**REACTOR ANALYSIS
APPR-1 CORE II**

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PREFACE

Subsequent to the analysis in the body of this report metallurgical developments indicated that it would not be feasible to build the APPR-1 Core II with the increased boron loading specified herein. At the time of the issuance of this report the Core II boron loading is to be the same as that for Core I. At the time of procurement of the core in the fall, if metallurgical developments warrant, the increased boron loading will be employed.

The loading discussed in the body of the report and the first three appendices is that designed to meet the specifications outlined in the introduction. Appendix IV discusses changes in the loading to account for the various methods of employing boron.

INTRODUCTION

This report contains the analysis to establish the specifications for the APPR-1 replacement core. This core is identified in this report as APPR-1 Core II.

APPR-1 Core I has performed satisfactorily to date but there are several areas for improvement. The reactivity of Core I is large (1)* so that the five rod bank (center plus four corner rods) is deeply inserted into the core. This results in a large fraction of the power being generated in the lower portion of the core. With the maximum reactivity in Core I it is not possible to make the reactor subcritical at 68°F with four of the seven rods present in APPR-1. The absorber sections in the APPR-1 Core I employ boron enriched in the B-10 isotope dispersed in an iron matrix clad with stainless steel. (2) The available irradiation data on small samples of this material are discouraging. (3) However, operation of the APPR-1 Core I to date has not indicated any difficulties with these absorber sections. Zero Power experiments on the APPR-1 showed that there is considerable thermal flux peaking in the water gap between the control rod absorber section and the control rod fuel elements. (4) This peaking was suppressed in Core I with 9 Haynes combs placed at the top of the control rod fuel element. This suppressor contributes to the activity buildup in the primary system. (5) The fuel plate clad side plates and end boxes of APPR-1 core were fabricated of Type 304L stainless steel with nominal cobalt and tantalum content.

The replacement core for the APPR-1 was designed to improve the above conditions. The general requirements for this core are given in Appendix B,

* Numbers in parenthesis refer to references listed at the back of the report.

Amendment No. 2, Contract No. AT(30-3)-278. Items 1 and 2 of this appendix are concerned with the cobalt, silicon and tantalum content of matrix and clad material. The effect of these changes from APPR-1 Core I are considered negligible as far as core analyses were concerned. Item 5 of the Appendix is concerned with end adaptor material and also has a negligible effect on core analysis. Items 3, 4 and 6 affected the core analysis and are reproduced below:

3. **Burnable Poison:** The boron loading of the core shall be specified so that:
 - a. The effective multiplication constant at startup with equilibrium xenon at temperature is equal to 1.02.
 - b. No major redesign or extensive investigation is required.
 - c. The core can be controlled throughout its lifetime by five (5) control rods; that is, the center line rod plus the four (4) corner rods.
 - d. The core can be completely shut down at any time during its lifetime to ambient temperature, a subcritical condition, by any four (4) of the five (5) rods identified in c. above, with the fifth rod stuck in its most reactive position.
 - e. The predicted core life, as compared with the present operating core, is not reduced by more than approximately 1 MWY.
 - f. If the intent of subparagraphs c., d., and/or e., cannot be established, core life will be increased as much as possible using the present APPR-1 system of seven (7) control rods.
4. **Composite Control Rod:**
 - a. **Absorber material:** Absorber sections will use europium oxide as the absorbing material in a Type 304L or Type 304 stainless steel matrix

using elemental powders. The silicon content of the matrix shall be nominal .02% by weight.

b. Flux suppressors: The external flux suppressors will be replaced by integral flux suppressors using europium oxide as the suppressing material in a Type 304 or 304L stainless steel matrix.

6. Fuel Inventory: Fuel inventory will be changed only within the fuel bearing core section of the control rod fuel elements and then only to the extent necessary to provide compatibility with the features changed above.

The results of the analyses performed to establish the specifications are summarized below:

APPR-1 Core II Loading*

Total kg U-235 in Core

22.38

Total gms B-10 in Core

33.8

This loading for APPR-1 Core II results in an effective multiplication at startup with equilibrium xenon at temperature of 1.023.

The core can be controlled throughout its lifetime by five control rods, i. e. the center line rod plus the four corner rods. It is marginal whether the core can be shut down at any time in its most reactive condition with any four of the five rods.

No means are available to extend the life with the present APPR-1 system of seven control rods without an extensive analysis program on the effect of rod programming on core life.

* This is based on employing a matrix steel content equal to that in the APPR-1 Core I and natural boron in boron carbide.

The control rod absorber sections will contain 151 grams of europium per plate contained as 38 w/o Eu_2O_3 in a stainless steel matrix. These rods will be worth slightly less than the Core I rods employing boron as absorbing material. The integral flux suppressors incorporated into the end of the control rod fuel plates are 0.020 x 0.875 x 2.281 in. They will contain 23.2 w/o Eu_2O_3 in a stainless steel matrix. These will eliminate the problem of flux peaking in the control rod fuel section.

Complete working fabrication specifications will be written for the stationary fuel elements and will be published as an ORNL report.

1.0 Calculation Model and Application to APPR-1

1.1 Description of APPR-1

The APPR-1 is a pressurized water reactor utilizing stainless steel, plate type fuel elements and operating at a power level of 10 megawatts of heat. The core consists of 38 fixed fuel elements and 7 control rod fuel elements, shown in cross section in Fig. 1.1.

The fixed fuel element consists of 18 fuel bearing plates brazed into side plates with end boxes welded to this assembly. The fuel bearing plates of the APPR-1 Core I contain UO_2 and B_4C in a type 302B stainless steel matrix clad with 304L steel. The boron is added to serve as a burnable poison to reduce the reactivity which must be controlled by the rods.

The control rods are of the MTR type, utilizing a fuel element section on the bottom and an absorber section above. (See Fig. 1.2) There is a 1.3" gap between the active fuel and absorber sections. Nine Haynes 25 combs have been inserted in this gap in Core I to reduce the thermal flux peaking. Insertion of the absorber displaces the fuel element section. Normally the core is controlled by a five rod bank consisting of the center rod and the four corner rods. The absorber section is made of four absorber plates formed into a moderator filled box. The control rod fuel element consists of 16 fuel bearing plates. These plates are the same thickness and length as the fixed fuel element plates but are of reduced width.

The APPR-1 Core I loading is 22.5 kilograms of U-235 and 19.5 grams of B-10. Other core constants for the APPR-1 Core I are listed in Appendix I.

FIGURE 1.1 REACTOR CORE CROSS SECTION

FIGURE 1.2 CONTROL ROD POSITIONS

1.2 Calculation Model

The calculation model employed in Core II analysis was developed and checked in the course of the analysis of the extended ZPE on the APPR (1). The principal features of this model are summarized here. The core was approximated by a cylinder of height 55.88 cm and radius 28.2 cm.

The calculation employed modified two group theory defined by the following set of equations:*

$$\begin{aligned} -D_f \nabla^2 \bar{\Phi}_f(\vec{r}) + (\Sigma_r - (1-\rho) \kappa_f \Sigma_r) \bar{\Phi}_f(\vec{r}) &= \kappa_{fh} \Sigma_{fh}^0 \bar{\Phi}_{fh}(\vec{r}) \\ -D_{fh} \nabla^2 \bar{\Phi}_{fh}(\vec{r}) + \Sigma_{fh}^0 \bar{\Phi}_{fh}(\vec{r}) &= \rho \Sigma_r \bar{\Phi}_f(\vec{r}) \end{aligned}$$

The solution to these equations was obtained from Valprod (7) and Windowshade (8), two IBM-650 programs which were used to treat the radial and axial directions respectively. The output of these codes includes power, thermal and fast flux distributions and K_{eff} . A uniform poison can be put into the Windowshade to represent the rod bank; the program will adjust the bank position to criticality.

Fast constants were prepared by the Muft III code (8) which computes few group constants based on solving the multigroup Fourier transform of the slowing down distribution from a point source. The approximation employed was the P-1 Selingut-Goertzel approximation. The thermal properties were evaluated by solving the one velocity transport equation to the third spherical harmonic approximation for plate type elements. This was done by the P-3 program (9) on the IBM-650.

The thermal properties were evaluated at hardened neutron temperatures to account for the effect of core absorption on the Maxwell distribution of

* Nomenclature is given at end of report.

neutron energies. The thermal cross sections were evaluated at neutron temperatures of 0.0331 eV at 68°F and 0.0549 eV at 440°F.

A new code, Nub I, was written to calculate the burnout distribution and to prepare input for Valprod and Windowshade. The equations programmed are included in Appendix II and described in more detail in APAE Memo 128 (10).

In certain calculations K_{eff} was found by using the equivalent bare model:

$$K_{eff} = \frac{K_{th} \rho}{(1 + \gamma B^2)(1 + L^2 B^2)} + \frac{K_R(1 - \rho)}{1 + \gamma B^2}$$

The cold reflector savings were measured in the Zero Power experiments (1). Calculations have shown that the cold reflector savings increase by 20% at 440°F. This factor was used to find the hot savings. Table I.6 in Appendix I lists the savings and bucklings used.

1.3 APPR-1 at 68°F

The calculation model discussed above was applied to the APPR-1 for a wide range of configurations measured in ZPE II (11). One problem encountered in application of the above calculation model is the presence of the control rod fuel elements in the core. This is because the control rod fuel elements contain a reduced number of fuel plates, a reduced fuel plate width and a surrounding steel basket. Measurements made in ZPE II (12)* indicate that the worth of substituting seven control rod elements for seven fixed elements is -1.93% ρ . The calculated reactivity at 68°F for 45 fixed elements is 15.47%. The calculated reactivity

* The experimental values in APAE 8 and 21 are reported either as $\Delta K/K$ in dollars and cents or as the integral of a measured reactivity worth curve ($\Delta K_e = \int \frac{\partial \rho}{\partial x} dx$). $\Delta K/K$ was converted to absolute units by multiplying dollars by $\beta = 0.0073$. The relationship $\rho = 1 - e^{-\Delta K_e \times 0.0073}$ (23) was used to convert the integral of a worth curve to reactivity.

for the mixed element core of 38 fixed and 7 control rod elements is 15.47-1.93=13.54%. For calculational purposes the substitution effect was included by computing a thermal absorption cross section, $\sum_{\text{sub}}^a$ in the following manner: All group constants are found for fixed elements (See Appendix I) so that

$$K_{\text{eff}}(45 \text{ Fixed elements}) = \frac{1}{1-\rho} = \frac{1}{1-.1547} = K_{\text{th}} H + J$$

$$K_{\text{th}} = \frac{U \sum_{\text{FE}}^F}{\sum_{\text{FE}}^O}$$

$$H = \frac{\rho}{(1+\tau B^2)(1+L^2 B^2)}$$

$$J = \frac{K_F(1-\rho)}{1+\tau B^2}$$

For 38 fixed and 7 control rod elements K_{eff} is

$$K_{\text{eff}} = \frac{1}{1-(\rho-.0193)} = \frac{1}{1-.1354} = K'_{\text{th}} H + J$$

$$K'_{\text{th}} = \frac{U \sum_{\text{FE}}^F}{\sum_{\text{FE}}^O + \sum_{\text{sub}}^O}$$

The value of $\sum_{\text{sub}}^a$ found at 68°F is 0.007846 cm⁻¹.

The measured cold reactivity for the mixed element core is 15.35%.

This is 1.81% ρ higher than the value calculated using K'_{th} . This difference was used as a constant correction applied to reactivity to account for the inadequacy of the calculation model at 68°F.

From the Zero Power experiments (4) the reactivity of the core with the five rod bank fully inserted can be inferred by integrating the five rod bank curve of worth per inch from the critical position to full insertion. This gives a measured reactivity of -4.63%. Since the calculated reactivity is 1.81% ρ lower than that measured, the value of the core reactivity with the rods full inserted is taken as -6.44%. A thermal poison cross section $\sum^P$, representing the bank in

the Windowshade calculation can be calculated in a similar manner as was done

for $\sum_{\text{sub}}^a$, i. e.

$$K_{\text{eff}}(\text{rods out}) = \frac{1}{1-\beta'} = \frac{1}{1-.1354} = K_{\text{th}}' H + J$$

$$K_{\text{eff}}(\text{rods in}) = \frac{1}{1-\beta''} = \frac{1}{1-(-.0644)} = K_{\text{th}}'' H + J$$

where

$$K_{\text{th}}'' = \frac{U \sum_{FE}^F}{\sum_{FE}^0 + \sum_{\text{sub}}^0 + \sum^P}$$

and the symbols K_{th} , H and J have the same meaning as above. The control rod

$\sum^P$ was found to be 0.088239 cm^{-1} at 68°F .

This value of $\sum^P$ used in a Windowshade calculation resulted in the absorber fuel interface at a position 4.6 inches above the bottom of the core for the critical reactor. The measurement of the bank position in the APPR-1 is a measurement of the distance from the bottom of the core to the top of the control rod fuel. As seen in Fig. 1.2 there is a 1.3 inch gap between the control rod fuel and absorber section. Therefore, there is no definite fuel absorber interface with which the Windowshade calculation can be compared. The measured bank position in the clean core at 68°F is 3.7". Therefore, the calculated absorber fuel interface is 0.9 inches above the measured bank position. This interface is within the 1.3 inch gap between fuel and absorber. In all calculations 0.9 inches was subtracted from the Windowshade absorber-fuel interface to convert to measured bank position.

1.4 APPR-1 at 440°F

At 440°F the core properties were calculated using the same model

employed at 68°F. It was assumed that $\sum_{\text{sub}}^a$ varies with temperature in the same manner as the U-235 cross section does. The $\sum_{\text{sub}}^a$ used at 440°F was 0.006131 cm⁻¹.

The cold to hot change in bank worth was calculated by solving the multigroup diffusion problem (Valprod) for a core with a central rod (water, absorber shell, core and water regions). The Valprods were run hot and cold, with and without the centerline rod inserted. An equivalent $\sum^p$ was calculated for the center rod at the two temperatures. The results are tabulated below:

Table 1.1

K_{eff} With Center Rod

	68°F	440°F
K_{eff} rod in	1.0757	1.0047
K_{eff} rod out	1.1566	1.0852
$\sum^p$ of center rod	0.0274	0.0240

The ratio of $\sum^p$ at 68°F to that at 440°F was 1.138. Using this ratio the $\sum^p$ of the 5 rod bank was taken to be 0.07756 cm⁻¹ at 440°F.

Windowshade calculations were done at 440°F for both the clean core and the core with equilibrium xenon. Some five rod bank positions measured in Core I at 440°F at Fort Belvoir, Va., (13) are listed below:

Table 1. 2**Measured Five Rod Bank Positions**

MWYR	Condition	Bank Position (in)
0	Clean	6. 77
0. 39	eq. xenon	8. 41
0. 78	eq. xenon	8. 54
1. 00	eq. xenon	8. 51
1. 40	eq. xenon	8. 50
1. 62	eq. xenon	8. 52
2. 21	eq. xenon	8. 63
2. 76	eq. xenon	8. 73
3. 16	eq. xenon	8. 82
3. 49	eq. xenon	8. 97

The measurements with accumulated fission products, including equilibrium xenon, were plotted and extrapolated to zero megawatt years. The first calculated bank position included equilibrium xenon but no other fission products. This introduces an uncertainty in the interpretation of the measured bank position (See figure). The initial bank position used in the calculations was 8. 3 inches. Because of the uncertainties in the extrapolation this 8. 3 inches may be high by one or two tenths of an inch.

With the rods inserted to the critical position at 8.3 inches (Window-shade interface at 9.2 inches) the Windowshade calculates the core reactivity of -0.3% , whereas the critical reactivity is actually zero. Therefore a reactivity correction of $+0.3\%$ was applied to the equilibrium xenon calculations to account for model error. This correction would be about to 0.45% if the initial position had been taken as 8.2 inches (at this insertion the bank is worth 1.5% per inch). The same calculation for the clean core at 440°F resulted in a model correction at $+0.6\%$.

The measured and calculated bank positions at startup for the APPR-1 Core I are summarized in the following table:

	Measured Bank Position	Measured Core Reactivity	Window-shade Fuel-Absorber Interface	Correction to Window-shade Position	Calculated Core Reactivity
68°F	3.7"	0	4.6"	-0.9	$-1.8\% \rho$
440°F Clean	6.8"	0	7.7"	-0.9	$-0.6\% \rho$
440°F Eq. Xe	8.3"	0	9.2"	-0.9	$-0.3\% \rho$

1.5 Expected Lifetime

The APPR-1 lifetime was calculated by methods outlined in Appendix II. The expected rod bank withdrawal curve is shown in Fig. 2.1 together with bank positions to date* as measured at Fort Belvoir. At a given bank position, the energy release as measured is about 80% of the calculated value. This correction was used to extrapolate the measured bank position. It is felt that the best estimate of the core life of APPR-1 Core I is 13 MWYR.

* The last point plotted was measured May 5, 1958.

FIG. 2.1

FIVE ROD SHIM BANK VS. ENERGY RELEASE
APPR-1 CORE I

2.0 APPR-1 Core II

2.1 Loading

APPR-1 type cores with higher B-10 loadings but no change in fuel content were calculated until the effective multiplication factor at 440°F with equilibrium xenon had dropped to 1.02. This resulted in a net loading of 22.3 kgms of U-235 and 33.8 gms. of B-10. Core constants for this loading are listed in Appendix I.

The same value of $\sum a_{\text{sub}}$, the effect of substituting control rod elements for fixed elements, was used for this core as was used for Core I. The calculated equivalent bare initial reactivity for this core at 440°F is 2.01% with equilibrium xenon. After applying the same model correction found from the Core I analysis, the best estimate of the initial reactivity for Core II at 440°F is 2.31% with equilibrium xenon.

The calculated initial equivalent bare reactivity at 68°F is 8.08%. Again if the same model corrections are assumed to hold for both cores, the measured value will be 9.89%. Other core constants for this loading are listed in Appendix I.

2.2 Lifetime

The burnout of Core II was calculated in the same manner as was done for Core I. All numerical corrections, including the value of $\sum \rho$ for the five rod bank, were assumed not to change from Core I to Core II. There is some evidence that the bank worth will decrease with increased core absorption (Sec. 4) and will be less for europium rods than for boron. However, this is not serious because the decrease is small and the reactivity at end of life, with the bank full out,

is independent of Σ^P . Thus the calculated life is independent of the bank worth.

The radial thermal and fast flux distribution of startup with equilibrium xenon is shown in Fig. 2.2. Fig. 2.3 shows the axial thermal and fast flux distribution with equilibrium xenon at startup. These distributions were used to burn the core out as outlined in Appendix II. The calculated curve of K_{eff} vs. megawatt years corrected for nonuniform radial and axial burnout is shown in Fig. 2.4. Figs. 2.5 and 2.6 show the axial and radial fuel distributions in the core as a function of burnout.

Fig. 2.7 is the expected bank position versus energy release. The expected initial bank positions (distance from bottom of core to top of control rod fuel element as calculated by the Windowshade model are listed in Appendix I.

The shape of the bank withdrawal curve for Core II is nearly the same as that for the first core loading after about 10 MWYR. The result is that both cores have about the same expected lifetime, i. e., 13 MWYR. It is not unexpected that the bank withdrawal should be similar for both cores for high burnups because the poison burns out so rapidly; for example, at the end of life the difference in boron loadings in the two cores is only 2.5 grams. The reason the two cores have the same expected life is probably due to the differences in non-uniformity.

2.3 Uncertainties in Core II Loading

All the calculations for the Core II loading have been made on the assumption that the second core will be manufactured the same as the APPR-1 and will have the same characteristics. However, changes in the core which will be within the manufacturer's tolerances can alter the reactivity.

The two points of most interest are initial hot equilibrium xenon

FIG. 2.2
 RADIAL FLUX DISTRIBUTION
 CORE II - 440° F Eq Xenon STARTUP

FIG. 2.3

AXIAL FLUX DISTRIBUTION CORE II - 440°F - Eq xenon

FIG.2.4 K_{eff} VS. ENERGY RELEASE APPR-I CORE II 440°F Eq Xe

FIG.2.5 AXIAL FUEL BURNOUT APPR-I CORE II

FIG. 2.6

RADIAL FUEL BURNOUT APPR-I CORE II

FIG. 2.7

BANK WITHDRAWAL APPR-I CORE II

440°F Eq Xe

— CALCULATED
--- EXPECTED

and maximum reactivity. The initial hot dirty reactivity is 2.3%. If the worst possible combination of manufacturer's errors and errors due to interpretation of experimental results should compile, the initial reactivity would still be +1.61%.

The maximum reactivity will occur at about 7.5 MWYR at 68°F.

At this point the core reactivity is 12.5% with a probable uncertainty of +1.39% and -1.03%. The important concern here is whether the control rod bank can hold down this much reactivity. The five rod bank is expected to be worth about 17.5% at 440°F (Sec. 4) which should be sufficient to control the core at all times. If one of the rods in the ring is stuck the four rod bank is worth 12% $\pm$ 1%. This means it is marginal whether the core can be made subcritical with an eccentric rod stuck.

The types of uncertainties which were considered were the following:

1. Manufacturer's tolerances
 - a. Material content uncertainties
 - b. Mechanical tolerances
 - c. Clad density uncertainty
 - d. Braze uncertainty
2. Calculational uncertainties
 - 2.3.1 Manufacturer's Uncertainties
 - a. Material content uncertainties

The following weighing specifications applied to

APPR-1 Core I. (2).

U-235	-0.57%	+0.38%
B-10	-5.00%	+1.30%
S. S.	-0.20%	+0.01%

These specifications will be maintained the same for Core II because they are not too tight from a practical standpoint, i. e. ORNL was able to make two cores to these specifications. Also these specifications are low enough to give reasonably good control on reactivity.

To calculate the effect of the material content uncertainties it was assumed that the xenon cross section would be directly proportional to the U-235 cross section. This tends to overestimate the xenon uncertainty, but since this contribution is rather small, the procedure is justified.

The case which will give the lowest reactivity is for the fuel to be measured low and everything else high. If it is assumed that K_{eff} is proportional to K_{th} , the uncertainty in reactivity is -0.17%.

b. Mechanical Tolerances

The plate thickness limits are $0.030 \pm .001$ in. (2) which introduces an uncertainty of ± 0.85 volume per cent metal in the core. It is expected that the steel content might be off by ± 0.42 volume per cent (14). This introduces an uncertainty of $\pm .000111 \text{ cm}^{-1}$ in the core thermal absorption cross-section. Again, if K_{eff} is proportional to K_{th} the error is -0.047%.

c. Uncertainty in density of stainless steel

Approximately 42% of the steel in the core is in the meat of the fuel plates. The remaining steel is contained in the fuel plate cladding, dead edges, side plates and etc. The worst expected density variation in the clad steel from Core I to Core II is $\pm 1.28\%$ ($7.8 \pm 0.1 \text{ gm/cm}^3$). The effect of this on the steel cross-section will cause a reactivity uncertainty of -0.085%.

d. Braze Uncertainty

Each element contains 35 grams of braze material. If the amount of braze material were as high as 40 grams per element there would be an additional absorption cross-section of $.00003 \text{ cm}^{-1}$ and a reactivity loss of 0.013%.

2.3.2 Calculational Uncertainties

Reactivity worth per gram of B-10 added

The following worth per gram was obtained from the zero power experiments at 68°F .

ZPE I (12)	-0.419 %/gm B-10
ZPE II (9)	-0.335 %/gm B-10

At present the reason for the difference between these numbers is not completely resolved. A large part of the difference is believed to be due to the different experimental techniques used in ZPE I and ZPE II. It is believed that the correct number is somewhere between the two and probably closer to 0.335 %/gm. At 68°F the calculated worth is -0.382 %/gm. If the ZPE results represent the two extremes the uncertainty in the calculation is

		-0.037
	-0.382	+0.047 %/gm
		-9.7%
or	-0.382 %/gm	+12.3%

The calculated worth per gram at 440°F is -0.409%/gm.

If it is assumed that the same percentage uncertainties are true for the hot

—

-0.409

This tended to overestimate the worth of the strips for two reasons: (1) The thermal neutron flux in the vicinity of the strips was higher than the flux in the fuel plate and (2) the plastic tapes displace water from the core causing a loss in reactivity.

Table 2. 1

Reactivity - best estimate		2.31%
Natural boron worth	-0.42%	
Material content error	-0.17%	
Plate thickness tolerance	-0.05%	
Braze uncertainty	-0.01%	
Clad density uncertainty	-0.08%	
	<u>-0.73%</u>	
Minimum credible reactivity		+1.58%

It must be remembered that this represents the worst possible combination of faults, i. e. all the materials are weighed wrong, the clad is everywhere too thick, and the braze content is high. However, the initial reactivity is still positive although all of the above assumptions tend to overestimate the uncertainties.

2.3.4 Maximum Reactivity

The maximum reactivity will occur at 68°F somewhere near midlife. For an uncertainty analysis this point was taken near the deepest bank insertions, i. e. 7.5 MWYR. The same uncertainties were considered as for the initial hot case. The results are tabulated below:

Table 2.2

Core II Maximum Reactivity

Radial nonuniform correction	-0.00%	-0.00%
Axial nonuniform correction	-0.02%	-0.02%
Natural boron worth uncertainty	+0.68%	-0.53%
Weighing uncertainties in meat	+0.46%	-0.25%
Clad thickness uncertainty	+0.08%	-0.08%
Clad density uncertainty	+0.14%	-0.14%
Braze uncertainty	<u>+0.01%</u>	<u>-0.01%</u>
	+1.39%	-1.03%

The best estimate of the maximum reactivity is 12.50% so the two limits are 13.89% and 11.47%. Again this represents the worst possible manufacturer's errors.

3.0 Internal Flux Suppressors

During the APPR-1 Zero Power experiments (4) it was discovered that the thermal flux peak in the water region between the control rod fuel element and absorber extended into the control rod fuel meat. This peaking resulted in the possibility of local boiling and high burnup at the tip of the element. Haynes 25 combs were employed at the end of the control rod fuel element in Core I. These Haynes combs have a high cobalt content.

In Core II the flux suppressors will be built into the fuel plates as shown in Fig. 3.1. These suppressors will contain one gram of europium as 23.2 w/o Eu_2O_3 in a stainless steel matrix. The dimensions of the suppressor are

Thickness	0.020 in.
Width	2.281 in.
Length	0.875 in.

In order to determine the variation of the effect of the suppressors with lifetime the burnout characteristics of europium were investigated.

3.1 Burnup of Europium

Europium will be used in the Core II for the control rod absorber material as well as for the internal flux suppressors. Europium has a high absorption cross-section and each absorption gives birth to a chain in which the daughters have good absorption cross-sections. Therefore, the ratio of the europium absorption cross-section at any burnup to the initial value should not fall off as rapidly as it would for another absorber, such as boron. This survey was intended to determine the cross section ratio as a function of burnup. Further details are reported in AP Note 97 (15).

Figure 3.1
A. Control Rod Fuel Element (Not to Scale)

B. Division of Control Rod Fuel Element into Regions For Valprod Calculation (Not to Scale)

3. 1. 1 Burnout Equations

If the starting material is natural europium there are seven isotopic states to be considered and the number of absorptions in europium must include absorptions in all seven. The isotopic states are Eu-151, Eu-153, Eu-154, Eu-155, Eu-156 and Eu-152 which has two isomeric states. The isomeric state of Eu-152 with a half-life of 9.2 hours appears in 16% of the transitions. Because of the short half-life, this state was not included. Because of the lack of cross section data, Eu-156 and higher isotopes were not considered. The differential equations that describe the burnout of europium are as follows:

$$\frac{dN^{151}}{dt} = -(\sigma_{th}^{a-151} \bar{\Phi}_{th} + \sigma_f^{a-151} \bar{\Phi}_f) N^{151} \quad (3-1)$$

$$\frac{dN^{152}}{dt} = 0.84(\sigma_{th}^{a151} \bar{\Phi}_{th} + \sigma_f^{a151} \bar{\Phi}_f) N^{151} - (\lambda^{152} + \sigma_{th}^{a152} \bar{\Phi}_{th} + \sigma_f^{a152} \bar{\Phi}_f) N^{152} \quad (3-2)$$

$$\frac{dN^{153}}{dt} = (\sigma_{th}^{a152} \bar{\Phi}_{th} + \sigma_f^{a152} \bar{\Phi}_f) N^{152} - (\sigma_{th}^{a153} \bar{\Phi}_{th} + \sigma_f^{a153} \bar{\Phi}_f) N^{153} \quad (3-3)$$

$$\frac{dN^{154}}{dt} = (\sigma_{th}^{a153} \bar{\Phi}_{th} + \sigma_f^{a153} \bar{\Phi}_f) N^{153} - (\lambda^{154} + \sigma_{th}^{a154} \bar{\Phi}_{th} + \sigma_f^{a154} \bar{\Phi}_f) N^{154} \quad (3-4)$$

$$\frac{dN^{155}}{dt} = (\sigma_{th}^{a154} \bar{\Phi}_{th} + \sigma_f^{a154} \bar{\Phi}_f) N^{154} - (\lambda^{155} + \sigma_{th}^{a155} \bar{\Phi}_{th} + \sigma_f^{a155} \bar{\Phi}_f) N^{155} \quad (3-5)$$

$$\frac{dA}{dt} = \sum_{i=151}^{155} (\sigma_{th}^{ai} \bar{\Phi}_{th} + \sigma_f^{ai} \bar{\Phi}_f) N^i \quad (3-6)$$

Here A is the number of absorptions as a function of time.

These equations were solved in terms of flux time on a analog computer for seven different cases.

3.1.2 Cross Sections

The cross sections used in all cases are given in Table 3.1. Thermal cross sections were evaluated from BNL-325 (16) for a Maxwell-Boltzman distribution about 0.0549 ev. It has been pointed out (17) that averaging over a Wigner Wilkens distribution will increase the europium cross section relative to U-235 because of the low lying europium resonance. The treatment using effective neutron temperature for a Maxwell-Boltzman distribution does not account for this effect, therefore the absorption of europium is underestimated. The fast one-group cross section for europium - 151 and europium - 153 were derived from 13-group cross sections calculated by R. L. Murray (18). Murray's cross sections were averaged over the fast flux as a function of lethargy to obtain a single fast group constant in the following manner:

$$\bar{\sigma}^0 = \frac{\int \sigma^0(u) \Phi(u) du}{\int \Phi(u) du}$$

The flux as a function of lethargy was obtained from a Muft III (8) calculation at 440°F. The fast cross sections for the other three europium isotopes are not known but were determined by assuming that the fast to thermal cross section ratios for all isotopes were the same as that for europium-153.

Table 3. 1

Microscopic Cross Sections and Decay Constants

Isotope	$\sigma_{th}^a (.0549 \text{ ev})$ cm ² /atom	$\sigma_{th}^o (.0549 \text{ ev})$ cm ² /atom	λ Sec ⁻¹	$\sigma_{th}^o (.0331 \text{ ev})$ cm ² /atom
Eu-151	3.5205×10^{-21}	0.4693×10^{-21}	Stable	7.575×10^{-21}
Eu-152	2.1514×10^{-21}	0.4434×10^{-21}	1.6892×10^{-9}	4.629×10^{-21}
Eu-153	0.1952×10^{-21}	0.04023×10^{-21}	Stable	0.354×10^{-21}
Eu-154	0.6971×10^{-21}	0.14367×10^{-21}	1.3725×10^{-9}	1.263×10^{-21}
Eu-155	6.5063×10^{-21}	1.34095×10^{-21}	1.2918×10^{-8}	11.784×10^{-21}
Natural Eu	1.7837×10^{-21}	0.2452×10^{-21}	---	3.803×10^{-21}

The following cases were considered:

Case	Isotopes in Which Fast Absorption Were Considered	Thermal Flux	Fast Flux Thermal Flux
I	---	1.3×10^{13}	9.16
II	151, 153	1.3×10^{13}	9.16
III	151, 152, 153, 154, 155	1.3×10^{13}	9.16
IV	---	1.9×10^{13}	6.32
V	151, 153	1.9×10^{13}	6.32
VI	151, 152, 153, 154, 155	1.9×10^{13}	6.32

Cases II and V were considered to give the lower limits of fast absorptions and Cases III and VI the upper limits. The fluxes in the first three cases are those expected at startup of Core II at 440°F, the next three at end of life and the last the initial flux using 68°F cross sections.

3.1.3 Results

The output of the computer is in the form of graphs where the concentration of each isotope and the number of absorptions is plotted against the flux time, $N(t) = \int_0^t \Phi(t') dt'$. For any γ , the concentrations of each isotope can be found for a given number of absorptions or a given number of absorptions per original atom of europium, A/No.

Cases I, IV and VII which include only thermal effects, were compared with similar curves derived by KAPL (19). Figure 3.2 compares the curves of absorption cross section versus neutrons absorbed per original europium atom obtained from Cases I, IV and VII with curves given by KAPL. As can be seen, the calculated cases compare favorably to that of KAPL. In Fig. 3.3, the thermal cross section ratios from Cases II, III, V, and VI were also compared to the KAPL curve.

Cases III and VI are comparable to the KAPL curve, while for Cases II and V the curve is somewhat higher. This is because only the thermal cross sections were considered at KAPL.

The results also tend to indicate that the cross section ratios are not very sensitive to a change in flux. As shown in Fig. 3.4, the fast cross section ratios are almost identical regardless of whether the startup or end of life fluxes are used. This seems also to be true for the thermal cross section ratio of Cases III and VI and Cases II and V, except that the thermal ratios are more sensitive to a change in flux than the fast ratios.

Cases II, III, V and VI were set up to determine the effect of fast absorption, i. e. Cases II and V included fast absorption in Eu 151 and 153

FIG. 3.2
THERMAL CROSS SECTION OF EUROPIUM
VS.
BURNOUT OF EUROPIUM
(NO FAST ABSORPTION)

FIG. 3.3
THERMAL CROSS SECTION RATIO OF EUROPIUM
VS.
BURNOUT OF EUROPIUM
(FAST ABSORPTION INCLUDED)

FIG. 3.4
FAST CROSS SECTION RATIO OF EUROPIUM
VS.
BURNOUT OF EUROPIUM

only, but Cases III and VI had fast absorption in all the isotopes. These two conditions were considered to give the maximum and minimum limits on fast absorption. Figs. 3.3 and 3.4 show the effect of the inclusion of all fast cross sections.

3.2 Burnout of Flux Suppressors

To determine the effect of the suppressor with burnout a one dimensional, two-group, multiregion model was set up to obtain the local flux and power distribution along the core centerline (20). The power distribution, obtained from the Windowshade program (7), was used as the source for an axial slab Valprod (6) calculation with regions as shown in Fig. 3.1.

From the way the model is set up, the suppressor calculation is dependent on the reactor as a whole only through the source term, in the two group equations. The source is of course dependent upon the properties of the core and therefore upon the bank position. However, the source distribution in the suppressor region does not change much in shape but essentially is only displaced from one position to another as the bank is moved. Therefore, the effectiveness of the suppressor is a local phenomena and not a strong function of the reactor as a whole. Even though the effect of the suppressor was not calculated for the most adverse condition (approximately midlife), it can be assumed that it would not be much different from the calculated case.

Inasmuch as self-shielding factors are difficult to estimate and fast cross sections for all europium isotopes are not known, two parallel cases were calculated to set limits on these uncertainties. Case I used available fast cross sections for Eu-151 and Eu-153 and assumed cross sections for Eu-152,

Eu-154 and Eu-155 based on the ratio of fast to thermal cross sections for Eu-153 and no self-shielding factors. Case II used available fast cross sections for Eu-151 and Eu-153 and self-shielding factors. Essentially this means that limits of maximum absorption and minimum absorption were calculated.

A one-step burnout calculation of the suppressor was performed. It was assumed that the source term from the axial Valprod was constant over the lifetime of the core. Since the flux distribution in the suppressor region is a local effect, this is a reasonable assumption. The relative fluxes from the Valprod can be normalized to absolute values by using the power density as a coupling quantity.

Since the Windowshade and axial Valprod give the axial flux distribution independently of the radial distribution, the absolute flux must be multiplied by the ratio of maximum radial thermal flux to the average radial flux in the core. Once the absolute flux is normalized to the relative flux distribution from the Valprod, the absolute average thermal and fast fluxes are determined for regions 5 through 11.

Once the absolute fluxes are known, the burnups in all regions can be calculated. The calculation of the europium burnups in regions 5 through 7 is a trial and error process. Equation 3.2-1 was used to obtain the burnups in the suppressor.

$$B_i = \frac{A}{N_o} = \left[\sigma_{th}^o \left(\frac{\bar{\Sigma}}{\bar{\Sigma}_o} \right)_{th} + \frac{\bar{\Sigma}_f}{\bar{\Sigma}_{th}} \left(\frac{\bar{\Sigma}}{\bar{\Sigma}_o} \right) \right] \frac{\bar{\Sigma}_{th} \Delta t}{N_o} \quad 3.2-1$$

Here the burnup is defined as the number of absorptions per original europium atom. Because of the chain of europium isotopes with high absorption cross sections it is possible to get burnups greater than one.

A burnup, B , was assumed and the average cross section ratios were taken from Figs. 3.2, 3.3, and 3.4. By using equation 3.3-1 the burnup was calculated. When the calculated burnup was equal to the assumed burnup, the average burnup in region i is then known.

The burnup in the fuel region was calculated in the same way as was done in the lifetime calculations, i. e.

$$\frac{N^{25}(t)}{N_0^{25}} = 1 - B_{25}$$

3.2.1 Relative Flux Distribution

The results from the Valprod calculations are plotted in the following figures. Figure 3.5 shows the relative thermal flux at startup with and without suppressors and at 30 per cent burnup with suppressor for the maximum absorption case (Case I). Figures 3.6 gives the relative power under the same conditions. Figures 3.7 and 3.8 show the relative thermal flux and the relative power for minimum absorption (Case II). All curves labeled 1 are at startup with suppressors. As can be seen from these curves, all distributions are lowered by the use of suppressors. The curves labeled 3 correspond to 30 per cent average fuel burnup in the core with suppressors. The fluxes and power are greater than at startup but they all show suppression due to the Eu_2O_3 . From these graphs, it can be seen that the suppressors will adequately keep the flux from peaking at the tip of the control rod fuel element.

3.2.2 Absolute Power Density Distribution

The absolute power density distribution along the axial centerline of the control rod fuel element and absorber box can be found by normal-

izing the axial power distribution from the Windowshade calculation and a power distribution from a radial Valprod calculation. The Windowshade gives the gross axial distribution and the Valprod the local effects due to the control rod. At the core centerline, $r = 0$. Therefore, the power distribution is:

$$P(0, z) = U P(z) P(r_0) \quad 3.2-2$$

Table 3.2 gives the appropriate values for equation 3.2.2 at startup.

Table 3.2

Normalization Factors For Absolute Power Density

Quantity	Startup
Total Power	10^7 watts
Reactor Volume	$1.39 \times 10^5 \text{ cm}^3$
U	71.6 watts/cm^3
$P(r)_0$	1.49
$UP(r)_0$	1.06 watts/cm^3

Curve 1 in Figure 3.9 gives the absolute power density distribution along the axial centerline for fixed fuel elements at startup, 440°F , and with equilibrium xenon. The other curves in Figure 3.9 are for Case I and Case II with a control rod element replacing the fixed fuel element with and without flux suppressors.

At startup, the use of the suppressor brings the power density below that of the distribution for fixed fuel elements. From this figure, it

is obvious that there is a need for flux suppressors and that the specified suppressor will be adequate for the job.

Figures 3.6 and 3.8 show that the suppressors are still adequate for suppressing the power near end of life (30% burnup).

3.2.3 Flux Peaking Around Water Gap

The integral flux suppressors adequately suppress the neutron flux in the tip of the control rod fuel elements but they do not suppress the flux in the water hole between the control rod fuel and the absorber. There was concern about the effect of this peaking on the adjacent fuel elements.

In ZPE I (4) axial traverses were taken 0.6 inches from the center of Rod C with both bare gold and cadmium-covered gold foils. These traverses were taken with no suppressors and with nine Haynes suppressors in the core. The bare minus the cadmium-covered flux is plotted in Fig. 3.10. Also plotted is the thermal flux in the center of the rod with europium integral flux suppressors. These results were obtained from a Valprod calculation at 440°F. As can be seen, the Haynes combs adequately depress the thermal flux in the control rod tip and considerably decrease the peak in the water gap. The internal suppressor is very effective at the fuel tip but does little in decreasing the water hole peak.

A radial traverse (4) was made 4.5 inches above the bottom of the core with the rod bank at 3.7 inches. This traverse includes the water gaps in control rods C and 2. The results of this traverse are plotted in Fig. 3.11 and the points tabulated in Table 3.3.

FIG. 3.10

FIG.3.II
THERMAL FLUX PEAKING DUE TO WATER HOLE

TRAVERSE 4.5 "
 BANK 3.7 "

Table 3.3

Radial Traverse Through Water Gaps

Radial Distance Inches	Bare Gold	Cd. Covered Gold (a)	Bare Minus Cd. Covered
0	4.972	--	--
1.5	2.344	1.285	1.059
2.93	1.976	1.245	0.731
4.3	2.218	1.197	1.021
5.86	4.152	--	--
7.25	1.703	0.907	0.796
8.80	1.109	0.699	0.410

(a) not measured - A bare gold and a cadmium-covered gold

traverse was made at 2.5 inches. The average cadmium fraction at center of the fuel elements 2.5 inches from the core bottom is 0.37. This value of the cadmium fraction was used to get the cadmium-covered activity at the center of the two fuel elements at 4.5 inches, i.e. $\text{cd. covered} = (1 - \text{cd fraction}) \times \text{bare gold}$. A smooth curve was drawn through these points and other cadmium covered activities read from this curve.

The thermal flux at the location of the radial traverse (0.8 inches above the tip of the control rod fuel) with and without suppressors can be read from Fig. 3.10. These points and the rest of the radial traverse are plotted in Fig. 3.11.

The average relative thermal flux in the fuel element with no suppressor is 0.829 and the peak to average ratio is 1.28. With integral suppressor for the calculated case of minimum absorption (fast absorption in Eu-151 and

153 only) the average is 0.826 and the peak to average reduced to 1.24; the internal flux suppressor calculations were done at 440°F. At 68°F the peak in the water hole is probably higher, however the case of minimum absorption in europium has been considered. If all the fast cross sections had been used, the peak would be lower. The important conclusion is that the integral flux suppressor decreases the flux in the adjacent elements, but only slightly.

The above traverses were made in a direction perpendicular to the fuel plates. Calculations made parallel to the fuel plate indicate that the peak to average thermal flux across the meat region is 1.2 at 440°F. A side plate peaking factor of 1.25 has been employed in APPR-1 and APPR-1a thermal analysis (21). Since the peaking due to the control rod water hole is less than this, and is not expected to reinforce it, no new problem is introduced.

4.0 Control Rods

The APPR-1 employs seven control rods. APPR-1 Core I utilizes B-10 as the absorber material. It is expected that seven rods will be inserted in APPR-1 Core II. The additional boron employed in Core II decreases the reactivity. It is the intent of the specifications that Core II be shut down with any four of the five rod bank. In Core II europium will be used as the absorbing material in place of enriched boron. At this time,* the control rods employing enriched boron have operated satisfactorily, however, the available radiation data on small samples of the boron absorber is discouraging. Upon irradiation europium forms other rare earths so the properties of the absorber matrix should not change drastically with burnout.

This chapter deals with the nuclear aspects of control rods. The comparison between experimental and analytical surveys for the determination of rod worth is given in Section 4.1. Section 4.2 discusses the change in rod worth with burnout while in Section 4.3, the rod worth required for shutdown is determined.

The specifications for the europium control rod absorber meat are as follows:

Table 4.1

Weight % Eu_2O_3 in meat	=	38%
Thickness	=	0.090 inches
Width	=	2.232 inches
Length	=	20.75 inches
Weight of Eu in meat/plate	=	151 grams

The maximum Eu content that appeared metallurgically feasible was

selected (22) since it was known that the maximum rod worth obtainable would be required to meet the stuck rod specification.

4.1 Experimental Measurements and Calculations

This section includes calculations which were made in order to try to predict the experimental results that were obtained from the extended ZPE on the APPR-1. It was felt that rod worth may be dependent upon core absorption, temperature and absorber concentration in control rod.

The rod worth is defined as

$$\rho_{\text{rod}} = \frac{K_{\text{out}} - K_{\text{in}}}{K_{\text{out}}} \quad (4.1-1)$$

This is not the same as defining the worth as the reactivity of the core with rods out minus the reactivity of the core with the rods in i. e.

$$\rho_{\text{out}} - \rho_{\text{in}} = \frac{K_{\text{out}} - K_{\text{in}}}{K_{\text{out}} K_{\text{in}}} \quad (4.1-2)$$

The experimental rod worths in APAE 8 and 21 are reported either as $\Delta K/K$ in dollars and cents or as the integral of a measured reactivity worth curve ($\Delta k_c = \int \frac{\partial \rho}{\partial x} dx$). $\Delta K/K$ was converted to absolute units by multiplying dollars by $\beta = 0.0073$. The relationship $\rho = 1 - e^{-\Delta K_e \times 0.0073}$ (23) was used to convert the integral of a worth curve to reactivity.

4.1.1 Rod Worth as a Function of Core Absorption

During the course of the Zero Power Experiments, measurements were made of the worth of the central control rod as a function of core absorption. In APAE 8 (4), Rod C was calibrated against the four-rod bank for various additions of boron steel to Core I. The results, plotted in Fig. 4.1, indicate

FIG. 4.1

¢ ROD WORTH VS. CORE ABSORPTION
(BORON-SS POISON)

FIG. 4.2

¢ ROD WORTH VS. CORE ABSORPTION
(STAINLESS STEEL POISONED)

that rod worth decreases markedly with an increase in core absorption. These measurements were made by adding boron steel strips, bringing the reactor to critical by adjusting the four-rod bank, then calibrating rod C. This means that during the measurements the bank moves from fully inserted to fully withdrawn and thus the flux distribution changes. Therefore it is possible that the observed decrease in rod worth is not due solely to change in core absorption but also to the change in flux distribution and rod-bank interaction.

In ZPE-II (11) rod C was calibrated in fully poisoned cores. In one case the core was poisoned with boron steel strips until the four rod bank was completely withdrawn. Rod C was then calibrated against the addition of more strips. The other case was performed by poisoning the core with stainless steel strips and calibrating Rod C in the same manner. The measurements give a center rod worth of 4.1% for the boron and 4.9% for the steel poisoned core. These points are plotted on Figs. 4.1 and 4.2.

In an attempt to duplicate the experimental values, analytic calculations were performed on rod worth by Scram and three group multiregion calculations. Scram (24) is a modified two group code which determines the gross rod worth of a central cylindrical control rod plus M identical rods in a ring concentric to the central rod.

The Scram program is a solution to two group diffusion theory with rods fully inserted. The model employs a boundary condition at the surface of the rod of $P'/\phi = d$ where d is the extrapolation length for thermal neutrons in a black cylinder (25). The values used for d are not a function of the core absorption and are derived for the case of a source which is at a distance from the control rod.

Dr. R. L. Murray (26) has used the P-3 approximation to transport theory to check the effects on extrapolation length of changes in the absorption to scattering ratio and of a uniform source distribution adjacent to the control cylinder. His conclusion was that the values of extrapolation length used in the program are low but that the net effect on control rod worth is less than one per cent of the rod value.

The values of reactivity obtained from the analytic calculations have to be corrected in order to compare them to the actual core. In the calculations the control rod replaced a fixed fuel element whereas in the actual core it replaces a control rod fuel element. For the central rod the difference is 67.3 cents and for the five rod bank it is 315 cents (12).

The Scram calculations give a center line rod worth of 3.8% for the fully poisoned boron steel core and 4.9% for the stainless steel poisoned core. These values are lower than the measured points but also show that the rod is worth more in a stainless steel poisoned core. The three-group theory calculation gives the worth of the rod in a core poisoned with boron steel as 4.22%.

The results of the Scram and three group calculations are plotted in Fig. 4.1 and summarized in Tables 4.2 and 4.3. Both of the calculations showed a slight decrease in rod worth with increased core absorption.

Table 4.2

Worth of Rod C in Poisoned Core - 68°F

	Measured	Scram	3 Group
Boron Steel Poisoned	4.1	3.8	---
Stainless Steel Poisoned	4.9	4.9	4.2

Dr. Murray (27) also computed the effect on rod worth of the addition of 8.2 grams of boron and found only a very small drop in rod value.

The indications are that the Scram calculations give the correct trends with approximate rod worths. From Fig. 4.1 it was concluded that for the Core II loading the rod worth will be about 99.5% of the Core I rod worth due to increased core absorption.

4.1.2 Rod Worth as a Function of Temperature

The Scram program was used in an attempt to determine if temperature has any effect on rod worth. These calculations did show a slight increase in rod worth with an increase in temperature. The results of the Scram calculations are given in Table 4.3.

Table 4.3

Rod Worth as a Function of Temperature and Increased Boron

Temperature		0 Grams	15 Grams	20 Grams
68°F	Center Rod	3.88%	3.82%	-
	5 Rod Bank	15.78%	15.61%	-
440°F	Center Rod	3.94%	3.91%	3.87%
	5 Rod Bank	15.89%	15.78%	15.63%

4.1.3 Rod Worth as a Function of Absorber Concentration in Control Rod

Measurements were made on the effect of absorber concentration and material upon the worth of a central control rod (12). The rod worth increases with an increase in B-10 concentration. The experimental values are shown in Fig. 4.3 as a function of the number of absorber atoms/rod.

FIG. 4.3

ROD WORTH AS A FUNCTION OF ABSORBER
CONCENTRATION IN CONTROL ROD

The measured value for a 28.5 volume per cent (27.1 w/o) Eu_2O_3 control rod containing 1.9×10^{24} atoms of europium is also plotted on this graph. A 38 w/o Eu_2O_3 (2.4×10^{24} atoms of Eu) rod is worth about 4% or .98 times the worth of the present rods. To check these measured values three group theory calculations were set up to determine the worth of rods containing 25 and 35 volumes per cent Eu_2O_3 in the absorber meat and the results are plotted in Figure 4.3.

The ratio between the calculated values and points on the measured curve is constant. Therefore, the change in europium rod worth with concentration is similar to the B-10 rods at least in this interval.

4.1.4 Worth of Five Rod Bank

In the Zero Power Experiments the worth of the 5 rod bank (Rods 1, 2, 3, 4, C) was determined (4, 11, 12). The probable errors associated with the period measurements on the bank are quite large, however all of the reported values are about 19% $\int$. The Core II rod worth will be about 99.5% of the Core I worth due to increased core absorption, 98% because of the change from boron to europium absorbers, and 101% because of the cold to hot change. This means the five rod bank in Core II will be worth about 18.5% at 68°F and 18.7% at 440°F.

4.1.5 Worth of Four Rod Bank

In order to determine if the core can be made subcritical with four of the five rod bank inserted it is necessary to know the worth of the four rods fully inserted. These four rods are referred to as the four rod bank. It may consist of rods 1, 2, 3, 4 (C stuck out) or 1, 2, 4, C (3 stuck out). There are several approaches to determining the worth of the four rod bank. The following was suggested by H. W. Geisler. In ZPE-II the critical positions of two four rod

bank configurations were measured. For these measurements the other three rods were withdrawn to 20.5 inches. The core was poisoned with an equivalent of 9.5 grams of B-10 which decreased the reactivity of the cold core to 11.85%. The four rod bank (Rods 1, 3, 4, C) full in would make the core subcritical by 0.8%. Using the expression $\rho = \frac{\Delta k}{k}$ for rod worth, this four rod configuration is worth 12.5%. The four rod bank, using rods 1, 2, 3, 4, shuts down the core by approximately 2.5%. Therefore this configuration has a worth of about 14.0%.

In ZPE I the stuck rod configurations were determined by withdrawing rods A and B, inserting four rods full in then noting the critical position of the last rod. The partially inserted rod was calibrated by period measurements. The results of the measurements are in Table 4.4. The worth of the four rod bank is found by assuming that the worth per inch curve is linear from the point at which the rod is calibrated to full insertion. Thus the integral of the curve ($\int \frac{\partial \rho}{\partial x} dx$) is just the area of a triangle with base of 22.0 inches minus the critical position and a height equal to the calibrated rod worth.

Table 4. 4

Stuck Rod Calibrations

Rod	Critical Position Inches	Rod Worth cents/inch	Four Rod Bank	Bank Worth % ρ
C	9.98	50.6	1, 2, 3, 4	12.7
3	9.84	50.0	1, 2, 4, C	12.8

In the ORNL critical experiments on the APPR-1 core (28) the worths of several configurations were measured. The four rod configuration 1, 3, 4, C is worth .61 times the worth of the five rod bank or 11.6% ρ and the

configuration 1, 2, 3, 4 is worth .65 of the five rod bank or 12.4% $\int$. These experiments were conducted by inserting the rods in a clean critical core then adding fuel plates to the core until criticality was again reached. The added fuel was interpreted in terms of rod worth. Thus the rods were actually calibrated in cores with different fuel loadings, none of which were exactly the same as the APPR-1 loading.

The ORNL measurements do show that the bank with rods 1, 2, 3, 4 in the core is worth more than the 1, 2, 4, C bank. This is not unexpected because with the center rod (C) stuck out there is still a symmetric bank in the core. However, with an eccentric rod stuck full out, half of the core is essentially a slab reactor with no internal control (See Fig. 1.1).

The results inferred from the zero power experiments are subject to large sources of error. In the direct calibration of the stuck rod condition in ZPE I, rods C and 3 were each calibrated at their critical positions but not calibrated as a function of withdrawal. This causes an extrapolation which could mean a large uncertainty in the total worth of the partially inserted rod. This uncertainty is probably why the ZPE I calculations do not show the expected difference between the four rod bank worths.

The ZPE II results are also subject to error because of extrapolations but have less uncertainty than the 4 rod bank worths as calculated from the ZPE I or the ORNL measurements. Thus in the APPR-1 the best estimate of the four rod bank worth is 12.5% for the 1, 3, 4, C configuration and 14.0% for the 1, 2, 3, 4 bank. These values have a probable error of $\pm 1.0\%$ $\int$.

Since the europium rods are worth about 98% of the present boron rods the Core II four rod banks at 68°F are worth about 12.2% with an eccentric rod stuck and 13.7% with the center rod stuck out.

4.2 Rod Worth Required for Shutdown

This section will discuss the shutdown margin of APPR-1 (Core II) under various configurations and operating conditions. The shutdown margin is defined as the rod worth minus the excess reactivity of the core. It is important to know this quantity if the question of whether the reactor can be shut down at any time can be answered. A positive shutdown margin means that the reactor can be shut down whereas with a negative margin it will not be possible to make the reactor subcritical under the operating conditions. However, because of the uncertainties in the calculations there is an interval between these extremes which can be considered marginal. This occurs when the shutdown margin is between plus and minus one.

4.2.1 Normal Shutdown

In Core II the maximum reactivity will occur at about 7.5 MWYRS. To evaluate the bank worth at 7.5 MWYRS and at end of life it is necessary to know the number of absorptions and the burnout in the control rods. Several methods of calculating the burnout in the absorber are presented in Appendix III. Probably the most accurate of the three methods is the excess reactivity method in which all excess neutrons ($K_{eff} - 1$) are absorbed in the control rods. Here it is assumed that all other absorptions and all leakages are treated properly. Because of the chain of europium isotopes, all of which contribute to the neutron absorption, it is not possible to calculate the decrease in the number of absorber atoms directly

from the burnout fraction. This decrease can be found from Fig. 3.3 which shows the absorber cross section, Σ/Σ_0 , as a function of burnout. Then Fig. 4.3 can be used to find the decrease in the center rod worth. This calculation is presented in Appendix III, Section V. The results are tabulated in Table 4.5.

Table 4.5
Five Rod Bank Worth - Core II

	68°F	440°F
OMWYR	17.5%	17.7%
7.5 MWYR	16.9%	17.1%
13 MWYR	16.5%	16.7%

The five rod bank worth is always greater than the reactivity in the core. Therefore, the reactor can be shut down at any time with the five rod bank. Fig. 4.4 shows the rod worth minus the reactivity in the core (shutdown margin) as a function of lifetime at 68°F. As can be seen, the shutdown margin is always positive and greater than 4%.

4.2.2 Stuckrod Condition

The stuckrod condition specification requires that the reactor can be shut down with any four of the five rod bank in and one rod stuck out. Under normal operation, the stuck rod would remain at the position it occupied during operation with the five rod bank. All bank worths and core reactivities are approximately $\pm 1\%$.

At 68°F, the four rod bank is worth 13.7 and 12.2% with rod C and rod 2 stuck out respectively. The shutdown margin with rod C full out is always positive but is less than one per cent near midlife. Here the ability to shut

FIG. 4.4
SHUTDOWN MARGIN AT 68°F

down the reactor with rods 1, 2, 3 and 4 may be considered marginal. However, it is still felt that this configuration will satisfy the stuckrod condition.

The second configuration, with an eccentric rod stuck out, is the worse case to be considered. With an eccentric rod stuck out, one-half of the core becomes essentially a slab reactor without control rods. Shutdown with an eccentric rod stuck out can be considered marginal with a good probability of satisfying the stuck rod condition between 0 and about 3 MWYR and after 9 MWYR. In these ranges, the shutdown margin is positive. As shown in Fig. 4.4, the shutdown margin becomes negative between 3 and 9 MWYRS because of the increase in core reactivity. In this interval, the shutdown of the core is definitely marginal with a good probability of not being able to shut down the reactor with an eccentric rod stuck out.

An uncertainty analysis was performed on the reactivity of Core II. If all the worse conditions would occur simultaneously, approximately 1.4% ρ would be added to the core reactivity. Under these conditions, the stuck rod condition could not be satisfied with an eccentric rod stuck out. With the central control rod stuck out, the situation would be marginal.

APPENDIX I **COMPARISON OF CORE I AND CORE II**

Table I. 1

Core Data

All constants in this appendix are for the design specified in the contract, i. e. 23.38 kgms of U-235 and 33.8 gms of B-10. This loading also reflects an 18.8 isotopic abundance B-10 in boron. Appendix IV discusses variations of this loading.

	Core I	Core II
Configuration	7 x 7 corners missing	7 x 7 corners missing
Equivalent diameter - in	22.2	22.2
Active core height - in	22.0	22.0
Number of fixed elements	38	38
Number of control rod elements	7	7
Fuel Plate dimensions		
Meat width - in		
Fixed element	2.500	2.500
Control rod element	2.281	2.281
Meat thickness - in		
Fixed element	0.020	0.020
Control rod element	0.020	0.020
Active length - in		
Fixed element	22.00	22.00
Control rod element	22.00	21.125

	Core I	Core II
Active volume per plate - in ³		
Fixed element	1. 100	1. 100
Control rod element	1. 004	0. 9637
Fuel Loading		
U-235 per plate - gms		
Fixed element	28. 62	28. 62
Control rod element	26. 11	25. 07
UO ₂ per plate - gms		
Fixed element	35. 09	35. 09
Control rod element	32. 01	30. 74
S-S per plate - gms		
Fixed element	99. 80	99. 80
Control rod element	91. 56	87. 40
B ₄ C per plate - gms		
Fixed element	0. 1880	0. 3273
Control rod element	0. 1717	0. 2868
B-10 per plate - gms		
Fixed element	0. 02480	0. 04321
Control rod element	0. 02267	0. 03786
B (natural) per plate - gms		
Fixed element	0. 1427	0. 2484
Control rod element	0. 1303	0. 2177
Total U-235 in core - Kgs	22. 5	22. 38
Total B-10 in core - gms	19. 52	33. 82

	Core I	Core II
Initial reactivities % ρ		
68°F no xenon	15.35	9.89
440°F no xenon	10.37	4.53
440°F eq. xenon	8.15	2.31
Initial bank positions in.		
68°F no xenon	3.7	6.8
440°F no xenon	6.8	11.5
440°F eq. xenon	8.3	14.4
Expected total energy release - MWYR	13	13
Thermal Flux center peak to average (startup)		
Radial	1.49	1.44
Axial	1.89	1.45
Thermal neutron flux - 440°F (neutrons/cm ² -sec)*		
0 MWYR	1.36×10^{13}	1.35×10^{13}
13 MWYR	2.06×10^{13}	1.99×10^{13}
Per cent thermal fissions (below 0.25 ev)		
0 MWYR	.751	.737
13 MWYR	.811	.807
Power peak to average (Start up, eq. xe., 440°F)		
Radial	1.49	1.49
Axial	1.71	1.45
Power peak to average (eq. xe., 440°F)	(0 MWYR)	(8.7 MWYR)
Radial	1.49	1.46
Axial	1.71	1.62

* Based on an energy release per fission of 193 Mev, hardened thermal spectrum, (Maxwellian about 0.0549 ev) and per cent thermal fissions below 0.25 ev.

	Core I	Core II
Average fuel burnup at 13 MWYR	0.28	0.28
Maximum fuel burnup at 13 MWYR	0.65	0.54
Composition of core at end of life		
Kg U-235	16.2	16.1
Original B-10 left - gms	2.6	5.0
Average atom % burnup in fuel matrix	0.78%	0.78%
Maximum atom % burnup in fuel matrix	1.81%	1.50%
Average burnup of control rod material- absorptions/original atom	37 % *	32 %
Temperature coefficient ($\Delta K/^\circ F$)		
68 $^\circ F$	-0.26×10^{-4}	-0.26×10^{-4}
440 $^\circ F$	-2.2×10^{-4}	-2.2×10^{-4}
Center line rod worth		
68 $^\circ F$	4.1%	4.0%
440 $^\circ F$	4.1%	4.0%
Five rod bank worth		
68 $^\circ F$	19.0%	18.5%
440 $^\circ F$	19.2	18.7

* Based on an average bank position of 12 inches and an average $K_{eff} = 1.072$.

Table I. 2
Initial Core Reactivities

		Core I	Core II
68⁰F	45 F E Calculated	15. 47	9. 98
	38 F E-7CRE Calculated	13. 54	8. 08
	38 F E-7CRE Measured	15. 35	9. 89 (Estimated)
440⁰F	Clean 45 F E Calculated	11. 82	5. 91
	38 F E-7 CRE Calculated	9. 84	3. 97
	38 F E-7 CRE Measured	10. 37	4. 53 (Estimated)
440⁰F	Eq. Xe 45 F E Calculated	9. 80	3. 92
	38 F E-7 CRE Calculated	7. 85	2. 01
	38 F E-7 CRE Calculated	8, 15	2. 31 (Estimated)

F E - Fixed elements

CRE - Control rod elements

Table I. 3
Initial Core Constants
440°F

	Core I	Core II
$\beta \Sigma^F$	0. 317149	0. 315183
$\Sigma_{th-core}^Q$	0. 207138	0. 220344
ρ	0. 729024	0. 717713
γ	42. 9631	42. 9162
K_F	1. 280967	1. 219928
Σ_{Xe}^0	0. 006272	0. 006272
Σ_{sub}^0	0. 006131	0. 006131
D_F	1. 491955	1. 492770
D_M	0. 245022	0. 242652
g'	0. 933348	0. 927558
K_{th}		
45 Fixed elements		
Clean	1. 53110	1. 430413
Eq. Xe	1. 486101	1. 390824
Mixed elements		
Clean	1. 487084	1. 391691
Eq. Xe	1. 444600	1. 354187
K_{eff}		
45 Fixed elements		
Clean	1. 134013	1. 062825
Eq. Xe	1. 108633	1. 040838

* Does not include Σ_{Xe}^0 or Σ_{sub}^0

Initial Core Constants (Cont'd)
440°F

		Core I	Core II
Mixed elements			
Clean		1. 109188	1. 041320
Eq. Xe	68°F	1. 085227	1. 020492
$\lambda \Sigma^F$		0. 405789	0. 402091
$\Sigma_{H_2\text{-Core}}^a$		0. 266733	0. 282408
ρ		0. 747790	0. 737526
τ		32. 3788	32. 2170
K_F		1. 286817	1. 223842
Σ_{Sub}^a		0. 007846	0. 007846
D_F		1. 321185	1. 320260
D_{th}		0. 170538	0. 168765
g'		0. 898091	0. 889906
K_{th}			
45 Fixed elements		1. 521330	1. 423794
Mixed elements		1. 477858	1. 385307
K_{eff}			
45 Fixed elements		1. 183101	1. 110916
Mixed elements		1. 156824	1. 087944

* Does not include Σ_{xe}^a or Σ_{Sub}^a

Table I. 4

Core Constants

APFR - 1

Number Densities for Muft III

Zero Burnout - Fixed Element

	440°F	68°F
	atoms/cm ³	atoms/cm ³
H	.044532 x 10 ²⁴	.053226 x 10 ²⁴
SS	.014653 x 10 ²⁴	.014653 x 10 ²⁴
O	.023211 x 10 ²⁴	.027474 x 10 ²⁴
C	.000047 x 10 ²⁴	.000047 x 10 ²⁴
B*	.000045946 x 10 ²⁴	.000045946 x 10 ²⁴
U-235	.0004242 x 10 ²⁴	.0004242 x 10 ²⁴

The only change for Core II is the number density of boron is .00080628 x 10²⁴.

* This is natural boron and not B-10 i. e. $N^{B-10} = .188N^B$

P-3 Input

APPR - 1

Fixed Elements - Zero Burnout

Dimensions:	1	=	0.038100 cm
	m	=	0.207256 cm
	p	=	3.175000 cm
	q	=	3.730625 cm

Region cross sections: 440°F

	Plate	Channel	Dead
Σ^a	1.296257	0.011052	0.057069
Σ^s	0.762252	1.728751	1.456698
Σ^{tr}	2.048792	1.258399	1.185148
$\nu\Sigma^f$	2.171908	0.000000	0.000000

Region cross sections: 68°F

	Plate	Channel	Dead
Σ^a	1.713152	0.017012	0.075372
Σ^s	0.762252	2.424332	1.926505
Σ^{tr}	2.465638	1.940469	1.660122
$\nu\Sigma^f$	2.888019	0.000000	0.000000

Relative Sources:

S active	=	0.816170
S dead	=	0.675409

For Core II the P-3 input for the plate region is the following:

	68°F	440°F
Σ^a	1.839433	1.394318
Σ^s	0.762252	0.762252
Σ^{tr}	2.591919	2.146853
$\nu\Sigma^p$	2.888018	2.171908

Table I. 5

Unshielded Microscopic Cross Sections (Barns)

	68°F (0.0331 ev)*	440°F (0.0549 ev)*
σ_{th}^{a-8}	3088.48	2398.13
σ_F^{a-8}	83.62	84.68
σ_{th}^{a-25}	512.66	385.54
σ_F^{a-25}	19.29	18.98
σ_{th}^{f-25}	434.45	326.73
σ_F^{f-25}	13.59	13.34
ν_{re}^0	2.26×10^6	1.82×10^6
$1 + \alpha_{th}$	= 1.18	
$1 + \alpha_f$	= 1.42	

Table I. 6

Reflector Properties (Pure Water)

	440°F	68°F
D_{th}	0.264886	0.172304
D_F	1.893739	1.596015
Σ_{th}^a	0.011052	0.016960
λ	47.5034	33.6945
P	0.988482	0.987116
Buckling (cm^{-2})		
	440°F	68°F
B^2 radial	.004465	.004895
B^2 axial	.002129	.002259
B^2 total	.006594	.007154

* Hardened neutron temperature

APPENDIX II

LIFETIME CALCULATIONS

The details of the lifetime calculations are in APAE Memo 126 (10). The following is a summary of that memo.

The first step in calculating the lifetime is the preparation of core constants. These constants can be obtained from the Muft III and P-3 codes described in the text; i. e. Muft III gives the fast diffusion coefficient D_f , the Fermi age τ , resonance escape probability ρ , and fast multiplication factor k_f ; P-3 calculates the thermal diffusion coefficient D_{th} , thermal diffusion length L , self-shielding factor g' , thermal absorption cross section Σ_a , and number of neutrons per fission times the fission cross section $\nu \Sigma_f$. The Muft III takes as input core atom concentrations and the P-3 takes region macroscopic cross sections. The input for these codes can be prepared by burning out the fuel and poison in the following manner:

$$N^{25}(t) = N^{25}(t_0)(1-B)$$

$$N^{B-10}(t) = N^{B-10}(t_0) [B]^g$$

$$g = \frac{\frac{\sigma_F^{a-B}}{\sigma_F^{b-15}} (1+\alpha_f)(1-\beta) + \frac{\sigma_{th}^{a-B}}{\sigma_{th}^{b-15}} (1+\alpha_{th})\beta}{(1+\alpha_f)(1-\beta) + (1+\alpha_{th})\beta}$$

$$N_{eq FP}^{B-10} = \gamma \times \frac{6.023 \times 10^{23}}{10} \times \text{Initial kg of U-235}$$

$N_{eq F.P.}^{B-10}$ is the amount of B-10 which accumulates as equivalent fission products, i. e. that amount of B-10 which would have the same thermal absorption cross section as the accumulated fission products. This includes samarium but not xenon.

γ is a conversion factor which is plotted as a function of burnout in KAPL 1501 (19) reproduced in APAE 11 (29).

The above equation for the fuel depletion defines the burnout fraction B. The boron burnout equation is derived in APAE Memo 126 (10).

The cross section for uniform xenon is

$$\Sigma_{xe}^q = \frac{\alpha \sigma_{xe}^q (\gamma_1 + \gamma_2) \frac{\delta P}{V}}{\lambda_1 + \sigma_{xe}^q \frac{\delta \beta P}{V \Sigma_o^f (1-B)}}$$

where

α	= 1.09	= Xenon nonuniform factor
σ_{xe}^0	= $1.82 \times 10^{-18} \text{ cm}^2$	= Xenon thermal absorption cross section at 0.0549 ev
γ_1	= .059	= Fractional fission yield of iodine
γ_2	= .003	= Fractional fission yield of xenon
λ_1	= $2.09 \times 10^{-5} \text{ sec}^{-1}$	= Decay constant of iodine
P	= 10^7 watts	= Reactor power
δ	= 3.24×10^{10}	Fissions/watt-sec.
V	= $1.39 \times 10^5 \text{ cm}^3$	= Core volume
B	=	Fraction burnout

If ρ is the reactivity of a core composed of all fixed elements and that of a core composed of fixed and control rod elements, the effect of substituting control rod for fixed elements can be found by

$$K_{\text{eff}} = \frac{1}{1-\rho} = K_{th} H + J = \frac{K_{th} P}{(1+L^2 B^2)(1+\tau B^2)} + \frac{K_f(1-\rho)}{(1+\tau B^2)}$$

$$K_{\text{eff}} = \frac{1}{1-\rho'} = K'_{th} H + J = \frac{K'_{th} P}{(1+L^2 B^2)(1+\tau B^2)} + \frac{K_f(1-\rho)}{1+\tau B^2}$$

where it is assumed that only K_{th} is effected by the substitution. Then

$$\frac{K_{th}}{K_{th}'} = \frac{\sum^a + \sum_{sub}^o}{\sum^a}$$

In this study it was assumed that $\sum_{sub}^a$ varies with temperature and burnout like the fuel cross section.

It is necessary to run Muft III and P-3 at only a few burnout points and then plot the core constants as functions of burnout or fit the constants with curves in the following manner:

First order polynomial - D_{th} , D_f , τ , ρ , thermal shielding factor g' and per cent thermal fissions β .

Second order polynomial - $U\Sigma^F$, $\sum_{th}^a$

Two second order polynomials - K_f (for regions of different concavity).

The bare equivalent core can now be burned out and a plot made of K_{eff} vs

MWYR.

$$K_{eff} = \frac{K_{th} P}{(1+L^2 B^2)(1+\tau B^2)} + \frac{K_f(1-P)}{(1+\tau B^2)}$$

$$MWYR = \left[\frac{\beta}{1+d_{th}} + \frac{1-\beta}{1+d_f} \right] \frac{N_0^{25} - N^{25}(t)}{1.07 \times 10^{24}}$$

$$\beta = \frac{K_{th} P}{(1+L^2 B^2)(1+\tau B^2) K_{eff}}$$

$$= \frac{K_{th} P}{K_{th} P + K_f(1-P)(1+L^2 B^2)}$$

A radial Valprod is run for the hot initial equilibrium xenon case to get the initial flux distribution. The core is divided into radial regions and the average flux per region found from

$$\bar{\Phi}'_{region} = \frac{\sum_r^r \Phi(r) r dr}{\sum_r^r r dr}$$

where Δr can be taken arbitrarily small; r is the radius at midpoint of Δr and r_1 and r_2 the limits of the region. The real average thermal flux in the core is

$$\bar{\Phi}_{real}^{th} = \frac{\delta \beta P}{V \Sigma^P}$$

and the real average thermal flux in each region

$$\bar{\Phi}_{region}^{th} = \bar{\Phi}'_{region} \frac{\bar{\Phi}_{real}^{th}}{\bar{\Phi}_{core}^{th}}$$

where the primes indicate the Valprod results.

Each region is now burned out as follows (these equations are programmed in NUB I)

$$N^{25}(t) = N^{25}(t_0) e^{-A(t-t_0)}$$

$$A = \sigma_f^a \bar{\Phi}_f + g' \sigma_{th}^a \bar{\Phi}_{th}$$

$$= \bar{\Phi}_{region}^{th} \left[\sigma_f^a \frac{\bar{\Phi}_f'}{\bar{\Phi}_{th}'} + g' \sigma_{th}^a \right]$$

$$N^{8-10}(t) = N^{8-10}(t_0) \left[\frac{N^{25}(t)}{N^{25}(t_0)} \right]^g$$

$$g = \frac{\frac{\sigma_{th}^{aB}}{\sigma_{th}^{a-25}} \beta (1+d_{th}) + \frac{\sigma_f^{a-8}}{\sigma_f^{a-25}} (1-\beta) (1+d_f)}{\beta (1+d_{th}) + (1-\beta) (1+d_f)}$$

$$\chi(t) = \frac{(\gamma_1 + \gamma_2) N^{25}(t) [\sigma_f^{f-25} \bar{\Phi}_f + g' \sigma_{th}^{f-25} \bar{\Phi}_{th}]}{\lambda_1 + \sigma_{ke}^a \bar{\Phi}_{th}}$$

For these calculations it is best to take small time intervals (of the order of a quarter of a year). From the burnout for each region, $B = 1 - e^{-A(t-t_0)}$ new region properties are found for each time step, put into Valprods and the output plotted, i. e. a graph of K_{eff} versus burnout, or MWYR, is obtained. The difference between this curve and the uniform burnout curve at any burnout point is the

radial nonuniform correction (See Fig. 2.4).

A value of Σ^P , the equivalent uniform poison which represents the rod bank, is required for the Windowshade calculations. For this calculation it is necessary to know the reactivities of the core with the rods in and with the rods out (these numbers are in the body of the report - Table 1.1). Σ^P can be found in a similar manner as Σ_{sub}^Q was calculated, i. e.

$$K_{eff}(\text{rods out}) = K'_{th} H + J$$

$$K_{eff}(\text{rods in}) = K''_{th} H + J$$

$$K''_{th} = \frac{U \Sigma_{FE}^P}{\Sigma_{FE}^Q + \Sigma_{sub}^Q + \Sigma^P}$$

For the APPR the cold to hot change in Σ^P was found by running Valprods with a central rod (water, absorbing shell, core, and water regions). These Valprods were run hot and cold, with and without rods. Thus the hot to cold change in a Σ^P representing a center rod was determined. This hot to cold change was assumed to be the same for the five rod bank.

The Windowshade will adjust the position to which Σ^P extends in the core by iterating to K_{eff} . To account for the radial burnout the input K_{eff} should be $1 + \Delta k$ where Δk is the difference between the uniform burnout and the radial nonuniformity. Also Δk may include other corrections, such as model correction.

The critical bank position is then plotted as a function of burnout or megawatt years. The Windowshade can be run with the bank out to find the axial non-uniform correction.

APPENDIX III

CALCULATION OF CONTROL ROD BURNUP

The burnup in the control rods was calculated by several methods. The results of each method are outlined in Table III. 1

Table III. 1

Control Rod Burnup

Method (Explained Below)	MWYRS	Avg. Bank Position (In)	Absorptions ($\times 10^{-24}$)	Burnup	Max. Burnup
I	13	13.35	1.27	0.318	---
	7.5	11.86	0.74	0.154	
II	13	12.91	0.92	0.217	0.586
III	13	13.35	1.10	0.275	0.526

Here the burnup is defined as the number of absorptions per original europium atom in the core i. e. $B = A/N_0$ where N_0 is the europium atomic concentration times the volume of absorber in the core at the average bank position. The burnup in the part of the rods not inserted in the core is zero.

The methods of calculating the above numbers and the change in rod worth with burnout are in the following sections.

I Excess Reactivity Method

In this approach it is assumed that all the excess neutrons ($K_{eff} - 1$) are absorbed in the control rod absorber material. This assumption implies that all other absorptions and leakages are treated properly.

$$\text{Absorptions } A = P \delta U \bar{K}_{ex} t$$

where

$$P = 10^7 \text{ watts} = \text{average power output}$$

$$\delta = 3.24 \times 10^{10} \text{ Fission/watt-sec}$$

$$\nu = 2.46 \text{ Neutrons/fission}$$

$$\bar{k}_{ex} = .039 = \text{average excess reactivity to be controlled}$$

$$t = 1.3 \text{ yr} = 4.1 \times 10^7 \text{ sec} = \text{core lifetime}$$

therefore

$$A = 1.27 \times 10^{24} \text{ absorptions}$$

The initial europium loading is 151 grams per plate. The absorber plate dimensions are

$$\text{Thickness} = .090 \text{ in.} = .229 \text{ cm}$$

$$\text{Width} = 2.232 \text{ in.} = 5.67 \text{ cm}$$

$$\text{Length} = 20.75 \text{ in.} = 52.70 \text{ cm}$$

$$\text{Volume} = 4.17 \text{ in.}^3 = 68.3 \text{ cm}^3$$

The initial europium concentration is then

$$\frac{151 \text{ gms} \times .6023 \times 10^{24}}{68.3 \text{ cm}^3 \times 152} = .00875 \times 10^{24} \text{ atoms/cm}^3$$

The rod bank will consist of 5 rods, each made of 4 plates. The total absorber volume is then $\frac{4 \text{ plates}}{\text{rod}} \times 5 \text{ rods} \times \frac{68.3 \text{ cm}^3}{\text{plate}} = 1366 \text{ cm}^3$

The average bank position in the Core II is 13.35 inches. Because of the 1.3 inch gap between the absorber and control rod fuel section, the average absorber insertion is $22 - 13.35 - 1.3 = 7.35 \text{ in.}$ The initial number of europium atoms in the core for this insertion is

$$N_0 = .00875 \times 10^{24} \frac{\text{atoms}}{\text{cm}^3} \times 1366 \times \frac{7.35}{22} \text{ cm}^3 = 3.99 \times 10^{24}$$

Thus the number of absorptions per original europium atoms

$$\frac{A}{N_0} = \frac{1.27 \times 10^{24}}{3.99 \times 10^{24}} = .318$$

It must be remembered that this is the burnout in terms of the part of the absorber which is in the core. The average burnout in terms of the total number of initial europium atoms is $\frac{A}{N_0^{total}} = \frac{1.27 \times 10^{24}}{11.94 \times 10^{24}} = .106$

II Windowshade Model

In this method the control rod burnup is calculated by finding the average thermal flux in the rodded region of the core. For this calculation only thermal absorptions are included. The calculations are as follows: (based on initial flux distribution (Fig. 2.3).)

$$\text{Average absolute thermal flux in core (averaged over time and geometry)} = \bar{\Phi}_{th} = 1.71 \times 10^{13} \text{ neut/cm}^2\text{-sec}$$

$$\text{Relative average flux in core (Windowshade)} = \bar{\Phi}_c = .7506$$

$$\text{Relative average flux in rod region (Windowshade)} = \bar{\Phi}_r = .2558$$

$$\text{Bank insertion} = 12.91 \text{ in}$$

The total number of absorptions

$$A = \bar{\Phi}_{th} \frac{\bar{\Phi}_r}{\bar{\Phi}_c} \Sigma^p V t$$

$$t = 4.1 \times 10^7 \text{ sec} = \text{Core lifetime}$$

$$\Sigma^p = 0.07755 \text{ cm}^{-1}. \text{ This } \Sigma^p \text{ is an equivalent uniform ab-}$$

sorption cross section which represents the 5 rod bank. The volume associated with this cross section is the volume of the portion of the core into which the absorber is inserted. The total core volume is $1.4 \times 10^5 \text{ cm}^3$ and the absorber is inserted 7.79 in. Thus the associated core volume is

$$V = 1.4 \times 10^5 \times \frac{7.79}{22} = 4.96 \times 10^4 \text{ cm}^3$$

Therefore

$$A = 1.71 \times 10^{13} \times \frac{.2558}{.7506} \times .07755 \times 4.96 \times 10^4 \times 4.1 \times 10^7 \\ = 0.92 \times 10^{24}$$

The initial number of europium atoms in 7.79 inches of absorber is

$$.00875 \times 10^{24} \times 1366 \times \frac{7.79}{22} = 4.23 \times 10^{24}$$

Thus the average burnup is

$$\frac{A}{N_0} = \frac{0.92 \times 10^{24}}{4.23 \times 10^{24}} = 0.217$$

This method can also be used to find the maximum burnup in the tip of the absorber section. The worst case will be for the center rod. The ratio of the flux at each rod to the core average is 1.44 for the center rod and 1.16 for the eccentric rods. Thus the number of absorptions in the center rod is

$$\frac{1.44}{1.44 + 4 \times 1.16} A = 0.237 A$$

$$= 0.218 \times 10^{24} \text{ absorptions}$$

The initial number of europium atoms in the center rod is

$$.00875 \times 10^{24} \text{ atoms/cm}^3 \times 68.3 \text{ cm}^3/\text{plate} \times 4 \text{ plates} = 2.39 \times 10^{24}$$

The average burnup per atom of absorber in the core is

$$\frac{A}{N_0} = \frac{.218 \times 10^{24}}{2.39 \times 10^{24} \times \frac{7.79}{22}} = 0.258$$

From the Windowshade the peak to average thermal flux in the rodded region of the core is $\frac{.5807}{.2558} = 2.27$ Therefore the maximum burnup in the tip of the center rod is 0.586.

III One Velocity Diffusion Theory

This method of finding the number of absorptions is presented in detail in an ORNL document CF 57 - 1 - 32 (3). The absorptions are calculated by a solution of the one velocity diffusion equation, i. e.

$$A = \frac{G(\alpha)}{4S} [\bar{\Phi}_0(0) + \bar{\Phi}_0(-S)]$$

$$G(\alpha) = G(\Sigma^a S) = \int_0^S (1 + \frac{3}{2} \alpha \mu) (1 - e^{-\alpha \mu}) \mu d\mu$$

S = Absorber thickness

$\bar{\Phi}_0(0)$ = Thermal neutron flux at the outside surface of the control rod

$\bar{\phi}_0(-s) =$ Thermal neutron flux at the inside surface of the rod

In ZPE II bare gold and cadmium covered gold traverses were made through a control rod absorber. The measurements give ratios of the thermal flux at the inside of the absorber to that at the outside of 2.7 at one side of the box and 4.6 at the other. Both of these numbers involve reading extrapolated curves. This is probably the reason for the difference. Valprod calculations on a core with central rod give the thermal flux ratio $\bar{\phi}(-s)/\bar{\phi}(0) = 2.6$

Thus

$$A = \frac{G(\alpha)}{4s} [\bar{\phi}_0(0) + 2.6\bar{\phi}_0(-s)]$$

$$= 0.9 \frac{G(\alpha)}{s} \bar{\phi}_0(0)$$

$\bar{\phi}_0(0)$ can be defined in terms of the unperturbed flux as

$$\bar{\phi}_0(0) = \frac{\bar{\phi}_\infty}{1 + \frac{G(\alpha)K_0(ka)}{4kDK_1(ka)}}$$

where

$$\bar{\phi}_\infty = 1.71 \times 10^{13} \times 1.44 = 2.46 \times 10^{13} \text{ neutrons/cm}^2\text{-sec.}$$

This is the unperturbed thermal flux at center control rod position averaged over core life.

$$a = 3.95 \text{ cm} = \text{rod radius}$$

$$k^2 = \frac{\Sigma^a}{D} = 0.96 \text{ cm}^{-1}$$

$$D = 0.2427 \text{ cm} = \text{Diffusion coefficient}$$

$$\alpha = \Sigma^a s = 3.552$$

$$G(\alpha) = \int_0^1 (1 + \frac{3}{2}\alpha u)(1 - e^{-\alpha u}) u du = 2.28$$

and K_0 and K_1 are modified Bessel functions of the second kind.

$$\therefore \bar{\phi}_0(0) = 8.0 \times 10^{12} \text{ neutrons/cm}^2\text{-sec}$$

$$A = 0.9 \frac{G(u)}{S} \bar{\Phi}(0) = 0.9 \times \frac{2.28}{.229} \times 8.0 \times 10^{12}$$

$$= 6.97 \times 10^{13} \text{ neutrons/cm}^3\text{-sec}$$

The total number of absorptions in the center rod with the absorber inserted an average of 7.35 inches into the core during the core life is then

$$A = 6.97 \times 10^{13} \frac{\text{neutrons}}{\text{cm}^3\text{-sec}} \times 4 \text{ plates} \times 68.3 \frac{\text{cm}^3}{\text{plate}} \times \frac{7.35}{22} \times 4.1 \times 10^7 \text{ sec}$$

$$= 2.61 \times 10^{23} \text{ absorptions}$$

The number of absorptions in the five rod bank is then

$$\frac{2.61 \times 10^{23}}{.237} = 1.1 \times 10^{24}$$

thus

$$\frac{A}{N_0} = \frac{1.1 \times 10^{24}}{3.99 \times 10^{24}} = 0.275$$

CF 57-1-32 also says that at the tip of the absorber the flux at the outside surface of the rod is practically unchanged but the flux at the inside surface is increased by a factor of 1.8 because of the water hole between the absorber and the control rod fuel. Thus

$$\bar{\Phi}_0(-S) = 1.8 \times 2.6 \times \bar{\Phi}_0(0)$$

and

$$A = 1.42 \frac{G(u)}{S} \bar{\Phi}_0(0)$$

The number of absorptions at the rod tip is then

$$A = 11.3 \times 10^{13} \text{ neutrons/cm}^3\text{-sec}$$

$$= 4.6 \times 10^{21} \text{ neutrons/cm}^3$$

The burnout at the tip is then

$$\frac{A}{N_0} = \frac{4.6 \times 10^{21}}{.00875 \times 10^{24}} = 0.526$$

IV. Burnup at 7.5 MWYR

The excess reactivity method will be used to calculate the number of absorptions at 7.5 MWYR because this method is felt to be the most accurate. At 7.5 MWYR

$$\begin{aligned}\bar{k}_{ex} &= 0.0391 \\ \bar{P} &= 10^7 \text{ watts} \\ t &= 2.36 \times 10^7 \text{ sec}\end{aligned}$$

and the average bank position is 11.86 inches. Thus the average length of absorber in the core is $22 - (11.86 + 1.3) = 8.84 \text{ in.}$

$$\begin{aligned}A &= \bar{P} \delta V \bar{k}_{ex} t \\ &= 10^7 \times 3.24 \times 10^{10} \times 2.46 \times .0391 \times 2.36 \times 10^7 = 0.737 \times 10^{24} \\ N_0 &= .00875 \times 10^{24} \frac{\text{atoms}}{\text{cm}^3} \times 1366 \times \frac{8.84}{22} \text{ cm}^3 = 4.8 \times 10^{24} \\ B &= \frac{A}{N_0} = \frac{.737 \times 10^{24}}{4.8 \times 10^{24}} = 0.154\end{aligned}$$

V. Decrease in Rod Worth with Burnout

The decrease in rod worth with burnout can be found by using Figures 3.2, 3.3, 3.4, and 4.1. The graphs in Section 3 show the decrease in absorption cross section (Σ/Σ_0) as a function of burnout. These curves are not linear because as europium burns out a chain of isotopes is formed, all of which contribute to the neutron absorption. In the present calculation Fig. 3.3 was used because it shows the decrease in the thermal absorption cross section when absorptions in the fast group are included. This is more realistic than considering only thermal absorptions because europium is a resonance absorber.

Fig. 4.1 shows the decrease in the center line rod worth as a function of absorber concentration. The number of absorptions in the center rod is 0.237 times

the total number of absorptions (Sec. II). However the number of atoms in the center rod is 1/5 the number in the five rod bank.

Therefore the burnout in the center rod is

$$B_d = \frac{.237 A}{.20 N_o} = 1.185 B_{avg}$$

The number of atoms in one rod is

$$.00875 \times 10^{24} \frac{\text{atom}}{\text{cm}^3} \times \frac{1366 \text{ cm}^3}{5 \text{ rods}} = 2.39 \times 10^{24}$$

Initially the center rod is worth 4.1% ρ and the 5 rod bank 17.5% at 68°F.

Thus the bank is worth 4.27 times as much as the center rod.

With the above information the following calculations were made

MWYR	Avg. Rod Burnup B_{avg}	Burnup in $\frac{1}{5}$ Rod	$\frac{\Sigma}{\Sigma_o}$ (Fig. 3.2)	# atoms left ($\frac{\Sigma}{\Sigma_o} \times N_o$)	ρ_d (Fig. 4.3.1)	5 rod bank $4.27 \times \rho_d$ (68°F)
7.5	15.4%	18.25%	.83	1.98×10^{24}	3.96%	16.9%
13.0	31.8%	37.68%	.65	1.55×10^{24}	3.87%	16.5%

These values for average burnout are those calculated by the excess reactivity method.

The change in bank worth at 68°F is - 0.6% ρ at 7.5 MWYRS and - 1.0% at end of life. The hot bank worths will then be $17.7 - 0.6 = 17.1\%$ at 7.5 MWYRS and 16.7% at 13 MWYRS.

APPENDIX IV

LOADING VARIATIONS

At an information meeting held at Oak Ridge (31) it was learned that the boron in Core II will not be in B_4C as in Core I because of the problem of diffusion of boron into the clad. The suggested method of employing boron was to use boron highly enriched in B-10 and included in a master alloy. Because of the master alloy, the net amount of matrix steel in the core would have to be increased. Also, it was learned that the nominal plate thickness in the APPR-1 is 29.5 mils rather than the 30 mils used in the calculations. The matrix steel content in Core II will be increased so the plates will roll to a nominal thickness of 30 mils.

Subsequent to the above meeting, plates containing the master alloy were rolled at Oak Ridge. Analysis of these plates indicated large boron losses (32) during the sintering of the fuel compact. The use of the master alloy would require an extensive investigation of the boron losses. Two alternate programs were suggested: (1) employ boron carbide containing boron enriched in B-10 and (2) build Core II with the same boron loading as that used in Core I.

The following questions are discussed in this appendix:

- A. What is the effect on the calculated core reactivity of a decrease of 0.5 mils in the plate thickness?
- B. If the steel content is increased to meet the specified nominal plate dimensions, how much will the boron loading have to be decreased to keep from changing the initial reactivity?
- C. What will the loading be if the master alloy is used?
- D. What will the loading be if enriched boron in B_4C is used?

E. What will the loading be if the Core I boron content is used but the amount of steel is changed to maintain nominal plate thickness?

(A) The largest effect on core reactivity caused by a .5 mil decrease in plate thickness will be due to the increased water content of the core. The added water volume is

$$5 \times 10^{-4} \text{ in} \times 2.78 \text{ in} \times 22 \text{ in} \times 810 \text{ plates} \times (2.54 \text{ cm/in})^3 = 406 \text{ cm}^3$$

The total water volume in the core (1) with 30 mil plates is 111330 cm^3 and $111330 + 406 = 111736 \text{ cm}^3$ with 29.5 mil plates. To account for this change, the Muft III water density was increased by the ratio of $\frac{111736}{111330}$. No other core concentrations were altered, i. e. it was assumed that the total amount of steel in the core did not change.

The P-3 input cross sections are cm^2 per cm^3 of region. The three regions are plate, channel, and dead edge. By decreasing the plate thickness the plate volume is decreased, but the material content is not changed. Thus, the plate cross sections were increased to a value $\frac{29.5}{30}$ times the value for the 30 mil plates. The channel is the water region in the gap between the fuel regions of the plates. This volume is increased but also the amount of water in the gap is increased so the region cross sections in the channel do not change. The dead edge consists of the dead ends of the fuel plates, the side plates and the water in the gap between the dead ends of the plates. The volume of this region is not changed and neither is the number of steel atoms. However, the amount of water in the region is increased. With the 30 mil plates the water gap is 0.133193 in. With 29.5 mil plates this gap is increased to 0.133693 inches. The water cross sections in the dead region were then increased to a value $\frac{.133693}{.133193}$ times the values for the 30

mil plate.

The P-3 input also includes source terms for the active and the dead regions. The active region is made up of the plate and channel regions discussed above. The source term is defined as the ratio of the water volume per region to the total region value. The total region volumes do not change with change in plate thickness, but the amount of water increases. The water volume increases by the same ratio as water gap increases. Thus, the source terms were multiplied by the ratio $\frac{.133693}{.133193}$.

The only dimensional change is 1, the plate half thickness. The P-3 input for a core with 29.5 mil plates is listed below

Table IV. 1

P-3 Input 440°F

29.5 Mil plates - Fixed elements

Dimensions.

l	=	0.037465
m	=	0.207256
p	=	3.175
q	=	3.730625

Region cross sections

	Plate	Channel	Dead
Σ^0	1.3182270	0.011052	0.057097
Σ^s	0.775171	1.728751	1.461080
Σ^{+r}	2.083517	1.258399	1.188338
$\nu \Sigma^f$	2.208714	0.000000	0.000000

Relative Sources

$$S_{\text{act}} = 0.819233$$

$$S_{\text{dead}} = 0.677944$$

The calculated reactivity of the APPR-1 with 29.5 mil plates at 440°F, equilibrium xenon is 7.9615. This is an increase of 0.1075% ρ over the core with 30 mil plates. A + 0.3% ρ correction was applied to the calculated reactivities to account for the difference between the calculated and the measured cores. Since the correction due to thickness of the plate is within this uncertainty, no further corrections were made.

(B) In order to maintain a nominal thickness of 30 mils, it is necessary to increase the steel loading in the meat. Correspondingly, the boron loading was decreased so that the initial calculated K_{eff} remained at 1.02.

In the APPR-1 there are 99.8 grams of matrix steel per fixed element plate or 1.796 kilograms per fixed element. The total volume occupied by the clad, dead edges and side plates is 308.277 cm³ (1). This steel has a density of 7.9 gms/cm³. Thus, there are 2.435 kilograms of clad, dead region and side plate steel. Also, there are 32.84 grams of steel per element in the braze, making a total of 4.263 kgm of steel per fixed element. An addition of 5 grams per plate corresponds to 90 grams per fixed element. To account for an addition of 5 grams of steel per plate, the Muft II steel input was multiplied by the ratio $\frac{4.263 + .090}{4.263}$.

To handle the input for the P-3, the steel must be broken down into its constituents because a different type of steel is used in the meat from that used in the clad. Table IV.2 lists the cross sections and the atomic concentrations per plate region for the APPR-1 at 440°F.

Table IV. 2
Plate Cross Sections - APPR-1 440°F
(30 Mil Plates)

	$\sum^s$ $\frac{\text{cm}^2}{\text{cm}^3}$	$\sum^a$ $\frac{\text{cm}^2}{\text{cm}^3}$	$\sum^{tr}$ $\frac{\text{cm}^2}{\text{cm}^3}$	N^* (in meat) $\frac{\text{atoms}}{\text{cm}^3} \times 10^{-24}$	N^* (in clad) $\frac{\text{atoms}}{\text{cm}^3} \times 10^{-24}$
U-235	.024402	1.045341	1.069672	.002711	-----
U-238	.001791	.000327	.002113	.000199	-----
O	.022963	-----	.022009	.006043	-----
B-10	.000243	.132377	.132605	.000055	-----
B-11	.001049	-----	.000984	.000238	-----
C	.001291	.000001	.001220	.000232**	.000037
Mn	.002691	.007326	.009984	.000344	.000577
Si	.005495	.000190	.005556	.001877	.000565
Ni	.096845	.015224	.110968	.003102	.002432
Cr	.058201	.023474	.080926	.007741	.005794
P	.000112	.000004	.000114	.000013	.000020
S	.000022	.000006	.000027	.000005	.000015
Fe	.547147	.071987	.612614	.029011	.019567
Total	.762252	1.296257	2.048792		

* Atoms per cm^3 of plate region

** Includes $.000084 \times 10^{24}$ atom/ cm^3 in B_4C

To calculate the effect of adding steel, the cross section for each constituent in the steel in the meat must be found, increased, then added to the cross section for that constituent in the clad. For example, the manganese scattering cross section in the meat is $.002691 \times \frac{.00344}{.00344 + .000577} = .001005 \text{ cm}^{-1}$

To account for the steel addition, this is increased to $.001005 \frac{104.8}{99.8} = .001055 \text{ cm}^{-1}$

The total manganese scattering cross section for the plate region with the addition of 5 grams of steel is then

$$s = .001055 + .002691 \times \frac{.000577}{.000344 + .000577} = .002741 \text{ cm}^{-1}$$

ZPE experiments (11) and analysis (1) were used to get an estimate of the amount of boron which should be removed to account for the addition of 5 grams of steel per plate (an addition of 3.91 kgs of steel for a core of 38 fixed and 7 control rod elements). APPR-1 has a cold clean reactivity of 15.35%. Full poisoning of this core requires 200 kgs of stainless steel or 220 grams of natural boron. Of the steel poisoning, 59.5% is due to the steel itself and the rest due to water displacement. Of the boron poisoning 88% is due to the boron. Thus

$$1 \text{ kgs SS} = \frac{-15.35 \times .88}{220} = -0.0614\%$$

An addition of 3.91 kgs of steel is worth -0.178% or 2.92 grams of natural boron. The initial specifications for Core II called for 33.8 grams of B-10. At an isotopic abundance 18.8% this is $\frac{33.8}{.188} \text{ gms} \frac{10.82}{10.02} = 194.14 \text{ gms natural boron}$. To account for a decrease of 2.96 grams of boron, all boron cross sections and atomic concentrations would be multiplied by the ratio $\frac{191.18}{194.14}$.

Several cases of increased steel and decreased boron were calculated. The boron loading was adjusted until the calculated initial K_{eff} was 1.0205. Fig. IV.1

FIG. IV.1
GRAMS OF BORON VS. STAINLESS STEEL
IN CORE II ($K_{eff} = 1.0205$)

is a plot of grams of natural boron per plate as a function of grams of steel per plate.

Subsequent to the information meeting, ORNL rolled several test plates with the following results (phone call between J. G. Gallagher and C. F. Leitten on May 22, 1958).

1. Several plates have been fabricated containing 18.454 grams of master alloy and 81.346 grams of matrix steel. This master alloy contains 0.26 w/o boron enriched to 92.075 w/o B-10. The plates all roll to 29 mil thickness with core lengths of 21-18/32", 21-13/32" and 21-12/32". If the nominal length is obtained, the plate thickness would be reduced to 28 mils.

2. Plates were then rolled containing 18.454 grams of master alloy and 86.5 grams of matrix steel. This resulted in a plate 30 mils thick with a meat section 2-1/2" wide and 21-3/4" long. This is an increase of 5.15 grams of steel over that employed in the meat of the APPR-1 fuel plates.

The effect of adding 5.15 grams of steel can be read directly from Fig. IV. 1. The final loading is 104.95 grams of steel per plate and 0.2465 grams of natural boron or 0.64460 grams of B-10.

(C) In the above reactivity calculations boron was calculated as natural boron and an isotopic abundance of 18.8 atom per cent (17.39 w/o) used to find the amount of B-10. Core II may employ boron highly enriched in the B-10 isotope and contained in a master alloy. Thus it is necessary to know accurately the amount of B-10 in the loading. Mr. Leitten performed a check of the isotopic abundance of B-10 in boron in the APPR-1. The results of three measurements are as follows:

Table IV. 3

B-10 Isotopic Abundance

	1	2	3
B-11	80.47	80.48	80.41
B-10	19.53	19.51	19.59

The original APPR-1 boron loadings (2) and values corrected to 19.54 atom per cent are in Table IV. 4.

Table IV. 4

APPR-1 Core I Boron Loadings (Fixed Elements)

Atom % B-10	18.8	19.54
B ₄ C - gms per plate (2)	0.188	0.188
w/o natural boron in B ₄ C (2)	75.9	75.9
Natural boron - gms per plate	0.1427	0.1427
w/o B-10 in boron	17.39	18.09
B-10 - gms per plate	0.02482	0.02581
Boron atomic weight	10.825	10.818
B-10 atomic weight (30)	10.016	10.016
B-11 atomic weight (30)	11.013	11.013

In core II the boron will be in a master alloy. This alloy contains 0.26 w/o boron enriched to 92.075 w/o B-10 (92.74 atom % B-10). The loading of natural boron calculated after making an adjustment for an increased steel loading is 0.2465 grams per fixed element plate. The increase in steel is necessary to maintain nominal plate dimensions with the master alloy.

The final loading specifications for the APPR-1 Core II if it employs master alloy are as follows:

Gms. of B-10 per fixed element plate	0.04460
Gms. of master alloy per fixed element plate	18.629
Gms. of matrix steel per fixed element plate	86.325
Total gms. of steel per fixed element plate	104.954

(D) The B-10 loading specified for Core II is 0.04460 grams per fixed element plate with 104.95 grams of matrix steel. Boron is available enriched to 38 atom (32) per cent (35.79 w/o) B-10 in natural boron. The B_4C in Core I contains 75.9 w/o natural boron. The final boron loading for Core II if 38% enriched boron is used is listed below.

Atom per cent B-10 in boron	38
Grams of B-10 per fixed element plate	0.0446
Grams of natural boron per fixed element plate	0.1246
Grams of B_4C per fixed element plate	0.1642

(E) If the Core II loading is a repeat of Core I with the exception of the flux suppressors, control rods, and steel loading the Core II loading is as follows:*

U-235 per plate - gms

Fixed element	28.62
Control rod element	25.07

UO₂ per plate - gms

Fixed element	35.09
Control rod element	30.74

* Based on the assumption that 104.95 grams of matrix steel will result in plates of nominal dimensions.

Stainless steel per plate - gms

Fixed element 104.95

Control rod element 91.94

B₄C per plate - gms

Fixed element 0.1880

Control rod element 0.1647

B-10 per plate - gms

Fixed element 0.02581

Control rod element 0.02261

B (natural) per plate - gms

Fixed element 0.1427

Control rod element 0.1250

The addition of 5.15 grams of steel over that in Core I will decrease the initial reactivity by about 0.25% ρ and cause a loss in the core lifetime of about 1/4 of a megawatt year.

NOMENCLATURE

B	Fraction burnout
$B^2 \text{ Cm}^{-2}$	Buckling
$D_f \text{ Cm}$	Fast diffusion coefficient
$D_{th} \text{ Cm}$	Thermal diffusion coefficient
g'	Thermal shielding factor
K_f	Fast multiplication factor
K_{th}	Thermal multiplication factor
$L \text{ cm}$	Diffusion length
$N \text{ atoms/cm}^3$	Atom concentrations
p	Resonance escape probability
$P \text{ MW}$	Reactor power
$P(r, z)$	Power Distribution
$t \text{ Sec}$	Time
u	Lethargy
$V \text{ cm}^3$	Core volume
α	Xenon nonuniform factor
α_f	Fast U-235 capture to fission ratio
α_{th}	Thermal U-235 capture to fission ratio
β	Fraction thermal fissions
γ_1	Iodine fraction fission yield
γ_2	Xenon fraction fission yield
δ	Conversions from fissions to watt-seconds

NOMENCLATURE (Cont'd)

λ	sec ⁻¹	Decay constant	
Δt	sec	Time interval	
ν		neutrons per fission	
ρ		Per cent reactivity	$\frac{K_{eff} - 1}{K_{eff}}$
σ	cm ²	Microscopic cross section	
Σ	cm ⁻¹	Macroscopic cross section	
τ	cm ²	Fermi age	
Φ	neuts/cm ² -sec	Neutron flux	

Subscripts

th	Thermal
f	Fast

Superscripts

a	Absorption
f	fission
tr	transport
B	boron-10
25	U-235
xe	xenon
151	europium 151
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