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**INTERIM REPORT OF NUCLEAR ANALYSIS
PERFORMED ON SM-2 CORE AND VESSEL**

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Edited By:
P. E. Bobe, Task Engineer

Analysis By:
P. E. Bobe B. J. Byrne
S. H. Birken E. F. Clancy
B. E. Fried

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Alco Products, Inc.
Post Office Box 414
Schenectady 1, New York

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ABSTRACT

This report contains a description of the nuclear analysis performed upon the SM-2 core and vessel for the period September 1, 1958 to December 31, 1959. Calculations are given for core reactivity, power distributions, lifetime, reactor control, kinetics, radiation problems, fuel and poison burn-ups, and the nuclear effects of poisons, temperature, and geometry. Wherever possible, experimental data is included in order to test the validity of the analytical models. The SM-2 nuclear analysis was performed by Alco Products, Inc. under Tasks 1, 8, and 10 of Contract No. AT(30-3)-326 for the Atomic Energy Commission.

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INTRODUCTION

This report contains a summary of the nuclear analysis performed on the SM-2 (formerly APPR-1B) core and vessel from September, 1958 to December 31, 1959.

The SM-2 Core and Vessel contract AT-(30-3)-326, lists the following general specifications for the core:

1. The core is to be of the APPR type.
2. The core is to be capable of approximately 25 MWT operation at 100% load factor for a minimum of one year. Growth potential in core life should be provided.
3. Fuel elements, both control and fixed as well as absorbers, are to be interchangeable with the SM-1 and SM-1A. Interchangeability is defined as follows: The active sections of the fixed and control rod elements are to be capable of installation in the SM-1, SM-1A, or SM-2 with end boxes and handles installed.
4. Temperature coefficient is to be sufficiently negative to insure inherent safety in plant response.
5. Control rod array must be capable of shutting down the core at room temperature, no xenon, with a reasonable and adequate stuck rod criteria.

The philosophy of determining a reasonable and adequate stuck rod criteria should be developed along the following guide lines:

1. On the basis of nuclear and design considerations, select a specific rod configuration and number of rods for the core type being studied.
2. Evaluate the probability that a rod or rods will stick and the most hazards conditions under which such events will occur.
3. Take into account the additional "backup" shutdown capabilities of the plant, such as the incorporation of a soluble poison injection system.
4. On the basis of these considerations, examine the consequences of one or more rods sticking at different times in core life under various operating conditions.

5. After an evaluation of the above items, determine a realistic stuck rod criteria for the specific core under consideration. This criteria should insure that the reactor can be shutdown at any time during core life, but it should not penalize the design and/or performance of the system through unnecessary conservatism.

This report contains a detailed account of the calculations performed, including a description of the IBM-650 and IBM-704 codes employed, basic equations, nuclear and geometrical data, and the calculated results. Wherever possible, the calculated results are compared to available experimental data in order to test the validity of the theoretical models employed.

The purpose of this report is to act as a cumulative data source for nuclear analysis work being performed under Task 10 (formerly Task 1) of the SM-2 Core and Vessel Program. It is hoped that the detailed nature of this report will be of value in future calculations made upon the SM-2 core and other similar APPR-type core designs.

Development of a consistent nuclear analysis of the core is difficult at this point during the core and vessel development program because of frequent design changes. Therefore, it is important to keep in mind the particular core design under consideration when one reviews the results presented in this report.

1.0 CALCULATIONAL MODELS

The calculational models employed in this report are based upon methods developed during the past few years. The approach has been to base calculations upon fundamental nuclear data rather than upon an empirical adjustment of parameters to fit experiments. The models are based upon modified two-group theory; a description of this theory and its application to various IBM machine codes are described in subsequent sections.

1.1 MODIFIED TWO-GROUP THEORY⁽¹⁾

Due to appreciable resonance absorption in the core, at least three neutron groups should be used to describe the neutron flux. However, the three groups may be reduced to two with multiplication in the fast group with little loss of generality. The general two-group equations, including resonance fission and absorption are:

$$D_f \nabla^2 \phi_f(\underline{r}) - \Sigma_R \phi_f + S_f = 0$$

$$D \nabla^2 \phi(\underline{r}) - \Sigma_a \phi + S_t = 0$$

where the source terms are defined as:

$$S_f = K_{th} \Sigma_a \phi(\underline{r}) + K_f (1-P) \Sigma_R \phi_f(\underline{r})$$

$$S_t = P \Sigma_R \phi_f(\underline{r})$$

and the effective removal cross section, Σ_R , is defined as:

$$\Sigma_R = D_f / \tau$$

where:

$\phi_f(\underline{r})$ = spatial dependent fast flux (neutrons/cm²-sec).

$\phi(\underline{r})$ = spatial dependent thermal flux (neutrons/cm²-sec).

D_f = fast diffusion coefficient (cm).

D = thermal diffusion coefficient (cm).

Σ_R	=	effective removal cross section from fast group (cm^{-1}).
Σ_a	=	thermal macroscopic absorption cross section (cm^{-1}).
P	=	resonance escape probability.
K_{th}	=	total neutrons produced by thermal fission per thermal neutron absorbed.
K_f	=	total neutrons produced by epithermal fission per epithermal neutron absorbed.
τ	=	neutron age (cm^2).
∇^2	=	the Laplacian operator.

1.1.1 Fast Group Constants

A major problem in two-group theory is the preparation of the fast group constants τ , D_f , K_f and P . Fortunately, there is available a machine code for computing few group constants based on solving the Fourier transform of the slowing down distribution from a point source. (2) The equations are solved within four approximations P-1, B-1, P-1 Selengut-Goertzel, and B-1 Selengut-Goertzel. The P-1 Selengut-Goertzel approximation has been used since the fast parameters obtained from this approximation yield reactivities which are in best agreement to SM-1 measured reactivities. Inelastic scattering is described by use of the Weisskopf statistical model. A description of this theory together with MUFT-III, an IBM-650 code employing this theory, is given in references (2) and (3). The code computes the Fourier transform of the slowing down density and then prepares few group summations and the second moment. Microscopic cross section data, atomic concentrations, and the buckling are the only input required for the calculation. The 59 group cross sections used in the MUFT-III calculations are shown in Appendix A of reference (4).

One problem is that of the transition energy between the fast and thermal groups. In this report, the lower limits of the fast group (or groups) is taken as 0.196 ev at 68°F. and 0.248 ev for both 440°F and 510°F. Fortunately, the cutoff energy has little effect on K_{eff} , as shown in Section 2.4.4 of reference (4).

1.1.2 Thermal Properties

Intercell measurements made in the SM-1 critical experiment⁽⁵⁾ and the SM-1 Zero Power Experiment ZPE-1⁽⁶⁾ indicate a significant variation of thermal flux within the fuel element cell. An accurate treatment of these flux variations is necessary if the thermal utilization is to be calculated accurately. The variation of innercell flux, both perpendicular and parallel to the fuel plates in the fixed elements was calculated using a one velocity P_3 approximation to the Boltzmann equation. The determination of the velocity to be used is discussed in Section 1.1.2.3. The calculational procedure was basically a superposition of several P_3 solutions to the one-dimensional single velocity transport equation. Murray gives a good description of the one-dimensional problem.⁽¹⁾ This theory has been programmed on the IBM-650 for APPR-type elements.⁽⁷⁾ For the control rod fuel elements, a one-velocity P_3 calculation in cylindrical geometry was employed. This theory has also been programmed on the IBM-650 for APPR-type control rod fuel elements.⁽⁸⁾

1.1.2.1 Fixed Fuel Elements

Figures 1.1 and 1.2 are drawings of a typical SM-2 fixed fuel element, showing the geometry and typical dimensions. A schematic diagram of the cross section of the element is shown in Fig. 1.3. The following nomenclature is used for the various regions:

Plate: This includes the meat and clad, but does not include the dead edges. Properties for this region are calculated as if the clad and meat are homogeneously mixed. The plate half-thickness is ℓ ; its half-width is p .

Channel: This includes material between the fuel plates. The half-width of this region is p , the same as for the fuel plates. Its half-thickness is $(a-\ell)$. Properties for this region are calculated on a homogenized basis.

Active: This is a mixed region consisting of the fuel plates and the channel. Its half-width is p .

Dead: This region includes the fuel plate dead edges, side plates, and all of the water in the element that is not in the channels. Its half-width is $(c-p)$. Properties are calculated as if all materials in the region are homogeneously distributed.

Fuel Element Cell: This includes all regions. It is made up of the active plus the dead regions.

The code⁽⁷⁾ first calculates the thermal flux distribution normal to the fuel plates. The flux is normalized to one in the active region. Average cross sections are calculated for the active region. Next the thermal flux distribution in the active and dead regions is calculated due to a uniform source of 1 neutron per cm^3 in the dead region. Then the thermal flux distribution in the two regions is calculated due to a uniform source of 1 neutron per cm^3 in the active region. The latter two solutions are weighted and superimposed to obtain the overall flux distribution parallel to the fuel plates. The relative source strength in each region is assumed proportional to the volume fraction of water. The relative flux is normalized to one in the entire element. From the thermal flux distributions normal and perpendicular to the fuel plates, the nuclear parameters for the whole fuel element are calculated.

1.1.2.2 Control Rod Fuel Elements

Figures 1.4 and 1.5 are drawings of a typical SM-2 control rod fuel element, showing the geometry and typical dimensions. A schematic diagram of the cross section of the element is shown in Fig. 1.6. The nomenclature for the plate, channel, and active region is the same as for the fixed elements. The dead region includes the fuel plate dead edges, side plates, control rod basket, and all the water not contained in the channels. The element is cylindricalized into an active region surrounded by a dead region. The relative sources in the active and dead regions were assumed equal to the volume fraction water in each region.

The code⁽⁸⁾ first calculates the thermal flux distribution normal to the fuel plates and normalizes it to one in the active region. Average cross sections are calculated for the active region. The properties of the dead region are calculated on a homogenized basis. The radial flux distribution through the active and dead regions are then calculated and normalized to one in the entire element. The nuclear parameters for the whole element are then calculated from the final thermal flux distribution.

1.1.2.3 Neutron Temperature

The absorption cross section of the SM-2 core is large compared to the scattering cross section (see Appendix A). This is known to harden the spectrum, or shift the spectrum to higher energy as a result of the preferential absorption of low energy neutrons. This problem has been treated by defining an effective neutron temperature which is higher than the physical temperature⁽⁹⁾ by an amount which depends on the ratio of absorption to scattering cross section in the assembly. The energy distribution of the thermal neutron flux was assumed to be a Maxwell-Boltzmann distribution about the effective neutron temperature.

The effective neutron temperature was calculated by the following equation:

$$T_n/T_o = 1 + C \left(\frac{\sum a}{\sum s} \right)$$

where:

T_o = temperature of the assembly

T_n = effective neutron temperature

$\sum a$ = macroscopic absorption cross section at $E_o = kT_o$

$\sum s$ = macroscopic scattering cross section at $E_o = kT_o$

C = a constant = 0.91

At the beginning of the SM-2 program, the actual atomic content of the core was not known. In addition, all through the program there have been changes made in the composition. However, it is known that the calculated effect on reactivity is small for small differences in the effective energy; this is shown in Section 11.1.4. Therefore, the atomic concentrations of the SM-1⁽¹⁰⁾ were used to calculate the effective neutron energies of the SM-2. These energies are calculated to be 0.0331 ev ($T_o = 68^\circ\text{F}$), 0.0549 ev ($T_o = 440^\circ\text{F}$), and 0.060 ev ($T_o = 510^\circ\text{F}$). A tabulation of the cross sections and their definitions is given in Appendix A.

1.2 CORE GEOMETRY CONSIDERATIONS

Figure 1.7 shows how the fixed and control rod elements are assembled into the core for a closed seven rod array, as was used in the SM-1. Fig. 1.8 shows an open seven rod array which was selected for the SM-2, as discussed in Section 8.6. The calculations of reactivity, flux and power distributions, control rod bank positions, etc., were performed by the methods described in the subsequent sections.

1.2.1 Equivalent Bare Model

The solutions of the core equations, given in Section 1.1, may be found by proposing that each flux obeys the wave equation, i. e.:

$$\nabla^2 \phi_f(\underline{r}) + B^2 \phi_f(\underline{r}) = 0$$

$$\nabla^2 \phi(\underline{r}) + B^2 \phi(\underline{r}) = 0$$

where B^2 , the buckling, is assumed to be common to both fluxes. These equations may be substituted into the core equations to yield:

$$\begin{aligned}\sum_R [K_f (1-P) - \tau B^2 - 1] \phi_f + K_{th} \sum_a \phi &= 0 \\ \sum_R P \phi_f - (DB^2 + \sum_a) \phi &= 0\end{aligned}$$

If these simultaneous equations in ϕ_f and ϕ are to be solved, the determinant formed by the materials - dependent coefficients must be zero. This implies that:

$$\frac{K_{th} P}{(1 + \tau B^2)(1 + L^2 B^2)} + \frac{K_f (1-P)}{(1 + \tau B^2)} = 1$$

where the square of the diffusion length, L^2 , is defined as:

$$L^2 = D/\sum_a$$

This equation has been derived for a critical reactor. For a non-critical reactor, the multiplication factor for the system, K_{eff} , may be expressed in terms of the factors ν and ν_c , the neutrons produced per fission and the neutrons produced per fission required for criticality respectively. Therefore:

$$K_{eff} = \nu/\nu_c = \frac{PK_{th}}{(1 + \tau B^2)(1 + L^2 B^2)} + \frac{K_f (1-P)}{(1 + \tau B^2)}$$

The effect of a reflector may be included in the bare reactor calculation by modifying the definition of the buckling. For a finite cylindrical core:

$$B^2 = \left(\frac{\pi}{H+2S_z}\right)^2 + \left(\frac{2.405}{R+S_r}\right)^2$$

where:

- H = height of the active core (cm).
- R = equivalent radius of the active core (cm).
- S_z = axial reflector savings (cm).
- S_r = radial reflector savings (cm).

The reactivity of the core due to a change in K_{eff} from $(K_{\text{eff}})_1$ to $(K_{\text{eff}})_2$ is computed by:

$$\rho = \frac{(K_{\text{eff}})_2 - (K_{\text{eff}})_1}{(K_{\text{eff}})_2 (K_{\text{eff}})_1}$$

Therefore, for a small change from a critical core:

$$\rho = \frac{K_{\text{eff}} - 1}{K_{\text{eff}}}$$

1. 2. 2 Multiregion One-Dimensional Calculations

In general, the core is approximated by an equivalent cylinder having a cross-sectional area equal to the cross-sectional area of the actual core array. Many of the reactivity calculations were based on one-dimensional multiregion computations using finite difference approximations. In general, the modified two-group equations were employed and solved to obtain the reactivity of the core, flux and power distributions. These calculations were conveniently performed by use of VALPROD⁽¹¹⁾ and Windowshade⁽¹²⁾ two IBM-650 programs which are used to treat the radial and axial directions respectively.

1. 2. 2. 1 VALPROD⁽¹¹⁾

The VALPROD code is used for the treatment of the radial direction where the height of the cylinder is accounted for by a perpendicular buckling. The code may also be used for axial calculations in which case the width or radius of the core is accounted for by a radial buckling. Input to the code includes nuclear parameters for each of the neutron groups in each region, configuration of the core, and number of calculational points in each region. Output includes the flux distribution for each neutron group, the power distribution, and K_{eff} of the core.

1. 2. 2. 2 Windowshade⁽¹²⁾

This code employs basically the same techniques as the VALPROD code. The code is a nine-region, two-group finite difference calculation in slab geometry for the IBM-650. The code solves the steady state modified two-group equations. A perpendicular buckling must be specified as well as nuclear properties for each neutron group in each region. Also included is the addition of a uniform thermal poison to one section of the reactor to represent the effect of control rods. The program has two options: the placement of the control rod bank may be adjusted until criticality is obtained

(the code may be made to iterate to $K_{eff} \neq 1$) or the location of the bank may be specified and the code will calculate K_{eff} . Output includes the flux distribution for each group, the power distribution, the bank position, and the K_{eff} of the core.

1. 2. 3 Analytical Two-Region, Two-Group Calculations

An analytical solution of the two-group, two-region equations for a finite cylindrical core has been programmed for the IBM-650.⁽¹³⁾ The FINK-1 code first solves the problem radially with an assumed axial savings and then axially with the computed radial savings. The process is repeated until satisfactory convergence is obtained. There are several limitations on this method. First, a finite radial reflector thickness must be used. This is not serious since experience has shown that there is no significant variation of reflector savings after several diffusion lengths of reflector. Another limitation is probably more serious. The same reflector properties must be used for the top and bottom reflector. The control rod absorber sections in the upper reflector probably reduce the reflector savings in this region. An additional limitation is that the reflectors must be considered on a homogenized basis; however, in the SM-2, there is a laminated steel reflector around the core.

1. 2. 4 Multiregion, Two-Dimensional Calculations

During the latter part of 1959, the IBM-704 code, PDQ,^{(14), (15), (16)} was made available to the program. PDQ is a two-dimensional, reactor-design code for the IBM-704 computer. It finds a discrete numerical approximation to the few-group, time independent, neutron diffusion equations for a heterogeneous reactor in a two-dimensional bounded region. It solves for one to four lethargy groups, with variables either x - y , in rectangular co-ordinates, or r - z , in cylindrical co-ordinates. Input to the code includes nuclear parameters for each neutron group for each region, core configuration, and number of calculational points in each region. Output includes flux and power distributions and the K_{eff} of the core. The code permits a better estimate of the detailed power distribution within the core than can be obtained with the one-dimensional codes.

1. 3 SPECIAL PREPARATION IBM-650 PROGRAMS

To facilitate the calculation of nuclear parameters for use in the various codes, special routines were devised and programmed on the IBM-650.

1.3.1 Program 50 (PROMPT-1)⁽¹⁷⁾

Program 50 (PROMPT-I) prepares input for the MUFT-III and Plate-Type P_3 codes for APPR-type fixed fuel elements, either brazed or welded. Input includes element and core geometry, material, and loading data. Temperature-dependent input data, including microscopic cross sections and densities, are obtained from specially prepared files. Output includes loading as a function of fuel burnup, weight and volume fractions, metal-water ratios, volumes, etc.

1.3.2 Program 51

Program 51 calculates K_{eff} of the core by use of modified two-group theory. Input includes nuclear two-group parameters for fixed fuel elements and core dimensions. The K_{eff} of a mixed element core (fixed and control fuel elements) is calculated by computation of a $\sum_{sub}^a$ which when added to the total absorption cross section of the core, yields an overall absorption cross section for the mixed element core. Various methods to calculate $\sum_{sub}^a$ have been used. This code is most useful in survey type of calculations in which some accuracy may be sacrificed.

1.3.3 Program 52

This program is similar to Program 50 except that the calculation is for APPR-type control rod fuel elements.

1.3.4 Program 53⁽¹⁸⁾

Program 53 (Medusa) is a generalized code combining Programs 50 and 52 designed primarily for calculation of the SM-2 Flexible Critical Experimental cores. Major variation is that the boron addition is in the form of a plate surface distribution (boron impregnated Mylar tapes) rather than the usual volume distribution within the fuel matrix of each plate.

1.3.5 Program 54⁽⁸⁾

This code prepares the thermal parameters for APPR-type control rod fuel elements. Program 54 is a three part program to be used in conjunction with one-dimensional plate P_3 ⁽¹⁹⁾ and cylindrical P_3 ⁽²⁰⁾ codes. Normal input consists primarily of control fuel element properties as calculated by Program 53. However, the data from Program 52 for regular APPR-type control elements may also be used as input. This code performs the operations listed in Section 1.1.2.2 to obtain the thermal parameters of the control rod fuel elements.

1.4 CORE BURNUP AND LIFETIME CALCULATIONS

Core burnup and lifetime calculations were performed utilizing the NUB-1, an IBM-650 code.⁽²¹⁾ This code calculates the fuel and burnable poison distributions in a multiregion core as it is non-uniformly burned out within the framework of modified two-group theory. Stored in the code are modified two-group constants as a function of burnup; these parameters are usually obtained by use of Program 50. Burnout is one-dimensional and can be done for either the axial or radial direction. The code is designed to punch out part of the input for either VALPROD or the Windowshade codes. Input includes nuclear parameters as a function of fuel burnup, average fast and thermal fluxes in each region of the core, and the volume of each region. The code calculates the burnup fraction in each region for a number of specified time intervals; from this the properties of each region are looked up in stored tables to be used in either VALPROD or Windowshade calculations. The latter codes permit the calculation of K_{eff} and bank position as a function of core life.

1.5 TEMPERATURE DEFECT

The temperature defect (cold to hot reactivity change) may be calculated by either VALPROD or equivalent bare calculations. The nuclear parameters are obtained by use of Programs 50, 52, 54, MUFT-III, and plate type P_3 calculations.

1.5.1 Equilibrium Xenon

The effect of equilibrium xenon is included by calculation of the macroscopic absorption cross section due to xenon, which is added uniformly to the thermal macroscopic cross section of the core. The formula for Σ_{xe}^a may be expressed as:

$$\Sigma_{xe}^a = \frac{\alpha \chi (\gamma_1 + \gamma_2) \bar{\sigma}_{a^{xe}} \left[\frac{\delta P_{mw}}{V} \right]}{\lambda_{xe} + \bar{\sigma}_{a^{xe}} \left(\frac{\beta \delta P_{mw}}{V} \right) \left[\frac{1}{(1-\beta) \Sigma_{fc}} \right]}$$

$$\chi = (1-\beta) + \beta g'(B)$$

where:

$g'(B)$ = average flux in fuel plate divided by the average flux in the entire fuel element

α = xenon non-uniform distribution factor -

- γ_1 = fractional fission yield of iodine
 γ_2 = fractional fission yield of xenon
 λ_{xe} = decay constant of xenon
 P_{mw} = reactor power, in watts
 $\bar{\sigma}_a^{xe}$ = xenon thermal absorption cross section
 δ = fissions/watt sec
 V = core volume
 Σ_{fo} = macroscopic fission cross section of the core at the start of core life
 β = fraction of total fissions which are thermal
 B = fuel burnup fraction ($B=0$ at start of core life)

The value of β may be estimated analytically by either:

$$\beta = \frac{PK_{th}}{(1 + L^2 B^2) (1 + \tau B^2) K_{eff}}$$

or

$$\beta = \frac{\nu \Sigma_f \bar{\phi}}{\nu \Sigma_f \bar{\phi} + \nu \Sigma_f^f \phi_f}$$

where the symbols are defined in Section 1.1. The constant parameters used in the calculation of xenon are listed in Appendix A.

1.6 CONTROL ROD WORTH CALCULATIONS

1.6.1 Scram I (22)

The control rod worth calculations are based on modified two-group theory. The theory employs cylindrical geometry for the core and rods. The boundary conditions at the rods are based on extrapolation lengths. The Scram I model treats the gross rod worth of a central rod plus M identical rods in a ring concentric to the central rod where $M \geq 0$. The calculation is applicable only for this rod configuration in a bare cylindrical reactor. The boundary conditions on the rods treat either a homogeneous rod or an absorbing shell filled with a moderating material. This material may be a good or

poor moderator. The absorbing shell is considered black to thermal neutrons and to have no effect on fast neutrons. Input includes the properties and geometry of the core, geometry and properties of the control rods, and the location of the rods. The calculation consists essentially of determining the reactivity of the core with the rods out and the reactivity of the core for the rods fully inserted. The rod worth is then defined by:

$$\rho_{\text{rods}} = \frac{K_{\text{eff}}^{\text{out}} - K_{\text{eff}}^{\text{in}}}{K_{\text{eff}}^{\text{out}}}$$

where:

$$\begin{aligned} \rho_{\text{rods}} &= \text{worth of the control rods} \\ K_{\text{eff}}^{\text{out}} &= \text{reactivity of core with rods withdrawn} \\ K_{\text{eff}}^{\text{in}} &= \text{reactivity of core with rods fully inserted} \end{aligned}$$

1. 6. 2 Scram II

Scram I has been superseded by Scram II, an IBM-650 code, which is identical to Scram I except that the revised code allows the calculation of a ring of M control rods without the central rod. This additional option makes the code more versatile than the Scram I code.

1. 6. 3 Scram III

Scram III is a control rod analysis developed for a cylindrical core with a radial reflector.

The code is quite similar to Scram II and has been combined with the latter code so that control rod worths may be calculated for either a bare or a radially reflected core. Additional input required includes the nuclear properties of the radial reflector.

1. 6. 4 Scram IV⁽²³⁾

Scram IV is an improved method for calculating the gross rod worth of a central cylindrical control rod, a ring of M rods, or a central rod plus the ring of M rods, in a bare cylindrical reactor. The theory was developed by R. L. Murray.⁽²⁴⁾ The rods may have different radii, different material properties and may be in an asymmetric array (different distances from the central axis and different angles between the rods). The code is applicable for a ring of up to a maximum of ten rods with a central rod and up to a maximum of seven rods in a ring when no central rod is used.

The Scram I, II, and III codes consist essentially of determining a fictitious number of neutrons produced per fission, ν_c , which will make the reactor critical. This number is changed by successive iterations until the criticality condition with rods inserted is satisfied. This method neglects changes in the resonance escape probability and neutron age as the number of neutrons produced per fission is changed. The Scram IV code iterates on loading until the critical determinant is zero. Input to the code includes tables of core nuclear parameters as a function of loading for the particular reactor under consideration.

The code was developed during the past year but has not been completely debugged and checked out. However, the theory involved has been applied to certain critical mass studies for the APPR reactor made at Oak Ridge National Laboratory.⁽⁵⁾ The agreement of calculated and measured rod worths was within ten percent.

1.7 CALCULATION OF INTEGRATED NEUTRON FLUX (nvt) ABOVE 1 MEV- ONE FAST GROUP METHOD⁽²⁵⁾

An estimate of the total integrated neutron flux (nvt) above 1 Mev is of considerable interest in the design of various reactor components, including reflectors, thermal shields, flow divider, and pressure vessel; a great deal of radiation damage data is related to the nvt above 1 Mev. Therefore, it is desirable to have available a method of calculation which yields fairly reliable results. This section describes a method based on theory developed in WAPD 45⁽²⁶⁾ for water moderated systems. This theory yields values of nvt which are slightly lower than that calculated by the use of two or more fast groups, as described in Section 7.10.1; however, the results are useful when a quick estimate of nvt is needed.

If elastic and inelastic scattering by heavy nuclei are neglected, the integral equation determining the fast flux can be written as:

$$\phi_H(E) \sum_H(E) = S(E) + \int_{E'=E}^{\infty} \sum_H(E') \phi_H(E') \frac{dE'}{E'}$$

where:

$\phi_H(E)$ = purely proton moderated flux per unit energy range.
(neuts/cm²-sec-Mev)

$\sum_H(E)$ = $N_H \sigma_H$ = proton macroscopic scattering cross section
as a function of energy. (cm⁻¹)

N_H = space mean proton number density (protons/cm³)

σ_H = microscopic proton scattering cross section. (cm²)

$S(E)$ = number of neutrons born per unit energy range. (neuts/cm³-sec-Mev).

This equation vigorously describes the fast flux, $\phi_H(E)$, in a purely proton moderated reactor. The source term, $S(E)$ may be taken as the fission spectrum, $F(E)$. The solution of the fast flux equation may be written as:

$$\phi_H(E) = \frac{\nu R_f}{N_H} \left[\frac{F(E)}{\sigma_H(E)} + \frac{1}{E \sigma_H(E)} \int_E^{\infty} F(E') dE' \right]$$

$$R_f = \delta P/V$$

where:

P = thermal power output of the core (watts)

ν = 2.46 = neutrons produced per fission

δ = 3.2175×10^{10} fissions per watt-sec

V = volume of active core (cm^3)

$F(E)$ = U-235 fission neutron energy spectrum normalized to unity.

R_f = space mean rate of fissioning in the reactor (fissions/ cm^3 -sec)

The multiplying constant, $\nu R_f/N_H$, is determined by the parameters of the particular reactor under consideration. The bracketed factor is a universal function of energy depending only upon the normalized fission spectrum and the proton cross section. The fission spectrum $F(E)$ may be written in a form suggested by B. E. Watt⁽²⁷⁾

$$F(E) = 0.475 \sinh \sqrt{2E} \times e^{-E}$$

The values of $F(E)$ and the integral fission spectrum, $\int_E^{\infty} F(E') dE'$

as a function of energy are tabulated in WAPD 45. (26) Analytical approximations to $\phi_H(E)$ were obtained over the whole energy range in this reference and are tabulated in Table 1.1.

TABLE 1. 1
ANALYTIC APPROXIMATIONS TO $\phi_H(E)$

Energy Range (Mev)	Analytic Approximation
Thermal $< E < 10^{-3}$	$4.762 \times 10^{22} (\sqrt{R_f/N_H} E)$
$10^{-3} < E < 10^{-2}$	$5.714 \times 10^{22} (\sqrt{R_f/N_H} E^{0.976})$
$10^{-2} < E < 10^{-1}$	$1.080 \times 10^{23} (\sqrt{R_f/N_H} E^{0.838})$
$10^{-1} < E < 1$	$2.369 \times 10^{23} (\sqrt{R_f/N_H} E^{0.497})$
$1 < E < 5$	$3.988 \times 10^{23} (\sqrt{R_f/N_H}) e^{-0.523 E}$
$5 < E < \infty$	$7.708 \times 10^{23} (\sqrt{R_f/N_H}) e^{-0.654 E}$

The total purely proton moderated flux

$$\phi_H = \int_{\text{thermal}}^{\infty} \phi_H(E) dE$$

can be computed by integrating over the analytic approximations. For all energies above 1 Mev, the purely proton moderated flux becomes:

$$\phi_H(> 1 \text{ Mev}) = 3.490 \times 10^{34} \left(\frac{P}{VN_H} \right) \text{ neutrons/cm}^2\text{-sec}$$

For a reactor operating at full power for t seconds, the integrated flux above 1 Mev becomes:

$$nvt(> 1 \text{ Mev}) = \phi_H(> 1 \text{ Mev}) t$$

It has been assumed in this derivation that the core is infinite and homogeneous. Therefore, the resulting spectrum should thus approximate that existing near the center of the core. In the usual case, the value of nvt above 1 Mev (or above any energy) is desired outside the core, such as at the inside surface of the pressure vessel. By multigroup diffusion calculations, (VALPROD), it has been shown in reference (28) and Section 7. 10. 1 of this report, that the fast distribution above 1 Mev has nearly the same shape as the total fast flux. Therefore, by a two-group VALPROD calculation, the value of the fast flux above 1 Mev may be found anywhere inside or outside of the core from the attenuation of the total fast flux:

$$\phi_H(R) = \frac{\phi_f(R)}{\phi_f(0)} \left[\phi_H(0) \right]$$

where:

$\phi_H(R)$ = fast flux above 1 Mev at any radial distance R.

$\phi_H(0)$ = fast flux above 1 Mev at center of core as calculated by the methods described in this section.

$\phi_f(R)$ = relative fast flux at any radial distance R as determined by a two-group VALPROD calculation.

$\phi_f(0)$ = relative fast flux at center of core as determined by a two-group VALPROD calculation.

In addition, a correction for the axial flux distribution may be made by multiplying the equation for ϕ_H by the average value of the maximum to average axial flux during the lifetime of the core.

The simple relation developed for ϕ_H in this section gives good agreement to more exact calculations made by use of multigroup (two fast groups) VALPROD's; this is described in Section 7.10.1 of this report.

1.8 TRANSIENT XENON CALCULATIONS

The chief source of changes in the amount of poisoning during reactor transients is the build-up of xenon. Xenon is formed in a reactor by the radioactive decay of the fission product I^{135} as well as being a direct fission product. If the reactor operates at a constant power for two or three days, an equilibrium state will occur in which the amount of xenon produced will equal that removed by neutron capture and radioactive decay. Due to the different decay rates and cross sections of xenon and iodine, when the reactor power is reduced, there is a period of time during which more xenon is produced by the decay of iodine than is removed by the flux. The analysis of the build-up and decay of xenon has been performed on an analog computer, using a model described in APAE No. 38. (29) The basic equations employed are derived as follows.

The concentrations of I^{135} and Xe^{135} in the core at any time t may be described by:

$$\begin{aligned}\frac{dN_I}{dt} &= \frac{\gamma_1 \sum_{\text{core}}^f \phi}{\beta} - \lambda_I N_I \\ \frac{dN_{Xe}}{dt} &= \frac{\gamma_2 \sum_{\text{core}}^f \phi}{\beta} + \lambda_I N_I - \sigma_{xe}^a \phi N_{Xe} - \lambda_{xe} N_{Xe}\end{aligned}$$

where:

- N_I = atomic concentration of I^{135} in the core (atoms/cm³)
 N_{xe} = atomic concentration of Xe^{135} in the core (atoms/cm³)
 γ_1 = cumulative fractional yield of I^{135} as a fission product
 γ_2 = fractional yield of Xe^{135} as a direct fission product
 λ_I = radioactive decay constant of I^{135} (sec⁻¹)
 λ_{xe} = radioactive decay constant of Xe^{135} (sec⁻¹)
 ϕ = average thermal flux within the core (neuts/cm²sec)
 β = ratio of thermal fissions to total number of fissions
 $\sum_{core}^f$ = macroscopic fission cross section of the core at operating temperature with no xenon (cm⁻¹)
 σ_{xe}^a = thermal microscopic absorption cross section of Xe^{135} (cm²)

The average thermal flux inside the core is related to the thermal power by:

$$\phi = \frac{\beta \delta P}{V \sum_{core}^f}$$

where:

- P = thermal power of the core in watts
 δ = fissions per watt-sec
 V = volume of the core

The reactivity is computed as a function of xenon absorption by the following relationship:

$$\rho_{xe} = -K_1 \sum_{xe}^a$$

and

$$\sum_{xe}^a = \alpha \sigma_{xe}^a N_{xe}$$

where:

K_1 = proportionality constant relating the change of reactivity with $\sum_{xe}^a$ from modified two-group theory.

α = weighting factor to account for the non-uniform distribution of Xe^{135} in the core.

Since the xenon is included as a uniform poison in the reactivity calculations, it is necessary to include a weighting factor, α , because of the actual non-uniform distribution of the poison. Investigations show that α varies not only with core lifetime but also during the transient condition. To determine the value of α that would best describe the change in reactivity throughout the life of the core, the reactivity calculated by the analog computer is converted into the change in rod bank position by using an average differential bank worth. The value of the distribution factor is then varied until the results are in good agreement with experimental data. The equation for the change in bank position due to xenon is:

$$\Delta(\text{bank position}) = -K_2 \rho_{xe}$$

where K_2 is the proportionality constant indicating bank worth for a particular bank position. To account for the change of α during the transient, a linear variation is assumed. In this report, experimental data obtained from the SM-1 is used to determine the magnitude of the changes in α during the transient.

1.9 HEAT GENERATION RATES

The heat generation rates in the thermal shields and pressure vessel of the SM-2 core were calculated in this report by use of the ROC shielding codes. (30) These codes calculate the core attenuation coefficient and equivalent core gamma volumetric source values for a specified core, then calculate the activation gamma source data in the shield, calculate the gamma dose rates in the shield due to gammas arising from activation of shield materials, and finally calculate the gamma dose rates in the shield due to gammas arising in the core. Gamma photon source values are obtained on the basis of two-group neutron flux distributions throughout the reactor core and shield. Basic input information consists primarily of material and geometry data plus normalized two-group neutron flux values. Output consists of five different energy groups of heating rates due to five distinct gamma groups.

1.10 SURFACE VS. VOLUME DISTRIBUTION OF BURNABLE POISON

The burnable poison (boron), as used in the SM-1 and SM-2 reference fuel plates, is contained within the meat of each fuel plate. The poison, UO_2 , and stainless steel compose a uniform structure which is bounded by a stainless steel "picture frame" and stainless steel cladding. In the SM-2 critical experiments, the poison is introduced in the form of boron-impregnated Mylar tapes which are glued to the surface of the fuel plates. The effect of a surface distribution as opposed to a volume distribution of burnable poison upon the thermal utilization and reactivity of the core was calculated by theory developed by R. L. Murray. ⁽³¹⁾ The theory will not be reproduced here but the working formulas are presented in a form convenient to the SM-2 analysis. The notation used is consistent to that employed in the Medusa (Program 53), IBM-650 code.

$$(\sum_a)_B = \frac{(N_3 y_o) N_A \bar{\sigma}_a^{B-10}}{(A_{B-10})(N_3 t_T)}$$

$$(\sum_a)_P = N_P^{25} \bar{\sigma}_a^{25} + N_P^{28} \bar{\sigma}_a^{28} + N_P^{SS} \bar{\sigma}_a^{SS} + N_P^O \sigma_a^O$$

$$(\sum_a)_{U-235} = \frac{Z_o N_A \bar{\sigma}_a^{25}}{(A_{U-235})V_m}$$

where:

N_3 = number of Mylar tapes per fuel plate

y_o = areal density of B-10 upon each Mylar tape (gms B-10/cm²)

N_A = Avagadros number (atoms / mol)

t_T = thickness of each Mylar tape strip (cm)

Z_o = gms. U-235 per fuel plate

V_m = volume of meat per plate (cm³)

A_{B-10} = atomic weight of B-10 (gms/mol)

A_{U-235} = atomic weight of U-235 (gms/mol)

$\bar{\sigma}_a^X$ = microscopic absorption cross section for $X = \text{U-235, U-238, oxygen, stainless steel and B-10 (cm}^2\text{)}$

N_P^X = atomic concentration of the elements in the fuel plates for
 $X = \text{U-235, U-238, oxygen, and stainless steel (atoms/cm}^3 \text{ of fuel plate)}$

$(\sum a)_B$ = macroscopic absorption cross section of B-10 (cm^{-1})

$(\sum a)_p$ = macroscopic absorption cross section of the fuel plate (cm^{-1})

$(\sum a)_{\text{U-235}}$ = macroscopic absorption cross section of U-235 (cm^{-1})

The effect upon the thermal utilization and reactivity is calculated by the following equations:

$$B = (\sum a)_B N_3 t_T$$

$$P = (\sum a)_p C$$

$$T = B + P$$

$$\frac{d}{d_0} = \frac{1 + 3 E_4(T)}{1 - 2 E_3(T)}$$

$$g = \frac{d}{d_0} \left[E_3(B) - E_3(P) \right] + \frac{3}{2} \left[E_4(B) - E_4(P) \right]$$

$$f_B = \frac{1}{2} (1 - g)$$

$$f_P = \frac{1}{2} (1 + g)$$

$$f_{BH} = B/T$$

$$f_{PH} = P/T$$

$$U = t_F (\sum a)_{\text{U-235}}$$

$$w = C/2$$

$$W = w + \frac{1}{2} \left[\frac{H' - N_1 C}{N_1} \right]$$

$$L_{\text{H}_2\text{O}}^2 = \frac{D_{\text{H}_2\text{O}}}{\sum a_{\text{H}_2\text{O}}} ; \quad K = \frac{1}{L_{\text{H}_2\text{O}}}$$

$$E = K(W - w) \coth K(W - w)$$

$$\frac{1}{f_{FP}} = (W - w) \left(\sum_a^{H_2O} \right) \left(\frac{2d}{d_0} \right) + E$$

$$f_U = f_{FP} f_P \left(\frac{U}{P} \right)$$

$$f_{UH} = f_{FP} f_{PH} \left(\frac{U}{P} \right)$$

$$\delta K_{eff} \cong \frac{\Delta F}{F} \cong \frac{f_{UH} - f_U}{f_{UH}}$$

where:

E_3, E_4 = Placzek functions tabulated in reference (32).

d = extrapolation distance from moderator (water) into the fuel plate.

d_0 = extrapolation distance into a void or black medium.

f_B = boron layer thermal utilization.

f_P = fuel plate thermal utilization.

f_{BH} = thermal utilization of boron in the homogeneous mixture of boron and fuel plate.

f_{PH} = thermal utilization of fuel plate in the homogeneous mixture of boron and fuel plate.

w = one-half the thickness of the fuel plate.

W = one-half of the sum of the plate thickness and the water channel thickness.

H' = the width of the entire fuel element (2.9375 in. for the SM-1 and SM-2).

N_1 = number of fuel plates per element.

D^{H_2O} = thermal diffusion coefficient for pure water.

$\sum_a^{H_2O}$ = thermal macroscopic cross section for pure water.

f_{FP} = fuel plate thermal utilization of the cell for boron on the side of the plate.

- f_u = true thermal utilization for boron on side of the plate,
defined by (absorptions in U/total absorptions)
- f_{UH} = true thermal utilization for boron homogeneously mixed,
defined by (absorptions in U/total absorptions).
- C = thickness of fuel plate, meat plus cladding (cm)

This theory was derived using diffusion theory in the moderator and attenuation in the fuel region in accordance with exponential reduction of the incident angular flux distribution; however, in reality, the neutrons entering the fuel plate are also subject to scattering. It would be expected that improvements in precision would be obtained by inclusion of at least the effect of once-scattered neutrons. This subject is presently under investigation by R. L. Murray.

It should be emphasized that the above derivation has taken no account of differences in the mode in which the boron is introduced in the two cases. If the particles distributed through the meat exhibit self-shielding by virtue of their large size, corrections must also be computed.

DATE	LET.	REVISION RECORD	AUTH.	DR.	CHK.
5-20-59	A	347 WAS 304L	J.K.

Figure 1.1

APPR-1b

ALCO		ALCO PRODUCTS, INC.			
		ATOMIC ENERGY DEPT.			
		SCHENECTADY, N. Y., U. S. A.			
SCALE	2x SIZE	REP.	DR.	12-3-58	
MATERIAL SPEC.		CLAD-TYPE 347 -S.S.	TR.		
		FRAME-TYPE 347 -S.S.	CHK.		
		CORE-AS PER SPEC.	APPR.	17	12-3-58
			APPR.		
			MET.		
NAME STATIONARY FUEL ELEMENT-CROSS SECTION.					
PART NO. AES-275					

BREAK SHARP EDGES
REMOVE ALL BURRS

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES.
TOLERANCES ON FINISHED
FRACTIONAL DIMENSIONS
TO BE ±

- ✓ FINISH AS INDICATED IN MICROINCHES.
- (/1) MACHINE FINISH - ROUGH
- (/10) FLAME CUT OR SAW

PART NO. **AES-275**

DATE	LET	REVISION RECORD	AUTH.	DR.	CHK.
7/1/59	A	2834 was 2800	JPC		
		Dimensions 2790	JPC		
		Removed	JPC

NOTE:
ACTIVE CORE MUST FALL WITHIN
DIMENSIONS MARKED "Z".

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES
TOLERANCES ON FINISHED
FRACTIONAL DIMENSIONS
TO BE 8

✓ FINISH AS INDICATED
IN DIMENSIONS
/1 MACHINE FINISH - BRUSH
/10 FLAME CUT OR SAW

ALCO		ALCO PRODUCTS, INC.	
		ATOMIC ENERGY DEPT.	
		SCHENECTADY, N. Y., U. S. A.	
SCALE	FULL	DR.	J. STAH 1-12-59
MATERIAL SPEC.	CLAD-TYPE 347 SS.	TR.	
	FRAME-TYPE 347 SS.	CHK.	
	CORE-AS SPECIFIED	APPR.	
		APPR.	
		NET.	19140 2-17-59
NAME FUEL PLATE, INNER			
(WELDED STATIONARY ELEMENT)			
PART NO.			
C9-13-2099			
AE-31-1PE

ONE QUARTER OF A FIXED ELEMENT

DATE	LET.	REVISION RECORD	AUTH.	DR.	CHK.

Figure 1.4

APPR-1 b

ALCO		ALCO PRODUCTS, INC. ATOMIC ENERGY DEPT. SCHENECTADY, N. Y., U. S. A.			
SCALE 2 X SIZE	REF.	DR.	French	12-3-58	
MATERIAL SPEC.		TR.			
CLAD-TYPE 347 -S.S.		CHK.			
FRAME-TYPE 304L-S.S.		APPR.	JFC	12/5/58	
CORE-AS PER SPEC.		APPR.			
		MET.			
NAME CONTROL ROD FUEL ELEMENT-CROSS SECTION					
PART NO.		AES-276			

BREAK SHARP EDGES
REMOVE ALL BURRS

UNLESS OTHERWISE SPECIFIED
DIMENSIONS ARE IN INCHES.
TOLERANCES ON FINISHED
FRACTIONAL DIMENSIONS
TO BE ±

- ✓ FINISH AS INDICATED IN MICROINCHES.
- (/1) MACHINE FINISH - ROUGH
- (/10) FLAME CUT OR SAW

CONTROL ROD FUEL ELEMENT

CLOSED SEVEN ROD ARRAY

(CONTROL RODS A,B,C,D,E,F,G)

OPEN SEVEN ROD ARRAY

(CONTROL RODS A,B,C,D,E,F,G)

Figure 1.8

2.0 NUCLEAR SURVEY TO DETERMINE REFERENCE LOADING #1

An extensive nuclear parameter survey was made to determine K_{eff} as a function of U-235 and B-10 loadings. The primary purpose of the survey was to aid in the selection of a reference loading for the SM-2 core. The survey was conducted on the basis of 45 fixed fuel elements having identical geometry as those used in the SM-1 core except that the meat was increased from 20 to 30 mils in thickness; the 30 mil meat was selected for metallurgical reasons, primarily to keep the weight percent of UO_2 in the meat approximately the same as in the SM-1. It was assumed that the various loadings would be achieved by adjustment of the stainless steel to UO_2 ratio in the meat of the plates. At the time of this survey, the mean coolant temperature was 440° F at a pressure of 1200 psia.

2.1 CALCULATIONAL MODEL

The calculations were performed using the modified two-group theory as described in Section 1.1. The SM-1 values of reflector savings were used;⁽¹⁾ therefore, the survey results are applicable to a water reflector core. The fast group parameters were calculated by the MUFT-III code using the P-1 Selengut-Goertzel slowing down approximation. Fifty-nine and fifty-eight neutron energy groups were used for the cold and hot cases respectively. The thermal properties were calculated by use of P_3 theory for APPR-type plate elements. The input to the latter two codes were facilitated by use of Programs 50 and 51.

Two "corrections" were applied to the calculation results. The first correction is the effect of replacing fixed fuel elements by control fuel elements; the latter contain less fuel, thereby causing a decrease in reactivity. The substitution effect was approximated by first calculating a thermal absorption cross section, $\sum_{sub}^a$, for the SM-1, which when added to the total absorption cross section of the core, yields a value of K_{th} such that the calculated K_{eff} (by the equivalent bare model) agrees with the actual measured value for a mixed element core. The effect at 440°F was calculated by assuming that $\sum_{sub}^a$ varies with temperature in the same manner as the U-235 cross section. This method is outlined in reference (2) and yields the following values for the SM-1:

$$\sum_{sub}^a \text{ (SM-1) } = 0.007846 \text{ cm}^{-1} @ 68^\circ\text{F}$$

$$\sum_{sub}^a \text{ (SM-1) } = 0.006131 \text{ cm}^{-1} @ 440^\circ\text{F}$$

It was assumed in this reference that $\sum_{\text{sub}}^a$ varies with temperature in the same manner as the U-235 cross section.

For the SM-2, it was assumed that $\sum_{\text{sub}}^a$ varies with loading in the same manner as the sum of the macroscopic thermal absorption cross section of U-235 and B-10. Therefore:

$$\sum_{\text{sub}}^a (\text{SM-2}) = \sum_{\text{sub}}^a (\text{SM-1}) \left(\frac{\sum_a (\text{SM-2})}{\sum_a (\text{SM-1})} \right)$$

$$\sum_{\text{sub}}^a (\text{SM-2}) = 0.032145 \left[\sum_a (\text{SM-2}) \right] @ 68^\circ\text{F}$$

$$\sum_{\text{sub}}^a (\text{SM-2}) = 0.033283 \left[\sum_a (\text{SM-2}) \right] @ 440^\circ\text{F}$$

The K_{eff} for a mixed element core was then expressed as:

$$K_{\text{eff}}' (\text{mixed}) = G K_{\text{th}}' + H$$

$$K_{\text{th}}' = \frac{\nu \sum_f}{\sum_a + \sum_{\text{sub}}^a}$$

$$G = \frac{P}{(1 + \tau B^2)(1 + L^2 B^2)}$$

$$H = \frac{(1-P) K_f}{(1 + \tau B^2)}$$

The second "correction" applied was a "model" correction to compensate for the difference between the calculated and measured values of reactivity. At the time of this survey, the differences between the measured and calculated values of the reactivity of the SM-1 core are given in Table 2.1. These were taken as the model correction factors, ρ_c . These factors were assumed to be applicable to the SM-2. Actually, it is now known that the calculated values were in error due to the following:

- (1) The actual boron content of the SM-1 core is approximately 22.4%⁽³⁾ below the specified values⁽⁴⁾ (5) due to losses during fabrication.
- (2) The elastic and inelastic scattering cross section files of stainless steel, as used in the MUFT-III calculations, have since been revised.

The final form of K_{eff} for the mixed element core was then expressed by:

$$K_{eff}'' = \frac{K_{eff}'}{1 - \rho_c K_{eff}'}$$

TABLE 2.1
MODEL CORRECTION FACTOR, ρ_c

Core Condition	T = 68°F	T = 440°F
Clean Core	0.0181	0.0056
With Equilibrium Xenon	-----	0.0030

2.2 RESULTS OF THE SURVEY

The results of the loading survey are shown in Fig. 2.1 for the core at 440°F, 1200 psia, and equilibrium xenon. The loading for the mixed core may be obtained by multiplying the loading for all fixed elements by 0.0956.

2.3 SELECTION OF A REFERENCE LOADING

Previous burnout analysis (6) performed during the SM-2 Core Development Planning stage had shown that a loading of 34 kg U-235 and 70 gm of B-10 in 30 mil plates was sufficient for a core life of 25 MWYR's. It was found that by increasing the plate thickness to 40 mils requires an additional 3.5 kg of U-235 to overcome the poisoning effect of the additional steel. This total loading of 37.5 kg U-235 in all fixed elements was selected as the reference loading. In order to minimize the reactivity to be controlled, the design objective was to poison the core out to obtain a K_{eff} of 1.03 at 440°F with equilibrium xenon. Figure 2.2 is a plot of K_{eff} vs. B-10 loading for a U-235 loading of 37.5 kg in all fixed elements. It is seen that 65.6 gms of B-10 yields the desired K_{eff} of 1.03. The B-10 loadings in Fig. 2.1 and 2.2 are based upon 18.8 atom percent B-10 in natural boron. More recent data (7) indicates that the abundance of B-10 in natural boron is more nearly 19.8 a/o. Therefore, the required B-10 loading becomes 69.089 gm for 45 fixed elements. The reference core loading for 7 control rod fuel elements and 38 fixed elements becomes 36.21 kg U-235 and 66.71 gms B-10, designated reference loading #1 in this report. It will be found in succeeding section of this report that design changes will necessitate changes in the boron loading; however, the U-235 loading is kept constant. The boron loading is the "effective" nuclear loading and does not reflect effects of particle self-shielding. (Section 7.7).

The core parameters for the reference loading as calculated by the MUFT-III and P₃ theory IBM-650 codes are tabulated in APAE Memo 160⁽⁸⁾ and are repeated in Tables 2. 2, 2. 3, and 2. 4. The dimensions and element geometry used to obtain these parameters are tabulated in Table 2. 5. It must be emphasized that these parameters are not to be considered valid for the final reference core. A tabulation of a later set of parameters is given in Section 6. 1.

TABLE 2. 2
REFERENCE CORE #1 - TOTAL CORE LOADINGS FOR 45 ELEMENTS

	<u>All Fixed Elements</u>	<u>Reference Core</u>
U-235 Loading, kgs	37. 50	36. 2095
UO ₂ Loading, kgs	45. 98	44. 3977
B-10 Loadings, gms	69. 08814	66. 71126

TABLE 2. 3
REFERENCE CORE #1 - INITIAL ATOMIC CONCENTRATIONS
(atoms/cm³) IN REFERENCE CORE
(SM-2 Fixed Elements)

<u>Atom</u>	<u>T = 68°F</u>	<u>T = 440°F</u>
H	0. 049264 x 10 ²⁴	0. 041200 x 10 ²⁴
O	0. 026105 x 10 ²⁴	0. 022073 x 10 ²⁴
SS	0. 017977 x 10 ²⁴	0. 017977 x 10 ²⁴
B-10	0. 002968 x 10 ²²	0. 002968 x 10 ²²
Nat. Boron*	0. 014989 x 10 ²²	0. 014989 x 10 ²²
U-235	0. 068620 x 10 ²²	0. 068620 x 10 ²²
U-238**	0. 005045 x 10 ²²	0. 005045 x 10 ²²

* Based on 19. 8 a/o B-10 in natural boron.

** Based on 87. 63 w/o U in UO₂ and 93. 07 w/o U-235 in U.

TABLE 2. 4
REFERENCE CORE #1 - REFERENCE CORE PARAMETERS
(SM-2 Fixed Elements; O Burnup)

Parameter		T = 68°F	T = 440°F
D_f	(cm)	1. 287671	1. 443776
Σ_R	(cm ⁻¹)	0. 015453	0. 014077
K_f		1. 274928	1. 270122
P		0. 610063	0. 583850
τ	(cm ²)	32. 491744	42. 680694
D	(cm)	0. 163136	0. 230765
Σ_a^*	(cm ⁻¹)	0. 417072	0. 333169
Σ_s	(cm ⁻¹)	2. 000734	1. 478375
Σ_{tr}	(cm ⁻¹)	2. 043290	1. 444472
$\nu \Sigma_f$	(cm ⁻¹)	0. 604880	0. 484474
L^2	(cm ²)	0. 391145	0. 692636
Σ_{xe}^a	(cm ⁻¹)	-----	0. 013367
Σ_{sub}^a	(cm ⁻¹)	0. 014106	0. 011055
K_{th}	(Fixed elements)	1. 450302	1. 454140
K_{th}	(Mixed elements)	1. 402854	1. 407438
β		0. 639608	0. 615228

* Does not include Σ_{xe}^a or Σ_{sub}^a

TABLE 2. 5
REFERENCE CORE #1 - SM-2 CORE DATA

	Fixed Element	Control Element
<u>Overall Core Data</u>		
Number of Elements	38	7
<u>Fuel Plate Dimensions</u>		
Meat width (in.)	2. 50	2. 281
Meat thickness (in.)	0. 03	0. 03
Active length (in.)	22. 0	21. 125
Active volume (in. ³)	1. 6500	1. 4456
Clad thickness (in.)	0. 005	0. 005
Plate thickness (in.)	0. 04	0. 04

TABLE 2.5 (CONT'D)

<u>Element Geometry</u>	<u>Fixed Element</u>	<u>Control Element</u>
Cell dimensions (in.)	2.9375 sq.	2.9375 sq.
Width of side plate (in.)	2.863	2.619
Thickness of side plate (in.)	0.05	0.05
Width of fuel plate (in.)	2.78	2.56
Fuel plates per element	18	16
Volume fraction of water	0.7383	--
Metal to water ratio	0.3545	--
<u>Plate Loadings</u>		
U-235/plate (gm)	46.296	40.561
UO ₂ /plate (gm)	56.765	49.733
B-10/plate (gm)	0.08529	0.07473
B ₄ C/plate (gm)*	0.64605	0.56602
SS/plate (gm)	144.697	126.772
Wt. % UO ₂ in meat (%)	28.087	28.087
Wt. % B ₄ C in meat (%)	0.319	0.319
Wt. % SS in meat (%)	71.594	71.594

* Based on 75.9 w/o B in B₄C and 19.8 a/o B-10 in natural boron.

2.4 CRITICAL MASS VS. CORE METAL TO WATER RATIO

Since it was expected that several design changes would be made before a final reference core was selected, critical masses were calculated as a function of changes in the number of plates per element, clad and meat thicknesses, meat width, and side plate thickness. The nuclear properties were prepared by the use of Program 50, MUFT-III, and Plate Type P₃ theory. K_{eff} was calculated for each case by the use of Program 51. From cross plots of K_{eff} vs. loading for various core metal-to-water ratios, the critical masses were determined for K_{eff} = 1.0. The calculations were performed at 440°F, 1200 psia, with equilibrium xenon, at 0 MWYR. The results of these calculations are given in Fig. 2.3, in which critical mass is plotted as a function of core metal-to-water ratio. The graph indicates that the critical mass at a given temperature is primarily dependent upon the metal-to-water ratio. Apparently, the critical mass is only slightly affected by the manner in which metal-to-water changes are made. Table 2.6 is a tabulation of the various cases studied. Table 2.6 and Figure 2.3 are not intended to be considered precise, but are useful in estimating the effect of making various design changes.

TABLE 2.6
CRITICAL MASSES (Kg U-235)

T = 440°F; P = 1200 psia.; Critical Mass Shown for 45 Fixed Elements)

$$\left[\frac{\text{Mixed Element Loading}}{\text{Fixed Element Loading}} = 0.9656 \right]$$

Plates Per Element	Clad Thickness (mils)	Meat Thickness (mils)	Meat Width (in.)	Side Plate Thickness (mils)	C. M. O MWYR Hot Clean	C. M. O MWYR Eq. Xenon	C. M. 25 MWYR Eq. Xenon	Metal-to- Water Ratio
18	5	20	2.50	50	10.5	11.70	13.65	0.2573
18	5	30	2.50	50	13.0	14.45	16.65	0.3545
20	5	30	2.50	50	-	15.825	-	0.4028
22	5	30	2.50	50	-	17.30	-	0.4546
18	5	30	2.50	25	-	13.57	-	0.3234
18	5	30	2.50	40	-	14.08	-	0.3411
18	7	30	2.50	50	-	15.75	-	0.3977
18	7	40	2.50	50	-	19.20	-	0.5185
18	5	30	2.55	25	-	13.70	-	0.3298

2.5 CORE TEMPERATURE VS. LIFETIME

The lifetime of the reference core, presented in Section 2.3, was calculated to be 28.8 MWYR as described in Section 4.0. This lifetime is based upon an average core temperature of 440°F. However, for thermodynamic reasons, the mean coolant temperature of the SM-2 was being considered to be increased over that in the SM-1. An estimate of the variation of lifetime with core temperature can be made with a knowledge of the effective mass utilization (MWYR/kg U-235) and the variation of critical mass with core temperature.

An estimate of the mass utilization may be made from an analysis of SM-1 data. The initial core loading of the SM-1 is 22.5 kg U-235. The estimate of critical mass at end of core life is approximately 12.5 kg U-235; therefore, the fuel available for burnup is 10 kg. The best estimate at the time of this report, of the SM-1 core life is 15 MWYR, or an actual burnup of 7.5 kg, based upon extrapolated data of bank position vs. total energy release.⁽⁸⁾ Therefore, the mass utilization is 1.5 MWYR per kg U-235. Burnup, as used in this report, includes both fission and capture by U-235.

The SM-2 critical mass was determined as a function of temperature by reevaluating the nuclear parameters at core temperatures of 440°F (1200 psia), 480°F (1600 psia), and 510°F (2000 psia). By use of Program 51, K_{eff} was calculated for various loadings for each of the three core temperatures. From cross plots to determine $K_{eff} = 1$, critical mass as a function of temperature was determined. These results are given in Table 2.7.

TABLE 2.7
CRITICAL MASS VS. AVERAGE CORE TEMPERATURE
(0 MWYR; 45 Fixed Elements)

System Temperature	System Pressure (psia)	Critical Mass (Kg U-235*)	
		Hot Clean	Eq. Xenon
440°F	1200	13.0	14.45
480°F	1600	13.41	14.95
510°F	2000	13.75	15.40

* Critical mass for mixed element core is 0.9656 times the all fixed element loading.

Using the "realistic" mass utilization of 1.5 MWYR per kg U-235 and an "expected" lifetime of 28.8 MWYR at 440°F, the following equation is obtained:

$$\text{Lifetime (MWYR)} = 28.8 - 1.5 \left[M(T) - M(440^\circ\text{F}) \right]$$

where: $M(T)$ = critical mass at any temperature T

$M(440^\circ\text{F})$ = critical mass at 440°F

The results of this calculation are given in Fig. 2.4 in which the estimated core lifetime is plotted as a function of the average core temperature. The loss in core lifetime will be partially compensated for, however, since higher system temperature will result in a more efficient thermodynamic system. From the results of an optimization study performed from the thermal and economic standpoint, the decision was reached to change the system operation from 440°F and 1200 psia, to 510°F and 2000 psia. (9) From Fig. 2.4, this reduces the expected lifetime from 28.8 MWYR to about 27.4 MWYR.

Figure 2.1

Figure 2.3

3.0 STEEL VS. WATER REFLECTOR

In an SM-1 type core with a water reflector, there is considerable thermal flux and power peaking at the edge of the core. ⁽¹⁾⁽²⁾ The reflector slows down a large number of the fast neutrons which escape from the core and return slow neutrons to the core. Since the diffusion length for thermal neutrons is relatively small, most of the neutrons reflected back into the core are absorbed within a distance of 1 in. from the core-reflector boundary. This peaking results in high fuel burnup in the fuel plates near the core edge and requires a higher coolant flow rate to avoid boiling.

Experiments ⁽¹⁾⁽³⁾ and calculations ⁽⁴⁾⁽⁵⁾ have shown that a steel reflector, either laminated or solid, placed around the SM-1 core, can essentially eliminate or substantially reduce the peaking effects at the core edge. The elimination of this peaking is especially important in the SM-2 since the power and flow requirements are quite stringent. Therefore, the effects of various thicknesses of steel placed at various distances from the core were studied.

3.1 CALCULATIONAL MODEL

The effect of a steel reflector upon flux and power distribution and upon the reactivity of the core was calculated by a series of VALPROD calculations. To test the validity of the model, calculations were compared with measurements ⁽⁴⁾ made during the Zero Power Experiments on the SM-1. ⁽¹⁾ Comparisons of measured and calculated flux distributions are shown in Fig. 3.1 through 3.3 for various geometries. The agreement is poorest within the core where the presence of the control rods distorts the flux distribution.

The actual core configuration was represented by an equivalent cylinder. The reflector was represented by annular rings of steel and water. For calculations on the SM-2 core, the core properties were taken from Table 2.4. The properties for water and stainless steel which were used are tabulated in Table 3.1. The effect of a flow divider is discussed in Section 6.3.

TABLE 3. 1
NUCLEAR PROPERTIES USED IN REFLECTOR EVALUATION
T = 440°F; P = 1200 psia)

Parameter	Water	Stainless Steel
D (cm)	0. 265809	0. 338232
D _f (cm)	1. 899871	1. 314459
Σ_a (cm ⁻¹)	0. 011014	0. 150037
τ (cm ²)	47. 8302	161. 6358
P	0. 98849	0. 000004
K _f	0	0
K _{th}	0	0

3. 2 EFFECT OF A STEEL REFLECTOR⁽⁶⁾

Figure 3. 4 shows the relative power distribution with an all-water reflector compared to the power distribution for the same core with no water gap and a 3/4-in. steel reflector. These two cases represent the extremes of power peaking for the cases studied and show the value of a steel reflector in suppressing the flux (or power) peaking at the edge of the core.

3. 2. 1 Effect of Water Gap and Slab Thickness

A survey was conducted for various thicknesses of steel slabs each separated by varying water gap thicknesses. The results of these calculations are shown in Fig. 3. 5, in which the relative power peak at the edge of the core is plotted as a function of slab thickness and water gap thickness. The effect upon core reactivity is shown in Fig. 3. 6. It is seen that the reactivity falls off rapidly for slabs up to about 1/4 in. thick. A net gain in reactivity requires steel reflectors of the order of 2-1/2 in. in thickness. ⁽⁷⁾

Figure 3. 5 indicates that the peak power at the core's edge is a function both of the water gap and slab thickness. As a basis for evaluation of the more optimum configuration, those cases were chosen which predicted a relative power of 0. 8 at the edge of the core. This leads to a number of configurations of water gap and reflector thicknesses which are "allowable" from thermal considerations. These results are shown in Fig. 3. 7.

3. 2. 2 Effect of Reflector Thickness and Metal-to-Water Ratio

As shown in the previous section, the loss in core reactivity due to a laminated steel reflector is a function of the slab thickness and the water gaps between the slab. A laminated steel reflector is less desirable from a mechanical and nuclear standpoint than a single solid steel reflector; however, a laminated reflector is required in order to reduce internal temperatures and thermal stresses to allowable limits. The thickness and spacing of the steel slabs are determined by core reactivity, thermal stress, and cooling requirements. An indication of the variation of core reactivity with the metal-to-water volume ratio in the laminated reflector may be seen in Fig. 3. 8. This calculation was performed during the SM-2 planning stage and is described in reference (7). The properties of the reflector were calculated as though the steel and water were homogeneously mixed. The fast constants were calculated by the MUFT-III code. It is seen that to gain maximum benefit from the reflector, the water gaps should be kept to a minimum, consistent with thermal and mechanical requirements. The effect of increasing the thickness of a typical laminated reflector is shown in Fig. 3. 9. It is shown that in order to avoid a loss of core reactivity, the reflector should be at least 2 in. or more in thickness. This variation of reactivity with thickness has been verified by experiment during the ZPE experiments on the SM-1⁽¹⁾ and in ANL experiments on a stainless steel core.

3. 3 SELECTION OF THE REFERENCE REFLECTOR

A thick laminated steel reflector was selected for the SM-2 principally on the basis of:

- 1) A possible reduction of radiation heating in the pressure vessel wall.
- 2) Improved power distribution.
- 3) Reactivity gain and consequently, a possible gain in core life.

The first characteristic is essential to employment of a relatively small stainless steel reactor vessel. Calculations involving a stainless steel vs. a carbon steel vessel are discussed in Section 7. 10. The actual configuration of the laminated reflector was determined by thermal calculations in which allowable reflector slab thickness was calculated as a function of inside diameter of the slab. The basis and description of these calculations are covered in reference (8). A diagram of the core and reflector configuration is shown in Fig. 3. 10.

CALCULATED AND MEASURED RADIAL
FLUX DISTRIBUTION FOR THE APPR-1
CORE (ZPE) WITH AN H₂O REFLECTOR.

Figure 3.1

CALCULATED AND MEASURED RADIAL FLUX DISTRIBUTION FOR APPR-1 CORE (ZPE) WITH A LAMINATED REFLECTOR NEXT TO CORE

**CALCULATED AND MEASURED RADIAL
FLUX DISTRIBUTION FOR APPR-1 CORE
(ZPE) WITH A LAMINATED REFLECTOR
PLACED 2 INCHES FROM CORE FACE.**

Figure 3.3

Figure 3.5

**ACCEPTABLE CONFIGURATIONS
FROM CONSIDERATION OF THE POWER
DISTRIBUTION**

Figure 3.7

REACTIVITY CHANGE (CENTS) VS. STEEL-WATER VOLUME RATIO IN A 5 INCH LAMINATED REFLECTOR

REACTIVITY CHANGE IN CENTS
VS. INCREASING THICKNESS
OF A LAMINATED STEEL
REFLECTOR OF M/W=12

Figure 3.9

Figure 3.10

LAYOUT			
SHEET NO. 1			
SHEET TOTAL 1			
MODEL	SM-2	DATE	11-1-54
REFERENCE		DATE	
AUTHORITY		DATE	
TITLE			
REACTOR CROSS SECTION			
LAYOUT NO.	AES-341		

4.0 NON-UNIFORM BURNOUT CALCULATION OF CORE⁽¹⁾

A one-shot, non-uniform burnup calculation of the reference core #1, described in Section 2.3, was performed. The core consisted of 45 fuel elements, including 7 control rods and was surrounded by a thick laminated steel reflector with a metal-to-water volume ratio of 6 to 1. The core considered was a one-pass core, containing no flow divider.

4.1 CALCULATIONAL MODEL

The calculation was performed by use of the NUB-1, Windowshade, and VALPROD, IBM-650 codes. The nuclear core parameters were calculated as a function of fuel burnup by the use of the Program 50, MUFT-III, and Plate Type P₃, IBM-650 codes. From previous analysis of the SM-1⁽²⁾, the rod bank position was found to be 0.91 in. below the Windowshade interface. Measurements on the SM-1 core indicated that the model underestimates the core reactivity by approximately 0.3% ρ at 440°F with equilibrium xenon.⁽²⁾ Therefore, a model correction of 0.3% was included. In addition, due to various uncertainties in determining lifetime by one-shot, non-uniform burnout calculations, a design factor of 94% was applied to the calculated values of energy release. This factor was based upon a calculated SM-1 lifetime of 16 MWYR and latest bank position measurements⁽³⁾ which indicated an extrapolated lifetime of 15 MWYR. This design factor was applied to the model corrected, bank position as a function of core energy release.

4.2 RESULTS OF CALCULATION

Figure 4.1 shows K_{eff} of the core as a function of average core burnup for both a uniform and non-uniform burnup calculations. The expected values at 68°F at startup and midlife are included for comparison. It is seen that the rapid burnup of B-10 during the first half of core life causes a large increase in reactivity. The maximum reactivity expected is 18.3%, occurring at approximately 22% fuel burnup.

Figure 4.2 shows the axial power distribution, both with and without equilibrium xenon, at the point of deepest bank insertion (at about 9.1 MWYR). It is seen that there is a large peaking at the bottom of the core. The presence of this peak has been verified experimentally.⁽⁴⁾ It was decided later to eliminate this peak by the use of Eu_2O_3 flux suppressors at the bottom of the fixed fuel plates. In addition, no correction for the rapid burnup of fuel in this region upon the power distribution has been included in the model employed. Figures 4.3 through 4.6 show the radial and axial distribution of U-235 and B-10 as a function of core energy release.

Figure 4.7 is a plot of calculated bank position vs. lifetime. It is seen that a corrected lifetime of 28.8 MWYR is indicated for this particular core.

There are several possible sources of error in determining lifetime by the one-shot, non-uniform burnout calculation; among these are:

- (1) Assumption of separability of fluxes and separability of axial and radial burnouts.
- (2) Assumption of constant flux distributions throughout life; however, the error due to this assumption is small.
- (3) The effect of replacing fixed elements by control fuel-elements was assumed to burnout as the fuel burnout.
- (4) Uncertainties in the fission product cross sections. The effect of the fission products was accounted by an equivalent amount of B-10 to yield the same poisoning effect. ⁽⁵⁾

In addition, later analysis and experimental data have revealed that the effect of the laminated steel reflector upon core reactivity has been overestimated by about 1% ρ . In addition, it has been found that due to boron losses during the SM-1 fabrication, calculations have been underestimating reactivity. In addition, the cross-sectional file for stainless steel, used in MUFT-III calculations, has been revised based on available experimental measurements; therefore, the "model correction" of $\sim 0.3\%$ would be modified.

4.3 MIDLIFE LOADINGS FOR THE SM-2⁽⁶⁾

The fuel element loadings of B-10 and U-235 for the SM-2 core at midlife were calculated for use in the SM-2 critical experiments. These loadings were based upon an operating temperature of 440°F while the latest reference core operates at 510°F; however, the calculations were considered sufficiently accurate for the initial mockup of the SM-2 core at midlife.

The 45 elements of the core were arranged into a series of 4 "rings". These "rings", the elements they contain, and the equivalent radius of each ring, are shown in Fig. 4.8. The concentrations of U-235 and B-10 at the calculated midlife of about 14.4 MWYR as a function of radial distance through the SM-2 core were obtained from results of the non-uniform calculations described in Section 4.2. Plots of $N^{25}(t)/N_0^{25}$ and $N^{B-10}(t)/N_0^{B-10}$ vs. radial distance for various core energy releases ⁽¹⁾ are shown in Fig. 4.3 and 4.5. Cross plots to obtain the distributions at 14.4 MWYR are shown in Fig. 4.9. The average concentrations of U-235 and B-10 at midlife in each "ring"

of elements were obtained by numerical integration, as follows:

$$\begin{aligned}
 \left[N(t)/N_0 \right]_{\text{avg}} &= \int_{R_0}^{R_n} \left[N(t)/N_0 \right]_i R_i dR \bigg/ \int_{R_0}^{R_n} R_i dR \cong \\
 &\left[\frac{\left[N(t)/N_0 \right]_0}{2} + \frac{\left[N(t)/N_0 \right]_n}{2} + \sum_{i=1}^{n-1} \left[N(t)/N_0 \right]_i R_i \right] \frac{\Delta R}{\frac{1}{2} (R_n^2 - R_0^2)}
 \end{aligned}$$

The results of these calculations are given in Table 4. 1. The initial (startup) loadings of U-235 (N_0^{25}) and B-10 (N_0^{B-10}) for the fixed and control fuel elements of the reference critical experiment core are given in Table 4. 2. It should be noted that the control element loadings for the critical experiment differ from the reference core as given in Table 2. 5; the elements of the latter are 3/4-in. shorter to allow for the use of Eu_2O_3 flux suppressors.

TABLE 4. 1
AVERAGE CONCENTRATIONS IN EACH "RING" OF ELEMENTS AT MIDLIFE

Ring	$\left[N^{25}/N_0^{25} \right]_{\text{avg}}$	$\left[N^{B-10}/N_0^{B-10} \right]_{\text{avg}}$
A	0. 709157	0. 149656
B	0. 729114	0. 174230
C	0. 775849	0. 250372
D	0. 853837	0. 440159

TABLE 4. 2
INITIAL (STARTUP) ELEMENT LOADINGS OF EXPERIMENTAL CORE

Element	U-235/Plate (Gm)	B-10/Plate (Gm)	U-235/Element (Gm)	B-10/Element (Gm)
Fixed	46. 296	0. 08529	833. 328	1. 53522
Control	42. 241	0. 07782	675. 856	1. 24512

Based upon Tables 4.1 and 4.2, the U-235 and B-10 loadings at midlife were calculated and are given in Table 4.3.

TABLE 4.3
MIDLIFE LOADINGS OF EXPERIMENTAL CORE

Ring	U-235/Element (Gm)		B-10/Element (Gm)	
	Fixed	Control	Fixed	Control
A	-	479.2880	-	0.18634
B	607.5950	-	0.26748	-
C	646.5408	524.3622	0.38438	0.31174
D	711.5308	-	0.67574	-

It should be noted that these calculations neglect the non-uniform axial burnup of fuel and burnable poison. However, as far as reactivity is concerned, neglect of the axial distribution will yield conservative results. Verification of this statement may be seen in Fig. 2.4 of reference (7) in which inclusion of the non-axial burnup distribution lowers the curve of K_{eff} vs. core energy release.

4.4 POWER SPIKE AT THE BOTTOM OF THE CORE (8)

The presence of the power spike at the bottom of the core has been verified by measurements made during the SM-2 Flexible Critical Experiments. (4) Measurements were taken by use of fission foils as a function of axial position in the core and have been compared to calculations made by use of the Windowshade model. The comparison is discussed in Section 9.0; it is found that the agreement is good with the analytical calculation slightly overestimating the power spike.

The power spike would impose severe flow requirements upon the core; therefore, it is desirable to either greatly suppress or eliminate it completely. From results of the non-uniform burnout calculation of the core, the gross axial power distribution was obtained as a function of lifetime. (1) Due to geometrical limitations in the Windowshade model, it is impossible to obtain a detailed description of axial fuel depletion, and hence, fine structure variation of power distribution with fuel burnup cannot be accurately predicted with these calculations. In an attempt to account for this greater rate of fuel depletion at the spike and its attendant reduction of specific power density, a local, one-shot, burnout calculation was performed for the bottom 2-1/2 in.

of the core. The "one-shot" assumption was made that:

$$[1 - B(Z)] = [1 - \overline{B}(\text{core})] A(Z)$$

where: $B(Z)$ = fuel burnout at axial position, Z .
 $\overline{B}(\text{core})$ = average axial burnout in the core
 obtained from NUB-1 calculations

α_{th} = thermal capture to fission ratio of U-235

α_f = fast capture to fission ratio of U-235

β = percent of thermal fissions

$$A(Z) = \left[\frac{(1+\alpha_{th}) \beta \bar{\phi}(Z) + (1+\alpha_f)(1-\beta) \bar{\phi}_f(Z)}{(1+\alpha_{th}) \beta + (1+\alpha_f)(1-\beta)} \right]$$

and where $\bar{\phi}(Z)$ and $\bar{\phi}_f(Z)$ are the axial, normalized fluxes as given in Fig. 4. 10. The Windowshade power distribution is calculated as:

$$P(Z) = \bar{N}_i^{25} \left[\bar{\sigma}_F^{25} \phi(Z) + \bar{\sigma}_{ff}^{25} \phi_f(Z) \right]$$

where: $\bar{N}_i^{25}$ = U-235 atom density in Windowshade region, "i"

$\phi(Z)$, $\phi_f(Z)$ = non-normalized Windowshade fluxes

As a first approximation, the local power may be expressed by:

$$P'(Z) = N^{25}(Z) \left[\bar{\sigma}_f^{25} \phi(Z) + \bar{\sigma}_{ff}^{25} \phi_f(Z) \right]$$

where now $N^{25}(Z)$ is the local atom concentration of U-235:

$$N^{25}(Z) = N_0^{25} [1 - B(Z)]$$

The local power may thus be related to the Windowshade distribution by:

$$P'(Z) = P(Z) \left[N^{25}(Z) / \bar{N}_i^{25} \right]$$

The quantities $\bar{N}_i^{25}$ and $P(Z)$ were obtained from results of the non-uniform burnout calculation described in Section 4. 2. The results of these calculations are given in Table 4. 4, in which the variation of the spike with core life may be seen. These results are given on the basis of the gross non-uniform burnout over the top 19-1/2 in. of the core with the local burnout calculation applied to the bottom 2-1/2 in. of the core. The local burnups corresponding to various average fuel burnups in the core are given in Table 4. 5.

TABLE 4.4
RESULTS OF GROSS AND LOCAL BURNOUT CALCULATIONS
(T = 440°F; With Eq. Xenon)

% Burnup	MWYRS	Fraction of Expected Life*	Calculated Bank Position (in.)	Spike Power	Max. Core Power	Spike/Core Max.
0	0	0	11.58	2.49	1.56	1.60
4.35	3.11	0.102	10.25	2.54	1.62	1.57
12.66	9.10	0.297	8.70	2.56	1.74	1.47
20.42	14.76	0.482	8.85	2.32	1.78	1.30
27.61	20.05	0.655	10.30	1.92	1.70	1.13
34.23	24.96	0.816	13.32	1.24	1.48	0.84
37.33	27.27	0.891	15.74	0.97	1.38	0.70

* Based on uncorrected bank curve in which expected lifetime is 30.6 MWYR.

TABLE 4.5
LOCAL AND GROSS FUEL BURNOUT IN CORE

<u>Average Fuel Burnup in Core</u>	<u>Fuel Burnup in Lower 4-1/2 in. of Core</u>	<u>Fuel Burnup at Tip of Meat</u>	<u>B-10 Burnup at Tip of Meat</u>
4.35%	5.50%	10.28%	44.93%
12.66%	15.98%	28.10%	83.71%
20.42%	25.71%	42.70%	95.32%
27.61%	34.66%	54.51%	98.69%
34.23%	42.81%	64.00%	99.64%
37.32%	46.59%	67.98%	99.81%

With the results of the local burnout calculations, the most adverse power distribution in the SM-2 was calculated. The uncorrected, most adverse axial distribution occurring at the point of deepest bank insertion under no xenon conditions is shown in Fig. 4.2. Local burning out the lower portion of the fuel yields the corrected distribution given in Table 4.6. The axial maximum to average power generation is 1.84 in the core.

TABLE 4.6
CORRECTED WINDOWSHADE (AXIAL) POWER DISTRIBUTION
(Deepest Bank Insertion; T = 440°F; No Xenon)

<u>Z (in.)*</u>	<u>Relative Power*</u>	<u>Z (in.)*</u>	<u>Relative Power*</u>
22	0.0038730	10	0.021950
21	0.0034236	9	0.025498
20	0.0045152	8	0.035331
19	0.0057233	7	0.039717
18	0.0069851	6	0.041590
17	0.0083316	5.5	0.041909
16	0.0098090	5	0.041758
15	0.011413	4	0.040878
14	0.013232	3	0.038199
13	0.014751	2	0.035542
12	0.016926	1	0.034493
11	0.019280	0	0.066420

* The relative axial power distribution normalized to a core average of 1 may be obtained by multiplying the values in this table by 44.

4.4.1 Methods of Spike Suppression (8)

At the date of this calculation, the maximum allowable ratio of spike power to maximum core power under steady state conditions was 1.36. From Table 4.4, it can be seen that some method of spike suppression is required over at least the first 45% of the expected core lifetime. Various methods were studied to suppress the spike, including the use of integral flux suppressors and reduced local fuel concentrations.

4.4.1.1 Integral Flux Suppressors

One method of spike suppression studied was the use of added B-10 to the bottom of the "picture frame" within the fuel plate. The boron was assumed to be mixed homogeneously with the stainless steel and to displace none of the steel. The axial power distribution as a function of w/o B-10 in the steel was calculated by use of the VALPROD code. The effect of 0.05, 0.10, and 0.20 w/o B-10 in the steel upon the axial power distribution is shown in Fig. 4.11. The effectiveness of B-10 on the spike suppression is shown in Fig. 4.12. It should be noted that since B-10 burns out at a faster rate than fuel, more B-10 would be required than that necessary to reduce the initial spike to core maximum power ratio below a value of 1.36. A more precise calculation would involve setting up and solving a set of differential equations which would include the variation of B-10 burnup and the local fuel burnup at the bottom of the core with time. Such a calculation was not undertaken since, for metallurgical reasons, other types of flux suppressors were more desirable; the use of B-10 would result in the evolution of large amounts of helium gas which could result in disruption of the cladding. The control rod fuel elements SM-1 Core II incorporate Eu_2O_3 in the form of integral flux suppressors. (9) Calculations (10) and measurements (11) indicate that the suppressors, containing 23.0 w/o Eu_2O_3 in a stainless steel matrix adequately depress the thermal flux (and consequently the power) at the tip of the fuel regions. Therefore, the use of Eu_2O_3 was incorporated as the reference spike suppressor for the SM-2.

4.4.1.2 Reduced Local Fuel Concentrations (8)

A second method of spike suppression studied was the use of a reduced local fuel concentration in the bottom of the core. This could be done either by rolling a two-section meat compact which contains a step gradient in the fuel concentration, or by rolling the plates such that the meat compact tapers off toward the bottom of the plate. The effect of fuel tapering was investigated using two different models.

The first method assumes that the original flux distributions remain unchanged upon insertion of a tapered plate, and that the power varies only as the U-235 atom density in the taper. Therefore:

$$P'(Z) = P(Z) \left[N^{25}(Z) / N_{\text{ref}}^{25} \right]$$

where: $P(Z)$ = original power level at Z (Fig. 4. 13)

$P'(Z)$ = new power level at Z

N_{ref}^{25} = original U-235 atom density in meat

$N^{25}(Z)$ = U-235 atom density in taper at Z

The quantities $P(Z)$ and N_{ref}^{25} were obtained from results of the non-uniform burnout results described in Section 4. 2. The adjusted power distributions for various lengths of taper are shown in Fig. 4. 14. The point of the taper for all cases was taken at 22 in. from the top of the meat.

A somewhat more elegant method of treating the taper was attempted by efforts to account for the perturbation of the original thermal flux distribution upon insertion of a decreased fuel concentration at the bottom of the core. The assumption is made that the original power distribution at the bottom of the core may be separated into two components. One of these components, the "return contribution," is due to the diffusing back into the core of neutrons which have become thermalized in the axial reflector. The other component, the "fundamental mode," (Fig. 4. 13) is essentially the contribution of epithermal fissions, and fissions due to neutrons thermalized in the core. It is further assumed that this "fundamental mode" contribution is independent of the taper, for small tapers. This assumption implies that a taper covering 1 in. of a 22-in. active core will not significantly change the axial buckling. The power distribution may then be expressed as:

$$P(Z) = \frac{N^{25}(Z)}{N_{\text{ref}}^{25}} \left[A e^{-\int_{Z_0}^Z K(Z) dZ} + P'(Z) \right]$$

$$K(Z) = \left[\sum_a^* + (N^{25}(Z)/N_{\text{ref}}^{25}) \sum_a^{25} \right]^{1/2} / \sqrt{D}$$

$$\sum_a^* = \sum_a^{\text{core}} - \sum_a^{25}$$

where: A = return contribution at bottom of core, $\cong 0.72$ (Fig. 4.13)

$e^{-\int_{Z_0}^Z K(Z) dZ}$ = attenuation coefficient which is dependent on the variation of U-235 atom density in a particular taper.

$P'(Z)$ = "fundamental mode" contribution at Z

$K(Z)$ = inverse of the diffusion length at Z

$\sum_{at}^{25}$ = U-235 macroscopic absorption cross section

D = thermal diffusion coefficient

The results of these calculations are shown in Fig. 4.15. The point of the taper again occurs at 22 in. from the top of the meat. It is seen that both methods of calculation of the effects of fuel tapering yield nearly similar results, indicating that the effect of the fuel tapering upon the flux distribution is small.

4.5 FUEL BURNOUT DISTRIBUTION IN ELEMENT 45

To determine the maximum fuel burnout expected in the SM-2, the local burnout distribution across the width of fuel plate number 9 of fixed fuel element 45 (Fig. 4.16) was calculated corresponding to 27 MWYR operation of the SM-2. At the time of this calculation, 27 MWYR was the best estimate of the core lifetime. The method used was essentially the one presented in reference (12) and used to calculate the burnout distribution in the SM-1. The formula employed is as follows:

$$B_j^m(Z) = A' B(Z) F(X) F(Y) P(j)$$

where: $B_j^m(Z)$ = specific local burnup in a fuel element at position (m, j, Z)

m = number of the fuel plate in the particular element under consideration

j = section of the fuel plate, A through D (Fig. 4.16)

A' = value of the power at the centerline of the core as calculated by VALPROD.

$F(X), F(Y)$ = geometrical power factors which result from an attempt to separate a radial power distribution into X and Y components. (13)

$P(j)$ = local side plate peaking factor obtained from plate type P₃ calculations.

$B(Z)$ = peak axial fuel burnout distribution.

The factors, A' and $B(Z)$, at 27 MWYR were obtained from results of the non-uniform burnup analysis presented in Section 4.2. Since the most adverse burnup distribution was desired, the axial point corresponding to 7 in. from the bottom of the core in element 45 was chosen. The resulting factors are shown in Table 4.7. It should be noted that these calculations assume that the innercell flux distribution does not change with time and therefore give an upper limit. The maximum fuel burnup (atom % U-235) is 92% for a 1/2-sq. in. area. A 1/2-sq in. is utilized to define local burnup since it is a convenient size for chemical analysis. This is an upper limit on the expected burnup.

TABLE 4.7
LOCAL BURNUP IN PLATE #9 OF FUEL ELEMENT #45

j	A'	F(X)	F(Y)	P(j)	B(Z)	$B_j^9(Z)$
A	1.675	0.970	1.00	1.08	0.526	0.923
B	1.675	0.954	1.00	0.97	0.526	0.815
C	1.675	0.913	1.00	0.97	0.526	0.781
D	1.675	0.898	1.00	1.06	0.526	0.841

4.6 AVERAGE, GROSS LOCAL, AND SPECIFIC LOCAL FUEL BURNUPS

The average fuel burnup over the volume of the core may be determined from a knowledge of the core energy release per Kg U-235 depleted by capture and fission. This value is generally taken as 2.00 ± 0.06 MWYR per kg U-235. This number is based upon 194 Mev released per fission, the effective capture to fission ratio is uniform throughout the core, and the fast neutron energy spectrum is similar to that found in the SM-1 or SM-2.

The gross local burnup is applicable over an area of approximately 2 in.² and is obtained by a synthesis of a one-dimensional axial burnout distribution and a gross radial burnout distribution which neglects such fine structure effects such as side plate peaking and corner peaking in a element.

One definition which may be used to calculate the gross local burnup may be expressed as:

$$B_{\text{gross}}^{\text{max}} = A' B(Z) F(X) F(Y)$$

where the symbols have been defined in Section 4.5. The specific local fuel burnup has also been defined in Section 4.5. The results of these calculations are given in Table 4.8. The values calculated for the SM-1 are also shown for comparison. These values represent upper limits since the effect of the control elements upon the overall and local flux distribution has been neglected; these elements depress the overall but increase the local flux distributions. In addition, multiplication of individual peaking factors generally overestimates the overall peaking factor.

TABLE 4.8
ESTIMATED PERCENT BURNUP OF U-235

	SM-1 (15 MWYR)	SM-2 (27 MWYR)
Average Burnup	33%	37%
Gross Local Maximum Burnup	85%	86%
Specific Local Maximum Burnup	90%	92%

4.7 BURNUP OF EUROPIUM ABSORBERS⁽¹⁾

The "burnup" of the control rod absorber sections was defined as the number of absorptions per original europium atom in the core. Therefore:

$$B^{\text{Eu}} = A/N_1$$

where N_1 is the product of the europium atomic concentration and the volume of the absorber in the core at an average bank position. It was assumed that all excess neutrons ($K_{\text{eff}}-1$) were absorbed in the europium, this implying that all other absorptions and leakage were treated correctly. The number of absorptions may be expressed as:

$$\text{Absorptions} = A = P \int \nu K_{\text{ex}} dt$$

where: P = average power (2.5×10^7 watts, at time of this calculation)

$\int$ = 3.24×10^{10} fissions/watt-sec.

ν = 2.46 neutrons produced per fission

K_{ex} = average excess reactivity in time period t .

t = time period in sec.

Data for the europium absorber section was taken from reference (7), Appendix III; it was assumed that the absorber sections would be similar to that used in the SM-1, Core II. This data is given in Table 4.9.

TABLE 4.9
EUROPIUM ABSORBER SECTION DATA

Initial Europium Loading	151 gms/plate
Plate Thickness	0.229 cm
Plate Width	5.67 cm
Plate Length	52.70 cm
Plate Volume	68.3 cm ³
Total Absorber Volume (7 rods)	1912.4 cm ³
Initial Europium Concentration	0.00875 x 10 ²⁴ atoms/cm ³
Initial Number of Europium Atoms	16.7335 x 10 ²⁴ atoms

Because of the gap between the absorber and the control rod fuel section, the absorber insertion was calculated as bank insertion (Fig. 4.7) minus 1.3 in. (7) Absorber "burnup" vs. core energy release is shown in Fig. 4.17 both as absorptions per total number of initial europium atoms, and as absorptions per number of europium atoms inserted in core at an average bank position.

The maximum "burnup" at the tip of the centerline rod was also calculated and is shown in Fig. 4.17. From VALPROD calculations, the ratio of thermal flux at each rod to the core average is 1.58 for the centerline rod and 1.20 for the eccentric rods. Therefore, the number of absorptions at the centerline rod is approximately

$$A_{\frac{1}{4}} = A \left[\frac{1.58}{1.58 + 6(1.20)} \right] = 0.1803 A$$

From Windowshade calculations at 0 MWYR, the peak to average thermal flux in the rodded region of the core is about 2.04; therefore,

$$B_{\frac{1}{4}}^{\text{Eu}} = \frac{(0.1803 A)(2.04)}{(N_i/7)} = \frac{2.755 A}{N_{\text{inserted}}}$$

4.8 TRANSMUTATION OF THE FUEL MATRIX

The atomic percent of the fuel compact matrix transmuted by fission of U-235 and capture in B-10 was calculated as a function of core energy release. Neutron capture in U-235 and in the stainless steel were not considered since it was assumed that transmutation implies a change in the atomic number of the capturing nuclide.

The initial atomic percents of U-235 and B-10 in the matrix are 6.0326% and 0.2477% respectively as calculated for the reference core #1 described in Section 2.3. The atoms of U-235 and B-10 depleted during lifetime were obtained from results of the non-uniform calculation described in Section 4.2. Figures 4.18 and 4.19 show the atomic percent of the matrix transmuted by fission of U-235 and capture in B-10 respectively, as a function of core energy release. For a core life of 27 MWYR, the maximum atomic percent of the matrix transmuted by fission of U-235 and capture in B-10 is 3.6% and 0.25% respectively; the average values are 1.8% and 0.23% respectively.

Figure 4.1

4-15

Figure 4.3

4-18

Figure 4.4

Figure 4.5

Figure 4.6

Figure 4.7

EQUIVALENT RINGS USED IN MIDLIFE LOADING CALCULATIONS

<u>Ring</u>	<u>Equivalent</u>	<u>Radius (cm.)</u>	<u>Fixed Elements</u>	<u>Control Elements</u>
A	4.209560		0	1
B	12.628677		8	0
C	21.047794		10	6
D	28.238577		20	0

Figure 4.9

Figure 4.11

Figure 4.12

Figure 4.13

Figure 4.14

Figure 4.15

CORE LATTICE AND PLATE DESIGNATIONS

Figure 4.17

Figure 4. 19

5.0 NUCLEAR DESIGN OF A TWO-PASS CORE

The original design of the SM-2 utilized a one-pass core as in the SM-1. From a plant optimization study, (1) it was decided to employ a two-pass core. Two methods of achieving two-pass flow were studied:

- 1) Use of boxed elements
- 2) Use of a flow divider

5.1 BOXED TYPE FUEL ELEMENTS⁽²⁾

A drawing of a boxed type element is shown in Fig. 5. 1; a schematic diagram, together with the dimensions of the element studied, is shown in Fig. 5. 2. The additional steel in such an element will absorb neutrons and result in increased flux peaking in the outer fuel plates.

The geometry and loading of the box-type element considered and the regular fuel element it was compared to, are tabulated in Table 5. 1. The regular fuel element is similar to the one described in Section 2. 3 except that the side plate thickness has been reduced from 50 to 30 mils; a drawing of this element is shown in Fig. 1. 1.

TABLE 5. 1
GEOMETRY AND LOADING CHARACTERISTICS

	<u>Box-Type Element</u>	<u>Regular Element</u>
<u>Geometry</u>		
Cell size (in.)	2. 94 sq.	2. 9375 sq.
Meat thickness (in.)	0. 03	0. 03
Clad thickness (in.)	0. 005	0. 005
Active length (in.)	21. 75	21. 75
Plate water gap (in.)	0. 111	0. 1232
Meat width (in.)	2. 54	2. 54
Plate width (in.)	2. 773	2. 773
Side plate width (in.)	2. 87	2. 863
Side Plate thickness (in.)	0. 03	0. 03
Width of inert plates (in.)	2. 773	-
Thickness of inert plates (in.)	0. 03	-
Plates per element	18	18
<u>Loading</u>		
U-235/plate (gm)	46. 296	46. 296
UO ₂ /plate (gm)*	56. 765	56. 765
B-10/plate (gm)	0. 08529	0. 08529
B ₄ C/plate (gm)**	0. 64605	0. 64605

* Based on 87. 63 w/o U in UO₂ and 93. 07 w/o U-235 in U.

** Based on 75. 9 w/o B in B₄C and 19. 8 a/o B-10 in B.

5.1.1 Nuclear Properties of the Fuel Elements

The fast group parameters for both types of fuel elements were obtained by homogenizing the fuel element and employing the MUFT-III, IBM-650 code. The atomic concentrations for the regular fuel elements were calculated by use of Program 50. The atomic densities calculated for both types of elements are given in Table 5.2. The fast group parameters as calculated by MUFT-III are tabulated in Table 5.3.

TABLE 5.2
ATOMIC DENSITIES (atoms/cm³) IN THE FUEL ELEMENTS

Atom	Box Type Element		Regular Element	
	T = 68°F	T = 440°F	T = 68°F	T = 440°F
N ^H x 10 ⁻²²	4.870228	4.073000	4.997825	4.179710
N ^O x 10 ⁻²²	2.583885	2.185270	2.647936	2.238879
N ^{SS} x 10 ⁻²²	1.882708	1.882708	1.719260	1.719260
N ²⁵ x 10 ⁻²⁰	6.929108	6.929108	6.940908	6.940908
N ²⁸ x 10 ⁻²⁰	0.509426	0.509426	0.510294	0.510294
N ^B x 10 ⁻²⁰	1.513531	1.513531	1.516111	1.516111

The thermal properties of the regular element were obtained by use of Program 50 and the Plate Type P₃ theory IBM-650 codes as described in Section 1.0. These properties are tabulated in Table 5.3. For the box-type element, the thermal properties of the active region (fuel plates plus the water between them) were obtained by use of a one-dimensional P₃ flux calculation⁽³⁾ perpendicular to the fuel plates. The code prints out the flux distribution and the average fluxes in the plate and channel regions. Then the flux is normalized such that the average flux in the active region (plate plus water channel) is unity. The normalization constant C_A is given by:

$$C_A = \frac{1}{m} \left[\ell N_O^{\text{Plate}} (\text{avg}) + (m - \ell) N_O^{\text{channel}} (\text{avg}) \right]$$

$$\Phi'_{\text{Plate}} = \frac{1}{C_A} N_O^{\text{Plate}} (\text{avg})$$

$$\Phi'_{\text{channel}} = \frac{1}{C_A} N_O^{\text{channel}} (\text{avg})$$

$$\sum_x^{\text{Active}} = \frac{1}{m} \left[\ell \sum_x^{\text{Plate}} \Phi'_{\text{Plate}} + (m - \ell) \sum_x^{\text{channel}} \Phi'_{\text{channel}} \right]$$

where: $\sum_x^{\text{Act}} = \sum_s^{\text{Act}}, \sum_a^{\text{Act}}, \sum_{\text{tr}}^{\text{Act}}$ and $\nu \sum_f^{\text{Act}}$ respectively.

$$D^{\text{Act}} = 1/3 \sum_{\text{tr}}^{\text{Act}}$$

$$L^2_{(\text{Act})} = D^{\text{Act}} / \sum_a^{\text{Act}}$$

$$K_{\text{th}}^{\text{Act}} = \nu \sum_f^{\text{Act}} / \sum_a^{\text{Act}}$$

where: $N_{\text{O}(\text{avg})}^{\text{plate}}, N_{\text{O}(\text{avg})}^{\text{channel}} =$ non-normalized average fluxes in the plate and channel respectively.

ℓ = half thickness of the fuel plate.

m = one-half of the thickness of the fuel plate plus one-half of the thickness of the water channel.

The properties of the dead region were calculated by homogenizing the steel and water contained in this region. The thermal properties of the entire fuel element were then obtained by cylindricalizing the element into two concentric regions and performing a cylindrical P_3 calculation ⁽⁴⁾ to obtain the flux distribution. The inner region is the active region while the outer region is the dead region; the equivalent radii of the two regions are 3.765306 cm. (R_1) and 4.213141 cm. (R_2) respectively. The code calculates the flux distribution and average flux in each region. The flux is normalized so that the average flux in the entire element is unity. The normalization constant C_E is given by:

$$C_E = \frac{1}{R_2^2} \left[R_1^2 N_{\text{O}(\text{avg})}^{\text{Act}} + (R_2^2 - R_1^2) N_{\text{O}(\text{avg})}^{\text{Dead}} \right]$$

$$\bar{\Phi}'_{\text{Act}} = \frac{1}{C_E} N_{\text{O}(\text{avg})}^{\text{Act}}$$

$$\bar{\Phi}'_{\text{Dead}} = \frac{1}{C_E} N_{\text{O}(\text{avg})}^{\text{Dead}}$$

$$\sum_x^{\text{EL}} = \left(\frac{R_1^2}{R_2^2} \right) \sum_x^{\text{Act}} \bar{\Phi}'_{\text{Act}} + \frac{(R_2^2 - R_1^2)}{(R_2^2)} \sum_x^{\text{Dead}} \bar{\Phi}'_{\text{Dead}}$$

where: $\sum_x^{EL} = \sum_s^{EL}, \sum_a^{EL}, \sum_{tr}^{EL}$ and $\nu \sum_f^{EL}$ respectively

$$D^{EL} = 1/3 \sum_{tr}^{EL}$$

$$L^2_{(EL)} = D^{EL} / \sum_a^{EL}$$

$$K_{th}^{EL} = \nu \sum_f^{EL} / \sum_a^{EL}$$

The thermal properties for the box type elements are tabulated in Table 5.3.

5.1.2 Reactivity Effect

To compare the reactivity of a core comprised of box-type elements as opposed to regular fuel elements, K_{eff} was calculated by the modified two-group model. Reflector savings and K_{eff} was computed by use of FINK-1, an IBM-850 code described in Section 1.2.3. These results are given in Table 5.4. The values of K_{eff} are for clean cores containing 45 fixed fuel elements and do not include any model correction. It is seen that a core of box-type elements results in a decreased reactivity of about 1.3% ρ ; this effect is due primarily to the additional steel in the box type elements.

5.1.3 Innercell Flux and Power Distribution

The P_3 flux distribution through the active and dead regions were calculated both parallel and perpendicular to the fuel plates in order to estimate power peaking factors; these factors were used in the thermal analysis of the box-type element. The cell was assumed to consist of a central active region of dimensions $a \times c$ (1.27 in. by 1.359 in.), surrounded by a dead region having the cell dimensions of $2b \times 2b$ (2.94 in. by 2.94 in.). The thermal properties of the active and dead region were calculated as described in Section 5.1.1. The flux distribution parallel to the fuel plates (X direction) was obtained by first calculating, by use of the one-dimensional P_3 flux code, ⁽³⁾ the flux distribution due to a source of 1 neutron/cm³-sec in the dead region.

TABLE 5.3
NUCLEAR PARAMETERS OF THE FUEL ELEMENTS

Parameter	Units	Box Type Fixed Fuel Element		APPR-1B Regular Fixed Element	
		T = 68°F (0.0331 ev)	T = 440°F (0.0549 ev)	T = 68°F (0.0331 ev)	T = 440°F (0.0549 ev)
τ	cm ²	32.440743	42.512115	32.276271	42.485361
D _f	cm	1.276468	1.428742	1.294196	1.453491
$\nu \Sigma_f^f$	cm ⁻¹	0.019818	0.017983	0.019949	0.018104
Σ_a^f	cm ⁻¹	0.015626	0.014239	0.015520	0.014132
K _f		1.268270	1.262939	1.285373	1.281064
P		0.602877	0.576326	0.612938	0.586920
Σ_a	cm ⁻¹	0.414447	0.332781	0.419156	0.335230
Σ_s	cm ⁻¹	1.997624	1.476640	2.027095	1.492168
Σ_{tr}	cm ⁻¹	2.039090	1.444011	2.063246	1.453390
$\nu \Sigma_f$	cm ⁻¹	0.598903	0.482384	0.612811	0.491039
K _{th}		1.445065	1.449553	1.462010	1.464784
D	cm	0.163471	0.230838	0.161558	0.229349
L ²	cm ²	0.394431	0.693663	0.385435	0.684154
Σ_{xe}^{a*}	cm ⁻¹	-	0.013288	-	0.013317

* Calculated at a power level of 28 MW

TABLE 5.4
REACTIVITY OF CORES CONTAINING 45
FIXED FUEL ELEMENTS

Parameters	Units	Box-Type Elements		Regular Elements	
		T = 68°F	T = 440°F	T = 68°F	T = 440°F
S _z	cm	6.499560	7.875301	6.566173	7.975429
S _r	cm	6.540821	7.983568	6.609954	8.084837
H	cm	55.245000	55.245000	55.245000	55.245000
R	cm	28.262615	28.262615	28.238582	28.238582
B _z	cm ⁻¹	0.046034	0.044250	0.045945	0.044126
B _r	cm ⁻¹	0.069096	0.066346	0.069007	0.066205
B ²	cm ⁻²	0.006893	0.006360	0.006873	0.006330
K _{eff}		1.121651	1.075915	1.138674	1.091601
ρ	%	10.845708	7.055854	12.178551	8.391436

The active region is a "source free" or "fuel" region and the dead region is a "source" or "moderator" region. This flux distribution is designated $\Phi(X)$. Then the flux distribution, due to a source of 1 neutron/cm³-sec in the active region was obtained. In this case, the dead region becomes a "source free" or "fuel" region and the active region becomes a "source" or "moderator" region; this flux distribution is designated $\Phi_1(X)$. The overall flux distribution was obtained by weighting and superimposing $\Phi_1(X)$ and $\Phi_2(X)$. Weighting is necessary because the source strength in the active and dead regions is not the same. The relative source strength in each of these regions was assumed proportional to the volume fraction of water in that region. Therefore:

$$\Phi'(X = X_1) = (VF)_w^{\text{dead}} \Phi_1'(X = X_1) + (VF)_w^{\text{active}} \Phi_2'(X = b - X_1)$$

The flux distribution was then normalized to the average flux in the active region.

$$\bar{\Phi}'(X) = \frac{1}{a} \int_0^a \Phi'(X) dX$$

$$\Phi(X) = \Phi'(X) / \bar{\Phi}'(X)$$

The overall flux distribution in the direction perpendicular to the fuel plates (y direction) was obtained in a manner similar to that described for the X direction. The following equations apply:

$$\Phi'(y = y_1) = (VF)_w^{\text{dead}} \quad \Phi'_1(y = y_1) + (VF)_w^{\text{Active}} \quad \Phi'_2(y = b - y_1)$$

$$\bar{\Phi}'(y) = \frac{1}{c} \int_0^c \Phi'(y) dy$$

$$\Phi(y) = \Phi'(y) / \bar{\Phi}'(y)$$

The flux distributions $\Phi(X)$ and $\Phi(y)$ at 440°F are plotted in Fig. 5. 3. The ratio of the maximum to the average flux in the active region in both directions are:

$$\Phi(X = a) / \bar{\Phi}(X) = 1.492$$

$$\Phi(y = c) / \bar{\Phi}(y) = 1.322$$

For comparison, the flux distribution parallel to the fuel plates in the regular fuel elements is plotted in Fig. 5. 4. This distribution was calculated by the P_3 theory code for plate type elements. The ratio of the maximum to average flux at 440°F in the active region, parallel to the fuel plates, is:

$$\left[\Phi_{\text{max}} / \Phi_{\text{avg}} \right]_x^{\text{Regular}} = 1.444$$

The power peaking factors may be estimated from the flux peaking factors by the following analysis. The relative power generation rate of two points within a unit cell may be expressed by:

$$\frac{P(X_1)}{P(X_2)} = \frac{\Phi(X_1) \sum_f + \phi_f(X_1) \sum_f^f}{\Phi(X_2) \sum_f + \phi_f(X_2) \sum_f^f}$$

If the thermal flux is normalized to the average thermal flux in the unit cell and if position X_2 corresponds to this point:

$$\frac{P(X_1)}{P(X_2)} = \frac{\bar{\Phi}(X_1) \sum_f + \phi_f(X_1) \sum_f^f}{\sum_f + \phi_f(X_2) \sum_f^f}$$

where $\bar{\phi}(X_1)$ is the thermal flux at X_1 normalized to the average thermal flux. The average percent of thermal fissions, $\bar{\beta}$, may be expressed by:

$$\bar{\beta} = \frac{\sum_f}{\sum_f + \sum_f^I} \phi_{f(\text{avg})}$$

In addition, if the assumption can be made that the fast flux is invariant over the cell:

$$\phi_f(X_1) = \phi_f(X_2) = \phi_{f(\text{avg})}$$

Therefore:

$$\frac{P(X_1)}{P_{\text{avg}}} = \bar{\beta} \bar{\phi}(X_1) + (1 - \bar{\beta})$$

Hot channel factors $F(\Delta T)$ and $F(\Delta \theta)$ were defined as:

$F(\Delta T)$ = Average power generated in the hottest plate relative to the average power in an average plate.

$F(\Delta \theta)$ = Maximum power generator at the hottest spot in the hottest plate relative to the average power in an average spot on average plate.

These factors were then expressed as:

Box Type Element

$$F(\Delta \theta) = \left[\frac{\bar{\phi}(X=a)}{\bar{\phi}(X)} \right] \left[\frac{\bar{\phi}(y=c)}{\bar{\phi}(y)} \right] \beta + (1 - \beta)$$

$$F(\Delta T) = \left[\frac{\bar{\phi}(y=c)}{\bar{\phi}(y)} \right] \beta + (1 - \beta)$$

Regular Element

$$F(\Delta \theta) = \left[\frac{\bar{\phi}_{\text{max}}}{\bar{\phi}_{\text{avg}}} \right]_{\text{Regular}} \beta + (1 - \beta)$$

$$F(\Delta T) = 1 \quad (\text{assumed to be unity})$$

The values of β and the power peaking factors as calculated for both the box-type and regular fuel elements are given in Table 5. 5. It is seen that use of the box-type element results in a significant increase in the power peaking factors as compared to the use of the regular elements. The value of $F(\Delta\theta)$ for the box-type element is probably overestimated since the overall maximum to average flux in the corner of the active region was assumed to be the product of the maximum to average fluxes parallel and perpendicular to the fuel plates.

TABLE 5. 5
HOT CHANNEL FACTORS AT 440°F AND 1200 psia

Parameter	Box-Type Element	Regular Element
$\beta(\%)$	80. 8526	61. 7972
$F(\Delta\theta)$	1. 592	1. 274
$F(\Delta T)$	1. 322	1. 000

5. 1. 4 Critical Mass

The additional steel in a core comprised of box-type fuel elements results in lower reactivity and hence requires a larger critical mass than a core containing the regular fuel elements. The critical mass of the box-type element core should properly be done by repeating the calculations made in Section 5. 1. 1 and 5. 1. 2 for various loadings of U-235 (and no boron), plotting K_{eff} as a function of kg. of U-235, and picking off the mass corresponding to $K_{eff} = 1$. Such a process is tedious and time consuming; therefore, since only an estimate of the effect upon critical mass was desired, use of Fig. 2. 3 was made. This graph of critical mass as a function of core metal-to-water ratio, was derived for APPR type elements, and is not strictly applicable to box-type elements; however, it is sufficient for preliminary estimates. The results of these calculations are given in Table 5. 6. It is seen that a core composed of box-type elements requires an increase in the critical mass of about 1. 0 kg U-235 over that of a core composed of the regular fuel elements.

TABLE 5. 6
CRITICAL MASS (Kg U-235) FOR CORES COMPRISED OF
45 FIXED ELEMENTS
(T = 440°F; With Eq. Xenon)

Parameter	Units	Box-Type Elements	Regular Elements
Metal volume per element	cm ³	832. 2874	772. 0668
Water volume per element	cm ³	2248. 4528	2303. 4362
Volume of the element	cm ³	3080. 7402	3075. 5030
Metal to water ratio		0. 3702	0. 3352
Critical Mass	Kg	14. 90	13. 90

5.2 EFFECT OF A FLOW DIVIDER

Another way in which two-pass flow may be accomplished is by the use of a flow divider. The effects of a flow divider on core reactivity and radial power distribution were calculated by use of the VALPROD, IBM-650 code. The properties of the core considered are tabulated in Section 2.3. A total of 9 flow divider configurations were examined consisting of gaps of 0.15, 0.20, and 0.25 in. containing 20, 40, and 60 volume percent steel. The properties of the latter region were calculated by homogenizing the steel and water contained in this region. These properties at 440°F were taken from Fig. 4.7 through 4.11 of reference (5). Radial VALPROD calculations were run for each configuration and were normalized to a no-divider or control VALPROD calculation.

The changes in reactivity as a function of both gap width and steel content are shown in Fig. 5.5. The "reference" flow divider configuration considered ⁽¹⁾ is shown in Fig. 5.6 and consists of a 0.094 in. thick flow divider contained in a 0.206 in. gap; therefore, from Fig. 5.5, the reactivity loss is about 0.6% ρ .

The radial power distribution was calculated and plotted for each configuration. A typical power distribution is shown in Fig. 5.7. To account for the power peaking at the core-gap interface, a constant local peaking factor for all the outer pass elements was defined as:

$$F_{\text{Outer}}^{\text{Gap}} = P_{\text{Gap}}' / P_{\text{Gap}}$$

where: P_{Gap} = control power level at gap radius

P_{Gap}' = configuration power level at outside interface of the gap

Peaking factors for each element in the inner pass were defined as:

$$F_{\text{Element}}^{\text{Gap}} = P_{\text{H.S.}}' / P_{\text{H.S.}}$$

where: $P_{\text{H.S.}}$ = control power level at hot spot of the element

$P_{\text{H.S.}}'$ = configuration power level at hot spot of the element

Peaking factors due to the flow divider apply to both $F(\Delta T)$ and $F(\Delta \theta)$; these latter factors are described in Section 5.1.3. Table 5.7 is a tabulation of the peaking factors for the various configurations examined. Also listed are the interpolated factors for the "reference" flow divider configuration. These values were used for thermal design calculations.

The increase in critical mass due to the loss in reactivity by use of the flow divider was estimated from graphs of reactivity as a function of U-235 loading, described in Section 2.4. The 0.6% ρ reactivity loss will result in an increase of critical mass of approximately 0.5 kg U-235.

5.3 SUMMARY

A summary of the results of the calculations for cores consisting of (a) all regular elements, (b) all regular elements with a flow divider, and (c) all box-type elements, is given in Table 5.8. It is evident that the use of either a flow divider or box-type elements in a two-pass core is disadvantageous, compared to the one-pass core, in reactivity, critical mass, and power peaking. However, since a two-pass core was decided upon, the use of a flow divider is more desirable, nuclear-wise, than use of the boxed elements. In addition, the flow requirements, due to the power peaking, would be significantly higher for the boxed element core. Therefore, nuclear-wise, the conclusion of this study was a preference for a core with a flow divider rather than use of the box-type elements. After careful consideration on a thermal, hydraulic, and mechanical basis, the box-type element was dropped from further consideration.

TABLE 5.7
GAP PEAKING FACTORS DUE TO A FLOW DIVIDER

Configuration	Outer Pass*	Element Numbers				
		24	25	34	35	44
0.15" Gap; 20% Steel	1.172	0.985	0.985	0.987	0.987	0.987
0.15" Gap; 40% Steel	1.093	1.000	1.000	1.000	1.000	1.000
0.15" Gap; 60% Steel	1.020	1.007	1.007	1.013	1.013	1.013
0.20" Gap; 20% Steel	1.215	0.985	0.985	0.987	0.987	0.987
0.20" Gap; 40% Steel	1.117	1.000	1.000	1.000	1.000	1.000
0.20" Gap; 60% Steel	1.020	1.011	1.011	1.022	1.022	1.022
0.25" Gap; 20% Steel	1.259	0.981	0.981	0.980	0.980	0.980
0.25" Gap; 40% Steel	1.146	1.000	1.000	1.000	1.000	1.000
0.25" Gap; 60% Steel	1.029	1.019	1.019	1.027	1.027	1.027
"Reference"***	1.095	1.003	1.003	1.005	1.005	1.005

* Outer pass elements include numbers 14, 15, 16, and 26.

** 0.094 in. flow divider in a 0.206 in. gap.

TABLE 5. 8
COMPARISON OF TWO-PASS CORES TO A ONE-PASS CORE

<u>Parameter</u>	<u>Core Composed of Box-Type Elements</u>	<u>Regular Elements with 2 Flow Divider</u>
Reactivity loss ($\% \rho$)	-1. 3	-0. 6
Critical Mass Increase (Kg U-235)	1. 0	0. 5
$\left[\frac{F(\Delta T)}{F(\Delta T)_{\text{Reference}}} \right]$	1. 322	1. 003-1. 005 (inner pass) 1. 095 (outer pass)
$\left[\frac{F(\Delta \theta)}{F(\Delta \theta)_{\text{Reference}}} \right]$	1. 250	1. 003-1. 005 (inner pass) 1. 095 (outer pass)

Figure 5.1

BOX TYPE FUEL ELEMENT

Figure 5.2

Figure 5.4

EFFECT OF FLOW DIVIDER ON REACTIVITY
AS A FUNCTION OF
GAP WIDTH
AND
VOL. % SS IN GAP

LAYOUT	
ALCO PRODUCTS INC.	
ROCKAWAY, NEW YORK	
ATOMIC ENERGY DIV.	
DATE: APR-13	BY: J. L. B.
CHECKED: J. L. B.	APPROVED: J. L. B.
TITLE: APP-13 CORE	
FLOW STUDY 7.6.72	
LAYOUT NO.: DEL-100	

Figure 5. 6

Figure 5.7

6.0 EFFECT OF DESIGN CHANGES UPON THE SM-2 CORE

6.1 REFERENCE CORE #2⁽¹⁾

The reference core described in Section 2.3 was based upon a very preliminary design, including an operating temperature of 440°F, operating pressure of 1200 psia and a power output of 25 tMW. From an optimization study performed upon the SM-2 secondary system⁽²⁾, the SM-2 operating conditions of 510°F, 2000 psia, and 28 tMW were selected. The core is to be a two-pass core. In addition, various design and metallurgical changes had been incorporated since the design described in Section 2.3. Therefore, the nuclear parameters for the new reference design (Reference Core #2) were calculated for various fuel burnups using the same techniques described in Section 2.3. These design parameters were used in much of the subsequent nuclear analysis of the SM-2 core and vessel and are tabulated in Tables 6.1 through 6.7. The B-10 and U-235 loadings in the fixed elements are identical to those used in Section 2.3.

TABLE 6.1
REFERENCE CORE #2 - PURE DENSITIES OF COMPONENTS IN
FUEL PLATE MEAT

<u>Component</u>	<u>Density</u>
Pure 347 SS (gm/cm ³)	7.85
Pure UO ₂ (gm/cm ³)	10.7
Pure ZrB ₂ (gm/cm ³)	6.09
Void Fraction of Meat (volume percent)	95.0

TABLE 6.2
REFERENCE CORE #2 - ATOMIC DENSITIES OF
FIXED FUEL ELEMENTS

Atom	T = 63°F (E = 0.0331 ev)			T = 510°F (E = 0.0600 ev)		
	B = 0	B = 0.22	B = 0.40	B = 0	B = 0.22	B = 0.40
$N_c^{25} \times 10^{-20}$	6.940908	5.413908	4.164545	6.940908	5.413908	4.164545
$N_c^{29} \times 10^{-20*}$	0.510294	→		0.510294	→	
$N_c^O \times 10^{-22}$	2.650026	→		2.106675	→	
$N_c^B \times 10^{-20**}$	1.516084	0.546094	0.273863	1.516084	0.546094	0.273863
$N_c^B \times 10^{-20}$	0.300185	0.108127	0.054225	0.300185	0.108127	0.054225
$N_c^{ss} \times 10^{-22}$	1.790806	→		1.790806	→	
$N_c^H \times 10^{-22}$	5.002005	→		3.915301	→	

* Based upon 87.63 w/o U in UO_2 and 93.07 w/o U-235 in U

** Based upon 19.17 w/o B in ZrB_2 and 19.8 a/o B-10 in B

TABLE 6.3
REFERENCE CORE #2 - ATOM DENSITIES OF
CONTROL FUEL ELEMENTS

Atom	T = 68°F (E = 0.0331 ev)	T = 510°F (E = 0.0331 ev)
	(B = 0)	(B = 0)
$N_c^{25} \times 10^{-20}$	5.629240	5.629240
$N_c^{28} \times 10^{-20*}$	0.413861	0.413861
$N_c^O \times 10^{-22}$	2.556809	2.027591
$N_c^B \times 10^{-20**}$	1.229602	1.229602
$N_c^{B-10} \times 10^{-20}$	0.243461	0.243461
$N_c^{ss} \times 10^{-22}$	2.032553	2.032553
$N_c^H \times 10^{-22}$	4.871895	3.813458

* Based upon 87.63 w/o U in UO_2 and 93.07 w/o U-235 in U

** Based upon 19.17 w/o B in ZrB_2 and 19.8 a/o B-10 in B

TABLE 6.4
REFERENCE CORE #2-NUCLEAR PARAMETERS (FIXED FUEL ELEMENTS)

Parameter		T = 68°F (E = 0.0331 ev)			T = 510°F (E = 0.06 ev)		
		B = 0	B = 0.22	B = 0.40	B = 0	B = 0.22	B = 0.40
τ	(cm ²)	31.694438	32.387968	32.927880	46.019895	47.218255	48.160097
D_f	(cm)	1.276164	1.279532	1.283060	1.490084	1.494624	1.499269
Σ_R	(cm ⁻¹)	0.040265	0.039506	0.038966	0.032379	0.031654	0.031131
Σ_a^f	(cm ⁻¹)	0.015630	0.012448	0.010279	0.014034	0.011252	0.009327
$\nu \Sigma_f^f$	(cm ⁻¹)	0.019972	0.016348	0.012981	0.017877	0.014686	0.011696
K_f		1.277800	1.313303	1.262866	1.273834	1.305190	1.253993
P		0.611830	0.684923	0.736194	0.566571	0.644538	0.700393
Σ_a	(cm ⁻¹)	0.420882	0.324118	0.260997	0.321151	0.241245	0.191565
Σ_s	(cm ⁻¹)	2.036880	2.027048	2.020881	1.380543	1.375479	1.372040
Σ_{tr}	(cm ⁻¹)	2.071936	1.967601	1.899500	1.353557	1.269845	1.217312
$\nu \Sigma_f$	(cm ⁻¹)	0.613246	0.505562	0.401986	0.468365	0.378499	0.297214
K_{th}		1.457050	1.559807	1.540196	1.458396	1.568942	1.551504
D	(cm)	0.160880	0.169411	0.175485	0.246265	0.262499	0.273827
L^2	(cm ²)	0.382246	0.522683	0.672363	0.767820	1.088103	1.429425
Σ_{xe}^{a*}	(cm ⁻¹)	---	---	---	0.016156	0.013233	0.010549
β (estimated)		0.620457	0.690257	0.747366	0.572948	0.637403	0.690139

* Σ_{xe}^a calculated at a power of 28 tMW.

TABLE 6.5
REFERENCE CORE #2 - NUCLEAR PARAMETERS
(CONTROL FUEL ELEMENTS, 0 MWYR)

<u>Parameter</u>		<u>T = 68°F (E = 0.0331 ev)</u>	<u>T = 510°F (E = 0.060 ev)</u>
τ	(cm ²)	31.838325	46.254915
D_f	(cm)	1.252334	1.450684
Σ_R	(cm ⁻¹)	0.039334	0.031363
Σ_a^f	(cm ⁻¹)	0.013062	0.012457
$\nu \Sigma_f^f$	(cm ⁻¹)	0.015619	0.014924
K_f		1.195758	1.198041
P		0.667927	0.602811
Σ_a	(cm ⁻¹)	0.327044	0.255422
Σ_s	(cm ⁻¹)	1.993773	1.367276
Σ_{tr}	(cm ⁻¹)	1.950634	1.286751
$\nu \Sigma_f$	(cm ⁻¹)	0.444748	0.351569
K_{th}		1.359902	1.376424
D	(cm)	0.170884	0.259050
L^2	(cm ²)	0.522510	1.014203

TABLE 6. 6
REFERENCE CORE #2 - REGION* PROPERTIES OF THE FIXED FUEL ELEMENTS

Parameter	Water Channel		Fuel Plate		Dead Region		Active Region	
	T = 68°F	T = 510°F	T = 68°F	T = 150°F	T = 68°F	T = 510°F	T = 68°F	T = 510°F
N ²⁵ atoms/cc	-	-	0. 003275	0. 003275	-	-	0. 000802	0. 000802
N ²⁸ atoms/cc	-	-	0. 000241	0. 00241	-	-	0. 000059	0. 000059
N ^B atoms/cc	-	-	0. 0007153	0. 0007153	-	-	0. 000175	0. 000175
N ^{B-10} atoms/cc	-	-	0. 0001416	0. 0001416	-	-	0. 000034	0. 000034
N ^{ss} atoms/cc	-	-	0. 068590	0. 068590	0. 025270	0. 025270	0. 016811	0. 013811
N ^H atoms/cc	0. 066890	0. 052358	-	-	0. 046985	0. 036778	0. 050494	0. 039524
N ^O atoms/cc	0. 033445	0. 026179	0. 007032	0. 007032	0. 023493	0. 018389	0. 026971	0. 021486
Σ _a (cm ⁻¹)	0. 017001	0. 009947	2. 249990	1. 613567	0. 069313	0. 049864	0. 517949	0. 385745
Σ _s (cm ⁻¹)	2. 426105	1. 569940	0. 775908	0. 775908	1. 967990	1. 366594	2. 055901	1. 383864
Σ _{tr} (cm ⁻¹)	1. 939212	1. 121773	3. 012930	2. 377113	1. 680211	1. 091500	2. 180089	1. 415954
νΣ _f (cm ⁻¹)	-	-	3. 488287	2. 474504	-	-	0. 782561	0. 579884

* See Section 1. 1. 2. 1

TABLE 6.7
REFERENCE CORE #2 - SM-2 CORE DATA

	<u>Fixed Element</u>	<u>Control Element</u>
<u>Overall Core Data</u>		
Number of elements	38	7
<u>Fuel Plate Dimensions</u>		
Meat width (in.)	2.54	2.3175
Meat thickness (in.)	0.03	0.03
Active length (in.)	21.75	21.75
Active volume (in. ³)	1.65735	1.512169
Clad thickness (in.)	0.005	0.005
Plate thickness (in.)	0.04	0.04
<u>Element Geometry</u>		
Cell dimensions (in.)	2.9375 sq.	2.9375 sq.
Width of side plate (in.)	2.863	2.619
Thickness of side plate (in.)	0.03	0.03
Width of fuel plate (in.)	2.784	2.564
Fuel plates per element	18	16
Volume fraction of water	0.7478	0.7283
Metal to water ratio	0.3373	0.3730
<u>Plate Loadings</u>		
U-235/plate (gm)	46.296	42.240
UO ₂ /plate (gm)	56.765	51.792
B-10/plate (gm)*	0.08529	0.07782
ZrB ₂ /plate (gm)*	2.42637	2.21386
SS/plate (gm)	157.8	144.0
W/O UO ₂ in meat (%)	26.164	26.164
W/O ZrB ₂ in meat (%)	1.118	1.118
W/O SS in meat (%)	72.718	72.718

* Does not include self-shielding effects due to lumps.

The relative P₃ thermal flux distributions at 510°F, both parallel and perpendicular to the fuel plates, as a function of fuel burnup, are shown in Fig. 6.1 and 6.2. These distributions show the local variations in flux within the individual fuel elements.

The values of the average fast and thermal fluxes in the SM-2 core as a function of fuel burnup were calculated from:

$$\bar{\Phi} = \frac{\beta \delta P_{mw}}{\sum_f V_{core}} \quad \text{neuts/cm}^2\text{-sec}$$

$$\bar{\Phi}_f = \frac{(1-\beta) \delta P_{mw}}{\sum_f^f V_{core}} \quad \text{neuts/cm}^2\text{-sec}$$

- where: β = % of thermal fissions at burnup B.
 δ = fissions/watt sec = 3.2175×10^{10}
 P_{mw} = power level = 28×10^6 watts
 V_{core} = core volume = $1.383976 \times 10^5 \text{ cm}^3$
 $\sum_f$ = macroscopic thermal fission cross section of core at burnup B.
 $\sum_f^f$ = macroscopic fast fission cross section of core at burnup B.

The calculated values of the fluxes, on the basis of all fixed elements, at 510°F and 68°F as a function of fuel burnup, are given in Table 6.8. The values at 68°F were also calculated at 28 MW power.

TABLE 6.8
REFERENCE CORE #2 - AVERAGE FAST AND THERMAL FLUXES

Burnup	$\bar{\Phi} \times 10^{-13}$ (neuts/cm ² -sec)		$\bar{\Phi}_f \times 10^{-13}$ (neuts/cm ² -sec)	
	T = 68°F	T = 510°F	T = 68°F	T = 510°F
0%	1.620	1.959	3.043	3.825
22%	2.186	2.697	3.034	3.954
40%	2.977	3.718	3.116	4.243

6.2 EFFECT OF WATER GAPS IN THE LAMINATED STEEL REFLECTOR

As seen in Section 3.2.1, the size of the water gaps between the steel rings making up the laminated steel reflector should be kept to a minimum in order to avoid loss of core reactivity. The effect of the reference laminated steel reflector⁽³⁾ upon the reactivity of the SM-2 core was calculated at 68°F and compared to measurements made during the SM-2 Flexible Critical Experiment.⁽⁴⁾ The reactivity was calculated by VALPROD, by cylindricalizing the core and reflector rings. The properties for the core were taken from Section 10.4.1 while the properties used for water and steel are tabulated in Table 6.9. The fast properties were calculated by MUFT-III while the thermal properties were calculated by MUFT-III while the thermal properties were evaluated at hardened neutron temperatures (Appendix A). The effect of reducing the first water gap in the reflector from 1/4 in. to 1/8 in. was also calculated and compared to the effect of reducing all water gaps to 1/8 in. The results of these calculations are given in Tables 6.10 and 6.11. It is seen that the model tends to underestimate the effect of the laminated reflector by about 0.2% ρ .

TABLE 6.9
STEEL AND WATER NUCLEAR PARAMETERS

Parameter	Pure Water		Pure 304L Stainless Steel	
	T = 68°F	T = 510°F	T = 68°F	T = 510°F
$N_H \times 10^{-24}$ (atoms/cc)	0.066730	0.052358	-	-
$N_O \times 10^{-24}$ (atoms/cc)	0.033365	0.026179	-	-
$N_{ss} \times 10^{-24}$ (atoms/cc)	-	-	0.085112	0.085112
τ (cm ²)	33.742736	54.253082	161.636403	161.635806
P	0.987035	0.988621	~ 0	0.000004
D_f (cm)	1.600095	2.015833	1.314456	1.314459
Σ_a (cm ⁻¹)	0.016960	0.009947	0.193228	0.144415
D (cm)	0.172303	0.297148	0.311170	0.326055

TABLE 6. 10
SM-2 EXPERIMENTAL CORE (T = 68°F)

Configuration	$\Delta\rho(\%)$	$\Delta\rho(\text{cents})^*$
All water reflector	-	-
Reference reflector		
Experimental value	+0. 59	+ 80
Calculated value	+0. 39	+ 53. 5

* Based on an effective delayed neutron fraction of 0. 0073.

TABLE 6. 11
SM-2 REFERENCE CORE #2 (T = 510°F)

Configuration	$\Delta\rho(\%)$	$\Delta\rho(\text{cents})^*$
All water reflector	-	-
Reference reflector	+0. 24	+ 32. 8
First water gap		
1/8 in	+0. 37	+ 50. 6
All water gaps		
1/8 in.	+0. 63	+ 86. 7

* Based on an effective delayed neutron fraction of 0. 0073.

6. 3 EFFECT OF A FLOW DIVIDER

From Section 5. 2, the reference flow divider would cause an expected reactivity decrease at 440°F of about 0. 60% ρ . This value is calculated for the reference core, described in Section 2. 3. The measured reactivity loss, as reported in reference (4), lies somewhat between -96. 6 cents and -128 cents, or -0. 7% ρ and -0. 93% ρ based on an effective delayed neutron fraction of 0. 0073. These latter values cannot be compared directly to the calculated value since the measured values were made at 68°F upon the midlife core mockup; however, for design purposes, a value of about -0. 8% ρ for the reactivity loss due to a flow divider seems reasonable and conservative.

6. 4 SIDE PLATE THICKNESS

The thickness of the side plates used in calculation of the nuclear parameters of reference core #2 (Section 6. 1) was taken as 30 mils. This thickness

was tentative, subject to metallurgical and hydraulic limitations. At the time of this report, the thickness had been changed to 40 mils. The effect of side plate thickness upon core reactivity and critical mass was calculated during the course of the nuclear survey described in Section 2.0. For various U-235 loadings, the change in reactivity between side plate thicknesses of 25, 40, and 50 mils was calculated. As described in Section 2.4, the nuclear properties were calculated by Program 50, MUFT-III, and Plate P₃ theory for various U-235 loadings and the reactivity was calculated by use of Program 51. The results of these calculations are given in Table 6.12. The effect upon critical mass was obtained by the use of Fig. 2.3, a plot of critical mass as a function of core metal-to-water ratio. These results are also given in Table 6.12. It is seen that the average reactivity loss per mil of side plate is about $-0.07\% \rho$ with an increase in the critical mass of about 0.034 kg per mil of side plate. Therefore, an increase in side plate thickness from 30 to 40 mils results in a reactivity loss of $-0.7\% \rho$ and an increased critical mass of +0.34 kg.

TABLE 6.12
EFFECT OF SIDE PLATE THICKNESS

Side Plate Thickness (mils)	$\Delta \rho(\%)$	Metal to Water Ratio	(Critical Mass) Kg U-235
25	-	0.3234	-
40	-1.05	0.3411	+ 0.51
50	-1.75	0.3545	+ 0.85

6.5 COATING OF THE BURNABLE POISON PARTICLES

If the fuel plates are sintered in a hydrogen atmosphere instead of in vacuum, it may be necessary to coat the ZrB₂ particles with some non-reactive material such as tungsten, chromium, niobium, etc. Coating thicknesses which would equal 10, 20, or 30% of the particle volume were contemplated. As support to the metallurgical program, preliminary calculations were made to determine the loss of core reactivity as a function of coating thickness and material. The total boron loading was held constant. For simplicity, due to the preliminary nature of the calculations, only thermal effects were considered.

For small changes in reactivity:

$$\Delta\rho \approx \frac{\Delta\Sigma_a}{\Sigma_a} = \frac{\sigma_a^{cm} N^{cm}}{\Sigma_a} \left[\frac{\epsilon V_{ZrB_2}}{V_P} \right]$$

where: $\Delta\Sigma_a$ = change in the plate total macroscopic absorption cross section due to coating of the particles

Σ_a = initial plate total macroscopic absorption cross section = 2.250 cm^{-1} at 68°F (0.0331 ev)

σ_a^{cm} = thermal microscopic absorption cross section of the coating material

N^{cm} = atoms/ cm^3 of the pure coating material

ϵ = coating material volume fraction of ZrB_2 particle volume

V_{ZrB_2} = volume of ZrB_2 particles per plate = 0.398344 cm^3

V_P = volume of a fuel plate = 36.212134 cm^3

The results of these calculations are given in Table 6.13.

TABLE 6.13
REACTIVITY LOSS ($\Delta\rho$) DUE TO COATING OF THE ZrB_2 PARTICLES

Coating Material	Coating Thickness *		
	0.10	0.20	0.30
Tungsten	0.046%	0.092%	0.138%
Chromium	0.009%	0.018%	0.027%
Niobium	0.002%	0.005%	0.007%

* Fraction of ZrB_2 Particle Volume

6.6 REFERENCE CORE #3

The loadings for reference cores #1 and #2 (Sections 2.3 and 6.1) are identical in the fixed elements even though various design changes have been made. The loadings for the control fuel elements are based upon their fuel matrix dimensions and upon the fixed element loadings; therefore, since the fuel matrix dimensions of the two reference core differ, the loadings for the control fuel elements differ.

6.6.1 U-235 Loading

The lifetime of the core is based upon the U-235 loading; the loading of 37.5 kg for all fixed elements was determined during the nuclear survey described in Section 2.3. This loading has remained invariant throughout this report since it was desirable to maintain a weight percent of UO_2 in the meat similar to that in the SM-1. In addition, the loading was sufficient to offset losses in core life due to primary system design changes since improvements in the secondary design resulted in reduced requirements on total core energy release.

6.6.2 Boron Loading

The boron loading has remained flexible to reflect changes in core design. The design requirement is for a K_{eff} of 1.03 at operating temperature and pressure, at 0 MWYR, and equilibrium xenon; the boron loading may be adjusted to meet this design requirement.

6.6.3 Determination of Reference Loading #3

A new reference boron loading was calculated for a core similar to that described in Section 6.1. The nuclear parameters of the fixed and control fuel elements were calculated as a function of boron content by use of the Program 50, Program 52, Program 54, MUFT-III, and Plate-Type P_3 Theory, IBM-650 codes. The reactivity of the core was calculated as a function of boron loading by use of the VALPROD, IBM-650 code. The control rod fuel elements were replaced by an "equivalent ring" as described in Appendix C; the flow divider was also replaced by an equivalent ring of steel having a cross-sectional area equal to that of the actual flow divider. The calculation was based upon the following:

1. Average core temperature of 510°F.
2. Primary system pressure of 2000 psia.
3. Two-pass core with a 94 mil flow divider.
4. Thick laminated steel reflector.
5. Design K_{eff} of 1.03 at 0 MWYR, with equilibrium xenon.

A self-shielding factor of 0.889 for boron particle size was included; this factor was obtained from Section 7.7. This factor represents an average value for the reference particle size range of 149 to 74 microns.

In addition, a correction factor to allow for a calculational model error was included. From results presented in Section 10.0, it is shown that the theoretical analysis of the SM-2 Flexible Critical Experiments tends to predict higher reactivities than the measured values. Experiments performed on cores containing 18 plates and 16 plates per fixed and control fuel elements, respectively, yielded reactivities greater than their respective calculated reactivities by the amounts shown in Table 6.14. It is shown that the range is from 0.563% to about 2% ρ . For the full size cores containing 45 fuel elements, the reactivity difference varies from 1 to 2% ρ . Therefore, a model correction of -1.5% ρ was used.

TABLE 6.14
COMPARISON OF MEASURED AND CALCULATED REACTIVITIES
(SM-2 Experimental Cores)

Experiment	Experimental ρ (%)	Calculated ρ (%)	$\Delta\rho$ (%)
Minimum Core Size	0.29	0.855	0.563
Full Size Core (No Boron)	23.0	25.0	2.0
Reactivity vs. Boron	-	-	1-2

The results of these calculations are shown in Fig. 6.3, in which K_{eff} of the core is plotted as a function of actual boron content of the core. It is shown that 63.4 grams of B-10 are needed to yield the desired K_{eff} of 1.03 at 510°F, at 0 MWYR, with equilibrium xenon. The total U-235 loading in the core is 36.2 kg. The reference B-10 and U-235 loadings in the fuel plates are tabulated in Table 6.15.

TABLE 6.15
REFERENCE CORE #3 - B-10 AND U-235 LOADINGS

<u>Description</u>	<u>Fixed Element</u>	<u>Control Fuel Element</u>
U-235 per plate, gm	46.30	40.56
UO ₂ per plate, gm	56.77	49.73
B-10 per plate, gm*	0.08106	0.07102
ZrB ₂ per plate, gm*	2.30603	2.02041
Wt. % UO ₂ in matrix	26.2	26.2
Wt. % natural boron in matrix	0.21	0.21
U-235 per element, gm	833.4	648.96
B-10 per element, gm*	1.45908	1.13632
Number of elements in the core	38	7

* Includes self-shielding effects due to lumps.

Figure 6.1

Figure 6.3

7.0 MISCELLANEOUS CORE CALCULATIONS

7.1 TEMPERATURE COEFFICIENT (1)

The temperature coefficient of the SM-2 core as a function of temperature was calculated from the slope of a calculated reactivity vs. temperature curve. The latter curve was obtained by calculating the reactivity of the core at various temperatures by use of modified two-group theory. The reflector savings and reactivities for both a pure water and a laminated steel-water reflected core were calculated by use of the FINK-1, IBM-650 code. The fast group parameters were calculated by use of the MUFT-III code, using input prepared by Program 50. The thermal parameters were calculated by the plate-type P_3 theory code also using input prepared by Program 50. The core considered was similar to the one described in Section 2.3, except that slight modifications were made in the assumed buckling (as used in the MUFT-III input), in the average thermal neutron energies as calculated for the various average core temperatures, and that 18.8 a/o B-10 in boron was assumed: therefore, the properties at 68°F and 440°F differ slightly from those presented in Section 2.3.

The nuclear parameters of the core and reflectors considered, calculated at temperatures of 68°, 440°, 480°, and 510°F, are given in Tables 7.1, 7.2, and 7.3. The properties of the core are for all fixed elements at 0 fuel burnup with no xenon. The properties for the laminated reflector were calculated on a homogenized basis; a steel-to-water volume ratio of 12 was assumed.

Curves of ρ vs. temperature are shown in Fig. 7.1 for both the water and the laminated steel reflected cores. Fig. 7.2 shows the temperature coefficient calculated from the slope of the ρ vs. temperature curves. Also shown in Fig. 7.2 is the latest experimental data of $(\Delta\rho/^\circ\text{F})$ for the SM-2 as measured during the SM-2 Flexible Critical Experiments. (2) In addition, the latest experimental data for the SM-1 core (3) are also shown for comparison; this curve was obtained over a period of 9.1 MWYR, but no significant change in $(\Delta\rho/^\circ\text{F})$ with lifetime was found. The experimental data, obtained from the SM-2 Critical Experiments, cannot be directly compared to the calculated curve due to the omission of certain effects which may influence the experimental values. The temperature coefficient may be a function of control rod or bank position. In addition, the value of the effective delayed neutron fraction, β_{eff} , used in converting experimental measurements expressed in cents to reactivity, is a function of the temperature of the core; in Fig. 7.2, a constant value of β_{eff} of 0.0073 has been used. Furthermore, the difference between the heating curve and the curve obtained by aluminum addition, Fig. 4.9 of APAE No. 54 (2), may increase with temperature. Therefore, the comparison between the calculated and measured temperature coefficient curves shown in Fig. 7.2 can be considered good. It is seen that the temperature coefficient at 510°F is expected to be about $-3.25 \times 10^{-4}/^\circ\text{F}$ and about $-0.75 \times 10^{-4}/^\circ\text{F}$ at 68°F.

TABLE 7.1
CORE NUCLEAR PARAMETERS AS A FUNCTION OF TEMPERATURE

(0 Burnup, 45 Fixed Elements, No Xenon)

Parameter	Units	T = 68°F E = 0.0331 ev P = 1200 psia	T = 440°F E = 0.0553 ev P = 1200 psia	T = 480°F E = 0.0581 ev P = 1600 psia	T = 510°F E = 0.0600 ev P = 2000 psia
τ	cm ²	32.200763	42.137103	44.747251	46.734276
P		0.612068	0.586996	0.575698	0.567397
K _f		1.283039	1.278617	1.278950	1.279119
D _f	cm	1.290197	1.445302	1.480516	1.506396
$\nu \Sigma_f$	cm ⁻¹	0.019943	0.018113	0.017954	0.017836
Σ_a	cm ⁻¹	0.420278	0.332834	0.323729	0.320565
K _{th}		1.460495	1.461812	1.462634	1.461873
D	cm	0.161503	0.228756	0.239088	0.247123
L ²	cm ²	0.384276	0.687297	0.738543	0.770898
Radius	cm	28.238582	28.238582	28.238582	28.238582
Height	cm	55.245	55.245	55.245	55.245

TABLE 7.2
WATER REFLECTOR PROPERTIES

Parameter	Units	T = 68°F E = 0.033 lev P = 1200 psi	T = 440°F E = 0.0553 P = 1200 psi	T = 480°F E = 0.0581 P = 1600 psi	T = 510°F E = 0.0600 P = 2000 psia
τ	cm ²	33.742736	47.505052	51.304443	54.253082
P		0.987035	0.988483	0.988563	0.988621
D _f	cm	1.600095	1.893770	1.963634	2.015833
Σ_a	cm ⁻¹	0.016960	0.011081	0.010398	0.009947
D	cm	0.172303	0.266751	0.282988	0.297148
L ²	cm ²	10.159375	24.072827	27.215618	29.873127
N _H x 10 ⁻²⁴	atoms/ cm ³	0.066730	0.056000	0.053859	0.052358
N _O x 10 ⁻²⁴	atoms/ cm ³	0.033365	0.028000	0.026930	0.026179
S _r	cm	6.591674	8.038932	8.298437	8.494191
S _z	cm	6.548372	7.930721	8.170819	8.349164

TABLE 7.3
LAMINATED STEEL REFLECTOR* PROPERTIES

Parameter	Units	T = 68°F E = 0.0331 ev P = 1200 psi	T = 440°F E = 0.0553 ev P = 1200 psi	T = 480°F E = 0.0581 ev P = 1600 psia	T = 510°F E = 0.0600 ev P = 2000 psia
τ	cm ²	108.573667	117.009633	119.121147	120.630317
P		0.127020	0.108106	0.101447	0.096830
D _f		1.092258	1.118522	1.125161	1.129939
Σ_a	cm ⁻¹	0.179667	0.139708	0.136268	0.134071
D	cm	0.293005	0.318868	0.321609	0.323629
L ²	cm ²	1.630804	2.282388	2.360121	2.413862
N _H x 10 ⁻²⁴	atoms/ cm ³	0.005133	0.004308	0.004143	0.004028
N _O x 10 ⁻²⁴	atoms/ cm ³	0.002567	0.002154	0.002072	0.002014
N _{SS} x 10 ⁻²⁴	atoms/ cm ³	0.078565	0.078565	0.078565	0.078565
S _r	cm	9.851297	11.193388	11.499057	11.729163
S _z	cm	6.609579	8.034537	8.287074	8.476231

* Properties given are for a metal-to-water volume ratio of 12.

7.2 PRESSURE COEFFICIENT (1)

The pressure coefficient as a function of temperature was calculated from the calculated curve of temperature coefficient vs. temperature (Fig. 7.2) and tables of thermodynamic properties of compressed water.⁽⁴⁾ It was assumed that the temperature coefficient is a function of water density alone; analysis of SM-1 measurements of temperature and pressure coefficients indicate that this is a good assumption. The pressure coefficient was then calculated from the equivalence of temperature and pressure changes on the water density. At a given temperature, T , a small increase in temperature at constant pressure causes a certain change in water density, $(\Delta T)_P$; the pressure change (at constant temperature) required to yield the same change in water density may then be found and is designated $(\Delta P)_T$. Therefore:

$$\left(\frac{\Delta \rho}{\Delta P} \right)_T = \left(\frac{\Delta \rho}{\Delta T} \right)_P \frac{(\Delta T)_P}{(\Delta P)_T}$$

The results of these calculations for the water reflected SM-2 core are given in Fig. 7.3. The pressure coefficient at 115°F as measured on the SM-1⁽³⁾ is also shown for comparison. At the SM-2 operating conditions of 510°F and 2000 psia, the pressure coefficient is $+ 2.77 \times 10^6 \Delta \rho / \text{psia}$, a factor of 100 less than the temperature coefficient evaluated at the same temperature. The pressure coefficient increases rapidly with higher temperatures; however, even at 600°F, it is still a factor of 80 less than the temperature coefficient.

7.3 HEAT FLUX IN THE FUEL PLATES

The heat flux through the SM-2 fuel plates as a function of average core temperature and thermal flux, was calculated both with and without the heat produced by fast fissions. The thermal cross sections were evaluated at hardened neutron energies of 0.0331 ev, 0.0549 ev, and 0.060 ev, corresponding to average core temperatures of 68°F, 440°F, and 510°F respectively. The heat flux was calculated from the following:

$$q'_A = \frac{1}{2} \left[\text{AEC} \sum_i P_i \bar{\Phi} \right] \quad \text{Btu/hr. -ft}^2$$

$$q_A = q'_A / \beta \quad \text{Btu/hr. -ft}^3$$

where:

q'_A = heat flux calculated for thermal fissions only

q_A = heat flux calculated with both thermal and fast fissions included

β	= fraction of total fissions which are thermal
E	= 174 mev released per fission, which is retained in the core
C	= fuel plate thickness = 0.1016 cm
$\sum_f^P$	= thermal macroscopic fission cross section of the fuel plate (cm ⁻¹)
$\bar{\phi}$	= average thermal flux (neutrons/cm ² -sec)
A	= 5.073621 x 10 ⁻¹⁰ (Btu/hr-ft ²)/(Mev/sec-cm ²)

The values of $\sum_f^P$ for the fuel plates at various temperatures were calculated by use of the Program 50 and Plate-Type P3 Theory, IBM-650 codes. The values of β were calculated by methods described in Section 1.5.1. These values are tabulated in Table 7.4. The results of these calculations are shown in Figs. 7.4 and 7.5 in which heat flux is plotted as a function of thermal neutron flux. Fig. 7.4 shows the heat flux due to thermal fissions only while Fig. 7.5 includes both thermal and fast fission effects.

TABLE 7.4
VALUES OF $\sum_f^P$ AND β USED IN HEAT FLUX CALCULATIONS

Temperature (°F)	β	$\sum_f^P$ (cm ⁻¹)
68	0.6205	1.48002
440	0.5968	1.066404
510	0.5729	1.005895

It is seen from Tables 6.8 and 6.4 of Section 6.1 that the calculated thermal flux for the SM-2 at startup is 1.96 x 10¹³ neutrons/cm²-sec and that approximately 43% of the fissions are thermal. From Fig. 7.5, this corresponds to an average heat flux of 1.58 x 10⁴ Btu/hr-ft² at 510°F. From Section 4.0, the calculated axial and radial peak to average power values are 1.84 and 1.95 respectively; the latter value being the value of Q($\Delta\theta$) in element 44. Therefore, the maximum thermal neutron flux is 7.03 x 10¹³ neutrons/cm²-sec; from Fig. 7.5, this corresponds to a heat flux of 5.44 x 10⁵ Btu/hr-ft² (for 2 sq. in.). Local values would be higher.

7.4 POWER DISTRIBUTION FOR A FULLY INSERTED CENTRAL CONTROL ROD

A flatter radial power distribution could be accomplished by fully inserting the central control rod and operating with a six rod bank. A comparison of the radial power distributions for a fixed element core, and a fixed element core

with the central control rod fully inserted was made for a laminated steel reflected core at 510°F, 2000 psia, 0% burnup, and no xenon; the power distributions were calculated by VALPROD. The control rod was represented by the following cylindrical configuration: a water region of equivalent radius 3.528144 cm. surrounded by a Eu-SS region of outer radius 3.980962 cm., which in turn was surrounded by a water - SS region of outer radius 4.209555 cm.; the latter region contains the control rod basket. The flow divider was not included in the calculation.

The properties of the fixed elements are given in Section 6.1 and the properties for pure water and the laminated steel reflector are given in Section 7.1. The properties for the absorber shell region used in the VALPROD calculations are tabulated in Table 7.5.

TABLE 7.5
PROPERTIES OF CONTROL ROD ABSORBER SHELL

(T = 510°F; 0% Burnup)

Parameter	Units	Value
τ	cm ²	0.602777
P		0.047183
K _f		-
D _f	cm	0.284204
Σ_a	cm ⁻¹	3.046194
K _{th}		-
D	cm	0.088619

The results of these calculations are shown in Fig. 7.6. It is seen from Fig. 7.6 that operation of the core with the central rod fully inserted results in a more uniform radial power distribution. In addition, the required coolant flow rate could be decreased due to the reduction in the gross peak to average value of the radial power distribution from 1.6 to 1.37, or a reduction in Q($\Delta\theta$) of 14%.

7.5 TRANSIENT XENON IN THE SM-2

A study of transient xenon in the SM-2 was made utilizing the model described in Section 1.8. The model was first applied to the SM-1. By varying the value of the distribution factor, α , until the results were in good agreement with experimental data obtained upon the SM-1 core. (3), (5), (6) The constants used in these calculations are tabulated in Tables 7.6 and 7.7.

TABLE 7.6
MISCELLANEOUS PLANT CONSTANTS

Parameter	Units	SM-1 (T = 440°F)	SM-2* (T = 510°F)
λ_I	sec ⁻¹	2.89×10^{-5}	2.89×10^{-5}
λ_{xe}	sec ⁻¹	2.11×10^{-5}	2.09×10^{-5}
γ_1		0.059	0.059
γ_2		0.003	0.003
$\bar{\sigma}_{xe}^a$	cm ²	1.8×10^{-18}	1.845×10^{-18}
V	cm ³	1.39×10^5	1.39×10^5
δ		3.24×10^{10}	3.2175×10^{10}
P	watts	10.3×10^6	28.0×10^6

* SM-2 constants are based upon the latest data presented in ANL 5800. (7)
The SM-1 constants are based upon previously used values.

TABLE 7.7
PLANT PARAMETERS AS A FUNCTION OF CORE LIFETIME

Parameter	SM-1 CORE (MWYR)					SM-2
	6.15	7.2	9.1	10.7	12.1	0 MWYR
$\sum_{\text{core}}^t (\text{cm}^{-1})$	0.11486	0.1120	0.1065	0.1021	0.0982	0.19039
β	0.778	0.7825	0.7711	0.7985	0.8048	0.573
K_1	3.332	3.350	3.40	3.445	3.482	1.16783
K_2	0.853	0.856	0.993	1.18	1.29	0.90
ϕ^*	0.0263P	0.0272P	0.0289P	0.0309P	0.0316P	0.01166P

* ϕ has been scaled as described in APAE No. 38. (8)

A comparison between the analog simulation and experimental data from the SM-1 is shown in Fig. 7.7 for the change in bank position as a function of time after shutdown. On the basis of this comparison, a value of $\alpha = 1.3 + 0.3 t/12$ where t is in hours, best satisfies the experimental data over a 12 hr. period during the transient condition. It was also found that α seems to decrease with core life.

The value of $\alpha = 1.3$ was taken to also apply to the SM-2 for which no experimental data is available. $\Delta\alpha$ was taken to be 0.3 and to increase linearly from 0 to 0.3 over a period of 12 hrs. after shutdown, the time required for maximum xenon concentration to occur. The results for the SM-2 are shown in Fig. 7.8, 7.9, and 7.10; this simulation was made for various changes in power for the core in the early stages of core life. From Fig. 7.8, it is seen that the reactivity tied down by equilibrium xenon and peak xenon is 2.7% ρ and 4.3% ρ respectively for the SM-2 core, near the beginning of core life.

7.6 EFFECTIVE DELAYED NEUTRON FRACTION

The reactivity of the core may be computed from the inhour equation:

$$\rho = \frac{\lambda}{T K_{\text{eff}}} + \sum_{i=1}^m \frac{\beta_i}{1 + \lambda_i T}$$

$$\beta_i = a_i \beta$$

where:

- ℓ = neutron lifetime (about 5×10^{-5} sec)
- T = reactor period in seconds
- K_{eff} = effective multiplication factor (assumed to be unity)
- β_i = delayed neutron fraction of the i th group
- β = total delayed neutron fraction
- λ_i = delay constant of the i th group
- a_i = relative abundance of the i th group

The above equation, although the one customarily used, is not completely exact. Another form of the equation is:

$$\rho = \frac{\ell}{T K_{\text{eff}}} + \sum_{i=1}^m \frac{\gamma_i \beta_i}{1 + \lambda_i T}$$

where γ_i is the effectiveness of the i th group delayed neutrons compared with prompt neutrons. However, because of lack of data, it is generally assumed that $\gamma_i = \beta_i$ and

$$\gamma_i = \beta_{\text{eff}} / \beta$$

where β_{eff} equals the effective total delayed neutron fraction. Therefore:

$$\rho = \frac{\ell}{T K_{\text{eff}}} + \beta_{\text{eff}} \sum_{i=1}^n \frac{a_i}{1 + \lambda_i T}$$

Because β_{eff} is not usually experimentally determined, experimental facilities usually report reactivity changes in cents. Therefore, in order to get an equation of cents as a function of the reactor period, by definition, the latter equation is divided through by β_{eff} :

$$\rho / \beta_{\text{eff}} = \frac{\ell}{T K_{\text{eff}} \beta_{\text{eff}}} + \sum_{i=1}^n \frac{a_i}{1 + \lambda_i T}$$

This equation gives the reactivity in dollars as a function of the reactor period. By definition, one dollar = 100 cents. The relation of converting from the experimentally obtained reactivity, expressed in cents, to reactivity, expressed in percent, becomes:

$$\rho(\%) = \rho(\text{cents}) \times \beta_{\text{eff}}$$

The value of β_{eff} which has been used through most of the SM-2 analysis has been taken as 0.0073. However, the value of β_{eff} not only varies with core composition, size, and temperature, but also probably varies with control rod bank position. Since the reliability of the calculational models is generally determined by a comparison between experimental and calculated results, the value which is used for β_{eff} is of considerable importance. It is of great value to have some method for calculation of β_{eff} as a function of core composition, core size, and temperature. R. L. Murray has derived an analytic expression for β_{eff} , using the modified two-group theory of a reflected spherical core. (9) In addition, methods are available to calculate β_{eff} approximately by the use of the MUFT-III code, employing a special source file for the delayed neutron energy groups. (10) The theory derived by R. L. Murray is based upon several simplifying assumptions, including:

- 1) A spherical core was assumed instead of the normally assumed shape of a right circular cylinder.
- 2) A water reflector was assumed instead of the actual configuration of a water axial reflector and a laminated steel radial reflector.

Therefore, when a more exhaustive analysis of the SM-2 critical experiments is undertaken, an evaluation of these assumptions upon the value of β_{eff} should be made.

Calculations performed upon the SPERT-III, a core similar to the SM-1, indicate a value of β_{eff} of about 0.0078. (11) Both values ($\beta_{\text{eff}} = 0.0073$ or 0.0078) have been used from time to time during the SM-2 nuclear analysis; the value used in a particular calculation is generally designated.

7.7 SELF-SHIELDING BY BORON PARTICLES IN FUEL

The effectiveness of the boron as a burnable poison is reduced due to self-shielding effects within the boron particles. R. L. Murray has described a method (12) to calculate the self-shielding effects for boron particles of appreciable size in the meat of the SM-2 fuel plates. This method is based on theory developed by K. M. Case et al, (13) which states that "the ratio of average flux within a body immersed in an isotropic neutron bath to the flux at large distances from the object is equal

to the escape probability for a uniform source of neutrons in the body". Therefore:

$$P_0 = \bar{\phi} / \phi_\infty$$

This ratio, P_0 , is also the ratio of the effective cross section to that based on the undisturbed or distant flux as can be seen from:

$$P_0 = \bar{\phi} / \phi_\infty = \frac{\bar{\phi} \Sigma_a}{\phi_\infty \Sigma_a} = \frac{\phi_\infty (\Sigma_a)_{\text{eff}}}{\phi_\infty \Sigma_a} = \frac{(\Sigma_a)_{\text{eff}}}{\Sigma_a}$$

With small particles separated by considerable distances, this theorem is applicable; this condition is met for the SM-2 fuel plates in which the ratio of separation of centers to radii is greater than 6. For the case of spherical particles of radius "a" with product $\Sigma_a a \ll 1$:

$$P_0 \approx 1 - 3/4 (\Sigma_a a) + 2/5 (\Sigma_a a)^2 - 1/6 (\Sigma_a a)^3$$

This equation is plotted in Fig. 7.11 in which the self-shielding factor P_0 is given as a function of the product $(\Sigma_a a)$. The reference boron particle size (ZrB_2) contemplated for the SM-2 is minus 100 plus 200 mesh or 149 to 74 microns. This corresponds to radii of 0.00745 to 0.0037 cm respectively. In addition, the density of ZrB_2 is taken as 6.09 gms/cm³. Therefore:

$$\Sigma_a = N_B \bar{\sigma}_a^B$$

$$N_B = \frac{(2)(6.09)(0.6024 \times 10^{24})}{(112.86)} = 0.065011 \times 10^{24} \text{ atoms/cm}^3$$

The values of $\bar{\sigma}_a^B$ are $580.6346 \times 10^{-24} \text{ cm}^2$ and $434.6316 \times 10^{-24} \text{ cm}^2$ at 68°F ($E = 0.0331 \text{ ev}$) and 510°F ($E = 0.060 \text{ ev}$) respectively. Substituting these numbers into the equation for P_0 , yields expected self-shielding factors at 510°F of 0.858 to 0.926 for the range of particles sizes; the average self-shielding factor is about 0.89. For comparison, the boron particles (B_4C) in the SM-1 are minus 325 mesh or 44 microns. Applying the same type of analysis to B_4C , having a density of 2.5 gms/cm³ and a molecular weight of 55.29 gms per mol, yields a maximum self-shielding factor at 510°F of 0.925.

7.8 GAS FORMATION IN THE FUEL MATRIX

Calculations were performed to obtain an estimate of the total amount of gases formed from the burnup of U-235 and B-10 in an SM-2 fuel plate. At the time of this calculation, the parameters of the core, operating at 440°F and 1200 psia, were:

U-235 loading = 37.5 kg

B-10 loading = 65.6 gm

β = percent of thermal fission = 61.528%

V = core volume = $1.39988 \times 10^5 \text{ cm}^3$

P = power level = 28×10^6 watts

Σ_f = initial macroscopic fission cross section of core = 0.196940 cm^{-1}

Fuel plates in the core = 796

By use of the equations in Section 6.1, the average thermal flux is calculated to be 2.01×10^{13} neutrons/cm²-sec at start of core life. The average thermal flux during core life was assumed to be about 1.25 times the value at startup. The radial and axial maximum to average flux ratios were taken to be 1.6 and 1.87 respectively; the latter numbers were based upon calculations performed on the SM-1 (14) and during the core planning stage of the SM-2 (15). Therefore, the average thermal flux over the core lifetime is 2.52×10^{13} neutrons/cm²-sec; for the centermost fuel plate the peak thermal flux is 7.55×10^{13} neutrons/cm²-sec. The core lifetime is about one year or about 3×10^7 seconds. Tables in ORNL 2127 (16) tabulate fission products as a function of thermal flux and irradiation time; Table 7.8 tabulates the atomic ratios of Kr and Xe to U-235 as a function of thermal flux at end of core life. The mols of U-235 per plate is about 0.205.

TABLE 7.8
ATOMIC RATIOS AT END OF CORE LIFE

Thermal Flux (neuts/cm ² -sec)	Kr/U-235	Xe/U-235
1×10^{13}	0.00691	0.043
3×10^{13}	0.0207	0.136
1×10^{14}	0.0691	0.469
2.52×10^{13}	0.0174	0.114
7.55×10^{13}	0.0522	0.354

Assuming that all of the B-10 is burned up to form helium:

The moles of B-10 per plate is 0.00825. The results of these calculations are given in Table 7.9. The He actually released is estimated to be in the range of 1 to 4%. The values in Tables 7.8 and 7.9 are conservative since the values given in ORNL-2127 are calculated for cross sections evaluated at 0.025 ev while these calculations used cross sections evaluated at 440°F (0.049 ev).

TABLE 7.9
TOTAL GASES FORMED AT END OF LIFE

<u>ϕ</u>	<u>Moles Kr/Plate</u>	<u>Moles Xe/Plate</u>	<u>Moles He/Plate</u>
Average	0.00349	0.0228	0.00825
Peak	0.0107	0.0725	0.00825

7.9 WIDTH OF FUEL MATRIX

In the SM-2 reference core described in Section 6.1, there is a considerable peaking of the power (and thermal flux) across the side plates; this local peaking is reflected in high coolant flow rate requirements. At the time of this report, the dimensions of the fuel matrix had been increased, over those described in Section 6.1, for metallurgical and thermal reasons. Increasing the width of the fuel matrix not only decreases the side plate peaking, but also decreases the weight percents of UO₂ and B-10 in the matrix.

The nuclear effects of changing the width of the fuel matrix were calculated by use of the plate-type P₃ theory, IBM-650 code; the input to this code was prepared by Program 50. The fixed element input to Program 50 was taken from the data presented in Section 6.1 except for the following modifications:

B-10 loading per plate = 0.08106 gm
 Length of core = 22 in. = 55.88 cm
 Fuel plate thickness = 0.04 in. = 0.1016 cm
 Width of clad fuel plate = 2.789 in. = 7.08406 cm
 Width of fuel matrix = variable

The effect upon the side plate peaking is given in Table 7.10 in which the ratio of the thermal flux at the edge of the active region (Section 1.1.2.1) to the average flux in the active region, is tabulated. The effect upon the power peaking was calculated by assuming a flat fast flux distribution across the cell;

therefore, as described in Section 5.1.3:

$$P_{\max}/P_{\text{avg}} = \left(\frac{\phi_{\max}}{\phi_{\text{avg}}} \right) \beta + (1 - \beta)$$

where β is the fraction of the total fissions which are thermal. The value of β at 510°F at 0 burnup of 0.573 given in Section 6.1 was assumed to hold for each case; in reality, this value is a function of the core parameters. In addition, an estimate of the effect upon core reactivity was obtained by assuming that the fast parameters were invariant for the various changes in fuel matrix width, and calculating the reactivity by use of the modified two group theory. These results are also given in Table 7.10. The effect upon the thermal parameters are tabulated in Table 7.11.

TABLE 7.10
EFFECT OF FUEL MATRIX WIDTH UPON FLUX,
POWER AND REACTIVITY

(T = 510°F; P = 2000 psia; No Xenon)

Width of Fuel Matrix	$\phi_{\max}/\phi_{\text{avg}}$	$P_{\max}/P_{\text{avg}}$	Reactivity Change+
6.4516* cm	1.4032	1.2310	-
6.7310** cm	1.2486	1.1424	0.15% ρ
6.84403 cm	1.2004	1.1148	0.25% ρ
6.95706 cm	1.1497	1.0858	0.33% ρ
7.08406 cm	1.0889	1.0509	0.41% ρ

* Calculated for reference core #2 described in Section 6.1.

** Latest reference width.

+ Calculated for an all fixed element core using a buckling of 0.006194 cm⁻²

TABLE 7.11
EFFECT OF FUEL MATRIX WIDTH UPON CORE THERMAL PARAMETERS

(T = 510°F; P = 2000 psia; No Xenon)

Parameter	Units	Width of Fuel Matrix (cm)				
		6.4516*	6.7310**	6.84403	6.95706	7.08406
Σ_a	cm ⁻¹	0.315808	0.327954	0.330776	0.333112	0.335074
Σ_s	cm ⁻¹	1.380611	1.374324	1.374512	1.374752	1.375114
Σ_{tr}	cm ⁻¹	1.348380	1.357603	1.360263	1.362479	1.364371
$\nu\Sigma_f$	cm ⁻¹	0.464204	0.483231	0.488243	0.492435	0.496033
K _{th}		1.469890	1.473471	1.476054	1.478288	1.480367
D	cm	0.247210	0.245531	0.245051	0.244652	0.244313

* Calculated for reference core #2 described in Section 6.1.

** Latest reference width.

7.10 NVT CALCULATIONS

As discussed in Section 1.7, the value of the integrated neutron flux (nvt) above 1 Mev is of importance in the determination of expected radiation damage to various reactor components.⁽¹⁷⁾ The determination of the nvt > 1 Mev for various SM-2 designs was calculated by two different methods and compared.

7.10.1 Two Fast Groups Method (18)

In this method, the neutron fluxes with energies greater than 1 Mev were obtained from the MUFT-III code using appropriate energy break points. The epithermal group was broken into two groups, one group with energies from 10 Mev to 1 Mev, and the second group from 1 Mev to above thermal energy. Various MUFT-III cases were run for the core and reflector materials (stainless steel and water) to obtain the fast two-group constants using a breakpoint of 1.05 Mev. The thermal parameters of the core, evaluated at 440°F, are tabulated in Section 2.3; the thermal parameters of the reflector materials were also evaluated at the hardened neutron energy of 0.0549 corresponding to the average core temperature of 440°F.

These constants were used in radial, three-group flux calculations (VALPROD) for various pressure vessel designs, AEL-386, AES-262, and AEL-395, Fig. 7.12 through 7.14 respectively. The design dimensions are listed in Table 7.13. The absolute flux on the vessel can be found by multiplying the flux at the vessel by the appropriate normalization factor. The fluxes from the VALPROD core were obtained from the formula:

$$\bar{\phi}_{\text{thermal}}^{\text{absolute}} = \frac{\delta P}{V} \left(\frac{\beta}{\Sigma_f} \right) = N_{\phi} \bar{\phi}_{\text{thermal}}^{\text{machine}}$$

where:

$$\delta = 3.24 \times 10^{10} \text{ fissions/watt-sec}$$

$$P = \text{power in watts}$$

$$V = \text{volume of the core (cm}^3\text{)}$$

$$\beta = \text{fraction of thermal fissions}$$

$$\Sigma_f = \text{thermal macroscopic fission cross section of core (cm}^{-1}\text{)}$$

$$N_{\phi} = \text{normalization factor for the fluxes}$$

An alternate method which gives the factor N_{ϕ} with much less calculation can be obtained from the VALPROD output of ν_c , K_{eff} , and ν , related by the equation:

$$K_{\text{eff}} = \nu / \nu_c$$

$$N_{\phi} = \nu_c \frac{\delta P}{V} = \frac{\phi_{\text{thermal}}^{\text{absolute}}}{\phi_{\text{thermal}}^{\text{machine}}}$$

TABLE 7.12
VARIOUS SM-2 PRESSURE VESSEL DESIGNS

(Core Radius = 11.118 in.)

<u>Design</u>	<u>Ref. Drawing</u>	<u>Reflector Composition</u>	<u>Vessel Thickness</u>
31" ID Pressure Vessel	AEL-386 (Fig. 7.12)	4.336" H ₂ O	2.75" SS
31" ID Pressure Vessel	AEL-386 (Fig. 7.12)	0.125" H ₂ O + 1.125" SS + 0.125" H ₂ O + 1.3" SS + 0.125" H ₂ O + 1.3" SS + 0.25" H ₂ O	2.75" SS
34" ID Pressure Vessel	AES-262 (Fig. 7.13)	0.125" H ₂ O + 1.125" SS + 0.125" H ₂ O + 1.3" SS + 0.125" H ₂ O + 1.3" SS + 0.125" H ₂ O + 1.3" SS + 0.25" H ₂ O	2.75" SS
45-1/2" ID Pressure Vessel	AEL-395 (Fig. 7.14)	0.25" SS + 9.1" H ₂ O + 2" SS + 0.25" H ₂ O	2" SS

The vessels were assumed to be used for 20-year service at 25 MW (assuming a 100% load factor); the 20-yr. nvt values above 1 Mev for each design are listed in Table 7.13. These values have been corrected for the non-uniform axial flux distribution. The point of greatest radiation damage to the pressure vessel will be directly opposite the maximum axial flux peak in the core. The fluxes obtained by the VALPROD calculations were increased by a factor of 1.6, obtained by averaging the maximum to average axial flux peak over the life of the core. From the results shown in Table 7.13, it is concluded that water is about 10% better than stainless steel in attenuating the flux above 1 Mev. It is also seen that the smaller vessels will suffer considerably more radiation damage, a factor which strongly affects material selection.

TABLE 7.13
"NVT" VALUES ABOVE 1 MEV ON SM-2 PRESSURE VESSEL DESIGNS

ID for Vessel	Reflector	nvt(neuts/cm ²) (Two Fast Groups)	nvt(neuts/cm ²) (One Fast Group)
31 in. (Fig. 7.12)	Water	9.12×10^{21}	-
31 in. (Fig. 7.12)	4.35 in. Laminated Steel-Water (86% steel volume)	1.10×10^{22}	7.18×10^{21}
34 in. (Fig. 7.13)	5.65 in. Laminated Steel-Water (86% steel volume)	6.37×10^{21}	4.33×10^{21}
45.5 in. (Fig. 7.14)	Skirt, Water and Thermal Shield	3.89×10^{20}	3.00×10^{20}

Figs. 7.15 and 7.16 are plots of the three-group and two-group radial flux distributions for the 31 in. ID pressure vessel design. It can be seen that the flux above 1 Mev shown in Fig. 7.15 has nearly the same shape as the total fast flux shown in Fig. 7.16. This is the basis of the assumption used in calculation of the nvt above 1 Mev by the one-group method described in Section 1.7.

7.10.2 One Fast Group Method (19)

The nvt > 1 Mev was also calculated for the various vessel designs by the one-group method described in Section 1.7; these results are also shown in Table 7.13. The one-group calculations, using these assumptions, were originally thought to be conservative; however, as seen in Table 7.13, they agree closely with the more detailed calculation by the two-fast group method. Therefore, the one-group method allows a good fast estimate of the nvt above 1 Mev.

The ratio of the fast flux above 1 Mev to the total fast flux was computed both by the one-group theory (Section 1.7) and from the VALPROD calculations. These results are tabulated in Table 7.14. It is seen that the fraction of the flux above 1 Mev to the total fast flux is about 0.3 inside the core. Outside the core, the ratio depends upon the reflector composition. This constitutes a source of error in use of the one fast group method; however, the error is small when one considers the approximations embodied in the derivation of the one fast group theory, as given in Section 1.7.

TABLE 7.14
RATIO OF ABOVE 1 MEV FLUX TO TOTAL FAST FLUX

Vessel Design	One Fast Group (At Core Centerline)	Two Fast Group Method	
		(At Core Centerline)	(At Pressure Vessel)
31"ID-H ₂ O Refl. (Fig. 7.12)	0.29	0.31	0.34
31"ID-SS Refl. (Fig. 7.12)	0.29	0.31	0.27
34"ID-SS Refl. (Fig. 7.13)	0.29	0.31	0.25
45-1/2"ID-SS Refl. (Fig. 7.14)	0.29	0.31	0.29

7.10.3 Nvt vs. Pressure Vessel Size

The results of the calculations for the various pressure vessel designs as given in Section 7.10.2 were extrapolated to obtain a relationship between nvt and inside pressure vessel diameter. The results for both a thick steel and a water reflected core are shown in Fig. 7.17.

7.10.3.1 Selection of Reference Reflector and Vessel Configuration

Two types of pressure vessel designs were contemplated: a carbon steel (SA-212) vessel with a water reflector or a stainless steel (Type 347) vessel with a laminated steel reflector. In reference (20), it is reported that the allowable $nvt > 1$ Mev for SA-212 is 4 to 5 x 10¹⁹. Examination of Fig. 7.17 indicates that a vessel ID of 55 to 56 in. would be required. The actual reflector on such a design is a combination of water and steel, due to the presence of thermal shields; therefore, a reference ID of 55 in. was selected for SA-212. This design may be seen in Dwg. AEL-451, (Fig. 7.18).

In reference (20), it was indicated that austenitic stainless steel could be used with confidence at a $nvt > 1$ Mev of 1×10^{21} . From Fig. 7.17, this nvt corresponds to a vessel ID of 41 in. This diameter was selected as the reference diameter for austenitic stainless steel vessel. This design may be seen in Dwg. 450 (Fig. 7.19).

A stainless steel reactor vessel was finally selected for the SM-2 reference design principally because of its reduced vessel diameter and expected better resistance to radiation damage. The latest design of the SM-2 core and vessel, at the time of this report, is shown in Fig. B.3 of Appendix B.

7.11 HEAT GENERATION CALCULATIONS

7.11.1 Heat Generation in Thermal Shields and Pressure Vessel⁽¹²⁾

The heat generation rates in the thermal shields and pressure vessel for a SM-2 core and vessel configuration (Dwg. AEL-451, Fig. 7.18 and Fig. B.2 of Appendix B) were calculated by use of the ROC shielding codes described in Section 1.9. The radial flux and power distributions for the core were calculated by use of the VALPROD code. The distributions were calculated both at start-up and end of life for the core operating at 510°F and 2000 psia. Core properties are tabulated in Section 6.1. Steel and water properties are tabulated in Section 6.2. The core contains a 94 mil flow divider and is reflected by a thick laminated steel reflector. The effects of the control rods were not included. The fast and thermal flux distributions for start-up and end of life are shown in Figs. 7.20 and 7.21 respectively. The relative radial power distributions are shown in Fig. 7.22.

The axial flux and power distributions were also calculated by VALPROD. A water reflector above and below the core was assumed. The distributions were calculated with rods fully withdrawn. The most adverse heat generation occurs at end of life due to the increase in the average thermal and fast flux with core lifetime. The axial flux distribution at end of life is shown in Fig. 7.23.

Utilizing the flux distributions obtained from VALPROD, the heat generation rate distributions were obtained at end of core life by the ROC, IBM-650 codes.⁽²²⁾ The results of these calculations are shown in Fig. 7.24.

7.11.2 Heat Generation in the Control Rod Racks⁽²¹⁾

By use of the axial flux distributions described in the previous section, the gamma heat generation within the control rod racks at various positions from the bottom of the core were calculated by use of the ROC codes. It was assumed that the gamma dose rates were attenuated entirely by water. These results are shown in Table 7.15. Actually, the water-to-metal ratio near the bottom of the core is estimated to be of the order of 10 to 1 and increases considerably as a function of distance from the bottom of the core.

TABLE 7.15
GAMMA HEAT GENERATION RATES WITHIN THE RACK ON THE
CORE CENTERLINE

<u>Distance From Bottom of Core (in.)</u>	<u>Heat Generation (Btu/hr. -ft³)</u>
6	1.18×10^6
8	8.71×10^5
10	6.22×10^5
12	4.79×10^5
14	3.61×10^5
16	2.80×10^5

7.11.3 Gamma Heating Within the Core(12)

A substance placed within the core is heated predominantly by gammas. This gamma heating can be computed easily if the assumptions are made that the reactor volume is large compared to the relaxation length for gamma absorption, and that the gamma production is uniform throughout the core. The SM-2 core operating at 28 MW has an average power density equal to 202 watts/cm³ throughout the core. Since about 7% (7) of the fission energy of U-235 is released through gamma ray emission, the heating rate is found to be 14 watts/cm³. Consideration of the effects of finite size of the core on the leakage of gammas from it, reduces this figure by approximately 20%. Furthermore, there will be a contribution of about 1 watt/cm³ from the (n,γ) reaction within the core materials. These latter two assumptions give a final value, for the average core gamma heat rate, of about 12 watts/cm³. Since the power generation varies with position within the core, applying typical radial and axial peak to average values of 1.95 and 1.84 results in a maximum heating rate of about 43 watts/cm³.

7.11.4 Most Adverse Heat Generation Across a Cell

The most adverse heat generation distribution across the SM-2 fuel elements in a direction perpendicular to the fuel plates was evaluated for use in a fuel element distortion study. The most adverse distributions in order of importance take place in fuel elements in the following positions. (See Fig. 1.8).

- (a) A fixed fuel element in the outside ring of the core; for example, number 41 or 47. This position would be of interest only if the fuel

plates of these outside elements were oriented parallel to the laminated steel reflector. The radial power distribution in this element is shown in Fig. 7.25 at 510°F, 0% burnup. This element contains the steepest gradient of power generation.

- (b) A fixed fuel element adjacent to the central control rod. This element could possibly have an adverse power distribution if the central control rod were fully inserted for rod programming. The radial power distribution across element 45 is shown for this condition in Fig. 7.26.
- (c) A fixed fuel element adjacent to the flow divider; for example, number 42 or 46.

The heat generation density used for this study is of the form:

$$G(x, y, z) = \bar{G} F_1(x, y) F_2(z) F_3(x, y)$$

where:

$G(x, y, z)$ = heat generation density at a particular point in the meat of a fuel element (Btu/hr-ft³)

$\bar{G}$ = average heat generation density in the meat of the core (Btu/hr-ft³). For the SM-2, this value at full power is 1.268×10^8 Btu/hr-ft³

$F_1(x, y)$ = radial power distribution factor based on a VALPROD calculation in which the radial power in the core is averaged to unity.

$F_2(z)$ = axial power distribution factor based on a Windowshade calculation in which the axial power in the core is averaged to unity. In the SM-2, this may conservatively be defined at a point near the bottom of the core at the time of deepest bank insertion. This value of $F_2(z)$ may be found in Section 4.2.

$F_3(x, y)$ = a local power distribution factor, in the direction of interest, perpendicular to the fuel plates. This value is usually taken as unity.

At the time of this calculation, the best estimate of $F_2(z)$ was taken as 3.05. This value was based upon a non-uniform burnup calculation performed during the SM-2 core planning stage.⁽¹⁵⁾ Therefore:

$$G(x, y, z) = 3.866 \times 10^8 \times F_1(x, y) \quad \text{Btu/hr - ft}^3$$

7.11.5 Gamma Heating as a Function of Position on the Pressure Vessel

The gamma heat generation rate throughout the inner surface of the pressure vessel of the reference SM-2 design was calculated at 510°F near the end of core life by extrapolating the results found in the radial direction as presented in Section 7.11.1. The heat generation rate along the inside pressure vessel wall was obtained by considering the effects of the increased amounts of reactor materials that attenuate the gamma flux from the core and secondary sources as a function of distance from the pressure vessel belt line. The calculations were made based on the important assumption that the ratio of the structural materials and water between the core and the vessel remains constant as a function of distance from the pressure vessel midpoint. This is very nearly the case from 6 to 8 in. from the top and bottom of the core. At larger distances, the structural material is cut off and the water fraction between the core and the vessel increases rapidly. Since the attenuation coefficient of water is smaller than that of the steel, the gamma heat generation on the inner wall of the pressure vessel will be somewhat higher than those based on the assumption stated above. This is compensated to some extent by a decrease in the secondary (n, γ) source in the structural materials. A lack of knowledge of the thermal and fast fluxes at oblique angles to the core prevented an estimate of their effect on the heat generation on the pressure vessel wall.

Consider a series of "n" adjacent regions having gamma attenuation coefficients $\mu_1, \mu_2, \mu_3, \dots, \mu_n$. If the heat generation rate at the inside surface of region 1 is H_0 , the heat generation rate at the outside surface of region n, along a reference axis, may be written as:

$$H_n(x, y = 0) = H_0(x = 0, y = 0) e^{-\sum_{i=1}^n \mu_i x_i}$$

where:

H_n = heat generation rate at outside surface of region n.

H_0 = heat generation rate at inside surface of region 1.

μ_i = attenuation coefficient of i^{th} region.

x_i = thickness of i^{th} region.

The heat generation rate at the outside surface of region n, along an axis which makes an angle θ with the reference axis, may be written as:

$$H_n(x, y) = H_0(x = 0, y = 0) e^{-\left(\sum_{i=1}^n \mu_i x_i\right) \cos^{-1} \theta}$$

Therefore, the heat generation rate at a point, lying on an axis which makes an angle θ with the reference axis, may be related to the heat generation rate along the reference axis by the following:

$$H_n(x, y) = H_n(x, y = 0) e^{\left(\sum_{i=1}^N \mu_i X_i\right) (1 - \cos^{-1} \theta)}$$

The values of $\mu_i X_i$ may be found from the slopes of the heat generation rate vs. distance graph (Fig. 7.24) when plotted on semi-log paper. The results of these calculations are shown in Fig. 7.27. The gamma heat generation on the flange, assuming that the gamma attenuation is by water alone, was found to be 2.1×10^{-5} Btu/in³-sec. Since this is a conservative value, no correction for any effects by structural material is included.

7.12 DISCRETE BURNABLE POISON

In the early stages of the SM-2 program, there was concern over the high weight percent of boron in the fuel matrix necessary to control the reactor during the first part of core life. From a metallurgical standpoint, little or no data was available on the effects of long irradiation of plates containing nearly 3.5 times as much B-10 as in SM-1. Therefore, thought was given to utilization of a discrete burnable poison loading. Several methods of adding discrete poisons to the element were suggested; two possible methods are shown in Fig. 7.28. The first method utilizes narrow strips of stainless steel containing B-10 which would fit between the fuel plates at the dead edges of the plate. The second method utilizes special side plates containing B-10 which would be adjacent to the regular side plates. Other possible methods include adding B-10 to the cladding or to the picture frame portion of the fuel plate.

From a metallurgical standpoint, it was believed that a maximum B-10 content in stainless steel of 0.2 w/o would be a conservative value. For a loading of 65.6 gms of B-10, this would require 32.735 kg of steel, or a volume per element of 5.619 in³. Therefore, for an element length of 21.75 in., 0.258 in.² of cross-sectional area of boron stainless steel is needed per element. In the reference fixed element described in Section 6.1, the cladding has a total area of 0.501 in.², while the picture frame portion of the fuel plates has a total area of 0.132 in.²; however, there was doubt that a B-10 concentration of 0.2 w/o could be tolerated in those locations. Two side plates, each 2.863 in. by 0.0451 in., would present the required area of 0.258 in.² if such a design would be feasible.

If it is not feasible to utilize the existing steel cross-sectional areas of the reference element, it would be necessary to add an extra 32.735 kg of steel (containing 0.2 w/o B-10) to the core. This would increase the metal-to-water ratio of the core. From Fig. 2.3, this would increase the critical mass and fuel loading by 1.65 kg U-235.

If the discrete poison would be incorporated in the dead region of the elements (in the region containing the side plates and fuel plate dead edges), side plate peaking would be reduced. This would be advantageous in regard to coolant flow requirements; however, self-shielding would decrease the rate of poison burnup, with a resultant decrease in core lifetime. The P_3 flux distributions parallel to the fuel plates were calculated by use of Program 50 and the plate-type P_3 theory, IBM-650 codes. Fig. 7.29 shows the calculated thermal flux distribution parallel to the fuel plates for the reference fuel element with all boron located in the meat of the plates; flux distributions for 0.25, and 50% fuel burnups are shown, together with some calculated SM-1 values for comparison. Fig. 7.30 presents the flux distributions at 0.25, and 50% fuel burnups for an element with all boron located in the dead region as a discrete poison. It was assumed that the boron was introduced as extra steel (containing 0.2 w/o B-10) in the dead region. It is seen that not only has the initial side plate peaking been eliminated, but the flux distribution is greatly flattened as the fuel burns up.

The increase in critical mass due to addition of discrete poison results in less mass available for burnup. For a "realistic" mass utilization of 1.5 MWYR/kg U-235 (Section 7.14.1), the loss in lifetime due to the 1.65 kg U-235 loss amounts to 2.48 MWYR.

7.13 EFFECT OF A SOLUBLE POISON

As part of the SM-2 program, the feasibility of chemical control was evaluated (23) both as a partial substitute for burnable poison in the fuel element meat and as a means of improving plant performance; boric acid was chosen as the reference soluble poison. The bulk of the analysis was made on a chemical basis while the nuclear analysis was limited to the effects of a soluble poison upon the temperature coefficient and the power distribution. Use of a soluble poison could also reduce the required number of control rods, provided that provision is made to increase the boric acid concentration during core lifetime as the burnable poison in the fuel plates is depleted.

7.13.1 Effect Upon Temperature Coefficient

Pressurized water reactors possess inherent stability during load changes due to their negative temperature coefficients of reactivity. During normal operation, no external control is needed. Changes in the average coolant temperature provide the necessary adjustment of power level. When soluble poison is introduced into the primary coolant, this inherent stability is decreased.

The temperature coefficient of reactivity may be expressed by the following formula:

$$\frac{1}{K_{eff}} \left(\frac{d K_{eff}}{dt} \right) = \frac{1}{K_{eff}} \left(\frac{\partial K_{eff}}{\partial t} \right)_{Bc} + \frac{1}{K_{eff}} \left(\frac{\partial K_{eff}}{\partial Bc} \right)_T \left(\frac{dBc}{dt} \right)$$

where:

K_{eff} = effective multiplication factor

t = time

T = temperature ($^{\circ}\text{F}$)

B_c = B-10 in the core (gm)

The total temperature coefficient may be considered as composed of two parts:

$$\gamma(T) = \gamma_T(T) + \gamma_B(T)$$

where:

$\gamma(T)$

$\gamma_T(T)$ = temperature coefficient without soluble poison

$\gamma_B(T)$ = effect of soluble poison on temperature coefficient

The total concentration of poison in the core is made up of boron dispersed in the fuel elements and soluble poison in the coolant. The term $\gamma_B(T)$ is due to the change of the concentration of soluble poison with a change in mean coolant temperature. Therefore:

$$\frac{d B_c}{d T} = \frac{d B_s}{d T} = \left(\frac{d \rho_w}{d T} \right) C_B V_W$$

where:

B_s = B-10 in the coolant (gm)

ρ_w = coolant density (gm/cm³)

C_B = gm B-10/gm coolant

V_W = volume of water in the core (cm³)

Therefore, the expression for $\gamma_B(T)$ becomes:

$$\gamma_B(T) = \frac{B_s}{\rho_w K_{\text{eff}}} \left(\frac{d \rho_w}{d T} \right) \left(\frac{\partial K_{\text{eff}}}{\partial B_c} \right)_T$$

The effective temperature coefficient for various concentrations of soluble poison, obtained from the nuclear data for the reference SM-2 core described in Section 2.3, is shown in Fig. 7.31. It is seen that the soluble poison decreases the temperature coefficient, which in turn would affect the plant stability.

7.13.2 Control Rod Requirements

Utilization of soluble poison would permit the addition of controlled quantities of absorbing material during plant operation. This would have the same effect as

that obtained by using additional control rods. In applications where a short response time is required (as for scram) it is desirable to have sufficient reactivity regulated by mechanical means. Compensation for fuel burnout could be made by the introduction of soluble poison during core life.

The effect of a soluble poison upon the hot to cold reactivity change was calculated for the reference core described in Section 2.3. In the first case (Case A), all of the boron was assumed to be in the meat of the fuel plates. In the second case (Case B), 29.5 gm of B-10 were assumed to be in the fuel plates with the other 37.2 gm contained in the soluble poison. The nuclear properties were calculated by the MUFT-III and Plate-Type P₃ Theory, IBM-650 codes using input calculated by methods described in Program 50. The reactivity was calculated by use of Program 51. The results of these calculations are given in Table 7.16.

TABLE 7.16
HOT TO COLD REACTIVITY CHANGES

SM-2 Core; P = 1200 psia)

Description	Case A		Case B	
	T = 68°F	T = 440°F	T = 68°F	T = 440°F
B-10 in fuel element (gm)	66.7	66.7	29.5	29.5
B-10 in coolant (gm)	0	0	44.9	37.2
Total B-10 content (gm)	66.7	66.7	74.4	66.7
K _{eff} , no xenon	1.147	1.074	1.123	1.074
Reactivity, no xenon (%)	12.8	6.9	10.9	6.9
Hot to cold reactivity change (%)	5.9%		4.0%	

Table 7.16 compares the reactivity change experienced when going from the operating temperature to room temperature for the core containing no soluble poison and the same core containing a combination of burnable and soluble poison. Because of the change in water density between these temperatures, the core containing a combination of burnable and soluble poison would effectively have a higher concentration of soluble poison at the lower temperature. The net effect is to decrease the amount of control necessary when going from the hot to cold condition. This reduces the control rod requirements.

7.13.3 Power Distribution

In the design of long-life cores, a combination of burnable poison and control rods must be used to control the large changes in reactivity. During operation, the B-10 burns out faster than the U-235; Fig. 7.32 indicates the relative amount of each as a function of U-235 burnout as calculated by Program 50 for the core described in Section 2.3. From the results in Section 4.2, the reactivity of the

core reaches a maximum at approximately 25% fuel burnup. At this time, the control rods must be inserted to their deepest point. This results in greater burnout of fuel in the lower region of the core and a consequent reduced core life. The ultimate in design would employ a burnable poison exactly matching the fuel burnout. The use of soluble poison is an advance in this direction.

The effect of soluble poisons on the axial power distributions for the core described in Section 2.3, was calculated by use of the Windowshade, IBM-650 code. These results are shown in Fig. 7.33 in which the effect upon the power spike at the bottom of the core is plotted as a function of soluble poison content. The reduction in the axial maximum to average power represents a decrease in the non-uniformity of burnout. Calculations performed on the SM-2 (Section 4.2) indicate a difference of about 6 MWYR between a core experiencing uniform burnup and one with non-uniform burnout. The improved power distribution would reduce the non-uniform burnup effect.

7.14 CORE LIFETIME OF REFERENCE CORE #2

The original non-uniform burnup calculation of the SM-2, as described in Section 4.0, was based upon a one-pass core at 440°F, 30 mil side plates, and with a thick steel reflector. This lifetime was calculated to be 28.8 MWYR; this lifetime was based upon the assumption that the thick steel reflector was worth 1.5% ρ in reactivity (Fig. 4.1). However, results of the SM-2 Flexible Critical Experiments (Section 6.2) indicate that the worth of the laminated steel reflector is closer to 0.6% ρ . This difference of 0.9% ρ results in a lifetime loss of about 0.7 MWYR so that the corrected lifetime is 28.1 MWYR.

Since the original calculations, there were several design changes, all of which affect the lifetime. However, a good estimate of the core lifetime may be made with knowledge of the effect of various design changes upon core reactivity. The effect of various design changes upon core lifetime is given in Table 7.17.

Therefore, the reference core, presented in Section 6.1, at 510°F, with 40 mil side plates, and with a flow divider, has an expected lifetime of about 25.7 MWYR.

TABLE 7.17
EFFECT OF DESIGN CHANGES UPON LIFETIME

Design Change	$\Delta\rho(\%)$	Δ (MWYR)
Flow divider	-0.75	-0.5
440°F to 510°F	-2.1	-1.4
All water reflector	-0.6	-0.4
Thin steel skirt	-1.2	-0.8
30 to 40 mil side plate	-0.7	-0.5

7.14.1 Empirical Estimate of Lifetime of Reference Core #2

Justification for confidence that the reference U-235 loading of 36.4 kg is adequate is based upon the following discussion of the SM-1, Core I burnout. At the time of this report, two elements had been replaced in SM-1 Core I by new ones; therefore, the lifetime will be slightly extended due to the addition of fresh fuel. However, based upon extrapolation of SM-1, Core I experimental data, available prior to the substitution of new elements, the following estimates of SM-1 lifetime are made:

Pessimistic Estimate	= 14 MWYR
Realistic Estimate	= 15 MWYR
Optimistic Estimate	= 16 MWYR

The end of life critical mass of the SM-1 at 440°F is estimated to be 12 to 13 kg U-235. If fully utilized, about 2 MWYR energy can be obtained from each excess kg U-235. Since non-uniform burnup increases the critical mass, full utilization of the excess fuel cannot be obtained. The following estimates of fuel utilization in the SM-1 may be made:

	<u>Pessimistic</u>	<u>Realistic</u>	<u>Optimistic</u>
Lifetime, MWYR	14	15	16
Critical Mass (end of life), kg U-235	12.0	12.5	13.0
Excess Mass, kg U-235	10.5	10.0	9.5
Mass Utilization, MWYR/kg U-235	1.333	1.50	1.684

The end of life critical mass for the SM-2 is estimated as follows:

Pessimistic Estimate	= 18 kg U-235
Realistic Estimate	= 17.5 kg U-235
Optimistic Estimate	= 16.5 kg U-235

From the reference core loading of 36.2 kg U-235, the expected lifetime is estimated as follows:

	<u>Pessimistic</u>	<u>Realistic</u>	<u>Optimistic</u>
Critical Mass (end of life), kg U-235	18	17.5	16.5
Excess Mass, kg U-235	18.4	18.9	19.9
Mass Utilization, MWYR/kg U-235	1.333	1.50	1.684
Lifetime, MWYR	24.5	28.4	33.5

Therefore, the reference core loading should be adequate to provide a core life at least somewhere between 24 and 28 MWYR. This crude model has the disadvantage of ignoring many aspects important in core design; however, it does have the advantage of being heavily based on the actual burnout behavior of the SM-1.

Figure 7.1

Figure 7.2

Figure 7.3

7-33

Figure 7.4

Figure 7.5

Figure 7.6

REACTIVITY TRANSIENTS IN THE SM-1 AS A FUNCTION OF CORE LIFETIME

Figure 7.7

REACTIVITY TRANSIENTS IN THE SM-2 AT START UP

REACTIVITY TRANSIENTS IN THE SM-2
AT START UP FOR REDUCTION OF POWER
FROM 23 MW $\rightarrow$ 14 MW

Figure 7.9

REACTIVITY TRANSIENTS IN THE SM-2
AT START UP FOR INDUCTION OF POWER
FROM 14 MW $\rightarrow$ 0 MW

Figure 7.11

7-41

Figure 7.12

Figure 7.14

Figure 7. 15

Figure 7.17

Figure 7.18

RADIAL NEUTRON FLUX AT STARTUP (T=510°F)

Figure 7.20

RADIAL NEUTRON FLUX AT END OF LIFE (510°F)

Figure 7.21

Figure 7.22

AXIAL FLUX DISTRIBUTION 40% BURNOUT

HEAT GENERATION AT END OF LIFE (510°F)

Figure 7.24

RADIAL POWER DISTRIBUTION FACTOR ACROSS FUEL ELEMENT 47

Figure 7.26

PART OF AES-234
DISCRETE POISON APPLICATION.

Figure 7.28

Figure 7.29

Figure 7.30

Figure 7.31

7-69

Figure 7.32

Figure 7.33

8.0 CONTROL ROD CALCULATIONS

The calculations involving control rods were made by use of the Scram IBM-650 codes described in Section 1.6. The most powerful code, Scram IV, was not completed in time to be used in the analysis contained in this report; Scram II, which was later superseded by Scram III, was used as the calculational model. Unfortunately, after an examination of the theory upon which these latter two codes are based, it was found that there were errors* contained which could result in control rod worths being in error by as much as $2\% \rho$; the amount of error in a specific calculation cannot be easily ascertained. There was not sufficient time available to revise the codes and rerun the calculations; therefore, the calculated values reported in this section cannot be considered precise. The results are important, however, in survey type of calculations and when trends of results are needed. In addition, there is experimental data available obtained during the SM-2 Flexible Critical Experiments and the SM-1 Critical Experiments, which can be used as backup information regarding the control rod worths, number of control rods needed to control the reactor under the most adverse condition, stuck rod criteria, operation of the reactor with a fully inserted central control rod, rod programming, etc.

8.1 EFFECTIVE CONTROL ROD RADIUS OF THE SM-1 (1)

APPR-type control rods are square in cross section while the Scram II calculational model uses cylindrical control rods; therefore, it is necessary to determine the effective radius of the elements to be used in the Scram calculations. There are various theoretical approaches to the problem; however, since experimental data on control rod worth was available for the SM-1, it is possible to determine the radius by comparison with reactivities or K_{eff} values computed by Scram II.

8.1.1 Calculational Model and SM-1 Differences

There are many apparent differences between the SM-1 and the model used in Scram II. These include:

1. The SM-1 rods are square in cross section with an absorber of finite thickness.
2. The SM-1 core is not a cylinder.
3. SM-1 rods are not fully inserted since the absorber sections are not a full 22 in. long.

* Principal error is neglect of change in ρ when artificial ν_c is used in calculation.

4. When a rod is withdrawn from the SM-1 core, it is replaced by a control rod fuel element which has different nuclear properties than the fixed elements.
5. The SM-1 has a 0.05 in. thick stainless steel skirt which introduces a negative reactivity not present in the model.
6. The presence of a source plate and corner bolts in the SM-1 will contribute a negative reactivity which is also not present in the model.

8.1.2 Theoretical Prediction of Rod Radius

There are a few schemes used to determine the equivalence of the various control rod shapes and the effective cylindrical rod. (2)

One method is the equivalence in cross-sectional area of the control rod and the cylindrical model rod. This method seems to be in agreement with experiment for a black absorber that does not moderate fast neutrons. However, it has been reported by A. M. Perry⁽³⁾ that this method is not applicable to the SM-1 since the rods do contain moderating material. He shows that there is good agreement between results on the ORNL-APPR critical experiments⁽⁴⁾ and the method of equivalent perimeters.

The method of equivalent perimeters is that the perimeter of an equivalent cylindrical rod is the same as the perimeter of the particular control rod geometry.

Fig. 8.1 is a diagram of the cross section of an SM-1 rod. The perimeter of the rod itself is 4×2.620 in. or 26.6192 cm. This yields an effective rod radius of 4.2366 cm.

There is an uncertainty applying the perimeter to the APPR-type rods since:

1. The perimeter calculated above includes the stainless steel cladding.
2. The corners of the control rods do not contain the boron absorber. Using the boron perimeter as a basis for getting an equivalent rod radius, we have a boron perimeter of $4(2.620 - 0.155 - 0.25) = 2.214$ in. which yields an equivalent radius of 3.58 cm. It is believed that this method is an underestimate of the effective rod radius and should only be used as the smallest possible value or bracket to the actual equivalent radius. Since the work of Perry⁽³⁾ indicated that the total perimeter (including cladding) was in good agreement with experiment, it is felt that the final results of SM-1 rod radius computed by comparison of experiments and Scram II should be close to the value of 4.2366 cm.

8.1.3 Experimental and Scram II Comparison

The values of K_{eff} for various rod configurations (e.g. 4 eccentric rods plus a central, 4 eccentric and no central, central rod only, etc.) can be deduced from the various experiments performed on the SM-1 core. (5) For any given rod configuration, it is possible to find a value of the control rod radius that yields a K_{eff} which is the same as the experimental value for rods fully inserted. In this manner, it is possible to obtain a value of the rod radius which is in agreement with experimental results. This is useful since it:

- a. Shows the accuracy of the theoretical method in obtaining the rod radius.
- b. Gives an empirical value of rod radius which may be used in other APPR type core, including the SM-2 core.
- c. Allows one to correct the differences in model and actual reactor which cannot be corrected by other means (e.g. effects of corner bolts, source plate, geometry, etc.)

It was decided that the best experimental data for fully inserted rods could be obtained from the case of the five rod bank (four eccentric rods plus a central). A survey was made on the SM-1, using Scram II, of rod radius versus K_{eff} for a five rod bank fully inserted; these values are plotted in Fig. 8.2. The nuclear constants of the control rods are those of pure water at 68°F and 1200 psia. The constants for the core (and water) were taken from APAE No. 32. (6)

The values of the bank were determined from the stuck rod experiments, cases H and I, described in APAE No. 8. (7) Case H shows that the five rod bank is critical with rod B at 9.84 in. and the other four rods inserted. Case I shows that the core is critical with the central rod at 9.98 in. Using the calibration curves for rods B and C, (7) the reactivity effect for full insertion of rod B and rod C was found. For Case H, the worth of the five rod bank is -435 cents; for Case I, -514 cents. The final value used for the worth of the five rod bank was the average value of -475 cents + 10%. To this value, a correction must be made for the effects listed in Section 8.1.1.

From Table 21 in APAE No. 8, (7) the value of the reactivity change for a control rod fuel element is 46 cents. Since there are two control rod fuel elements in the actual core when a five rod bank is inserted, the correction to the experimental results is +92 cents. The correction for the effect of the steel skirt is reported as 70 cents. (5) The final worth of the five rod bank is 313 cents + 10% with corresponding K_{eff} values of 0.9774 with error limits of +0.0022 or -0.0023. Using these values and the curve of rod radius versus K_{eff} , as shown in Fig. 8.2, the value of the rod radius with a K_{eff} of 0.9774 is 4.246 cm. The dashed lines in Fig. 8.2 are for the indicated error in determining K_{eff} .

It can readily be seen that this value is in good agreement to the theoretical prediction of equivalent radius of 4.2366 cm, using the perimeter method. It was decided that the empirical value of 4.246 cm would be used for the rod radius for future APPR-type elements in Scram II calculations, including the SM-2; the perimeter method determining rod radius can be used for APPR-type control elements (absorber with a moderating material), where empirical data is not available.

8.1.4 Effect of Corrected Boron Loading Upon Rod Radius

The rod radius of 4.246 cm. was based upon the "as specified" boron loading of the SM-1. This value of rod radius was used in the control rod calculations made upon the SM-2 core. It was later determined that the actual boron content of the SM-1 was about 22.4% lower than the "as specified" amount. This raises the calculated values of K_{eff} as shown by the dashed curve in Fig. 8.2, yielding an effective control rod radius of about 4.35 cm. This will not strongly affect the values of the calculated control rod worth but should be kept in mind for future control rod calculations.

8.2 EFFECT OF INCREASING ABSORBER THICKNESS (8)

The Scram codes were derived by a two-group approach which considered the control rod as a homogeneous mixture of moderator and black thermal absorber.

In the APPR design, the control rod consists of stainless steel-clad boron plates with an internal water moderator. The boron thickness is about 0.23 cm. Since the relaxation length of thermal neutrons in boron is 0.01 cm., the rod is justly considered black to thermal neutrons. Increasing the boron thickness will yield a negligible effect on the change in thermal absorptions in the rod. However, increasing the boron thickness decreases the water gap in the rod which in turn changes the number of fast neutrons slowed down. Since less neutrons are slowed down, less fast neutrons will be absorbed in the rod. Consequently, it is expected that the rod worth will decrease with increased boron thickness.

8.2.1 Development of the Analysis

Since the thermal neutrons see only the boron, and the fast neutrons are only affected by the water (assuming no fast neutron scattering in boron), the two-group equations in the Scram analysis may be rederived on the basis of two separate rod radii; one radius for the fast neutrons and one for the thermal neutrons.

There are two points in the control rod analysis where the rod radius is used. First, in the determination of the fast and thermal extrapolation distances, d_1 and d_2 , and second, in the determination of the ψ_a function.

The ψ_a function is: (9)

$$\psi_a = \frac{a_{31} a_{34} - a_{41} a_{34}}{a_{22} a_{44} - a_{42} a_{34}}$$

where:

$$a_{31} = J_0(\bar{\mu} a)/d_1 + \bar{\mu} J_1(\bar{\mu} a)$$

$$a_{32} = Y_0(\bar{\mu} a)/d_1 + \bar{\mu} Y_1(\bar{\mu} a)$$

$$a_{34} = K_0(\bar{\nu} a)/d_1 + \bar{\nu} K_1(\bar{\nu} a)$$

$$a_{41} = S_1 [J_0(\bar{\mu} a)/d_2 + \bar{\mu} J_1(\bar{\mu} a)]$$

$$a_{42} = S_1 [Y_0(\bar{\mu} a)/d_2 + \bar{\mu} Y_1(\bar{\mu} a)]$$

$$a_{44} = S_2 [K_0(\bar{\nu} a)/d_2 + \bar{\nu} K_1(\bar{\nu} a)]$$

and:

$$d_1 = \frac{D_1 I_0(\bar{\mu}_{ir} a)}{D_{ir} \bar{\mu}_{ir} I_1(\bar{\mu}_{ir} a)}$$

The thermal extrapolation distance is obtained graphically and is a function of the rod radius.

The analysis will now be altered so that the worth of a control rod consisting of a cylindrical shell of thermal absorber with an internal moderator may be evaluated. The outside radius of the thermal absorber is designated a_2 and the radius of the internal moderator (water) is designated a_1 . In order to rederive the ψ_a function, it is assumed that the thermal absorber is a void to fast neutrons. The following derivation is based on work performed by Dr. R. L. Murray for Alco. (10)

The total positive neutron current at a_1 is equal to the positive current at a_2 :

$$2 \pi a_2 j_+(a_2) = 2 \pi a_1 j_+(a_1)$$

Similarly for the negative current:

$$2 \pi a_2 j_-(a_2) = 2 \pi a_1 j_-(a_1)$$

The familiar current equations are:

$$j_+ = \frac{\phi}{4} - \frac{\lambda}{6} \frac{d\phi}{dr}$$

$$j_- = \frac{\phi}{4} + \frac{\lambda}{6} \frac{d\phi}{dr}$$

Combining the last four relations:

$$a_1 \phi(a_1) = a_2 \phi(a_2)$$

$$a_1 D_{ir} \phi'(a_1) = a_2 D_{ic} \phi'(a_2)$$

Since the fast extrapolation distance is defined as:

$$d_1 = \frac{\phi(a_2)}{\phi'(a_2)}$$

and the solution for the flux in the rod is:

$$\phi_1 = A I_0(\bar{\mu}_{ir} r)$$

then:

$$d_1 = \frac{D_{ic}}{D_{ir}} \frac{\phi(a_1)}{\phi'(a_1)} = \frac{D_{ic}}{D_{ir}} \frac{I_0(\bar{\mu}_{ir} a_1)}{\bar{\mu}_{ir} I_1(\bar{\mu}_{ir} a_1)}$$

The terms in the ψ_a function become:

$$a_{31} = J_0(\bar{\mu} a_2)/d_1 + \bar{\mu} J_1(\bar{\mu} a_2)$$

$$a_{32} = Y_0(\bar{\mu} a_2)/d_1 + \bar{\mu} Y_1(\bar{\mu} a_2)$$

$$a_{34} = K_0(\bar{\nu} a_2)/d_1 + \bar{\nu} K_1(\bar{\nu} a_2)$$

$$a_{41} = S_1 [J_0(\bar{\mu} a_2)/d_2 + \bar{\mu} J_1(\bar{\mu} a_2)]$$

$$a_{42} = S_1 [Y_0(\bar{\mu} a_2)/d_2 + \bar{\mu} Y_1(\bar{\mu} a_2)]$$

$$a_{44} = S_2 [K_0(\bar{\nu} a_2)/d_2 + \bar{\nu} K_1(\bar{\nu} a_2)]$$

and:

$$d_1 = \frac{D_{ic}}{D_{ir}} \frac{I_0(\bar{\mu}_{ir} a_1)}{\bar{\mu}_{ir} I_1(\bar{\mu}_{ir} a_1)}$$

and d_2 is dependent on a_2 . The thermal rod radius, a_2 , is the same as the overall radius, a , in the previous analysis.

8.2.2 Computational Results

Calculations were performed to determine the effect of increasing boron thickness by factors of 2, 3, 5 and 7 for the APPR-type absorber section. The

change in the fast radius was determined by equating the inside water perimeter with a cylinder of the same outside perimeter.

From Fig. 8.1, the water perimeter in the rod is:

$$P_1 = [2.62 - 2(0.156)] \times 4 = 9.322 \text{ in.}$$

Doubling the boron thickness yeilds a perimeter of:

$$P_2 = 9.232 - 4(2)(0.09) = 8.512 \text{ in.}$$

The corresponding water perimeters for 3, 5, and 7 times the original boron thickness are:

$$P_3 = 8.152 \text{ in.}$$

$$P_5 = 7.432 \text{ in.}$$

$$P_7 = 6.712 \text{ in.}$$

The corresponding fast rod radii are:

$$a_1 = 3.732 \text{ cm.}$$

$$a_2 = 3.441 \text{ cm.}$$

$$a_3 = 3.295 \text{ cm.}$$

$$a_5 = 3.004 \text{ cm.}$$

$$a_7 = 2.713 \text{ cm.}$$

The results of the calculations, for the change in rod worth with increased boron thickness for the SM-2 seven rod bank, are shown in Fig. 8.3. These results were obtained by the Scram code using SM-2 core parameters tabulated in Table 6.4 and rod parameters (water) tabulated in Table 7.2.

8.3 EFFECT OF HIGHER ORDER TERMS ON ROD WORTH PREDICTIONS⁽¹¹⁾

The expression for $1/\psi_r$ in the control rod analysis is given by R. L. Murray⁽¹²⁾ as:

$$\frac{1}{\psi_r} = M \sum_{s=-\infty}^{+\infty} \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)}$$

where M is the number of eccentric control rods. The factor, $1/\psi_r$ is approximated in the Scram II analysis as:

$$\frac{1}{\psi_r} \approx M \frac{J_0^2(\bar{\mu} d) Y_0(\bar{\mu} R)}{J_0(\bar{\mu} R)}$$

This approximation does not give a large error in the final rod worth if there are at least four eccentric rods, since the ratio of the second term of the expansion for $1/\psi_r$ (which contains Bessel functions of order four) is small compared to the zero order term. However, the error increases with decreasing number of ring rods and is greatest for one eccentric rod.

The purpose of this calculation was to determine the number of terms in the $1/\psi_r$ expansion that are needed to keep the analytical error in the Scram analysis small.

The error is always small, for practical problems, since there will always be four or more rods in a ring; however, the additional terms in the $1/\psi_r$ expansion should be evaluated since they are simple to calculate and would eliminate a source of analytical error.

8.3.1 Development of the Analysis

The equation for $1/\psi_r$ may be rewritten as:

$$\frac{1}{\psi_r} = M \frac{J_0^2(\bar{\mu} d) Y_0(\bar{\mu} R)}{J_0(\bar{\mu} R)} + M \sum_{s=-\infty}^{-1} \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)} +$$

$$M \sum_{s=1}^{\infty} \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)}$$

or:

$$\frac{1}{\psi_r} = M \frac{J_0^2(\bar{\mu} d) Y_0(\bar{\mu} R)}{J_0(\bar{\mu} R)} + M \sum_{s=1}^{\infty} \frac{J_{-ms}^2(\bar{\mu} d) Y_{-ms}(\bar{\mu} R)}{J_{-ms}(\bar{\mu} R)} +$$

$$M \sum_{s=1}^{\infty} \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)}$$

Using the identities:

$$J_{-n}(x) = (-1)^m J_n(x)$$

$$Y_{-n}(x) = (-1)^m Y_n(x)$$

we have:

$$\frac{J_{-ms}^2(\bar{\mu} d) Y_{-ms}(\bar{\mu} R)}{J_{-ms}(\bar{\mu} R)} = \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)}$$

Therefore:

$$\frac{1}{\psi_r} = M \frac{J_0^2(\bar{\mu} d) Y_0(\bar{\mu} R)}{J_0(\bar{\mu} R)} + 2M \sum_{s=1}^{\infty} \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)}$$

8.3.2 Computational Results

Surveys of rod worth vs. number of terms in the expansion of ψ_r were made to determine the number of terms necessary for convergence of the rod worth. The core constants of the SM-2 are tabulated in Table 6.4 and the control rod (water) constants are tabulated in Table 7.2. The results of the Scram calculations are shown in Table 8.1.

For the case of a single eccentric rod, it is evident that six terms plus the zero order terms is sufficient for the accuracy needed. Since the Scram program then had incorporated into it the subroutines for calculating Bessel functions up to order six, it was necessary to see if higher order functions were necessary for the case of more than one eccentric rod. Calculations were performed for $M=2, 4$ and 6 . For $M=2$, three higher order terms were calculated. In the case of four eccentric rods, an additional higher order term was calculated to see if one higher order term was sufficient. Since the second higher order term (Bessel functions of order eight) predicted a rod worth within four place agreement with the prediction with one higher order term, it was concluded that only one additional term be calculated for $M=4$. The results of these calculations are shown in graphical form in Fig. 8.4

TABLE 3.1
EFFECT OF HIGHER ORDER TERMS ON ROD WORTH

$$\frac{1}{\psi_r} = M \frac{J_0^2(\bar{\mu} d) Y_0(\bar{\mu} R)}{J_0(\bar{\mu} R)} + 2M \sum_{s=1}^{S_m} \frac{J_{ms}^2(\bar{\mu} d) Y_{ms}(\bar{\mu} R)}{J_{ms}(\bar{\mu} R)}$$

For M=1:

<u>M S_m</u>	<u>ρ(Rods) %</u>	<u>ρ(Rods) - 2.2484552%</u>
0	2.2484552	0
1	2.1018097	0.1466455
2	2.1228819	0.1255733
3	2.1260579	0.1223973
4	2.1263303	0.1221249
5	2.1263952	0.1220600
6	2.1264266	0.1220286

For M=2:

<u>M S_m</u>	<u>ρ(Rods) %</u>	<u>ρ(Rods) - 4.9613702 %</u>
0	4.9613702	0
2	5.0651295	0.1027593
4	5.0664196	0.1050494
6	5.0665050	0.1051348

For M=4:

<u>M S_m</u>	<u>ρ(Rods) %</u>
0	11.73397
4	11.75261
8	11.74780

For M=6:

<u>M S_m</u>	<u>ρ(Rods) %</u>
0	16.1038760
6	16.1040190

8-10

8.4 COMPARISON OF SCRAM II AND SCRAM III⁽¹³⁾

Scram III calculates the rod worth in a bare core while Scram III calculates the rod worth in a side reflected core. To determine the effect of a side reflector upon the calculated rod worth, various configurations of rods were calculated for a core similar to the one described in Section 2.0. The constants used are tabulated in Table 8.2.

K_{eff} versus rod radius was calculated as a function of various rod configurations:

- a. Rods B, G, E, and F plus the central rod C, for operation as a 5 rod bank. (Fig. 8.5)
- b. Seven rod bank operation for the open array as seen in Fig. 8.5.
- c. Six rod ring operation for the open seven array as seen in Fig. 8.5.

Rods B, G, E, and F lie at a distance of 16.6839 cm from the center of the core, while rods A and D lie at a distance of 14.9225 cm from the center of the core. The effective ring radius for the six rod bank was calculated by finding the average distance of an eccentric rod from the central axis. Thus, the effective ring radius, $\bar{d}$, for a ring of six rods is:

$$\bar{d} = \frac{4 (16.6839) + 2 (14.9225)}{6} = 16.0968 \text{ cm.}$$

TABLE 8.2
CORE CONSTANTS USED IN SCRAM II AND SCRAM III COMPARISON
(Water Reflected Core)

Parameter	Units	T = 68°F	T = 440°F
$R + S_T$	cm	34.807264	36.257304
B_z^2	cm ⁻²	0.002116	0.001954
B^2	cm ⁻²	0.006889	0.006353
D_f	cm	1.281126	1.435227
D	cm	0.162304	0.229470
τ	cm ²	32.393216	42.499314
P		0.605285	0.578865
L^2	cm ²	0.376860	0.670219
K_f		1.273012	1.268035
K_{th}		1.454730	1.456858
Σ_a	cm ⁻¹	0.430673	0.342380
$\nu \Sigma_f$	cm ⁻¹	0.626513	0.498799
D_f (rods)	cm	1.600095	1.899871
τ (rods)	cm ²	33.742736	47.830185

The results of these calculations are shown in Figs. 8.6 through 8.8. It is seen that by including the effect of the side reflector reduces the calculated rod worths.

8.5 ROD WORTH CALCULATIONS FOR THE SM-2 CORE (13) (14)

Calculations were made using the Scram III code to predict the rod worth of the SM-2 for both the open seven array (Fig. 1.8) and closed seven array (Fig. 1.7), at startup and midlife (0 and 22% fuel burnup), with a steel or water reflector. The constants for the reflector and control rods (water) used in the calculations are tabulated in Table 8.3. The constants for the core are tabulated in Table 6.4 of Section 6.1.

TABLE 8.3
CONSTANTS USED IN SCRAM III ANALYSIS

Parameter	Units	Control Rods		Water Reflector		Steel T = 68°F
		T = 68°F	T = 510°F	T = 68°F	T = 510°F	
D_f	cm	1.600095	2.015833	1.600095	2.015833	1.087203
D	cm	-	-	0.172303	0.297148	0.293005
τ	cm ²	33.742736	54.253082	-	-	-
Σ	cm ⁻¹	-	-	0.016960	0.009947	0.179669
$\Sigma^* R$	cm ⁻¹	-	-	0.047420	0.037156	0.010123
P		-	-	0.987035	0.988621	0.129429
R + S _r	cm	-	-	34.830256	36.732773	38.089879
B_z^2	cm ⁻²	-	-	0.002113	0.001907	0.002106
* $\Sigma_R = D_f / \tau$						

Since the Scram II and III program compute K_{eff} values for a core with all fixed elements, it was necessary to correct the K_{eff} values computed (both rods-in and rods-out) for the presence of control rod fuel elements which have different nuclear properties than the fixed elements. The K_{eff} rods-out computed by the bare equivalent model must be corrected for the seven control rod elements. The K_{eff} value for the rods-in does not have to be corrected for control rod fuel elements in the seven bank case since there are none present. However, when five rods are inserted, the K_{eff} rods-in value has to be corrected for two control rod fuel elements at a distance of 14.9225 cm. from the core centerline. Similarly, in the ring of six rods case, K_{eff} rods-in must be corrected for the central control rod fuel element.

The change in reactivity due to replacing a central fuel element by a control rod fuel element was calculated by running a VALPROD for a core of 45 fixed fuel elements and another VALPROD for the same core with the central element replaced by a control fuel element (based on a core similar to that described in Section 6.1); the change in reactivity was calculated to be $0.276838\% \rho$. The worth of a central control rod fuel element plus a ring of eccentric control rod fuel elements was determined by flux weighting the eccentric rods. Thus, the change in reactivity associated with replacing the central fuel element and M eccentric fuel elements at a ring radius, d cm., from the center of the core with control rod fuel elements is:

$$\Delta \rho_{\text{total}} = 0.276838 [1 + M J_0^2 (B_r d)]$$

where B_r^2 is the radial buckling. The change in reactivity for a seven rod bank of control rod fuel elements calculated by this method is $-1.19218\% \rho$; for two eccentric rods 14.9225 cm. from the center, the change in reactivity from a fixed element to a mixed element core is $-0.3379\% \rho$.

In addition, a model correction, based upon a comparison of SM-1 experimental data and calculation, must be added to the Scram III predictions. This correction is described in Section 8.5.1.

Using the value of K_{eff} rods-in computed from the Scram code, the rod worth is determined from the relation:

$$\rho(\text{rods}) = - \left[\frac{K_{\text{eff}}(\text{rods-out}) - K_{\text{eff}}(\text{rods-in})}{K_{\text{eff}}(\text{rods out})} \right]$$

Since the final value of K_{eff} rods-in was determined for a fixed element core, the $\rho(\text{rods})$ for the five rod bank was corrected for the two control rod fuel elements ($0.3379\% \rho$) and the ring of six rods was corrected for the worth of the central control rod fuel element ($0.276838\% \rho$).

8.5.1 Accuracy of the Scram Code

The errors in the rod worth calculations were determined by comparison of Scram predictions with available experimental values; correction factors could be obtained to apply to the calculated worths. The largest source of error is due to an underestimate of the equivalent rod radius. The equivalent rod radius was determined by comparison of experimental estimates of the five rod bank worth in the SM-1 with Scram II predictions; this is described in Section 8.1. It was found that based upon the "as specified" boron content of the SM-1, an equivalent rod radius of 4.246 cm. was calculated. This value was used in the rod worth calculations performed upon the SM-2 core; however, it was later found, as explained in Section 8.1.4, that the actual boron content of the SM-1 is approximately 22.4% less than the "as specified" content. This yields an equivalent

radius of about 4.35 cm. Since the equivalent rod radius was underestimated, the rod worth calculations, based upon the old radius, will also be underestimated.

The best experimental estimates for the five rod bank worth in the SM-1 is 19.5% ρ and 24.6% ρ for the seven rod bank. The Scram III code, using core parameters based upon a 22.4% loss of boron in the SM-1, predicted the rod bank worth as 16.9% ρ and the seven rod bank worth as 22.9% ρ which are underestimates of 2.5% ρ for the five rod bank and 1.7% ρ for the seven rod bank. Evidence of underestimating the rod worth for the open array was achieved by using data on the SM-2 core mockup. It was found experimentally, as described in Section 10.1.3, that the fully inserted open seven rod array was critical when the core loading was 10.1 plates per element, with no boron in the 45 element core. The Scram prediction for this case was a "k rods in" values of 1.036 which is a 3.5% ρ underestimate of the rod worth. Utilization of the new MUFT-III files for stainless steel, as described in Section 11.3.1, would still yield an underestimate of about 2% ρ . Because of the preceding examples of consistent underestimates of rod worth, the following corrections were made for the control rod worth calculations performed for the SM-2 core; for the five rod bank closed array, 2.5% ρ was added to the predicted rod worth; for the seven rod bank closed array, 1.7% ρ was added; for all other cases, a conservative value of 2% ρ was added to the predicted rod worth.

8.5.2 Calculational Results

The results of the calculations of rod worth of the SM-2, including the corrections described in Section 8.5.1, are given in Table 8.4. It is seen that the open seven rod worth, at 68°F, at 22% fuel burnup (approximately the midlife of the core), for a steel reflected core, is predicted to be 25.2% ρ . The six rod ring worth for the core under the same conditions, is predicted to be 19.6% ρ .

8.6 SELECTION OF SM-2 ROD ARRAY

The open seven rod array was selected for the reference SM-2 configuration. A diagram of the location of the rods for both the open and the closed seven rod arrays may be seen in Figs. 1.8 and 1.7. The choice of the open versus closed array is dependent not only upon nuclear considerations, but also upon mechanical design problems. A preliminary analysis of the data obtained during the SM-2 Flexible Critical Experiments, which was available at the time of this analysis, was made regarding stuck rod conditions, total rod worth, etc. A more detailed analysis of the later data obtained upon the final SM-2 mockup core will be made and reported in the SM-2 final design report. An analysis of the available data was made, and coupled with the mechanical problems associated with finding space in the closed seven array for the large diameter control rod racks or large caps, the open seven rod array was selected. The nuclear considerations are discussed in the following sections.

TABLE 8.4
CONTROL ROD WORTH OF SM-2 CALCULATED BY SCRAM III

Reflector	Fuel Burnup (%)	Temp. (°F)	Array	Excess (%) ρ	Rod Worth (% ρ)		
					7 Rod Bank	6 Rod Ring	5 Rod Bank
Water	0	68	Open	12.065	23.988	18.075	17.102*
Steel	0	68	Open	13.882	24.028	18.696	17.159*
Water	22	68	Open	17.280	24.992	18.813	17.794*
Steel	22	68	Open	19.035	25.239	19.581	18.010*
Water	22	510	Open	11.987	-	20.137	-
Water	0	68	Closed	12.065	23.79	21.518	18.793**
Steel	0	68	Closed	13.882	23.289	21.311	18.579**
Water	22	68	Closed	17.280	24.635	22.374	17.513**
Steel	22	68	Closed	19.035	24.391	22.371	19.492**

* Rods E, F, C, B, and G in Fig. 18.

** Rods A, B, C, D, and E in Fig. 17.

8.6.1 Stuck Rod Conditions

Critical experiments (15) were performed with a core, containing the reference loading of U-235 and about 7/8 the reference B-10 loading, on both the open and closed seven rod arrays; the core contained an excess reactivity of about 12% ρ . These experiments showed that of 21 possible combinations of two rods stuck out, 15 were subcritical in the closed array while only 9 were subcritical in the open array. However, for the worst case of a single eccentric stuck rod, the open array has a marginal advantage over the closed array. The worst eccentric stuck rod condition for the open array, for the core measured, was when rod D (Fig. 1.8) was stuck out; the core was critical with rod E and 2.6 in. out of the core. The worst case for the closed array was rod A (Fig. 1.7) stuck out and the core was critical with rod F at 11.6 in. out of the core. This difference of 1-in. in position is small and experimental evidence(16) indicates that this difference decreases with core burnout. Consideration of the stuck rod condition for any single rod, including the central rod, shows no significant difference in the open vs. closed array. If rod C were stuck out in the open array, the core is critical with rod D at 11.9 in. Since the worst stuck rod condition for the closed array is the same (the central rod is relatively unimportant in the closed as compared to the open array), the stuck rod condition would be the same. However, if the core is operated with a five rod bank or if rod programming were used, the central rod might not be fully withdrawn; therefore, the open array would have a marginal advantage over the closed array. It should be noted that the positions mentioned in this section only have meaning when referred to the core used in these experiments.

8.6.2 Total Rod Worth

The total rod worth calculated for both the open and closed rod array, is approximately 25% in the midlife core, as given in Table 8.4. This may be compared to an experimental value of about 24% ρ . Experimental results obtained during the SM-2 critical experiments verify that the total worth of an open seven rod array is nearly the same as that of the closed seven rod array.

8.6.3 Rod Programming

If rod programming were employed, to obtain a more uniform burnout of fuel, the open array could be used more effectively than a closed array since the control rods are further apart.

APPR-1 BORON ABSORBER CROSS SECTIONAL VIEW

Figure 8.1

CHANGE IN ROD WORTH WITH INCREASED BORON THICKNESS FOR SEVEN ROD BANK IN SM-2

Figure 8.3

ROD GEOMETRY

SM-1 (APPR-1)
5 ROD BANK
CLOSED ARRAY

SM-2 (APPR-1B)
5 ROD BANK
OPEN ARRAY

SM-1 (APPR-1)
ROD GEOMETRY
(CLOSED ARRAY)

SM-2 (APPR-1B)
OPEN ARRAY
 $d_1 = 16.6839$
 $d_2 = 14.9225$

CONTROL ROD

CONTROL ROD FUEL ELEMENT

BLANK PORTION OF CORE CONTAINS FIXED ELEMENTS ONLY

Figure 8.5

Figure 8.7

9.0 FLUX AND POWER DISTRIBUTIONS

During the design of the SM-2 Core, a method was devised for calculating the power distribution for APPR-type cores⁽¹⁾. The technique is based upon a combination of analytical calculations and empirical correction factors. The first method is dependent upon various one-dimensional IBM-650 codes, such as the Windowshade, VALPROD, and plate-type P₃. The Windowshade code calculates the axial flux and power distribution, the VALPROD code calculates the gross radial flux and power distributions, and the P₃ theory code calculates the local flux peaking within each element. The second method employs the two-dimensional PDQ, IBM-704 code in conjunction with the Windowshade code.

9.1 FLUX AND POWER DISTRIBUTIONS USING ONE-DIMENSIONAL IBM-650 CODES

The power generation within the core must be known to perform a thermal analysis to determine surface temperatures, bulk coolant temperature rise, film temperature rise, coolant rates, etc. To perform this analysis, certain nuclear factors must be supplied; these include:

$F_N (\Delta T)$	= nuclear hot spot factor to allow for uncertainty in calculated power distribution in calculation of bulk coolant temperature rise.
$F_N (\Delta \theta)$	= nuclear hot spot factor to allow for uncertainty in calculated power distribution in calculation of the film temperature rise.
$Q (\Delta T)$	= power generation rate for worst channel in each element, normalized to an average radial power of 1 over the whole core.
$Q (\Delta \theta)$	= power generation rate at the hottest spot in each element normalized to an average radial power of 1 over the whole core.
$\psi (\Delta T)$	= fraction of total reactor power generated in core.
$\psi (\Delta \theta)$	= fraction of total reactor power generated in fuel plate.
N	= total number of space increments within the core, in the Windowshade calculations.
L	= core height

q_k = axial power distribution printed out by Windowshade calculation, normalized such that $\sum_{k=0}^N q_n = 1$

The power distribution in the core can be expressed by an equation:

$$P(x, y, z) = A F(x) F(y) P(z) P_{\text{local}} F_{\text{rods}}$$

where:

$P(x, y, z)$ = local power distribution at x, y, z .

A = normalization constant.

$F(x)$ = calculated power distribution in x direction normalized to $F(x = 0) = 1$.

$F(y)$ = calculated power distribution in y direction normalized to $F(y = 0) = 1$.

$P(z)$ = calculated power distribution in z (axial) direction, normalized to an average axial value of unity.

P_{local} = local peaking factor.

F_{rods} = correction factor, mostly due to control rod effects.

The constant A is obtained from a radial VALPROD calculation and is equal to the value of $P(r)$ at the core centerline ($r = 0$) when $P(r)$ is normalized to an average radial value of unity. The functions $F(x)$ and $F(y)$ are constructed from the radial VALPROD power distribution. They are normalized such that $F(x)$ and $F(y) = 1$, at $x = 0$ and $y = 0$, respectively. The axial distribution $P(z)$ is obtained from a Windowshade calculation. It is normalized such that $P(z) = 1$.

The axial and radial calculations are based upon homogenized core properties and do not account for local power peaking. The factor P_{local} is included to correct for local peaking effects. This local peaking is generally due to side plate peaking but local peaking due to other factors must be carefully considered; for example, there is local peaking in APPR- type elements in the water gap between the control rod absorbers and the control rod fuel elements.

The correction factor, F_{rods} , is required to account for the fact that the power distribution $P(x, y, z)$ cannot be constructed from separable functions of x, y, z . Calculations on the SM-1 show that F_{rods} is a function of radial location with respect to control rods and a function of rod bank position. The factor, F_{rods} , must be found empirically. This was done for the SM-1 and a set of factors for each fuel element was determined for two bank positions. These factors are taken as the basis for the factors of other cores similar to the SM-1, such as the SM-2, in lieu of more conclusive available information. Extrapolation of the factors to other bank positions and reflector configurations may be necessary.

9.1.1 Factors A, $F(x)$, and $F(y)$

The actual core cross section is irregular; for the SM-1 and SM-2, it is a 7 x 7 array with the corners missing. The radial VALPROD calculation assumes a circular cross-section having the same area as the actual core, as shown in Fig. 9.1. This represents a reasonable approximation for calculating reactivity, as discussed in Section 11.4. The power distribution in the central part of the core is not influenced much by this approximation; however, the power distribution in the outer elements is poorly represented. Accurate prediction of power distribution in each element is required if flow tailoring is used. The actual x-y power distribution is non-separable into x and y components; however, for purposes of analysis, it is assumed to be separable. The distribution is given by:

$$P(x, y) = A F(x) F(y)$$

The radial VALPROD distribution is:

$$P(r) = AP'(r)$$

where the functions $F(x)$, $F(y)$, and $P'(r)$ are normalized such that:

$$F(x = 0) = F(y = 0) = 1$$

$$P'(r) = 1$$

$$P(r = 0) = A$$

The construction of the functions $F(x)$ and $F(y)$ from $P(r)$ is described in APAE Memo 195(1).

9.1.2 Factor $P(z)$

The axial power distribution $P(z)$ is found from a Windowshade calculation. This code is described in Section 1.2.2.2.

9.1.3 Factor P_{local}

This factor is obtained from use of P_3 theory calculations of the thermal flux distribution parallel to the fuel plates; because of epithermal fissions, the power distribution does not peak as much as the thermal flux distribution. In APPR-type cores, this peak must be investigated carefully, for example, the peaking between two adjacent fixed elements, the peaking between a fixed and a control fuel element, and the peaking between various elements separated by a flow divider (as in the SM-2).

9.1.4 Factor F_{rods}

This factor is chiefly required to account for the effect of the control rods on the radial power distribution. The factor F_{rods} was determined for the SM-1 as described in reference (1); this factor was obtained for each element from the equation:

$$F_{rods} = \frac{p_{max}(x, y)}{AF(x) F(y) p_{max}(z)}$$

where:

- x, y = coordinates of the centerline of each fuel element.
- $p_{max}(x, y)$ = measured maximum activity in each element.
- $p_{max}(z)$ = axial maximum to average power obtained from Windowshade calculations.

The values for F_{rods} for the SM-1 are shown in Table 9.1 as a function of bank position. In the SM-1 at 440°F, the minimum bank position is 6.8 in. (no xenon, 0 MWYR). These values of F_{rods} , at a bank position of 6.8 in., were used for thermal design calculations in lieu of better calculational methods for estimating power distribution. The values of F_{rods} for all center locations were estimated by interpolation or extrapolation; these values are shown in Fig. 9.2.

TABLE 9.1
F_{rods} FACTORS FOR SM-1 CORE

(T=440°F; P=1200 psia; No Xenon; p MWYR)

Element*	F _{rods} Factor**		
	Bank at 3.7 in.	Bank at 6.3 in.	Bank at 6.8 in.
44 (Rod C)	0.76 (0.76)	0.84 (0.84)	(0.86)
34	0.73 (0.75)	0.84 (0.83)	(0.85)
35	0.81 (0.80)	0.88 (0.88)	(0.91)
24 (Rod A)	0.82 (0.82)	0.90 (0.90)	(0.92)
25	0.86 (0.88)	1.00 (0.98)	(1.00)
26	1.10 (1.01)	1.02 (1.11)	(1.13)
14	0.84 (0.91)	1.08 (1.01)	(1.03)
15	0.99 (0.99)	1.09 (1.09)	(1.11)
16	1.04 (1.03)	1.13 (1.14)	(1.15)

* See Fig. 1.7.

**Factor for 3.7 in. and 6.3 in. bank positions were calculated directly. Values in parenthesis are smoothed values. Factors for 6.8 in. bank position were estimated by extrapolation.

9.1.5 SM-2 Flux and Power Distributions

In the calculations of the SM-2 thermal flux and power distributions, the following assumptions were made:

1. It was assumed that a steel reflector would be used so that power peaking at the core-reflector interface would be eliminated.
2. It was assumed that a flow divider would be used which would affect the local peaking factors in the elements adjacent to the flow divider.
3. For the rest of the core, it was assumed that the gross radial power distribution would be the same as the SM-1. This is a reasonable assumption since the sizes of both cores are identical, and the change in reflector savings will have only a slight effect.

9.1.5.1 Factors A, F (x), and F (y)

The factors A, F (x), and F (y) were calculated for the SM-2 reference core, described in Section 6.1, by the use of a radial VALPROD calculation and the methods described in Section 9.1.1. These factors are tabulated in Table 9.5 and 9.6 for the calculation of the factors $Q(\Delta T)$ and $Q(\Delta\theta)$ respectively.

9.1.5.2 Factor P (z)

The axial power distribution was taken to be the same as calculated for the reference core described in Table 4.6 of Section 4.2. This assumption should not be serious since the axial distribution is not a strong function of temperature or slight changes in boron content. This distribution was taken at the deepest bank insertion (approximately 9.1 MWYR) at 440°F with no xenon; the distribution has been corrected for the high local burnup at the bottom of the active meat.

9.1.5.3 Factor F_{rods}

The factor F_{rods} for the SM-2 elements were based upon those calculated for the SM-1 since no experimental measurements of axial power distribution was available at the time of this calculation. The values of F_{rods} for the SM-2 were taken at the off-center locations as shown in Fig. 9.2, and are tabulated in Tables 9.5 and 9.6 for the factors $Q(\Delta T)$ and $Q(\Delta\theta)$ respectively. The off-center locations correspond to the hottest portion of each element, where the flux peaking is the greatest. At the time of this calculation, it was not known whether an open or closed seven rod array (Fig. 1.8 and 1.7) would be used; the values of F_{rods} given in Tables 9.5 and 9.6 are based upon a closed seven rod array. The latest SM-2 reference core employs an open seven rod array; such an array results in a more symmetrical core configuration, which in turn, would probably alter the values of F_{rods}.

9.1.5.4 Factor P_{local}

The factor P_{local} was obtained by calculating the intercell P₃ flux distributions for the various element configurations, converting to a power distribution, and correcting these power distributions to agree with experimental measurements. From these corrected power distributions, the local peaking factors may be obtained. The fuel elements considered are described in Section 4.0, except that the operating temperature had been changed from 440°F to 510°F. Drawings of the core configuration (including a flow divider), fixed elements and control fuel elements, considered, are shown in Fig. 3.10, 1.1, and 1.4. The thermal properties and innercell P₃ flux distributions for the fixed and control rod fuel elements were obtained by methods described in Sections 1.1.2.1 and 1.1.2.2 respectively.

The properties of the active region in the control rod element were assumed equal to those in the fixed fuel element. As described in Section 1.1.2.2, the control fuel elements are cylindricalized into an active and surrounding dead region of radii 3.502 cm. and 4.213 cm. respectively. The thermal parameters calculated are tabulated in Tables 9.2 and 9.3 for the active and dead regions of the control fuel elements, respectively, for various fuel burnups.

TABLE 9.2
THERMAL PARAMETERS OF ACTIVE REGION OF
CONTROL FUEL ELEMENTS

Parameter	Units	T=68°F 0% Burnup	T=510°F 0% Burnup	T=510°F 22% Burnup
Σ_a	cm ⁻¹	0.516744	0.384719	0.281034
Σ_s	cm ⁻¹	2.048788	1.376591	1.370010
Σ_{tr}	cm ⁻¹	2.171918	1.407782	1.299599
$\nu\Sigma_f$	cm ⁻¹	0.783020	0.580107	0.459523

TABLE 9.3
THERMAL PROPERTIES OF DEAD REGION OF
CONTROL FUEL ELEMENTS

Parameter	Units	T=68°F 0% Burnup	T=510°F 0% Burnup	T=510°F 22% Burnup
Σ_a	cm ⁻¹	0.076281	0.055180	0.055180
Σ_s	cm ⁻¹	1.908901	1.340738	1.340738
Σ_{tr}	cm ⁻¹	1.985182	1.395918	1.395918

As seen in Fig. 3.10, there are several combinations of material regions surrounding a fixed fuel element. These are:

1. Two fixed fuel elements separated by a nominal spacing.
2. A fixed fuel element adjacent to a control rod element.
3. Two fixed fuel elements separated by a flow divider.
4. A fixed fuel element separated from a control rod element by the flow divider.

Each of these cases can be modified by aligning the fixed fuel element plates parallel or perpendicular to the interface under consideration. A total of seven combinations involving the fixed fuel element were investigated. Each of these are slightly different because of the dimensions and material properties of the region surrounding the fixed element change. Thermal flux distributions were obtained from a one-dimensional P_3 calculation for infinite slab geometry. (2) The unit cell consisted of:

1. An active region - consisting of the active portion of the fuel plates plus the water between them. These properties are the same as those for the active region of the control elements and are given in Table 9.2.
2. A dead region - containing the side plates, fuel plate dead edges, and the water not contained in the active region. Thermal properties were obtained by combining the individual macroscopic cross sections weighted by their volume fractions. The microscopic cross sections for water and stainless steel are tabulated in Appendix A.

Two P_3 calculations were made for each configuration investigated, with the sources alternately in the active and dead regions. The results were weighted to account for differences in source strength and combined to form a relative thermal flux through the cell. (See Sections 1.2.2.1 and 5.1.3). The relative thermal flux for the cell was normalized to an average of unity in the active region. The results of these calculations are shown in Fig. 9.3 through 9.9. It is seen that, in general, a higher thermal flux peaking is obtained when the fuel plates are aligned perpendicular rather than parallel to the cell interface. The highest relative thermal flux peaking is found in a fixed fuel element separated from a control rod fuel element by the flow divider.

To obtain an experimental correction to the calculated power distributions, the power distribution for the SM-2 critical experimental core was calculated and compared to the experimental measurements made on the initial SM-2 mock-up cores. (3) Thermal flux distributions were calculated for a fixed fuel element separated from a control rod element and for a fixed element separated by another fixed element, using the calculational model described above. The thermal properties of the active region were calculated by the Medusa, IBM-650 code (Program 53) and are tabulated in Table 9.4. The flux distributions are shown in Fig. 9.10 and 9.11.

TABLE 9.4
THERMAL PROPERTIES OF ACTIVE REGION OF CRITICAL EXPERIMENT CORE
($T=68^\circ\text{F}$; 0% Burnup)

Parameter	Units	Value
$\sum a$	cm^{-1}	0.50625
$\sum s$	cm^{-1}	2.04010
$\sum \text{tr}$	cm^{-1}	2.15676

The power distributions were calculated from the thermal flux distributions by assuming a flat fast flux distribution across a cell. This method is described in Section 5.1.3 and may be stated as:

$$P(x_1)/P_{avg} = \phi(x_1)\beta + (1-\beta)$$

where:

$$\begin{aligned}\phi(x_1) &= \text{normalized flux at position } x_1 \\ P(x_1) &= \text{power at position } x_1 \\ P_{avg} &= \text{average power} \\ \beta &= \text{fraction of thermal fissions}\end{aligned}$$

Relative power distributions were obtained in the SM-2 critical experiments for the initial SM-2 mockup core.⁽³⁾ Small, bare, fission foils were attached to each edge and the center of fuel plate k in fixed fuel elements 45, 46, and 47 in a plane 5 in. above the bottom of the core. The measure activities were normalized to unity at a point 2.884 in. radially in the 5-in. axial plane as shown in Fig. 9.12. When comparing the measured power to the calculated distributions, the latter must be corrected for the radial overall decrease. This correction is of the form;

$$\left[P(r) \right]_{\text{measured}} = P'(r) J_0 \left[\frac{2.4048 r}{\tilde{R}} \right]$$

where:

$$\begin{aligned}\left[P(r) \right]_{\text{measured}} &= \text{actual measured value of power at position } r \\ &\quad \text{from center of core.} \\ P'(r) &= \text{revised measured value of power at position } r, \\ &\quad \text{with the normal radial fall-off removed.} \\ \tilde{R} &= \text{equivalent radius of core plus the radial reflector} \\ &\quad \text{savings.}\end{aligned}$$

The calculated maximum to minimum power ratios are considerably smaller than those measured in the critical experiment. To insure agreement, a correction factor was applied where:

$$\begin{aligned}\left[\phi(\text{actual}) - 1 \right]_{\text{max}} &= k \left[\phi(\text{calculated}) - 1 \right]_{\text{max}} \\ \left[\phi(\text{actual}) - 1 \right]_{\text{min}} &= k \left[\phi(\text{calculated}) - 1 \right]_{\text{min}} \\ A \left[P(\text{actual}) \right]_{\text{max}} &= \beta \left[\phi(\text{actual}) \right]_{\text{max}} + (1-\beta) \\ A \left[P(\text{actual}) \right]_{\text{min}} &= \beta \left[\phi(\text{actual}) \right]_{\text{min}} + (1-\beta)\end{aligned}$$

These were solved simultaneously and used to correct the calculated thermal flux distributions at 510°F and for various burnups.

The power distributions as a function of core lifetime were calculated by assuming non-uniform burnup. The cell was divided into ten regions and the average thermal flux in each region obtained by investigation of a P_3 flux distribution. This flux value was converted to an "actual" flux using the first of the above latter four equations. The burnup in each segment was related to the average cell burnup by:

$$\frac{N^{25}(x, t)}{N^{25}(x, 0)} = \left[\frac{\bar{N}^{25}(t)}{N^{25}(t)} \right] A(x, t)$$

where:

$$A(x, t) = \left\{ \frac{(1 + \alpha_{th}) \phi^{actual}(x) \beta(t) + (1 + \alpha_f) [1 - \beta(t)]}{(1 + \alpha_{th}) \beta(t) + (1 + \alpha_f) [1 - \beta(t)]} \right\}$$

α_{th} = thermal U-235 capture to fission ratio

α_f = fast U-235 capture to fission ratio

The relative power in each segment was then obtained from:

$$P(x, t) = \frac{N^{25}(x, t)}{\bar{N}^{25}(t)} \left[\phi^{actual}(x, t) \beta(t) + [1 - \beta(t)] \right]$$

Figures 9.13 and 9.14 show the relative power at the edge of the cell for two configurations as a function of burnup.

The results of these calculations are given in Tables 9.5 and 9.6 for the factors $Q(\Delta T)$ and $Q(\Delta \theta)$ respectively.

9.1.5.5 Summary of Power Peaking Factors for the SM-2

The results of the calculations for the radial peaking factors are given in Tables 9.5 and 9.6 for the factors $Q(\Delta T)$ and $Q(\Delta \theta)$ respectively. The axial power distribution is described in Table 9.12. These results were used as the reference factors for thermal calculations until they were superseded by the two-dimensional calculations described in Section 9.2.

The nuclear uncertainty factors and heat release factors were taken to be the same as used for the SM-1, that is:

$$\begin{aligned} F_N(\Delta T) &= 1.05 \\ F_N(\Delta \theta) &= 1.10 \\ \psi(\Delta T) &= 1.00 \\ \psi(\Delta \theta) &= 0.95 \end{aligned}$$

TABLE 9.5
SM-2 RADIAL POWER DISTRIBUTION (510°F; 0% BURNUP)

$$Q(\Delta T) = A \overline{F(x)} \overline{F(y)} \overline{F_{local}} \overline{F_{rods}}$$

where $A = 1.63$ and $\overline{F_{local}} = 1.00$

Element No.	$\overline{x}$ (in.)	$\overline{y}$ (in.)	$\overline{F(x)}$	$\overline{F(y)}$	$\overline{F_{rods}}$	$Q(\Delta T)$
44	0.56	1.22	0.997	0.988	0.86	1.3808
34	0.61	1.55	0.996	0.983	0.86	1.3725
35	2.94	1.55	0.944	0.983	0.88	1.3310
24	0.56	4.65	0.997	0.849	0.89	1.2278
25	2.94	4.48	0.944	0.861	0.96	1.2718
26	5.87	4.48	0.738	0.861	1.06	1.0919
14	0.61	7.42	0.996	0.763	1.00	1.2387
15	2.94	7.42	0.944	0.763	1.06	1.2444
16	5.87	7.42	0.738	0.763	1.14	1.0463

TABLE 9.6
SM-2 RADIAL POWER DISTRIBUTION (510°F; 0% BURNUP)

$$Q(\Delta\theta) = A F(x) F(y) F_{local} F_{rods}$$

where $A = 1.63$

Element No.	x (in.)	y (in.)	$F(x)$	$F(y)$	F_{local}	F_{rods}	$Q(\Delta\theta)$
44	1.11	1.22	0.989	0.988	1.44	0.87	1.98
34	1.22	1.55	0.988	0.983	1.45	0.87	1.99
35	1.38	1.55	0.981	0.983	1.23	0.87	1.75
24	1.11	4.65	0.989	0.849	1.44	0.92	1.81
25	1.38	4.48	0.981	0.861	1.45	0.92	1.84
26	4.81	4.48	0.841	0.861	1.45	1.01	1.73
14	1.22	7.42	0.988	0.763	1.45	1.02	1.82
15	1.75	7.42	0.977	0.763	1.45	1.02	1.80
16	4.65	7.42	0.848	0.763	1.28	1.10	1.48

9.2 FLUX AND POWER DISTRIBUTIONS USING TWO-DIMENSIONAL IBM-704 CODES

Near the end of 1959, the IBM-704 code, PDQ, became available to the project. This code permits a two dimensional flux and power analysis of the core. The core was broken up into various active and dead regions surrounded by a reflector region. Figures 9.15 and 9.16 show the various regions considered for the SM-2 Mockup Core, as used during the SM-2 Flexible Critical Experiments, and for the SM-2 reference core design respectively. The two-group nuclear properties of the various regions were calculated by use of the MUFT-III and plate-type P_3 theory IBM-650 codes. The fast group properties were averaged over 56 neutron groups for both at 68°F and 510°F; the cut-off for the fast group has only a slight effect upon the values of the fast parameters. Fifty-six groups were taken since this cutoff at 0.400 ev corresponds closely to the cadmium cutoff energy; it was felt that this would yield the best agreement to experimental measurements of the flux and power distributions.

9.2.1 SM-2 Experimental Core

A fairly extensive flux and power mapping of the SM-2 Mockup Core was performed during the SM-2 Flexible Critical Experiments.⁽³⁾ The PDQ code was used to obtain analytical predictions of the x-y power distribution to be compared to the measurements. This would permit an estimate of correction factors to be applied to calculations upon the reference SM-2 core at 510°F to obtain the best estimate of the reference SM-2 core at 510°F to obtain the best estimate of the reference power distribution. Axial predictions of power distributions were obtained by use of the Windowshade, IBM-650 code.

The atomic concentrations and two-group nuclear parameters of the various regions of the SM-2 Experimental Core are given in Table 9.7. The properties for pure water and pure 304L stainless steel are given in Table 9.8. The latter properties were used for the reflector regions.

TABLE 9.7
NUCLEAR PROPERTIES OF SM-2 EXPERIMENTAL CORE

(Elements at 68°F and 1 atm.)
($B^2=0.006896 \text{ cm}^{-2}$)

Parameter	Units	Fixed Elements		Control Fuel Elements*	
		Active Region	Dead Region	Dead Region #1 ⁺	Dead Region #2 ⁺⁺
$N^{25} \times 10^{24}$	atoms/cm ³	0.000818	-	-	-
$N^{28} \times 10^{24}$	atoms/cm ³	0.000059	-	-	-
$N^{B-10} \times 10^{24}$	atoms/cm ³	0.000034	-	-	-
$N^{ss} \times 10^{24}$	atoms/cm ³	0.017242	0.022490	0.029387	0.027984
$N^H \times 10^{24}$	atoms/cm ³	0.050046	0.043877	0.040303	0.044650
$N^O \times 10^{24}$	atoms/cm ³	0.025900	0.015239	0.015445	0.022325
τ	cm ²	31.738459	43.154199	41.097850	33.948244
D_f	cm	1.307954	1.439150	1.300459	1.192521
$\nu \Sigma_f^f$	cm ⁻¹	0.018674	-	-	-
Σ_a^f	cm ⁻¹	0.014458	0.003020	0.003861	0.003784
K_f		1.291573	-	-	-
P		0.649165	0.909445	0.977972	0.892292
Σ_a	cm ⁻¹	0.522375	0.062210	0.076960	0.074880
Σ_s	cm ⁻¹	2.041601	1.800086	1.750235	1.911608
Σ_{tr}	cm ⁻¹	2.173952	1.530698	1.521148	1.646655
$\nu \Sigma_f$	cm ⁻¹	0.797463	-	-	-
K_{th}		1.526610	-	-	-
D	cm	0.153330	0.217765	0.219132	0.202430

* Active region of control fuel elements assumed to be identical to the active region of the fixed elements.

+ Includes the side plates.

++ Includes the dead region bounded by dead Region #1 (parallel to the side plates).

TABLE 9.8
NUCLEAR PROPERTIES OF PURE WATER AND PURE STEEL USED
IN SM-2 EXPERIMENTAL CORE AT 68°F and 1 atm.

($B^2 = 0.006896 \text{ cm}^{-2}$)

Parameter	Units	Pure Water	Pure 304L Stainless
$N^H \times 10^{24}$	atoms/cm ³	0.066730	-
$N^O \times 10^{24}$	atoms/cm ³	0.033365	-
$N^{SS} \times 10^{24}$	atoms/cm ³	-	0.085112
τ	cm ²	33.429716	161.908623
D_f	cm	1.621547	1.316621
$\sum_a^f$	cm ⁻¹	0.000486	0.008132
P		0.989989	0.000019
$\sum_a$	cm ⁻¹	0.016960	0.193228
$\sum_s$	cm ⁻¹	2.420297	0.888572
$\sum_{tr}$	cm ⁻¹	1.934569	1.071224
D	cm	0.172303	0.311170

The measurements of radial and axial power distributions in the SM-2 Experimental Core at 68°F were taken with uranium and gold foils. Figures 9.17 and 9.18 show comparisons between calculated (PDQ-x, y) and experimental radial distributions, for a water and a steel reflected core respectively. The agreement is seen to be good except at the edge of the elements, where the calculated value underestimates the power peak considerably. This can be expected since the PDQ code employs diffusion theory, just as the VALPROD code, which is inaccurate near boundaries.

Figures 9.19, 9.20, 9.21 and 9.22 show the comparison between calculated and measured axial power distributions. The analytical values were obtained

by use of the Windowshade, IBM-650 code. The homogenous bank position was at 7.95 in. out while the measured bank position was 6.97 in. out; this is a difference of 0.98 in. In the SM-1, the difference between the Windowshade bank and measured bank position was 0.90 in. The Windowshade axial power distribution represents an average for all the elements and is not expected to correspond to the measurements made in a particular element. However, the agreement between the measured and calculated axial power distributions seems quite good over a wide range of fixed fuel element positions. The experimental peak to average power is lower than the calculated value since most of the measured points fall above the calculated curve. Each of the experimental axial traverses was taken down the center of the element.

Fig. 9.23 shows a two-dimensional power contour of the SM-2 Experimental Core at 68°F with a stainless steel reflector. Local power peaking in the corners is included. The worst power peak in the core occurs in the corners of the fixed fuel element adjacent to the central control rod fuel element.

9.2.2 SM-2 Reference Core

The properties of the various active and dead regions of the SM-2 reference core at 510°F were calculated by the same methods used in the previous section and are tabulated in Table 9.9; the properties were calculated for no xenon at 0 MWYR. The properties of pure water and pure type 347 stainless steel are given in Table 9.10; these properties were used for the reflector regions.

The PDQ code, with the use of the properties tabulated in Table 9.9 and 9.10, was used to calculate the flux and power distribution at 510°F for the reference core. In order to obtain the power generation for the hot spot and worst plate in each element, $Q(\Delta T)$ and $Q(\Delta \theta)$ respectively, it was necessary to first obtain correction factors to bring the calculated values into agreement with the measured distributions.

TABLE 9.9
NUCLEAR PROPERTIES OF SM-2 REFERENCE CORE
ELEMENTS AT 510°F and 2000 psia

$$(B^2=0.006194 \text{ cm}^{-2})$$

Parameter	Units	Fixed Elements		Control Fuel Elements*	
		Active Region	Dead Region	Dead Region #1 ⁺	Dead Region #2 ⁺⁺
$N_{25} \times 10^{24}$	atoms/cm ³	0.000802	-	-	-
$N_{28} \times 10^{24}$	atoms/cm ³	0.000059	-	-	-
$N_{B-10} \times 10^{24}$	atoms/cm ³	0.000034	-	-	-
$N_{SS} \times 10^{24}$	atoms/cm ³	0.016811	0.025270	0.028877	0.026010
$N^H \times 10^{24}$	atoms/cm ³	0.039524	0.036778	0.034554	0.036322
$N^O \times 10^{24}$	atoms/cm ³	0.020624	0.018389	0.017277	0.018161
τ	cm ²	46.175812	48.152052	48.650984	48.228750
D_f	cm	1.532036	1.390490	1.343863	1.380385
$\nu \Sigma_f^f$	cm ⁻¹	0.017642	-	-	-
Σ_a^f	cm ⁻¹	0.013636	0.003381	0.003803	0.003468
K_f		1.293793	-	-	-
P		0.589005	0.882921	0.862335	0.878847
Σ_a	cm ⁻¹	0.385745	0.049864	0.055562	0.051033
Σ_s	cm ⁻¹	1.383864	1.366594	1.337568	1.360650
Σ_{tr}	cm ⁻¹	1.415954	1.091500	1.087177	1.090620
$\nu \Sigma_f$	cm ⁻¹	0.579884	-	-	-
K_{th}		1.503283	-	-	-
D	cm	0.235412	0.305389	0.306604	0.305636

* Active region of control fuel elements assumed to be identical to the active region of the fixed elements.

+ Includes the side plates.

++ Includes the dead region bounded by Dead Region #1 (parallel to the side plates).

TABLE 9.10
NUCLEAR PROPERTIES OF PURE WATER AND PURE STEEL USED
IN SM-2 REFERENCE CORE AT 510°F and 2000 psia

$$(B^2 = 0.006194 \text{ cm}^{-2})$$

Parameter	Units	Pure Water	Pure 347 Stainless
$N^H \times 10^{24}$	atoms/cm ³	0.052358	-
$N^O \times 10^{24}$	atoms/cm ³	0.026179	-
$N^{SS} \times 10^{24}$	atoms/cm ³	-	0.084922
τ	cm ²	53.949312	162.000958
D_f	cm	2.040232	1.314557
Σ_a^f	cm ⁻¹	0.000364	0.008114
P		0.990373	0.000021
Σ_a	cm ⁻¹	0.009947	0.144092
Σ_s	cm ⁻¹	1.569940	0.886586
Σ_{tr}	cm ⁻¹	1.121773	1.020039
D	cm	0.297148	0.326784

The complete expression for the radial hot spot factor at the start of core life is:

$$Q(\Delta\theta) = P(x,y)_{\max} P(z)_{\max} F_{\text{rods}} F_{\text{local correction}}$$

where the factors are similar to those described in Section 9.1 but have been slightly modified to take advantage of the knowledge of the two-dimensional power distribution. The use of the two-dimensional code permits a better estimate of the effect of the control rods upon the power distribution and the power peaking in the corner of the elements. The factors in the equation for $Q(\Delta\theta)$ are defined in this case as:

$Q(\Delta\theta)$ = Power generation rate at the hottest spot in each element, normalized to an average radial power of 1 over the core.

$$F_{\text{rods}} = \frac{\left(\frac{P(x, y, z) \text{ measured at the center of each element}}{P \text{ measured core average of 1.0}} \right)}{\left(\frac{P(x, y) P(z) \text{ calculated at the center of each element}}{P \text{ calculated core average of 1.0}} \right)}$$

$$F_{\text{local correction}} = \frac{\left(\frac{P(x, y, z) \text{ measured at the edge of each element}}{P(x, y, z) \text{ measured at the center of each element}} \right)}{\left(\frac{P(x, y) \text{ calculated at the edge of each element}}{P(x, y) \text{ calculated at the center of each element}} \right)}$$

$P(x, y)_{\text{max}} P(z)_{\text{max}}$ = maximum calculated power generation in each element.

The local peaking correction factor, $F_{\text{local correction}}$, was calculated at the edge of the element instead of the corner simply because no measurements were taken at corner points. The correction factors, on the calculated power, F_{rods} and $F_{\text{local correction}}$, were assumed to remain constant with temperature.

Since the worst hot spot occurs near core midlife when the rods are furthest in, the axial power distribution was taken at this point. The radial peaks decrease with core burnup; the correction for burnup was obtained from Section 9.1. This correction, calculated at the edge of the elements was also applied to the hot spot factor at the corner of the elements.

To obtain the power generation rate in the hottest plate in each element, $Q(\Delta T)$, the factor $Q(\Delta\theta)$ was multiplied by the ratio of the average power in the hottest plate to the power at the hottest spot in each element. The factors $Q(\Delta T)$ and $Q(\Delta\theta)$ are tabulated in Table 9.11. The radial power distributions in Table 9.11 must be multiplied by the worst maximum to average axial power distribution in order to obtain the overall peaking factors. The axial power generation was taken from the one described in Table 4.6 of Section 4.2 and is given in Table 9.12. The values in Table 9.12 were found by multiplying the values in Table 4.6 by the number of flux points ($N = 44$) within the core; the values in Table 4.6 were normalized to the number of flux points within the core. The use of flux suppressors would eliminate the spike at the bottom of the core.

The recommended nuclear uncertainty factor, $F_N(\Delta\theta)$, has been reduced from 1.10 (as described in Section 9.1.5.5) to 1.05, since local peaking factors have been predicted more accurately by the PDQ - x, y code and corrected to agree with the experimental measurements. Some conservatism has been introduced in the local burnup correction since the burnup at the element edge was used. The burnup factor in the corner of the element will be larger due to greater fuel depletion.

TABLE 9. 11
SM-2 RADIAL POWER DISTRIBUTION

(SM-2 Reference Core - 510°F - S. S. Reflector -- Midlife)

<u>Element Number</u>	<u>Q (ΔT)</u>	<u>Q ($\Delta \theta$)</u>
44	1. 66	1. 95
45	1. 62	1. 88
46	1. 19	1. 57
47	1. 15	1. 37
34	1. 41	1. 93
35	1. 37	1. 93
37	1. 08	1. 35
25	1. 37	1. 80
26	1. 16	1. 57
27	0. 71	1. 01
14	0. 89	1. 51
15	0. 88	1. 49
16	0. 62	1. 07

$$P(z)_{\max} = 1.84$$

$$F_N(\Delta T) = 1.05$$

$$F_N(\Delta \theta) = 1.05$$

$$\psi(\Delta T) = 1.00$$

$$\psi(\Delta \theta) = 0.95$$

where:

$Q(\Delta T)$	= power generation for worst channel in each element normalized to a core average of 1.0.
$Q(\Delta \theta)$	= power generation at the hottest spot in each element, normalized to a core average of 1.0.
$P(z)_{\max}$	= power generation at worst axial point in the core, normalized to a core average of 1.0.
$F_N(\Delta T)$ $F_N(\Delta \theta)$	= nuclear hot spot factors to allow for uncertainty in the calculated power distributions.
$\psi(\Delta T)$	= fraction of total reactor power generated in the core.
$\psi(\Delta \theta)$	= fraction of total reactor power generated in the fuel plates.

TABLE 9.12
CORRECTED WINDOWSHADE (AXIAL) POWER DISTRIBUTION AT THE
DEEPEST ROD BANK POSITION

(T = 440°F; No Xenon)

<u>Inches from Bottom of Core</u>	<u>P(z)/P core average of 1.0</u>
0	2.92 (Bottom spike)
2	1.56
4	1.80
5.5	1.84 (axial max)
6	1.83
8	1.55
10	0.97
12	0.74
14	0.58
16	0.43
18	0.31
20	0.20
22	0.17

9.3 COMPARISON OF PDQ AND VALPROD CALCULATIONS OF POWER DISTRIBUTION

The power distribution within the SM-2 Experimental Core for all fixed elements with a water reflector was calculated by use of the PDQ, IBM-704 code, a radial VALPROD for a cylindrical core, and a slab VALPROD through the central plane of the core. The comparison is shown in Table 9.13; it is

seen that the VALPROD code shows good agreement to the PDQ for one-dimensional calculations. This is expected since both codes are based upon diffusion theory. Figure 9.24 shows a comparison of a one-dimensional slab VALPROD calculation and a PDQ (x, y) calculation through the core central plane. The two curves are identical when normalized to the same value at the core centerline. However, the centerline values are considerably different. If the slab VALPROD calculation is corrected by the centerline value of the radial VALPROD calculation, which is in good agreement to the PDQ values, then good agreement is obtained.

Intercell thermal flux distributions which were obtained from the plate-type P₃ theory, IBM-650 code were compared to PDQ (x, y) calculations; these results are shown in Figs. 9.25 and 9.26. In Fig. 9.25, at 68°F, the PDQ code which utilized P₁ or diffusion theory, predicts higher peaking than the P₃ theory. This is in contrast to the results of Section 11.13 in which P₃ predicts higher peaking than the P₁ theory; however, it is felt that the P₃ theory would still predict higher peaking if it were employed in the two-dimensional PDQ code instead of the presently used P₁ theory. In Fig. 9.26 at 510°F, both calculations seem to be in good agreement and show a decrease in flux peaking with an increase in temperature.

TABLE 9.13
COMPARISON OF POWER DISTRIBUTION CALCULATED BY VARIOUS CODES
SM-2 Experimental Core - 68°F - All Fixed Elements - H₂O Reflector)

Code	$\left(\frac{P_{\text{core center}}}{P_{\text{core avg of 1.0}}} \right)$	$\left(\frac{P_{\text{core edge}}}{P_{\text{core avg of 1.0}}} \right)$	$\left(\frac{P_{\text{edge}}}{P_{\text{center}}} \right)$
PDQ-through central plane of the core	1.56	1.71	1.10
PDQ-through plane where the distance from the center of the core to the edge of the core is equal to the equivalent radius of the cylindrical core	1.56	1.44	0.92
Radial VALPROD - cylindrical core geometry	1.58	1.51	0.96
Radial VALPROD - extrapolated to the central plane of the core.	1.58	1.74	1.10
Slab VALPROD - through central plane of the core	1.28	1.41	1.10

RELATIONSHIP BETWEEN ACTUAL CORE
AND
CYLINDRICAL APPROXIMATION

Figure 9.1

F_{RODS} , FACTORS FOR SM-I, SM-IA

				1.06	1.12	1.16	1.19	1.21				
	12	13		14	X	X	15	X	X	16		
				<u>1.03</u>	X	X	<u>1.11</u>	X	X	<u>1.16</u>		
				<u>1.00</u>	X	X	<u>1.06</u>	X	X	<u>1.14</u>		
21	22	23		24			25			26		27
				<u>0.92</u>	0.96		<u>1.00</u>	1.06	<u>1.13</u>			<u>1.16</u>
				<u>0.89</u>	X	X	<u>0.96</u>	X	<u>1.06</u>			
31	32	33		34			35			36		37
				<u>0.85</u>	0.88		<u>0.91</u>			<u>1.00</u>		<u>1.11</u>
				<u>0.86</u>	X	X	<u>0.88</u>	0.92				
41	42	43		44	X		45			46		47
				<u>0.86</u>	0.86		<u>0.85</u>			<u>0.92</u>		<u>1.03</u>
					0.87							
51	52	53		54			55			56		57
61	62	63		64			65			66		67
	72	73		74			75			76		

Figure 9.2

Figure 9.3

Figure 9.5

9-27

Figure 9.6

Figure 3.7

Figure 9.9

Figure 9.10

**THERMAL FLUX DISTRIBUTION IN A FIXED FUEL ELEMENT ADJACENT
TO ANOTHER FIXED FUEL ELEMENT IN THE CRITICAL EXPERIMENT
CORE**

Figure 9.11

Figure 9.13

Figure 9.14

*DIAGRAM OF SM-2 EXPERIMENTAL
CORE AS SEEN BY THE
PDQ (X,Y) CODE*

Figure 9. 15

**DIAGRAM OF SM-2 REFERENCE
CORE AS SEEN BY THE
PDQ (X,Y) CODE**

Figure 9.17

9-40

Figure 9.18

Figure 9.19

9-42

Figure 9.20

Figure 9.21

9-44

Figure 9.22

SM-2 POWER CONTOUR
68°F STAINLESS STEEL REFLECTOR

Figure 9.23

Figure 9.25

10.0 ANALYSIS OF CRITICAL EXPERIMENTS

The following sections contain miscellaneous calculations performed during the course of the SM-2 Flexible Critical Experiments⁽¹⁾ and, in general, are based upon preliminary information available at the time of the calculation; therefore, the calculations cannot be considered precise but are included to illustrate the methods employed. Some analysis of the experimental measurements has been described in previous Sections.

For the SM-2 Flexible Experiments, the stationary fuel elements contain up to 18 stainless steel UO_2 matrix fuel plates each loaded with 46.3 gm U-235. Control rod fuel elements contain up to 16 similar fuel plates each loaded with 42.24 gm of U-235. The fuel plates are of the SM-1 type with increased thickness of the matrix from 0.020 in. to 0.030 in. to accommodate the higher UO_2 loading; the matrix is clad with 0.005 in. stainless steel.

Flexibility of number and distribution of fuel plates in both the stationary and control rod fuel elements is provided by extruded polystyrene grooves which hold the fuel plates erect and act as fuel plate spacers.

The uniform burnable poison (in the form of B_4C) was added to the core in the form of Mylar film, loaded with boron dispersed in ferrous oxide. This film, containing various areal densities of boron, may be added in a number of layers by gluing the tapes to the fuel plates. Boron, in the form of B_4C , in particle sizes of one to three microns, was added to 0.005 in. Mylar film.

A special IBM-650 program was written to prepare input to the MUFT-III and P₃ code, for the calculation of the fast and thermal parameters, respectively, both for the fixed and control rod fuel elements. This program (Medusa or Program 53) is described in Section 1.3.4.

The polystyrene was assumed to have the chemical formula of $(\text{C}_6\text{H}_5\text{CHCH}_2)_n$; the theoretical weight percents of carbon and hydrogen become 92.2568% and 7.7432% respectively. Therefore, the atomic concentrations of carbon and hydrogen become 0.048950×10^{24} atoms/cm³, for polystyrene having a density of 1.058 gm/cm³.

10.1 CLEAN CORE (NO BORON) CALCULATIONS

The first series of experiments performed, during the SM-2 Flexible Critical Experiments, was concerned with a cold clean core containing no burnable poison (boron). Analysis of such a core is simplified due to the absence of problems associated with use of a burnable poison; including: no uncertainties in the actual boron content, and no self-shielding problems in the poison.

The basic geometry and loading data for the elements, used in the clean core calculations, are tabulated in Table 10.1. The nuclear properties, as calculated by the Program 53, Program 54, MUFT-III, and Plate-Type P₃, IBM-650 codes, are tabulated in Table 10.2 for 18 and 16 plates in the fixed and control fuel elements respectively.

10.1.1 Minimum Core Size

Criticality was attained by a 3 x 3, 18 plate per element core with a single central control rod. The control rod, containing 16 fuel plates, was about 17 in. withdrawn, with the balance worth about 40 cents or about 0.29% ρ . This array contained a critical mass of 7.2 kg of U-235.

In order to compare theoretical to experimental predictions of reactivity, a two-group, multiregion calculation was made utilizing the VALPROD, IBM-650 code. The control fuel element was included by the equivalent ring method described in Appendix C. The nuclear properties of the elements were taken from Table 10.2. The calculated reactivity of the core for the control rod fuel element fully inserted was calculated to be 0.855% ρ . The experimental value, for the control rod fully withdrawn, is about 0.29% ρ . Therefore, the model overpredicts the reactivity by about 0.56% ρ ; this is considered good agreement.

Some of the assumptions used in the calculation are as follows. The square cross-sectional area of the core array was approximated by a circle of equivalent area. This assumption becomes worse as the size of the core decreases. A cylindrical core has less leakage than the corresponding rectangular core; therefore, the VALPROD calculation would tend to overestimate the core reactivity. In addition, the fast nuclear properties are a function of the geometric buckling. The properties were calculated for a buckling corresponding to a 7 x 7 array; however, the error due to this assumption is small.

10.1.2 Critical Mass of the 7 x 7 Core

For the 7 x 7 core array, criticality was achieved with three plates per fixed element and four plates per control fuel element; the seven rod bank critical position was at 17.884 in. withdrawn with the balance worth about 1.0% ρ in reactivity.

The nuclear properties of the elements were obtained by the methods described in Section 11.1 and a VALPROD calculation was made. The control fuel elements were accounted for by the equivalent ring method described in Appendix C. The VALPROD calculation predicts a reactivity of 4.3% ρ for the control fuel elements fully inserted; the reactivity for the control elements withdrawn 17.884 in. becomes 3.3% ρ . This seemingly large discrepancy may be partially explained by use of a cylindrical model and inaccuracies introduced into the P₃ calculations due to the large separation of fuel plates.

TABLE 10.1
GEOMETRIC AND LOADING DATA USED IN CLEAN CORE CALCULATIONS

	<u>Fixed Element</u>	<u>Control Element</u>
<u>Element Geometry</u>		
Element size (in.)	2.9375 sq.	2.9375 sq.
Width of side plate (in.)	2.848	2.621
Width of active region (in.)	2.9375	2.6111
Width of clad fuel plate (in.)	2.777	2.554
Width of meat (in.)	2.51	2.32
Length of active region (in.)	21.6	21.6
Meat thickness (in.)	0.0306	0.0301
Clad thickness (in.)	0.0051	0.005
Fuel plates per element	18	16
Polystyrene volume per element (in ³)	7.4244	7.5837
Outside dimensions of control rod basket (in.)	-	2.797 sq.
Thickness of control rod basket (in.)	-	0.054
<u>Loading Data</u>		
U-235 per fuel plate (gm)	46.30	42.24
Uranium per fuel plate (gm)	49.69	45.36
UO ₂ per fuel plate (gm)	56.64	51.72
S. S. in meat per fuel plate (gm)	157.35	141.53
"Void" fraction in meat of fuel plate	0.065061	0.089044
<u>Material Densities (pure material)</u>		
Polystyrene (gm/cm ³)	1.058	1.051
UO ₂ (gm/cm ³)	10.8	10.8
Side plate stainless steel (gm/cm ³)	7.751	7.902
Fuel plate stainless steel (gm/cm ³)	7.74	8.18
Control rod basket stainless steel (gm/cm ³)	-	7.85

TABLE 10.2
NUCLEAR PROPERTIES FOR CLEAN CORE ELEMENTS

(No Boron; $T=68^{\circ}\text{F}$; $B^2 = 0.006896 \text{ cm}^{-2}$)

Parameter	Units	Fixed Elements	Control Elements
τ	cm^2	33.232621	33.650480
D_f	cm	1.305786	1.277892
Σ_R	cm^{-1}	0.039292	0.037975
$\nu \Sigma_f^f$	cm^{-1}	0.020547	0.016773
Σ_a^f	cm^{-1}	0.013870	0.012163
K_f		1.481395	1.378989
P		0.646997	0.679706
Σ_a	cm^{-1}	0.356861	0.286660
Σ_s	cm^{-1}	1.996915	1.955155
Σ_{tr}	cm^{-1}	1.977490	1.881086
$\nu \Sigma_f$	cm^{-1}	0.633743	0.471763
K_{th}		1.775879	1.645723
D	cm	0.168564	0.177202
L^2	cm^2	0.472351	0.618160

10.1.3 Critical Mass with Seven Control Rods Fully Inserted

The minimum critical mass for the full size core with no boron and seven control rods full-in was obtained experimentally by extrapolation of data of bank position vs. number of plates per element. (1) This value was found to be 10.1 plates per element. Scram III calculations were performed, using core properties corresponding to 10.1 plate per element, and predicted a reactivity of $3.5\% \rho$. However, as described in Section 8.5.1, inaccuracies in the Scram III code yields results which may be several percent high in reactivity; therefore, the model probably overestimates the reactivity by the order of about 1 to $2\% \rho$.

10.1.4 Excess Reactivity of Full Size Core

The experimental value of the excess reactivity for the full size core with no boron was obtained by determining the area under the curve of seven rod bank worth per inch versus bank position, extrapolated for 18 plates per element.⁽¹⁾ This value was found to be $23\% \rho$, based upon an effective delayed neutron fraction of 0.0073. The calculated value was determined by a VALPROD calculation using the element properties described in Section 10.1 and replacing the control rod fuel elements by the equivalent ring method described in Appendix C; the calculated value was found to be $25\% \rho$. There are always errors inherent in analyzing cores with large excess reactivities; therefore, it is difficult to reach definite conclusions regarding the accuracy of calculations performed on such cores.

10.1.5 Summary

It is seen that for the above calculations the model overestimates the reactivity in each case by as much as nearly $3\% \rho$. As described above, there are errors inherent in each calculation which may affect the results; in addition, there are two other important sources of error which should be mentioned. The value for the effective delayed neutron fraction, used in converting experimental values expressed in dollars to analytical values expressed in percent reactivity, is not known exactly. The value of 0.0073, as used in the SM-1 experimental analysis,⁽²⁾ was used above; however, this value is a function of rod position and core composition. In addition, as described in Section 11.3, the elastic and inelastic scattering parameters of iron and stainless steel, as used in the MUFT-III file, were based on outmoded data. Use of the latest data for iron and stainless steel reduces the calculated reactivity by as much as 1 to $2\% \rho$. Therefore, it is seen that by use of the new files, good agreement between measured and calculated values of reactivity may be obtained. This problem will be studied further when a more detailed analysis of the SM-2 Flexible Critical Experiments is made.

10.2 COMPARISON WITH ORNL CRITICAL EXPERIMENTS FOR THE SM-1

In conjunction with the development of the SM-1 Core, critical experiments were performed at ORNL in which the effect of percent metal and boron loading on critical mass were studied.⁽³⁾ It was found that the variation of critical mass with percent metal and with boron loading was approximately linear. Assuming both of these effects to be additive, the ORNL data for a 7 x 7 core (45 elements) is correlated by the following formula:

$$\text{Critical Mass (kg U-235)} = 2.70 + 0.395 (\% \text{ metal}) + 0.0378 (\text{gm B})$$

Table 10.3 shows the material content of three configurations measured in the SM-2 critical experiments.

TABLE 10.3
SM-2 CRITICAL CONFIGURATIONS

(45 Element Core)

$\left(\frac{\text{gms. B}}{\text{kg U-235}}\right)$	38 Fixed Elements			7 Movable Elements			Reactivity (Cents)
	No. Plates	gms U-235	% Metal *	No. Plates	gms U-235	% Metal *	
0.00	3	138.9	6.62	4	169.0	14.18	136
7.05	4.632**	214.4	8.81	5	211.2	15.40	70
14.10	11	509.3	17.17	11	424.6	22.55	18

* % Metal = 100 $\left[1 - \frac{\text{Hydrogen Number Density}}{0.06673 \times 10^{24}} \right]$

** 4 plates in 14 elements, 5 plates in 24 elements.

In order to make a reasonable comparison with the ORNL experiments, the material content of the SM-2 elements should be weighted by their relative importance. Assuming a radial savings of 6.7 cm. and a $J_0^2(B_{rr})$ variation of the radial importance function, the seven control rod element positions were found to be worth 1.50 times as much as an average position. This means that the 45 element core is equivalent to a uniform mixture of 10.5 movable elements and 34.5 fixed elements.

Table 10.4 shows a comparison of the SM-2 critical masses (corrected for $J_0^2(B_{rr})$ weighting and for excess reactivity) with the critical mass calculated by the formula based on the ORNL experiments.

In view of the approximate nature of the corrections employed in the comparison, the critical masses for the SM-2 experiments appear to be reasonably consistent with the ORNL experiments on the SM-1.

TABLE 10.4
COMPARISON OF MEASURED AND CALCULATED CRITICAL MASS

(45 Element SM-2 Core)

$\left(\frac{\text{gms B}}{\text{kg U-235}}\right)$	$J_0^2(B_{rr})$ Weighted Material Content			Excess $\rho(\%)$	Critical Mass, kg U-235	
	% Metal	gms. B	kg U-235	Correction	Measured	Calculated
0.00	8.38	0	6.57	-0.15	6.42	6.02
7.05	10.34	62	9.62	-0.17	9.45	9.34
14.10	18.42	317	22.45	-0.09	22.36	21.98

10.3 SURFACE VS. VOLUME DISTRIBUTION OF BURNABLE POISON

Using the equations described in Section 1.10, the reactivity change due to use of boron as a surface distribution as opposed to a volume distributed was calculated.

The effect was calculated at 68 F for a core composed of all fixed elements using the data tabulated in Table 6.4. The microscopic cross sections are tabulated in Appendix A. Based on this information, the change in reactivity was calculated as a function of $B = (\sum a)_B N_3 t_T$. These results are shown in Fig. 10.1. The value of B may be expressed by:

$$B = (\sum a)_B N_3 t_T = 6.013312 \times 10^{22} \times N_3 y \bar{\sigma}_a \text{ B-10}$$

It is seen that the maximum loss of reactivity expected due to using a surface rather than a volume distribution of boron is about 0.75% ρ .

For the SM-2 critical experiments, the boron used is in the form of B_4C having a size range from one to three microns. The self-shielding corrections for particles of this size are quite small (Section 7.7) and were not included in the analysis of the critical experiments.

10.4 REACTIVITY AS A FUNCTION OF BORON LOADING

The excess reactivity, with fully withdrawn control rods, was measured as a function of number of fuel plates per fixed fuel element and various B-10 loadings, during the SM-2 Flexible Critical Experiments.⁽¹⁾ By cross plotting, the reactivity of the core containing 18 plates per fixed element (and consequently, 16 plates per control rod fuel element) may be obtained as a function of B-10 loading. These results may be seen in Fig. 10.2. A value of 0.0073 for the effective delayed neutron fraction was assumed. To compare the calculational model to the experimental results, the reactivity of the core was calculated as a function of B-10 loading.

10.4.1 Nuclear Parameters of the Elements

The nuclear parameters of the flexible critical experiment fuel elements were calculated as a function of B-10 content per plate. The U-235 loading and geometry of the elements are tabulated in Table 10.1. The parameters were calculated at 68 F and atmospheric pressure through use of the following IBM-650 codes; Program 53, Program 54, MUFT-III, and Plate Type P3 Calculations. The results of these calculations for fixed elements and control elements containing 18 and 16 plates per element respectively are shown in Fig. 10.3 through 10.9.

10.4.2 Reactivity Vs. Boron Loading

The reactivity of the core as a function of B-10 content was calculated by the use of VALPROD. The nuclear parameters of the elements were taken from the calculations described in Section 10.4.1. The core considered was water reflected; the control fuel elements were included by the "equivalent ring method" described in Appendix C. The effect of boron introduced as a surface distribution (boron-impregnated mylar tape) instead of the usual volume distribution with the fuel matrix was included in the calculated results. The results of these calculations are shown in Fig. 10.2. It is seen that the calculated values of reactivity are consistently higher by about $1.5\% \rho$ in reactivity than the experimental values. This error could be attributed partly to the value of the effective delayed neutron fraction assumed (0.0073); however, a more significant correction could be made if the revised MUFT-III cross-sectional files for iron and stainless steel had been used. As described in Section 11.3, use of new files reduces the calculated reactivity of the SM-2 by the order of $1.5\% \rho$; this would result in good agreement between the measured and calculated values, provided the errors due to the choice of the effective delayed neutron fraction and uncertainties in the actual material content of the experimental core are small.

10.5 REACTIVITY CALCULATIONS BY USE OF THE PDQ CODE

10.5.1 Reactivity of the SM-2 Experimental Core

The reactivity of the SM-2 Experimental Core was calculated for various configurations by the PDQ code and compared to measured values; these results are shown in Table 10.5. The reference core (stainless steel reflector) reactivity of 68 F is $13.0\% \rho$ with seven control rod fuel elements. This is a gain of $0.7\% \rho$ over the reactivity of the core with a water reflector. The experimental value of the reactivity of the reference core is $11.5\% \rho$; the gain in reactivity using a large steel reflector instead of water was experimentally extrapolated to $0.6\% \rho$.

The effect of substituting fixed fuel elements in place of control rod fuel elements was calculated as $+1.0\% \rho$. Measured values were obtained by substituting for one control rod fuel element at a time, the total effect being $2.0\% \rho$. The total effect was obtained by adding up the individual effects. This measured value may be lower if all seven elements could be replaced at one time, since there are "interaction effects" on the flux when just one element is replaced. In this case, the core array becomes non-symmetrical and there is a shifting of the flux to the side containing the additional fixed fuel element. Adding the effects of each individual element tends to overestimate the effect of replacing all seven control rod fuel elements.

The experimental reactivities must be corrected for the use of boron in mylar rather than dispersed (in rather large lumps) in the matrix. This

correction has been estimated analytically to be a maximum of $0.75\% \rho$. This will be checked experimentally using substitution plates.

The reactivity change in replacing the central control rod fuel element alone was calculated as $+0.2\% \rho$ and measured as $+0.3\% \rho$, which is in much better agreement than that calculated for seven control rod elements.

TABLE 10.5
REACTIVITY OF VARIOUS CONFIGURATIONS
OF THE EXPERIMENTAL CORE - 68°F

Core Configuration	Calculated Reactivity ($\% \rho$)	Measured Reactivity ($\% \rho$)
Seven Control Rod Fuel Elements - Stainless Steel Reflector	13.0	11.5*
All Fixed Fuel Elements - Stainless Steel Reflector	14.0	13.5**
Seven Control Rod Fuel Elements - Water Reflector	12.3	10.9
All Fixed Fuel Elements - Water Reflector	13.3	12.9+

* This value obtained using total K_{excess} in cents and $\rho = 1 - e^{-0.0078 \times K_{\text{excess}}}$ and integral rod worth curve presented in APAE No. 54, Fig. 4.4. (1)

** This value obtained using $\Delta \rho$ due to substitution of fixed fuel elements of $+2.0\% \rho$.

+ This value obtained using $\Delta \rho$ due to a steel to water reflector change of $-0.6\% \rho$.

10.5.2 Reactivity of the SM-2 Core at 510°F

The reactivity of the reference SM-2 core at the operating temperature of 510°F with a stainless steel reflector and flow divider and no xenon was calculated to be $6.7\% \rho$ as seen in Table 10.6.

The stainless steel reflector is now calculated to be worth $0.8\% \rho$ more

than a water reflector. The effect of substituting fixed fuel elements for the control fuel elements was calculated to be $+ 1.3\% \rho$.

The calculated temperature defect from 68°F to 510°F is $6.2\% \rho$ for the SM-2 core with a stainless steel reflector. The measured value is $6.7\% \rho$.

At present, the best estimate of the reactivities of the SM-2 core is given in Table 10.7. The loss in core reactivity due to a flow divider was extrapolated from experimental data⁽¹⁾ and is found to be $0.7\% \rho$. From Section 7.5, the zero to equilibrium and zero to peak xenon reactivity changes are calculated to be $2.7\% \rho$ and $4.3\% \rho$ respectively.

TABLE 10.6
REACTIVITY OF VARIOUS CONFIGURATIONS OF THE SM-2
REFERENCE CORE - 510°F - NO XENON

Core Configuration	Calculated Reactivity ($\% \rho$)
Seven Control Rod Fuel Elements - Stainless Steel Reflector	6.7
Seven Control Rod Fuel Elements - Water Reflector	5.9
All Fixed Fuel Elements - Stainless Steel Reflector	8.0
All Fixed Fuel Elements - Water Reflector	7.1

TABLE 10.7
BEST ESTIMATE OF SM-2 REFERENCE CORE REACTIVITY

Condition	Reactivity ($\% \rho$)	
	Calculated	Measured
68°F, Clean, 0% Burnup	12.3 [*]	11.5 ⁺
510°F, Clean, 0% Burnup, No Xenon	6.1 [*]	4.8 ⁺⁺
510°F, Eq. Xenon, 0% Burnup	3.4 ^{**}	2.1 ^{**}
510°F, Max. Xenon, 0% Burnup	1.8 ^{**}	0.5 ^{**}

* Calculated by PDQ, corrected for flow divider loss of 0.7% ρ or 96.6 cents.

** Calculated zero to equilibrium and zero to peak xenon reactivity changes are 2.7% ρ and 4.3% ρ respectively.

+ Taken from integral worth curve of seven rod bank, corrected for flow divider loss of 0.7% ρ or 96.6 cents.

++ The measured hot to cold reactivity change is 6.7% ρ or 89.0 cents.

Figure 10.1

Figure 10.3

Figure 10.5

Figure 10.7

Figure 10.9

11.0 REVIEW OF THE CALCULATIONAL MODELS

As described in Section 1.0, the basic model employed in the nuclear analyses performed at Alco is the use of modified two-group theory. Fast parameters are calculated by use of the MUFT-III code, employing the P-1 Selengut - Goertzel approximation; inelastic scattering is described by use of the Weisskopf statistical model. The thermal properties are calculated by use of P_3 theory using cross sections averaged over a Maxwell - Boltzmann distribution evaluated at a hardened neutron temperature. The innercell power distribution is generally calculated from the P_3 thermal flux distribution and a knowledge of the fraction of thermal fissions within the element.

It has been found during the nuclear analysis of the SM-2 Flexible Critical Experiments (Section 10.0) that the calculational models overestimate the reactivity of the core and underestimate the thermal flux and power distribution within the elements; therefore, a study of the calculational models was made to attempt to reduce the discrepancy between the experimental data and theoretical predictions.

11.1 EFFECT OF CALCULATIONAL MODEL AND NEUTRON TEMPERATURE UPON THERMAL CALCULATIONS⁽¹⁾

11.1.1 P_3 Theory With Maxwell-Boltzmann Averaged Cross Sections at a Hardened Neutron Temperature

The method presently used at Alco for the calculation of the innercell thermal flux distributions and thermal parameters for APPR-type elements involves the use of P_3 theory. This theory is explained adequately in Section 1.1.2.1 and has been programmed for the IBM-650. This program calculates the thermal flux distribution both perpendicular and parallel to the fuel plates, and from these distributions, computes the thermal parameters of the fuel element. The element considered for study is described in Section 6.1. The thermal parameters of this element, at 68°F, calculated by this theory, are tabulated in Section 6.1 and again in Table 11.5. As stated in Section 6.1, the thermal cross sections have been evaluated at a hardened neutron energy of 0.0331 ev. The calculated P_3 flux distributions parallel and perpendicular to the fuel plates are shown in Fig. 11.1 and 11.2 respectively. The thermal microscopic cross sections evaluated at the kT energy value of 0.0331 ev are tabulated in Table 11.2.

11.1.2 P_3 Theory With Wigner-Wilkins Averaged Cross Sections

In this case, the thermal microscopic cross sections used in the plate-type P_3 theory, IBM-650 code, were averaged over a Wigner-Wilkins neutron spectrum. WAPD-185⁽²⁾ contains a tabulation of cross sections, at various core temperatures, calculated by the SOFOCATE code⁽³⁾ for mixtures of

hydrogen, Pu-239, U-235, and a $1/v$ absorber. The tables are presented as a function of $1/v$ barns per hydrogen atom at 0.0253 ev, U-235 atoms per hydrogen atom, and Pu-239 atoms per hydrogen atom. For the SM-2 fixed elements described in Section 6.1, these parameters are calculated to be:

3.676	=	$1/v$ barns per hydrogen atom
0.01388	=	U-235 atoms per hydrogen atom
0	=	Pu-239 atoms per hydrogen atom

By cross plotting the data presented in WAPD-185, the Wigner-Wilkins averaged cross sections were obtained; these values are tabulated in Table 11.3. By use of the IBM-650 code for type-type elements, employing P_3 theory, the flux distributions and thermal parameters were calculated. The P_3 flux distributions parallel and perpendicular to the fuel plates are shown in Fig. 11.1 and 11.2 and may be compared to the distributions obtained by use of Maxwell-Boltzmann averaged cross sections evaluated at a hardened neutron energy. It is seen that the use of Wigner-Wilkins theory results in a decreased thermal flux peaking. Data obtained upon the SM-2 Flexible Critical Experiment ⁽⁴⁾ indicates that calculations using the hardened, Maxwell-Boltzmann averaged cross sections underestimate the measured flux and power peaking (Section 9.1.5). Therefore, the use of Wigner-Wilkins theory offers no advantage from the power distribution viewpoint.

The thermal parameters calculated by the Wigner-Wilkins theory are given in Table 11.5. It is seen that the effect upon the thermal multiplication factor, K_{th} is small as compared to the use of hardened Maxwell-Boltzmann averaged cross sections.

11.1.3 P_1 Theory With Maxwell-Boltzmann Averaged Cross Sections at a Hardened Neutron Temperature

The innercell flux distributions parallel to the fuel plates and the thermal parameters were calculated by P_1 (diffusion) theory using the same techniques as used in the P_3 treatment described in Section 1.1.2.1. The element is divided into an active and a dead region. The flux distribution is first found due to a source in the dead region; then the flux distribution is found due to a source in the active region. These flux distributions are then superimposed and weighted to obtain the overall flux distribution. From the flux distributions, the thermal parameters of the element are calculated.

Consider an active region containing a fuel mixture and an adjacent dead region which contains no fuel. A picture of the model used, for a source in the dead region, is shown in Fig. 11.3. In this figure, W is the half-width of the active plus the half-width of the dead region while w is the half-width of the active region alone. In this case, the "fuel" region is the active region and the "moderator" is the dead region. The flux distribution due to a source of

1 neutron/cm³-sec in the dead region was then obtained. Letting the subscripts 0 and 1 designate the "fuel" and "moderator" respectively, the diffusion equations for each region may be written as: (5)

$$\begin{aligned} D_0 \nabla^2 \phi_0(z) - \Sigma_0 \phi_0(z) &= 0 \\ D_1 \nabla^2 \phi_1(z) - \Sigma_1 \phi_1(z) + 1 &= 0 \end{aligned}$$

The solutions may be written as:

$$\begin{aligned} \phi_0(z) &= A \cosh K_0 z && \text{(Fuel)} \\ \phi_1(z) &= C \cosh K_1 (W - z) + \frac{1}{\Sigma_1} && \text{(Moderator)} \end{aligned}$$

where:

$$\begin{aligned} K_0^2 &= \Sigma_0 / D_0 \\ K_1^2 &= \Sigma_1 / D_1 \\ A &= \frac{1}{\Delta} \left[D_1 K_1 \sinh K_1 (W - w) \right] \\ C &= -\frac{1}{\Delta} \left[D_0 K_0 \sinh K_0 w \right] \\ \Delta &= \Sigma_1 \left[D_1 K_1 (\cosh K_0 w) \sinh K_1 (W - w) + \right. \\ &\quad \left. + D_0 K_0 (\sinh K_0 w) \cosh K_1 (W - w) \right] \end{aligned}$$

$$\begin{aligned} \text{and: } w &= 3.2258 \text{ cm.} && ; W = 3.730625 \text{ cm.} \\ D_1 &= D_{\text{dead}} && ; D_0 = D_{\text{active}} \\ K_1 &= K_{\text{dead}} && ; K_0 = K_{\text{active}} \end{aligned}$$

The flux distribution due to a source in the active region is next obtained. In this case, as shown in Fig. 11.4, the "moderator" is the active region and the "fuel" is the dead region. Then, the flux distribution, due to a source of 1 neutron/cm³-sec in the active region is obtained. The form of the equations and their solutions are similar to those described above, except that in this case:

$$\phi'_0(z') = A' \cosh K'_0 z' \quad (\text{Fuel})$$

$$\phi'_1(z') = C' \cosh K'_1 (W - z') + \frac{1}{\Sigma_1} \quad (\text{Moderator})$$

and

$$w' = 0.504825 \text{ cm.} ; \quad W = 3.730625 \text{ cm.}$$

$$D'_0 = D_{\text{dead}} ; \quad D'_1 = D_{\text{active}}$$

$$K'_0 = K_{\text{dead}} ; \quad K'_1 = D_{\text{active}}$$

The overall flux distribution is obtained by weighting and superimposing

$\phi_0(z)$, $\phi'_0(z')$, $\phi_1(z)$, and $\phi'_1(z')$. Weighting is necessary because the source strength in the active and dead regions is not the same. The relative source strength in each of these regions was assumed proportional to the volume fraction of water in that region (S_{active} and S_{dead}). In addition, it is desirable to normalize such that the average flux in the element is unity. The normalization constant, γ , is:

$$\gamma = \frac{w}{W} \left[S_{\text{dead}} \bar{\phi}_0 + S_{\text{active}} \bar{\phi}'_1 \right] + \frac{W-w}{W} \left[S_{\text{active}} \bar{\phi}'_0 + S_{\text{dead}} \bar{\phi}_1 \right]$$

where:

$$w/W = \text{volume fraction of the active region} = 0.864680$$

$$\frac{W-w}{W} = \text{volume fraction of the dead region} = 0.135320$$

Therefore, in the active region:

$$\bar{\phi}(z)_{\text{active}} = \frac{1}{\gamma} \left[S_{\text{dead}} \phi_0(z) + S_{\text{active}} \phi'_1(z = W - z) \right]$$

and in the dead region:

$$\bar{\phi}(z)_{\text{dead}} = \frac{1}{\gamma} \left[S_{\text{active}} \phi'_0(z' = W - z) + S_{\text{dead}} \phi_1(z) \right]$$

The average fluxes, $\bar{\phi}_0$, $\bar{\phi}'_0$, $\bar{\phi}_1$, and $\bar{\phi}'_1$ may be calculated by:

$$\begin{aligned}
\bar{\phi}_0 &= \frac{A}{w K_{\text{active}}} \left[\sinh w K_{\text{active}} \right] \\
\bar{\phi}_1 &= \frac{C}{(W-w) K_{\text{dead}}} \left[\sinh (W-w) K_{\text{dead}} \right] + \frac{1}{\Sigma_{\text{dead}}} \\
\bar{\phi}'_0 &= \frac{A'}{w' K_{\text{dead}}} \left[\sinh w' K_{\text{dead}} \right] \\
\bar{\phi}'_1 &= \frac{C'}{(W-w') K_{\text{active}}} \left[\sinh (W-w') K_{\text{active}} \right] + \frac{1}{\Sigma_{\text{active}}}
\end{aligned}$$

The constants used in these equations are tabulated in Table 11.1. These constants were calculated for the fixed elements described in Section 6.1. The thermal properties for the whole fuel element are calculated by:

$$\Sigma_x^{\text{Element}} = \left(\frac{w}{W} \right) \Sigma_x^{\text{active}} \bar{\phi}_{\text{active}} + \left(\frac{W-w}{W} \right) \Sigma_x^{\text{dead}} \bar{\phi}_{\text{dead}}$$

where: $\Sigma_x = \Sigma_a, \Sigma_s, \Sigma_{\text{tr}}, \text{ and } \nu \Sigma_f$

$$\bar{\phi}_{\text{active}} = \frac{1}{\gamma} \left[s_{\text{dead}} \bar{\phi}_0 + s_{\text{active}} \bar{\phi}_1 \right]$$

$$\bar{\phi}_{\text{dead}} = \frac{1}{\gamma} \left[s_{\text{active}} \bar{\phi}'_0 + s_{\text{dead}} \bar{\phi}'_1 \right]$$

$$K_{\text{th}}^{\text{Element}} = \frac{\nu \Sigma_f^{\text{Element}}}{\Sigma_a^{\text{Element}}}$$

$$D^{\text{Element}} = \frac{1}{\left(3 \Sigma_{\text{tr}}^{\text{Element}} \right)}$$

$$L^2_{\text{Element}} = \frac{D^{\text{Element}}}{\Sigma_a^{\text{Element}}}$$

These parameters are tabulated in Table 11.5. It should be noted that the active and dead region properties were taken to be those calculated in Section 11.1.1, which were calculated by P_3 theory. A more accurate

comparison between P_1 and P_3 theory would have been obtained by also calculating the nuclear parameters of the active and dead regions using the same kind of equations developed in this section. In this case, the flux distributions perpendicular to the fuel plates would first be calculated due to a source in the dead region (water channel) alone. This would be the only distribution required since there would be no thermal source in the active region (the fuel plate). The thermal parameters of the active and dead regions as determined by P_1 theory could then be calculated and used in the calculation of the flux distribution parallel to the fuel plates.

The thermal flux distribution, parallel to the fuel plates, as calculated by P_1 theory is compared to P_3 theory in Fig. 11.5. It is seen that the peaking is less than that predicted by P_3 theory, therefore, as in the use of Wigner-Wilkins theory, P_1 theory offers no advantage from the standpoint of thermal flux and power distribution. It is seen in Table 11.5 that the effect upon the thermal multiplication factor, K_{th} , is small as compared to the use of P_3 theory with the same microscopic cross sections employed. A further discussion of P_1 , P_2 , and P_3 flux distribution in APPR-type elements may be found in reference (6).

TABLE 11.1
CONSTANTS USED IN P₁ THEORY CALCULATIONS

(T = 68°F; E = 0.0331 ev)

Parameter	Units	Value
w	cm	3.2258
w'	cm	0.504825
$\Sigma_1 = \Sigma'_0 = \Sigma_a^{\text{dead}}$	cm ⁻¹	0.069313
D ₁ = D' ₀ = D _t ^{dead}	cm	0.198387
K ₁ = K' ₀ = K _{dead}	cm ⁻¹	0.591085
$\Sigma_0 = \Sigma'_1 = \Sigma_a^{\text{active}}$	cm ⁻¹	0.517949
D ₀ = D' ₁ = D ^{active}	cm	0.152898
K ₀ = K' ₁ = K _{active}	cm ⁻¹	1.840528
Δ	cm ⁻¹	4.327134
Δ'	cm ⁻¹	32.334936
A	cm	0.008205
A'	cm	1.648684
C	cm	-12.319969
C'	cm	0.001098
W	cm	3.730625
S _{active}		0.754894
S _{dead}		0.702430
Φ_0	neuts/cm ² -sec	0.261796
Φ'_0	neuts/cm ² -sec	1.673298
Φ_1	neuts/cm ² -sec	1.923403
Φ'_1	neuts/cm ² -sec	1.895659
γ		1.750141

11.1.4 P₃ Theory With Maxwell-Boltzmann Averaged Cross Sections at a Non-Hardened Neutron Temperature

To find the effect of using a hardened neutron temperature upon the innercell flux distribution and the thermal parameters, the calculations were repeated using non-hardened cross sections evaluated at 0.0253 ev (T = 68°F). A tabulation of these microscopic cross sections are given in Table 11.4.

The flux distributions and thermal parameters were calculated by use of the P₃ theory for APPR-type elements as described in Section 1.1.2.1. The flux distribution parallel to the fuel plates using non-hardened as compared to hardened Maxwell-Boltzmann averaged cross sections, are shown in Fig. 11.6. It is seen that the former predicts a higher peaking as compared to the use of hardened cross sections; however, it is questionable whether non-hardened cross sections should be used merely to get better agreement with experimental results since some hardening is certainly expected due to the relatively high preferential absorption of low energy neutrons. The thermal parameters are tabulated in Table 11.5; as before, the effect of calculational model, or neutron temperature, upon the thermal multiplication factor, K_{th}, is small.

TABLE 11.2
THERMAL MICROSCOPIC CROSS SECTIONS (BARNS) AVERAGED OVER
A HARDENED MAXWELL-BOLTZMANN DISTRIBUTION

(T = 68°F; E = 0.0331 ev)

Atom	$\bar{\sigma}_s$	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	512.656442	521.630	1065.139411
U-238	9.0	2.118036	11.093	-
H	34.32	0.254164	27.17	-
O	3.9	0	3.642	-
B-10	4.4	2932.4980	2936.6051	-
SS	10.44	2.270275	12.586	-

TABLE 11.3
THERMAL MICROSCOPIC CROSS SECTIONS (BARNS) AVERAGED
OVER A WIGNER-WILKINS SPECIMEN AT 68°F

Atom	$\bar{\sigma}_s$	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	352.5	361.474	736.770
U-238	9.0	1.51218	10.4870	-
H	33.90	0.183762	22.05	-
O	3.9	0	3.642	-
B-10	4.4	2110.496	2114.602	-
SS	10.44	1.631414	11.9461	-
11-8				

TABLE 11.4
THERMAL MICROSCOPIC CROSS SECTIONS (BARNs) AVERAGED OVER
A NON-HARDENED MAXWELL-BOLTZMANN DISTRIBUTION

(T = 68°F; E = 0.0253 ev)

Atom	$\bar{\sigma}_s$	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	599.210	608.1845	1249.4183
U-238	9.0	2.43794	11.41275	-
H	38.8386	0.29433	32.8223	-
O	3.9	0	3.642	-
B-10	4.4	3380.3191	3384.4263	-
SS	10.44	2.61299	12.92771	-

TABLE 11.5
TABULATION OF THERMAL PROPERTIES OF SM-2 FIXED ELEMENT

(0 Burnup, T = 68°F, P = 2000 psia)

Parameter	Units	P ₃ Theory M.B. Hard- ened*	P ₃ Theory W.W. Aver- aged**	P ₁ Theory M.B. Hard- ened+	P ₃ Theory M.B. Aver- aged++
Σ_a	cm ⁻¹	0.420882	0.309712	0.427266	0.471971
$\nu\Sigma_f$	cm ⁻¹	0.613246	0.450926	0.624381	0.690359
K _{th}		1.457050	1.455953	1.461340	1.462714
Σ_s	cm ⁻¹	2.036880	2.006063	2.038132	2.273421
Σ_{tr}	cm ⁻¹	2.071936	1.698522	2.079049	2.413584
D ²	cm	0.160880	0.196249	0.160329	0.138107
L ²	cm ²	0.382246	0.633650	0.375243	0.292618

* Cross sections averaged over a hardened Maxwell-Boltzmann neutron distribution.

** Cross sections averaged over a Wigner-Wilkins neutron spectrum.

+ Cross sections averaged over a hardened Maxwell-Boltzmann neutron distribution.

++ Cross sections averaged over a non-hardened Maxwell-Boltzmann neutron distribution.

11.1.5 Effect Upon Reactivity

The reactivity, ρ , was calculated by modified two-group theory using the thermal parameters tabulated in Table 11.5 and the fast parameters tabulated in Section 6.1. The reactivities were calculated for cores consisting

of 45 fixed fuel elements, using a buckling of 0.006881 cm^{-2} ; these values are tabulated in Table 11.6. It is seen that the reactivity of the core is only slightly dependent upon the method used to obtain the thermal parameters.

TABLE 11.6
REACTIVITY FOR 45 FIXED SM-2 ELEMENTS

(0 Burnup, $T = 68^{\circ}\text{F}$, $P = 2000 \text{ psia}$)

Calculational Method	K_{eff}	$\rho(\%)$
P_3 Theory; Maxwell-Boltzmann Averaged Cross Sections at a Hardened Energy	1.137135	12.0596
P_3 Theory; Wigner-Wilkins Averaged Cross Sections	1.135329	11.9198
P_1 Theory; Maxwell-Boltzmann Averaged Cross Sections at a Hardened Energy	1.139318	12.2281
P_3 Theory; Maxwell-Boltzmann Averaged Cross Sections at a Non-Hardened Energy	1.140423	12.3900

11.1.6 Summary

The change in reactivity as calculated by the various methods is only a few tenths of a percent. Therefore, we may conclude that the method normally used at Alco (P_3 Theory using hardened, Maxwell-Boltzman averaged cross sections) for the calculation of the thermal parameters are just as good as the more sophisticated methods, such as use of Wigner-Wilkins theory. In fact, from the standpoint of innercell thermal flux distributions, the reference method yields better comparison to experimental results than use of Wigner-Wilkins theory. Since some hardening of the neutron spectrum is certainly expected, due to the preferential absorption of low energy neutrons, the use of non-hardened cross sections does not seem justified, merely to obtain better agreement with experimental intercell results. The possibility of errors in the experimental measurements in addition to a possible spectrum shifting within the fuel element were not considered in this study. Therefore, until a more detailed analysis of the thermal treatment of APPR-type fuel elements is made, the presently used methods seem adequate and offer a good compromise between the accurate prediction of innercell thermal flux distributions and calculated core reactivity.

11.2 CALCULATION OF LOCAL POWER PEAKING FACTORS FROM INNERCELL THERMAL FLUX DISTRIBUTIONS

From a thermal standpoint, the power generation within an element is of more importance than the thermal flux distribution. As derived in Section 5.1.3, the power peaking may be expressed in terms of the thermal flux peaking and the fraction of fissions which are thermal by the following equation:

$$\frac{P(X_1)}{P_{avg}} = \bar{\beta} \bar{\phi}(X_1) + (1 - \bar{\beta})$$

This expression is the one normally used in converting from the P_3 calculated thermal flux distributions to power distributions. To test the validity of the expression, a multiregion, three-group, one-dimensional VALPROD was calculated through the SM-2 core parallel to the fuel plates. The properties for the various regions were taken from Section 6.1. The three-group flux distributions are shown in Fig. 11.7. The fraction of thermal fissions at any point, r_1 , may be calculated by:

$$\beta(r_1) = \frac{\sum_f \phi(r_1)}{\sum_f \phi(r_1) + \sum_{f_1}^f \phi_{f_1}(r_1) + \sum_{f_2}^f \phi_{f_2}(r_1)}$$

The macroscopic fission cross sections for each group in the active region of the fixed elements were calculated to be:

$$\sum_f = 0.318114 \text{ cm}^{-1}$$

$$\sum_{f_1}^f = 0.001363 \text{ cm}^{-1}$$

$$\sum_{f_2}^f = 0.026176 \text{ cm}^{-1}$$

Using these values, β was calculated as a function of position inside a fixed element; these results are shown in Fig. 11.8. The average value as calculated by the flux distributions, β_{Flux} , is found to be 0.64773. This value is calculated for the active region of the element only.

In general, the multigroup flux distributions are not available, since normally, only the P_3 thermal flux distribution is obtained. In this case, the average β for the whole fuel element is calculated by:

$$\bar{\beta}_{\text{Element}} = \frac{PK_{th}}{K_{eff}(1 + \tau B^2)(1 + L^2 B^2)}$$

Using the parameters listed in Section 6.1, for the fixed elements, this value of β is calculated to be 0.6419.

Another possible β to be used may be found from a knowledge of the calculated thermal flux and power peaking within the element. Fig. 11.9 is the relative power distribution obtained from the VALPROD calculation of the three-group flux distribution; the max to average power within the active region is 1.363152. From Fig. 11.7, the maximum thermal flux normalized to the average flux in the active region is 1.554533. Therefore:

$$\begin{aligned} P_{\max}/P_{\text{avg}} &= 1.363152 = \frac{\phi_{\max}}{\bar{\phi}} \bar{\beta} + (1 - \bar{\beta}) \\ &= 1.554533 \bar{\beta} + (1 - \bar{\beta}) \\ \therefore \bar{\beta}_{\text{cal}} &= 0.654878 \end{aligned}$$

It is seen that all of the β 's, calculated by the various methods, are less than the maximum β (at the edge of the active region) as given in Fig. 11.8. The max to average power as calculated by use of the various β 's, are:

$$\begin{aligned} (P_{\max}/P_{\text{avg}})_{\text{Flux}} &= 1.35919 \\ (P_{\max}/P_{\text{avg}})_{\text{Element}} &= 1.35601 \\ (P_{\max}/P_{\text{avg}})_{\text{cal}} &= 1.363152 \end{aligned}$$

11.2.1 Summary

In general, results obtained from the SM-2 flexible critical experiments indicate that the calculated power peaking factors are considerably less than the experimentally measured values. Therefore, an experimental correction factor is needed. However, no additional factor is necessary to allow for errors in the selection of β since the maximum change in $(P_{\max}/P_{\text{avg}})$, as calculated by the three methods discussed in this section is only 0.52%. This error is well within the nuclear uncertainty factor of 5 to 10% normally used in reporting power distributions. (Section 9.0)

Therefore, it appears that the methods presently used in converting

from the innercell thermal flux distribution to power distribution are adequate provided that the fast flux is fairly flat (constant) across the element. An experimental correction factor is needed primarily because the calculated value of β , the fraction of thermal fissions, is probably too low. The calculated fraction of thermal fissions for the SM-2 core is considerably smaller than measurements of the cadmium ratio, as made during the flexible critical experiment, indicate.⁽⁴⁾ For example, for the SM-2 at 510°F, the peak to average flux ratio is about 1.45. For a β of 0.57 (Section 6.1), the power ratio is 1.257. If β is actually of the order of 0.75, as in the SM-1, the power ratio becomes 1.338; therefore the accuracy of β and its dependence upon position within the element is of more importance than the use of a multiregion VALPROD as opposed to P_3 theory. After a more complete analysis of the data obtained from the SM-2 Flexible Critical Experiments has been made, the problem of accurately calculating the value of β will be studied further.

11.3 REVIEW OF MUFT-III CROSS SECTION FILES⁽⁷⁾

Recent criticality calculations have shown disagreement with SM-1, ZPE measurements⁽⁸⁾ and SM-2 Flexible Critical Experiments (Section 10.0). For the SM-1, the calculated values have been lower than the experimental values by about 1.6% ρ ^{(8), (9)} whereas, for the SM-2, the calculated values have been higher than the experimental values by about 1.6% ρ . The original calculations made upon the SM-1 core⁽⁸⁾ were based upon the "as specified" boron content. Using the best known actual boron content,^{(10), (11)} at the time of this report, the calculated reactivity became greater than the measured values by about 0.6% ρ . As one possibility to reduce the discrepancies between measured and calculated values of reactivity, the cross-sectional files used in the MUFT-III code were reevaluated by comparison to the latest known data. A tabulation of the MUFT-III files, used during the entire nuclear analysis of the SM-2 to date, is given in reference⁽⁸⁾. As a result of this study, changes in the cross sections for Fe (and stainless steel) were made.

More recent determinations of iron slowing down parameters are significantly different from those previously employed in the MUFT-III program. Slowing down properties by reactor materials are determined solely by inelastic and elastic collisions of neutrons with these materials. The previous nuclear data was based on the use of the Weisskopf statistical model.⁽¹²⁾ This model assumes the inelastically scattered neutrons have a Maxwellian distribution in energy (see Appendix A). The degraded neutron energy distribution of inelastically scattered neutrons is thus determined by a characteristic nuclear temperature. Since the model is statistical in nature, it cannot be valid unless the impinging neutrons are energetic enough to leave the target nuclei in levels high about the ground state. Previous neutron energy distributions used as input to the MUFT-III code were determined by this model for all incident neutron energies. The characteristic neutron temperature for the

previous calculations was based on old data. A new inelastic scattering matrix has been redetermined by using known cross sections for the (n, n', γ) reaction to various levels in iron from the threshold at 0.85 Mev to about 4 Mev. ⁽¹³⁾ At higher incident neutron energies, the slowing down distributions have been redetermined by using more recent measured values of the "nuclear temperature." ⁽¹⁴⁾ The degraded neutron energies have a spectrum, for example, similar to the energy spectrum of water molecules evaporating from a pond. Hence, this model is sometimes referred to as the "Evaporation Model." The new inelastic scattering matrix for iron is shown in Table 11.7.

The behavior of neutrons scattered elastically is described by two nuclear parameters. The first of these, $\bar{\mu}$, is the average cosine of the angle between the direction of the incident neutron and the direction of the scattered neutron. The second of these, ξ , is the average loss in the logarithmic energy decrement. The previous values of these parameters were computed by assuming isotropic scattering in the center of mass system. Since this assumption was made for neutrons at all energies, the values of $\bar{\mu} = 0.01064$ and $\xi = 0.03529$ were used for neutrons elastically scattered from all groups. In order to account for anisotropic scattering events in the center of mass system, the elastic slowing down distribution of scattered neutrons have been redetermined by using known angular distributions of scattered neutrons reported in the literature. The average cosines of the scattered neutrons were then used to compute the average logarithmic energy loss by means of transport theory. ⁽¹⁵⁾ The results of the age and reactivity calculations using the new input data are given in the next section. The new values of $\bar{\mu}$ and ξ for iron are shown in Fig. 11.10 and 11.11. The new cross-sectional file for iron, as used in the MUFT-III code, is tabulated in Appendix A.

11.3.1 Results and conclusions

The new inelastic scattering matrix for iron was used to calculate the neutron age to Po-Be neutrons in various iron-water mixtures, and compared to measured values. ⁽¹⁶⁾ These values are shown in Table 11.8; it is seen that the calculated ages show poor agreement for high metal-water ratios. By including both the elastic and inelastic scattering changes, the ages were calculated and are also given in Table 11.8; it is seen that the agreement between the calculated and measured neutron ages is very good. The Po- α -Be neutron spectrum used in the MUFT-III calculations is shown in Fig. 11.12. The spectrum was determined by averaging a number of spectrums collected and reported in graphical form in reference (16).

TABLE 11.7
Fe INELASTIC SCATTERING PARAMETERS

Group Scattered	Group Scattered Out Of →									
Into	1	2	3	4	5	6	7	8	9	10
2	.001	-	-	-	-	-	-	-	-	-
3	.005	.003	.001	-	-	-	-	-	-	-
4	.018	.010	.006	.056	-	-	-	-	-	-
5	.004	.030	.018	.353	-	-	-	-	-	-
6	.078	.061	.045	.075	.463	-	-	-	-	-
7	.110	.095	.079	.157	.197	.365	-	-	-	-
8	.134	.125	.114	.151	.037	.535	.063	-	-	-
9	.134	.136	.133	.107	.060	-	.563	-	-	-
10	.120	.126	.131	.062	.076	-	.333	.179	-	-
11	.101	.112	.121	.038	.044	.020	-	.461	-	-
12	.078	.090	.100	.001	.034	.017	-	.360	.184	-
13	.068	.079	.092	-	.035	.016	-	-	.357	-
14	.030	.037	.042	-	.020	.010	-	-	.182	-
15	.028	.033	.041	-	.016	.009	-	-	.191	-
16	.019	.023	.027	-	.010	.013	-	-	.086	.111
17	.011	.014	.018	-	.007	.004	-	-	-	.203
18	.008	.010	.012	-	.001	.004	.007	-	-	.163
19	.005	.006	.007	-	-	.003	.010	-	-	.125
20	.003	.004	.005	-	-	.002	.009	-	-	.098
21	.003	.004	.005	-	-	.001	.010	-	-	.137
22	.001	.001	.002	-	-	.001	.005	-	-	.083
23	.001	.001	.001	-	-	-	-	-	-	.050
24	-	-	-	-	-	-	-	-	-	.030
	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000

TABLE 11.8
NEUTRON AGES OF Po-Be NEUTRONS IN Fe-H₂O MIXTURES

Metal/Water Volume Ratio	Measured Age (cm ²)	Calculated Age (cm ²) (Inelastic Change Only)	Calculated Age (cm ²) (Elastic and Inelastic Changes)
0	55.4	54.7	54.7
0.5	56.2	40.8	54.5
1.0	58.8	42.9	58.4
2.0	72.1	49.9	68.0

The reactivity of the SM-1 core at 440°F, using the "as specified" boron content, was calculated using the new iron elastic and inelastic MUFT-III files and compared to the results obtained using the old files. The effect upon the fast parameters and reactivity is shown in Table 11.9. The reactivity of the SM-2 mockup core (as part of the SM-2 Flexible Critical Experiments) containing no boron was also calculated at 68°F with the new iron files and compared to the results obtained using the old files; these results are shown in Table 11.9. The values of reactivity have been calculated by use of modified two-group theory using thermal parameters calculated by use of the Program 50 and plate-type P_3 theory, IBM-650 codes. The experimental values were obtained by converting from the measured values expressed in cents to reactivity expressed in percent. The value of the effective delayed neutron fraction was taken to be 0.0073. It is seen that use of the new iron files reduces the calculated reactivity by 1.42% ρ and 1.73% ρ for the SM-1 at 440°F and the SM-2 at 68°F respectively. The discrepancy between the calculated and measured values of reactivity becomes 0.02% ρ and 0.24% ρ for the SM-1 and SM-2 respectively. The calculated reactivity of the SM-2 core at the operating temperature of 510°F was also calculated both with the new and old iron files; these results are shown in Table 11.10. It is seen that the difference in reactivity as calculated by the two files is not a strong function of temperature.

It is seen that use of the new iron files results in much better comparison between experimental and calculated values of reactivity; however, care must be exercised in making broad conclusions from these comparisons due to uncertainties in the proper value of the effective delayed neutron fraction, β_{eff} . This parameter, which is discussed further in Section 7.6, is a function of core composition, temperature, and bank position. In addition, errors are inherent in the interpretation of large reactivities, such as in the case of the SM-2 critical experiment core containing no boron. Measurements made on cores which are nearly critical would be a more reliable basis of comparison.

On the basis of the excellent agreement of calculated neutron ages in iron-water mixtures to experimentally measured values, and in view of the fact that the new iron files yield reactivities for the SM-1 and SM-2 more in line to the measured values, it was decided to utilize the new iron files for all future SM-1 and SM-2 calculations made after January 1, 1960. All of the calculations in this report have been made on the basis of the old iron MUFT-III files as tabulated in reference (8).

TABLE 11.9
REACTIVITY OF SM-1 CORE USING OLD
AND NEW IRON FILES FOR MUFT-III* CALCULATIONS

(T = 68°F; E = 0.0331 ev)

Parameter	Units	Old Fe Files	New Fe Files
τ	cm ²	42.988889	46.224590
D _f	cm	1.492717	1.613791
Σ_R	cm ⁻¹	0.034723	0.034912
Σ_a^f	cm ⁻¹	0.009410	0.009444
$\nu \Sigma_f^f$	cm ⁻¹	0.012053	0.012117
K _f		1.280871	1.283036
P		0.729012	0.729493
B ²	cm ⁻²	0.006593	0.006593
K _{th}		1.531100	1.531100
D	cm	0.245022	0.245022
L ²	cm ²	1.182892	1.182892
ρ (cal.)	%	11.77	10.35
ρ (meas.)**	%	10.37	10.37

* Reactivity based upon core containing "as specified" boron.

Thermal parameters and number densities taken from APAE No. 32. ⁽⁹⁾

** Based upon an effective delayed neutron fraction of 0.0073. Value of reactivity taken from APAE No. 32. ⁽⁹⁾

TABLE 11.10
REACTIVITY OF SM-2 CORE USING OLD AND NEW IRON FILES
FOR MUFT-III CALCULATIONS

Parameters	Units	Critical Experiment Core at 68°F*		Reference Core at 510°F	
		Old Fe Files	New Fe Files	Old Fe Files	New Fe Files
τ	cm ²	33.232621	35.901573	46.019894	49.950951
D_f	cm	1.305786	1.417132	1.490084	1.626988
$\sum_a^f$	cm ⁻¹	0.013870	0.013920	0.014034	0.014102
$\sum_f^f$	cm ⁻¹	0.020547	0.020642	0.017877	0.017981
K_f		1.481395	1.482926	1.273834	1.275067
P		0.656997	0.647362	0.566571	0.567046
B^2	cm ⁻²	0.006896	0.006896	0.006194	0.006194
K_{th}		1.775879	1.775879	1.458396	1.458396
D	cm	0.168564	0.168564	0.246265	0.246265
L^2	cm ²	0.472351	0.472351	0.766820	0.766820
ρ (cal.)	%	26.32	25.24	6.5076	4.7789
ρ (meas.)	%	25.0	25.0	4.8 ⁺	4.8 ⁺

* Critical experiment core containing no boron. Thermal parameters calculated by use of Program 53 and Plate-Type P₃ theory, IBM-650 codes.

** Reference core #2 as described in Section 6.1.

+ This value based upon a measured value at 68°F of 11.5% ρ and a hot to cold change of 6.7% ρ . A value of 0.0078 for the effective delayed neutron fraction was used as reported in APAE Memo 235. (17)

11.4 COMPARISON OF PDQ TO FINK-1 AND VALPROD CALCULATIONS OF REACTIVITY

The reactivity of the SM-2 Experimental Core for all fixed elements with a water reflector was calculated using three different machine codes, namely:

1. The PDQ code, a two-dimensional code (x, y or r, z) for the IBM-704 machine.
2. The VALPROD code, a one-dimensional code for the IBM-650 machine, used for radial calculations with cylindrical geometry.
3. The FINK-1 code, a two-group, two-region code for the IBM-650 which solves for reactivity and the radial and axial reflector savings by iterating between radial and axial calculation, assuming a cylindrical core.

The equivalence of these calculations can be seen in Table 11.11. The excellent agreement between these calculations shows that the assumption, that the 7 x 7 array with no corner elements can be calculated as a cylindrical core, is good for reactivity calculations. In all other comparisons made, the difference between the one-dimensional radial calculations and the two-dimensional, x, y, calculation was less than 0.2%.

TABLE 11.11
EQUIVALENCE OF ANALYTICAL CODES

(SM-2 Experimental Core - 68°F - Water Reflector - All Fixed Elements)

Code	K _{eff}	ρ (%)
PDQ-x, y	1.155	13.4
VALPROD	1.153	13.3
FINK-1	1.153	13.3

Figure 11.1

Figure 11.2

SOURCE IN DEAD REGION

Figure 11.3

SOURCE IN ACTIVE REGION

Figure 11.4

Figure 11.6

11-25

11-26

Figure 11.7

Figure 11.8

Figure 11.9

Figure 11.10

11-29

11-30

Figure 11.11

Figure 11.12

APPENDIX A MICROSCOPIC CROSS SECTIONS

1.0 THERMAL GROUP

1.1 Absorption Cross Sections

The average absorption cross section can be obtained by integrating over the thermal neutron energy spectrum:⁽¹⁾

$$\bar{\sigma}_a = \frac{\int_0^{0.2 \text{ ev}} \sigma_a(E) \Phi(E) dE}{\int_0^{0.2 \text{ ev}} \Phi(E) dE}$$

By extending the upper limit of integration to infinity and assuming a Maxwell-Boltzmann distribution about the effective neutron temperature, T_N , the average cross section for a $1/v$ absorber is given by:

$$\bar{\sigma}_a = 0.886 \sigma_a(E_n) = 0.886 \sqrt{\frac{0.025}{E_n}} \sigma_a(0.025 \text{ ev})$$

where $\sigma_a(E_n)$ is the absorption cross section at a neutron energy, E_n .

For a non - $1/v$ absorber:

$$\bar{\sigma}_a = 0.886 \sigma_a(E_n) \times F(E_n)$$

where $F(E_n)$ is the non - $1/v$ factor. The only non - $1/v$ factor in the SM-1 or SM-2 is U-235.

1.2 Transport and Scattering Cross Sections

For all atoms except hydrogen, the scattering cross section and average scattering angle can be assumed constant in the thermal group. Scattering cross sections were taken from BNL-325.⁽²⁾ The average cosine of the scattering angle in the laboratory system is given by:

$$\bar{\mu} = 2/3A$$

where A is the atomic weight of the nuclide. The transport cross section is given by:

$$\bar{\sigma}_{tr} = \bar{\sigma}_s (1 - \bar{\mu}) + \bar{\sigma}_a$$

The transport cross section of hydrogen is complicated by chemical binding in the water molecule. The effective mass of hydrogen in the water molecule is given by:⁽³⁾

$$\frac{1}{M_{eff}(E)} = \sqrt{\frac{80}{\sigma_s^H(E)}} - 1$$

where $\sigma_s^H(E)$ is the microscopic scattering cross section of hydrogen, in barns. The average values of the cosine of the scattering angle in the laboratory system are:

$$\bar{\mu}_H(E) = \frac{2}{3 M_{eff}(E)}$$

The transport cross section of hydrogen is given by:

$$\sigma_{tr}^H(E) = \sigma_a^H(E) + \sigma_s^H(E) \left[1 - \bar{\mu}_H(E) \right]$$

The average transport cross section can be obtained by integration over the thermal neutron energy spectrum:⁽¹⁾

$$\frac{1}{\bar{\sigma}_{tr}^H} = \frac{\int_0^{0.2 \text{ ev}} \frac{1}{\sigma_{tr}^H(E)} \bar{\Phi}(E) dE}{\int_0^{0.2 \text{ ev}} \bar{\Phi}(E) dE}$$

The transport cross section for hydrogen is very nearly $1/v$ between 0.008 ev and 0.2 ev. The flux distribution was assumed to be a Maxwell-Boltzmann about the neutron temperature, T_N . An analytical integration was made by extending the upper limit of integration to infinity. This gives:

$$\bar{\sigma}_{tr}^H = 0.752 \sigma_{tr}^H(E_n)$$

where $\bar{\sigma}_{tr}^H(E_n)$ is the transport cross section for hydrogen at a neutron energy, E_n .

The average scattering cross section for hydrogen was found by:

$$\bar{\sigma}_s^H = \frac{\bar{\sigma}_{tr}^H - \bar{\sigma}_a^H}{1 - \bar{\mu}_H(E_n)}$$

1.3 Stainless Steel Cross Sections

The major constituent in stainless steel is iron. Addition of Ni, Cr, and Mn tends to increase the absorption cross section per atom while the scattering cross section per atom decreases. A weighted average of the thermal cross section for the Fe, Ni, Cr, and Mn contained in type 304 L stainless steel was calculated. It was found that the absorption cross section for stainless steel was 1.165 times that of iron, while the scattering cross section was 0.908 times that of iron. Therefore, it was assumed that:

$$\bar{\sigma}_a^{ss} = 1.165 \bar{\sigma}_a^{Fe}$$

$$\bar{\sigma}_s^{ss} = 0.908 \bar{\sigma}_s^{Fe}$$

1.4 Neutron Temperature

Using the atom densities for the SM-1(4) and cross sections evaluated at the kT values, where T is the average core temperature, the effective thermal neutron temperature was calculated by use of the equation described in Section 1.1.2.3.

$$1.4.1 \quad T_0 = 68^\circ\text{F}; P = 1200 \text{ psia:}$$

$$\left(\frac{T}{T_0}\right) = 1 + 0.91 \frac{\sum_a}{\xi \sum_s} = 1 + 0.91 \left[\frac{0.3869 \text{ cm}^{-1}}{1.0794 \text{ cm}^{-1}} \right] = 1.326$$

$$\text{or: } E_n = (0.025) (1.326) = 0.0331 \text{ ev.}$$

1.4.2 $T_O = 440^\circ\text{F}$: $P = 1200$ psia:

$$\left(\frac{T}{T_O}\right) = 1 + 0.91 \left[\frac{0.2831 \text{ cm}^{-1}}{0.9068 \text{ cm}^{-1}} \right] = 1.284$$

or: $E_n = (0.0428) (1.284) = 0.0549 \text{ ev.}$

1.4.3 $T_O = 480^\circ\text{F}$: $P = 1600$ psia:

$$\left(\frac{T}{T_O}\right) = 1 + 0.91 \left[\frac{0.2760 \text{ cm}^{-1}}{0.8724 \text{ cm}^{-1}} \right] = 1.288$$

or: $E_n = (0.0451) (1.288) = 0.0581 \text{ ev.}$

1.4.4 $T_O = 510^\circ\text{F}$: $P = 2000$ psia:

$$\left(\frac{T}{T_O}\right) = 1 + 0.91 \left[\frac{0.2696 \text{ cm}^{-1}}{0.8482 \text{ cm}^{-1}} \right] = 1.289$$

or: $E_n = (0.0465) (1.289) = 0.0600 \text{ ev.}$

1.5 Tabulation of Thermal Microscopic Cross Sections

A tabulation of the thermal microscopic cross sections averaged over a Maxwell-Boltzmann distribution at 68°F ($E_n = 0.0331 \text{ ev}$), 440°F ($E_n = 0.0549 \text{ ev}$), 480°F ($E_n = 0.0581 \text{ ev}$), and 510°F ($E_n = 0.0600 \text{ ev}$) are given in Tables A. 1 through A. 4 respectively.

TABLE A. 1
THERMAL MICROSCOPIC CROSS SECTIONS

(T = 68°F; E_N = 0.0331 ev)

Atom	$\bar{\sigma}_s$	(1 - $\bar{\mu}$)	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	0.997165	512.6564	521.630	1065.1394
U-238	9.0	0.997201	2.118036	11.093	-
H	34.32	0.784	0.254164	27.170	-
O	3.9	0.958334	0	3.7375	-
B-10*	4.4	0.933441	2932.4980	2936.6051	-
S.S.	10.44	0.988	2.270275	12.586	-

TABLE A. 2
THERMAL MICROSCOPIC CROSS SECTIONS

(T = 440°F; E_N = 0.0549 ev)

Atom	$\bar{\sigma}_s$	(1 - $\bar{\mu}$)	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	0.997165	385.5384	394.512	801.0342
U-238	9.0	0.997201	1.644605	10.620	-
H	28.97	0.7059	0.197353	20.65	-
O	3.9	0.958334	0	3.7375	-
B-10*	4.4	0.933441	2277.0146	2281.1217	-
S.S.	10.44	0.988	1.762814	12.078	-

* Based upon 19.8 a/o of B-10 in natural boron.

TABLE A. 3
THERMAL MICROSCOPIC CROSS SECTIONS

(T = 480°F; E_N = 0.0581 ev)

Atom	$\bar{\sigma}_s$	(1- $\bar{\mu}$)	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	0.997165	365.8882	374.8627	765.7425
U-238	9.0	0.997201	1.608775	10.5836	-
H	28.389	0.6969	0.193053	19.977	-
O	3.9	0.958334	0	3.7375	-
B-10*	4.4	0.933441	2230.7161	2234.8232	-
S. S.	10.44	0.988	1.724285	12.039	-

TABLE A. 4
THERMAL MICROSCOPIC CROSS SECTIONS

(T = 510°F; E_N = 0.0600 ev)

Atom	$\bar{\sigma}_s$	(1- $\bar{\mu}$)	$\bar{\sigma}_a$	$\bar{\sigma}_{tr}$	$\bar{\nu}\bar{\sigma}_f$
U-235	9.0	0.997165	362.2284	371.2029	775.5832
U-238	9.0	0.997201	1.583095	10.5579	-
H	28.035	0.6908	0.189971	19.5563	-
O	3.9	0.958334	0	3.7375	-
B-10*	4.4	0.933441	2195.1091	2199.2162	-
S. S.	10.44	0.988	1.696762	12.0115	-

* Based upon 19.8 a/o of B-10 in natural boron.

2.0 FAST GROUP CROSS SECTIONS

2.1 Materials

The microscopic data for U-235, U-238, hydrogen, oxygen, carbon, and natural boron were obtained from WAPD. ⁽⁵⁾ The absorption cross section for natural boron was assumed to be $1/v$ from thermal energy to 10 Mev; the scattering cross section was neglected. The cross sections for B-10 were calculated by assuming 19.8 a/o of B-10 in natural boron. ⁽⁶⁾

The total cross section curve for iron indicates that there are many unresolved resonances between 0.008 Mev and 10 Mev. These are believed to be primarily scattering resonances since there are no sharp peaks. However, the measured resonance integral ⁽⁷⁾ for iron (2.1 barns) is higher than the value to be expected if the iron absorption cross section followed a simple $1/v$ behavior (1.2 barns). It appears that there must be some resonance absorption cross section distributed somewhere between 10 Mev and cadmium cut-off energy. The iron cross section was assumed to be $1/v$ from thermal out to 20 ev. Beyond 20 ev, the iron cross section was taken to be 0.082 barns. This procedure gives a value of 2.1 barns for the iron resonance integral. Inelastic cross sections for iron were obtained by averaging values given in BNI-325 ⁽²⁾ over the various group widths. Elastic scattering cross sections were obtained from the KAPL Lost File. ⁽⁸⁾ The latest elastic and inelastic files for iron, which supersede the files used in this report, are described in Section 11.3.

The resonance integral for stainless steel was calculated from the published values for Fe, Cr, Ni, and Mn. ⁽⁷⁾ This showed that the resonance integral (R. I.) of stainless steel is 1.12 times the resonance integral of iron. Within the accuracy with which resonance integrals can be measured, this is practically the same ratio as the thermal value of 1.165 (Section 1.3 of Appendix A). Therefore, it was assumed that for each group:

$$\bar{\sigma}_a^{ss} = 1.165 \bar{\sigma}_a^{Fe}$$

Since elastic scattering cross sections are nearly the same in the thermal group and in most of the fast groups for Fe, Ni, Cr, and Mn, it was assumed that in every group:

$$\bar{\sigma}_s^{ss} = 0.908 \bar{\sigma}_s^{Fe}$$

It was assumed that the mixture of Fe, Ni, Cr, and Mn was effectively equal to Fe as far as inelastic scattering cross section was concerned. Therefore:

$$\bar{\sigma}_{in}^{ss} = \bar{\sigma}_{in}^{Fe}$$

The latest elastic and inelastic files for stainless steel were also calculated based upon the information presented in Section 11.3 and upon the equations above.

2.2 MUFT-III Cross Section File

The MUFT-III energy-lethargy group structure is given in Table A.5. The MUFT-III microscopic files used in the fast group calculations, presented in this report, are tabulated in APAE-27. (4) The new files for Fe and stainless steel, which contain the latest data concerning elastic and inelastic scattering, are listed in Tables A.7 and A.8 respectively; these new files were not used in any calculations presented in this report, except those presented in Section 11.3. Table A.6 lists the MUFT-III file for B-10, based on 19.8 a/o of B-10 in natural boron.

TABLE A. 5
MUFT-III
ENERGY - LETHARGY GROUP STRUCTURE

(Energy - Lethargy Values Correspond to Bottom of Each Group)

Group	Energy (ev)	Lethargy, μ	$\Delta \mu$
0	10×10^6	0.00	---
1	7.79	0.25	0.25
2	6.07	0.50	0.25
3	4.72	0.75	0.25
4	3.68	1.00	0.25
5	2.86	1.25	0.25
6	2.23	1.50	0.25
7	1.74	1.75	0.25
8	1.35	2.00	0.25
9	1.05	2.25	0.25
10	821×10^3	2.50	0.25
11	639.	2.75	0.25
12	498.	3.00	0.25
13	387.	3.25	0.25
14	302.	3.50	0.25
15	235.	3.75	0.25
16	183.	4.00	0.25

TABLE A. 5 (CONT'D)

<u>Group</u>	<u>Energy (ev)</u>	<u>Lethargy,</u>	
17	143.	4. 25	0. 25
18	111.	4. 50	0. 25
19	86. 5	4. 75	0. 25
20	67. 4	5. 00	0. 25
21	40. 9	5. 50	0. 50
22	24. 8	6. 00	0. 50
23	15. 0	6. 50	0. 50
24	9. 12	7. 00	0. 50
25	5. 53	7. 50	0. 50
26	3. 35	8. 00	0. 50
27	2. 03	8. 50	0. 50
28	1. 23	9. 00	0. 50
29	750 x 10 ⁰	9. 50	0. 50
30	454.	10. 00	0. 50
31	275.	10. 50	0. 50
32	167.	11. 00	0. 50
33	130.	11. 25	0. 25
34	101.	11. 50	0. 25
35	78. 7	11. 75	0. 25
36	61. 3	12. 00	0. 25
37	47. 8	12. 25	0. 25
38	37. 2	12. 50	0. 25
39	29. 0	12. 75	0. 25
40	22. 6	13. 00	0. 25
41	17. 6	13. 25	0. 25
42	13. 7	13. 50	0. 25
43	10. 7	13. 75	0. 25
44	8. 32	14. 00	0. 25
45	6. 50	14. 25	0. 25
46	5. 10	14. 50	0. 25
47	3. 97	14. 75	0. 25
48	3. 06	15. 00	0. 25
49	2. 38	15. 25	0. 25
50	1. 855	15. 50	0. 25
51	1. 440	15. 7538	0. 2538
52	1. 125	16. 00	0. 2462
53	0. 835	16. 30	0. 3000
54	0. 625	16. 5884	0. 2884
55	0. 500	16. 8114	0. 2230
56	0. 400	17. 0350	0. 2236
57	0. 315	17. 2730	0. 2380
58	0. 248	17. 5110	0. 2380
59	0. 196	17. 750	0. 2390

TABLE A.6
MUFT-III FILE FOR BORON-10

A-10	FILE NO 09		B-10						
	GROUP	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_s$	$\bar{\sigma}_c$	$\bar{\sigma}_{in}$	$\bar{\sigma}_f$	ν	Δu
	1	.000000	.000000	.00000	.00000	.000	.00000	.0000	.25000
	2	.000000	.000000	.00000	.00229	.000	.00000	.0000	.25000
	3	.000000	.000000	.00000	.00260	.000	.00000	.0000	.25000
	4	.000000	.000000	.00000	.00294	.000	.00000	.0000	.25000
	5	.000000	.000000	.00000	.00333	.000	.00000	.0000	.25000
	6	.000000	.000000	.00000	.00378	.000	.00000	.0000	.25000
	7	.000000	.000000	.00000	.00428	.000	.00000	.0000	.25000
	8	.000000	.000000	.00000	.00485	.000	.00000	.0000	.25000
	9	.000000	.000000	.00000	.00551	.000	.00000	.0000	.25000
	10	.000000	.000000	.00000	.00624	.000	.00000	.0000	.25000
	11	.000000	.000000	.00000	.00706	.000	.00000	.0000	.25000
	12	.000000	.000000	.00000	.00801	.000	.00000	.0000	.25000
	13	.000000	.000000	.00000	.00908	.000	.00000	.0000	.25000
	14	.000000	.000000	.00000	.01029	.000	.00000	.0000	.25000
	15	.000000	.000000	.00000	.01165	.000	.00000	.0000	.25000
	16	.000000	.000000	.00000	.01321	.000	.00000	.0000	.25000
	17	.000000	.000000	.00000	.01495	.000	.00000	.0000	.25000
	18	.000000	.000000	.00000	.01694	.000	.00000	.0000	.25000
	19	.000000	.000000	.00000	.01933	.000	.00000	.0000	.25000
	20	.000000	.000000	.00000	.02177	.000	.00000	.0000	.25000
	21	.000000	.000000	.00000	.02610	.000	.00000	.0000	.50000
	22	.000000	.000000	.00000	.03351	.000	.00000	.0000	.50000
	23	.000000	.000000	.00000	.04306	.000	.00000	.0000	.50000
	24	.000000	.000000	.00000	.05531	.000	.00000	.0000	.50000
	25	.000000	.000000	.00000	.07097	.000	.00000	.0000	.50000
	26	.000000	.000000	.00000	.09172	.000	.00000	.0000	.50000
	27	.000000	.000000	.00000	.11784	.000	.00000	.0000	.50000
	28	.000000	.000000	.00000	.15046	.000	.00000	.0000	.50000
	29	.000000	.000000	.00000	.19303	.000	.00000	.0000	.50000
	30	.000000	.000000	.00000	.24759	.000	.00000	.0000	.50000
	31	.000000	.000000	.00000	.31819	.000	.00000	.0000	.50000

	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_s$	$\bar{\sigma}_c$	$\bar{\sigma}_{in}$	$\bar{\sigma}_f$	ν	Δu
32	.000000	.000000	.000000	.40862	.000	.000000	.0000	.50000
33	.000000	.000000	.000000	.49562	.000	.000000	.0000	.25000
34	.000000	.000000	.000000	.56204	.000	.000000	.0000	.25000
35	.000000	.000000	.000000	.63721	.000	.000000	.0000	.25000
36	.000000	.000000	.000000	.72193	.000	.000000	.0000	.25000
37	.000000	.000000	.000000	.81770	.000	.000000	.0000	.25000
38	.000000	.000000	.000000	.92653	.000	.000000	.0000	.25000
39	.000000	.000000	.000000	.04980	.000	.000000	.0000	.25000
40	.000000	.000000	.000000	.18927	.000	.000000	.0000	.25000
41	.000000	.000000	.000000	.34720	.000	.000000	.0000	.25000
42	.000000	.000000	.000000	.52660	.000	.000000	.0000	.25000
43	.000000	.000000	.000000	.72927	.000	.000000	.0000	.25000
44	.000000	.000000	.000000	.95895	.000	.000000	.0000	.25000
45	.000000	.000000	.000000	.21852	.000	.000000	.0000	.25000
46	.000000	.000000	.000000	.50757	.000	.000000	.0000	.25000
47	.000000	.000000	.000000	.83636	.000	.000000	.0000	.25000
48	.000000	.000000	.000000	.22222	.000	.000000	.0000	.25000
49	.000000	.000000	.000000	.65262	.000	.000000	.0000	.25000
50	.000000	.000000	.000000	.15050	.000	.000000	.0000	.25000
51	.000000	.000000	.000000	.70555	.000	.000000	.0000	.25000
52	.000000	.000000	.000000	.33333	.000	.000000	.0000	.25380
53	.000000	.000000	.000000	.10656	.000	.000000	.0000	.24620
54	.000000	.000000	.000000	.07373	.000	.000000	.0000	.30000
55	.000000	.000000	.000000	.06565	.000	.000000	.0000	.28840
56	.000000	.000000	.000000	.02020	.000	.000000	.0000	.22300
57	.000000	.000000	.000000	.12474	.000	.000000	.0000	.22360
58	.000000	.000000	.000000	.41010	.000	.000000	.0000	.23800
59	.000000	.000000	.000000	.84696	.000	.000000	.0000	.23900

TABLE A.7
MUFT-III FILE FOR IRON

A-12

FILE NO 03		IRON						
GROUP	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_s$	$\bar{\sigma}_c$	$\bar{\sigma}_{in}$	$\bar{\sigma}_f$	ν	Δu
1	.850000	.005400	1.72000	.08200	1.380	.00000	.0000	.25000
2	.822000	.006400	2.03000	.08200	1.370	.00000	.0000	.25000
3	.751000	.008900	2.25000	.08200	1.350	.00000	.0000	.25000
4	.651000	.016000	2.29000	.08200	1.310	.00000	.0000	.25000
5	.542000	.016000	2.12000	.08200	1.180	.00000	.0000	.25000
6	.456000	.020000	2.20000	.08200	1.000	.00000	.0000	.25000
7	.380000	.022000	2.20000	.08200	.800	.00000	.0000	.25000
8	.220000	.028000	2.18000	.08200	.620	.00000	.0000	.25000
9	.220000	.028000	2.03000	.08200	.470	.00000	.0000	.25000
10	.230000	.028000	2.59700	.08200	.200	.00000	.0000	.25000
11	.150000	.032000	2.39600	.08200	.000	.00000	.0000	.25000
12	.210000	.028000	3.09600	.08200	.000	.00000	.0000	.25000
13	.150000	.030000	4.04400	.08200	.000	.00000	.0000	.25000
14	.094000	.032000	3.04500	.08200	.000	.00000	.0000	.25000
15	.089000	.033000	3.09600	.08200	.000	.00000	.0000	.25000
16	.068000	.033000	3.54500	.08200	.000	.00000	.0000	.25000
17	.054000	.034000	3.39500	.08200	.000	.00000	.0000	.25000
18	.045000	.034000	3.44600	.08200	.000	.00000	.0000	.25000
19	.040000	.034000	4.39400	.08200	.000	.00000	.0000	.25000
20	.034000	.035000	3.24500	.08200	.000	.00000	.0000	.25000
21	.021000	.035000	5.29300	.08200	.000	.00000	.0000	.50000
22	.017000	.035000	4.64300	.08200	.000	.00000	.0000	.50000
23	.014000	.035000	4.84300	.08200	.000	.00000	.0000	.50000
24	.013000	.035000	6.96500	.08200	.000	.00000	.0000	.50000
25	.012000	.035000	6.99000	.08200	.000	.00000	.0000	.50000
26	.012000	.035000	6.09100	.06200	.000	.00000	.0000	.50000
27	.012000	.035000	5.89100	.08200	.000	.00000	.0000	.50000
28	.012000	.035000	6.34100	.08200	.000	.00000	.0000	.50000
29	.012000	.035000	7.14000	.08200	.000	.00000	.0000	.50000
30	.012000	.035000	8.28800	.08200	.000	.00000	.0000	.50000
31	.012000	.035000	9.07700	.08200	.000	.00000	.0000	.50000

	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_s$	$\bar{\sigma}_c$	$\bar{\sigma}_{in}$	$\bar{\sigma}_F$	ν	Δu
32	.012000	.035000	10.44400	.08200	.000	.00000	.0000	.50000
33	.012000	.035000	10.85200	.08200	.000	.00000	.0000	.25000
34	.012000	.035000	10.83500	.08200	.000	.00000	.0000	.25000
35	.012000	.035000	10.93600	.08200	.000	.00000	.0000	.25000
36	.012000	.035000	11.00300	.08200	.000	.00000	.0000	.25000
37	.012000	.035000	11.08400	.08200	.000	.00000	.0000	.25000
38	.012000	.035000	11.01000	.08200	.000	.00000	.0000	.25000
39	.012000	.035000	11.01700	.08200	.000	.00000	.0000	.25000
40	.012000	.035000	11.22200	.08200	.000	.00000	.0000	.25000
41	.012000	.035000	11.39400	.08960	.000	.00000	.0000	.25000
42	.012000	.035000	11.27300	.10160	.000	.00000	.0000	.25000
43	.012000	.035000	11.04000	.11500	.000	.00000	.0000	.25000
44	.012000	.035000	10.88700	.13030	.000	.00000	.0000	.25000
45	.012000	.035000	10.83800	.14760	.000	.00000	.0000	.25000
46	.012000	.035000	11.39700	.16680	.000	.00000	.0000	.25000
47	.012000	.035000	11.66200	.18870	.000	.00000	.0000	.25000
48	.012000	.035000	11.53600	.21440	.000	.00000	.0000	.25000
49	.012000	.035000	11.07600	.24370	.000	.00000	.0000	.25000
50	.012000	.035000	11.24400	.27610	.000	.00000	.0000	.25000
51	.012000	.035000	11.18600	.31310	.000	.00000	.0000	.25380
52	.012000	.035000	11.64600	.35480	.000	.00000	.0000	.24620
53	.012000	.035000	11.39000	.40670	.000	.00000	.0000	.30000
54	.012000	.035000	10.87500	.47110	.000	.00000	.0000	.28840
55	.012000	.035000	10.87500	.53530	.000	.00000	.0000	.22300
56	.012000	.035000	10.92600	.59850	.000	.00000	.0000	.22360
57	.012000	.035000	10.91200	.67180	.000	.00000	.0000	.23800
58	.012000	.035000	10.89500	.75710	.000	.00000	.0000	.23800
59	.012000	.035000	10.87500	.85240	.000	.00000	.0000	.23900

TABLE A.8
MUFT-III FILE FOR STAINLESS STEEL

A-14

FILE NO 04		STAINLESS STEEL						
GROUP	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_s$	$\bar{\sigma}_c$	$\bar{\sigma}_{in}$	$\bar{\sigma}_f$	ν	Δu
1	.010640	.035290	1.56200	.09550	1.380	.00000	.0000	.25000
2	.010640	.035290	1.84300	.09550	1.370	.00000	.0000	.25000
3	.010640	.035290	2.04300	.09550	1.350	.00000	.0000	.25000
4	.010640	.035290	2.07900	.09550	1.310	.00000	.0000	.25000
5	.010640	.035290	1.92500	.09550	1.180	.00000	.0000	.25000
6	.010640	.035290	1.99800	.09550	1.000	.00000	.0000	.25000
7	.010640	.035290	1.99800	.09550	.800	.00000	.0000	.25000
8	.010640	.035290	1.97900	.09550	.620	.00000	.0000	.25000
9	.010640	.035290	1.84300	.09550	.470	.00000	.0000	.25000
10	.010640	.035290	2.35800	.09550	.200	.00000	.0000	.25000
11	.010640	.035290	2.17600	.09550	.000	.00000	.0000	.25000
12	.010640	.035290	2.81100	.09550	.000	.00000	.0000	.25000
13	.010640	.035290	3.67200	.09550	.000	.00000	.0000	.25000
14	.010640	.035290	2.76500	.09550	.000	.00000	.0000	.25000
15	.010640	.035290	2.81100	.09550	.000	.00000	.0000	.25000
16	.010640	.035290	3.21900	.09550	.000	.00000	.0000	.25000
17	.010640	.035290	3.08300	.09550	.000	.00000	.0000	.25000
18	.010640	.035290	3.12900	.09550	.000	.00000	.0000	.25000
19	.010640	.035290	3.99000	.09550	.000	.00000	.0000	.25000
20	.010640	.035290	2.94600	.09550	.000	.00000	.0000	.25000
21	.010640	.035290	4.80600	.09550	.000	.00000	.0000	.50000
22	.010640	.035290	4.21600	.09550	.000	.00000	.0000	.50000
23	.010640	.035290	4.39700	.09550	.000	.00000	.0000	.50000
24	.010640	.035290	6.32400	.09550	.000	.00000	.0000	.50000
25	.010640	.035290	6.34700	.09550	.000	.00000	.0000	.50000
26	.010640	.035290	5.53100	.09550	.000	.00000	.0000	.50000
27	.010640	.035290	5.34900	.09550	.000	.00000	.0000	.50000
28	.010640	.035290	5.75800	.09550	.000	.00000	.0000	.50000
29	.010640	.035290	6.48300	.09550	.000	.00000	.0000	.50000
30	.010640	.035290	7.52500	.09550	.000	.00000	.0000	.50000
31	.010640	.035290	8.24200	.09550	.000	.00000	.0000	.50000

	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_s$	$\bar{\sigma}_c$	$\bar{\sigma}_{in}$	$\bar{\sigma}_F$	ν	Δu
32	.010640	.035290	9.48300	.09550	.000	.00000	.0000	.50000
33	.010640	.035290	9.85300	.09550	.000	.00000	.0000	.25000
34	.010640	.035290	9.83800	.09550	.000	.00000	.0000	.25000
35	.010640	.035290	9.93000	.09550	.000	.00000	.0000	.25000
36	.010640	.035290	9.99100	.09550	.000	.00000	.0000	.25000
37	.010640	.035290	10.06400	.09550	.000	.00000	.0000	.25000
38	.010640	.035290	9.99700	.09550	.000	.00000	.0000	.25000
39	.010640	.035290	10.00300	.09550	.000	.00000	.0000	.25000
40	.010640	.035290	10.18900	.09550	.000	.00000	.0000	.25000
41	.010640	.035290	10.34600	.10440	.000	.00000	.0000	.25000
42	.010640	.035290	10.23600	.11840	.000	.00000	.0000	.25000
43	.010640	.035290	10.02400	.13400	.000	.00000	.0000	.25000
44	.010640	.035290	9.88500	.15180	.000	.00000	.0000	.25000
45	.010640	.035290	9.84100	.17200	.000	.00000	.0000	.25000
46	.010640	.035290	10.34800	.19430	.000	.00000	.0000	.25000
47	.010640	.035290	10.58900	.21990	.000	.00000	.0000	.25000
48	.010640	.035290	10.47400	.24980	.000	.00000	.0000	.25000
49	.010640	.035290	10.05700	.28390	.000	.00000	.0000	.25000
50	.010640	.035290	10.20900	.32170	.000	.00000	.0000	.25000
51	.010640	.035290	10.15700	.36480	.000	.00000	.0000	.25380
52	.010640	.035290	10.57400	.41340	.000	.00000	.0000	.24620
53	.010640	.035290	10.34200	.47380	.000	.00000	.0000	.30000
54	.010640	.035290	9.87400	.54890	.000	.00000	.0000	.28840
55	.010640	.035290	9.87400	.62370	.000	.00000	.0000	.22300
56	.010640	.035290	9.92100	.69730	.000	.00000	.0000	.22360
57	.010640	.035290	9.90800	.78270	.000	.00000	.0000	.23800
58	.010640	.035290	9.87400	.88210	.000	.00000	.0000	.23800
59	.010640	.035290	9.87400	.99310	.000	.00000	.0000	.23900
60	.000000	.000000	.00000	.00000	.000	.00000	.0000	.00000

2.3 Inelastic Scattering Matrices

The inelastic scattering matrix for scattering from j energy groups into i energy groups was calculated by the Weisskopf statistical model⁽⁹⁾ for the old Fe, U-235 and stainless steel MUFT-III files. (The new files for Fe and stainless steel, based upon a different model, are described in Section 11.3. The matrix elements were calculated from:⁽¹⁰⁾

$$X_{ji} = (\text{const.}) \left[e^{-E_i/\theta(E_i+\theta)} - e^{-E_{i+1}/\theta(E_i+\theta)} \right] \quad E_{i+1} \geq E'$$

$$X_{ji} = (\text{const.}) \sqrt{\theta E'} \left[\sqrt{\frac{E_i}{\theta}} e^{-E_i/\theta} - \sqrt{\frac{E_{i+1}}{\theta}} e^{-E_{i+1}/\theta} + \frac{\sqrt{\pi}}{2} \text{EI} \sqrt{\frac{E_i}{\theta}} - \frac{\sqrt{\pi}}{2} \text{EI} \sqrt{\frac{E_{i+1}}{\theta}} \right] \quad E_{i+1} < E'$$

where:

$$\theta = \sqrt{\frac{\Delta\theta_{ji}}{\alpha}}$$

$$\Delta\theta_{ji} = \frac{1}{2} \left[E_j + E_{j+1} - E_i - E_{i+1} \right]$$

EI = error integral

α = value dependent on atomic number

= 12.6 for U-235. (9)

= 1.4 for S.S. (9)

E' = 0.498 Mev (10)

The values of X_{ji} are normalized such that:

$$\sum_{i=j+1}^I X_{ji} = 1$$

Tables A.9 and A.10 list the values of $g_{j \rightarrow i} = \sigma_{in} X_{ji}$ for the MUFT-III files for iron and U-235 respectively. For each group, i , the values of $g_{j \rightarrow i}$ are listed in order from $g_{1 \rightarrow i}$ through $g_{i-1 \rightarrow i}$. In the MUFT-III listing for U-235, two lines are required for some of the groups.

The new MUFT-III file for Fe and stainless steel as calculated by the new model, described in Section 11.3, is tabulated in Table A.11.

TABLE A.9

OLD INELASTIC FILE-STAINLESS STEEL

GROUP

2	.021000	.000000	.00000	.00000	.000	.00000	.0000	.00000
3	.123800	.000000	.00000	.00000	.000	.00000	.0000	.00000
4	.209201	.054200	.00000	.00000	.000	.00000	.0000	.00000
5	.237202	.183100	.00000	.00000	.000	.00000	.0000	.00000
6	.212602	.238202	.94200	.00000	.000	.00000	.0000	.00000
7	.170602	.227902	2.02101	.00000	.000	.00000	.0000	.00000
8	.129601	.191102	2.22901	.03700	.000	.00000	.0000	.00000
9	.090001	.141201	1.87301	.57800	.000	.00000	.0000	.00000
10	.059900	.097701	1.39001	.41801	.787	.00000	.0000	.00000
11	.040400	.067600	1.00501	.11901	.947	.00000	.0000	.00000
12	.026000	.044400	.67800	.79200	.767	.00000	.0000	.00000
13	.017700	.030700	.47800	.57600	.592	.00000	.0000	.00000
14	.012500	.021800	.34300	.42100	.447	.00000	.0000	.00000
15	.008900	.015700	.25000	.31100	.336	.00000	.0000	.00000
16	.006200	.011100	.17800	.22300	.243	.00000	.0000	.00000
17	.004300	.007700	.12400	.15600	.172	.00000	.0000	.00000
18	.003100	.005500	.08900	.11300	.125	.00000	.0000	.00000
19	.002100	.003800	.06100	.07900	.087	.00000	.0000	.00000
20	.001500	.002600	.04300	.05400	.061	.00000	.0000	.00000
21	.001700	.003100	.05000	.06400	.072	.00000	.0000	.00000
22	.000800	.001500	.02400	.03100	.034	.00000	.0000	.00000
23	.000400	.000700	.01200	.01500	.017	.00000	.0000	.00000
24	.000200	.000300	.00500	.00700	.008	.00000	.0000	.00000
25	.000100	.000200	.00300	.00300	.004	.00000	.0000	.00000
26	.000000	.000100	.00100	.00200	.002	.00000	.0000	.00000
27	.000000	.000000	.00100	.00100	.001	.00000	.0000	.00000

TABLE A. 10

INELASTIC FILE U-235

A-18

GROUP

3	.001800	.000000	.000000	.000000	.0000	.000000	.0000	.000000
4	.022600	.000100	.000000	.000000	.0000	.000000	.0000	.000000
5	.086800	.009600	.000000	.000000	.0000	.000000	.0000	.000000
6	.175101	.061100	.100000	.000000	.0000	.000000	.0000	.000000
7	.247402	.151300	3.08000	.000000	.0000	.000000	.0000	.000000
8	.284102	.240001	11.95005	.54000	.0000	.000000	.0000	.000000
9	.268502	.277802	21.41015	6.83001	.0000	.000000	.0000	.000000
10	.224102	.264702	26.03022	16.22008	1.200	.000000	.0000	.000000
11	.179402	.232102	26.57026	22.92016	7.510	.000000	.0000	.000000
12	.131701	.182002	23.06024	23.95020	13.210	2.14000	.0000	.000000
13	.099601	.144401	19.64022	22.91021	16.240	8.10003	.0000	.000000
14	.075500	.113401	16.21019	20.39020	16.620	12.14009	.0326	.000000
15	.057400	.088501	13.11015	17.37018	15.430	13.49012	.0833	.00334
16	.042100	.066200	10.07012	13.85014	13.020	12.60012	.1031	.00705
17	.030200	.048200	7.47009	10.55011	10.310	10.61011	.0989	.00791
18	.022200	.035900	5.64007	8.13009	8.160	8.75009	.0878	.00752
19	.015500	.025200	4.02005	5.87006	6.020	6.63007	.0695	.00617
20	.010900	.017900	2.88004	4.25005	4.420	4.97005	.0536	.00486
21	.013000	.021500	3.48004	5.21006	5.490	6.29007	.0696	.00642
22	.006300	.010500	1.71002	2.59003	2.770	3.22004	.0364	.00340
23	.003000	.005100	.83001	1.26001	1.360	1.60002	.0183	.00172
24	.001400	.002400	.39000	.60001	.650	.77001	.0088	.00083
25	.000700	.001200	.19000	.29000	.310	.37000	.0043	.00041
26	.000300	.000500	.09000	.14000	.150	.18000	.0021	.00019
27	.000200	.000300	.04000	.07000	.070	.08000	.0010	.00009
28	.000100	.000100	.02000	.03000	.030	.04000	.0005	.00004
29	.000000	.000100	.01000	.01000	.020	.02000	.0002	.00002
30	.000000	.000000	.01000	.01000	.010	.01000	.0001	.00001
31	.000000	.000000	.00000	.00000	.000	.00000	.0001	.00001

TABLE A.11

NEW INELASTIC FILE-FE AND S.S.

GROUP

2	.001000	.000000	.000000	.000000	.000	.000000	.00000	.000000
3	.005000	.001000	.000000	.000000	.000	.000000	.00000	.000000
4	.018000	.006000	.000000	.000000	.000	.000000	.00000	.000000
5	.044000	.018000	.000000	.000000	.000	.000000	.00000	.000000
6	.078000	.045000	4.63000	.000000	.000	.000000	.00000	.000000
7	.110000	.079001	1.97003	.000000	.000	.000000	.00000	.000000
8	.134001	.114001	.37005	.63000	.000	.000000	.00000	.000000
9	.134001	.123001	.60000	.63000	.000	.000000	.00000	.000000
10	.120001	.131000	.76000	.33001	.000	.000000	.00000	.000000
11	.101001	.121000	.44000	.000004	.000	.000000	.00000	.000000
12	.078000	.100000	.34000	.000003	1.840	.000000	.00000	.000000
13	.068000	.092000	.35000	.000000	1.820	.000000	.00000	.000000
14	.030000	.042000	.20000	.000000	1.820	.000000	.00000	.000000
15	.028000	.041000	.16000	.000000	1.910	.000000	.00000	.000000
16	.019000	.027000	.10000	.000000	.860	.000000	.00000	.000000
17	.011000	.018000	.07000	.000000	.000	.000000	.00000	.000000
18	.008000	.012000	.01000	.07000	.000	.000000	.00000	.000000
19	.005000	.007000	.00000	.10000	.000	.000000	.00000	.000000
20	.003000	.005000	.00000	.09000	.000	.000000	.00000	.000000
21	.003000	.005000	.00000	.10000	.000	.000000	.00000	.000000
22	.001000	.002000	.00000	.05000	.000	.000000	.00000	.000000
23	.001000	.001000	.00000	.00000	.000	.000000	.00000	.000000
24	.000000	.000000	.00000	.00000	.000	.000000	.00000	.000000

3.0 XENON PARAMETERS

Calculation of equilibrium xenon in the SM-1 or SM-2 cores is performed by the following equation:

$$\sum_{xe}^a = \frac{\alpha \chi (\gamma_1 + \gamma_2) \bar{\sigma}_a^{xe} \left(-\frac{\delta P_{mw}}{V} \right)}{\lambda_{xe} + \bar{\sigma}_a^{xe} \left[\frac{\beta \delta P_{mw}}{V} \right] \left[\frac{1}{(1-B) \sum f_o} \right]}$$

where the symbols are defined in Section 1.5.1. Table A.12 tabulates the parameters which were used during the SM-2 nuclear analysis. Set 1, corresponding to values used during the nuclear analyses of the SM-1, were used in the early stages of the SM-2 analysis. Later, better values were more available through the literature⁽¹¹⁾ and are designated Set 2 in Table A.12.

The value of $\bar{\sigma}_a^{xe}$ used in conjunction with the Set 1 parameters, was calculated by the formula:(12)

$$\bar{\sigma}_a^{xe} = (3.04 - 25.5 E + 58.3 E^2) \times 10^6 \text{ barns}$$

where E is the effective, hardened, average thermal neutron energy expressed in ev. For example, the values at $T = 440^\circ\text{F}$ (corresponding to a hardened energy of 0.0549 ev) yields $\bar{\sigma}_a^{xe} = 1.8158 \times 10^{-18} \text{ cm}^2$; the value at 510°F ($E = 0.0600 \text{ ev}$) becomes $\bar{\sigma}_a^{xe} = 1.72 \times 10^{-18} \text{ cm}^2$.

The value of $\bar{\sigma}_a^{xe}$ used in conjunction with the Set 2 parameters was based upon the newest data presented in ANL 5800⁽¹¹⁾ by C. H. Westcott for a spin factor $g = 5/8$ or $J = 2$. This reference tabulates $g(T)$ factors as a function of temperature by which the Maxwellian averaged cross section, for a temperature T, may be obtained:

$$\bar{\sigma}(T) = g(T) \sigma_0 \sqrt{\frac{\pi}{4} \left(\frac{T_0}{T} \right)}$$

where $T_0 = 293.6^\circ\text{K}$. The value of σ_0 for xenon (at 0.0253 ev) may be taken as 2.72×10^6 barns from BNL-325.⁽⁶⁾ The appropriate temperature, T, in degrees absolute, corresponding to the averaged hardened thermal neutron energy, E, expressed in ev. may be found by the relation:

$$E (\text{ev}) = 8.63 \times 10^{-5} \times T(^{\circ}\text{K})$$

Based upon the factors $g(T)$ as presented in ANL-5800, the absorption cross section of xenon was calculated as a function of temperature; these results are given in Fig. A. 1. For example, for averaged hardened, thermal neutron energies of 0.0549 ev and 0.0600 ev corresponding to temperatures of 636°K and 695°K respectively, the values of $\bar{\sigma}_a^{xe}$ become 1.98×10^6 barns and 1.845×10^6 barns respectively

TABLE A. 12
XENON PARAMETERS

Parameter	Units	Set 1	Set 2
λ_I	sec ⁻¹	2.89×10^{-5}	2.89×10^{-5}
λ_{xe}	sec ⁻¹	2.11×10^{-5}	2.09×10^{-5}
$\gamma_I = \gamma_1$		0.059	0.060
$\gamma_{xe} = \gamma_2$		0.003	0.0029
δ	fissions/watt-sec	3.24×10^{10}	3.2175×10^{10}

Figure A. 1

APPENDIX B

SM-2 REACTOR CORE AND VESSEL DESIGN DATA AND DRAWINGS

1.0 SM-2 CORE AND VESSEL DESIGN PARAMETERS

1.1 Reference Core #4

At the time of this report, the configuration of the fuel elements had been changed from those described in Section 1.1.2, by increasing the size of the fuel matrix in the fuel plates. This change was made too late to be analyzed from a nuclear standpoint, but the following general comments may be made:

1. Increasing the width of the fuel matrix will decrease the side plate power peaking. (See Section 7.9.) This will reduce the overall core max-to-average power ratio, thereby reducing the coolant flow requirements. From Section 7.9, increasing the width of the fuel matrix from 2.54 in. to 2.65 in. (6.4516 cm. to 6.731 cm.) reduces the local power peaking factor from 1.231 to 1.1425. Referring to Section 9.0, this reduces the hot spot factor, $Q(\Delta\theta)$, by about 7.2%. This is a reduction in the overall calculated max to average power from 3.86 to 3.58.
2. Increasing the dimensions of the fuel matrix decreases the weight percents of UO_2 and ZrB_2 in the meat. It is possible to add additional UO_2 and ZrB_2 to the matrix to bring the weight percent back to the previous value; this would result in additional fuel for burn-up, thereby increasing the lifetime of the core.

The design parameters for this core are tabulated in Table B.1, as taken from APAE Memo 235. (1) The new matrix dimensions are given for the fixed elements only, "since these dimensions had not been determined for the control fuel elements at the time of this report. Therefore, it was assumed that the loadings and geometry for the control fuel elements were the same as given for reference core #3 described in Section 6.6.

TABLE B. 1
DESIGN PARAMETERS OF REFERENCE CORE #4(1)

Geometrical - Core

Assembly drawing	AES-319
Core configuration	7 x 7 - Corners missing
Cell size, in.	2.9375 x 2.9375
Active core height, in.	22
Number of cells	45
Number of control rod cells	7
Gap for flow divider, in.	0.218
Flow divider thickness, in.	0.094
Equivalent diameter without flow divider, in.	22.24
Equivalent diameter with flow divider, in.	22.74

Geometrical - Elements

Assembly drawing - fixed element	R9-13-1017
Assembly drawing - control rod	AEL-445
Number of fuel plates	
Fixed element	18
Control fuel element	16
Fuel plate meat thickness, in.	0.030
Cladding thickness, in.	0.005
Total fuel plate thickness, in.	0.040
Side plate thickness, in.	0.040
Plate width, in.	
Fixed element	2.839
Control fuel element	2.589
Meat width, in.	
Fixed element (min.)	2.650
Control element (min.)	2.440 (tentative)
Active length, in.	
Fixed element	22
Control fuel element	21.5
Total plate length, in.	
Fixed element	23.5
Control fuel element	23.0
Length of europium flux suppressor, in.	0.50

Material Composition - Core

Fuel	Highly enriched UO ₂
Burnable poison	Natural boron in ZrB ₂
Wt. % UO ₂ in matrix	24.5
Wt. % natural boron in matrix	0.191
Cladding	Type 347 SS (0.025 w/o Co and 0.01 w/o Ta)
Absorber material	Eu ₂ O ₃
Flux suppressor material	Eu ₂ O ₃

TABLE B. 1 (CONT'D)

Loading - Core

U-235 per plate, gms	
Fixed element	46.30
Control fuel element	40.56
B-10 per plate, gms (actual loading)	
Fixed element	0.08106
Control fuel element	0.07102
Atom % U-235 in meat	5.2
Atom % B-10 in meat	0.21
Total U-235 in core, kg	36.2
Total B-10 in core, gm	63.4

Reflector and Vessel

Drawing number (reflector)	AES-341
Drawing number (vessel)	AEL-460
Material	Type 304 SS (0.04 w/o Co)
Thickness of reflector, in.	8
Nvt = 1 Mev on vessel, neutrons/cm ²	1.0×10^{21}
Reactor vessel thickness (2200 psia design), in.	4-1/2
Volume of reactor vessel, ft. ³	101.0
Weight of reactor vessel, lb.	31,300

Nuclear Characteristics

Reactivities, % ρ	
68°F, clean, 0% burnup	11.5
510°F, clean, 0% burnup	4.8
510°F, eq. xenon, 0% burnup	2.1
510°F, max. xenon, 0% burnup	0.5
68°F, no. xenon, 22% burnup	15.8
Bank positions, inches from bottom of core	
68°F, clean, 0% burnup	7.0
510°F, clean, 0% burnup	11.3
510°F, eq. xenon, 0% burnup	14.1
510°F, max. xenon, 0% burnup	17.3
68°F, no xenon, 22 in. burnup	4.9
Temperature coefficient (at 510°F), °F ⁻¹	
Core lifetime, MWYR	25.7
Control rod worth, (%) ρ	
7 rod bank	22.8
6 rod ring	17.9
Calculated max. to average radial power (including local peak)	2.10

TABLE B. 1 (CONT'D)

Calculated max. to average axial power	1. 84
Initial average thermal flux in core, neutrs/ cm ² -sec	2. 0 x 10 ¹³
<u>Thermal-Hydraulic Characteristics</u>	
Number of coolant passes	2
Reactor power (reference), MW	28
Heat flux, Btu/hr. -ft. ²	
Average	1. 68 x 10 ⁵
Maximum	6. 50 x 10 ⁵
Burnout ratio (Pass II)	2. 63
Maximum surface temperature, °F	635
Maximum plate centerline temperature, °F	697
Inlet coolant temperature, °F	500
Outlet coolant temperature, °F	530
Total volume flow, gpm	7800
lb/hr x 10 ⁻⁶	3. 05
Maximum coolant flow through fixed elements, gpm	
First pass	218
Second pass	316
Coolant volume flow through control rods, gpm	331

2. 0 SM-2 CORE AND VESSEL DRAWINGS

2. 1 Stationary and Control Fuel Elements

The cross sections of the stationary and control rod fuel elements as used in reference core #1 calculations (Section 2. 0) are shown in Fig. 1. 1 and 1. 4 respectively. A more detailed drawing of both the welded stationary and control fuel elements is shown in Fig. 1. 2 and 1. 5 respectively. These figures are incorporated in Section 1. 1. 2 of this report.

2. 2 Box-Type Fuel Elements

A drawing of a box-type element, similar to the one described in Section 5. 1, is shown in Fig. 5. 1 of Section 5. 1. The element shown is a stationary element containing 18 fuel bearing plates and two inert steel plates.

2.3 Discrete Burnable Poisons

Two methods of incorporating a discrete burnable poison within the fuel element were considered as described in Section 7.12. The first method incorporates boron-steel strips placed between the fuel plates and attached to the side plates. The second method incorporates special boron-steel side plates which are attached to the regular side plates. These are shown in Fig. 7.28.

2.4 Miscellaneous Pressure Vessel Designs

During the course of selecting a reference pressure vessel design, several concepts were devised. Three of these designs were studied during calculations of nvt upon the pressure vessel (Section 7.10). Drawing AEL-386 (Fig. 7.12) is a 31 in. ID pressure vessel with a thick laminated steel reflector. Drawing AES-262 (Fig. 7.13) is a 34 in. ID pressure vessel with a thick laminated steel reflector. Drawing AEL-395 (Fig. 7.14) is a 45-1/2 in. ID pressure vessel with a thin steel skirt, a thick water reflector, and a thermal shield inside the vessel.

2.5 Stainless Steel Vs. Carbon Steel Pressure Vessel Designs

In Section 7.10.3.1, the relative merits of a stainless vs. a carbon steel vessel are discussed. The reference carbon steel vessel considered is shown in AEL-451 (Fig. 7.18). The reference stainless steel vessel considered is shown in AEL-450 (Fig. 7.19).

2.6 Flow Divider

A preliminary core design containing a flow divider is shown in AEL-400 (Fig. 5.6). The reference flow divider configuration and a fuel element spacing study performed upon the core containing this flow divider are shown in AES-319 (Fig. B.1). It is noted that the elements in the outer rows of this core are oriented such that the fuel plates are perpendicular to the steel reflector. This drawing was used in the analysis discussed in Sections 5.2 and 9.1.5.

2.7 Reference Laminated Steel Reflector Configuration

The reference laminated steel reflector configuration is shown in AES-341 (Fig. B.10). This configuration was used in the gamma heating calculations described in Section 7.11. This drawing, including the configuration of the core itself, represents the latest reference design of the SM-2 at the time of this report.

2.8 Reference Stainless Steel Reactor Vessel

The reference stainless steel reactor vessel is shown in AEL-460 (Fig. B.2). This drawing represents the latest reference design of the SM-2 at the time of this report.

Figure B.1

APPENDIX C

METHODS FOR TREATING CONTROL ROD FUEL ELEMENTS IN THE CORE

1.0 CALCULATIONAL MODEL

In radial VALPROD calculations, it is necessary to be able to include the control rod fuel elements within the core. This is easily done for a central control rod fuel element alone, but cannot be done for eccentric rods directly when using a one-dimensional code. One method of estimating the effect of the eccentric control fuel elements is by use of a J_0^2 weighting function. A radial VALPROD calculation for a core consisting of all fixed elements and another VALPROD calculation for a core containing a control fuel element in the center of the core, with the remainder of the elements being fixed elements, yields the reactivity loss, designated $\Delta\rho_c$, due to the substitution of the central element along. Therefore:

$$\Delta\rho_{\text{total}} = \Delta\rho_c \left[1 + \sum_{i=1}^N J_0^2(B_r d_i) \right]$$

$$B_r = \left(\frac{2.4048}{R + S_r} \right)$$

where:

$\Delta\rho_{\text{total}}$ = total reactivity change due to substitution of the N control rods in place of fixed elements.

B_r = radial buckling of the core (cm^{-1}).

R = effective radius of the core (cm).

S_r = radial reflector savings of the core (cm)

d_i = radial distance from center of core to center of i th control rod. (cm)

N = number of control rod elements.

Another method for treating the control rod fuel elements, which may be called the "Equivalent Ring Method," is more convenient to use in VALPROD calculations and yields results comparable to the previous method within a tenth of a percent in reactivity. In this method, a group of control fuel elements lying at the same, or nearly the same distance from the center

of the core is replaced by an "equivalent ring" having an area equal to the elements making up the ring, and lying at an average distance $\bar{d}_n$ from the center of the core; this distance is calculated by:

$$J_0^2 (B_R \bar{d}_n) = \left(\frac{1}{n} \right) \left[\sum_{i=1}^n J_0^2 (B_R d_i) \right]$$

$$A_R = n A_E$$

$$R_{in} = \sqrt{\bar{d}_n^2 - \frac{A_R}{2\pi}}$$

$$R_{on} = \sqrt{\bar{d}_n^2 + \frac{A_R}{2\pi}}$$

where:

n = number of control fuel elements making up the equivalent ring.

A_E = cross-sectional area of each control fuel element (cm^2).

A_R = total cross-sectional area of the equivalent ring (cm^2).

R_{in} = inner radius of the equivalent ring (cm).

R_{on} = outer radius of the equivalent ring (cm).

1.1 Closed Seven Rod Array (Equivalent Ring Method)

The SM-1 core contains a closed seven control rod array as shown in Fig. 1.7. In this case, the six eccentric rods could be replaced by a single equivalent ring; however, since rods F and G are much closer to the core center than rods A, B, D, and E, it is probably best to form two equivalent rings. In this case, rods F and G and rods A, B, D, and E lie at distances d_1 and d_2 from the core centerline. The fuel element size is 2.9375 in. by 2.9375 in. (7.46125 cm. by 7.46125 cm), therefore:

Rods F and G (Inner Ring):

$$J_0^2 (B_r \bar{d}_{FG}) = \frac{1}{2} \left[J_0^2 (B_r d_1) + J_0^2 (B_r d_1) \right] = J_0^2 (B_r d_1)$$

$$\bar{d}_{FG} = d_1 = 10.551805 \text{ cm.}$$

$$A_R = (2) (7.46125 \text{ cm})^2 = 111.340504 \text{ cm.}^2$$

$$R_{iFG} = 9.675753 \text{ cm.}$$

$$R_{oFG} = 11.360501 \text{ cm.}$$

Rods A, B, D, and E (Outer Ring)

$$J_0^2 (B_r \bar{d}_{ABDE}) = \left[\frac{1}{4} \right] \left[J_0^2 (B_r d_2) + J_0^2 (B_r d_2) + J_0^2 (B_r d_2) + J_0^2 (B_r d_2) \right] = J_0^2 (B_r d_2)$$

$$\bar{d}_{ABDE} = d_2 = 14.922500 \text{ cm.}$$

$$A_R = (4) (7.46125 \text{ cm.})^2 = 222.681008 \text{ cm}^2$$

$$R_{iABDE} = 13.683575 \text{ cm.}$$

$$R_{oABDE} = 16.066169 \text{ cm.}$$

1.2 Open Seven Rod Array

The SM-2 core will contain an open seven rod array as shown in Fig. 1.8. To test the equivalence of the methods used to treat the control fuel elements, a VALPROD calculation was performed, using both methods, upon the SM-2 Experimental Core at 68°F containing no boron. The nuclear properties of the fixed and control fuel elements are given in Table 9.7.

1.2.1 Weighting by a J_0^2 Distribution

In this case, control fuel elements B, E, F, and G lie at a distance $d_1 = 16.683863 \text{ cm}$ from the center of the core and elements A and D lie at a distance $d_2 = 14.922500 \text{ cm.}$ from the center of the core. By a FINK-1 calculation, the radial reflector savings (S_r) is calculated to be 6.703837 cm.

Therefore; for an equivalent core radius of 28.238577 cm., $B_r = 0.068821$ and:

$$B_r d_1 = 1.148200 \quad ; \quad J_0^2(B_r d_1) = 0.485376$$

$$B_r d_2 = 1.026981 \quad ; \quad J_0^2(B_r d_2) = 0.567309$$

The reactivity loss in replacing the central element in an all-fixed element core by a control rod fuel element was calculated by the VALPROD code and resulted in $\Delta \rho_{\frac{1}{2}} = 0.258026 \% \rho$.

$$\begin{aligned} \Delta \rho_{\text{total}} &= (0.258026) [1 + 4(0.485376) + 2(0.567309)] \% \rho \\ &= 1.051745 \% \rho \end{aligned}$$

1.2.2 Equivalent Ring Method

In this case, all six eccentric rods lie at nearly the same distance from the center of the core so that they may be replaced by a single equivalent ring.

$$\begin{aligned} J_0^2(B_r d) &= \frac{1}{6} [4 J_0^2(B_r d_1) + 2 J_0^2(B_r d_2)] \\ &= \frac{1}{6} [4(0.485376) + 2(0.567309)] \\ &= 0.512687 \end{aligned}$$

$$\bar{d} = \frac{1.1075}{B_r} = 16.09247 \text{ cm.}$$

$$A_R = 6(7.46125 \text{ cm})^2 = 334.021512 \text{ cm.}^2$$

$$R_i \cong 14.346 \text{ cm.}$$

$$R_o \cong 17.667 \text{ cm.}$$

The VALPROD calculation of reactivity from an all-fixed element core and for a core containing a central control element and an equivalent ring of 6 control fuel elements yields reactivities of 26.129371% ρ and 25.029557% ρ respectively. Therefore:

$$\Delta \rho_{\text{total}} = 1.099814 \% \rho$$

It is seen that this method agrees within a tenth of a percent to the J_0^2 weighting method which was calculated in Section 1.2.1, of this Appendix.

The equivalent ring method is very convenient to use in a series of VALPROD calculations, such as in a NUB-1 non-uniform burnup calculation. The flux and power distributions resulting from this calculation cannot be considered accurate; however, the method may be used for reactivity calculations. The location of the ring may be considered constant during core lifetime even though S_r is a function of core composition; this is true since the value of $\bar{d}$ is not a strong function of the radial reflector savings. In addition, the calculated reactivity of the core is not a strong function of the location of the ring; therefore, slight errors in $\bar{d}$, R_i , and R_o will not appreciably affect the results.

It should be noted that there may be interaction effects between the various control fuel elements due to distortion of the flux distribution; neither the J_0^2 weighting or equivalent ring methods include these effects.

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