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SUBJECT: APPR-1 TYPE ABSORBER ROD IRRADIATION TEST--
IRRADIATION REQUEST ORNL MTR-29

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FROM: E. E. Gross and L. D. Schaffer

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APPR-1 TYPE ABSORBER ROD IRRADIATION TEST-- IRRADIATION REQUEST ORNL MTR-29

I. Introduction

In order to evaluate the behavior of an APPR type absorber rod, an irradiation test program has been established. The program presently is being operated under two MTR irradiation requests, ORNL MTR-28, Phase I[1] and II[2], and ORNL MTR-29[3].

The ORNL MTR-28 Irradiation Test Program consists of in-pile testing of miniature control rod samples in order to determine radiation damage as a function of boron-10 burn-up. In Phase I five (5) samples have been prepared with boron loading comparable to that specified in the APPR-1 control rods. In Phase II two (2) samples have been prepared with approximately 60% and 2 samples with approximately 120% of the APPR-1 design boron loading. Approximately 21 more samples are planned for testing under this request.

The ORNL MTR-29 Irradiation Request proposes testing a full size APPR-1 type control rod in the MTR. The objective of the test is to better evaluate the neutron absorbing material proposed for the APPR-1 control rod. Of specific interest in this test is determining whether

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1. ORNL CF 56-6-173, Neill, F. H. and C. F. Leitten, Jr., Phase I Absorber Rod Sample Irradiation Request ORNL MTR-28, June 27, 1956.
2. ORNL CF 56-9-118, Leitten, C. F., Jr., Phase II Absorber Rod Sample Irradiation Request ORNL MTR-28, September 27, 1956.
3. ORNL CF 56-9-79, Leitten, C. F., Jr., Fabrication of APPR Type Control Rod in the MTR, September 21, 1956.

or not there are any limiting differences in operational behavior or dimensional stability to that of the irradiated miniature samples. This report will cover this testing program.

The change in rod worth of the APPR-1 boron absorber rod and the MTR cadmium absorber rod as a function of time has been investigated by Gross and Neill[4]. The conclusion of this study was that the APPR-1 type rod could be expected to be at least equal if not better than the cadmium MTR type rod in attenuation of thermal neutrons over the performance life of the rod.

II. Design and Fabrication

The control rod which has been prepared, has been designed in accordance with APPR-1 control rod specifications. In order to adapt the control rod for use in the MTR in a fuel shim rod position, the length of the active absorber section was increased to 30-3/4 in. The only significant dimensional difference between the cadmium and boron section is in thickness. The boron section has a total thickness of 0.156 in. compared to the 0.080 in. thickness of the cadmium section. The boron section is composed of 0.033 in. stainless steel clad plates and a 0.090 in. core composed of enriched boron-10 dispersed in electrolytic iron.

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4. ORNL CF 56-5-6, Gross, E. E. and F. H. Neill, Test of APPR Type Control Rod in the MTR, May 1, 1956.

The boron control section after fabrication was placed in an MTR shim rod aluminum extrusion housing. The upper section and an MTR shim rod fuel section was then attached by welding, after which the lower section was attached to the fuel section.

The detailed fabricational procedure and design of the rod has been previously reported by Leitten[5].

III. Heat Generation in Absorber Rod

When in-pile, there will be two sources of heat generation in the boron absorber section. One source arises from the exothermic reaction of a boron-10 atom absorbing a neutron. This reaction is expressed by the following equation:

$$B^{10} + n \longrightarrow He^{4} + Li^{7} + 2.792 \text{ Mev} \qquad (1)$$

The other source arises from gamma heating of the material which is a function of the material location in the reactor.

In order to determine the maximum heat generation contribution from the neutron-boron-10 reaction, the maximum ($n\text{-}B^{10}$) reaction rate must be determined. To make such a determination, the following assumptions are made:

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5. ORNL CF 56-9-79, Leitten, C. F., Jr., Fabrication of APPR Type Control Rod in the MTR, September 21, 1956.

1. The boron shim rod will be placed in fuel-shim position S.R.#3[6], between fuel positions C-23 and C-25.

2. The boron shim rod will at start of reactor life be required to control 8% excess multiplication of the reactor when the power level is at 40 Mw.

3. All neutrons resulting from the 8% excess multiplication will be absorbed by the boron shim rod.

4. The number of neutron-boron-10 reactions are equally divided among each of the 4 plates in the rod and that the number of reactions which occur per plate are divided equally on each side of the plate.

5. The reaction rate along the length of the plate surface is proportional to the flux in an adjacent fuel position.

6. The shim fuel rods are ganged and are at a level of 1 in. below the core centerline when the reactor initially reaches full power.

7. The flux distribution does not change with power level.

The maximum flux that the shim rod will be in, will be the flux at the tip of the rod when the reactor initially reaches full power. Therefore, the maximum rate of reaction will occur at the shim tip

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6. IDO-16047, Bright, G. O. and Schroeder, F., Neutron Flux Distributions in the Material Testing Reactor, Part I, p. 9, February 16, 1953.

when the reactor initially reaches a power level of 40 Mw. This reaction rate on the entire rod is:

$$40 \text{ Mw} \times 10^6 \text{ w/Mw} \times 0.08 \times 3.1 \times 10^{10} \frac{\text{fission}}{\text{sec w}} \times 2.5 \text{ n/fission} \times 1 \frac{(n\text{-}B^{10})}{n} = 2.48 \times 10^{17} \frac{(n\text{-}B^{10})}{\text{sec}}$$

The reaction rate per shim surface is:

$$\frac{2.48 \times 10^{17} \frac{(n\text{-}B^{10})}{\text{sec}}}{8 \text{ surface}} = 3.1 \times 10^{16} \frac{(n\text{-}B^{10})}{\text{sec surface}}$$

The reaction rate of the rod surface is considered proportional to the flux in an adjacent fuel position. Shown in Figure 1 is a plot of the relative activity obtained by a flux measurement of position C-25[7], an assumed adjacent fuel position of the shim rod.

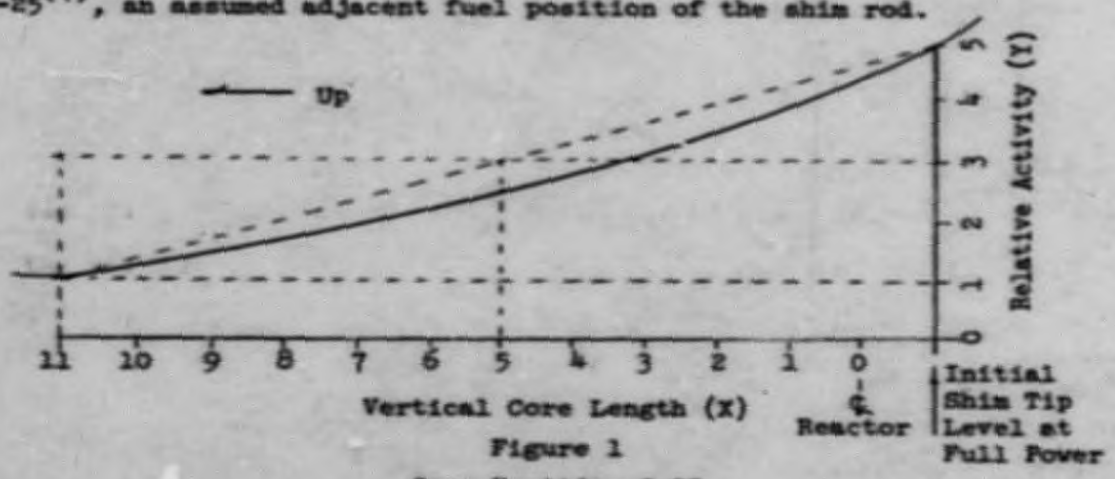

Figure 1

Core Position C-25

Relative Activity Vs. Vertical Core Length

7. IDO-16047, Bright, G. O. and F. Schroeder, Neutron Flux Distributions in the Material Testing Reactor, Part I, p. 102, February 16, 1953.

Here it is assumed that the neutron-boron-10 reactions take place only on that portion of the rod which is in the active core. Considering the relative activity as a straight line function of the rod length, the reaction rate at the tip is expressed by:

$$\frac{\left(\frac{n\text{-}B^{10}}{\text{sec in.}}\right)_{Av}}{\left(\frac{n\text{-}B^{10}}{\text{sec in.}}\right)_{Max}} = \frac{\bar{Y}}{Y_{Max}} \qquad (2)$$

where Y is equal to the relative activity. The maximum reaction rate is then:

$$\frac{3.1 \times 10^{16}\ (n\text{-}B^{10})}{12\ \text{in. sec}} \quad \frac{5}{3} = 4.3 \times 10^{15} \quad \frac{(n\text{-}B^{10})}{\text{sec in.}}$$

Since this absorber section is black and therefore, neutrons are absorbed only on the core surface, the reaction rate can be expressed in units of core area. With the absorber core width equal to 1.940 in.[8], the reaction rate per unit of surface area is:

$$4.3 \times 10^{15} \quad \frac{(n\text{-}B^{10})}{\text{sec in.}\cdot\text{(plate)}} \times \frac{1}{1.940\ \text{in.}} \times \frac{0.06\ \text{in.}^2}{\text{cm}^2} =$$

$$1.33 \times 10^{14} \quad \frac{(n\text{-}B^{10})}{\text{sec cm}^2}$$

The heat flux contribution due to the neutron-boron-10 reaction can now be written as:

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8. ORNL CF 56-9-79, Leitten, C. F., Jr., Fabrication of APPR Type Control Rod in the MTR, p. 6, September 21, 1956.

$$\frac{Q}{A} = RE_R \qquad (3)$$

where E_R = energy release given in Eq. (1).

R = rate of the reaction written in Eq. (1).

Therefore, the maximum heat flux due to neutron absorption of the absorber surface is:

$$\frac{Q}{A} = 1.33 \times 10^{14} \frac{(n\text{-}B^{10})}{sec\ cm^2} \times 2.792\ Mev \times 1.602 \times 10^{-13} \frac{Watt\text{-}sec}{Mev}$$

$$\times \frac{929\ cm^2}{ft^2} \times 3.413 \frac{Btu}{Watt\text{-}hr} = 188{,}000\ Btu/hr\text{-}ft^2$$

The heat generation contribution from nuclear heating in the absorber plate and its aluminum housing was determined by using a nuclear heating value of 14 w/gm[9]. An example of this calculation is given for one of the stainless steel cladding plates. The heat generation calculated is:

$$\frac{Q}{A} = 0.033\ in. \times 144\ in.^2 \times 16.38 \frac{cm^3}{in.^3} \times 7.93\ g/cm^3 \times 14.0\ w/gm$$

$$\times 0.0569 \frac{Btu}{watt\text{-}min} \times 60 \frac{min}{hr} = 29{,}500\ Btu/hr\text{-}ft^2$$

In Table I, a summary is given for the heat flux generated in each material by nuclear heating as well as properties of each material used in calculating the nuclear heating.

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9. AED-R-1049, MTR Heat Generation Pattern at Reactor Midplane.

TABLE I

NUCLEAR HEATING OF ABSORBER ROD AND HOUSING

Material	Function	Thickness (in.)	Density gm/cm^3	Heat Flux $\frac{Btu}{hr\text{-}ft^2}$
304-L SS	Clad	0.033 x 2	7.93	59,000
3.23 w/o enriched B-10 in iron	Core	0.090	7.87	79,500
2S aluminum	Housing	0.45	2.70	136,500

It should be noted that the thickness used for the aluminum housing is the maximum dimension on the convex side.

Now, before the heat flux at each surface of the plate can be determined, the direction of heat flow must be established. Since the absorber section was fabricated with a maximum 0.036 in.[10] clearance between the cladding surface and the aluminum housing, it is assumed that the air gap between these surfaces constitutes an insulated boundary. This assumption is supported in Appendix A, where air conduction was assumed across the gap having a spacing of 0.018 in. Under these conditions, the maximum temperature at the inner wall of the absorber section is reduced only by 1° F compared to the inner wall temperature when the gap was considered an absolute barrier. Therefore, all heat generated in the absorber plate flows to the inner channel and all heat generated in the aluminum housing flows out to

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10. Leitten, C. F., Jr., Private communication on Control Rod Fabrication, October 1956.

the interstice between the aluminum shim rod housing and the adjacent fuel element. With the above concept, the heat fluxes are summarized in Table II.

TABLE II

HEAT FLUX FROM ABSORBER ROD

Internal Clad Surface

Material	Source	Heat Flux Btu/hr-ft^2
2-SS clad plates	Nuclear Heating	59,000
Fe-B^{10} core	Nuclear Heating	79,000
Internal and External Reaction Faces, Fe-B^{10} core	(n-B^{10}) Reaction	376,000
Total		514,000

External Aluminum Housing Surface

Aluminum Housing	Nuclear Heating	136,500

IV. Absorber Rod Temperatures

The temperature distribution at the tip of the rod, the point of maximum heat generation, is determined by assuming the bulk water temperature at this point to be 115°F.

The heat transfer coefficient calculated for this set of conditions was found to be 3420 Btu/hr-ft^2-°F. The method of calculation for this value is given in Appendix B.

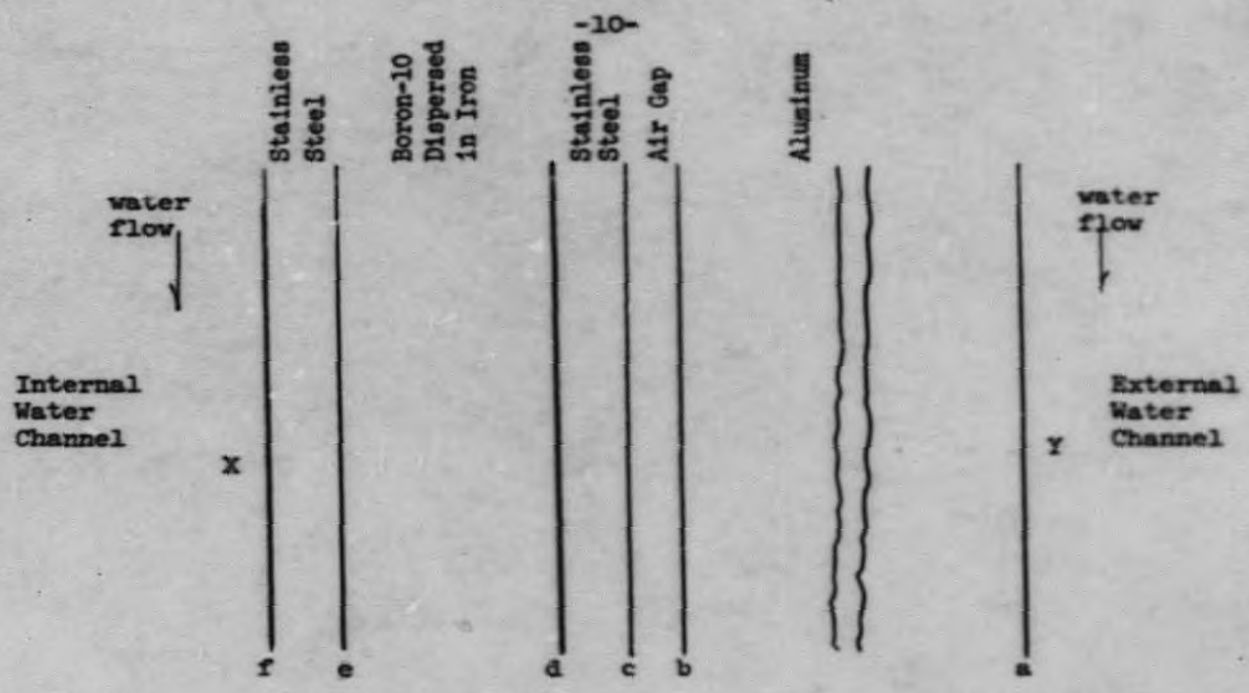

Figure 2
Cross Section of Absorber Plate and Aluminum Housing

In order to simplify the temperature distribution calculations, it is assumed that the heat flux developed by a uniform heat generating volume, would be considered a plane source located at the boundary of the heat generating volume which is fartherest removed from the heat sink. The plane of heat divergences is considered to be the air gap which is assumed as an insulated surface, represented by cb in Figure 2. Heat is assumed to flow from c to f and from b to a. The heat flux distribution over the absorber rod cross section is summarized in Table III.

TABLE III

HEAT FLUX DISTRIBUTION IN ABSORBER ROD

Sink	Plane	Total Heat Flux Btu/hr-ft^2
Inner channel	c	29,500
	d	296,500
	e	514,000
	f	514,000
Outer interstice	b	136,500
	a	136,500

The temperature drop between two boundaries can be expressed by the following:

$$\Delta T_{c-d} = \frac{Q_c}{A} \quad \frac{1}{K} \; t_{c-d} \qquad (4)$$

where K = thermal conductivity of the material.

t = thickness of the material.

Hence, the temperature drop across c-d is:

$$\Delta T_{c-d} = 29{,}500 \text{ Btu/hr-ft}^2 \times \frac{1}{0.029 \frac{\text{Btu in.}}{\text{sec ft}^2\text{F}}} \times \frac{1 \text{ hr}}{3600 \text{ sec}} \times 0.033 \text{ in.} = 9.3^\circ\text{F}$$

The temperature drop from the rod surface to the coolant stream can be written:

$$\Delta T = \frac{Q}{A} \quad \frac{1}{h} \qquad (5)$$

where h = heat transfer coefficient.

As mentioned previously the heat transfer coefficient was calculated to be 3420 Btu/hr-ft^2-°F. Therefore, the ΔT between the metal surface and the water can be calculated. The drop from the inner surface to the water is:

$$\Delta T_{f\text{-}H_2O} = \frac{514{,}000 \text{ Btu/hr-ft}^2}{3420 \text{ Btu/hr-ft}^2\text{-°F}} = 150°F$$

In Table IV, the boundary temperatures, the temperature drops across each material, and the data used in the calculation of ΔT values are given.

TABLE IV

TEMPERATURE DISTRIBUTION IN ABSORBER ROD

	t in.	K Btu in./sec-ft^2-°F	ΔT °F	T °F
Water (x)	-	-	-	115
f-x	-	-	150	-
f	-	-	-	265
e-f	0.033	0.029	162	-
e	-	-	-	427
d-e	0.090	0.090	82	-
d	-	-	-	509
c-d	0.033	0.029	9.3	-
c	-	-	-	518.3
bc	0.030	0	0	-
Water (y)	-	-	-	115
a-y	-	-	40	-
a	-	-	-	155
b-a	0.45	0.43	9.9	-
b	-	-	-	164.9

Of primary concern in this calculation is the establishment of the maximum wall temperature in order to determine whether or not nucleate boiling is probable. In order to avoid any underestimation of the maximum surface temperature which has been shown to be the absorber section tip inner surface, the conditions chosen and assumptions which have been made are conservative.

With the maximum surface temperature established, it is next necessary to establish the boiling temperature of the water at this point. From the MTR Handbook[11], the pressure drop through the shim-fuel rod is normally 40 psi which maintains the flow through the fuel section at a velocity of 30 fps. The flow through the shim rod bank is 3380 gpm, assuming 8 shim rods in the bank. The water enters the control rod in 4 water slots 7.75 in. x 0.69 in. The water flows down the control section, through the fuel section and out 4 similarly shaped water slots in the fuel shim lower section. In order to estimate the water pressure to determine the water saturation temperature, it will be assumed that the water flowing out of the bottom water slots is at a gage pressure of 75 psi[12].

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11. TID-7001, Materials Testing Reactor Project Handbook, May 7, 1956.

12. TID-5275, Selected Reference Material United States Atomic Energy Program, Research Reactors, p. 236.

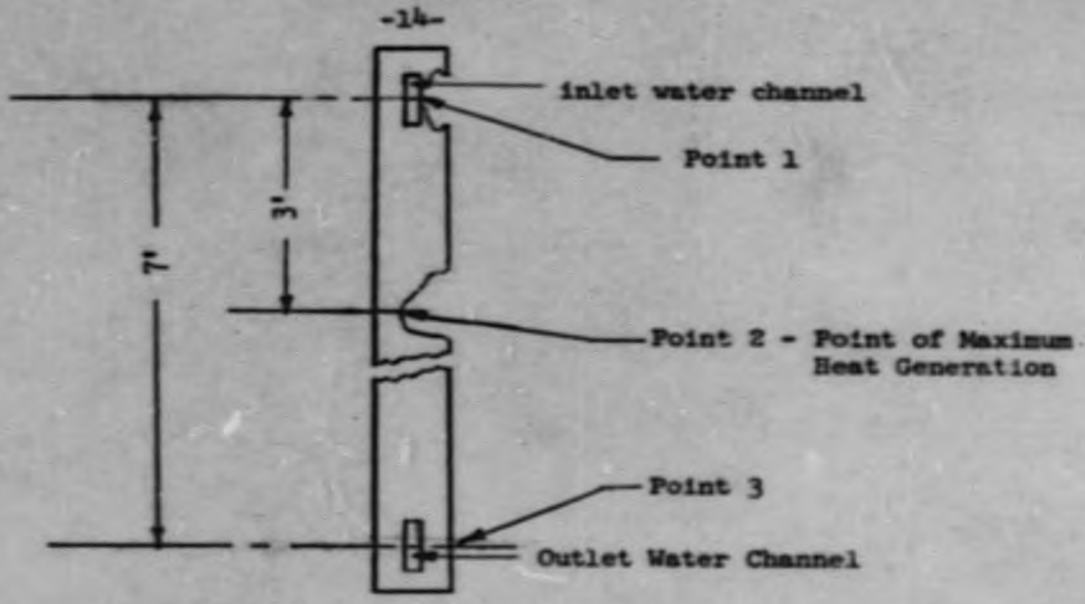

Figure 3
MTR Shim Fuel Assembly

Before determining the pressure at point 2, the inner channel pressure 3 ft below the water inlet, the pressure at point 1 must be found. By finding the pressure drop, at the water inlet, the pressure at point 1 can be determined. Therefore, the frictional energy drop due to contraction at the water inlet slots can be written:

$$Fc = \frac{Ke\, V_2^2}{2g} \qquad (6)$$

where V_2 = average linear velocity downstream.

K = loss coefficient.

Assuming $\frac{A_2}{A_1}$ = 0.4 which is the ratio of flow areas, K_c = 0.315, and F_c may be calculated.

$$F_c = \frac{0.315}{2 \times 32.2 \text{ ft/sec}^2} \left[\frac{\frac{3380 \text{ gal/min}}{8 \text{ flow paths} \times 7.48 \text{ gal/ft}^3 \times \frac{\text{min}}{60 \text{ sec}}}}{\frac{7.75 \text{ in.} \times 0.69 \text{ in.} \times 4}{144 \text{ in.}^2/\text{ft}^2}} \right]^2 = 0.189 \text{ ft}$$

The pressure drop across the inlet water slots is then:

$$\Delta P_{inlet} = \frac{0.189 \text{ ft} \times 61.7 \text{ lb/ft}^3}{144 \text{ in.}^2/\text{ft}^2} = 0.081 \text{ lb/in.}^2$$

Now, the gage pressure at point 1 can be expressed:

$$P_1 = P_3 + \Delta P_{shim\ rod} - (H_3 - H_1)\rho_w - P_c \qquad (7)$$

where H = water head.

ρ_w = density of water.

P_c = pressure loss due to contraction at inlet.

Thus, pressure at point 1 is:

$$P_1 = 7.5 \text{ lb/in.}^2 + 40 \text{ lb/in.}^2 - \left(\frac{7 \text{ ft} \times 61.7 \text{ lb/ft}^3}{144 \text{ in.}^2 \times \text{ft}^2} \right) - 0.08 \text{ lb/in.}^2$$

$$= 44.4 \text{ lb/in.}^2$$

Then, the pressure at point 2 can be expressed by:

$$\frac{1}{}(P_1-P_2) - (H_2-H_1) - \frac{f\,V^2(L_2-L)}{2g\,De} = 0 \qquad (8)$$

where L = length of channel.

De = $4A/P = 4\ell^2/4\ell = \ell$ = absorber box inner width-equivalent diameter.

f = pipe roughness coefficient.

To obtain a value for f, the Reynolds' number has to be determined.

$$Re = \frac{D_e V}{\mu} \qquad (9)$$

where μ = water viscosity.

$$Re = \frac{\frac{2.038 \text{ in.}}{12 \text{ in./ft}} \times 30 \text{ ft/sec} \times 61.7 \text{ lb/ft}^3 \times 3600 \text{ sec/hr}}{1.57 \text{ lb/hr-ft}} = 7.21 \times 10^5$$

Now, the pressure at point 2 can be found; using f = 0.019.

$$P_2 = \left\{\left[3 \text{ ft} - \left(\frac{(0.019)(900 \text{ ft}^2/\text{sec}^2)(36 \text{ in.})}{(2)(32.2 \text{ ft/sec}^2)(2.038 \text{ in.})}\right)\right] \frac{61.72 \text{ lb/ft}^3}{144 \text{ in./ft}^2}\right\} + 44.4 \text{ lb/in.}^2$$

P_2 = 43.7 lb/in.^{2} (gage)

Assuming an absolute pressure of 14.7 lb/in.^{2}, the absolute pressure at point 2, the point where the absorber rod metal surface temperature is 265° F, is 58.4 lb/in.^{2} which corresponds to a water saturetion temperature of approximately 291° F. The absorber rod peak surface temperature, therefore, is 26° F below the point of boiling.

V. Burn-up

The maximum percent burn-up of the boron-10 atoms desired at the rod tip is 80%. However, since this experiment is to be carried on in one of the shim-fuel rod positions, assurance must be gained on the operational behavior and dimensional stability of the control material being used in this absorber rod. Therefore, the burn-up which will initially be requested in this report will be 47% burn-up of the boron-10 atoms in the rod tip, since the maximum burn-up attained on a miniature sample No. 8 of the ORNL MTR-28 test was successfully irradiated to this burn-up. As other miniature absorber samples with higher burn-up are removed and found dimensionally stable, it is desired that the requested burn-up on this absorber rod be raised to the burn-up value of the successfully irradiated absorber sample, until the absorber rod has received as much burn-up as possible in regard to the absorber unit's operational limitation or until the maximum specified tip burn-up has been obtained.

According to Bartz[13], the MTR cadmium shim rod No. 163 which was used in position C-26 to an exposure of 1406 MWD, showed an approximate transmission of 100% thermal neutrons at the rod tip. Therefore, since the neutron absorption rate of the cadmium and boron rods should

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13. TID-7515, Pt.I, Papers Prepared for Radiation Effects Review Meeting, Congress Hotel, Chicago, July 31-August 1, 1956, p. 37, August 1956.

be the same for constant power operation, the number of Cd-113 atoms burned out in the tip of shim rod No. 163 should represent the same number of B-10 reactions that occur in the APPR-1 rod irradiated under comparable circumstances. This assumes that the cadmium and boron rods are both black to thermal neutrons, that the only reacting nuclides are Cd-113 and B-10, that the boron rod will be substituted in a comparable lattice position, that the boron rod would be programmed in a similar manner to that of the cadmium rod and that when the cadmium rod ceases to attenuate thermal neutrons, it is completely burned out.

The only dimensional variation between the two type absorber rods is in the thickness. The cadmium rod No. 163 is 0.020 in. thick[14], and the APPR-1 boron rod is 0.090 in. thick. The Cd-113 density in the MTR rod is 5.67×10^{21} atom/cm^3 and the B-10 density in the APPR-1 type rod is 1.28×10^{22} atom/cm^3.

The atoms per unit of area in each rod are:

MTR Shim Rod (Cd-113)

$$5.67 \times 10^{21} \frac{\text{atom}}{\text{cm}^3} \times 0.020 \text{ in.} \times 2.54 \frac{\text{cm}}{\text{in}} = 2.88 \times 10^{20} \frac{\text{atom}}{\text{cm}^2}$$

APPR-1 Type Control Rod (B-10)

$$1.28 \times 10^{22} \frac{\text{atom}}{\text{cm}^3} \times 0.090 \text{ in.} \times 2.54 \frac{\text{cm}}{\text{in}} = 2.92 \times 10^{21} \frac{\text{atom}}{\text{cm}^2}$$

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14. N. B. Van Sice, private communication, December 12, 1956.

In assuming that MTR rod No. 163 after approximately 1400 MWD has all its Cd-113 atoms burnt out at the rod tip, the approximate number of reactions or percent burn-up which will occur at the tip of the boron rod after two MTR cycles at 40 Mw will be:

$$\% \text{ B-10 Burn-up} = \frac{2.88 \times 10^{20} \frac{\text{atoms}}{\text{cm}^2}}{2.92 \times 10^{21} \frac{\text{atoms}}{\text{cm}^2}} \times 100 = 9.9\%$$

Therefore, the burn-up that can be estimated from the above data is 4.9% burn-up of the B-10 atoms per MTR cycle. This rate can be assumed to be linear to a relatively high degree of burn-up.

The above figure for the burn-up of the control rod tip was based on the assumption that a test control rod would have to absorb close to the same number of neutrons as the MTR control rod that it replaces, otherwise the reactor would not operate at constant power. An independent estimate of the rod burn-up may be obtained from a knowledge of the reactor flux in the neighborhood of the control rod and the nuclear parameters of the MTR core and of the control rod.

Assuming one velocity diffusion theory, the volume rate of absorptions in the control rod is given by (see Appendix C for derivation):

$$A = \frac{G(\alpha)}{4s} \left[\phi_o(0) + \phi_o(-s)\right] \qquad (10)$$

where $\phi_o(0)$ is the flux at the outside surface of the rod, $\phi_o(-s)$ is the flux at the inside surface of the rod, "s" is the thickness of absorber in the rod, and $G(\alpha) = G(\Sigma_a s)$ is plotted in Figure 1C.

From experimental data on the effect of the water hole on the reactivity value of a rod[15] it is concluded that the addition of water to the APPR control rod water hole increases the reactivity worth of the rod by 45% (see Fig. 8 of reference 14). This suggests that when the central hole is filled with water the inside surface contributes an absorption rate of about 45% the absorption rate of the outside surface. To account for the effect of the water hole, $\phi_o(-s)$ may then be replaced by $0.45\ \phi_o(0)$ in Eq. 10.

From MTR data[16] it appears that the rod will be in an average unperturbed flux of about 3.12×10^{14} neutrons/cm^{2}sec. However, measurements[17] along the side of an APPR control rod and along the water hole within the rod indicate a flux peaking in the region between the end of the fuel rod and the beginning of the control rod. The data indicate that the flux at the outside surface of the rod tip is practically unchanged but that the flux at the inside surface of the rod tip is increased by a factor of about 1.8 over what it would have been in the absence of flux peaking. To account for the effect of flux peaking at the rod tip as well as the water hole effect discussed above, $\phi_o(-s)$ may therefore be replaced by $0.45 \times 1.8\ \phi_o(0)$.

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15. R. L. Murray and J. W. Niestlie, Nucleonics, 13, #2, p. 18, February, 1955.

16. IDO-16047, Bright, G. O. and Schroeder, F., Neutron Flux Distributions in the Material Testing Reactor, Part I, February 16, 1953.

17. Private communication from C. H. Harvey, November 9, 1956.

In applying Eq. 10 to the tip of the APPR rod we then have:

$$A = 0.452 \frac{\phi_o(0)\, G(\alpha)}{s} \tag{11}$$

As shown in Appendix C, the flux at the outside rod surface $\phi_o(0)$, may be put in terms of the unperturbed flux, ϕ_∞ (Eq. 15C of Appendix C):

$$\phi_o(0) = \frac{\phi_\infty}{1 + \frac{G(\alpha)\, K_o(ka)}{4kD\, K_1(ka)}} \tag{12}$$

where K_o and K_1 are modified Bessel functions of the second kind, "a" is the rod radius, $k^2 = \Sigma_a/D$ for the MTR core and D is the diffusion coefficient for the MTR core. The burn-up for the APPR rod tip as a function of time, B(t), in terms of ϕ_∞ is (Eq. 18C of Appendix C):

$$B(t) = \frac{0.452\, \phi_\infty}{N_o\, s} \int_o^t \frac{G(\alpha)}{1 + \frac{G(\alpha)\, K_o(ka)}{4kD\, K_1(ka)}}\, dt \tag{13}$$

where N_o is the original B-10 atom density for the APPR rod.

Replacing the APPR rod by a cylinder with the same perimeter as the perimeter of absorber in the rod we obtain a radius a=3.14 cm for the equivalent cylinder. Reactor constants for the MTR[18] are k= 0.667 cm^{-1} and D=0.263 cm. For the APPR control rod, N_o= 1.28×10^{22} B-10 atoms/cm^3. With these values we obtain the burn-out as a function of time pictured in Figure 4.

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18. J. H. Buck and C. F. Leyse, Materials Testing Reactor Project Handbook, Vol. 1, 1952.

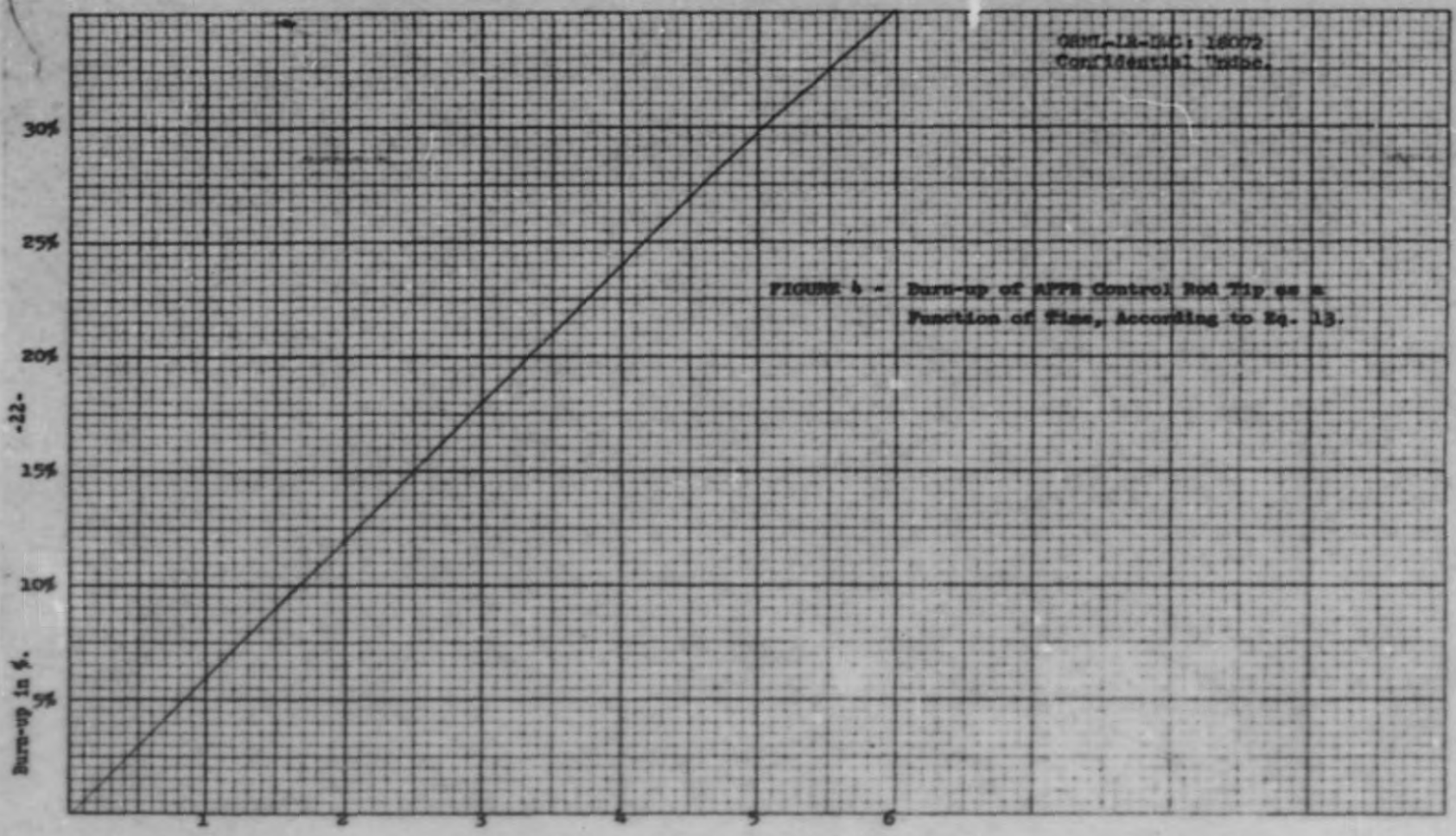

FIGURE 4 - Burn-up of APPR Control Rod Tip as a Function of Time, According to Eq. 13.

APPENDIX A

RECALCULATION OF INNER PLATE SURFACE TEMPERATURE

A recalculation was made on the inner plate surface temperature in order to determine if this temperature was appreciably reduced when air conduction was assumed through the gap between the absorber section and its aluminum housing. The original calculation assumed this gap to be insulated and the maximum inner wall temperature was found to be 265°F. This calculation will assume air conduction through the gap which will be considered to be 0.018 in.

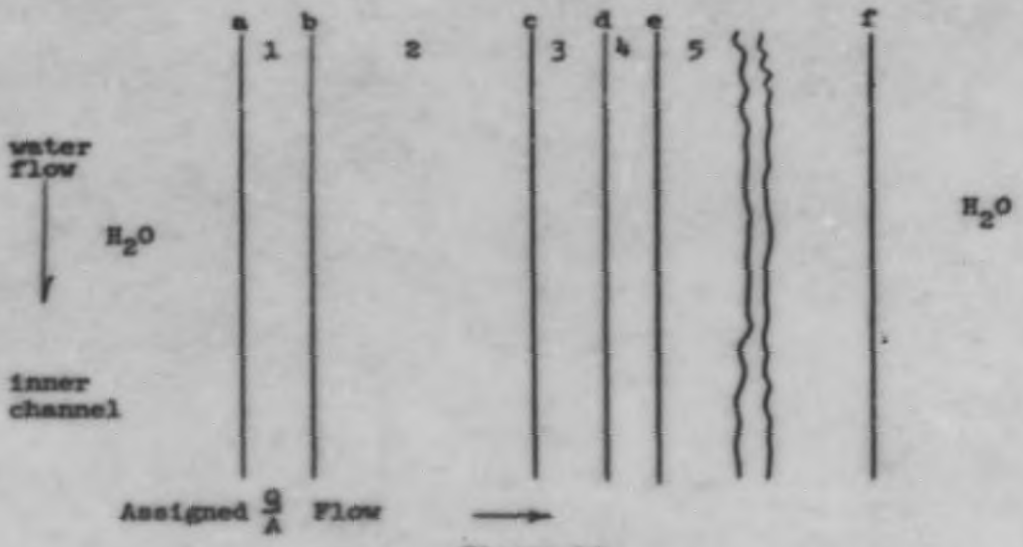

Figure 1A

Cross Section Through the Boron Absorber Plate Wall

Writing temperature drop equations for all boundaries:

$$T_f - T_{H_2O} = \frac{\left(\frac{Q_e}{A} + \frac{Q_d}{A} + \frac{Q_c}{A} + \frac{Q_b}{A}\right)}{h} + (T_{H_2O} - T_a) \quad (1A)$$

$$T_e - T_f = \left(\frac{Q_e}{A} + \frac{Q_d}{A} + \frac{Q_c}{A} + \frac{Q_b}{A}\right) R_5 + (T_{H_2O} - T_a)\ hR_5 \quad (2A)$$

$$T_d - T_e = \left(\frac{Q_e}{A} + \frac{Q_c}{A} + \frac{Q_b}{A}\right) R_4 + (T_{H_2O} - T_a)\ hR_4 \quad (3A)$$

$$T_c - T_d = \left(\frac{Q_c}{A} + \frac{Q_b}{A}\right) R_3 + (T_{H_2O} - T_a)\ hR_3 \quad (4A)$$

$$T_b - c = \left(\frac{Q_b}{A}\right) R_2 + (T_{H_2O} - T_a)\ hR_2 \quad (5A)$$

$$T_a - T_b = (T_{H_2O} - T_a)\ hR_1 \quad (6A)$$

where h = heat transfer coefficient.
R = thermal resistivity.
Q/A = heat flux.
T = temperature.

In order to simplify, let:

$$X = \frac{Q_e}{A} + \frac{Q_d}{A} + \frac{Q_c}{A} + \frac{Q_b}{A}$$

$$Y = \frac{Q_d}{A} \quad \frac{Q_c}{A} \quad \frac{Q_b}{A}$$

$$Z = \frac{Q_c}{A} \quad \frac{Q_b}{A}$$

$$W = \frac{Q_b}{A}$$

Then, by solving equations 1A-6A for T_a, the following equation is obtained:

$$T_a = X(1/h + R_5) + Y R_4 + ZR_3 + WR_2 + T_{H_2O}(hR_1 + hR_2 + hR_3 + hR_4 + hR_5 + 2) - T_a h(R_1 + R_2 + R_3 + R_4 + R_5) - T_a$$

Using the following values,

$X = 650{,}500$ Btu/hr-ft^2
$Y = 514{,}000$ Btu/hr-ft^2
$Z = 484{,}500$ Btu/hr-ft^2
$W = 217{,}500$ Btu/hr-ft^2
$h = 3{,}420$ Btu/hr-ft^2-F
$R_3 - R_1 = 0.000316$ hr-ft^2-F/BTU
$R_2 = 0.000268$ "
$R_4 = 0.062$ "
$R_5 = 0.00029$ "
$T_{H_2O} = 115°$ F

The temperature at point a is:

$$T_a = 264° \text{ F.}$$

This surface temperature is only 1°F lower than that calculated assuming no conductance through the air gap. Therefore, the air gap very probably represents the point of heat divergences. This, of course, will vary with the length of the gap or the heat transfer coefficient if the plate and the housing are in contact.

APPENDIX B

CALCULATION OF HEAT TRANSFER COEFFICIENT

According to a recent report by Rainwater and Walker[15], the formula which is presently being used at the MTR to calculate film heat transfer coefficients is:

$$\frac{hD}{K} = 0.020\,(Re)^{0.8}\,(Pr)^{0.33} \qquad (1B)$$

where h = film coefficient.
D = equivalent diameter of the cooling channel.
K = thermal conductivity.
Re = Reynolds' number
Pr = Prandtl modulus.

Reynolds number can be expressed:

$$Re = \frac{DV\rho}{\mu} \qquad (2B)$$

where μ = fluid viscosity.
ρ = fluid density.

Prandtl modulus can be given as:

$$Pr = \frac{C_p \mu}{K} \qquad (3B)$$

where C_p = specific heat at constant pressure.

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15. PTR-54, Rainwater, J. H. and V. A. Walker, Preliminary Report on Apparent Boiling in MTR, p. 6, April 3, 1956.

Substituting equations 2B and 3B into 1B and simplifying gives the film coefficient:

$$h = \frac{0.020\, K^{0.67}\, V^{0.8}\, \rho^{0.8}\, C_p^{0.33}}{D^{0.2}\, \mu^{0.47}}$$

Substituting the following values:

K = 0.368 Btu/hr-ft-F
ρ = 61.7 lb/ft^3
V = 30 ft/sec
C_p = 1.0 Btu/lb-F
D = 2.038 in.

The following is obtained:

$$h = \frac{0.020\left(0.368 \frac{\text{Btu}}{\text{hr-ft-}^\circ\text{F}}\right)^{0.67} \left(30 \frac{\text{ft}}{\text{sec}} \times 3600 \frac{\text{sec}}{\text{hr}}\right)^{0.8} \left(961.7 \frac{\text{lb}}{\text{ft}^3}\right)^{0.8} \left(1.0 \frac{\text{Btu}}{\text{lb-}^\circ\text{F}}\right)^{0.33}}{\left(\frac{2.038 \text{ in.}}{12 \text{ in./ft}}\right)^{0.2} (1.57 \text{ lb/hr-ft})^{0.47}}$$

h = 3420 Btu/hr-ft^2-F

APPENDIX C

BURNOUT OF A NEUTRON ABSORBER ACCORDING TO ONE VELOCITY DIFFUSION THEORY

The neutron current density into the surface of an absorber from the upper half space ($Z \geq 0$) is given by[19]:

$$J_-(0) = \int_0^{\pi/2} J_-(\theta)\, d\theta = 1/2 \int_0^1 \left[\phi_0 + 3D \left(\frac{\delta\phi}{\delta Z}\right)_0 \mu\right] \mu d\mu \quad (1C)$$

where only the first two terms in a Taylor's expression of the flux near the absorber boundary ($Z=0$) are retained. The quantity D is the diffusion coefficient of the one velocity neutrons in the medium adjacent to the absorber and $\mu = \cos\theta$. If the absorber has a thickness s, the current out the other side is then:

$$J_-(-s) = 1/2 \int_0^1 \left[\phi_0 + 3D \left(\frac{\delta\phi}{\delta Z}\right)_0 \mu\right] e^{-\alpha/\mu}\, \mu d\mu \quad (2C)$$

where $\alpha = \Sigma_a s$ and Σ_a is the microscopic absorption cross section of the absorber. In obtaining Eq. 2C neutrons are assumed to travel in straight lines and $e^{-\alpha/\mu}$ then represents the attenuation of neutrons having the direction cosine μ.

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19. Glasstone and Edlund, "Elements of Nuclear Reactor Theory", Van Nostrand Co., 1952, p. 94.

The net current into the absorber is equal to the surface absorption rate:

$$As = J_-(0) - J_-(-s) + J_+(-s) - J_+(0) = J_Z(-s) - J_Z(0)$$

$$= -D\left(\frac{\delta\phi}{\delta Z}\right)_{-s} + D\left(\frac{\delta\phi}{\delta Z}\right)_0 \qquad (3C)$$

where A is the volume rate of absorptions. If the flux is the same on both sides of the absorber then $J_-(0) = J_+(-s)$; $J_-(-s) = -J_+(0)$; $J_Z(0) = -J_Z(-s)$; $D\left(\frac{\delta\phi}{\delta Z}\right)_0 = -D\left(\frac{\delta\phi}{\delta Z}\right)_{-s}$. Eq. 3 then becomes:

$$As = 2\left[J_-(0) - J_-(-s)\right] = -2J_Z(0) = 2D\left(\frac{\delta\phi}{\delta Z}\right)_0 \qquad (4C)$$

In this treatment, the flux gradiant term, $\left(\frac{\delta\phi}{\delta Z}\right)_0$, is regarded as a correction term in Eq. 1C. To obtain an approximate expression for $\left(\frac{\delta\phi}{\delta Z}\right)_0$ let us apply diffusion theory to the absorber. In the absorber:

$$D_a \vec{\nabla}\cdot\vec{\nabla}\phi_a = \Sigma_a \phi_a \qquad (5C)$$

where ϕ_a is the flux in the absorber and D_a is the diffusion coefficient in the absorber. Now integrate Eq. 5C over the volume of the absorber:

$$D_a \int_V \vec{\nabla}\cdot\vec{\nabla}\phi_a \, dV = \int_V \Sigma_a \phi_a \, dV \qquad (6C)$$

By Green's theorem the left hand volume integral may be converted into a surface integral:

$$D_a \int_S \vec{\nabla}\phi_a \cdot d\vec{S} = D \int_S \vec{\nabla}\phi \cdot d\vec{S} = \int_V \Sigma_a \phi_a \, dV \qquad (7C)$$

where $D_a \vec{\nabla}\phi_a = D \vec{\nabla}\phi$ since the current must be continuous across the boundary. Assuming no variation of ϕ or $\vec{\nabla}\phi$ along the surface, Eq. 7C reduces to:

$$2D \left(\frac{\partial \phi}{\partial z}\right)_o = \int_{-s}^{o} \Sigma_a \phi_a \, dz \approx \Sigma_a \phi_o \, s = \alpha \phi_o \qquad (8C)$$

With this approximation for $\left(\frac{\partial \phi}{\partial z}\right)_o$, Eq. 4C for the volume rate of the absorptions becomes:

$$A = \frac{2}{s}\left[J_-(0) - J_-(-s)\right] = \frac{\phi_o}{2s}\int_o^1 \left[1 + \frac{3}{2}\alpha\mu\right]\left[1 - e^{-\alpha/\mu}\right] \mu d\mu$$

$$= \frac{\phi_o}{2s} G(\alpha) \qquad (9C)$$

The function

$$G(\alpha) = \int_o^1 \left[1 + \frac{3}{2}\alpha\mu\right]\left[1 - e^{-\alpha/\mu}\right] \mu d\mu \qquad (10C)$$

is plotted in Fig. 1C. Eq. 9C represents the volume absorption rate in an absorber of thickness s, macroscopic absorption cross section $\Sigma_a = \alpha/s$ and with the same surface flux, ϕ_o, at both surfaces.

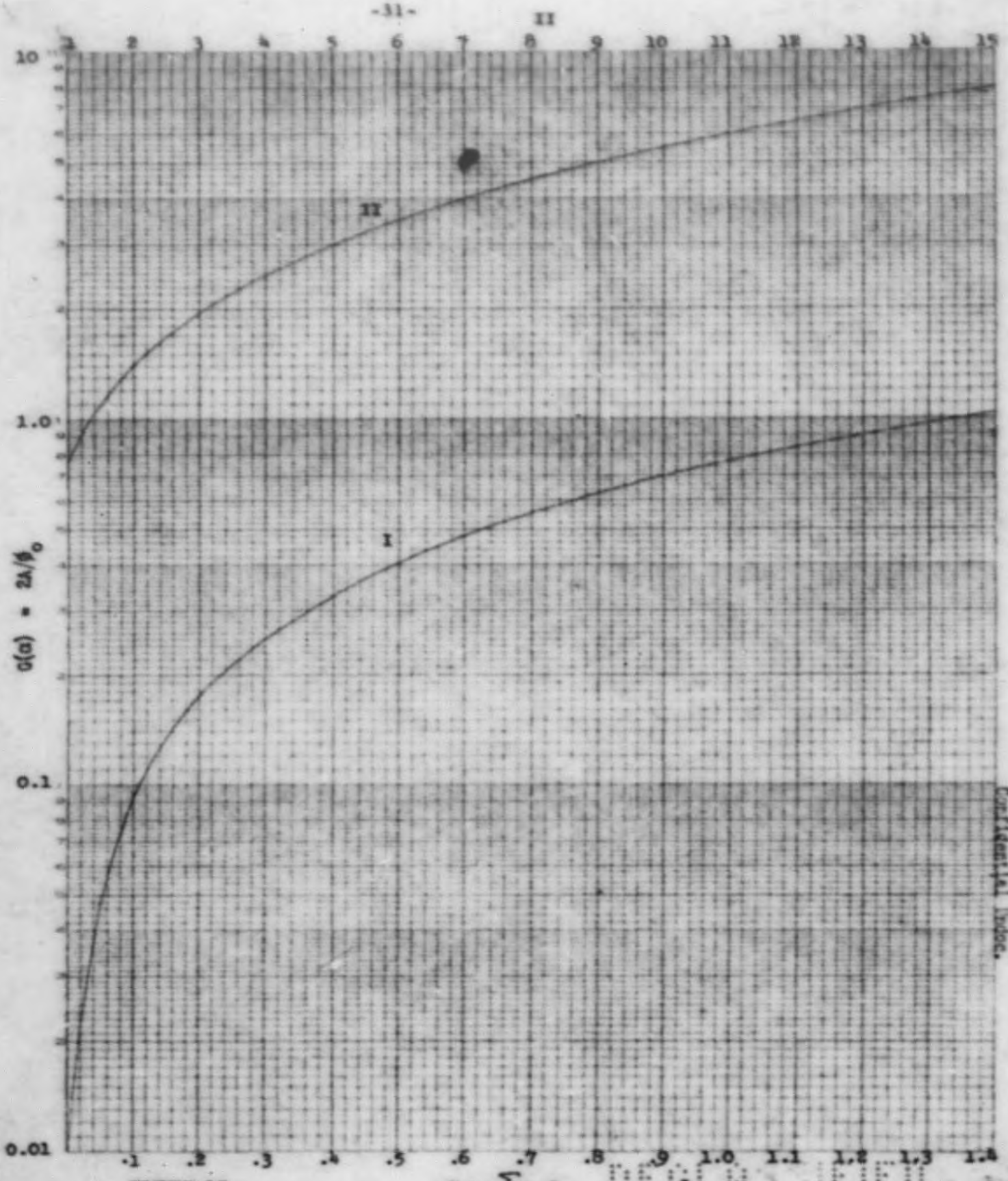

FIGURE 1C

In most cases the flux at the absorber surface, ϕ_o, is not known whereas the unperturbed flux, ϕ_∞, is usually obtainable. The dependence of ϕ_o upon ϕ_∞ may be obtained by applying diffusion theory to the material surrounding the absorber:

$$\nabla^2 \phi - k^2\phi + k^2\phi_\infty = 0 \qquad (11C)$$

where $k^2 = \left(\frac{\Sigma_a}{D}\right)_{Reactor}$ and $k^2 \phi_\infty$ is the uniform source term. Eq. 11C has the solution:

$$\begin{aligned} \phi(Z) &= \phi_\infty + C_1 e^{-kZ} && \text{slab geometry} \\ \phi(r) &= \phi_\infty + C_2 K_o(kr) && \text{cylindrical geometry} \\ \phi(r) &= \phi_\infty + C_3 \frac{e^{-kr}}{r} && \text{spherical geometry} \end{aligned} \qquad (12C)$$

where K_o is the modified Bessel function of the second kind and order zero. The constants C_1, C_2, and C_3 may be related to the gradiant of the flux at the absorber surface:

$$\begin{aligned} C_1 &= -\frac{1}{k}\left(\frac{\partial\phi}{\partial Z}\right)_o \\ C_2 &= -\frac{1}{k K_1(ka)}\left(\frac{\partial\phi}{\partial r}\right)_a \\ C_3 &= -\frac{a^2 e^{ka}}{1+ka}\left(\frac{\partial\phi}{\partial r}\right)_a \end{aligned} \qquad (13C)$$

From Eqs. 4C and 9C we have $\left(\frac{\partial \phi}{\partial z}\right)_o$ in terms of ϕ_o:

$$\left(\frac{\partial \phi}{\partial z}\right)_o = \frac{\phi_o}{4D} G(\alpha) \qquad (14C)$$

Substituting Eqs. 14C and 13C into Eq. 12C and evaluating 12C at the absorber boundary we have:

$$\phi_o = \frac{\phi_\infty}{1 + \frac{G(\alpha)}{4kD}} \qquad \text{slab geometry}$$

$$\phi_a = \frac{\phi_\infty}{1 + \frac{G(\alpha) K_o(ka)}{4kD\, K_1(ka)}} \qquad \text{cylindrical geometry} \qquad (15C)$$

$$\phi_a = \frac{\phi_\infty}{1 + \frac{G(\alpha) ka}{4kD[1+ka]}} \qquad \text{spherical geometry}$$

These values for the flux at the absorber surface may now be used in Eq. 9C to obtain the volume absorption rate. The fractional burnup of the absorber with time may then be described by:

$$B(t) = \frac{\int_o^t A(t)\,dt}{N_o}$$

$$= \frac{\phi}{2s\,N_o} \int_o^t \frac{G(\alpha)}{1 + \frac{G(\alpha)}{4kD}}\,dt \qquad \text{slab geometry}$$

$$= \frac{\phi}{2s\,N_o} \int_o^t \frac{G(\alpha)\,dt}{1 + \frac{G(\alpha)\,K_o(ka)}{4kD\,K_1(ka)}} \qquad \text{cylindrical geometry (16C)}$$

$$= \frac{\phi}{2s\,N_o} \int_o^t \frac{G(\alpha)\,dt}{1 + \frac{G(\alpha)\,ka}{4kD(1+ka)}} \qquad \text{spherical geometry}$$

Since α itself is a function of time as the absorber burns out, the calculation of B(t) by Eq. 16C has to be done in piecemeal fashion.

END