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**DEVELOPMENT & APPLICATION OF DIAGNOSTIC INSTRUMENTATION FOR
MEASUREMENT OF ELECTRON DENSITY & CONDUCTIVITY**

Final Technical Progress Report

**By
Leslie E. Bauman**

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FOR MEASUREMENT OF ELECTRON DENSITY & CONDUCTIVITY**

Final Technical Progress Report

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NOMENCLATURE

Symbol	Unit(s)	Abbreviation
Roman Alphabet		
A spectral line integrated absorptivity	per second per meter	$s^{-1} \cdot m^{-1}$
B magnetic induction	Tesla	T
c speed of light in a vacuum, 2.9979×10^8	meter per second	$m \cdot s^{-1}$
C_2 Planck's 2nd radiation constant, 1.43388×10^7	nanometer °Kelvin	$nm \cdot K$
E_i ionization energy	electron volts	eV
e electron charge, 1.6021×10^{-19}	Coulomb	C
F(v) Maxwellian electron velocity distribution	per meter per second to the sixth power	$(m/s)^{-6}$
g_i atomic partition function for level i	dimensionless	
h Planck's constant, 6.624×10^{-34}	Joule second	J·s
I intensity	arbitrary	
I current	Amperes	A
k Boltzmann's constant, 1.3805×10^{-23}	Joule per °Kelvin	J/K
k_v atomic absorption coefficient	per meter	m^{-1}
L path length	meter	m
m_e electron mass, 9.108×10^{-31}	kilogram	kg
n real index of refraction	dimensionless ratio	
n_e electron number density	number per cubic meter	m^{-3}
n_i ion number density	number per cubic meter	m^{-3}
n_s neutral seed atom number density	number per cubic meter	m^{-3}
N complex index of refraction	dimensionless	
N_i mole fraction of specie i	dimensionless	
P laser power	Watts	W
Q_d electron-molecule collision transfer cross section	meter squared	m^2
T temperature	°Kelvin	K
v velocity	meter per second	m/s
Greek Alphabet		
α plasma absorption coefficient	per meter	m^{-1}
χ polarization ellipticity	dimensionless ratio	
δ standard deviation		
ϵ_0 permittivity of free space, 8.854×10^{-12}	farads per meter	$C^2 \cdot s^2 \cdot kg^{-1} \cdot m^{-3}$
λ wavelength	micron	μm
μ_e electron mobility	Coulomb second per kilogram	$C \cdot s \cdot kg^{-1}$
μ_i ion mobility	Coulomb second per kilogram	$C \cdot s \cdot kg^{-1}$
ν collision frequency	collisions per second	s^{-1}
ν_t total collision frequency	collisions per second	s^{-1}
π pi, 3.14159		
θ polarization angle	radians or degrees	rad or °
σ electrical conductivity	per ohms per meter	$mhos \cdot m^{-1}$
τ decay constant	second	s
ω laser angular frequency	radians per second	$rad \cdot s^{-1}$
	and polarizer rotation speed	$rad \cdot s^{-1}$
ω_p plasma frequency	radians per second	$rad \cdot s^{-1}$
ω_c cyclotron frequency	radians per second	$rad \cdot s^{-1}$

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EXECUTIVE SUMMARY

The purpose of this contract was to assemble and demonstrate in the laboratory a Faraday rotation system for measurement of electron density and conductivity, with the intent to produce a system suitable for diagnostic support of the development of pulsed, space-based magnetohydrodynamic (MHD) power systems. Two system configurations were tested: (1) a rotating polarizer and (2) a beam splitting polarizer. Due to the short path length plasma produced in the laboratory flame, the long wavelength 496 μm methyl fluoride laser line was used and only the more sensitive rotating polarizer configuration was used for the demonstration experiments. Electron number densities from 2×10^{19} to 9×10^{19} were measured with good agreement to statistical equilibrium (Saha) calculations using emission absorption-measured flame temperatures and neutral seed atom number densities. The electron collision frequencies were measured by transmission measurements. Combining these two measurements gave measured electron conductivities of between 4 and 12 mohms/m. These results compared reasonably well with those found with an electron collision frequency model combined with chemical equilibrium calculations and the emission absorption measurements. Ellipticity measurements of electron collision frequency were not possible due to the short path length of the laboratory plasma.

The system in a rotating polarizer configuration is capable of measuring rotation angles to within $\pm 0.3^\circ$. The time period for these measurements was just over 4 minutes, but could be shortened to around 2 minutes. For a nominal 3 Tesla (T) magnet, the system could be used to measure electron density times path length as low as $2.7 \times 10^{17} \text{ m}^{-2}$ using the 496 μm laser line. With the 118 μm methanol laser line, the maximum measurable density time path length would be $1.4 \times 10^{20} \text{ m}^{-2}$. In the beam splitting configuration the uncertainty in the angle is $\pm 0.6^\circ$ and these minimum and maximum values are 5.4×10^{17} and $7.1 \times 10^{19} \text{ m}^{-2}$, respectively. A beam splitting measurement can provide time resolution of 0.1 s which should provide adequate time resolution for diagnostics on an MHD pulse. Recommendations for configuring the system performance on larger scale facilities have been made. The system should prove to be a useful diagnostic for MHD development.

INTRODUCTION

The technique of Faraday rotation was proposed by this author in 1987 as the most practical method for a direct determination of the conductivity of large-scale, pulsed magnetohydrodynamic (MHD) plasmas to support the Department of Energy program for Development of Space-Based, Multi-Megawatt, MHD Power Systems. The laser-based technique, typically using far infrared wavelengths, is non-intrusive; however, it requires opposed windows for electromagnetic wave transmission along the magnetic field lines. What the technique gives up in sensitivity (compared to the typical, far infrared interferometry) is gained in practicality due to immunity to vibration, simplified optics, and unambiguous results. The measurement inherently provides a spatially averaged result over the path length of the beam through the plasma. The demonstration measurements presented here utilized a rotating polarizer system for collection of the maximum possible information on the polarization of the transmitted laser beam. Development of a beam splitting data collection routine provides a faster time response that will allow measurements of the temporal variations of electron density during an MHD pulse.

The essential system components are the far infrared laser, which provides a linearly polarized source of electromagnetic radiation, a grid polarizer for separation of polarization components after traversing the plasma, and detector(s). The rotation of the polarization is linearly proportional to the electron density and the induced ellipticity is related to the collision frequency of the electrons; the combination of density and collision frequency gives the electron conductivity. The laser wavelength must be chosen so the beam will (1) transmit through the plasma without appreciable losses and (2) produce a measurable angle of rotation. Polarization component separation using a grid polarizer was chosen to provide a fast time response; an optically chopped system was selected in order to account for background radiation. This necessitates phase sensitive detection, which is accomplished with pyroelectric detectors and lock-in amplifiers. Preliminary experiments presented in this report were run with a single detector and rotation of the grid polarizer in order to obtain the most complete data for analysis during system development. Data collection and calculation of electron density and conductivity from the data is performed by computer.

Well defined flows were required, along with independent measurements of conductivity in order to validate the Faraday rotation measurements. Because of availability, ease of operation, and low cost, seeded flames were chosen to provide such flows. The

flows, which are best described as weakly ionized gases, did provide an adequate demonstration of the Faraday rotation system for conductivity measurements.

PROJECT DESCRIPTION

The project has been split into six tasks:

1. A. Analysis and Characterization of Flow Properties for Choice of Spectral Line;
1. B. Design and Construction of Optical Components;
1. C. System Assembly;
1. D. Sensitivity Analysis;
2. A. Measurements on MHD Flows;
3. A. Analysis of System Performance.

PROJECT STATUS

Task 1. A. Analysis and Characterization of Flow Properties for Choice of Spectral Line

Faraday rotation involves the rotation of the polarization of a beam of linearly polarized electromagnetic radiation, typically produced by a laser, as it passes through matter in the direction of an applied magnetic field. The rotation of the polarization scales directly with the electron density and the applied field strength. An elliptical component will also be introduced into the polarized laser beam, linear with the electron collision frequency; the electron mobility is related to the collision frequency. Knowledge of electron density and electron mobility provides electron conductivity, the predominant mechanism for current transport across an MHD channel.

Brief Theory of Plasma Conductivity. Considering a plasma to be an ionized gas, we can view it as being composed of electrons, positively charged ions, and neutral atoms or molecules. In a direct current electric field, the electrons and ions will, in opposite direc-

tions, repetitively accelerate until suffering a collision, and then accelerate again. This results in a drift velocity for the motion of the charged particles that provides a conductivity

$$\sigma = -n_e e \mu_e + n_i e \mu_i, \quad (1)$$

where n_e is the electron density, $-e$ is the electron charge, μ_e is the electron mobility, and μ_i is the ion mobility.

The electron mobility depends upon particle charge, mass and collision frequency as

$$\mu_e = e / (m_e \nu), \quad (2)$$

where m_e is the electron mass and ν is the collision frequency. A similar expression could be written for the ions in the plasma. Because of the ratio of masses, the mobility of the ions will be negligibly small compared to that of the electrons; we can, for a very good approximation, consider the conductivity to arise solely from the electrons.

Collisions will occur between the various particles and at different impact velocities, depending upon the temperature of the plasma. The important velocity changing collisions will occur between the electrons and the heavy particles in the plasma. For the seeded flame used for these demonstration experiments, the neutral particles vastly exceed the number of ions. In this case, the collision frequency is primarily the electron neutral collision frequency averaged over the electron velocities and the neutral species. The velocity average is denoted by an overbar and the conductivity of a weakly ionized gas can be written as

$$\sigma = n_e e^2 / (m_e \bar{\nu}). \quad (3)$$

Thus, a measurement of the electron density and collision frequency will provide a measure of the plasma conductivity.¹

Brief Theory of Polarized Light Transmission Across a Plasma . Sensitive and direct measures of the electron density and collision frequency can be made by detecting the polarization changes in a linearly polarized "light" beam that traverses a plasma in the direction of an applied magnetic field. The induced rotation of the polarization is related to the electron density, and the induced ellipticity is related to the electron density and collision frequency.

The rotation can be explained by decomposing the linearly polarized light into equal amplitude, left and right hand circularly polarized components, LHCP and RHCP. The index of refraction differs for the two polarization components so that the waves travel at different velocities through the plasma suffering different losses through absorption.^{2,3}

The complex index of refraction is given by

$$N_{\pm} = \left[1 - \frac{\omega_p^2(\omega \pm \omega_c)}{\omega [(\omega \pm \omega_c)^2 + \bar{\nu}^2]} - i \frac{\bar{\nu} \omega_p^2}{\omega [(\omega \pm \omega_c)^2 + \bar{\nu}^2]} \right]^{1/2}, \quad (4)$$

where N is the complex index of refraction with subscripts $+$ and $-$ to denote LHCP and RHCP components; ω is the probe laser angular frequency; ω_p is the plasma frequency, $\omega_p = [n_e e^2 / \epsilon_0 m_e]^{1/2}$; ω_c is the cyclotron frequency, ω_c (radians/s) = eB/m_e ; and $\bar{\nu}$ is the electron collision frequency with an overbar denoting the velocity average.

Writing the complex index of refraction in the form $N_{\pm} = n_{\pm} - i c \alpha_{\pm} / 2\omega$ allows one to see by inspection $n_{\pm}$, the real component which determines the speed of propagation of the LHCP and RHCP waves in the plasma, and $\alpha_{\pm}$, the complex component which is related to the absorption coefficient of the plasma.^{4,5} To do this we will assume (1) that the cyclotron frequency is less than $\omega/2$ to avoid a singularity at $\omega = \omega_c$, and (2) that the laser frequency is much greater than the plasma frequency or the collision frequency, $\omega^2 \gg \omega_p^2, \bar{\nu}^2$. Then,

$$N_{\pm} \approx 1 - \frac{\omega_p^2(\omega \pm \omega_c)}{2\omega [(\omega \pm \omega_c)^2 + \bar{\nu}^2]} - i \frac{\bar{\nu} \omega_p^2}{2\omega [(\omega \pm \omega_c)^2 + \bar{\nu}^2]}, \quad (5)$$

so that

$$n_{\pm} \approx 1 - \frac{\omega_p^2(\omega \pm \omega_c)}{2\omega [(\omega \pm \omega_c)^2 + \bar{\nu}^2]} \quad (6)$$

and

$$\alpha_{\pm} \approx \frac{\bar{\nu} \omega_p^2}{c [(\omega \pm \omega_c)^2 + \bar{\nu}^2]}. \quad (7)$$

After traversing the plasma, the induced relative phase difference between the two components is evident in the altered angle of linear polarization. And if the collision frequency is significant, then $\alpha_+ < \alpha_-$ and the circularly polarized components will have different amplitudes. The resultant is a dominant linearly polarized wave with a small left circularly polarized wave, i.e. a slightly elliptically polarized wave as illustrated here.

Figure 1. Illustration of the polarization changes in a laser beam after traversing a plasma in the direction of a magnetic field.

Faraday Rotation Angle Measurements. The polarization rotation induced over a path length L is given by

$$\theta_{\text{rot}} = \frac{\omega}{2c} (n_+ - n_-) L. \quad (8)$$

If $\omega^2 \gg \bar{\nu}^2$, then

$$\theta_{\text{rot}} = \frac{\omega_p^2 \omega_c L}{2c (\omega^2 - \omega_c^2)}. \quad (9)$$

For constant properties across the line of sight of the laser beam and for the case that $\omega^2 \gg \omega_c^2$, the rotation angle is given by

$$\theta_{\text{rot}} (\text{radians}) = 2.63 \times 10^{-13} (\text{C}\cdot\text{s}/\text{kg}) \lambda^2 n_e B L, \quad (10)$$

where the laser wavelength, λ , the electron density, n_e , magnetic induction, B , and path length through the plasma, L , are given in meter-kilogram-second (MKS) units. If the plasma or magnetic field is not uniform then the observed rotation angle is an integral across the path length as

$$\theta_{\text{rot}} (\text{radians}) = 2.63 \times 10^{-13} (\text{C}\cdot\text{s}/\text{kg}) \lambda^2 \int n_e B dx \text{ where } \int dx = L. \quad (11)$$

These expressions assume that the laser frequency is much greater than the plasma cyclotron frequency and the plasma frequency, in order for the plasma to be transparent to the laser light. These requirements, along with optimizing the rotation angle for signal-to-noise considerations, determine the selection of the laser wavelength. The optimum wavelengths will be in the far infrared region.

Two basic methods are used to measure Faraday angles: (1) a rotating polarizer and (2) a beam splitting polarizer. In the rotating polarizer method, the transmitted laser power (ignoring ellipticity) is given by

$$P = P_0 \exp(-2\alpha L) \cos^2(\theta_{\text{rot}} - \theta_0), \quad (12)$$

where P is the transmitted laser power, P_0 is the laser power without the flame, and θ_0 is the polarization angle without the flame. Depending upon the polarizer's rotation speed, hysteresis will be part of the θ_0 term (see page 21). Fitting the transmitted power curve to

$$P = A \cos^2(\theta_{\text{rot}} - \theta_0) + D \quad (13)$$

allows for imperfection in the polarizer, offset in the signal level, and any measurable ellipticity. The rotation angle can be found by searching for a minimum or maximum in the curve, although this is not a very sensitive method because the slope of P versus θ is small at the minima or maxima. A more sensitive method used in this study was least squares fitting of the transmitted laser power curve for adjustable parameters A , D and θ_{rot} .

Kuzmenko and Self ⁶ propose a relationship for the optimum wavelength for a null seeking rotating polarizer experiment as

$$\lambda_{\text{opt}} (\mu\text{m}) = 8.18 \times 10^{17} / (\bar{\nu} n_e L)^{1/2}, \quad (14)$$

where $\bar{\nu}$, n_e , and L are in MKS units.

In a beam splitting experiment the polarizer is set at an angle to the laser beam to reflect one polarization component to one detector and transmit the other component to a second detector. The polarizer orientation is initially adjusted to equalize the signals on the two detectors, and then left fixed. The Faraday rotation angle is determined by relative measurement of the transmitted and reflected power. The ratio of the transmitted power to the reflected power is insensitive to small changes in rotation angle; the typically used measurement scheme is to ratio the difference and sum of the transmitted and reflected laser power as

$$R = \frac{P_0 \cos^2\theta - P_0 \sin^2\theta}{P_0 \cos^2\theta + P_0 \sin^2\theta}, \quad (15)$$

where θ is $\theta_{\text{rot}} - \theta_0$. Again the Faraday rotation angle is found as a difference. The ratio is insensitive to Faraday rotation angles approaching 45° , setting the upper limit for detection.

Electron Collision Frequency Measurements. A measure of the ellipticity introduced into the beam will give a measure of the collision cross section. If χ is the ratio of the minor axis of the ellipse to the major axis, then⁷

$$\tan \chi = \frac{\exp[-(\alpha_- - \alpha_+)L] - 1}{\exp[-(\alpha_- - \alpha_+)L] + 1} = \tanh[(\alpha_- - \alpha_+)L/2], \quad (16)$$

so that if $\omega^2 \gg \bar{\nu}^2$, then

$$\bar{\nu} = \frac{c (\omega^2 - \omega_c^2)^2}{\omega_p^2 \omega \omega_c L} \tanh^{-1}(\tan \chi). \quad (17)$$

Since the collision frequency is proportional to the inverse of the plasma frequency squared, and therefore proportional to the inverse of the electron density, one must first measure the electron density to determine collision frequency from measures of ellipticity. Also, as a consequence, any bias in the measurement of electron density will be amplified in the electron conductivity.

If the magnetic induction is uniform across the plasma, but the plasma is not, then

$$\tanh^{-1}(\tan\chi) = \frac{e^2 \omega \omega_c}{\epsilon_0 m_e c (\omega^2 - \omega_c^2)^2} \int \bar{v} n_e dx, \quad (18)$$

and the measured collision frequency is a density-weighted average across the plasma.

Or alternatively, measuring the absorption of the linearly polarized light across the plasma will provide a measure of the electron collision frequency:

$$\bar{v} = \frac{-2 \pi^2 m_e \epsilon_0 c^3}{n_e e^2 \lambda^2 L} \ln [P/P_0] \quad (19)$$

where P_0 and P are, respectively, the laser power before the plasma, and after traversing the plasma. If the plasma is not uniform across the beam path length then the measurement of transmission provides a density-weighted average of the electron mobility

$$-\ln [P/P_0] = \frac{e^2 \lambda^2}{2 \pi^2 m_e \epsilon_0 c^3} \int \bar{v} n_e dx. \quad (20)$$

If the induced ellipticity is small, a transmission measurement may provide a more certain measure of the collision frequency than an ellipticity measurement. In general, for shorter path lengths and consequent longer wavelength radiation, a transmission measurement may be the best choice. For longer path lengths and shorter wavelength, radiation the transmission may be so small that a measure of the ellipticity may be best. As discussed in Task 1. D., only transmission measurements could be used for our laboratory flame experiments.

Electromagnet. The magnet affects not only the measured angle of rotation but also the cyclotron frequency, which is determined by the magnetic induction as

$$\omega_c \text{ (rad/s)} = \frac{eB}{m_e}, \quad (21)$$

where e is the electron charge and m_e is the electron rest mass. An electromagnet was obtained from the Department of Physics and Astronomy; although, the magnet had not been operated for many years, simple replacement of resistors and capacitors made it operational.

For safety considerations, a ground strap was attached to the power supply cabinet and the electrical plug was replaced. The magnetic induction was characterized for several operating currents, and found to be very stable and uniform along the axial line between the pole faces. At the maximum current the induction was found to be 0.8 T. The original pole faces were cylindrical with a diameter of 17.8 cm (7"). The pole separation, i.e. pole gap, is 3.18 cm (1.25"). To provide for the axial access for the laser beam and to increase the field strength, the original pole faces were replaced. The design for the new pole faces is shown in Figure 2.

Figure 2. Design for replacement pole faces for the Harvey Wells model L-75B 7" laboratory electromagnet.

The presumption that the new pole faces would double the magnetic induction from 0.8 T to about 1.5 T was born out as shown in Figure 3. At a direct current of 50 A the new pole faces provide a field strength at the center of the gap of 1.43 T. At the maximum current the measured field strength is 1.50 T. At a magnetic induction of 1.5 T, the cyclotron frequency ($\omega/2\pi$) is 42 GHz. The field strength was measured with an Applied Magnetics⁸ model GM1A Micropower Gaussmeter with an axial probe model PA74-30. The meter has an internal calibration; the manual lists the accuracy of this probe as 1.5% of the reading.

Figure 4 shows the spatial variation of magnetic induction along the path of the laser beam, i.e., parallel to the magnetic field lines. The measured Faraday rotation angle is a spatial integral along this line, linear with the induction, so any variation in the induction along the laser path must be taken into account in the analysis. The magnetic induction in the center 1.5 cm of the total 3.2 cm distance between the poles is uniform to within 7%.

Therefore analysis of rotation angle results was not complicated by a non-uniform field since for the stable flame conditions the flame width was 1.25 to 1.5 cm.

Figure 3. Magnetic induction at the center of the pole gap versus direct current to the electromagnetic coils.

Figure 4. Magnetic induction at 50 A versus axial position across the pole gap.

Line Selection. An Apollo model 122 CO₂ laser-pumped, molecular, far infrared laser was purchased for the Faraday rotation system.⁹ This laser can be operated with many different molecular gases providing wavelengths from 118 to 1000 μm . As shown below, the electromagnetic wave frequency will be much greater than the cyclotron frequency for all of the reasonably strong far infrared laser lines. So, the condition of the laser frequency being much greater than the cyclotron frequency is easily satisfied.

Table 1. Wavelength and frequency of some strong, far infrared laser lines.¹⁰

Wavelength (μm)	Laser Medium	Frequency (GHz)
118.8	Methanol	2525
164.3	Methanol	1840
192.8	Methyl Fluoride	1556
372.7	Methyl Fluoride	805
392.3	Methanol	765
496.1	Methyl Fluoride	605
570.5	Methanol	526
890.0	1,1-difluoroethene	337

The plasma angular frequency depends upon the square root of the electron number density as

$$\omega_p = \left[\frac{n_e e^2}{\epsilon_0 m_e} \right]^{1/2}, \quad (22)$$

where the permittivity of free space, ϵ_0 , is $8.85 \times 10^{-12} \text{ C}^2\text{-s}^2\text{-kg}^{-1}\text{-m}^{-3}$. It is necessary to estimate the range of electron densities that can be produced at atmospheric pressure in the laboratory to verify the requirement that the plasma frequency should be much less than the laser frequency. An upper limit on electron density for an atmospheric pressure medium is estimated to be $1 \times 10^{21} \text{ m}^{-3}$; shown here for several densities are the associated plasma frequencies. For all but the longest wavelength laser lines, this condition will be met.

Table 2. Plasma frequency for a range of electron densities.

Electron Density (m^{-3})	Plasma Frequency (GHz)
1×10^{18}	8.97
1×10^{19}	15.5
1×10^{20}	89.7
1×10^{21}	155.

With these conditions met, selection of laser wavelength is dependent upon producing a measurable rotation angle. The table below shows expected Faraday rotation angles for six different laser lines and several electron densities for the assumptions of a path length of 2 cm and a magnetic induction of 1.5 T. The dimension of 2 cm was chosen because of the pole gap dimension and observations of the flame produced by the burner. However, operation of the burner with the nitrous oxide-acetylene slot burner produced a more narrow flame than expected. The electron density of $1 \times 10^{21} \text{ m}^{-3}$ is included as the outside limit of achievable laboratory densities, so it is clear that the very long, far infrared laser lines must be used for a measurable Faraday rotation in the laboratory.

Table 3. Predicted Faraday rotation angle versus electron number density for various wavelength laser lines, including the 496.1 μm methyl fluoride line.

Electron Density (Number / m^3)	Faraday Rotation Angle ($^\circ$)					
	Laser Wavelengths (μm)					
	118.8	163.0	229.1	392.3	496.1	570.5
1×10^{18}	0.0064	0.012	0.024	0.070	0.11	0.15
1×10^{19}	0.064	0.12	0.24	0.70	1.1	1.5
1×10^{20}	0.64	1.2	2.4	7.0	11.	15
1×10^{21}	6.4	12.	24.	70.	111.	150

Using Eq. 14, the optimal wavelength would be approximately 1000 μm for a path length of 2 cm, a typical collision frequency for an atmospheric plasma of $3 \times 10^{11} \text{ Hz}$ and an electron density of $1 \times 10^{20} \text{ m}^{-3}$, as shown in Table 4.

Table 4. Optimal laser wavelengths for a null polarimeter measurement of Faraday rotation angles for various plasma path lengths.

L (m)	λ (μm)
0.02	1056
0.10	472
0.20	334
0.50	211

However, the shorter wavelength laser lines are desirable because of increasing difficulties with longer wavelengths, for instance increasing beam divergence, increasing absorptions, difficulty in detection, etc. The strength of the laser line must be considered also. The 392.3 μm line is a good compromise between these two considerations; not requiring a change in the laser gas, it was selected as the prime wavelength for design crite-

ria. However, efforts to achieve lasing on this wavelength were unsuccessful as discussed in Task 1. C. and experimentation was completed using the 496.1 μm methyl fluoride line.

Task 1. B. Design and Construction of Optical Components

The system design chosen for the Faraday rotation angle measurement is that of a polarizing beamsplitter with two detectors for determining the orthogonal components. This allows for a reasonably rapid time response for the system. The polarizing beamsplitter is a crucial component of the system and typically requires specialized construction. With increasing use of the far infrared wavelengths, commercially available grid polarizers are now available. The polarizer purchased was a Molelectron IGP224, which became available in 1988.¹¹ It is a metallic grid of approximate 10 μm spacing deposited upon a polyester substrate that transmits wavelengths longer than 50 μm . So it should be suitable for any of the far infrared laser lines.

Polarizer Characteristics. Any detailed treatment of Faraday rotation data must include the polarizer characteristics. The manufacturer lists the extinction (or polarization) ratio as greater than 15 and the transmission as about 87% for 100 to 1000 μm . This was verified by testing on the 118 and 496 μm wavelength lines. One method which allows the treatment of polarized light with simple matrices is Jones calculus.¹² In this representation, an elliptically polarized beam with the major axis along the x-axis is shown as the following standard matrix

$$\vec{E}_0 = \begin{pmatrix} 1 \\ i c \end{pmatrix}, \quad (23)$$

where c is a measure of the ellipticity in the beam. The matrix for a perfect polarizer is a 2×2 identity matrix, while the matrix for an imperfect polarizer is given by

$$\vec{P} = \begin{pmatrix} (1-\beta)^{1/2} & 0 \\ 0 & \alpha^{1/2} \end{pmatrix}, \quad (24)$$

where β and α are measures of the imperfection of the polarizer in transmitting the desired polarization and leaking the unwanted polarization, respectively. The output after one polarizer is then the square of the matrix formed by multiplying $\vec{P} \times \vec{E}_0$:

$$P = (1 - \beta + c^2 \alpha) \cos^2 \theta + [\alpha + c^2(1 - \beta)] \sin^2 \theta. \quad (25)$$

Continuing this analysis with a second polarizer in the beam and measuring the minimum and maximum transmitted laser power for rotation of one polarizer, alternately gives enough information to recursively solve for the polarizer characteristics and the laser

beam ellipticity. The results showed that the laser beam at 118.8 and 496 μm is perfectly polarized within the measurement uncertainty, giving the following polarizer characteristics:

Table 5. Measured polarizer imperfection parameters at 496.1 and 118.8 μm .

	496.1 μm	118.8 μm
Polarizer 1		
α	$(1.07 \pm 0.57) \times 10^{-3}$	$(8.12 \pm 0.37) \times 10^{-3}$
β	$(8.75 \pm 0.65) \times 10^{-2}$	$(5.87 \pm 0.53) \times 10^{-2}$
Polarizer 2		
α	$(2.30 \pm 0.88) \times 10^{-4}$	$(7.78 \pm 0.31) \times 10^{-3}$
β	$(7.28 \pm 0.64) \times 10^{-2}$	$(3.42 \pm 0.68) \times 10^{-2}$

Waveguides for Laser Beam Transmission. The far infrared laser beam diverges much more rapidly than a visible laser beam and provisions must be made for beam transport over any but the shortest distances. Preliminary work on far infrared beam transport has shown that a hollow Pyrex waveguide can provide efficient transmission without significant loss of laser beam polarization at the 496 μm . The waveguide is small enough in diameter to be placed through the axial holes in the magnet pole faces. Using waveguides for all but the pole gap, gives adequate transmission without the use of lenses. After transmission through 17" of waveguide on the laser side of the magnet, the laser beam ellipticity is measured to be 0.119 ± 0.018 at 496.1 μm and 0.218 ± 0.021 at 118.8 μm . For operation on the 118.8 μm line, a search for a better waveguide would be appropriate.

Task 1. C. System Assembly

System Components. System components were selected for wavelengths of about 400 μm and longer. Figure 4 illustrates the system diagram for the two configurations developed for this work.

A survey of commercially available far infrared detectors was made. Pyroelectric detectors were chosen because of good responsivity, low noise, room temperature operation, spectral range, and time response. Molectron P4-45 detectors with silicon windows and a condenser cone for input were chosen. The silicon window will allow operation at wavelengths from 22 to 1000 μm .

Figure 5. Faraday rotation system component diagrams: (a) beam splitting polarizer experiment and (b) rotating polarizer experiment.

A CO₂ laser line analyzer and infrared and far infrared laser power meters are necessary for efficient operation of the laser. A model 16-A CO₂ laser spectrum analyzer was purchased from Optical Engineering.¹³ Several companies provide CO₂ power meters. Based upon bid price and performance, a model 201 power meter was ordered from Coherent.¹⁴ This power meter is based upon a thermal sensor and meets the following specifications: spectral range of 0.3 to 30 μm ; maximum continuous wave (cw) power of 100 W; an active surface area of 2.54 cm² with an aperture diameter of 1.90 cm; and time response of less than 1 s.

The laser manufacturer estimates in their literature a continuous wave output for the selected line of the methanol molecule (392.3 μm) of 35 milliwatt. This laser beam power was used as a maximum in selecting the far infrared power meter and the far infrared detectors. Bearing in mind a factor of 2 loss in chopping the beam, transmission losses for the beam which can be quite high, and losses in splitting the beam at the polarizer, a maximum beam power criterion of 5 milliwatts was determined for selecting the detectors. A model LPM-20 laser power meter was ordered from Laser Photonics.¹⁵ This power meter is an absorbing thermopile disc and was purchased with a thermally insulating shell to decouple the meter from ambient temperature changes. Input is through a 60 mm entrant cone reducing to a 25 mm diameter disc, to accommodate the large diameter far infrared beam. The power meter is calibrated for the 0.4 to 300 μm spectral range but will operate at longer wavelengths. It has a maximum power input of 20 W, a resolution of 100 μW , and a time response of 5 seconds.

To account for background radiation from the MHD flow, an optically chopped system was selected and, therefore, lock-in amplifiers were also necessary for the system. Lock-in amplifiers were selected for the system again based upon price and performance, requiring output suitable for computerized data collection. Stanford Research Systems model SR510 lock-in amplifiers were selected.¹⁶ A model MK II light chopper from Rofin was used for optical chopping.¹⁷ Data collection and calculation were computerized with a Data Translation Lab Datax (LSI-11/23) computer.¹⁸

System components selected for the design wavelength of 392 μm included the grid polarizer and the pyroelectric detectors. There is no appreciable change in performance for either of these components at the slightly longer methyl fluoride wavelength of 496 μm or at the shorter 118 μm methanol line. The system, as assembled, should be capable of measurements at any far infrared wavelength.

Laboratory Arrangement. The laser was mounted on a 4' x 8' optical table for stability. The electromagnet was positioned on a wooden table between the laser table and a 4' x 6' optical table for the detector-side components. Adequate table space was left between the laser and magnet for insertion of an optical chopper, power meter, etc., as needed. Hoses were run from a sink in the laboratory for cooling water for the laser and magnet. The magnet power supply sat in a niche between the tables and the arrangement was reasonably compact. The computer and lock-in amplifiers were situated on a nearby table with cables from the detectors and chopper run overhead.

Metal rails were mounted between the two optical tables for the optics of the multi-wavelength emission absorption system that was used for independent measurements of conductivity. This system is typically used for measurements on the potassium D-line and is referred to as the Potassium Emission Absorption System [PEAS].¹⁹ The lamp and transfer optics were positioned on the 4' x 6' table with mirrors on the rails, to direct the beam between the magnet poles perpendicular to and at the same height as the laser beam. A fiber optic cable transfers the light signal to the detector and computer in the upstairs laboratory. The fiber was run across the ceiling, out a window, upstairs, and back in a window to the system; coordination for data collection was done over the telephone.

Far Infrared Laser Wavelength Adjustment. As reported in Task 1. A. the 392 μm methanol line was selected as the design wavelength. Considerable work was done in an attempt to achieve convenient laser operation at the 392 μm methanol line or any methanol line with a wavelength long enough to produce measurable Faraday rotation angles with a laboratory burner. As the 118 μm methanol line will most likely be used for measurements on any MHD flows larger than those in the laboratory, the initial desire to also use a methanol line for the laboratory measurements was one of convenience. Previous work that apparently achieved operation of the far infrared laser at methanol wavelengths longer than the 118 μm line (strong line specified and delivered by the manufacturer) was not reproducible. After considerable efforts it was decided to switch to methyl fluoride as the laser gas.

Switching laser gas entails a day or two of pumping the laser back down. This is the case of using methyl fluoride rather than methanol. When methyl fluoride was received, lasing on the 496.1 μm line was achieved immediately.

Far infrared laser tuning is accomplished by scanning the position of the output mirror to adjust the round trip cavity length to an integer multiple of half the laser wavelength. The laser output is monitored to search for output corresponding to published laser

lines at the appropriate cavity lengths. This technique identifies the precise cavity mirror position for the desired line as well as measuring the wavelength. Raw data of far infrared laser power versus cavity length adjustment was obtained using computerized data collection routines. The chopped laser output is detected by a pyroelectric detector and processed using a lock-in amplifier. The output of the lock-in is digitized and stored in a data file on the LSI-11/23 computer. Figure 6 represents a laser cavity scan done this way.

Figure 6. Far infrared laser power versus laser cavity length adjustment.

Methods of autocorrelation and maximal entropy spectral estimation²⁰ have been applied to the data and indicate that autocorrelation should provide a suitable method for precise cavity length adjustment for laser tuning. If a single wavelength is present, autocorrelation is the easiest method to interpret since it contains peaks at multiples of the laser wavelength. See Figure 7 for an autocorrelation of the Figure 6 laser cavity scan data showing the 164.3 μm line of methanol. When more than one wavelength is present, the autocorrelation is more difficult to interpret and the maximal entropy spectral analysis does a better job of determining the wavelengths present. Figure 8 shows the maximal entropy method applied to the data of Figure 6. The computer data collection and analysis is quick and either of these methods is a useful tool in searching for longer wavelengths from the far infrared molecular laser.

Figure 7. Autocorrelation analysis of the laser cavity length scan shown in Figure 6.

Figure 8. Maximal entropy analysis of the cavity length scan shown in Figure 6.

Instrumentation Control and Data Collection. Instrumentation control and data collection was developed using an LSI-11/23 computer. The computer serves well for this project as it was configured for general purpose laboratory control and measurements. It includes the following: a 16-bit digital input/output interface board; a 12-bit, 16 channel analog-to-digital interface board; a 4 channel digital-to-analog interface board; and a real time clock board.

The primary measurement needed for a Faraday rotation experiment is laser power. The laser beam is optically chopped with phase sensitive detection of the laser power, through lock-in amplification of the output of pyroelectric detectors. The lock-in amplifiers allow for either analog output or digital output through an IEEE-488 GPIB interface. Input of direct digital data through the GPIB interface was determined to be slower than the analog-to-digital conversion. The data from the two pyroelectric detectors is brought into memory with a temporal spacing between the two channels of only 40 μ s. Using the real time clock to set the data collection frequency allows for a collection frequency of 5 kHz, i.e. a set of two measurements every 200 μ s so that the data collection is essentially simultaneous to within the response time of the detectors. The data is read into memory and then written onto disk storage for computation following the experiment. Using this method of data collection, the amount of data that can be collected is limited by memory. This method will maximize the amount of information that can be collected during MHD pulses.

Though the desired end result of these experiments is demonstration of a fast time response measurement system through beam splitting detection and a fixed polarizer, measurement using a rotating polarizer and single detector can provide the most complete data for analysis of system performance and measurement of electron mobility. In this method, the the orientation of the grid polarizer is rotated while measuring laser power versus orientation angle. The resulting power curve can be fit to a theoretical curve to provide a measure of the Faraday rotation angle for a measure of electron number density as well as the ellipticity which gives the electron mobility. This method is the one used for the laboratory experiments reported below and is the method of choice for stable laboratory MHD flows, but would be too slow to time resolve an MHD pulse. Figure 5b illustrates the system for a rotating polarizer experiment.

The orientation of the grid polarizer is controlled through a rotational stage (Newport model 472A),²¹ a computer-controlled stepper motor (Rapid Syn motor 23 D 6102 C and driver model M-62).²² The current hardware gives 100 steps per degree, which is much finer than necessary considering the experimental uncertainty limits.

Time Response of the System. The response of the pyroelectric detectors is very fast; however, the system time response is determined by the chopping frequency and the settings of the lock-in amplifier. A simple experiment with insertion of a beam block and rapid data collection during the signal decay revealed that the time response of the system is determined to within three significant figures as expected by the sum of the pre and post-time constants selected on the front control panel of the lock-in amplifiers. Most rotating polarizer experiments were run with a pre-time constant of 0.1 s and a post-time constant of 0.3 s for a total time constant of 0.4 s. Figure 9 shows measured signal-to-noise ratios versus selected time constants for 100 reads of the pyroelectric detector output voltage using the 496 μm line. The signal-to-noise ratio is crucial in determining the sensitivity of a beam splitting system for measurement of electron density; the graph shows that a signal-to-noise ratio of 30 or greater can be achieved with a time resolution of 0.1 s.

Figure 9. Signal-to-noise ratio for various lock-in amplifier time constants.

Hysteresis. In a rotating polarizer experiment, hysteresis can be introduced by the rate and direction of the angular positioning of the polarizer. The rotation stage has a precision of 0.005° with no perceptible backlash; however, as a matter of precaution, data was collected while scanning in only a single direction.

The hysteresis introduced by the decay constant of the detector system, τ , and the angular speed for turning the polarizer, ω , shows up in the measured laser power curve as

$$P(\theta) = 1/\tau \int_0^\infty \cos^2(\theta - \omega t) \exp[-t/\tau] dt \quad (26)$$

$$P(\theta) = \cos^2(\theta + \phi) + \frac{-\omega^2 \tau^2}{1 + 2\omega^2 \tau^2} \cos[2(\theta + \phi)], \text{ where} \quad (27)$$

$$\phi = 1/2 \sin^{-1} [2\omega\tau / \sqrt{1 + 4\omega^2 \tau^2}].$$

If $\omega\tau \ll 1$, then

$$P(\theta) \approx \cos^2(\theta + \omega\tau) - \omega^2 \tau^2 \cos[2(\theta + \omega\tau)]. \quad (28)$$

As an example, for a decay constant of 0.4 s and an angular speed of 5°/s, the hysteresis introduces a shift in the measured rotation angle of 1.3° and an amplitude of a $\cos(2\theta)$ term of 5×10^{-4} . The shift in the angle will be consistent from experiment to experiment as long as $\omega\tau$ is constant. The shift should introduce no error into the angle measurement since the Faraday rotation angle is found as a difference.

Task 1. D. Sensitivity Analysis

Applying an uncertainty analysis to Eq. 10 and using the square root of the sum of the squares analysis, which is appropriate for independent errors, the measurement uncertainty in measuring the electron density is

$$\frac{\delta n_e}{n_e} = \left[2 \left(\frac{\delta \theta}{\theta} \right)^2 + \left(\frac{\delta L}{L} \right)^2 + \left(\frac{\delta B}{B} \right)^2 \right]^{1/2}. \quad (29)$$

The uncertainty in the laser wavelength is assumed to be negligible and is certainly much smaller than the other uncertainties. The magnetic field measurement should be accurate to within 1.5% according to the gaussmeter manufacturer. The path length through the potassium formate (KHCO_2) flame was determined to be 1.25 ± 0.25 cm and provides the largest component of the electron density uncertainty for these laboratory measurements. For the cesium hydroxide (CsOH) seeded flames, this was 1.50 ± 0.25 cm. For measurements on a confined plasma this term would provide a very small contribution to the total uncertainty.

Uncertainties in measuring the Faraday rotation angles were typically 0.3° for a rotating polarizer and 0.6° for a beam splitting experiment. Uncertainties in the background angle and the flame angle must be combined in a root mean square relationship to give the total uncertainty in the Faraday rotation angle, increasing the angular uncertainty. This gives for a rotating polarizer measurement of around $\theta = 6^\circ$, $\delta n_e/n_e = 0.21$. For a beam splitting

measurement the value would be 0.24. For a confined plasma, the error in the path length would be considerably smaller. So for reference, the relative uncertainty in the electron number density without the path length uncertainty would be 0.072 and 0.142. Choosing a maximum relative uncertainty in the angle measurement as 10%, the minimum electron density that can be measured in our laboratory would be 4.3×10^{19} and $8.6 \times 10^{19} \text{ m}^{-3}$ for rotating and beam splitting experiments, respectively. The maximum measurable electron density for the rotating polarizer experiment is determined by the ambiguity of fitting the laser power versus angle curve for angles beyond 90° and, therefore, for our laboratory conditions has a maximum measurable electron density of $1.3 \times 10^{21} \text{ m}^{-3}$. A beam splitting experiment becomes much less sensitive at angles approaching 45° beyond the initial setting of 45° , and so has a maximum of $6.4 \times 10^{20} \text{ m}^{-3}$. These maxima are greater than achievable densities with laboratory atmospheric plasmas. The seeded flames in our laboratory were much closer to the minimums than the maximums.

The only significant bias in a rotating polarizer experiment is the hysteresis, which presents no problem as long the angular speed is kept constant. However, significant bias can be introduced into a beam splitting experiment by the initial orientation of the polarizer. This bias could be quite large if positioning of the polarizer is done by hand with only $\pm 1^\circ$ accuracy. With fine adjustment rotation stages and computer control of both the polarizer transmission axis, as well as the angle between the laser beam and the polarizer, this bias can be kept to less than 0.3° .

The uncertainty expression for the collision frequency measured by the ellipticity method is

$$\frac{\delta \bar{\nu}}{\bar{\nu}} = \left[2 \left(\frac{\delta \theta}{\theta} \right)^2 + \left(\frac{\delta L}{L} \right)^2 + \left(\frac{\delta B}{B} \right)^2 + \left(\frac{\delta \chi}{\cos 2\chi \tanh^{-1}(\tan \chi)} \right)^2 \right]^{1/2} \quad (30)$$

For a short path length benchtop flame, χ will be on the order of 10^{-2} and the ellipticity uncertainty, third term in the square root bracket, can be approximated as $(\delta \chi / \chi)^2$. For such a small value of ellipticity the ratio between the intensities of the minor and major axes of the ellipse is on the order of 10^{-4} . This value is within the noise of the laser power measurements. The collision frequency could not be measured with any precision in these laboratory experiments through measurement of ellipticity. An intensity ratio would need to be on the order of 10^{-2} for 10-20% uncertainty; this could be realized by a similar flame only with a ten fold path length increase. So for path lengths of greater than 10 cm, an ellipticity measurement may be feasible from a rotating polarizer experiment.

What can be done instead is to measure the collision frequency through an absorption measurement. This measurement gives

$$\frac{\delta \bar{\nu}}{\bar{\nu}} = \left[2 \left(\frac{\delta \theta}{\theta} \right)^2 + \left(\frac{\delta L}{L} \right)^2 + 2 \left(\frac{\delta P}{-\ln(P/P_0) P} \right)^2 \right]^{1/2}, \quad (31)$$

where P_0 is the measured laser power before the flame is on and P is the laser power with the flame present. The uncertainty in P_0 is assumed to be the same as for P , hence the factor of 2. A typical measurement of laser power has a 2% uncertainty arising in the pyroelectric detectors. For the typical values given above, the expected transmission through the flame would be 58% and the expected uncertainty would be 22%. The combined uncertainty in the electrical conductivity is then

$$\frac{\delta \sigma}{\sigma} = \left[4 \left(\frac{\delta \theta}{\theta} \right)^2 + \left(\frac{\delta L}{L} \right)^2 + 2 \left(\frac{\delta B}{B} \right)^2 + 2 \left(\frac{\delta P}{-\ln(P/P_0) P} \right)^2 \right]^{1/2}. \quad (32)$$

For the same individual uncertainty values, this gives an estimate for the conductivity uncertainty of 29%.

Task 2. A. Measurements on MHD Flows

Sources under consideration for these laboratory demonstration experiments included seeded flames, small rocket motors, and arcs. Seeded flames were the first choice because of low cost, ease of operation, and convenient use of atomic resonance lines for independent measurement of the seed atom density, temperature and, indirectly, electron density. Since the laboratory does not currently have the capability for rocket motors and arcs, experiments were limited to seeded flames. Early experiments run with potassium as the seed compound gave small rotation angles; later experiments were run with cesium as the seed.

To provide an independent measure of the electron density, measurements of flame temperature and seed atom density were made by emission absorption measurements at the 766.49 nm potassium D_2 line or the 455.53 nm cesium secondary D_2 line. From these measurements the electron density can be inferred by application of the Saha equation:¹

$$\frac{n_e n_i}{n_s} = \frac{(2\pi m_e kT)^{3/2}}{h^3} \frac{2g_i}{g_0} \exp \left(-\frac{E_i}{kT} \right), \quad (33)$$

where g_i/g_0 is 1/2 for the alkali metals. The ionization energies for potassium and cesium are 4.340 and 3.893 eV, respectively. The assumption of equilibrium should be valid since the measurements were made well out of the flame reaction zone. Emission absorption methods

cannot provide a measure of the electron mobility in order to provide the conductivity, so chemical equilibrium calculations of the composition of the flame gases, coupled with an electron mobility model were used to calculate the electron collision frequency. The measured electron density and calculated collision frequency were then combined to give an independent measure of conductivity.

Seeded Flames. A Perkin-Elmer atomic absorption burner with several burner heads and a nebulizer were borrowed for use on this contract.²³ A nitrous oxide-acetylene burner head was chosen since this flame provides the highest temperature available, around 2900K, and the electron density in an equilibrium plasma varies exponentially with temperature. The burner operation was straightforward and the stable flame provided a useful demonstration plasma. The burner is a single slot-type with a 1 mm by 5 cm long slot. The width of the burner head is 2.5 cm (1 inch) and fits between the poles of the magnet. An extra length of pipe extended the neck of the burner to place it properly between the poles. The burner was positioned with the center of the axial hole for the laser beam transmission 2.7 cm above the burner head.

Figure 10 shows the measured flame temperature for various fuel and oxidizer pressures. The temperatures were measured by potassium D-line modified line reversal through the long dimension of the flame, perpendicular to the far infrared laser beam path and at the same height above the burner. An aqueous solution of potassium hydroxide was nebulized into the flame for these measurements.

Figure 10. Flame temperature versus fuel and oxidizer pressures.

Stable operation of the burner was difficult at the highest temperatures, i.e. lowest fuel pressure, so the flows were set for a slightly higher fuel pressure and the flame temperatures were around 2800 K for the Faraday rotation experiments reported below.

Both Faraday rotation and emission absorption are line-of-sight techniques so the temperature and density profile of the flame were investigated using emission absorption measurements on a flame with nebulization of a 50% weight aqueous solution of cesium hydroxide. Figures 11 and 12 show the lengthwise profiles for the flame at a height of 2.7 cm above the burner head. This height above the burner was fixed to coincide with the height of the far infrared laser beam, a height determined by the tables holding the magnet and laser. The light beam was focused through an achromat lens to a lamp image in the center of the flame and then imaged onto the tip of a 1 mm core diameter optical fiber for transmission to the detector-side components of the instrument. The collimated light beam was apertured to 1 cm size to match the far infrared laser. The beam would be slightly larger at the edges of the flame and in addition the flame positioning accuracy was probably only ± 0.2 cm. So, the measurements shown in Figures 11 and 12 are spatial averages in height and width and only accurate in the width to ± 0.4 cm.

The figures show the average and standard deviation for 40 samples as bars. The increased standard deviation at -2.25 cm indicated that the flame is flickering in and out of view at this position. The standard deviations were larger for these lengthwise profile measurements than for the crosswise profile measurements below, because of the shorter path length for absorption. There was no detectable flame absorption at positions ± 2.75 cm. Within the measurement uncertainty the flame is uniform along its length and does not have a significant thermal boundary layer at the edges that would complicate the emission absorption analysis. From these results and visual observations, a path length of 5.0 cm was assumed for analyzing the emission absorption data.

Figures 13 and 14 show the crosswise profiles for the flame at a height of 2.7 cm above the burner head, i.e. along a path length of approximately 5 cm. The increased uncertainties at positions of ± 1.5 cm shows that the flame is flickering in and out of the light beam path. Similar optics were used for this study as for the lengthwise profiles and the beam diameter at the edges of the flame was approximately 0.3 cm. Because of the width of the beam, this study could not be used to determine the width of the flame. The width of the flame was determined by placing a ceramic piece into the flame at the height of the laser beam and measuring the burned area. This area was found to be 1.25 ± 0.25 cm wide for the potassium seeded flames and is consistent with these flame measurements. The burner head

was repositioned before the final CsOH experiments and the path length was estimated as 1.5 cm. There was no measurable cesium line emission at positions of ± 1.5 cm.

Figure 11. Lengthwise profile of flame temperature (across the narrow dimension of the flame) at a height of 2.7 cm above the 5 cm slot nitrous oxide-acetylene burner.

Figure 12. Lengthwise flame profile of cesium atom and electron densities at a height of 2.7 cm above the 5 cm slot nitrous oxide-acetylene burner.

Figure 13. Crosswise profile of flame temperature (across the long dimension of the flame) at a height of 2.7 cm above the 5 cm slot nitrous oxide-acetylene burner.

Figure 14. Crosswise flame profile of cesium atom and electron densities at a height of 2.7 cm above the 5 cm slot nitrous oxide-acetylene burner.

Table 6 gives the results of experiments to investigate the nebulization into the acetylene-nitrous oxide flame of different salt solutions, potassium formate²⁴ or potassium fluoride²⁵ in either water, ethanol, or ethanol. The potassium salt solutions were prepared by mixing a weighed amount of salt into a measured volume of distilled water or alcohol. The results of the table reflect the relative effects of solubility and nebulizer efficiency. The flame operation was constant for all of the measurements and, as determined by PEAS measurements, was considerably under the maximum attainable temperature. After these experiments, only aqueous solutions were used for the final experiments.

Table 6. Measured Faraday rotation angles for a nitrous oxide-acetylene flame with nebulization of various salts.

Rotation Angle $\theta \pm \delta\theta$ (°)	Salt Solution
1.37 $\pm$ 0.24	Potassium Fluoride in Water
1.02 $\pm$ 0.28	Potassium Fluoride in Ethanol
1.37 $\pm$ 0.27	Potassium Fluoride in Methanol
1.44 $\pm$ 0.24	Potassium Formate in Water
0.58 $\pm$ 0.24	Potassium Formate in Ethanol
0.71 $\pm$ 0.27	Potassium Formate in Methanol
0.06 $\pm$ 0.26	Water

Trying to maximize the electron density in the flame led to the use of cesium salts due to the lower ionization energy compared to potassium. Cesium hydroxide was purchased both as a solution, 50% by weight aqueous solution²⁶ which was used directly, and as a solid²⁷, which was dissolved in water for nebulization. A cesium chloride²⁸ aqueous solution was also used for some preliminary experiments.

To predict electron collision frequency in the flame, accurate mass flows were needed for chemical equilibrium calculations. The mass flows for the flame, (acetylene, nitrous oxide, water and salt) were measured as follows. The gas flow volumes were measured with model VFB volume flow meters from Dwyer Instruments and were corrected to mass flows by correction for pressure and specific gravity. The estimated uncertainty in the measurement of gas flow was less than $\pm 5\%$. The total aqueous solution flow was measured as a total volume change over approximately 20 minutes of flame operation.

Table 7 lists the mass flows for three flames used in the final experiments. Experiments for flame 1 were performed on December 18, 1989, and for flame 3 with 61.7%

cesium by weight on February 16, 1990. Flame 2 flows were measured on January 26 for a 50% cesium solution. Experiments on January 11, 18 and 26 also used a 50% cesium solution. The solution flow was only 3% different on the 18th. No measurement was made on the 11th.

Table 7. Mass flows for seeded nitrous oxide-acetylene flames.

Flame # / Salt	Mass Flows (g/s)		
	1 / KCHO ₂	2 / CsOH	3 / CsOH
N ₂ O	1.046	1.046	1.046
C ₂ H ₂	0.118	0.118	0.118
Seed	0.00371	0.00959	0.00819
H ₂ O	0.00599	0.00959	0.00384

Emission Absorption Measurements. A brief summary of the methods used to measure flame temperature and atomic number density through emission absorption of an atomic resonance line is given here. Line reversal is a well known and much used technique for measuring flame temperatures.^{29,30} Even with the development of more sophisticated laser techniques which can provide spatial resolution, line reversal is still routinely used for verification.³¹ Applied in its most general form and referred to as a modified line reversal method,³² the technique falls into the category of emission absorption. In a modified line reversal measurement the reference lamp temperature is held constant and the gas or flame temperature is found by recording the intensity of the pre-flame lamp, I_L , the transmitted lamp, I_{L+F} , and the flame emission, I_F , according to

$$\frac{1}{T_g} = \frac{1}{T_L} - \frac{\lambda}{C_2} \ln \left[\frac{I_F}{I_L - I_{L+F} + I_F} \right], \quad (34)$$

where λ is the wavelength of the absorption line, C_2 is Planck's second radiation constant, T_g is the gas temperature, and T_L is the lamp temperature. The intensities are line-of-sight path integrals and hence the measured gas temperature is a spatial average.

The emission absorption intensities may be either spectrally resolved or integrated. The cesium and potassium D-lines could not be resolved with the multi-channel emission absorption system (PEAS)¹⁹ so integrated measurements were made. Wing measurements would have been preferable in order to have minimized thermal boundary layer errors. On large-scale MHD facilities where the line centers would certainly be opaque because of

longer path lengths and higher seed atom loadings, wing measurements would be necessary and could be easily made with the PEAS instrument.*

Absorption measurements of an unresolved atomic line can also be used to provide a measure of the atomic density.³³ The integrated absorptivity is given by

$$A = \int_{\Delta\nu} [1 - \exp(-k_\nu L)] d\nu = \frac{\int_{\Delta\nu} I_L d\nu}{\int_{\Delta\nu} I_L d\nu - \int_{\Delta\nu} I_{L+G} d\nu + \int_{\Delta\nu} I_G d\nu}, \quad (35)$$

where $\Delta\nu$ is the width of the spectral line, whose absorption coefficient is k_ν . The absorption coefficient is a linear function of atomic number density with a shape given by a Voigt profile, so a numerical integration coupled with an iterative adjustment of a guessed number density yields the atomic number density. An approximate form of the Voigt profile was used to speed the calculations.³⁴ The collisional cross sections used for the shape computations were those of Hofmann and Kohn³⁵ for potassium and Chen and Garret.³⁶ for cesium. The cesium values were for collisions with argon. Any error in the cesium line shape should affect the measurement very little since the flame is optically thin and the curve of integrated absorption versus cesium number density is nearly linear in the regime of these measurements. These atomic number densities are for the neutral atoms only (this emission line is not present in the ion).

The seed ion number density was assumed to be equal to the electron density for the Saha computation, although chemical equilibrium computations predict as much as a 10% deviation from equality because of CO_2^- and OH^- ions; see Appendix B. Chemical equilibrium calculations are only accurate to within 10 to 20%, and the accuracy of the emission absorption measurements is less than 10 or 25%, respectively, for potassium or cesium, due to the uncertainty in the absolute absorption coefficient from the oscillator strength.^{37,38}

* The PEAS instrument uses an Instruments SA model HR-320 0.32 m Czerny-Turner monochromator with either a 600, 1200, or 1800 groove/mm grating to give a dispersion across the channels of a vidicon detector of approximately 0.12, 0.06, or 0.03 nm/channel. The vidicon detector suffers from a signal bloom of at least three channels FWHM (full-width-half-maximum) so that the effective instrument resolution with these gratings is about 0.4, 0.2, and 0.1 nm, respectively. The highest resolution grating could not be used for the potassium D-lines since its maximum wavelength is 700 nm while the potassium D-lines are at 768 nm so the 1200 groove/mm grating was used for all of the measurements reported here. A Voigt model for temperature, pressure, path length and atomic density as experimentally observed predicts FWHM for the emission intensity of 1.0 and 0.1 nm for potassium and cesium respectively. These predictions agree well with the observed FWHM of approximately 1.1 and 0.35 nm, respectively, which include vidicon effects. The curves of optical density versus wavelength are even steeper and accurate measurement of absolute optical depth versus wavelength could not be made in the line wings.

The measurement uncertainty for determining the electron number density through the Saha calculations is given by

$$\frac{\delta n_e}{n_e} = \left[\left(\frac{1}{2} \frac{\delta n_s}{n_s} \right)^2 + \left(\left(\frac{3}{4} + E_i / 2kT \right) \frac{\delta T}{T} \right)^2 \right]^{1/2} \quad (36)$$

Considering the relative measurement uncertainties for seed atom density and temperature, the percentage uncertainty for measurements of electron density would be expected to be similar to those for the seed atom density. However, the spectrally integrated emission absorption measurement of temperature and seed atom density are correlated--any fluctuation in the flame absorptivity between the temporally-separated emission and transmitted lamp signals will affect the temperature and number density in the opposite direction. These errors nearly cancel in the electron density calculation and the relative measurement uncertainty for electron density is smaller than for seed atom density. No bias is introduced into the average measurements by such fluctuations.*

Electron Collision Frequency Model. The electron collision frequency was modeled by the method presented by Frost³⁹ with the modification of using numerical integration for momentum transfer with neutral molecules, rather than the approximate expressions given by Frost. The expression is an integral over electron velocities as

$$\mu_e = \frac{4\pi e}{3m_e} \int F(v) \frac{d}{dv} \frac{v^3}{v_t} dv, \quad (37)$$

where $F(v)$ is a Maxwellian velocity distribution which should be applicable for these conditions and v_t is the total collision frequency at a particular electron velocity. This expression is valid for no magnetic field or in the direction of the field; in the perpendicular direction the mobility is reduced by electron cyclotron effects by approximately $\mu_{\perp}/\mu = (1 + B^2\mu^2)^{-1}$. The mobilities and conductivities in this report will always refer to those without a magnetic field or in the field direction.

The total collision frequency is the sum of the collision frequency with neutral molecules, ν_0 , and the collision frequency with ions, μ_i . The ν_0 is found as a sum over the species in the flame

$$\nu_0 = \sum_i v N_i Q_d(i), \quad (38)$$

* In two-wavelength emission absorption measurements, the standard PEAS measurement scheme, this error is cancelled in the seed atom number density determination and nearly cancelled in the temperature determination.

where N_i is the i th species mole fraction (excluding ions and electrons) and $Q_d(i)$ is the species electron-molecule momentum collision transfer cross section.

The electron-ion collision frequency is found as an analytic expression,

$$\nu_i = 0.476 K \eta^{-1}, \quad (39)$$

where η is the dimensionless electron energy variable $\frac{1}{2} m_e v^2 / kT$ and

$$K = \frac{2\pi m_e e^2 \ln \Lambda}{(4\pi\epsilon_0)^2} (2e/m_e)^{1/2} (kT/e)^{-3/2}, \quad (40)$$

where

$$\Lambda = \frac{3}{(2\pi n_e)^{1/2}} (4\pi\epsilon_0 kT/e^2)^{3/2}. \quad (41)$$

The numerical integration was started at a small non-zero value in order to avoid problems with an infinite electron-ion cross section at zero electron energy. A polynomial interpolation was used to find the cross sections from a table of values (see Appendix A on page 48). A Simpson's rule integration was used and integration continued until the Maxwellian velocity distribution dropped below 10^{-5} or an electron energy of 10 eV was reached. The integration step size was reduced until convergence was achieved.

Figure 15 shows the predicted electron number density from equilibrium calculations, versus temperature for the three mass flows reported in Table 7. These predicted mole fractions are listed in Appendix B. The values are about double those measured by emission absorption/Saha calculation and also by Faraday rotation.

Figure 16 shows the predicted collision frequency found by modeling, using the predicted mole fractions. The slight irregularity in the calculations arises from the different step sizes in the integration and the interpolation routine used for finding the collision cross sections. Any overestimation of the electron density will also mean an overestimation of the alkali ion and, hence, a bias introduced into the ion collision frequency term. This is not a large effect and the results are in qualitative agreement with the microwave measurement of a collision frequency of $2.6 \times 10^{11} \text{ s}^{-1}$ for a 2480 K acetylene-air flame.⁴⁰

Figure 17 shows the predicted electron mobility versus temperature and Figure 18 shows the predicted electron conductivity versus temperature.

Figure 15. Chemical equilibrium calculation of electron density versus temperature for the three flames whose mass flows are given in Table 7.

Figure 16. Model calculation of electron collision frequency versus temperature for the three flames whose mass flows are given in Table 7.

Figure 17. Model calculation of electron mobility versus temperature for the three flames whose mass flows are given in Table 7.

Figure 18. Model calculation of electron conductivity versus temperature for the three flames whose mass flows are given in Table 7.

Faraday Rotation Measurements. The rotating polarizer experimental data were analyzed for Faraday rotation angles by least squares fitting of the laser power versus polarizer orientation angle to the function

$$I(\theta) = A (1 - \alpha\theta) \cos^2(\theta - \theta_0) + D, \quad (42)$$

where θ is the polarizer orientation angle, θ_0 is the angle of maximum transmission through the polarizer, A is the amplitude of the laser, α allows for a linear drift of the laser power in θ which is linear in time, and the constant D allows for polarizer leakage, background noise, and/or polarization ellipticity introduced by the flame or waveguide. The Faraday rotation angle is the difference between the value of θ_0 for a background file, and that of the seeded flame.

The fitting process minimizes the sum of the square of the differences between experiment and function as

$$F = \sum_{i=1}^n [I(\theta_i) - I_i]^2, \quad (43)$$

where I_i is the measured laser power at θ_i . The computer program uses a polytope-type⁴¹ algorithm and the fitting continues until the function F does not change significantly. Because the magnitude of the data changes appreciably from maximum to minimum, a more appropriate fitting is the Levenberg-Marquardt method⁴² which weights the fitting with the standard deviations, δ_i , as

$$F = \sum_{i=1}^n \left(\frac{I(\theta_i) - I_i}{\delta_i} \right)^2. \quad (44)$$

Experimental Results. Table 8 lists the results of rotating polarizer Faraday rotation experiments for three seeded flames: KHCO_2 and CsOH aqueous solutions of 50 and 62% nebulized into a nitrous oxide-acetylene flame. A path length value of 1.25 cm was used for analyzing the potassium-seeded flame measurements and 1.5 cm for the cesium-seeded flames.

The error limits in angle were determined by the 99% confidence interval for the least squares fitting, combining the background and measured angles in a square root of the sum of the squares fashion. Measurements of the polarizer transmission angle of a second polarizer indicates that the system operating on the 496 μm line can repetitively measure polarization angles with an error less than $\pm 0.3^\circ$. Some of the flame measurements are double this value, most likely representing real fluctuations in the electron density and/or the flame

width. As the flame appears to be stable by temperature measurement, the most likely explanation is fluctuations in the flame width.

Table 8. Faraday rotation results for experiments on seeded flames.

Clock Time hh:mm:ss	Rotation Angle (°)	Electron Density ($\times 10^{19} / \text{m}^{-3}$)	Laser Beam Transmission (%/100)	Collision Frequency ($\times 10^{11} / \text{s}$)	Electron Conductivity (mhos/m)
September 28, 1989 -- KHCO_2 Seed, 38.2% weight aqueous solution					
14:21:42	1.44 ± 0.42	2.07 ± 0.73	0.94 ± 0.02	1.62 ± 0.99	3.6 ± 2.5
December 18, 1989 -- KHCO_2 Seed, 38.2% weight aqueous solution					
16:16:24	3.03 ± 0.63	4.36 ± 1.26	0.83 ± 0.03	2.32 ± 0.89	5.3 ± 2.5
16:37:20	3.02 ± 0.47	4.34 ± 1.10	0.86 ± 0.03	1.88 ± 0.70	6.5 ± 2.9
16:45:45	2.80 ± 0.29	4.03 ± 0.91	0.86 ± 0.02	2.03 ± 0.72	5.6 ± 2.3
January 11, 1990 -- CsOH Seed, 50% weight aqueous solution					
14:52:20	6.63 ± 0.59	7.95 ± 1.51	0.57 ± 0.03	3.20 ± 0.83	7.0 ± 2.2
14:57:32	7.02 ± 0.72	8.42 ± 1.65	0.52 ± 0.03	3.52 ± 0.92	6.7 ± 2.2
15:17:10	6.72 ± 0.47	8.06 ± 1.46	0.52 ± 0.03	3.67 ± 0.92	6.2 ± 1.9
January 18, 1990 -- CsOH Seed, 50% weight aqueous solution					
11:32:02	5.75 ± 0.42	6.89 ± 1.26	0.64 ± 0.02	2.93 ± 0.75	6.6 ± 2.1
11:37:16	4.94 ± 0.44	5.92 ± 1.12	0.62 ± 0.02	3.65 ± 0.95	4.6 ± 1.5
January 26, 1990 -- CsOH Seed, 50% weight aqueous solution					
11:36:00	6.82 ± 0.32	8.18 ± 1.42	0.66 ± 0.02	2.30 ± 0.58	10.0 ± 3.0
11:44:48	7.40 ± 0.29	8.87 ± 1.52	0.63 ± 0.02	2.36 ± 0.58	10.6 ± 3.2
11:50:38	6.47 ± 0.74	7.76 ± 1.57	0.57 ± 0.03	3.28 ± 0.88	6.7 ± 2.2
February 16, 1990 -- CsOH Seed, 61.7% weight aqueous solution					
15:30:38	6.14 ± 0.43	7.36 ± 1.34	0.78 ± 0.02	1.53 ± 0.42	13.6 ± 4.4
15:37:49	6.13 ± 0.43	7.35 ± 1.33	0.76 ± 0.02	1.69 ± 0.45	12.2 ± 4.0
15:47:08	5.55 ± 0.40	6.65 ± 1.21	0.74 ± 0.02	2.05 ± 0.54	9.2 ± 2.9
15:55:44	5.08 ± 0.34	6.09 ± 1.10	0.72 ± 0.02	2.44 ± 0.64	7.0 ± 2.2

The error limits on the transmission measurements are estimated as 2-3% since the signal-to-noise ratio for these measurements was 100 or greater. The limits on the other parameters were determined by propagating the error limits in angle, transmission, path length ($\pm .25$ cm), and in magnetic field strength ($\pm 1.5\%$), in a square root of the sum of the squares relationship as given in Equation 31. The primary contribution to the error limits is that due to the path length uncertainty of approximately 20%. This error limit for electron density and collision frequency is doubled in the conductivity calculation because of the square dependence on the measured electron density. The smaller angle, lower electron density potassium flames were to a greater extent affected by the angular uncertainty.

The trends in Faraday rotation results shown in Table 8 qualitatively match the time variation observed in the emission absorption measurements. Figures 19 through 23 show the emission absorption measured temperature and number densities versus time for the various experiments. The horizontal lines show the time period for averaging results for 50 samples and the vertical bars show the $\pm$ one standard deviation error limits. The heavy bars are for the densities, upper points for neutral seed atom density and lower for electron density, while the lighter bars are for temperature.

Figure 19. Potassium emission absorption measurements on December 18, 1989 of temperature and neutral seed atom density and calculated electron density.

Unfortunately only scattered data were taken on December 18, 1989 and only one Faraday rotation measurement coincided with those measurements. On the 11th and the 26th of January the emission absorption results indicated a very stable flame operation, while the 18th of January data showed steady operation for only the first two thirds of the measurements. Measurements on the 16th of February show an almost step function change in flame temperature, and consequently, electron density while the seed atom density remains fairly constant until the end of the measurements. These variations, over time periods shorter than the time needed for a rotating polarizer experiment, indicate the importance of monitoring the flame operation and the need for faster rotation measurements.

Figure 20. Cesium emission absorption measurements on January 11, 1990 of temperature and neutral seed atom density and calculated electron density.

Figure 21. Cesium emission absorption measurements on January 18, 1990 of temperature and neutral atom density and calculated electron density.

Figure 22. Cesium emission absorption measurements on January 26, 1990 of temperature and neutral atom density and calculated electron density.

Figure 23. Cesium emission absorption measurements on February 16, 1990 of temperature and neutral atom density and calculated electron density.

Figure 24 illustrates the comparison between electron densities measured with Faraday rotation and with a Saha calculation using the emission absorption measurements of temperature and neutral atom density. The comparison shows no obvious bias between the direct measurements of electron density with the Faraday rotation system and the indirect emission absorption measurement.

The uncertainty in the Faraday rotation measurements are shown as horizontal bars, while the uncertainty in the emission absorption/Saha measurements are shown as vertical bars. The emission absorption uncertainty is slightly underestimated since the instrument uses a data rejection scheme for points two standard deviations away from the average; typically only 38 data points were kept out of 40 measurements. The uncertainty in the Faraday rotation measurements were calculated as in Equation 29.

Figure 24. Comparison between electron number densities measured by Faraday rotation and by emission absorption/Saha calculation.

Figure 25 shows a similar comparison for electron conductivity. Again the error limits are shown as horizontal and vertical bars. The emission absorption measured temperature was used along with the curves shown in Figure 16 to estimate the electron collision frequency, which with the electron number density gives conductivity. The uncertainty in the emission absorption/Saha/modeling calculation was found as the square root of the sum

of the squares for the calculated electron density and a 20% error limit for the collision frequency model. The error limits on the Faraday rotation measurements were found as in Equation 32.

Figure 25. Comparison between electron conductivity measured by Faraday rotation and by emission absorption/Saha/collision frequency modeling.

The measurements of electron number density and electron conductivity agree for most of the points within error limits with the independent measurements. Some of the differences could be due to a lack of coincidence between the two measurements, although Faraday rotation data points where the flame did not appear to be stable were not included in the comparison.

All the results are within the ranges expected for seeded flames. The measured electron number density values are generally around a factor of 2 below that predicted by chemical equilibrium calculations. This factor of two is consistent with the higher predictions of neutral seed atom density as compared to the emission absorption measurements. Several scenarios were studied through chemical calculations in an attempt to explain this discrepancy. First, air entrainment in the flame would lower the electron density, but a calculation with a very fuel rich flame and large air entrainment only lowered the prediction of electron density at the measured flame temperature by 30%. Varying the values of the

measured gas flows to give a flame from very fuel rich to very fuel lean only changed the electron density at the measured flame temperature by 20%. A subsequent potassium formate experiment revealed a 10% higher concentration of salt in the waste, than in the starting solution. This translates into a 30% decrease in the salt nebulized into the flame, and to a decrease in electron density of about 15%.

These results can be understood by examining the form of the Saha calculation of electron density for incremental changes in neutral seed atom density and temperature, showing that the change in electron density is not a strong function of neutral seed atom density because of the square root relationship. However, the electron density is a strong function of temperature because of the exponential relationship, but the error in the measured temperature is certainly less than 50 K as line reversal is a very sensitive, accurate technique for measurements of flame temperature. An error of 50 K out of 2700 K is less than 2% and even with a value of 8 to 9 for the $E_i/2kT$ term, this translates to an error in electron density of less than 20%.

It is possible that an important chemical reaction was omitted from the chemical equilibrium calculations. Addressing this possibility is beyond the expertise of this author, but hopefully any error from omitting a reaction should be encompassed by the ± 10 to 20% in the stated accuracy of chemical equilibrium calculations.

Also investigated as a possible source of bias between the measurements and prediction was a temperature profile across the flame. Numerical calculations with a parabolic flame temperature profile, and chemical equilibrium calculation predicted profiles of seed atom and electron density, revealed less than 5% difference between the predicted Faraday rotation and emission absorption measurements of electron density and the chemical equilibrium calculation predicted electron density at the emission absorption measured average temperature.

Summing all of these possible sources of error cannot account for a factor of two. This leaves only the possibility of a lack of chemical equilibrium. With a measurement height of 2.7 cm above the burner surface and a reaction zone that extends less than 0.5 cm above the surface, it was assumed that equilibrium would be established. If chemical equilibrium is not established, the results are not changed appreciably. The chemical equilibrium calculation were only used as input to the mobility model and the electron mobility does not vary appreciably due to the chemical constituents. In fact, referring to Figures 15 and 17, the

mobility increases by less than 10% over a 400 K increase in temperature, corresponding to a predicted increase in electron density of over a factor of 6.

However, if chemical equilibrium is not achieved, the question of whether ionizational equilibrium is established is raised; this question is very important to this study. Failure to achieve chemical equilibrium while ionizational equilibrium is established is very conceivable in light of the much smaller mass for the electron and, therefore, several orders of magnitude higher velocity and collision frequency as compared to the atoms and molecules in the flame. Work by Hofmann, Kohn, and Schneider⁴³ on an acetylene-air flame supported ionizational equilibrium at a position of 0.8 cm above a slot burner very similar to the one used in this study. The close agreement between the direct measurement of electron density through Faraday rotation and the indirect measurement by emission absorption implies that ionizational equilibrium has been established.

Task 3. A. Analysis of System Performance

These experiments demonstrated far infrared Faraday rotation measurements of electron number densities ranging from 2×10^{19} to $9 \times 10^{19} \text{ m}^{-3}$, almost an order of magnitude. The results for cesium as well as potassium-seeded flames were in good agreement with Saha calculations using emission absorption measured flame temperatures and neutral seed atom number densities. The electron collision frequency, measured by laser beam transmission, gave measured electron conductivities of between 4 and 12 mohs/m. These results compared reasonably well with those found with an electron collision frequency model using chemical equilibrium calculations and the emission absorption measurements of temperature and electron density. Ellipticity measurements of electron collision frequency were not possible due to the short path length of the laboratory plasma.

The measurement uncertainties were about 30% for the electron number density and 35-45% for the conductivity. Close to half of this uncertainty arises from the uncertainty in the plasma width. A confined plasma rather than an open flame would provide a better test of the system performance; some time was spent at the start of the contract looking at different plasma sources. However, money or facilities for other sources were not available.

The overwhelming constraint in designing the experiment was the narrow pole gap of the available magnet, which limited the maximum plasma path length to less than 3 cm and also physically limited the type of plasma source that would fit between the poles. A magnet with an axial access and a larger pole gap would have cost at least \$20,000⁸ and plasma

torches between \$10,000 and \$20,000.²³ Use of the atomic absorption burner was the only available option for producing a stable, reproducible plasma. Due to the short path length allowed by the pole gap, a very hot flame was necessary to achieve an acceptable rotation angle; at temperatures of 2750 to 2850 K the flame could not be contained to provide a more certain path length. Although the path length uncertainty is large, the burner did provide a reasonably stable plasma, $\pm 10\%$, over the 25 minutes or so of solution nebulization for seeding and a reproducible plasma, $< \pm 20\%$ in electron density, from day to day.

The system in a rotating polarizer configuration is capable of measuring rotation angles to within $\pm 0.3^\circ$. This is an angular resolution that would translate into a very good measure of the average electron density for a plasma which gives larger rotation angles. By maximizing the flame temperature and using cesium as the seed, the 1.5 cm flame width did provide an acceptable test since the rotation angles were greater than 10 times the angular measurement uncertainty. The time period for these measurements was just over 4 minutes, but could be reduced to around 2 minutes by shortening the delays in the data collection.

In addition, because of the short path length and small angles, the experiments were performed on the 496 μm laser line. This line is not as strong or as stable as the 118 μm methanol line, which presumably would be used for measurements on large-scale MHD flows. So, the relative error for these demonstration experiments will no doubt be greater than that for actual MHD testing. However, these experiments were performed with a rotating polarizer and this method is expected to give lower uncertainties than the beam splitting method; a beam splitting system will be necessary for time-resolved measurements. A time resolution approaching 0.1 s can be achieved with the beam splitting system.

Recommendations for configuring the system for measurements on larger scale MHD flows are shown in Table 9. Applying the system to other MHD flows requires only a change in the laser wavelength—all of the system components work well at wavelengths from 118 to 496 μm . For path lengths shorter than 5 cm, a long wavelength such as the 496 μm methyl fluoride line is necessary, while for path lengths longer than 20 cm the short wavelength 118 μm methanol line will be necessary. At intermediate path lengths, wavelengths between these two values are necessary for reasonable angles and transmission values. Measurement of electron collision frequency through ellipticity is recommended only for situation of small transmission. These recommendations would need to be adjusted for the actual magnetic field value and expected electron densities; the system should prove to be a useful diagnostic for MHD development because of its applicability to a almost any size flow.

Table 9. Recommended laser wavelengths for application of the Faraday rotation system to large-scale MHD flows. The calculations were performed for a magnetic field strength of 3 T and a collision frequency of $3 \times 10^{11} \text{ s}^{-1}$.

Path Length (m)	Laser Wavelength (μm)	Faraday Rotation ($^\circ$)	Transmission (%)	Ellipticity ($^\circ$)
0.01	496	11	64	1.7
0.02	496	22	41	3.5
0.05	373	31	29	7.4
0.10	192	17	51	1.0
0.20	164	24	38	1.3
0.50	118	32	28	1.2
0.70	118	45	17	1.7
1.00	118	64	8	2.4

The range of electron densities that can be measured by the Faraday rotation system is limited by angles of 3 to 90° for a rotating polarizer system. For a nominal 3 T magnet, such a system could be used to measure electron density times path length as low as $2.7 \times 10^{17} \text{ m}^{-2}$ using the 496 μm laser line. With the 118 μm laser line, the maximum measurable density time path length would be $1.4 \times 10^{20} \text{ m}^{-2}$. However, the laser beam transmission would be quite low at the higher densities and longer path lengths. Path lengths greater than 0.5 m may make measurements unfeasible. In the beam splitting configuration, the uncertainty in the angle is $\pm 0.6^\circ$ and these minimum and maximum values are 5.4×10^{17} and $7.1 \times 10^{19} \text{ m}^{-2}$, respectively, for angles of 6 and 45° , and laser lines of 496 and 118 μm .

While Faraday rotation is predicted to be an appropriate diagnostic instrument for large-scale MHD flows, there are still a few practical problems and engineering difficulties which must be addressed. The most difficult engineering problem is the opposed optical windows which must be far infrared transparent. This problem was solved for the small-scale research MHD facility at Stanford and their work should provide a good starting point for optical access on larger facilities.⁴⁴ Another problem may be laser stabilization because of thermal variations, mechanical vibrations, and stray magnetic fields near a large-scale MHD facility. Operating the laser away from the facility by the use of long waveguides, magnetic shielding, and vibration isolation will likely be necessary. Similarly concerns need to be addressed for the detectors as well. Only short lengths of waveguide were used in these experiments and work to optimize the waveguide transmission will be necessary. Customized far infrared lenses of polyethylene or TPX may be necessary for proper beam transmission, although such was not required for these laboratory experiments. These

concerns would all be facility dependent. Testing on a larger scale and pulsed MHD flow would be necessary to verify the prediction of the suitability of Faraday rotation as an MHD diagnostic for support of innovative channel development efforts.

SUMMARY

All of the tasks of this contract have been completed. The demonstration experiments show measurements of electron density that agree with an independent measurement by resonance line emission absorption and a Saha calculation. The measurements of electron collision frequency agree well with modeling calculations, giving electron conductivity results that are reasonable for the flame system used to provide the plasma. The uncertainty in the measurements were larger than desired for a demonstration of the system performance; however, a magnet with a larger pole gap would have been required for a longer path length in order to increase the angles and lower the percentage uncertainty.

The system in a rotating polarizer configuration is capable of measuring rotation angles to within $\pm 0.3^\circ$. For a nominal 3 T magnet, the system could be used to measure electron density times path length as low as $2.7 \times 10^{17} \text{ m}^{-2}$ using the $496 \mu\text{m}$ laser line. With the $118 \mu\text{m}$ methanol laser line, the maximum measurable density time path length would be $1.4 \times 10^{20} \text{ m}^{-2}$. In the beam splitting configuration the uncertainty in the angle is $\pm 0.6^\circ$ and these minimum and maximum values are 5.4×10^{17} and $7.1 \times 10^{19} \text{ m}^{-2}$, respectively. A beam splitting measurement can provide a time resolution of 0.1 s which would provide adequate time resolution for diagnostics on an MHD pulse. Recommendations for configuring the system performance on larger scale facilities have been made. The system is generally applicable to MHD flows over a wide range of size and plasma conditions because of the many laser wavelengths that can be used. Recommendations were made for laser wavelengths to use when applying the system to MHD flows. The system should prove to be a useful diagnostic for MHD development.

APPENDIX A. ELECTRON MOMENTUM COLLISION CROSS SECTIONS

Table 10 tabulates the electron collision cross sections for the molecules considered in the electron mobility model presented on page 32. Most of the values were derived from the compilation presented by Spencer and Phelps.⁴⁵

Table 10. Electron momentum transfer collision cross sections for various neutral molecules as a function of electron energy in electron volts (eV). The values given are in units of meter squared times 10^{20} .

eV	H2OCO2	CO	O2	OH	H2	O	H	N2	K	Cs	Ar	NO	CsOH	KOH	
0.00	50000	600	60	0.35	40000	6.4	4.71	39.6	1.1	676	35000	7.5	120	149000	180000
0.001	33000	540	40	0.35	26400	6.4	4.71	40.4	1.36	350	30000	7.5	94	149000	180000
0.002	16500	380	25	0.36	13200	6.5	4.71	40.6	1.49	243	25600	7.1	58	75000	90000
0.003	11000	307	17.7	0.40	8800	6.6	4.71	40.8	1.62	189	21700	6.7	38	5000	60000
0.005	6600	237	14.0	0.50	5280	6.8	4.71	41.1	1.81	200	15600	6.1	22.5	30000	36000
0.007	4710	200	9.8	0.58	3800	7.1	4.71	41.3	2.0	433	11700	5.4	16.3	21300	25700
0.085	3880	182	8.6	0.64	3100	7.2	4.71	41.4	2.1	680	10200	5.05	13.3	17500	21200
0.010	3300	170	7.8	0.70	2640	7.3	4.71	41.5	2.19	1776	10000	4.6	12.0	14900	18000
0.015	2120	138	6.5	0.87	1700	7.7	4.715	41.6	2.55	7200	7100	3.75	8.1	9900	12000
0.02	1610	120	5.9	0.99	1300	8.0	4.72	41.8	2.85	4300	5550	3.25	6.2	7500	9000
0.03	1060	97	5.4	1.24	850	8.5	4.73	41.84	3.4	1840	3530	2.5	4.65	5000	6000
0.04	830	85	5.2	1.44	660	8.96	4.74	41.8	3.85	1220	2340	2.05	3.85	3700	4500
0.05	650	76	5.4	1.6	520	9.28	4.75	41.7	4.33	960	1620	1.73	3.4	2980	3600
0.07	456	64	6.1	2.1	360	9.85	4.77	41.3	5.1	720	800	1.13	3.0	2130	2570
0.10	318	52	7.3	4.2	257	10.5	4.8	40.6	5.95	550	305	0.59	3.1	1490	1800
0.12	265	46	7.9	4.38	214	10.85	4.82	39.7	6.45	500	307	0.4	3.45	1240	1500
0.15	210	40	8.8	4.65	170	11.4	4.85	38.8	7.1	435	650	0.23	4.05	990	1200
0.17	187	36	9.4	4.8	151	11.6	4.87	38.2	7.4	405	1020	0.16	4.55	870	1060
0.20	153	31.5	10	5.1	122	12.0	4.9	37.3	7.9	360	1570	0.103	5.3	750	900
0.25	124	25.0	11.2	5.3	100	12.5	4.92	36.0	8.5	313	1793	0.091	7.0	600	720
0.30	102	20.0	12	5.46	82	13.0	4.95	35.0	9.0	270	1490	0.153	11.1	500	600
0.35	89.0	16.5	12.7	5.65	71	13.45	4.97	33.7	9.4	243	790	0.235	17.5	425	514
0.40	78.0	14.0	13.5	5.8	62	13.9	5.0	32.7	9.7	215	458	0.33	22.5	373	450
0.50	63.5	10.7	14.3	5.9	51	14.7	5.05	31.0	9.9	180	202	0.51	29.0	298	360
0.70	46.3	7.6	15.5	6.1	37	16.3	5.15	27.4	10.0	136	82	0.69	22.5	213	257
1.0	33.1	5.8	17	7.2	26	17.4	5.2	25.2	10.0	96	80	1.38	16.1	149	180
1.2	28.0	5.3	23	7.9	22	17.8	5.25	23.8	10.4	81	114	1.37	14.0	124.5	150
1.3	26.0	5.1	31	7.9	21	18.0	5.3	23.0	11.0	76	134	1.82	13.7	115	138
1.5	22.9	5.05	35	7.6	18.3	18.25	5.35	21.7	12.0	66	156	2.1	12.0	99.3	120
1.7	20.0	5.1	30	7.4	16.1	18.25	5.4	20.5	13.8	51	161	2.32	11.7	87.8	106
1.9	18.2	5.2	20	6.9	14.5	18.1	5.43	19.4	19.6	67	158	2.58	11.1	78.4	94.7
2.1	16.6	5.35	15	6.6	13.2	17.9	5.46	18.5	27.0	66	154	2.85	10.7	71	85.7
2.2	16.0	5.65	14	6.5	12.8	17.7	5.49	18.1	28.5	63	152	2.97	10.4	67.7	81.8
2.5	14.4	6.5	12.6	6.1	11.5	17.2	5.6	16.8	30.0	54	146	3.37	9.9	59.6	72
2.8	13.2	7.5	12.1	5.8	10.6	16.7	5.65	15.8	28.0	47	141	3.8	9.3	53.2	64.3
3.0	12.4	8.9	12.0	5.7	9.9	16.1	5.7	15.2	21.7	43	137	4.1	9.1	49.7	60
3.3	11.6	11.1	11.8	5.5	9.3	15.6	5.75	14.3	17.2	39.5	132	4.5	8.6	45.1	54.5
3.6	10.8	14.3	11.7	5.45	8.7	15.0	5.85	13.4	14.7	36.5	129	4.9	8.3	41.4	50
4.0	10.0	15.3	11.7	5.5	8.1	14.3	6.0	12.4	12.6	33.5	125	5.4	8.1	37.3	45
4.5	9.3	14.8	11.7	5.55	7.5	13.6	6.1	11.3	11.3	32	121	6.1	8.1	33.1	40
5.	8.6	13.0	11.6	5.6	6.9	12.9	6.2	10.2	10.9	31	105	6.7	8.1	29.8	36
6.	7.55	10.3	11.5	6.0	6.9	11.8	6.4	8.4	10.4	31	100	8.1	8.2	24.8	30
7.	7.05	9.8	11.4	6.6	6.9	11.0	6.6	7.1	10.1	31	95	9.6	8.3	21.3	25.7
8.	6.7	10.8	11.2	7.1	6.9	10.3	6.75	6.1	10.0	31	85	11.7	8.4	18.6	22.5
10.	6.6	12.0	11.0	8.0	6.9	9.0	7.2	6.1	10.2	31	68	15.0	8.8	14.9	18

APPENDIX B. CHEMICAL EQUILIBRIUM CALCULATIONS

Chemical equilibrium calculations were performed for each of the three seeded flames used for the final experiments. The mass flows listed in Table 7 were converted to mole fractions for input to the computer program.⁴⁶

Table 11. Chemical equilibrium predictions of mole fractions for a nitrous oxide-acetylene flame with nebulization of an aqueous solution of 38.2% potassium formate by weight.

Input Values					Moles	Energy/State	Temp	Density	
Chemical Formula						Cal/Mole	K	g/cm ³	
Oxidant	N	2.00000	O	1.00000	23.774	19609.803 G	298.15	0.0000	
Fuel	C	2.00000	H	2.00000	4.543710	54189.375 G	298.15	0.0000	
Fuel	K	1.00000	C	1.00000	0.044120	0.000 S	298.15	0.0000	
Oxidant	H	2.00000	O	1.00000	0.332880	-68317.453 L	298.15	0.0000	
O/F= 8.6245 % Fuel= 10.3901 Equivalence Ratio= 0.9582					Phi= 0.9574	Reactant Density= 0.0000			
Thermodynamic Properties									
P, atm	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
T, K	2800	2750	2700	2650	2600	2550	2500	2450	2400
Rho, g/cm ³	1.2423e-4	1.2759e-4	1.3101e-4	1.3448e-4	1.3800e-4	1.4158e-4	1.4521e-4	1.4890e-4	1.5266e-4
H, CAL/g	146.8	89.3	34.9	-16.6	-65.0	-110.4	-152.8	-192.4	-229.2
S, CAL/g·K	2.3953	2.3746	2.3546	2.3354	2.3170	2.2993	2.2825	2.2665	2.2514
M, Mol. Wt.	28.543	28.792	29.026	29.242	29.442	29.624	29.788	29.935	30.064
(DLV/DLP)T	-1.02190	-1.01958	-1.01738	-1.01531	-1.01337	-1.01156	-1.00990	-1.00838	-1.00701
(DLV/DLP)P	1.5044	1.4614	1.4190	1.3774	1.3370	1.2980	1.2608	1.2257	1.1929
CP, CAL/g·K	1.1794	1.1196	1.0591	0.9983	0.9379	0.8782	0.8198	0.7633	0.7092
Gamma, s	1.1257	1.1262	1.1271	1.1285	1.1303	1.1327	1.1357	1.1393	1.1437
Sonic Vel,m/s	958.2	945.7	933.7	922.1	911.0	900.4	890.2	880.5	871.3
Mole Fractions									
CO	9.1274e-2	8.2369e-2	7.3502e-2	6.4814e-2	5.6440e-2	4.8495e-2	4.1079e-2	3.4269e-2	2.8119e-2
CO ₂	1.3066e-1	1.4150e-1	1.5218e-1	1.6256e-1	1.7248e-1	1.8184e-1	1.9054e-1	1.9849e-1	2.0564e-1
CO ₂ -	1.8980e-6	1.8196e-6	1.7174e-6	1.5952e-6	1.4574e-6	1.3089e-6	1.1552e-6	1.0011e-6	8.5129e-7
e ⁻	3.6998e-5	3.0118e-5	2.4256e-5	1.9314e-5	1.5193e-5	1.1797e-5	9.0336e-6	6.8142e-6	5.0574e-6
H	8.0032e-3	6.2949e-3	4.8976e-3	3.7677e-3	2.8641e-3	2.1500e-3	1.5924e-3	1.1626e-3	8.3581e-4
HO ₂	5.1384e-6	4.4860e-6	3.8676e-6	3.2944e-6	2.7739e-6	2.3103e-6	1.9049e-6	1.5564e-6	1.2615e-6
H ₂	9.6832e-3	8.5702e-3	7.5196e-3	6.5396e-3	5.6351e-3	4.8086e-3	4.0605e-3	3.3898e-3	2.7945e-3
H ₂ O	9.4269e-2	9.8504e-2	1.0239e-1	1.0593e-1	1.0911e-1	1.1196e-1	1.1449e-1	1.1670e-1	1.1862e-1
K ⁺	7.2414e-4	7.0271e-4	6.7690e-4	6.4689e-4	6.1292e-4	5.7533e-4	5.3454e-4	4.9109e-4	4.4561e-4
K ⁺	4.0919e-5	3.3636e-5	2.7384e-5	2.2067e-5	1.7589e-5	1.3858e-5	1.0783e-5	8.2782e-6	6.2647e-6
KO	1.8020e-5	1.7196e-5	1.6235e-5	1.5160e-5	1.3998e-5	1.2777e-5	1.1527e-5	1.0275e-5	9.0486e-6
KOH	2.8922e-4	3.2813e-4	3.6992e-4	4.1447e-4	4.6158e-4	5.1096e-4	5.6224e-4	6.1495e-4	6.6853e-4
N	2.6279e-6	1.8127e-6	1.2328e-6	8.2593e-7	5.4466e-7	3.5322e-7	2.2504e-7	1.4071e-7	8.6250e-8
NO	1.5276e-2	1.3867e-2	1.2502e-2	1.1196e-2	9.9619e-3	8.8089e-3	7.7445e-3	6.7732e-3	5.8968e-3
NO ₂	4.3142e-6	3.9675e-6	3.6158e-6	3.2674e-6	2.9296e-6	2.6085e-6	2.3093e-6	2.0354e-6	1.7892e-6
N ₂	5.7017e-1	5.7593e-1	5.8134e-1	5.8638e-1	5.9104e-1	5.9530e-1	5.9915e-1	6.0261e-1	6.0566e-1
O	1.1577e-2	9.1819e-3	7.2013e-3	5.5841e-3	4.2809e-3	3.2446e-3	2.4315e-3	1.8019e-3	1.3208e-3
OH	2.1910e-2	1.9416e-2	1.7047e-2	1.4830e-2	1.2785e-2	1.0925e-2	9.2536e-3	7.7712e-3	6.4721e-3
OH ⁻	1.4734e-6	1.2773e-6	1.0918e-6	9.2003e-7	7.6407e-7	6.2516e-7	5.0373e-7	3.9954e-7	3.1180e-7
O ₂	4.6049e-2	4.3239e-2	4.0290e-2	3.7275e-2	3.4267e-2	3.1332e-2	2.8528e-2	2.5904e-2	2.3498e-2

Additional products which were considered but whose mole fractions were less than 0.10000e-05 for all assigned conditions: C(s), C, C⁺, C⁻, CH, CH⁺, CH₂, CH₂O, CH₃, CH₄, CN, CN⁺, CN⁻, CNN, CN₂, C₂, C₂⁻, C₂H, C₂H₂, C₂H₄, C₂H₆, C₂N, C₂N₂, C₂O, C₃, C₃O₂, C₄, C₅, H⁺, H⁻, HCN, HCO, HCO⁺, HNCO, HNO, HNO₂, HNO₃, H₂O(s), H₂O(l), H₂O₂, K(s), K(l), KO⁻, KOH(s), KOH(l), KOH⁺, K₂, K₂CO₃(s), K₂CO₃(l), K₂O(s), K₂O₂H₂, NCO, NH, NH₂, NH₃, NO⁺, NO₂⁻, NO₃, N₂H₄, N₂O, N₂O⁺, N₂O₄, N₂O₅, N₃, O⁺, O⁻, OH⁺, O₂⁻, O₃, C₃H₈, C₆H₆, C₆H₆(l), C₇H₈, C₈H₁₆, C₈H₁₈, C₈H₁₈(l), C₁₂H₂₆, CH₄O, C₂H₅OH, C₃H₈O

Table 12. Chemical equilibrium predictions of mole fractions for a nitrous oxide-acetylene flame with nebulization of an aqueous solution of 50% cesium hydroxide by weight.

Input Values					Moles	Energy / State Temp.			Density
Chemical Formula						Cal/Mole	K		g/cm ³
Oxidant	N	2.00000	O	1.00000	23.774	19609.803	G 298.15		0.0000
Fuel	C	2.00000	H	2.00000	4.543710	54189.375	G 298.15		0.0000
Fuel	Cs	1.00000	O	1.00000	0.063980	-99597.750	S 298.15		0.0000
Oxidant	H	2.00000	O	1.00000	0.533210	-68317.453	L 298.15		0.0000
O/F= 8.2563 % Fuel= 10.8035 Equivalence Ratio= 0.9567					Phi= 0.9556	Reactant Density= 0.0000			
Thermodynamic Properties									
P, atm	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
T, K	2800	2750	2700	2650	2600	2550	2500	2450	2400
Rho, g/cm ³	1.2459e-4	1.2798e-4	1.3141e-4	1.3489e-4	1.3843e-4	1.4202e-4	1.4566e-4	1.4937e-4	1.5314e-4
H, CAL/g	144.0	86.5	32.0	-19.4	-67.8	-113.1	-155.4	-194.9	-231.6
S, CAL/g·K	2.3890	2.3683	2.3483	2.3291	2.3107	2.2931	2.2763	2.2604	2.2453
M, Mol. Wt.	28.626	28.878	29.114	29.332	29.533	29.716	29.881	30.029	30.158
(DLV/DLP)T	-1.02213	-1.01977	-1.01753	-1.01542	-1.01344	-1.01161	-1.00992	-1.00838	-1.00699
(DLV/DLT)P	1.5081	1.4644	1.4213	1.3791	1.3382	1.2987	1.2611	1.2256	1.1925
CP, CAL/G·K	1.1806	1.1199	1.0588	0.9975	0.9365	0.8764	0.8178	0.7610	0.7067
Gamma, s	1.1256	1.1261	1.1270	1.1284	1.1302	1.1326	1.1356	1.1393	1.1436
Sonic Vel, m/s	956.8	944.2	932.2	920.6	909.5	898.9	888.8	879.1	869.9
Mole Fractions									
CO	9.0310e-2	8.1487e-2	7.2699e-2	6.4088e-2	5.5785e-2	4.7908e-2	4.0556e-2	3.3806e-2	2.7714e-2
CO ₂	1.2942e-1	1.4018e-1	1.5077e-1	1.6106e-1	1.7091e-1	1.8019e-1	1.8881e-1	1.9669e-1	2.0377e-1
CO ₂ ⁻	4.5185e-6	4.3186e-6	4.0598e-6	3.7529e-6	3.4105e-6	3.0462e-6	2.6734e-6	2.3045e-6	1.9504e-6
Cs	6.6257e-4	6.1729e-4	5.6912e-4	5.1913e-4	4.6837e-4	4.1782e-4	3.6837e-4	3.2080e-4	2.7577e-4
Cs ⁺	9.8364e-5	8.0598e-5	6.5345e-5	5.2399e-5	4.1540e-5	3.2542e-5	2.5177e-5	1.9225e-5	1.4479e-5
CsO	1.7994e-5	1.6457e-5	1.4842e-5	1.3202e-5	1.1583e-5	1.0025e-5	8.5623e-6	7.2176e-6	6.0064e-6
CsOH	7.6810e-4	8.4629e-4	9.2407e-4	1.0004e-3	1.0745e-3	1.1455e-3	1.2127e-3	1.2756e-3	1.3336e-3
e ⁻	8.8920e-5	7.2157e-5	5.7875e-5	4.5861e-5	3.5884e-5	2.7708e-5	2.1098e-5	1.5830e-5	1.1693e-5
H	8.1498e-3	6.4079e-3	4.9839e-3	3.8326e-3	2.9124e-3	2.1854e-3	1.6180e-3	1.1807e-3	8.4835e-4
HO ₂	5.2439e-6	4.5788e-6	3.9487e-6	3.3648e-6	2.8347e-6	2.3626e-6	1.9498e-6	1.5948e-6	1.2944e-6
H ₂	1.0041e-2	8.8808e-3	7.7867e-3	6.7671e-3	5.8268e-3	4.9682e-3	4.1916e-3	3.4960e-3	2.8791e-3
H ₂ O	9.7860e-2	1.0221e-1	1.0620e-1	1.0983e-1	1.1311e-1	1.1603e-1	1.1862e-1	1.2089e-1	1.2287e-1
N ⁻	2.6211e-6	1.8081e-6	1.2297e-6	8.2387e-7	5.4330e-7	3.5234e-7	2.2448e-7	1.4036e-7	8.6037e-8
NO	1.5254e-2	1.3850e-2	1.2491e-2	1.1191e-2	9.9618e-3	8.8138e-3	7.7540e-3	6.7868e-3	5.9141e-3
NO ₂	4.3125e-6	3.9680e-6	3.6186e-6	3.2725e-6	2.9369e-6	2.6179e-6	2.3207e-6	2.0487e-6	1.8042e-6
N ₂	5.6723e-1	5.7299e-1	5.7840e-1	5.8344e-1	5.8809e-1	5.9234e-1	5.9619e-1	5.9963e-1	6.0267e-1
O	1.1589e-2	9.1943e-3	7.2132e-3	5.5954e-3	4.2916e-3	3.2545e-3	2.4405e-3	1.8100e-3	1.3280e-3
O ⁻	1.0939e-6	8.2271e-7	6.0778e-7	4.4089e-7	3.1395e-7	2.1939e-7	1.5042e-7	1.0115e-7	6.6705e-8
OH	2.2336e-2	1.9791e-2	1.7375e-2	1.5116e-2	1.3033e-2	1.1138e-2	9.4367e-3	7.9275e-3	6.6049e-3
OH ⁻	3.6101e-6	3.1193e-6	2.6553e-6	2.2268e-6	1.8396e-6	1.4970e-6	1.1997e-6	9.4684e-7	7.3569e-7
O ₂	4.6149e-2	4.3355e-2	4.0423e-2	3.7427e-2	3.4438e-2	3.1523e-2	2.8740e-2	2.6137e-2	2.3754e-2

Additional products which were considered but whose mole fractions were less than 0.10000e-05 for all assigned conditions: C(s), C, C⁺, C⁻, CH, CH⁺, CH₂, CH₂O, CH₃, CH₄, CN, CN⁺, CN⁻, CNN, CN₂, C₂, C₂⁻, C₂H, C₂H₂, C₂H₄, C₂H₆, C₂N, C₂N₂, C₂O, C₃, C₃O₂, C₄, C₅, Cs(s), Cs(l), CsOH(s), CsOH(l), CsOH⁺, Cs₂, Cs₂O, Cs₂O₂H₂, H⁺, H⁻, HCN, HCO, HCO⁺, HNCO, HNO, HNO₂, HNO₃, H₂O(s), H₂O(l), H₂O₂, NCO, NH, NH₂, NH₃, NO⁺, NO₂⁻, NO₃, N₂H₄, N₂O, N₂O⁺, N₂O₄, N₂O₅, N₃, O⁺, OH⁺, O₂⁻, O₃, C₃H₈, C₆H₆, C₆H₆(l), C₇H₈, C₈H₁₆, C₈H₁₈, C₈H₁₈(l), C₁₂H₂₆, CH₄O, C₂H₅OH, C₃H₈O.

Table 13. Chemical equilibrium predictions of mole fractions for a nitrous oxide-acetylene flame with nebulization of an aqueous solution of 61.7% cesium hydroxide by weight.

Input Values:		Moles		Energy State		Temp.		Density	
Chemical Formula				Cal/mole		K		g/cm ³	
Oxidant	N 2.00000 O 1.00000	23.774		19609.803	G	298.15		0.0000	
Fuel	C 2.00000 H 2.00000	4.543710		54189.375	G	298.15		0.0000	
Fuel	Cs 1.00000 O 1.00000 H 1.00000	0.041250		-99597.750	S	298.15		0.0000	
Oxidant	H 2.00000 O 1.00000	0.213200		-68317.453	L	298.15		0.0000	
O/F= 8.4359 % Fuel= 10.5978 Equivalence Ratio= 0.9561 Phi= 0.9556 Reactant Density= 0.0000									
Thermodynamic Properties									
P, atm	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
T, K	2900	2850	2800	2750	2700	2650	2600	2550	2500
Rho, g/cm ³	1.1821e-4	1.2148e-4	1.2480e-4	1.2818e-4	1.3161e-4	1.3509e-4	1.3863e-4	1.4221e-4	1.4586e-4
H, CAL/g	272.3	209.3	149.3	92.2	38.1	-13.0	-61.1	-106.2	-148.4
S, CAL/g·K	2.4275	2.4056	2.3844	2.3638	2.3439	2.3248	2.3065	2.2890	2.2723
M, Mol. Wt	28.129	28.409	28.674	28.924	29.158	29.375	29.575	29.757	29.921
(DLV/DLP)T	-1.02685	-1.02430	-1.02187	-1.01955	-1.01734	-1.01526	-1.01331	-1.01150	-1.00983
(DLV/DLT)P	1.5898	1.5465	1.5033	1.4604	1.4180	1.3764	1.3359	1.2969	1.2596
CP, Cal/g·K	1.2873	1.2301	1.1717	1.1124	1.0523	0.9919	0.9318	0.8723	0.8142
Gamma, s	1.1259	1.1257	1.1259	1.1264	1.1272	1.1286	1.1304	1.1328	1.1358
Sonic Vel, m/s	982.4	969.0	956.1	943.6	931.6	920.1	909.0	898.4	888.3
Mole Fractions									
CO	1.0844e-1	9.9875e-2	9.1066e-2	8.2152e-2	7.3277e-2	6.4585e-2	5.6207e-2	4.8262e-2	4.0848e-2
CO ₂	1.0916e-1	1.1989e-1	1.3075e-1	1.4160e-1	1.5228e-1	1.6265e-1	1.7258e-1	1.8193e-1	1.9062e-1
CO ₂ ⁻	3.8223e-6	3.7821e-6	3.6819e-6	3.5257e-6	3.3199e-6	3.0732e-6	2.7962e-6	2.5001e-6	2.1960e-6
Cs	4.7655e-4	4.5556e-4	4.3081e-4	4.0300e-4	3.7287e-4	3.4118e-4	3.0865e-4	2.7597e-4	2.4379e-4
Cs ⁺	1.1399e-4	9.5561e-5	7.9294e-5	6.5106e-5	5.2879e-5	4.2469e-5	3.3715e-5	2.6443e-5	2.0480e-5
CsO	1.3300e-5	1.2578e-5	1.1722e-5	1.0765e-5	9.7441e-6	8.6946e-6	7.6494e-6	6.6369e-6	5.6802e-6
CsOH	3.8392e-4	4.3387e-4	4.8505e-4	5.3678e-4	5.8838e-4	6.3916e-4	6.8851e-4	7.3586e-4	7.8074e-4
e ⁻	1.0483e-4	8.7196e-5	7.1722e-5	5.8318e-5	4.6858e-5	3.7188e-5	2.9136e-5	2.2522e-5	1.7166e-5
H	1.2380e-2	9.9404e-3	7.9005e-3	6.2134e-3	4.8337e-3	3.7178e-3	2.8256e-3	2.1205e-3	1.5701e-3
HO ₂	9.0639e-7	7.8777e-7	5.1025e-6	4.4572e-6	3.8453e-6	3.2778e-6	2.7623e-6	2.3030e-6	1.9011e-6
H ₂	1.1741e-2	1.0572e-2	9.4362e-3	8.3499e-3	7.3245e-3	6.3678e-3	5.4847e-3	4.6778e-3	3.9474e-3
H ₂ O	8.2854e-2	8.7651e-2	9.2136e-2	9.6289e-2	1.0010e-1	1.0357e-1	1.0670e-1	1.0950e-1	1.1197e-1
N ⁻	5.3197e-6	3.7669e-6	2.6335e-6	1.8166e-6	1.2354e-6	8.2769e-7	5.4581e-7	3.5396e-7	2.2551e-7
NO	1.8243e-2	1.6794e-2	1.5355e-2	1.3943e-2	1.2575e-2	1.1267e-2	1.0030e-2	8.8744e-3	7.8079e-3
NO ₂	4.9933e-6	4.6824e-6	4.3492e-6	4.0023e-6	3.6503e-6	3.3016e-6	2.9633e-6	2.6419e-6	2.3424e-6
N ₂	5.6017e-1	5.6655e-1	5.7263e-1	5.7840e-1	5.8382e-1	5.8887e-1	5.9353e-1	5.9780e-1	6.0166e-1
O	1.7850e-2	1.4475e-2	1.1611e-2	9.2122e-3	7.2278e-3	5.6072e-3	4.3010e-3	3.2619e-3	2.4463e-3
OH	2.6839e-2	2.4244e-2	2.1693e-2	1.9228e-2	1.6886e-2	1.4694e-2	1.2672e-2	1.0832e-2	9.1793e-3
OH ⁻	3.6012e-6	3.2158e-6	2.8280e-6	2.4493e-6	2.0893e-6	1.7552e-6	1.4523e-6	1.1834e-6	9.4952e-7
O ₂	5.1202e-2	4.8905e-2	4.6322e-2	4.3525e-2	4.0587e-2	3.7585e-2	3.4589e-2	3.1667e-2	2.8876e-2

Additional products which were considered but whose mole fractions were less than 0.10000e-05 for all assigned conditions: C(s), C, C⁺, C⁻, CH, CH⁺, CH₂, CH₂O, CH₃, CH₄, CN, CN⁺, CN⁻, CNN, CN₂, C₂, C₂⁻, C₂H, C₂H₂, C₂H₄, C₂H₆, C₂N, C₂N₂, C₂O, C₃, C₃O₂, C₄, C₅, Cs(s), Cs(l), CsOH(s), CsOH(s), CsOH(l), CsOH⁺, Cs₂, Cs₂O, Cs₂O₂H₂, H⁺, H⁻, HCN, HCO, HCO⁺, HNCO, HNO, HNO₂, HNO₃, H₂O(s), H₂O(l), H₂O₂, NCO, NH, NH₂, NH₃, NO⁺, NO₂⁻, NO₃, N₂H₄, N₂O, N₂O⁺, N₂O₄, N₂O₅, N₃, O⁺, O⁻, OH⁺, O₂⁻, O₃, C₃H₈, C₆H₆, C₆H₆(l), C₇H₈, C₈H₁₆, C₈H₁₈, C₈H₁₈(l), C₁₂H₂₆, CH₄O, C₂H₅OH, C₃H₈O.

REFERENCES

1. Rosa, R.J., Magnetohydrodynamic Energy Conversion, Hemisphere Publishing Corporation, New York, NY, 1987, pp. 17-35.
2. Uman, M.A., Introduction to Plasma Physics, McGraw-Hill Book Company, New York, NY, 1964, pp. 86-100.
3. Heald, M.A. and Warton, C.B., Plasma Diagnostics with Microwaves, Wiley, New York, NY, 1965.
4. Tanenbaum, B.S., Plasma Physics, McGraw-Hill, New York, NY, 1967.
5. Hecht, E. and Zajac, A., Optics, Addison Wesley, New York, NY, 1974.
6. Kuzmenko, P.J. and Self, S.A., "Measurements of Plasma Conductivity Using Faraday Rotation of Submillimeter Waves," Journal of Energy, 7, 176-181 (1983).
7. Born, M. and Wolf, E., Principles of Optics, 5th Ed., Pergamon Press, New York, 1975.
8. Applied Magnetics Labs, Inc., 1404 Bare Hills Road, Baltimore, MD 21209.
9. Laser Photonics, 12351 Research Parkway, Orlando, FL 32826. Purchased under Department of Energy Contract DE-AC02-80ET-15601.
10. Gallagher, J.J., Blue, M.D., Bean, B. and Perkowitz, S., "Tabulation of Optically Pumped Far Infrared Laser Lines and Applications to Atmospheric Transmission," Infrared Physics, 17, 43-55 (1977).
11. Molelectron Detector, Inc., 1520B Dell Avenue, Campbell, CA 95008.
12. Shurcliff, W. A., Polarized Light, Harvard University Press, Cambridge, MA, 1962.
13. Optical Engineering, Post Office Box 696, Santa Rosa, CA 95402.
14. Coherent, Components Group, 2301 Lindbergh Street, Auburn, CA 95603.
15. Laser Photonics, Inc., 5145 Avenida Encinas, Carlsbad, CA 92008.
16. Stanford Research Systems, Inc., 1290 D Reamwood Avenue, Sunnyvale, CA 94089.
17. Rofin Sinar, Inc., 3333 North First Street, San Jose, CA 95134-1995.
18. Data Translation, Inc., 100 Locke Drive, Marlboro, MA 01752-1192.
19. Bauman, L.E. and Wolverton, M.K., "Time-Resolved Line Reversal Gas Temperature Measurements using a Multi-Channel Detector," Proc. 9th International Conf. on MHD Electrical Power Generation, Tsukuba, Japan, Nov. 1986.

-
20. Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T., Numerical Recipes, Cambridge University Press, New York, NY, 1986.
 21. Newport Corporation, 18235 Mt. Baldy Circle, Fountain Valley, CA 92728-8020.
 22. Computer Devices, Santa Fe Springs, CA.
 23. Perkin-Elmer Corp., Analytical Instruments, Main Ave. (MS-12), Norwalk, CT 06856.
 24. Potassium formate, 99% pure, hygroscopic salt. Aldrich Chemical Company, Inc., 1001 West Saint Paul Avenue, Milwaukee, WI 53233.
 25. Potassium fluoride, >99% pure. American Scientific Products, 155 Brookhollow Esplanade, P.O. Box 23628, Harahan, LA 70123.
 26. Cesium hydroxide monohydrate, 99.9% pure, 50 weight % water solution. Aldrich Chemical Company, Inc., 1001 West Saint Paul Avenue, Milwaukee, WI 53233.
 27. Cesium hydroxide monohydrate, 99% pure based on metals analysis. Aldrich Chemical Company, Inc., 1001 West Saint Paul Avenue, Milwaukee, WI 53233.
 28. Aldrich Chemical Co., Inc., 1001 West Saint Paul Avenue, Milwaukee, WI 53233.
 29. Fery, C., "Thermométrie. — Sur la température des flammes," *Compt. Rend.* **137**, 909-912 (1903).
 30. Griffiths, E., and Awbery, J.H., "The Measurement of Flame Temperatures," *Proceedings of the Royal Society A*, **123**, 401-421 (1929).
 31. Drake, M.C., Grabner, L.H., and Hastie, J.W., "A Comparison of Spectroscopic Flame Temperature Measurements: Na-D Line Reversal, Rotational and Vibrational Raman, and OH Absorption," Proc. 10th Materials Research Symp. on Characterization of High Temperature Vapors and Gases, N.B.S. Publication 561, Gaithersburg, MD, 1978.
 32. Thomas, D.L., "An Automatic Remotely Operated Sodium D-Line Reversal Temperature Measuring Technique," *Combustion and Flame*, **12**, 569-574 (1968).
 33. Mitchell, A.C.G., and Zemansky, M.W., Resonance Radiation and Excited Atoms, Cambridge Press, New York, NY, 1934 and reprinted 1971.
 34. Humlicek, J., "Optimized Computation of the Voigt and Complex Probability Functions," *Journal of Quantitative Spectroscopy and Radiation Transfer*, **4**, 437-444 (1982).

-
35. Hofmann, F.W. and Kohn, H., "Optical Cross Section of Resonance Lines Emitted by Flames under Conditions of Partial Thermal Ionization," *Journal of the Optical Society of America*, **51**, 512-521 (1961).
 36. Ch'en, S.Y. and Garrett, R.O., "Pressure Effects of Foreign Gases on the Absorption Lines of Cesium. I. The Effects of Argon on the First Two Members of the Principal Series," *Physical Review*, **144**, 59-65 (1966).
 37. Wiese, W.L. and Martin, G.A., Wavelengths and Transition Probabilities for Atoms and Atomic Ions. Part II. Transition Probabilities, National Bureau of Standards, Washington, D.C., 1980.
 38. Radzig, A.A. and Smirnov, B.M., Reference Data on Atoms, Molecules, and Ions, Springer-Verlag, New York, NY, 1985.
 39. Frost, L.S., "Conductivity of Seeded Atmospheric Pressure Plasmas," *Journal of Applied Physics*, **32**, 2029-2036 (1961).
 40. Schneider, J. and Hofmann, F.W., "Absorption and Dispersion of Microwaves in Flames," *Physical Review*, **116**, 244-249 (1959).
 41. Gill, P.E., Murry, W., and Wright, M.H., Practical Optimization, Academic Press, New York, NY, 1981.
 42. Press, W.H., Flannery, B.P., Teukolsky, S.A., and Vetterling, W.T., Numerical Recipes, Cambridge University Press, 1989.
 43. Hofmann, F.W., Kohn, G., and Schneider, J., "Microwave Determination of Electron Concentrations in Flame Gases Used as a Spectroscopic Light Source," *Journal of the Optical Society of America*, **51**, 508-511 (1961).
 44. Kuzmenko, P.J., "Far Infrared Diagnostics of Electron Concentrations in Combustion MHD Plasmas using Interferometry and Faraday Rotation," Topical Report 249, High Temperature Gasdynamics Laboratory, Stanford University, Stanford, CA, June 1985.
 45. Spencer, Jr., F.E. and Phelps, A.V., "Momentum Transfer Cross-Sections and Conductivity Integrals for Gases of MHD Interest," Proc. 15th Symposium on Engineering Aspects of MHD, Philadelphia, PA, 1976.
 46. Gordon, S. and McBride, B.J., "Computer Program for Calculation of Complex Chemical Equilibrium Composition, Rocket Performance, Incident and Reflected Shocks, and Chapman-Jouget Detonations," NASA/Lewis Research Center, NASA SP-273, 1976.

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