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CAMBRIDGE ELECTRON ACCELERATOR

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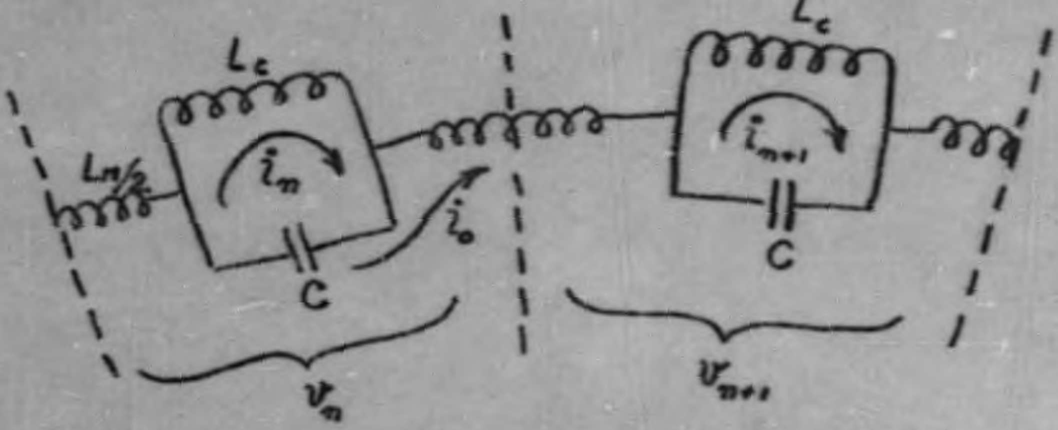
ANALYSIS OF SPURIOUS MODES IN MAGNET POWER SUPPLY

Abstract

The resonant frequencies of the magnet power supply are investigated. It is found that there are eleven resonant modes in addition to the desired mode of operation with six additional resonant frequencies. From measurements of the choke model, it appears that the frequencies of some of the spurious modes may lie in the same range as the desired mode of operation.

The basic circuit of the magnet power supply is shown below.



In the desired mode of operation, the voltage of each complete unit is zero ($v_n=0$). The currents in each unit are identical ($i_n=i_m$).

Then $$i_o\left[i\omega L_M + \frac{1}{i\omega C}\right] - i_n \frac{1}{i\omega C} = 0$$

and $$i_o \cdot i\omega L_M + i_n \cdot i\omega L_c\left[1 + \sum_{m\neq 0}^{N} K_m\right] = 0$$

K_m is the coupling coefficient between current i_m and voltage in choke i_{n+m}.

N is number of complete units.

The resonant frequency of the fundamental mode is given by:

$$\begin{Vmatrix} \omega L_{11} - \frac{1}{\omega C} & \frac{1}{\omega C} \\ \omega L_M & \omega L_c\left[1+\sum_{m\neq 0}^{N} K_m\right] \end{Vmatrix} = 0$$

$$\omega L_M\left[\omega L_c - \frac{1}{\omega C} + \omega L_c \sum_{m\neq 0}^{N} K_m\right] - \frac{L_c}{C}\left[1+\sum_{m\neq 0}^{N} K_m\right] = 0$$

$$\omega_{oo}^2 = \frac{1}{L_c C}\left[\frac{1}{1+\sum_{m\neq 0}^{N} K_m} + \frac{L_c}{L_M}\right]$$

For a spurious mode the voltages of each unit will be related in phase so as to make the total ring voltage zero. In the spurious modes there will be no current in the magnet.

$$i_o = 0$$

$$i_{n+m} = i_n e^{im\theta_p} \qquad v_{n+m} = v_n e^{im\theta_p}$$

$$\theta_p = \frac{2\pi p}{N}$$

Then

$$i_n\left[i\omega L_c + \frac{1}{i\omega C}\right] + i\omega L_c \sum_{m\neq 0}^{N} i_{n+m} K_m = 0$$

$$i_n\left\{\omega L_c - \frac{1}{\omega C} + \omega L_c \sum_{m\neq 0}^{N} K_m e^{im\theta_p}\right\} = 0$$

Since $K_m = K_{N-m+1}$ by symmetry

$$\omega L_c - \frac{1}{\omega C} + \omega L_c \sum_{m\neq 0}^{N} K_m \cos m\theta_p = 0$$

$$\omega_p^2 = \frac{1}{L_c C} \frac{1}{1 + \sum_{m\neq 0}^{N} K_m \cos m\theta_p}$$

By inspection it is seen that a possibility exists of having $\omega_p = \omega_{b\nu}$ for some values of K_m and L_m.

For the numerical interpretation of the analysis, we assume the values fixed by the present power supply design : $C = 313.5\,\mu F$, $L_M = 140\,mH$ and $L_c' = L_c\left(1 + \sum_{m\neq 0}^{N} K_m\right) = 250\,mH$;

and furthermore we use the following measured coupling coefficients: $K_0 = 1$, $K_{1,11} = .66$, $K_{2,10} = .5$ $K_{3,9} = .42$, $K_{4,8} = .36$, $K_{5,7} = .33$, $K_6 = .32$

valid for our 12-unit choke model with reduced gap width (= .34" instead of 1").

It is obvious that we can perform the frequency mapping of the spurious modes. Therefore, we calculate the ratio

$$\frac{\omega_p}{\omega_o} = \frac{1}{\left(1 + \sum_{m=1}^{6} k_m \cos m\theta_p\right)^{1/2}}$$

where $(\omega_o/2\pi) = \frac{1}{2\pi}(L_c C)^{-1/2}$ represents the resonant frequency for one choke unit with L_c, an arbitrary C, and where all the other coils are open circuits. This is only useful for the present choke model where we do not need to insert L_M in order to investigate the spurious mode frequencies. For the final design, the ratio of the spurious mode frequencies ω_p to the fundamental frequency ω_{oo} is needed, and we have the equation

$$\frac{\omega_p}{\omega_{oo}} = \frac{\omega_p}{\omega_o}\,\frac{1}{\left(\frac{L_c}{L_M} + \frac{1}{1+\sum_{m=1}^{6} k_m}\right)^{1/2}}$$

Using both equations and the above values, we get the following table:

p	1,11	2,10	3,9	4,8	5,7	6
$\frac{\omega_p}{\omega_o}$	.85	1.28	1.57	1.8	1.99	2.13
$\frac{\omega_p}{\omega_{oo}}$	1.23	1.85	2.28	2.61	2.88	3.09

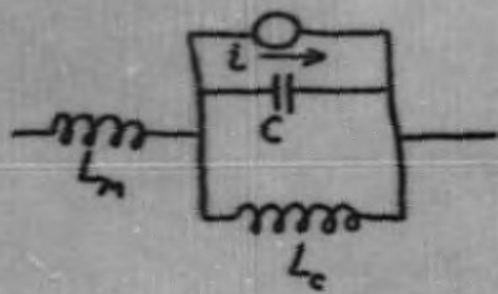
Enlarging the choke's gap from 0.34" to 1", we try to estimate the new coupling coefficient. In order to do this, we assume that for our ring shaped choke the leakage flux ϕ varies like $\frac{d\phi}{ds} \sim (AW)$ (AW is a magnetomotive force) around the circumference, and we know that $\frac{d(AW)}{ds} \sim R_m \phi$. So we find the expression $K = \cosh \alpha m'$ for the coupling coefficient as a function of position m' around the ring, and α is a constant proportional to magnetic reluctance or gap width. We find fairly good agreement with the measured K-values. The new gap is 3 times bigger, so we multiply α by a factor of three, and the new set of K-values is : $K_0 = 1$, $K_{1,11} = .515$, $K_{2,10} = .27$ $K_{3,9} = .14$, $K_{4,8} = .076$, $K_{5,7} = .047$ $K_6 = .038$.

Using the new K-values and the above values for circuit elements, we have again

p	1,11	2,10	3,9	4,8	5,7	6
$\frac{\omega_p}{\omega_{oo}}$	.755	1.07	1.4	1.36	1.95	1.88

If the 1" gap choke model was scaled from full size choke, we notice the close coincidence of the $p = 2$ or 10 modes with the fundamental one.

If a spurious mode is degenerate with the fundamental mode, appreciable power may be produced in the spurious mode by relatively small imperfections in the system.



The effect of a variation in the capacity C may be calculated as a first order effect. A variation of capacity ΔC is equivalent to adding a current generator $i = V_o \omega \Delta C$ at that point. V_c is the voltage across the capacity due to the fundamental mode. By a Fourier analysis, the component of the current generator in each mode is $i_p = \frac{i}{N} = \frac{V_o \omega \Delta C}{N}$. The voltage produced in a degenerate mode is then

$$V_p = \frac{i_p Q_p}{\omega C} = V_o \frac{Q_p}{N} \frac{\Delta C}{C}$$

Q_p is the Q of the spurious mode.

The ratio of power in the spurious mode to power in fundamental mode is given by

$$\frac{P_p}{P_o} = \left(\frac{V_p}{V_o}\right)^2 \frac{Q_o}{Q_p} = \frac{1}{N^2} \left(\frac{\Delta C}{C}\right)^2 Q_o Q_p$$

If all condensers are changed by a r.m.s. value ΔC in a random manner, the power in the spurious mode will be increased by a factor N. Also there will probably be two spurious modes degenerate with the fundamental mode, which will increase the additional power required by a factor of 2.

Then

$$\frac{\Delta P}{P_0} = 2\,\frac{Q_u Q_o}{N}\left(\frac{\Delta C}{C}\right)^2$$

It is seen that if the Q values are high, small changes in capacity may significantly increase the power dissipatinn.

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