISKS

Schlichting⁸ has a picture of a spinning disk (p. 508) that shows the features of the laminar-to-turbulent transition on a disk. At the center of the disk is a region of laminar flow. At a radius such that the local Reynolds number, Re_r (see the Nomenclature section of this report) is about 1.9×10^5 , a clear pattern of spirals appears. By the radius such that Re_r is about 2.8×10^5 , the spirals have broken up so that the flow is fully turbulent. The moment coefficient data reported by Theodorsen and Regier³ show transition to full turbulence for Re_R ($Re_R = Re_d/2$) about 3.1×10^5 , which is within 10% of Schlichting's value for full turbulence. Kobayashi et al.⁹ show patterns of flow on rotating disks that are similar to those from Schlichting. Their value of the local Reynolds number for a transition to full turbulence is 3.2×10^5 , which is certainly consistent with the results from the other sources just cited. However, for the onset of spirals, they measured a local Reynolds number of 8.8×10^4 , a value that is about half that shown in Schlichting.

As is the case with the cylinder, an objective of this study of the fluid flow effects on electroplating is to determine how to prevent the spiral region on a disk. From the evidence I have seen in Schlichting and in Kobayashi et al., it seems to me that, if a disk were rotated at $Re_d < \text{about } 1.8 \times 10^5$, the spirals should not appear. [Note that the change from Reynolds numbers based on the radius (Re_R) as reported in Schlichting and in Kobayashi et al. to those based on a diameter (Re_d) causes a factor of two increase in Reynolds numbers for determining transition thresholds. Note also that I have taken the value from Kobayashi et al. as a more conservative estimate for the onset of spirals.] For Reynolds numbers higher than about 1.8×10^5 it is apparently not possible to prevent the spirals without some sort of boundary layer trip. In Theodorsen and Regier, a "small high pressure air jet" (which was presumably turbulent) aimed at the center of the disk was able to reduce the turbulent transition to Re_d about 1.8×10^5 , or less. I think it interesting that this is the Reynolds number needed to make the spirals appear according to Kobayashi et al.

My expectation, which is based more on gut feeling than any definite knowledge, is that it should be possible to prevent ever having spirals on a disk by hitting the axis with a turbulent jet. If the disk were spinning too slowly to form spirals, then the turbulence would probably decay without making spirals. If the disk were spinning fast enough to make spirals, then the small eddies in the turbulent jet would already be present to absorb the energy that would otherwise go into the spirals. I suspect that it will be necessary to rotate a part at least as fast as the spiral point, which is Re_d about 1.8×10^5 , if it were considered desirable that the boundary layer on a disk that had a jet for a boundary layer remain turbulent.

In electroplating processes, it is fairly common to have one or more jets aimed at the part. My recommendation of the use of a jet as a boundary layer trip is not a recommendation for a new electroplating practice but rather an additional justification of a feature that is already common practice.

In summary, my conclusions for the disk are as follows:

1. If a disk is rotated at $Re_d < \text{about } 1.8 \times 10^5$, spirals should not appear.
2. Spirals probably cannot be prevented for $Re_d > \text{about } 1.8 \times 10^5$ without the use of a boundary layer trip.
3. With the use of a suitable boundary layer trip—such as a jet aimed at the axis—it should be possible to prevent spirals by going directly to full turbulence for $Re_d > 1.8 \times 10^5$.

SPIRALS ON SPHERES

A geometry that is particularly applicable to the electroplating work I studied is that of a rotating sphere. Kohama and Kobayashi¹⁰ had given a detailed description of the transition from laminar to turbulent flow on rotating spheres. As was the case with rotating disks, the spirals appeared as an intermediate flow pattern between the laminar and the fully turbulent. The onset of spirals did not occur at the same spin Reynolds number for all of the tests. Instead, there was a diameter effect. For spheres of 150-mm diameter or greater, the minimum Reynolds number for the onset of spirals was Re_d about 2.4×10^4 . For a 70-mm sphere, the onset occurred at Re_d about 7×10^3 . Kim¹¹ found spiral patterns on hemispheres of 7–9 cm diameters in aqueous solutions for $Re_d > \text{about } 1.4 \times 10^4$.

Kohama and Kobayashi¹⁰ have suggested that the minimum rotation rate for the onset of spirals is that for which the onset occurs at the equator, a location where the instabilities are much like those of a cylinder. Therefore, I think it probable that the minimum Reynolds number for the onset of spirals in a sphere of a very small diameter would be close to the value for a cylinder which, as was stated earlier, is Re_d about 550. The experiments cited by Kobayashi and Kohama were done for air. I think it unlikely that the diameters for which the diameter effect is important would be the same for air as they are for water. Instead, a scaling rule could probably be used, but I do not know what this rule should be. Thus, I believe that the only reasonably reliable criterion for guaranteeing that there will be no spirals for a sphere should be the same as that for a cylinder (i.e., $Re_d < 550$). As was the case with the disk, there seems to be no Reynolds number that will give a fully turbulent flow field without some boundary layer trip. The minimum Reynolds number needed to achieve turbulent transition in Kohama's and Kobayashi's experiments was Re_d about 1.4×10^5 . The heat transfer experiments of Kreith et al.¹² showed turbulent transition at Re_d about 2.5×10^4 . These two different sets of measurements give transition values smaller than that needed to achieve turbulence for a disk, but considerably larger than the value that was estimated to be needed for a cylinder with a wide gap.

As was the case with cylinders and disks, criteria for preventing spirals on spheres are desired. The criterion for guaranteeing that the flow will remain laminar with no spirals is $Re_d < 550$, which, according to Kim's¹¹ results, is extremely conservative for spheres of any but minuscule size in water. For high Reynolds numbers there seems to be no way to prevent spirals on a sphere without using some sort of boundary layer trip. As was the case

for the disk, I think it probable that a turbulent jet aimed at the axis (the pole) will be sufficient to prevent spirals from forming. As was also the case for the disk, the sphere should probably be turned fast enough to reach into the region for which spirals might appear or else the boundary layer might not be able to remain turbulent. The criterion for this is $Re_d >$ about 2.4×10^4 .

In summary, my conclusions for the sphere are as follows:

1. It should be possible to guarantee that no spirals will appear if the sphere is limited to $Re_d < 550$.
2. Spirals probably cannot be prevented for $Re_d > 550$ without the use of a boundary layer trip.
3. With the use of a suitable boundary layer trip—such as a jet aimed at the axis—it should be possible to prevent spirals by going directly to full turbulence for $Re_d > 2.4 \times 10^4$.

SPIRALS ON CONES

The last geometry I studied for fluid flow effects on electroplating was the spinning cone. Part of the relevance of the cone is that a cylinder is a cone of zero half-angle. Thus, the limit as the cone half-angle becomes zero should be a good check on estimates for the cylinder. In addition, it is possible that someone might want to plate a conical object.

Kobayashi and Izumi¹³ had shown data for cones in air with half-angles from 15 to 75°. Figure 1 shows the measured values of Re_d for which the spirals were detected and for which the flow reached full turbulence. As was the case with both disks and spheres, the spirals appeared as an intermediate state between the laminar flow field near the pole and the turbulent field at the edge. For the 15° cone, the onset of spirals was at a local Reynolds number about 1×10^3 , and the turbulent transition was at a local Reynolds number about 4.9×10^3 . Thus, it is probable that, for a conical part with a 15° half-angle, spirals could be prevented by rotating at $Re_d < 1 \times 10^3$. Note that this value is about twice as large as the criterion for preventing spirals on a cylinder. Using the same criteria that were applied to disks and spheres, the part should be able to remain fully turbulent if it has a turbulent jet aimed at its axis to act as a boundary layer trip and if it is rotating at $Re_d > 1 \times 10^3$.

In summary, my conclusions for cones are as follows:

1. It should be possible to guarantee that no spirals will appear if the cone is limited to values of Re_d as shown in Fig. 1.
2. Spirals probably cannot be prevented for larger values of Re_d without the use of a boundary layer trip.
3. With the use of a suitable boundary layer trip—such as a jet aimed at the axis—it should be possible to prevent spirals by going directly to full turbulence for $Re_d >$ the values for the onset of spirals, as shown in Fig. 1.

Fig. 1. Reynolds number for onset of spirals and transition to turbulence on cones as a function of cone half-angle. Source: R. Kobayashi and H. Izumi, "Boundary-Layer Transition on a Rotating Cone in Still Fluid," *J. Fluid Mech.*, 127, 353-364, (1983).

GENERAL CRITERIA FOR PREVENTING SPIRALS

As was previously stated, one of the objectives of the work described herein is to determine how to prevent spirals on real electroplated parts. An example of a typical real part is a spherical cap attached to the end of a cylinder.¹⁴ The flow field around such a part will begin as that around a sphere. There will be some sort of transition at the joint between the sphere and the cylinder. As the flow proceeds down the cylinder, it should eventually approach that from a pure cylinder. The gaps around the parts of interest are large so that the ratio of gap width to part height is not small compared to unity and may be much larger than 1.0. As a consequence, the flow will probably never reach the asymptote given by the flow around a cylinder. It is probable that one way to ensure laminar, spiral-free plating is to rotate the part at a speed that is less than the lowest criterion for the onset of spirals. This lowest criterion is the one for cylinders and is $Re_d < \text{about } 550$. This is the lowest of all the onset criteria and should be all right for parts that have shapes that are more complex than a spherical cap on a cylinder. This criterion is probably very conservative. However, there is no obvious way to get a better estimate except by experiment. One small bit of confirmation exists, which is that Keyes et al.¹⁴ noted that their flow visualization experiments began to show "approach to turbulence" for $Re_d > \text{about } 540$.

If it is desired that the flow around a part be turbulent, then it would probably be necessary to aim a turbulent jet at the axis to use as a boundary layer trip. To ensure that the flow field remains turbulent, the part should be rotated at a speed greater than the highest criterion for the onset of spirals. The Reynolds number above which it is probable that spirals will occur is usually larger than the value below which it is probable that spirals will not occur. For a sharp, pointed object with a small cone angle, the most reasonable criterion seems to be that for the cylinder: $Re_d > \text{about } 2 \times 10^3$. However, if the end of the part is flat, is a partial arc of a circle, or is a cone blunter than a half-angle of about 30° , then determining the criterion is more difficult, and Fig. 1 may be of some help in determining the criterion. I conclude that the most conservative criterion for ensuring that a part will remain in turbulent flow is that for the rotating disk: $Re_d > 1.8 \times 10^5$.

JETS FOR TURBULENT TRIP

One of the subjects that was examined as part of this study of the fluid flow effects on electroplating was that of the characteristics of the jet needed to ensure that all of the boundary layer on a spinning part would be turbulent. I wanted to see if it would be possible to estimate the maximum value for the distance between the origin of the jet and the part before the jet's ability to act as a trip was in doubt. According to the usual diagrams of friction factor in pipes, a flow in a pipe with pipe Reynolds number = 4000 (the pipe Reynolds number is based on the average velocity in the pipe and the inside diameter of the pipe) is just barely turbulent. A case was run on the FLUENT/PC fluids code for which a fully developed, turbulent flow exiting from a 3/4 in. O.D. (0.652 in. I.D.) tube with pipe Reynolds number = 4000 into a chamber was simulated. Both the jet

and the fluid in the chamber were water at 24°C. The flow rate through the tube was about 2.5 gpm. The decay of the turbulent kinetic energy as the jet penetrated the still water was examined. The result desired is an estimate of how far the jet should penetrate before it is no longer "turbulent enough" to act as a boundary layer trip.

There is no well-known criterion that I can use for determining this. However, I believe that a jet could still cause turbulence in the rotating boundary layer if its level of turbulence (as indicated by the average turbulent kinetic energy) was as great as that in the tube, remembering that a pipe Reynolds number of 4000 is just barely large enough for the flow to be turbulent. Figure 2 shows contours of turbulent kinetic energy for the FLUENT/PC case. The outermost contour is the lowest value of turbulent kinetic energy. The highest values are found in the oval region (i.e., a torus of oval cross section) extending outward from the inside corner of the tube exit. The high shear rates where the jet exits into the still fluid create a large amount of additional turbulent kinetic energy (above the values in the flow inside the tube) which takes a certain distance to dissipate. For this case, the estimate of the distance from the jet exit for which the average level of turbulent kinetic energy is back down to the average in the tube is about 7 in., which is about 11 times the inner diameter of the tube. It should be emphasized that this case represents a minimum. That is, this is a jet that is just barely turbulent and the criterion is that its level of turbulence after penetration should be as great as the turbulence level of the flow in the tube. A more turbulent jet should penetrate even farther before the jet is no longer "turbulent enough." I conclude from this calculation that any jet issuing from a tube with pipe Reynolds number ≥ 4000 and less than about 10 tube diameters from the part will probably be able to trip the boundary layer.

CHARACTERISTICS OF SPIRALS

Another thing I examined in the study of fluid flow effects on electroplating was the velocities in the spirals. A sample spiral pattern for a cone had been shown in Kobayashi and Izumi¹³ and one for a disk in Kobayashi et al.⁹ The velocity field may be crudely simulated by a driven cavity whose driving velocity is approximately $0.25 \Omega r$, where r is the local radius. The only difference between the simulation of one of these vortices and the usual driven cavity problem is that the sidewalls of the cavity should be free-slip in the vertical direction rather than in the customary non-slip condition. (In the mathematical free-slip boundary condition, there is no shear at the surface; in the non-slip condition, the velocity at the surface is constrained to be that of the surface.) Figure 3 shows the velocity vectors from such a driven cavity vortex simulation on the FLUENT/PC code. The vortex should first appear at a radius that is approximately $1 \times 10^5 \sqrt{\nu/\Omega}$. The height of the vortex should be approximately equal to the thickness of the laminar boundary layer, which is roughly $5\sqrt{\nu/\Omega}$. As best can be told from ref. 9, the vortex should be more or less circular. None of the photographs and drawings of vortices in any of the documents I examined show vortices that are very far from being circular.

Fig. 2. Turbulent kinetic energy for turbulent submerged jet entering a cavity in water, FLUENT/PC results.

Fig. 3. Velocity vectors from driven cavity simulation of shed vortex.

CONCENTRATION BOUNDARY LAYER THICKNESSES

An objective of this study of the fluid flow effects on electroplating was determining a concentration boundary layer thickness to put in the electroplating equations. Some fairly successful calculations of the thickness of the plating have been made¹⁵ using a turbulent fit from Eisenberg et al.¹ for cylindrical sections and a laminar solution from Der-Tau Chin² for spherical caps. The fact that the calculations cited in ref. 15 match experiments as closely as they do indicates that the effect of the thickness of the concentration boundary layer on electroplating rates (at least in the experiments cited in ref. 15) is probably second order.

The concentration boundary layer thickness is a construct. The idea is that the mass flux equals the diffusivity, multiplied by the difference between the concentrations in the bulk fluid and on the surface, and finally divided by the concentration boundary layer thickness. The concentration boundary layer thickness is the number that makes this equation work. This is a somewhat naive concept. The reason is that the equation defining the concentration boundary layer thickness contains the tacit assumption that the gradient of the concentration is constant throughout the thickness of the layer, whereas the gradient at the wall, which determines the mass flux, is higher than the average gradient across the layer. Thus, the constructed concentration boundary layer thickness will be smaller than the boundary layer thickness that can be found on a plot of concentration vs distance from the wall.

For the Blasius solution for laminar momentum transfer on a flat plate, it can be demonstrated that a momentum boundary layer thickness constructed by dividing the shear stress into the product of the laminar viscosity times the free stream velocity is 0.6 times the velocity boundary layer thickness (i.e., the thickness that represents the height above the plate for which the local velocity is 99% of the free stream value). For the usual fits for laminar forced convection heat transfer on a flat plate, it can be demonstrated that the thermal diffusion boundary layer thickness (which is usually defined as the height above the plate for which the difference between the local temperature and the free stream temperature is 1% of the difference between the plate temperature and the free stream temperature) is the velocity boundary layer thickness multiplied by the same 0.6 and then further divided by the Prandtl number raised to the 1/3 power. Figure 4 is a simplified diagram that shows the difference between the thicknesses of the boundary layers for velocity and for concentration. (The case shown, for which the velocity and momentum boundary layers are much thicker than the diffusion layer, is for a Schmidt number quite a bit larger than 1.0.) The figure also shows the difference between the true shapes of the boundary layers and the simplified line-segment shapes that are used to construct the concentration and momentum boundary layer thicknesses.

It is well known that turbulent boundary layers are thicker than laminar ones. It is also well known that turbulent shear stresses and heat transfer film coefficients are larger than laminar ones. Thus, the momentum and thermal diffusion boundary layer thicknesses for turbulent flow are smaller than those for laminar flow. The ratio of the momentum diffusion boundary layer thickness to the velocity boundary layer thickness for flat plates is roughly proportional to a Reynolds number to the minus about 0.7

Fig. 4. Comparison of velocity and concentration boundary layers.

to 0.75 power. The largest value of the ratio is for a layer that is just barely turbulent and is about 0.04. The fact that this thickness ratio is so much smaller for turbulent layers than it is for laminar layers (0.6) may be explained by the fact that the effective transport coefficients for the turbulent cases are not the laminar coefficients but rather the turbulent eddy coefficients.

Data for use in estimating concentration boundary layer thicknesses in electroplating are somewhat sparse. However, electroplating is just one of the known types of mass transfer from a body to a fluid. Other such processes include evaporation from a body into the surrounding fluid. In principle, mass transfer data from other mechanisms can be used to estimate electroplating concentration boundary layer thicknesses. Furthermore, the well-known Reynolds and Chilton-Colburn analogies may be used to estimate mass transfer effects from the results of momentum and heat transfer experiments. In its most general form, the Reynolds analogy between momentum and heat transfer states that the skin friction coefficient is proportional to the Nusselt number multiplied by a function of the Prandtl number. Similarly, the Chilton-Colburn analogy between momentum and mass transfer states that the skin friction coefficient is proportional to the Sherwood number multiplied by a function of the Schmidt number. These analogies may be used to derive more specific relationships between the concentration boundary layer thickness (i.e., the constructed boundary layer thickness for mass transfer) and the similarly constructed boundary layer thicknesses for momentum and heat transfer. These relationships are as follows:

$$\delta_c = \delta_m Sc^{-1/3} \quad (1a)$$

$$\delta_c = \delta_T Pr^{1/3} Sc^{-1/3} \quad (1b)$$

The symbols in Eqs. (1a) and (1b) were defined in the Nomenclature section of this document. The 1/3 power exponent for the Prandtl and Schmidt numbers is approximate. Values have been seen that range from 0.25 to 0.4 for this parameter. Some experimenters have determined that the values of the exponent for both laminar and turbulent flow in a particular system are the same; others have found different values for laminar and turbulent flow. In this study, I found no consistent pattern. To the precision that the values of these exponents can be believed, it is probable that the values are nearly the same and are all approximately equal to 1/3 for both laminar and turbulent flows.

Mass, heat, and momentum transfer results are usually presented as nondimensional coefficients: the Nusselt number, Sherwood number, and skin friction coefficient. A transformation to diffusion boundary layer thickness is therefore needed. For heat transfer, the Nusselt number is defined by the following equation:

$$Nu = h x/k$$

where h is the film coefficient, k is the thermal conductivity, and x is some scale length. The thermal diffusion boundary layer thickness is defined by the following equation:

$$q/A = k \Delta T/\delta_T$$

where q/A is the heat flux and ΔT is the temperature difference. The film coefficient h is defined by the following equation:

$$q/A = h/\Delta T$$

By substitution among the above three definitions, it can be demonstrated that the equivalence between Nu and δ_T is as follows:

$$\delta_T = x/Nu \quad (1c)$$

A similar derivation will give the following equivalence between Sh and δ_o :

$$\delta_o = x/Sh \quad (1d)$$

The situation with the momentum diffusion boundary layer thickness is a bit different because the skin friction coefficient has a form unlike the Nusselt and Sherwood numbers. By definition,

$$c_f = \tau / (\frac{1}{2} \rho u^2)$$

where τ is the shear stress, ρ is the fluid density, and u is a scale velocity. The definition of the momentum diffusion boundary layer thickness is as follows:

$$\tau = \mu u / \delta_m$$

where $\mu = \rho \nu$ is the viscosity. By substitution among the above two definitions, it can be demonstrated that the equivalence between c_f and δ_m is as follows:

$$\delta_m = 2 \nu / u / c_f \quad (1e)$$

With the use of Eqs. (1c)-(1e), it is possible to convert the customary nondimensional parameters for mass, momentum, and heat transfer to equivalent diffusion boundary layer thicknesses.

I prefer to write the concentration boundary layer thickness in a form that contains the base diameter of the part, multiplied by functions of the Reynolds and Schmidt numbers, and, if appropriate, a function of the location on the part. In the remainder of this document, I show how thicknesses would be written in this form for the four geometries studied (cylinders, disks, spheres, and cones). The form I use is somewhat different from those published by Der-Tau Chin² and by Eisenberg et al.¹ so that their fits may not be immediately recognizable.

CONCENTRATION BOUNDARY LAYER THICKNESS ON CYLINDERS

The available information for long cylinders (i.e., cylinders whose lengths are much greater than their diameters) suggests that the boundary layer thickness on a cylinder is more or less uniform. This is certainly the case for the turbulent fit of Eisenberg et al.¹ There are torque measurements

in Theodorsen and Regier³ that can be used to estimate the concentration boundary layer thickness for long cylinders with a wide gap using the Chilton-Colburn analogy. For $Re_d < \text{about } 20$, the flow is simple Couette flow with no Taylor vortices. For $20 < Re_d < 2000$, the flow forms Taylor vortices followed by spirals and eventually turbulence. For $Re_d > 2000$, the flow is turbulent and the form from Eisenberg et al. is applicable.

I propose the following equations for concentration boundary layer thickness on a cylinder:

$$\delta_c = d/4 Sc^{-1/3} \quad (Re_d < 20) \quad (2a)$$

$$\delta_c = d .392 Re_d^{-0.15} Sc^{-1/3} \quad (20 < Re_d < 264) \quad (2b)$$

$$\delta_c = d 2.76 Re_d^{-0.5} Sc^{-1/3} \quad (264 < Re_d < 2000) \quad (2c)$$

$$\delta_c = d 12.64 Re_d^{-0.7} Sc^{-1/3} \quad (Re_d > 2000) \quad (2d)$$

The symbols in Eqs. (2a)-(2d) were defined in the Nomenclature section of this document.

Equations (2b) and (2c) come from a rather crude attempt at curve fitting that matches the drag coefficient data of Theodorsen and Regier to within a few percent (which is less than the scatter of the data). There is no science involved in the sudden change in exponent at Re_d about 264. The curve of drag coefficient vs Reynolds number in Theodorsen and Regier is smooth. The sudden break occurs because I fit Theodorsen's and Regier's plot of drag coefficient vs the Reynolds number using two straight line segments for $20 < Re_d < 2000$ rather than finding a smooth curve to fit the data. Finding the exact shape of the drag coefficient curve in this range does not seem to be important enough to spend a number of additional hours (and dollars) working out a smooth transition in the fit of the data.

Equation (2d) is a rearrangement and slight modification of the form from Eisenberg et al.¹ The exponent for the Schmidt number, which has a value of about 0.35 in Eisenberg et al., has been altered to a value of 1/3. In general, the boundary layer thickness for a cylinder is dependent on the diameter of the cylinder. It is true that, for Eq. (2c), the exponents of diameter cancel (Re_d is proportional to d^2), so that δ_c is independent of diameter for $264 < Re_d < 2000$. However, this is an artifact due to the very crude fit for drag coefficient in that range so that it is unlikely that the boundary layer thickness is independent of cylinder diameter for any range of Reynolds numbers. Figure 5 shows comparisons of the boundary layer thickness equations for a particular cylinder that was studied by Keyes et al.¹⁴ The difference between the line given by the Eisenberg et al. fit and that from Eq. (2d) is almost entirely due to the difference between the exponents of the Schmidt number.

The concentration boundary layer thickness in the Eisenberg et al. turbulent fit is proportional to the Schmidt number to the minus approximately 0.35 power. Most fits for turbulent heat transfer show the Nusselt number, and thus the film coefficient, proportional to the Prandtl number to about the 1/3 power, which is the theoretical value from the Chilton-Colburn analogy. The fit from Kreith et al.¹² for turbulent heat transfer on rotating spheres shows the Nusselt number proportional to the

Fig. 5. Diffusion boundary layer thickness for cylinder of diameter 1.5 in., Schmidt Number 912 (showing proposed fits and turbulent fit from Eisenberg et al.).

Prandtl number to the 0.4 power. The thermal diffusion boundary layer thickness is inversely proportional to the Nusselt number and is thus proportional to the Prandtl number to the minus between 0.4 and 1/3 power. To the precision that one can believe the mass transfer fit of Eisenberg et al. and the heat transfer fits, 0.35, 0.4, and 1/3 are the same number. Most of the usual fits for forced convection turbulent heat transfer show a Reynolds number to the between 0.75 and 0.8 power dependence for the Nusselt number. This means that the thermal diffusion boundary layer thickness will be proportional to the Reynolds number to the minus 0.75 to 0.8. The fit from Kreith et al. for turbulent heat transfer on rotating spheres shows the Nusselt number proportional to the Reynolds number to the 0.67 power. The mass transfer fit for turbulent rotating cylinders from Eisenberg et al. shows the concentration boundary layer thickness proportional to the minus 0.7 power. Again, to the precision of the fits, this is indistinguishable from the minus 0.67 to 0.8 powers from heat transfer.

Unfortunately, the concentration boundary layer thicknesses derived for long cylinders may not be particularly relevant to actual electroplating problems. The end of the part, whether spherical or not, will drive some additional circulation that may turn out to be dominant. If there is a rotating collar in the electrolyte, that too will drive an additional circulation.

It is not clear if there is a good way to make an *a priori* estimate for the concentration layer thickness on a short cylindrical section. A possible assumption is that the layer is a constant whose thickness is the Der-Tau Chin value for the equator. However, the evidence from Keyes et al. is that the layer thickens after it makes the turn from the arc of the sphere to the cylinder. Some useful information about transfer in a circulation driven by a rotating collar might be obtained from calculations in a code such as FLUENT. That may be the best that can be done with limited resources.

CONCENTRATION BOUNDARY LAYER THICKNESS ON DISKS

The Levich solution for laminar flow on a rotating disk is well known.^{3,8,16} The laminar boundary layers have a constant thickness independent of the location on the disk. The concentration boundary layer thickness from that fit is given by the following equation:

$$\delta_c = d \, 1.14 \, Re_d^{-1/2} \, Sc^{-1/3} \quad (3a)$$

The symbols in Eqs. (3a) and (3b) were defined in the Nomenclature section of this document. Note that the exponents on the diameter in Eq. (3a) cancel so that the thickness is proportional to Ω/ν but independent of d .

The von Karman solution for a turbulent rotating disk is also well known. (See refs. 3, 8, and 16.) The concentration boundary layer thickness from that fit is shown by the following equation:

$$\delta_c = d \, 51.3 \, Re_d^{-0.9} \, Sc^{-1/3} (2 \, r/d)^{-0.8} \quad (3b)$$

This thickness is a function of the location on the disk. The thickness has a dependence on Re_d raised to the -0.9 power, which is larger than the -0.7

power of the dependence of the turbulent boundary layer on the cylinder. The exponents on the diameter in Eq. (3b) cancel leaving a dependence on Ω/ν and on r , but not on d .

CONCENTRATION BOUNDARY LAYER THICKNESS ON SPHERES

The form of Der-Tau Chin's² solution for the laminar flow on a sphere that I prefer to use is as follows:

$$\delta_c = d \, 1.14 \, Re_d^{-1/2} \, Sc^{-1/3} / (1 - 0.207 \, \phi^2) \quad (4a)$$

The symbols in Eqs. (4a) and (4b) were defined in the Nomenclature section of this document.

Der-Tau Chin has noted that his concentration boundary layer thickness is the velocity boundary layer thickness multiplied by 0.4 and then divided by the Schmidt number to the 1/3 power. The difference between the 0.6 factor for a flat plate (see the introduction to this section) and the 0.4 factor for the rotating sphere is probably due to the difference in development of the boundary layers in the two different geometries. Note that the exponents on the diameter in Eq. (4a) cancel so that the thickness is proportional to Ω/ν but independent of d .

The only data that I have found so far on transfer rates for rotating spheres is that of Kreith et al.¹² Kreith shows data for heat transfer from a number of different spheres rotating in air, in water, and in an oil. He gives fits for the average Nusselt number for both laminar and turbulent flow. He does not give any information about the velocity fields or the thickness of the velocity boundary layer nor does he give any information about the variation in heat transfer coefficient between the pole and the equator. An equivalence between the Nusselt number and the thermal diffusion boundary layer thickness has already been derived [Eq. (1c)]. With the use of Eq. (1b), the thermal diffusion boundary layer can be converted to a mass transfer diffusion layer thickness, which is equivalent to the concentration boundary layer thickness. According to this conversion, the average mass transfer diffusion layer thickness for a turbulent sphere using the Kreith et al. data is as follows:

$$\delta_{T(\text{average})} = 9.5 \, d \, Re_d^{-0.67} \, Sc^{-0.4}$$

The Eisenberg et al.¹ correlation for turbulent mass transfer along a cylinder has a similar form, but the Schmidt number is raised to the minus 0.35 power, times the Reynolds number to the minus 0.7 power, and the constant factor is 12.6. As I have said before, the difference between the 0.4 and 0.35 powers for the Schmidt number and the difference between the 0.67 and 0.7 powers for the Reynolds number seem trivial.

The only fits I have found for local mass or heat transfer (as distinguished from average transfer over the whole object) in turbulent flow around objects that are not cylinders are for the turbulent disk and cone. (The cone solution is a modification of the disk solution; see ref. 16.) The

following equation for the concentration boundary layer thickness for a turbulent sphere is therefore proposed:

$$\delta_c = d \, 51.3 \, \text{Re}_d^{-0.8} (\cos\theta)^{-0.1} (\sin\theta)^{-0.8} \text{Sc}^{-1/3} \quad (4b)$$

This fit is based on a simple modification of the cone fit [Eq. (5b)] with the spherical character of the body accounted for by a change in the apparent cone angle with the polar angle of the sphere. Note that in this context, the cone angle α is $90^\circ - \theta$ so that $\sin\alpha = \cos\theta$. The exponents on both the Reynolds and Schmidt numbers for Eq. (4b) are different from those given by Kreith et al. for the average heat transfer on a sphere as well as for the Eisenberg et al. fit for mass transfer on a cylinder. However, a selection of numerical values for average concentration boundary layer thickness using the Kreith et al. fit and an integral of Eq. (4b) over the surface of the sphere for Reynolds numbers ranging from 5×10^4 to 3×10^5 (the range of the experiments reported in Kreith et al.) shows that the results are the same to within about 30%. The dependence of thickness on diameter is a bit complicated. If the substitution

$$\sin\theta = 2 \, r/d$$

is made, then the exponents of the diameter cancel leaving a dependence on Ω/ν and on r but not on d . However, since

$$\cos\theta = (1 - 4 \, r^2/d^2)^{1/2}$$

there remains a weak dependence on d .

Figure 6 shows a plot of the thickness for a turbulent sphere [Eq. (4b)] vs angle from the pole, together with the equivalent for a disk and the constant value for a turbulent cylinder [Eq. (2d)]. The fit for a turbulent sphere matches the disk fit [Eq. (3b)] for small values of polar angle α , which is to be expected. Surprisingly, the sphere and disk fits are close to one another until very near to the equator ($\theta = 90^\circ$). The fit for the sphere also matches the cone fit [Eq. (5b)] for large cone half-angle α . However, that is not necessarily a desirable condition. It is expected that the proposed fit for turbulent spheres may be reasonably accurate for $\theta < 60^\circ$. However, the thickness from both Eqs. (4b) and (5b) becomes infinite as $\theta \rightarrow 90^\circ$ (or $\alpha \rightarrow 0^\circ$). The curve of α vs θ from Eq. (4b) has a minimum at $\alpha = 70.5^\circ$. It seems unlikely that the thickness in a real flow field will have a minimum. My best estimate is that the cylinder solution [Eq. (2d)] should be used as a limit for large θ . If the value given by Eq. (4b) at $\theta = 70.5^\circ$ is greater than that given by Eq. (2d), then the simplest solution is to linearly interpolate on θ so that δ_c goes from the Eq. (4b) value at $\theta = 70.5^\circ$ to the Eq. (2d) value at $\theta = 90^\circ$. However, if the Eq. (4b) value at $\theta = 70.5^\circ$ is smaller than the Eq. (2d) value, then an alternative is to use the maximum of either Eq. (4b) or Eq. (2d) for all values of θ . The methods for estimating concentration boundary layer thickness at the high θ just given are crude approximations that are included so there will be some way of getting values for the thickness under those conditions. There is no more science in them than a gut feeling that the thickness on the sphere probably should not be greater than the value given

Fig. 6. Diffusion boundary layer thickness for sphere of diameter 1.5 in., Reynolds Number 125,000, Schmidt Number 912 (showing proposed turbulent fit together with fits for disk and cylinder).

by the cylinder, and thus the best I can say is that the numbers may be in the right ballpark.

The fit given in Eq. (4b) was developed for spheres rotating in a still fluid. A natural question arises regarding the usefulness of such a fit for a system for which the boundary layer at the pole has been tripped by a jet. I have found no information about heat or mass transfer from spinning objects with jets. There is a very small amount of data from Theodorsen and Regier³ that give torque measurements for spinning disks with a jet aimed at the axis. The data suggest that the jet increased the total torque by, at most, 5%. Using the analogies between shear stress and other forms of transfer, it could be argued that the jet should decrease the average concentration boundary layer thickness by 5% and that it is probable that the effect of the jet should be greatest near the pole. Thus, the expected increase in mass transfer is likely to be accomplished mainly by a considerable lowering of the very large boundary layer thickness at the pole. I am not aware of any analytical solutions of the problem of a turbulent jet impinging on a flat plate (or disk), much less one that includes the effect of rotation. If such a solution could be found, an estimate is that the boundary layer thickness might be the value given by the jet solution at the pole and might change to the solution of the rotating disk (or sphere) as the radius gets large compared to the radius of the jet. It is likely that this is a problem that will require numerical solutions.

CONCENTRATION BOUNDARY LAYER THICKNESS ON CONES

Kapesser et al.¹⁶ propose a solution for laminar flow on a cone that is a modification of the well-known Levich solution for laminar flow on a rotating disk. As was the case for the disk, the boundary layers have a constant thickness independent of the location on the cone. The concentration boundary layer thickness from that fit is

$$\delta_c = d \, 1.14 \, Re_d^{-1/2} (\sin \alpha)^{-1/2} Sc^{-1/3} \quad (5a)$$

where α is the half-angle of the cone. (A disk is a cone with $\alpha = 90^\circ$ while a cylinder has $\alpha = 0^\circ$.) The remaining symbols in Eqs. (5a) and (5b) were defined in the Nomenclature section of this document. Note that the exponents of the diameter in Eq. (5a) cancel so that the thickness is proportional to Ω/ν but independent of d .

The boundary layer thickness in this fit becomes infinite for $\alpha = 0^\circ$ (i.e., a cylinder). From Eqs. (2a), (2b), and (2c), it is clear that this is not a correct result. At some value of cone angle, the thickness from Eq. (5a) becomes larger than that for a cylinder as given by Eqs. (2b) and (2c). Comparing the results from Eq. (5a) to those from Eq. (2c) (assuming $Re_d > 264$), one finds that the curve of concentration boundary layer thickness vs α for the cone crosses the value for a cylinder at $\alpha = 9.5^\circ$.

The solution of Kapesser et al. for a turbulent cone is a modification of the von Karman solution for a turbulent rotating disk. The concentration boundary layer thickness from that fit is given by the following equation:

$$\delta_c = d \, 51.3 \, Re_d^{-0.8} (\sin \alpha)^{-0.1} (2 \, r/d)^{-0.8} Sc^{-1/3} \quad (5b)$$

The fit proposed by Kapesser et al. uses a $1/4$ exponent for the Schmidt number, but I have somewhat arbitrarily changed this to $1/3$ to be consistent with Eqs. (1a) and (1b) and to match other data on turbulent rotating objects. The exponents of the diameter in Eq. (5b) cancel leaving a dependence on Ω/ν and on r , but not on d . As was the case for laminar flow, the boundary layer thickness in this fit becomes infinite for $\alpha = 0$ (i.e., a cylinder). From Eq. (2d), it is clear that this is not a correct result. At some value of cone angle, the thickness from Eq. (5b) becomes larger than that for a cylinder given by Eq. (2d). Comparing the results from Eq. (5b) to those from Eq. (2d) at $Re_d = 2000$, one finds that the curve of concentration boundary layer thickness vs α for the cone crosses the value for a cylinder at $\alpha = 17.6^\circ$. For $Re_d = 56,000$, the curves cross at 0.022° , which seems to be a bit small. Kapesser et al. point out that their average mass transfer rates for $\alpha = 9.6^\circ$ for turbulent flow were larger than had been predicted by their model. The model was able to predict mass transfer rates for $\alpha = 15^\circ$ much more accurately.

Figures 7(a) and 7(b) show the diffusion boundary layer thickness for a turbulent cone from Eq. (5b) for a few different cone angles and also the constant thickness for the turbulent cylinder [Eq. (2d)]. Figure 7(b) is a replot of the results shown in Fig. 7(a) on an expanded vertical scale. It is surprising that the thickness is not very sensitive to cone angle until the angle becomes quite small (less than about 5°). Both the laminar and turbulent fits for a cone suffer from the defect that the limit of the concentration boundary layer thickness as the cone approaches a cylinder is infinite. At some small angle, the thickness from the fits for the cone becomes greater than the thickness for a cylinder. A crude way to possibly overcome this defect would be to compare the concentration boundary layer thickness for the cone and for the cylinder and use whichever value is smaller.

Fig. 7a. Diffusion boundary layer thickness for cones of base diameter 1.5 in., Reynolds Number 125,000, Schmidt Number 912 (showing turbulent fit from Kapesser et al. together with fit for cylinder).

Fig. 7b. Diffusion boundary layer thickness for cones of base diameter 1.5 in., Reynolds Number 125,000, Schmidt Number 912 [showing turbulent fit from Kapesser et al. together with fit for cylinder (expanded scale)].

CONCLUSIONS

I have developed some criteria for preventing spirals on objects in turbulent flow and for maintaining a spiral-free laminar flow. Table 1 summarizes the recommendations for the Reynolds numbers needed to prevent the onset of spirals. The most stringent criterion is that found for cylinders, which is $Re_d < \text{about } 550$.

Table 1. Recommended Reynolds numbers (Re_d) for the onset of spirals and transition to turbulence for spinning parts

Part geometry	Onset of spirals	Turbulent trip (with jet at pole)	Full turbulence (no jet)
cylinder	550		2×10^3
disk	1.8×10^5	1.8×10^5	6×10^5
sphere	550	2.4×10^4	2.5×10^4
cone	The values for cones are strongly dependent on cone angle. [See Fig. 1 for a plot of Re_d vs cone half-angle for the onset of spirals and transition to turbulence (in the absence of jets)]		

Criteria for being able to keep the whole part in turbulent flow have also been developed, and they are summarized in Table 1. If the cap on the part is spherical, it is probably sufficient that the part be rotated at $Re_d > \text{about } 2.4 \times 10^4$. However, if the end of the part is flat, is a partial arc of a circle, or is a cone that is blunter than a half-angle of about 30° , the most conservative criterion is that for the disk, which is that the part should rotate at about $Re_d > 1.8 \times 10^5$. In all cases, one additional requirement must be met, which is that the part must have a boundary layer trip such as a turbulent jet (from a tube with a pipe Reynolds number > 4000) aimed at its pole. It is likely that such a jet will be able to act as a boundary layer trip if the outlet of the tube is within about 10 tube diameters of the pole of the part.

The solution of Der-Tau Chin [Eq. (4a)] is probably adequate for laminar concentration boundary layer thickness on a spherical part section. Equations (2a) and (2b) give concentration boundary layer thickness for cylinders in laminar and Taylor vortex flow (including the transition to spirals). However, at Reynolds numbers in the laminar range, circulations from the cap and the rotating collar (if any) will probably make a

significant contribution to the overall flow pattern. It is possible that some further insight on the subject might be developed by doing some numerical calculations of flow patterns on specific part configurations.

The fit for turbulent concentration boundary layer thickness from Eisenberg et al. [Eq. (2d)] is probably adequate for cylindrical part sections in turbulent flow. For spherical sections, Eq. (4b) seems like a reasonable estimate for the concentration layer thickness for turbulent flow. Unfortunately, an estimate for the thickness of the layer near the pole of a sphere that has a jet for a boundary layer trip has not yet been found.

Equations (5a) and (5b) should be usable for conical sections in laminar and turbulent flow, respectively. However, the concentration boundary layer thickness for the cone should be compared with the value given by the appropriate fit for a cylinder and the smaller of the two used. This should not be necessary except for cone half-angles of 10° or less.

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