**ANALYSIS
OF
EXTENDED ZERO POWER EXPERIMENTS
ON THE
ARMY PACKAGE POWER REACTOR
ZPE-2**

Contract No. AT(30-3)-278

May 7, 1958

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INTRODUCTION

This report is principally concerned with analysis of measurements taken on the APPR-1 core during the course of the extended Zero Power Experiments (ZPE-2). The bulk of these measurements are reported in APAE No. 21 ⁽¹⁾. There are some additional measurements reported in APAE Memo 115 ⁽²⁾. In addition to the analysis of the ZPE-2 data some re-evaluation has been made of a few of the results obtained from the first set of Zero Power Experiments (ZPE-1). The ZPE-1 measurements are reported in APAE No. 8 ⁽³⁾.

During the course of the analysis work it became apparent that a considerable amount of basic experimental data had been taken on the APPR-1 core. It seemed worthwhile to organize this report in such a fashion that other investigators could make maximum use of this data. It provides an excellent opportunity for individuals and groups interested in basic reactor analysis problems to check calculational techniques.

An attempt has been made to include all of the fundamental information concerning the material content and geometry of the APPR-1. This material is included in the Appendices. In addition, cross-section files and group constants have been listed rather extensively in order that other investigators could compare the results presented in this report with those obtained by other techniques. Listings are generally given to six decimal places although not all of these places are significant. This was done so as to avoid round off error.

ACKNOWLEDGEMENTS

The analysis of the ZPE-2 data has been performed under the direction and supervision of Mr. J. G. Gallagher. Assistance has been received on various analysis problems from Messrs. T. G. Williams, M. J. Leibson, R. C. DeYoung, P. E. Bobe and E. M. Reiback. Messrs. J. W. Noaks, H. W. Giesler and S. D. MacKay have contributed greatly by discussion of experimental techniques and methods of interpretation. Mr. A. N. Crouch has been particularly helpful with many of the computations and with checking the final proof of this document. We are indebted to Dr. R. L. Murray, North Carolina State College, for preparation of the multi-group cross-sections and for helpful discussions on reactor analysis techniques. Capt. R. L. Harris, Project Officer for this Contract, has rendered considerable assistance in guidance of this project.

SUMMARY AND CONCLUSIONS

In analyzing the ZPE-2 data it was necessary to work with a comparatively simple calculational model. Except for one multi-group calculation, the bulk of the analysis described in this report is based on the modified two-group model. Analysis has been limited to separate, axial and radial, one-dimensional calculations.

In spite of the limitations inherent in the calculation techniques, good results were obtained for most of the analysis. There are some analytical results which are not in good agreement with experimental data. These are discussed in particular sections of the report. The following conclusions can be made.

1. A calculation model based on microscopic nuclear data and modified 2-group theory gave results within 2% reactivity for the majority of the experimental configurations. The modified 2-group theory employs fast constants prepared by the WAPD Muft-III code and thermal constants prepared by a P_3 code written for plate geometry.
2. The calculated thermal neutron flux distribution based on the P_3 approximation is in good agreement with the measured distribution.
3. Calculated percent thermal fissions in the clean core is in good agreement with the measured value.
4. The variation of reflector savings with core size is very small both on the basis of measurements and calculations.
5. The calculated effect of replacing a fixed fuel element by a control rod fuel element and basket in the centerline position compares favorably with measured values. The substitution effect at off center positions varies as J_0^2 .

6. Both calculation and measurements (which do not utilize the bank for calibration) indicate the variation in rod worth is only a slowly varying function of core absorption.

7. Measurements at the tip of the control rod fuel element indicate that peaking in this region is not a strong function of absorber fuel element insertion and that local flux suppression must be employed in this type of rod.

8. Measurements with non-uniform boron additions indicate that significant reductions in maximum to average power generation can be obtained by poisoning either the lower half of the core or the central fuel elements.

9. Measurements indicated that asymmetric rod programming increases the maximum to average power generation as compared to five rod bank control. Symmetric rod programming decreases the maximum to average power generation as compared to bank control. The effect of rod programming on core life cannot be established by the measurements.

10. Measurements of the temperature coefficient for various clean and poisoned configurations indicated only a slight variation of temperature coefficient with method of control, that is, uniform poison addition versus control rod bank. Comparatively poor agreement was observed between calculations and measurements.

11. The steel reflector experiments indicated that solid steel reflectors could make significant contributions to core reactivity. The effect of coolant passages on the reflectors causes a reduction in this reactivity increase. The steel reflectors placed adjacent to the core resulted in an increased radial maximum average power generation. The measurements indicated that the optimum gap between a laminated steel water reflector and the core is about one

to two inches. The calculations of the reactivity effect of the solid and laminated reflectors gave agreement with the trends in the measurements. The magnitudes were in only fair agreement, this due in part to interpretation of the experiment.

12. The measurements of the fast and thermal neutron flux from the reactor centerline through the pressure vessel and into the steel shield rings was in good agreement with those calculated by means of modified 2-group theory.

13. The measured gamma attenuations through the thermal shield, pressure vessel and shielding ring mockup was in agreement with the calculated attenuation.

14. The measurements indicated that there was no significant difference in gamma dose at the pressure vessel with either a laminated or solid steel reflector adjacent to the core. In all cases, measured dose at the pressure vessel was less than that with only water present in the reflector.

ANALYSIS

Introduction

Fig. 1.1 shows a cross section of the APPR-1 core, pressure vessel and shield rings. The normal loading consists of 38 fixed fuel elements and 7 control rod fuel elements. Each fixed element contains 515 grams U-235 and 0.4468 grams B-10. Each control rod has a lower section containing 418 grams U-235 and 0.3627 grams B-10. It also has an upper section consisting of four absorber plates in the form of a square box which is filled with water. Fig. 1.2 shows the control rod positions relative to that of the fixed fuel elements.

A 45 element core has an excess reactivity of about 15 per cent at 68°F. This situation permitted considerable flexibility in the ZPE-2 experiments since various measurements could be taken on a number of clean and poisoned cores of different size and shape. The calculation model used in the nuclear analysis of these cores is described in the sections which follow.

FIGURE 1.1 REACTOR CORE CROSS SECTION

FIGURE 1.2 CONTROL ROD POSITIONS

1.0 Computational Model

Previous calculations ⁽⁴⁾ ⁽⁵⁾ for the APPR-1 have employed semi-empirical methods based on modified two-group theory. The discrepancy between predicted and measured reactivity in the APPR-1, ⁽⁶⁾ together with the necessity of placing the analysis of stainless reactors on firmer theoretical grounds have necessitated a revision of the analytical approach. An effort has been made to employ methods developed principally at Westinghouse Atomic Power Division (WAPD) and the Knolls Atomic Power Laboratory (KAPL) to the analysis of stainless steel-water systems. The approach has been to base calculations on fundamental nuclear data rather than upon an empirical adjustment of parameters to fit experiment.

The starting point for all the analysis methods was the energy dependent Boltzmann equation ⁽⁷⁾.

$$\vec{\Omega} \cdot \nabla \Phi(\vec{r}, u, \vec{\Omega}) + \Sigma(u) \Phi(\vec{r}, u, \vec{\Omega}) = S(\vec{r}, u, \vec{\Omega}) + \int_{u'} \int_{\vec{\Omega}'} \Phi(\vec{r}, u', \vec{\Omega}') \Sigma^s(u') f(u', u, \vec{\Omega}', \vec{\Omega}) d\vec{\Omega}'$$

where

$\vec{\Omega}$ = solid angle

$\vec{r}$ = position vector

u = lethargy = $\ln E_0/E$ where E is energy

$\Phi(\vec{r}, u, \vec{\Omega})$ = neutron flux

$\Sigma(u)$ = total cross section

$S(\vec{r}, u, \vec{\Omega})$ = neutron source

$\Sigma^s(u')$ = scattering cross-section

$f(u', u, \vec{\Omega}', \vec{\Omega})$ = scattering kernel

This equation has been solved using two approaches. The first method involved a solution using the multigroup approximation.⁽⁸⁾ The second method used the two-group approximation in which fast group constants were determined by taking the Fourier transform of the slowing down distribution from a point source.⁽⁹⁾ Each of these methods is described in subsequent sections.

Both methods have the following in common.

1. A hardened thermal neutron temperature was used to evaluate thermal cross-sections.
2. Relative thermal fluxes were calculated by one velocity transport theory. (P-3 approximation)
3. Inelastic scattering was based on the Weisskopf statistical model.
4. The slowing down process was described by the Goertzel-Selengut approximation.

1.1 Multigroup Method

The age diffusion approximation with the Selengut-Goertzel term for scattering by hydrogen is given by the following set of equations⁽⁸⁾

$$-\nabla \cdot \frac{1}{3\Sigma^s(\vec{r}, u)} \nabla \Phi(\vec{r}, u) + [\Sigma^a(\vec{r}, u) + \Sigma^f(\vec{r}, u)] \Phi(\vec{r}, u) = -\frac{\partial q(\vec{r}, u)}{\partial u} - \frac{\partial \eta(\vec{r}, u)}{\partial u} + \chi(u) P(\vec{r}) + Q(\vec{r}, u), \quad 0 < u < u_T$$

$$q(\vec{r}, u) = \int \Sigma^s(\vec{r}, u) \Phi(\vec{r}, u)$$

$$\frac{\partial \eta(\vec{r}, u)}{\partial u} = \Sigma^H(\vec{r}, u) \Phi(\vec{r}, u) - \eta(\vec{r}, u)$$

$$\Phi(\vec{r}, 0) = \eta(\vec{r}, 0) = q(\vec{r}, 0) = 0$$

$$-\nabla \cdot \frac{\nabla \Phi(\vec{r}, u_T)}{\Sigma^t(\vec{r}, u_T)} + \Sigma^a(\vec{r}, u_T) \Phi(\vec{r}, u_T) \\ = \lim_{u \rightarrow u_T} [\eta(\vec{r}, u) + \chi(\vec{r}, u)] + \chi(u_T) \rho(\vec{r}) + Q(\vec{r}, u_T)$$

$$\rho(\vec{r}) = \int_0^{u_T} \nu \Sigma^f(\vec{r}, u) \Phi(\vec{r}, u) du + \nu \Sigma^f(\vec{r}, u_T) \Phi(\vec{r}, u_T)$$

$$Q(\vec{r}) = \int_0^u \sum_m \chi_m(u' \rightarrow u) \Sigma_m^n(\vec{r}, u) \Phi(\vec{r}, u') du'$$

where

$\rho(\vec{r})$ = fission source

$Q(\vec{r})$ = inelastic scattering source

u_T = $\ln E_0/E_t$ where E_t is thermal energy

$\Phi(\vec{r}, u)$ = neutron flux

$\Sigma^t(\vec{r}, u)$ = macroscopic transport cross-section

$\Sigma^a(\vec{r}, u)$ = macroscopic absorption cross-section

$\eta(\vec{r}, u)$ = slowing down density due to elastic scattering by materials other than hydrogen.

$\Sigma^H(\vec{r}, u)$ = macroscopic hydrogen scattering cross-section

$\eta(\vec{r}, u)$ = slowing down density due to scattering by hydrogen

$\chi(u)$ = fission spectrum normalized such that

$$\int_0^{u_T} \chi(u) du + \chi(u_T) = 1$$

ν = number of neutrons emitted per fission

$\Sigma^f(\vec{r}, u)$ = macroscopic fission cross-section

$\chi_m(u' \rightarrow u)$ = distribution function for inelastic scattering by material m at lethargy u' such that

$$\int_{u'}^{u_T} \chi_m(u' \rightarrow u) du + \chi_m(u' \rightarrow u_T) = 1$$

$\Sigma_m^n(\vec{r}, u)$ = macroscopic inelastic scattering cross-section for material m .

These equations were solved by dividing the lethargy range $0 \leq u \leq u_T$ into 13 groups and assuming that the flux and cross-sections do not change radically over any of them. In this approach it is always necessary to postulate an additional relation between the average slowing down density in each group and the slowing down density at the bottom of the lethargy group⁽¹⁰⁾. In the KAPL approach this is taken as

$$g_i(\vec{r}) = f_i(\vec{r}) g(\vec{r}, u_i)$$

The spatial integration was handled by the usual finite difference equations after reducing the problem to one dimension by introducing the perpendicular buckling.

The multigroup approach was applied only to the clean APPR-1 at 68°F because of the computation time required on an IBM-650. This calculation affords a comparison with the two-group approach which is described subsequently. The 13 group cross-sections used in the calculations are given in Appendix A.

1.2 Two-Group Method

The two-group equations which include absorption and fission in both groups are

$$\begin{aligned}-D_f \nabla^2 \Phi_f + \Sigma^r_f \Phi_f &= \nu (\Sigma^f_f \Phi_f + \Sigma^f_t \Phi_t) \\ -D_t \nabla^2 \Phi_t + \Sigma^a_t \Phi_t &= \rho \Sigma^r_f \Phi_f\end{aligned}$$

where

- ν = neutrons released per fission
- D_f = fast diffusion coefficient
- D_t = thermal diffusion coefficient
- Φ_f = fast flux
- Φ_t = thermal flux
- Σ^r_f = fast removal cross-section
- Σ^a_t = thermal absorption cross-section
- Σ^f_f = fast fission cross-section
- Σ^f_t = thermal fission cross-section
- ρ = resonance escape probability

The principal problem in two-group theory is the preparation of fast group constants. Fortunately, there is available a machine code for computing few group constants based on solving the Fourier transform of the slowing down distribution from a point source ⁽⁹⁾. The equations are solved within four approximations P-1, B-1, P-1 Selengut-Goertzel, and B-1 Selengut-Goertzel. The P-1 Selengut-Goertzel approximation was used extensively because it was known to give the best agreement with the age in water ⁽¹¹⁾. The equations solved are

$$[\Sigma^g(u) + \Sigma^h(u)] F_0(u) - iBF_1(u) = f_0(u) - \frac{\partial \eta}{\partial u} - \frac{\partial g}{\partial u} + \omega(u) + \Sigma^a(u) F_1(u) - \frac{iB}{3} F_0(u)$$

where $\frac{\partial \eta}{\partial u} = -\eta(u) + \Sigma^{SH} F_0(u)$

$$g(u) = \sum_i \xi^i(u) \Sigma^{Si}(u) F_0(u)$$

$$\gamma = 1 = \frac{\alpha^2 \tan^{-1} \alpha}{3(\alpha - \tan^{-1} \alpha)}$$

where $\alpha = \frac{B}{\Sigma(u)}$

$$\eta(u) = \int_0^u du' \Sigma^{SH}(u') e^{-(u-u')} F_0(u')$$

$$\xi(u) = \int_0^u du' \Sigma^{SH}(u') e^{-\frac{3}{2}(u-u')} F_1(u')$$

$$\omega(u) = \int_0^u du' \Sigma^{in}(u \rightarrow u') F_0(u')$$

Where symbols not previously defined are

$F_0(u)$ = Fourier transform of flux

$F_1(u)$ = Fourier transform of current

B^2 = buckling

$f_0(u)$ = fission spectrum

In the WAPD approach the average slowing down density in each group is related to the slowing down density at the top of the group by

$$\bar{g}_i = \frac{g_{i-1} + g_i}{2}$$

It is possible to use this approximation when a large number of groups are employed.

The code ⁽⁹⁾ computes the Fourier transform of the slowing down density and then prepares few-group summations and the second moment. Microscopic cross-section data, atomic concentrations, and the buckling are the only input required for the calculation. The 59 group cross-sections used in the Muft III Calculations are shown in Appendix A.

1.3 Thermal Group Cutoff

A problem common to all the calculation procedures is that of transition energy between the fast and thermal group. In all the analyses of this report the lower limit of the fast group (or groups) is taken as 0.196 ev (at 68°F). This compares with 0.625 ev used at WAPD ⁽⁹⁾. Fortunately, the cutoff energy has little effect on K_{eff} as shown in Section 2.4.4.

1.4 Thermal Group

Intercell measurements made in the APPR-1 critical experiment ⁽¹²⁾ and the APPR-1 Zero Power Experiment ZPE-1 ⁽³⁾ indicate a significant variation of thermal flux within the fuel element. Thermal neutron traverses perpendicular to fuel plates indicates a rise in flux between plates. In addition, traverses parallel to fuel plates indicates a flux peak in the vicinity of the side plates. An accurate treatment of these flux variations is necessary if the thermal utilization is to be calculated accurately.

1.4.1 Neutron Temperature

The absorption cross section of the APPR-1 is large compared to the scattering cross section. This is known to harden the spectrum, or shift the spectrum to higher energy as a result of the preferential absorption of low energy neutrons. This problem has been treated by defining an effective neutron temperature which is higher than the physical temperature ⁽¹³⁾ by an amount which depends on the ratio of absorption to scattering cross section in the assembly. The energy distribution of the thermal neutron flux was assumed to be a Maxwell-Boltzmann distribution about the effective neutron temperature.

A more accurate approach actually would be to calculate the hardened spectrum using methods developed by Wigner-Wilkins and others. ^{(14) (15) (16) (17)}

The important question in regard to the neutron temperature problem is its effect on core reactivity. The magnitude of this effect is seen by the fact that in an assembly with about the same $\Sigma^a / \xi \Sigma^s$ ratio as the APPR-1, the effect of averaging the cross sections over a Wigner-Wilkins spectrum rather than over a Maxwellian about the moderator temperature was of the order of 0.3% increase in K_{eff} ⁽¹⁸⁾. Since the neutron temperature concept at least gives a first order estimate of the effect of neutron hardening, it would be expected that the actual error involved in this approximation should be small.

Effective neutron temperature was calculated by the following equation ⁽¹³⁾

$$\frac{T_N}{T_0} = 1 + C \frac{\Sigma^a}{\xi \Sigma^s}$$

where

T_0 = temperature of assembly

T_w = effective neutron temperature

Σ^a = macroscopic absorption cross section at $E_0 = kT_0$

Σ^s = macroscopic scattering cross section

C = a constant = 0.91

1.4.2 Flux Distribution

The variation of intercell flux both perpendicular and parallel to the fuel plates was calculated using a one velocity P_3 approximation to the Boltzmann equation.

The calculation procedure was basically a superposition of several P_3 approximation solutions to the one dimensional single velocity transport equation. Murray gives a good description of the one dimensional problem, ⁽¹⁹⁾

The equations used for each region were ⁽⁷⁾

$$\frac{d\bar{\Phi}_1}{dz} + \Sigma^a \bar{\Phi}_1 = 5$$

$$2 \frac{d\bar{\Phi}_2}{dz} + \frac{d\bar{\Phi}_1}{dz} + 3 \Sigma^t \bar{\Phi}_1 = 0$$

$$3 \frac{d\bar{\Phi}_3}{dz} + 2 \frac{d\bar{\Phi}_2}{dz} + 5 \Sigma \bar{\Phi}_2 = 0$$

$$3 \frac{d\bar{\Phi}_4}{dz} + 7 \Sigma \bar{\Phi}_3 = 0$$

where

Σ = macroscopic total cross-section

Σ^a = macroscopic absorption cross-section

Σ^t = macroscopic transport cross section

$\bar{\Phi}_i$ = coefficients of the expansion of the neutron flux in a series of Legendre Polynomials

Φ_0 = corresponds to the total flux

Φ_1 = corresponds to the current

S = neutron source

At symmetric boundaries the odd harmonics Φ_1 and Φ_3 must be zero and the slopes of the even harmonics Φ_0 and Φ_2 must be zero. At the interface between regions the Φ_s must be equal. The source in any region was assumed proportional the volume fraction of water in that region since the principal slowing down material is hydrogen.

1.4.2.1 Fuel Element Geometry

The geometry and dimensions used in the intercell calculations are given in Appendix B. A schematic diagram of a fixed fuel element is given in Fig. 1.3. The following nomenclature is used for various regions.

Plate: This includes the meat and clad, but does not include the dead edges. Properties for this region are calculated as if the clad and meat are homogeneously mixed. The plate half-thickness is l ; its half-width is p .

Channel: This includes material between fuel plates. The half-width of this region is p , the same as for the fuel plates. Its half-thickness is $(m-l)$. The channel may contain water plus other material such as steel strips. Properties for this region are calculated as if the water and other material are homogeneously mixed.

Active: This is a mixed region consisting of the fuel plates and the channel. Its half-width is p . Properties for this region are obtained in Part I of the calculation.

FIGURE 1.3
ONE QUARTER OF A FIXED ELEMENT

Dead: This region includes the fuel plate dead edges, side plates, braze, and all of the water in the fuel element that is not in the channels. Its half-width is (q-p). Its properties are calculated as if all materials in the region are homogeneously distributed.

Fuel Element Cell: This includes all regions. It is made up of the active plus the dead regions.

1.4.2.2 Calculation Procedure

The two-dimensional problem is broken down into five parts.

In Part I the flux distribution normal to the fuel plates is calculated. Fluxes are normalized such that

$$\bar{\Phi}'_{active} = 1 = \frac{L \bar{\Phi}'_{plate} + (m-L) \bar{\Phi}'_{channel}}{m}$$

Average cross-sections are calculated for the active region. These cross-sections are used for later parts of the calculation

$$\Sigma^x_{active} = \frac{L \Sigma^x_{plate} \bar{\Phi}'_{plate} + (m-L) \Sigma^x_{channel} \bar{\Phi}'_{channel}}{m}$$

This is done for $\Sigma^x = \Sigma^s, \Sigma^a, \nu \Sigma^f$, and Σ^{cr} respectively.

In Part II, a uniform source of 1 neutron per $\text{cm}^3 \text{ sec}$ is placed in the dead region. The flux distribution in the active and dead regions is calculated.

In Part III, a uniform source of 1 neutron per $\text{cm}^3 \text{ sec}$ is placed in the active region. The flux distribution in the active and dead regions is calculated.

In Part IV, the solution obtained in Parts II and III are weighted and superimposed to obtain the overall flux distribution parallel to the fuel plates. Weighting is necessary because the source strength in the active and dead regions is not the same. The relative source strength in each of these regions is assumed proportional to the volume fraction of water.

In Part V, the flux distributions normal to the fuel plates and parallel to the fuel plates are used to compute nuclear parameters for the whole fuel element. The solutions from Parts I and IV are used to calculate these constants.

Fluxes are normalized such that

$$\bar{\Phi}_{core} = 1 = \frac{p \bar{\Phi}_{active} + (q-p) \bar{\Phi}_{dead}}{q}$$

Average cross sections for the core are calculated

$$\Sigma_{core}^x = \frac{p \Sigma_{active}^x \bar{\Phi}_{active} + (q-p) \Sigma_{dead}^x \bar{\Phi}_{dead}}{q}$$

This is done for $\Sigma^x = \Sigma^a, \Sigma^s, \nu \Sigma^f, \Sigma^t$ respectively.

$K_{th}, D,$ and L^2 are calculated by the equations

$$K_{th} = \nu \Sigma_{core}^f / \Sigma_{core}^a$$

$$D_{core} = 1 / 3 \Sigma_{core}^t$$

$$L_{core}^2 = D_{core} / \Sigma_{core}^a$$

1.5 Geometry Considerations

1.5.1 Multi-region One-Dimensional Calculations

A majority of the reactivity calculations were based on one-dimensional multi-region computations using finite difference approximations. These calculations can be conveniently performed with an available IBM-650 Code (Valprod). (20)

Each of the various arrays of elements was approximated by a cylinder except for a few cases in which perfectly square arrays had been measured. The geometry and dimensions of the configurations used in the calculations are given in Appendix C. The height of the cylinder was accounted for by a perpendicular buckling.

1.5.2 Analytical Two-Region, Two-Group Calculations

An analytical solution of the two-group, two-region equations for a finite cylindrical core has been programmed for the IBM-650. ⁽²¹⁾ The problem is first solved radially with an assumed axial savings and then axially with the computed radial savings. The process is repeated until satisfactory convergence is obtained.

There are two limitations on this method. First, a finite radial reflector thickness must be used. This limitation is not serious since experience has shown that there is no significant variation of reflector savings for water reflector thicknesses greater than 30 cm.

The second limitation probably introduces significant errors. The same reflector properties must be used for the top and bottom reflector. Measurements on the APPR-1 core indicate that the control rod absorber sections in the upper reflector reduce the reflector savings in this region.

A comparison of the reactivity calculated by this method with that obtained by use of the Valprod code and experimental axial savings is given in Section 3.1.4

1.5.3 Equivalent Bare Core

The two-region problem can be reduced to an equivalent one-region problem by use of reflector savings. The reflector savings were computed by least squared

fitting of experimental flux traverses measured with bare and cadmium covered foils.

The experimental data were fitted by an equation of the form

$$\Phi(r) = AF(\mu, r)$$

where

$\Phi(r)$ = activity at r

$F(\mu, r)$ = function to be fitted

A = multiplying constant

μ = argument constant

The best values of A and μ were determined by least squared analysis using $\sum_i [\phi_i - AF(\mu, r_i)]^2 = \text{a minimum.}$

The radial distributions were fitted by

$$F(\mu, r) = J_0(\mu, r)$$

where

$$\mu = \frac{2.405}{a + S_r}$$

a = core radius

S_r = radial reflector savings.

The axial distributions were fitted by

$$F(\mu, r) = \cos \mu r$$

where

$$\mu = \frac{\pi}{2} \frac{1}{b + S_e}$$

b = core half height

S_e = axial reflector savings (one end)

1.5.4 Fixed and Control Rod Fuel Elements

The APPR-1 consists of both fixed and control rod fuel elements. The basic cell dimensions are the same for both elements but the fuel, boron, and steel content are different although the fuel to boron ratio is the same. The treatment of the APPR-1 is complicated by the distribution of these control rod fuel elements.

Measurements have been made to determine the increase in reactivity obtained by substituting a fixed fuel element for a control rod fuel element. From these substitution measurements the measured reactivity for a core consisting of a mixture of fixed and control rod fuel elements was corrected to yield an experimental value for the reactivity of a core containing only fixed fuel elements. With this correction a valid comparison could be made between the "measured" reactivity (corrected for control rod fuel elements) and the calculated reactivity for a fixed element core.

In Section 4.2 a procedure is outlined for calculating the increase in reactivity due to substitution of fixed elements for control rod elements. This involved making a three-region calculation of the reactivity of a fixed element array except for one control rod fuel element at the center of the core (Rod C). This value was compared with the reactivity of a fixed element core in order to determine the substitution worth of the central fuel element. The substitution worth of the other elements could then be found by a $J_0^2(\mu r)$ weighing factor.

1.6 Control Rods

The worth of control rods in the APPR-1 has been calculated by several methods. The most common procedure uses two-group theory with special boundary conditions at the core rod interface (22) (23) (24). The theory employs

an effective extrapolation length defined by the equations

$$d_s = \frac{\bar{\Phi}_s(a)}{d\bar{\Phi}_s(a)/dz}$$

$$d_s = \frac{\Phi_s(a)}{d\Phi_s(a)/dz}$$

where

a = radius of rod

d_s = extrapolation lengths for the thermal group

d_f = extrapolation length for the fast group

The fast extrapolation length is given by

$$d_f = \frac{D_{fc} I_0(\gamma_{fr} a)}{D_{fr} I_1(\gamma_{fr} a)}$$

where

$$\gamma_{fr} = \left(\frac{1}{\tau_r}\right) + \left(\frac{\pi}{\tilde{H}}\right)^2$$

τ_r = age in rod

$\tilde{H}$ = equivalent bare height of core

The thermal extrapolation length is based on the solution of the transport equation for a black medium adjacent to a scattering medium (25). These calculations were performed using an IBM-650 Code. (Scram I) (26)

An attempt was made to calculate the variation of rod worth with core absorption by treating the case of the centerline rod fully inserted. The absorbing shell and the control rod water hole are two additional regions which are incorporated into the multi-region, two-group calculation.

2.0 Clean Core Calculations

The calculation of the various configurations employing the clean APPR-1 fuel elements is of fundamental importance to the understanding of the APPR-1 core at Ft. Belvoir, Va.* The analytical methods discussed in Chapter 1.0 have been applied.

2.1 Thermal Group Properties

2.1.1 Neutron Temperature

The thermal neutron temperature existing in the clean fixed fuel elements was found using the atom densities given in Appendix B and the microscopic cross-sections at 0.025 ev. The result of this calculation for the fixed fuel element positions (38 out of 45 elements) is as follows for saturated water at 68°F.

$$\frac{T}{T_0} = 1 + 0.91 \frac{\sum a}{\sum s}$$

$$\frac{T}{T_0} = 1 + 0.91 \times \frac{0.3869 \text{ cm}^{-1}}{1.0794 \text{ cm}^{-1}} = 1.326$$

or $E_N = 0.025 (1.326) = 0.0331 \text{ ev}$

It should be noted that the cross sections used in computing the neutron temperature were based on a uniform mixture of materials. The minor effect of neutron temperature on K_{eff} does not warrant repeating the neutron temperature evaluation after the intercell distribution is computed.

2.1.2 Flux Distribution Normal to Fuel Plates

Table 2.1 shows the input data used for this P_3 calculation. Macroscopic cross-sections for these regions were calculated from the atomic concentrations given in Appendix B and the microscopic cross-sections given in Appendix A. Output data from this calculation are also listed in Table 2.1

* The core used in ZPE-2 (and ZPE-1)⁽¹⁰⁾ is identical with that presently being burned out at Ft. Belvoir.

Table 2.1 Flux Distribution Normal to Fuel Plates

Input Data

Cross-Sections, cm^{-1}	Plate	Channel
Σ^a	1.713152	0.016960
Σ^s	0.762252	2.419949
Σ^{tr}	2.465638	1.934561
$\nu\Sigma^f$	2.888019	0.000000

1/2 plate thickness, $\ell = 0.038100$ cm

1/2 plate to plate spacing, $m = 0.207256$ cm

Output Data

Average Fluxes (normalized to $\bar{\Phi}'_{\text{active}} = 1$)

$\bar{\Phi}'_{\text{plate}} = 0.950739$

$\bar{\Phi}'_{\text{channel}} = 1.011095$

Active Region Cross-Sections cm^{-1}

$$\Sigma^a = 0.313412$$

$$\Sigma^s = 2.130225$$

$$\Sigma^{tr} = 2.027380$$

$$\nu\Sigma^f = 0.504753$$

Fig. 2.1 shows the flux distribution obtained from the P_3 calculation and from the measured bare minus cadmium covered gold activity. Fluxes have been normalized to unity at the surface of the plate for this comparison. The excellent agreement is probably fortuitous in view of the difficulty of obtaining good experimental measurements and in view of the limitations inherent in the P_3 approximation.

A higher order approximation, such as a P_5 calculation would tend to increase the thermal flux peaking in the water channel. ⁽²⁶⁾

2.1.3 Flux Distribution Parallel to Fuel Plates

Table 2.2 shows the input data used for this P_3 calculation. Macroscopic cross-sections for the dead region were calculated from the atomic concentrations given in Appendix B and the microscopic cross-sections given in Appendix A. Output data from this calculation are listed in Tables 2.2 and 2.3.

The calculated flux distribution is shown on Fig. 2.2 together with the distribution obtained from the measured bare minus cadmium covered gold activity. Fluxes have been normalized to unity at the center of the element. Agreement between calculations and measurements is within the uncertainty in the experimental measurements.

Table 2.2 Flux Distribution Parallel to Fuel Plates

Input Data

Cross-Sections cm^{-1}	Active Region	Dead Region
Σ^a	0.313412	0.075337
Σ^s	2.130225	1.921506
Σ^t	2.027380	1.656121
$\nu\Sigma^f$	0.504753	0.000000
Relative Volumetric Source		
(Volume Fraction Water)	0.816170	0.675409
1/2 Thickness of active region	3.175000 cm	
1/2 Thickness of fuel element	3.730625 cm	

FIGURE 2.1
THERMAL FLUX TRAVERSE PERPENDICULAR TO FUEL PLATES
APPR-1 68° F

FIGURE 2.2
THERMAL FLUX TRAVERSE
PARALLEL TO FUEL PLATE
APPR-1 68° F

Output Data

Average Fluxes (normalized to $\bar{\Phi}_{\text{core}} = 1$)

$$\bar{\Phi}_{\text{active}} = 0.944624$$

$$\bar{\Phi}_{\text{dead}} = 1.316436$$

Table 2.3 Thermal Group Constants for Fixed Fuel Elements

$$\Sigma^a = 0.266734$$

$$\Sigma^s = 2.089303$$

$$\Sigma^c = 1.954589$$

$$1/2\Sigma^r = 0.405789$$

$$K_m = 1.521326$$

$$D_g = 0.170539$$

$$L^2 = 0.639360$$

2.2 Fast Group Properties

The fast group constants in the two-group formulation were calculated for the fixed element composition using the Goertzel-Selengut, P-1 slowing down approximation. The atomic concentrations and bucklings used are given in Table 2.4 along with the results of the Muft III calculation.

Table 2.4 - Fixed Element Fast Group Properties

(G-S, P-1 Approximation)

Atomic Concentrations	(atoms/cm ³ x 10 ⁻²⁴)
U ²³⁵	0.0004242
B	0.000045947
H	0.053064
O	0.027477
C	0.000047
SS	0.014653

Fast Group Constants	$B^2=0.007154 \text{ cm}^{-2}$ $B = 0.08458 \text{ cm}^{-1}$	$B^2=0.011209 \text{ cm}^{-2}$ $B = 0.10587 \text{ cm}^{-1}$	$B^2=0.013394 \text{ cm}^{-2}$ $B = 0.11573 \text{ cm}^{-1}$
$D_f \text{ cm}$	1.321185	1.321585	1.322159
$\Sigma_f^f \text{ cm}^{-1}$	0.040804	0.040153	0.039800
$\Sigma_a^f \text{ cm}^{-1}$	0.010291	0.010175	0.010112
$\lambda \Sigma_a^f \text{ cm}^{-1}$	0.013343	0.013079	0.012990
ρ	0.747790	0.746601	0.745937
$\tau \text{ cm}^2$	32.378844	32.913631	33.219709
K_f	1.286817	1.285405	1.284612

There is a problem with regard to the correct interpretation of the group constants with buckling when using the Muft III calculation. ^(X7) The problem arises from the fact that the neutron energy spectrum is slightly space dependent and changes with a change in buckling. The geometrical buckling of the configurations of interest varied from 0.007154 cm^{-2} to 0.013394 cm^{-2} . Table 2.4 lists group constants for this range of buckling.

2.3 Water Reflector Properties

The reflector properties were based on water at 68°F , saturated. The thermal group assumed an unhardened Maxwell-Boltzmann distribution.

There was some problem involved in choosing a proper value of the buckling for Muft III input. For a slab of large thickness, the geometric buckling is zero. However, the Muft III Code will give zero values for the fast diffusion coefficient and age if a value of $B = 0$ is used. Consequently, a very small buckling ($B = 0.001 \text{ cm}^{-1}$ or $B^2 = 0.000001 \text{ cm}^{-2}$) was used as input. Properties of the reflector are given in Table 2.5.

Table 2.5 Water Reflector Properties

Atomic Concentrations (atoms/cm³) x 10⁻²⁴

$$H = 0.066730$$

$$O = 0.033365$$

Buckling, cm⁻¹

$$B = 0.001$$

Group Properties

Fast (10 Mev-0.196 ev) (G-S, P-1)

$$D_f = 1.671769 \text{ cm}$$

$$\Sigma_f = 0.047814 \text{ cm}^{-1}$$

$$\Sigma_{af} = 0.00691 \text{ cm}^{-1}$$

$$\tau = 34.963812 \text{ cm}^2$$

$$k = 0.985556$$

Thermal (0.196 ev - 0.0 ev)

$$D = 0.1434 \text{ cm}$$

$$\Sigma_a = 0.01947 \text{ cm}^{-1}$$

$$L^2 = 7.365 \text{ cm}^2$$

2.4 Reactivity vs Core Size

Experiments were performed on a 5 x 5 and a 7 x 7 array (corners missing). In each experiment there were 7 control rod fuel elements. The excess reactivity in these cores was determined from bank calibrations. The effect of substituting fixed fuel elements for control rod elements in these cores was also measured. This permits an estimate to be made of the "measured" reactivity of a core containing all fixed fuel elements. The method used for estimating "measured" reactivity is shown in Appendix C.

In a special experiment,⁽²⁾ a 4 x 4 array of fifteen fixed fuel elements plus one control rod element in an outside row was made critical. This experiment provides one of the best checks on the calculation methods since there is minimum

excess reactivity in the core and since a minimum correction must be made for the substitution effect.

A comparison of the "measured" reactivity and the reactivity calculated by several different methods is described in the sections which follow.

2.4.1 Equivalent Bare versus Two-Region Calculations

Table 2.6 lists the "measured" reactivity for three clean core arrays. The reactivity as calculated by the equivalent bare core model and by a two region Valprod calculation is also shown. The Goertzel-Selengut, P-1 slowing down approximation was used for both calculations. Group properties used in the calculations are given in Tables 2.3, 2.4, and 2.5.

Table 2.6 Reactivity of Fixed Element Cores

Array	Type	$B^2_{\perp}$ (*)	B^2_{tot} (**)	Calculated $\% \rho$		Measured $\% \rho$
				Two Region	Equivalent Bare	
4 x 4	Slab	0.007636	0.013203	0.74	1.01	2.67
5 x 5	Cylinder	0.002259	0.011209	5.80	5.82	7.66
7 x 7	Cylinder	0.002259	0.007154	15.47	15.47	17.22

(*) Used for two region calculations

(**) Used for equivalent bare core calculations

Both calculation methods underestimate reactivity. The equivalent bare core calculations give slightly better agreement with the measured reactivity than do the two-region calculations but the differences cannot be considered very significant.

Figure 2.3 shows a plot of reactivity versus total buckling. This curve is useful for interpolation since equivalent bare core reactivities can be found without running additional Muft III calculations.

2.4.2 Two-Group vs. Four-Group Calculations

A four-group, equivalent bare core calculation was made for the 7 x 7 array. In both cases fast group constants were based on the Goertzel-Selengut, P-1 approximation. The four-group calculation gave $\rho = 14.91\%$ as compared with the two-group value of 15.47% and the "measured" value of 17.22%

This result is somewhat consistent with the work of other investigators who found that 4-group calculations gave about 2% lower reactivity than two-group calculations.⁽¹⁸⁾ However, in their work measured reactivities were about 3% lower than two-group calculations and about 1% lower than four-group calculations.

2.4.3 Effect of Slowing Down Approximation

Equivalent bare core reactivities of the 7 x 7 and 5 x 5 arrays were calculated using each of the slowing down approximations available with the Muft III code. These results are shown in Table 2.7

Table 2.7 Effect of Slowing Down Approximation

Array	No. Elements	Measured ρ	P-1	B-1	SG, P-1	SG, B-1
7 x 7	45	17.01	17.01	17.53	15.47	16.04
5 x 5	21	7.66	8.09	9.09	5.82	7.13

$$\% \Delta \rho = 9.35 \quad \% \Delta \rho = 8.92 \quad \% \Delta \rho = 3.44 \quad \% \Delta \rho = 9.65 \quad \% \Delta \rho = 8.91$$

The P-1 approximation gives the best check with the measured reactivity. However, the Goertzel-Selengut, P-1 approximation gives the best estimates of the reactivity change between the 7 x 7 and 5 x 5 arrays.

2.4.4 Effect of Thermal Cut-off Energy

In connection with burnout calculations being made on the APPR-1 core, a

calculation was made to determine the effect of thermal cut-off energy on reactivity. Increasing the thermal cut-off energy from 0.196 ev to 0.625 e.v. decreased the reactivity by only 0.07%⁽²⁸⁾.

2.5 Critical Water Height

In order to eliminate the problem of estimating the reactivity of the arrays containing a large number of fuel elements, critical water heights were determined. In calculating these assemblies total axial reflector savings must be estimated in order to determine the axial buckling. Two approaches were used to estimate axial reflector savings:

1. Interpretation of variation of water worth with critical water height.
2. Least squared fit of an axial flux traverse.

2.5.1 Variation of Water Worth with Critical Water Height

The equivalent bare core K_{eff} equation can be approximated by the expression,⁽²⁹⁾

$$K_{eff} = \frac{k_{\infty}}{1 + M^2 B^2}$$

where k_{∞} = infinite multiplication constant

M^2 = migration area

The water worth at the critical water height is

$$\frac{\Delta \rho}{\Delta h} = \frac{2\pi^2 M^2}{k_{\infty}} \frac{1}{(h + S_z)^3}$$

where h = critical water height

S_z = total axial reflector savings

If the experimental measurements of core height are plotted versus $\left(\frac{\Delta h}{\Delta \rho}\right)^{1/3}$, the line best fitting the data will have an intercept, $-S_z$, and a slope equal to $\left(\frac{2\pi^2 M^2}{k_{\infty}}\right)^{1/3}$. Figure 2.4 shows such a plot. This yields a value of 11.7 cm for S_z and 19.07 cm² for $\frac{M^2}{k_{\infty}}$.

The value of $\frac{M^2}{k_\infty}$ was also estimated from the group constants given in Tables 2.3 and 2.4 (the fast group properties vary slightly with buckling. A value for $B^2 = 0.013394 \text{ cm}^{-2}$ was used since this corresponds rather closely to the buckling in the critical water height experiments.)

$$M^2 = \tau + L^2 = 33.86 \text{ cm}^2$$

$$k_\infty = K_{th} p + K_f (1-p) = 1.461$$

$$\frac{M^2}{k_\infty} = 23.2$$

2.5.2 Axial Flux Traverse

A least squared cosine fit was performed on the axial traverse taken in a fixed element with bare and cadmium covered gold foils. The experimental points from 4.5 to 8.5 in. above the bottom of the core were used in the fitting. This yielded a value of 2.95 cm for the air-fuel element end. Using a value of 6.59 cm for the bottom reflector, the sum of the savings is 9.54 cm by this method.

2.5.3 Comparison of Calculated and Measured Reactivity

It was felt that the method employing water worth measurements gave a better estimate of the total axial savings. A total savings of 11.70 cm was used in calculating the experimental buckling. Table 2.8 shows a comparison of the "measured" and calculated reactivities. "Measured" reactivities are calculated in Appendix D. Bucklings for these experiments are tabulated in Appendix C. Calculated reactivities were found from the graph shown on Fig. 2.3 and are based on the equivalent bare model.

Table 2.8 Reactivity for Critical Water Height Experiments

All fixed element cores					
Array	No. Elements	Critical Water Height, in.	B^2 Total	Measured % ρ	Calculated % ρ
5 x 5	21	14.20	0.013293	2.82	0.84
5 x 5	25	12.41	0.013332	2.72	0.74
6 x 6	32	10.12	0.013507	2.57	0.32
7 x 7	44	8.82	0.013476	2.30	0.39
7 x 7	45	8.64	0.013611	2.28	0.06

2.5.4 Interpretation of Critical Water Height Measurements

It appears that the calculated value for $\frac{M^2}{k_\infty}$ is somewhat high. Since the value of k_∞ is not very sensitive to the slowing down model used, it seems that the value for M^2 as calculated by the Goertzel-Selengut approximation is a little high. This result is consistent with the fact that calculated reactivities are lower than measured values by about 2%.

Further calculations were made on the APPR-1 at 440°F in connection with another study.⁽²⁸⁾ These calculations showed that at 440°F the Goertzel-Selengut, P-1 approximation gave the best estimate of core reactivity. All of the approximations underestimated the hot to cold change, with the Goertzel-Selengut, P-1 approximation giving the least error.

It is evident that none of the slowing down approximations can be said to give consistently better agreement with measurements than do the other approximations. The Goertzel-Selengut slowing down approximation was selected for all further Muft III calculations because it appeared to give the best estimates of reactivity changes.

2.6 Multigroup Calculation

A multigroup calculation was performed for the 7 x 7 array of fixed elements. This calculation used the group cross-sections and inelastic scattering matrices given in Appendix A. The Goertzel-Selengut, P-1 slowing down approximation was used. The thermal group cross-sections were adjusted until the calculated K_{th} checked that given in Table 2.3. This accounted for the intercell variation of the thermal flux. The atomic concentrations are given in Table 2.4. A radial two region calculation was performed with a perpendicular buckling of 0.002259 cm^{-2} .

Table 2.9 gives the calculated and experimental reactivity.

Table 2.9 Reactivity for 7 x 7 Array, All Fixed Elements

	% Reactivity
Multigroup, P-1 GS, Two-Region	15.23
Two-Group, P-1 GS, Two-Region	15.47
Experimental	17.22

There is a difference of only 0.24% reactivity between the multigroup and two-group calculation. The agreement is probably fortuitous in view of the fact that different cross-section data was used in the two methods. In addition, some differences would be expected due to the different methods of averaging the slowing down density in each group and due to the different number of fast groups used in the multigroup and Muft III calculations.

Considering all of the limitations inherent in the calculational models and the uncertainties in basic cross-section data, it can be concluded that the difference of about 2% between calculated and measured reactivities is as good agreement as can be expected.

2.7 Flux as a Function of Lethargy

The variation of flux with lethargy was obtained from both the multigroup and Muft III calculations. The fluxes from these calculations were normalized to the actual thermal flux. In modified two-group theory, the coupling coefficient for the fundamental mode is,

$$S_1 = \frac{\Phi}{\Phi_f} = \frac{\rho \Sigma^f}{DB^2 + \Sigma^a}$$

The thermal flux can be calculated from,

$$\Phi = 3.24 \times 10^{10} \frac{\text{fissions}}{\text{wall sec}} \times \frac{\beta P}{V \Sigma^f}$$

where P = reactor power, 10^7 watts

V = core volume, $1.39 \times 10^5 \text{ cm}^3$

Σ^f = thermal fission cross section, cm^{-1}

β = fraction thermal fissions

The fraction of thermal fissions can be calculated from modified two-group theory,

$$\beta = \frac{\rho K_m}{\rho K_m + (1 - \rho) K_f (1 + L^2 B^2)}$$

The normalizing constant for the fluxes is the ratio of the actual thermal flux to the thermal flux found from the coupling coefficient. Figure 2.5 shows the normalized flux for both the Muft III and the multigroup calculations. Both calculations give nearly the same distribution.

The cadmium fraction at the center of the core for indium and gold foils was computed using the fluxes shown on Fig. 2.5 and the cross-sections given in Appendix A. The cadmium fraction is defined as the ratio of bare foil activity

minus cadmium covered foil activity to bare foil activity. The cross sections for the resonance groups included a self-shielding factor to account for depression of the resonance flux within the foil. Table 2.10 compares the calculated and experimental values.

Table 2.10 Calculated and Experimental Cadmium Fractions

Dectector	Calculated	Experimental
Gold	0.296	0.359 - 0.412
Indium	0.222	0.443 - 0.469

The fission distribution in lethargy is shown in Fig. 2.6 for both the multi-group and Muft III calculations. The cutoff energy for 0.020 inch cadmium covered foils is 0.414 e.v.⁽³⁰⁾ The calculated fraction of fissions below a cadmium cutoff of 0.414 e.v. is 0.80. The experimental values reported are 0.801 at the center line and 0.840, 5.81 inches from the center line.

There is fairly good agreement between the calculated and measured values of the fraction of fissions below cadmium cutoff. However, the calculated cadmium fractions for gold and indium activation are much lower than the measured values. This indicates that additional self-shielding corrections should be made for the resonance flux in the vicinity of the foil.⁽²⁶⁾

2.8 Importance Function

Reactivity coefficients were measured in symmetric elements 23, 36, 52, and 65 for a number of materials. (The element number designations are shown on Fig. 6.1) The measurements are listed in Table 2.11.

Table 2.11 - Reactivity Coefficient of Various Materials.

Elements 23, 36, 52, 65.

Material	Length in.	Density gm/cm ³	Reactivity Coefficient		
			Σ^a cm ⁻¹	cents/gm	cents/cm ³
A1	23	2.742	0.0108	-0.0210	-0.0527
SS	23	7.765	0.191	-0.0169	-0.1315
1.01% B-SS	22	7.57	2.648	-0.144	-1.092
1.16% B-SS	22	7.81	3.113	-0.166	-1.298

Notes - (Measured values for 23 in. strips were increased by 4.55%. This puts all reactivity coefficients on same basis - cents/(gm) or (cm³) in core.)

(Σ^a evaluated from number density and thermal cross-sections at 0.0331 e.v.)

The reactivity change for each of these materials includes the loss due to water displacement and the loss due to absorption in the material. Fig. 2.7 shows a plot of reactivity coefficient (cents/cm³) versus Σ^a . By extrapolating to $\Sigma^a=0$, the reactivity loss caused by water displacement was found to be 0.0533 cents/cm³. Using the stainless steel measurement, it is possible to break down the reactivity loss of the boron strips into contributions due to water displacement, stainless steel poisoning, and boron poisoning. These breakdowns are shown in Table 2.12.

FIGURE 2.7
REACTIVITY COEFFICIENT VS $\sum^a$
ELEMENTS 23, 36, 52, 65

Table 2.12 Reactivity Coefficient Breakdown

(Elements 23, 36, 52, and 65)

Reactivity Coefficients (cents/cm³)

Material	Total	Water Displacement	SS Poison	Other Poison
A1	0.0577	0.0533 (92.4%)	-----	0.0044 (7.6%)
SS	0.1315	0.0533 (40.5%)	0.0782 (59.5%)	-----
1.01% B-SS	1.092	0.0533 (4.9%)	0.0774 (7.1%)	0.961 (88.0%)
1.16% B-SS	1.298	0.0533 (4.1%)	0.0773 (6.0%)	1.167 (89.5%)

Reactivity coefficients were also measured for aluminum and for uranium - aluminum alloy in several different elements. The first measurement (ρ_1) gave the increase in reactivity due to the addition of 133.63 gm A1 + 30.54 gm U (28.57 gm U-235). This material was arranged to cover the 22 inch active length of the fuel element and displaced 49.6 cm³ water. The second measurement (ρ_2) gave the decrease in reactivity due to the addition of 129.2 gm A1. This material was in the form of 23 inch length strips and displaced 47.11 cm³ water. In this second measurement there were 123.6 gm A1 in the core and 45.06 cm³ water was displaced from the core.

From Table 2.12 it can be seen that the reactivity loss of an aluminum strip is 7.6% due to aluminum poisoning and 92.4% due to water displacement. This permits a breakdown of the U-A1 measurements.

$$\begin{aligned}
 \text{Water Displacement (cents/gm)} &= \frac{0.924}{45.06} \rho_2 = 0.0205 \rho_2 \\
 \text{Aluminum Poison (cents/gm)} &= \frac{0.076}{123.6} \rho_2 = 0.000615 \rho_2 \\
 \text{U-235 Addition (cents/gm)} &= \frac{\rho_1 - 49.6 (0.0205 \rho_2) - 133.63 (0.000615 \rho_2)}{28.57} \\
 &= \frac{\rho_1 - 1.099 \rho_2}{28.57}
 \end{aligned}$$

The reactivity coefficient breakdown is shown in Table 2.13 and on Fig. 2.8

Table 2.13 Reactivity Coefficient Breakdown vs Position

Element	Reactivity Coefficients (cents/gm)				
	ρ_i (cents)	ρ_z (cents)	Water Displacement	Alum. Poison	U-235 Addition
34	+3.339	-2.556	-0.052	-0.0016	+0.215
35	+3.617	-3.573	-0.073	-0.0022	+0.264
25	+6.945	-2.872	-0.059	-0.0018	+0.354
26	+0.517	-2.556	-0.052	-0.0016	+0.116
14	+1.927	-1.258	-0.076	-0.0008	+0.116
15	+2.695	-0.760	-0.016	-0.0005	+0.124
16	+1.570	-1.346	-0.028	-0.0008	+0.107

3.0 Poisoned Cores

3.1 Fully Poisoned

Boron steel and stainless steel were added to the 5 x 5, 6 x 6, and 7 x 7 arrays until the cores were essentially completely poisoned. These cores provide excellent checks on the calculation model since there is no problem evaluating large amounts of excess reactivity.

The fully poisoned cores were computed by the equivalent bare core model, by an analytical two-group two region calculation (Fink 1)⁽²¹⁾, and by a one dimensional two-group, two-region calculation (Valprod)⁽²⁰⁾. Cases calculated are listed in Table 3.1.

Table 3.1 Fully Poisoned Cores

Case	Size	No. Elements	Core Radius, cm	Strips per Element
1	5 x 5	21	19.29	4.715 (1.01% B-SS)
2	6 x 6	32	23.81	12.123 (1.01% B-SS)
3	7 x 7	45	28.24	17.340 (1.01% B-SS)
4	5 x 5	21	19.29	6.6667 (SS) -
5	6 x 6	32	23.81	17.2814 (SS) -
6	7 x 7	45	28.24	24.0923 (SS)

The boron poisoned cores present a little difficulty in preserving the material balance in the core due to the fact that two types of strips were used in the measurements. These strips have a different amount of steel associated with each gram of boron. In order to avoid the necessity of keeping track of two types of strips it was decided to convert the 1.16% B-steel strips to equivalent strips of 1.01% B-steel.

As shown in Appendix B, it was found that a 1.16% B-steel strip was equivalent to 1.187 strips of 1.01% B steel.

The methods used for obtaining the atomic concentrations required for calculating the thermal and fast group constants are shown in Appendix B. Calculation of the group constants are shown in the following section.

3.1.1 Thermal Group Constants

The thermal neutron temperature for the poisoned cores was assumed equal to that of the clean core. This was due to the small effect of neutron temperature on K_{th} and the labor involved in preparing new thermal cross-sections for different neutron temperatures.

There is some difficulty in calculating thermal group properties due to two uncertainties. The position of the poison strips in the water gap is not known precisely. Furthermore, self-shielding of the strips must be estimated.

In the ZPE-2 experiments the poison strips were more or less randomly distributed in the water gaps between the fuel plates. The strips were not positioned in the dead region due to interference from the side plates. It was felt that the most representative distribution for the strips was a uniform distribution in the water channel. Consequently, thermal properties for the channel were calculated assuming a uniform mixture of water and strips in the channel.

A series of P_3 calculations had previously been made on the variation of self-shielding factor with $\sum \epsilon$. From these results it was estimated that the self-shielding factor for a boron strip was 0.982; whereas, the self-shielding factor for a stainless steel strip was 0.999. Consequently, it was assumed that as far as thermal group properties were concerned, one boron strip was effectively 0.982 strip.

In the stainless steel poisoned cores, no self-shielding correction was used since the use of a correction factor is not warranted.

The methods described in section 1.4 were followed in calculating thermal group properties. Results are shown in Table 3.2

Table 3.2 Thermal Group Constants for Fully Poisoned Cores

Input Data

Geometry factors (half thicknesses, cm)

$$l = 0.038100$$

$$\eta = 0.207256$$

$$p = 3.175000$$

$$q = 3.730625$$

Relative neutron source in dead region 0.675409

Dead Region Cross-Sections, cm^{-1}

$$\Sigma^a = 0.075337$$

$$\Sigma^s = 1.921506$$

$$\Sigma^{tr} = 1.656121$$

$$\nu \Sigma^f = 0.000000$$

Plate Region Cross-Sections, cm^{-1}

$$\Sigma^a = 1.713152$$

$$\Sigma^s = 0.762252$$

$$\Sigma^{tr} = 2.465638$$

$$\nu \Sigma^f = 2.888019$$

Channel Region Cross - Sections, cm⁻¹

Case	Σ^a	Σ^s	Σ^{tr}	$\gamma \Sigma^f$
1	0.037701	2.404737	1.946611	0.000000
2	0.070284	2.385545	1.965533	0.000000
3	0.093231	2.372029	1.978859	0.000000
4	0.029023	2.309844	1.870583	0.000000
5	0.048233	2.139322	1.768728	0.000000
6	0.060558	2.029910	1.703377	0.000000

Active Region

Case	Relative Source
1	0.809750
2	0.799866
3	0.792564
4	0.759435
5	0.669102
6	0.611140

Output Data

Relative Fluxes

($\bar{\Phi}'$ values normalized to $\bar{\Phi}'_{\text{active}} = 1$, $\bar{\Phi}$ values normalized to $\bar{\Phi}_{\text{core}} = 1$)

Case	$\bar{\Phi}'_{\text{plate}}$	$\bar{\Phi}'_{\text{channel}}$	$\bar{\Phi}_{\text{active}}$	$\bar{\Phi}_{\text{dead}}$
1	0.950501	1.011149	0.940477	1.340129
2	0.949999	1.011262	0.933996	1.377165
3	0.949663	1.011338	0.929443	1.403185
4	0.951860	1.010843	0.937862	1.355077
5	0.953596	1.010452	0.925508	1.425669
6	0.954705	1.010202	0.916288	1.478357

Active Region Cross-Sections, cm^{-1}

Case	Σ^a	Σ^s	Σ^{tr}	$2/\Sigma^f$
1	0.330455	2.117744	2.037301	0.504627
2	0.357193	2.107055	2.052871	0.504370
3	0.376032	2.090999	2.063840	0.504182
4	0.323714	2.039044	1.974706	0.505349
5	0.340094	1.897920	1.890896	0.506270
6	0.350595	1.807431	1.837157	0.506859

Thermal Group Properties							
Case	Σ_a	Σ_s	Σ_{tr}	$\nu\Sigma_f$	K_{eff}	D	L^2
1	0.279535	2.078576	1.961220	0.404319	1.446400	0.169962	0.608018
2	0.292994	2.065022	1.971494	0.401549	1.343000	0.169077	0.564753
3	0.313191	2.055577	1.978633	0.400352	1.278300	0.168567	0.537903
4	0.273586	2.015323	1.910409	0.403359	1.474339	0.174483	0.637761
5	0.283877	1.902929	1.841045	0.398772	1.404730	0.181057	0.637799
6	0.289988	1.832548	1.797296	0.395258	1.363014	0.185464	0.639556

3.1.2 Fast Group Properties

The fast group constants for the poisoned fixed elements were calculated using the Muft III Code. Table 3.3 lists the atomic concentrations and fast group constants.

Table 3.3 Fast Group Properties for Fully Poisoned Cores.

(All fixed elements)

Input Data

Case	B, cm^{-1}	Atomic Concentrations (atoms/cm ³ x 10 ⁻²⁴)					
		U-235	H	S.S.	O	C	B
1	0.10587	0.0004242	0.052701	0.015096	0.027293	0.000047	0.000069188
2	0.09333	0.0004242	0.052131	0.015793	0.027004	0.000047	0.000105704
3	0.08458	0.0004242	0.051729	0.016283	0.026801	0.000047	0.000131421
4	0.10587	0.0004242	0.049844	0.018706	0.025864	0.000047	0.000045947
5	0.09333	0.0004242	0.044717	0.025160	0.023295	0.000047	0.000045947
6	0.08458	0.0004242	0.041427	0.029301	0.021647	0.000047	0.000045947

Output Data

Case	τ	$D^{\frac{1}{2}}$	Σ_{rf}	Σ_{af}	$\nu \Sigma_{ff}$	ρ	K_f
1	32.946822	1.317056	0.039975	0.0105245	0.0130088	0.736724	1.236048
2	32.644771	1.309219	0.040105	0.0111456	0.0130021	0.722091	1.166570
3	32.458714	1.303742	0.040166	0.0115752	0.0129903	0.711817	1.122480
4	33.410900	1.272200	0.038228	0.0105890	0.0129413	0.722995	1.222145
5	34.134390	1.216083	0.035626	0.0113148	0.0128306	0.682403	1.133966
6	34.810198	1.182533	0.033971	0.0117711	0.0127550	0.653494	1.083584

3.1.3 Calculated and Measured Reactivity

Using the thermal group properties from Table 3.2, the fast group properties from Table 3.3, and the reflector properties from Table 2.5, K_{eff} was calculated for the B-SS and SS poisoned arrays consisting of fixed fuel elements only. Table 3.4 lists the calculated and experimental reactivities. The measurements were made in cores containing seven control rod fuel elements. The experimental correction factors reported in Appendix D were used to find the reactivity of a fixed element core which should be compared with the calculated value.

Table 3.4 Reactivity of Fully Poisoned Cores

(All fixed elements)

Case	Measured $\% \rho$	Calculated $\% \rho$		Fink I
		Equivalent Bare	Valprod	
1	2.97	1.05	0.91	1.12
2	2.83	0.36	0.31	0.57
3	2.43	-0.22	-0.22	0.09
4	2.93	1.60	1.27	1.44
5	2.61	1.23	0.86	1.04
6	2.42	1.04	0.72	0.92

It can be seen that all of the calculated reactivities are lower than the measured values. (This was also true for the clean cores as shown in Table 2.6) Furthermore, the difference between measured and calculated reactivities seems to vary only slightly with steel content but increases with increased boron content. This indicates that the calculation procedure overestimates the effect of boron addition while the calculated effect of steel addition appears to be more nearly correct.

3.1.4 Reflector Savings

The axial and radial flux traverses for the poisoned arrays were used to estimate reflector savings. Both bare gold and cadmium-covered gold activities were available for axial and radial traverses. Since axial traverses were made along a coolant channel which minimizes heterogeneity effects, total flux measurements (bare gold activity) were used in determining axial savings. Bare-gold traverses made radially showed considerable peaking due to heterogeneity effects, therefore radial savings were determined from cadmium covered gold traverses.

The experimental data were fitted by least squared analysis (see Section 1.5.3) with the axial flux distribution being fitted by a cosine function and the radial distribution by a J_0 Bessel function. It should be noted that since a least squares analysis based on a cosine or J_0 fit effectively weights more heavily those values which are the furthest from the core centerline, careful judgement must be exercised in selecting outermost points to avoid effects of the reflector.

Table 3.5 lists the axial and radial reflector savings determined by least squares analysis. These values are also plotted in Figures 3.1 and 3.2.

Table 3.5 Reflector Savings From Least Squares Analysis
(Cadmium-Covered Gold Traverses in Poisoned Arrays)

Configuration	Poison	Reflector Savings - cm		
		Axial		Radial
		Top	Bottom	
5 x 5	B-SS	3.51	7.02	6.43
6 x 6	B-SS	3.98	6.07	5.79
7 x 7	B-SS	3.53	7.28	6.33
5 x 5	SS	3.45	7.08	6.08
6 x 6	SS	3.63	6.00	5.87
7 x 7	SS	3.60	6.07	6.26
Average		3.62	6.59	6.13

The reflector savings were also found by an analytical solution of the two-group, two-region problem. A program was written for the IBM-650 computer that solves the two group equations for a cylindrical core reflected radially by an infinite

reflector and axially by a finite reflector (Fink I)⁽²¹⁾. This approach was applied to all the fully poisoned arrays using the reflector and core properties given in Tables 2.5, 3.2, and 3.3. The results are given in Table 3.6. For comparison with fitted experimental data, the analytical results are also plotted on Figures 3.1 and 3.2.

Table 3.6 Calculated Reflector Savings for Poisoned Arrays

Configuration	Poison	Reflector Savings-cm	
		Axial	Radial
5 x 5	B-SS	5.80	6.04
6 x 6	B-SS	5.92	6.06
7 x 7	B-SS	6.01	6.08
5 x 5	SS	5.68	5.93
6 x 6	SS	5.66	5.80
7 x 7	SS	5.68	5.73

The results of axial curve fitting indicate a difference between the savings at the top of the core and at the bottom. This difference is probably due primarily to the presence of control rod absorbers in the top reflector. The analytical solution makes no such distinction and considers top and bottom reflectors to be identical.

Within the accuracy of the data, there is no consistent variation of the experimental reflector savings with core size or type of poisoning. The calculated reflector savings also show only slight variation with core size and type of poisoning. The differences between the calculated and measured radial savings are within

the accuracy of the experimental data. The differences between the calculated and measured axial savings are probably due to the fact that the calculated values do not account for the effect of control rod absorbers in the upper reflector.

3.1.5 Neutron Flux Distributions

The radial flux distributions in the core and reflector were obtained from the Valprod calculations described in Section 3.1. These values are shown on Figures 3.3 through 3.8 together with the experimental traverses. The calculated fast flux was normalized to the measured bare gold activity at the center of the core. The calculated thermal flux was normalized to the measured bare minus cadmium-covered gold activity at the center of the core.

There is excellent agreement between the measured and calculated fast flux distributions in all cases. This is consistent with the fact that there is little difference between the calculated and measured radial reflector savings.

The differences between the calculated and measured thermal flux distributions are largely due to two factors. Within the core itself there is a large scatter of the experimental data because of the variation of thermal flux within a fuel element cell. In addition, the calculated and measured thermal flux peaks in the reflector do not occur at the same distance from the center of the core. This is due to the fact that the calculations were based on the equivalent radius of the core, while the actual core is a square array with corners missing. Consequently, the location of the core-reflector boundary is not quite the same for the calculations and the measurements.

FIGURE 3.6
RADIAL TRAVERSE
7 X 7 B-SS

FIGURE 3.6
RADIAL TRAVERSE
7 X 7 B-SS

FIGURE 3.7
RADIAL TRAVERSE
6X6 B-SS

3.2 Critical Water Height

The critical water height was measured in the 6 x 6 and 7 x 7 arrays partially poisoned with B-SS and SS. Using the equivalent bare model and a total axial savings of 11.7 cm (Section 1.5) K_{eff} was calculated for the critical configurations, using group constants for the fixed fuel elements.

Group constants, calculated reactivities and measured reactivities are shown in Table 3.7. Measured reactivities were corrected for the effect of control rod fuel elements.

Table 3.7 Critical Water Height Analysis
(Partially Poisoned Cores)

Array	6 x 6	7 x 7	6 x 6	7 x 7
No. Elements	32	44	32	44
Strips per Element	8 B-SS	8 B-SS	11 SS	11 SS
Critical Water Height, in.	15.30	12.33	15.02	12.05
B^2 axial, cm^{-2}	0.003861	0.005335	0.003972	0.005513
B^2 radial, cm^{-2}	0.006451	0.004988	0.006451	0.004988
B^2 total, cm^{-2}	0.010312	0.010323	0.010423	0.010501
K_{th}	1.398193	1.398193	1.445189	1.445189
L^2 cm^2	0.588882	0.588882	0.637403	0.637403
ρ	0.730150	0.730146	0.706921	0.706887
K_f	1.204151	1.204152	1.184355	1.184312
τ cm^2	32.843253	32.844680	33.704810	33.71950
Calculated K_{eff}	1.000738	1.000452	1.007914	1.005781
Calculated β	0.07	0.05	0.79	0.57
Measured β	2.57	2.30	2.57	2.30

The calculated reactivities are lower than measured reactivities in all cases. The differences are larger for the boron steel poisoned cores than for the stainless steel cores. This is consistent with the comparisons made on the fully poisoned cores.

4.0 Substitution of Control Rod Elements for Fixed Elements

4.1 Measurements

The reactivity correction for the control rod fuel elements was measured experimentally in the 7 x 7 and 5 x 5 fully poisoned boron cores. These reactivity coefficients are given in Table 4.1. Assuming these measurements can be added directly, the substitution of fixed fuel elements in all of the control rod positions results in an increase in reactivity of 392 cents in the 5 x 5 array and 315 cents in the 7 x 7 array. The replacement of fixed elements by control rod elements will decrease the reactivity by a corresponding amount.

Table 4.1 Reactivity Coefficient of a Fixed Fuel Element in a
Control Rod Fuel Element Position

Control Rod Position	Reactivity Coefficient, cents	
	5 x 5 array	7 x 7 array
C	115 $\pm$ 10	67.3 $\pm$ 10
1	34.3 $\pm$ 10	37.0 $\pm$ 10
A	70.1 $\pm$ 10	49.7 $\pm$ 10
A, B, C, 1, 2, 3, 4	392 $\pm$ 50	315 $\pm$ 50

Figure 4.1 shows curves of ρ lost by substitution of a control rod element versus radial position based on $J_0^2 (\mu r)$ weighting,

$$\text{where } \mu = \frac{2.405}{R + S_r}$$

$$R = \begin{array}{l} 19.29 \text{ cm for } 5 \times 5 \text{ array} \\ 28.24 \text{ cm for } 7 \times 7 \text{ array} \end{array}$$

$$S_r = 6.13 \text{ cm}$$

The curves are normalized to the measured value for the ϕ element. These curves show that if the substitution effect can be found for a center line element, the corresponding value for other elements can be found by a $J_0^2 (\mu r)$ weighting.

4.2 Calculations

It is important that an analytical method be established to account for the control rod fuel elements. The model can then be used to calculate the reactivity correction as a function of temperature and burnout.

Calculations were made to determine the increase in reactivity due to substitution of a fixed element for a control rod fuel element in position C for the eight cases shown in Table 4.2. A radial, three region calculation was made for a core consisting of all fixed fuel elements except for a control rod fuel element in position C (ϕ rod). The reactivity of this core was compared with that of a core with all fixed elements. Group constants for the fixed fuel elements are given in Tables 2.3, 2.4, 3.2, and 3.3. Group constants for the water reflector are given in Table 2.5. The methods used to calculate group constants for the control rod fuel element are shown in the following sections.

Table 4.2 Cases for Substitution Calculations

Case	Strips per Element		Size	No. Elements
1	Clean		5 x 5	21
2	Clean		7 x 7	32
3	4.715 B-SS(1.01%B)		5 x 5	21
4	12.123	"	6 x 6	32
5	17.340	"	7 x 7	45
6	6.6667	SS	5 x 5	21
7	17.2814	"	6 x 6	32
8	24.0923	"	7 x 7	45

FIGURE 4.2
CONTROL ROD FUEL ELEMENT

4.2.1 Thermal Group Constants for Control Rod Elements

To obtain thermal group constants, a knowledge of the intercell flux distribution was required. This was determined from a one velocity P_3 calculation in cylindrical geometry using an available IBM 650 code.⁽³¹⁾ In this calculation thermal neutron sources may be in either or both regions.

The active portion of the fuel element (2.281 in. x 2.611 in.) was treated as an equivalent cylinder of 3.497306 cm radius. The total fuel element (2.9375 in. x 2.9375 in.) was treated as an equivalent cylinder of 4.209562 cm radius. The relative sources in the active and dead regions were assumed equal to the volume fraction water in each region. Cross-sections in the dead region were obtained from the number densities given in Appendix B and the cross-sections given in Appendix A. Fig. 4.2 shows a typical control rod fuel element.

In the clean cores, the active region has the same composition in both fixed and control rod fuel elements. Therefore, the active region cross-sections given in Table 2.1 were used for the clean core calculations.

In the poisoned cores, there were as many strips in the control rod fuel elements as in the fixed elements. However, the fixed elements have an active region volume 1.23301 times that of the control rod fuel elements. Therefore, a control rod fuel element with one poison strip will have the same active region properties as a fixed element containing 1.23301 strips. Active region properties for the control rod fuel elements were obtained by interpolating the results of calculations made on the fixed elements.

The input and output data for the P_3 calculation are shown in Table 4.3. Relative fluxes were normalized to $\bar{\Phi}_{\text{core}} = 1$ using the same basic procedure described in Section 1.4.2.

Table 4.3 Input and Output Data for P₃ Calculations (Control Rod Fuel Elements)

*

Case	Active Region				Relative Source
	Σ^a, cm^{-1}	Σ^s, cm^{-1}	$\Sigma^{\tau}, \text{cm}^{-1}$	$\gamma \Sigma^f, \text{cm}^{-1}$	
1, 2	0.313423	2.127740	2.027380	0.504770	0.816170
3	0.334063	2.115625	2.039400	0.504596	0.808254
4	0.366471	2.096604	2.058267	0.504275	0.795815
5	0.389307	2.083207	2.071567	0.504059	0.787056
6	0.326107	2.018394	1.962440	0.505475	0.746770
7	0.346299	1.844420	1.559121	0.506611	0.636270
8	0.359268	1.732844	1.792876	0.507374	0.565369

Dead Region (All Cases)

$$\begin{aligned}\Sigma^a &= 0.079104 \\ \Sigma^s &= 1.805634 \\ \Sigma^{\tau} &= 1.557778 \\ \gamma \Sigma^f &= 0.000000\end{aligned}$$

Relative Source = 0.654325

Note * - For machine calculations, cross-sections input is actually specified as

$\Sigma^a, \Sigma, \Sigma', \text{ and } \Sigma^2$

where,

$$\begin{aligned}\Sigma &= \Sigma^a + \Sigma^s \\ \Sigma' &= \bar{\mu} \Sigma^s = \Sigma - \Sigma^{\tau} \\ \Sigma^2 &= 0\end{aligned}$$

Output Data	(Fluxes normalized to	$\bar{\phi}_{\text{core}} = 1$)
Case	$\bar{\phi}_{\text{active}}$	$\bar{\phi}_{\text{dead}}$
1, 2	0.891938	1.240783
3	0.882461	1.261900
4	0.867903	1.294338
5	0.857861	1.316713
6	0.877272	1.273462
7	0.851113	1.331748
8	0.832065	1.374192

Core Properties							
Case	Σ^a	Σ^s	Σ^{τ}	$\nu\Sigma^f$	K^{th}	D	L^2
1, 2	0.223360	2.003930	1.844579	0.310757	1.391283	0.180707	0.809039
3	0.234400	1.994448	1.848789	0.307350	1.311220	0.180298	0.769189
4	0.251251	1.979939	1.855188	0.302087	1.202331	0.179674	0.715117
5	0.262781	1.969989	1.859626	0.298464	1.135789	0.179245	0.682107
6	0.228669	1.934055	1.800440	0.306075	1.338559	0.185139	0.809669
7	0.236071	1.828418	1.732329	0.297616	1.260705	0.192417	0.815081
8	0.240006	1.763828	1.690243	0.291393	1.214107	0.197208	0.821679

4.2.2 Fast Group Constants for Control Rod Elements

Fast group constants were obtained from a Muft III calculation using the Geortzel-Selengut, P-1 approximation. Atomic densities and group constants are given in Table 4.4.

Table 4.4 Fast Group Properties
(Control Rod Fuel Elements)

Input Data		Atomic Concentrations (atom/cm ³ x 10 ⁻²⁴)					
Case	B, cm ⁻¹	U ²³⁵	H	O	S.S.	B	C
1	0.10587	0.0003440	0.051116	0.026325	0.017452	0.000037264	0.000046
2	0.08458	0.0003440	0.051116	0.026325	0.017452	0.000037264	0.000046
3	0.10587	0.0003440	0.050753	0.026141	0.017895	0.000060506	0.000046
4	0.09333	0.0003440	0.050183	0.025852	0.018592	0.000097022	0.000046
5	0.08458	0.0003440	0.049781	0.025649	0.019082	0.000122738	0.000046
6	0.10587	0.0003440	0.047896	0.024712	0.021505	0.000037264	0.000046
7	0.09333	0.0003440	0.042769	0.022143	0.027959	0.000037264	0.000046
8	0.08458	0.0003440	0.039479	0.020495	0.032100	0.000037264	0.000046

Output Data							
Case	τ	D_f	Σ_{rf}	Σ_{af}	$\nu \Sigma_{ff}$	ρ	K_f
1	33.369090	1.290516	0.038674	0.0091512	0.0107350	0.763375	1.173070
2	32.733650	1.288253	0.039356	0.0092599	0.0108835	0.764713	1.175340
3	33.415221	1.286486	0.038499	0.0095119	0.0106760	0.752936	1.122380
4	33.076291	1.278398	0.038650	0.0101448	0.0106760	0.737521	1.052360
5	32.873511	1.272910	0.038721	0.0105828	0.0106688	0.726693	1.008126
6	34.022637	1.250491	0.036755	0.0095834	0.0106260	0.739260	1.108787
7	34.950356	1.194163	0.034167	0.0103316	0.0105481	0.697619	1.020952
8	35.775919	1.163147	0.032512	0.0108020	0.0104929	0.667755	0.971389

4.2.3 Reactivity Calculations

Results of the reactivity calculations are compared with experimental measurements in Table 4.5. Agreement is fairly good although the calculation appears to overestimate the substitution effect in each case.

Table 4.5 Reactivity Coefficient of Rod C Fuel Element

Core	% ρ calc.	% ρ meas.	% ρ calc.	% ρ meas.	% $\Delta\rho$ calc.	% $\Delta\rho$ meas.
1	5.80	7.66	4.99	6.88	0.81	0.78
2	15.47	17.22	15.09	16.85	0.38	0.37
3	0.91	2.97	0.00	2.15	0.91	0.82
4	0.31	2.83	-0.40	2.17	0.71	0.66
5	-0.22	2.43	-0.77	1.95	0.55	0.48
6	1.27	2.93	0.37	2.11	0.90	0.82
7	0.86	2.61	0.15	1.95	0.71	0.66
8	0.72	2.42	0.16	1.94	0.56	0.48

5.0 Control Rod Measurements and Calculations

5.1 Control Rod Worth

The worth of control rods was measured under various core conditions. The centerline rod worth was measured in ZPE-1 for a core with various boron steel additions by calibration against the five rod bank. In ZPE-2, the centerline rod was measured against boron steel and stainless steel additions. These experiments differed in the method of balancing the centerline rod. The former method employed four rods of the five rod bank and the latter method used uniform poison addition. When the bank was used to balance the remaining rod, there was no change in core absorption while when the uniform poison was used, the core absorption was changed throughout the calibration. Calibration against the four-rod bank does, however, markedly change the flux distribution during the calibration.

5.1.1 Comparison of Calculations and Measurements

The results obtained from the calibration of the central control rod with the four-rod bank are plotted in Figures 5.1 and 5.2. In Figure 5.1, the ratio of the rod worth with strips inserted into the core to the rod worth with no additions, ρ_0 , is shown as a function of the number of strips added to each fuel element. Figure 5.2 shows the absolute value of the rod worth as a function of the number of strips added.

It is important to be able to predict the variation of rod worth with core absorption. In an effort to fit analytical methods to the experimental values, Scram I, which employs Murray's method,⁽³²⁾⁽³³⁾⁽³⁴⁾ was used to compute the central control rod worth as a function of the number of strips added to the core. These results are also plotted in Figures 5.1 and 5.2 in a similar manner to the experimental

FIGURE 5.2
CENTERLINE ROD WORTH AS
A FUNCTION OF THE NUMBER
OF BORON STEEL STRIPS
ADDED TO THE CORE

results. If it is assumed that the control rod absorber can be treated as a region with properties evaluated by the methods already described, then the case of a core with a centerline rod fully inserted can be calculated. This model should, within the approximation involved, give the variation of rod worth with core absorption. The properties of the regions used in the Scram calculations are given in Table 5.1.

Similar calculations were also made to determine the rod worth as a function of the number of stainless steel strips added to the core. The results are plotted in Figure 5.1 and the group constants are listed in Table 5.2.

Table 5.1 Properties Used in Scram Calculation with Boron Steel Addition

Geometry	Number of Boron Steel Strips Added/Element			
	<u>0</u>	<u>12.491</u>	<u>17.494</u>	<u>28</u>
*Rod Radius, cm	3.954640	3.954640	3.954640	3.954640
$R + S_r$, cm	34.324000	34.324000	34.324000	34.324000
B^2 , cm^{-2}	0.002259	0.002259	0.002259	0.002259
Core Constants				
D_f , cm	1.321185	1.308522	1.303625	1.293702
D_s , cm	0.170539	0.169034	0.168449	0.167237
τ , cm^2	32.378844	32.434570	32.460675	32.528810
ϵ	1.000000	1.000000	1.000000	1.000000
p	0.747790	0.728075	0.711500	0.690031
L^2 , cm^2	0.639339	0.562768	0.537149	0.490231
K_f	1.286818	1.163695	1.120988	1.041163
K_{th}	1.521340	1.334276	1.271548	1.157295
Water Hole Constants**				
D , cm	1.596016	1.596016	1.596016	1.596016
τ , cm	33.694514	33.694514	33.694514	33.694514

* The rod radius was calculated using the model set up by Hurwitz and Roe,
(35)
KAPL-1336.

** Evaluated at a buckling, $B = 0.08458$.

Table 5.2 Properties Used in Scram Calculations with Stainless Steel Addition

Geometry	Number of Stainless Steel Strips Added/Element			
	<u>0</u>	<u>9</u>	<u>16.7577</u>	<u>24.2866</u>
Rod Radius, cm	3.954640	3.954640	3.954640	3.954640
$R + S_r$, cm	34.324000	34.324000	34.324000	34.324000
B^2 , cm^{-2}	0.002259	0.002259	0.002259	0.002259
Core Constants				
D_f , cm	1.298980	1.259480	1.216353	1.181714
D_s , cm	0.172290	0.175896	0.180724	0.185592
τ , cm^2	32.509790	32.921150	33.731180	34.842100
ϵ	1.000000	1.000000	1.000000	1.000000
p	0.737553	0.715913	0.685389	0.652611
L^2 , cm^2	0.638475	0.637508	0.637733	0.639619
K_f	1.258153	1.203901	1.139532	1.082244
K_{th}	1.499840	1.458511	1.408024	1.36153
Water Hole Constants				
D_f , cm	1.596016	1.596016	1.596016	1.596016
τ , cm	33.694514	33.694514	33.694514	33.694514

A multiregion three-group calculation was performed for the clean core, with and without the centerline rod inserted. This was repeated for a series of boron steel poisoned cores. The rod worth was based on the following equation,

$$\rho = \frac{K_{eff}^{rod\ out} - K_{eff}^{rod\ in}}{K_{eff}^{rod\ out}}$$

The results are plotted in Figure 5.1.

In the ZPE-2 experiments, the core was fully poisoned out with boron steel and stainless steel strips. The center line rod was then calibrated by the addition of more strips. For each of these cases, Scram calculations were performed and the results compared in Table 5.3.

Table 5.3 Rod Worth in the Fully Poisoned Core*

	17.34 Strips Boron Steel	24.0923 Strips Stainless Steel
Experimental Values	578.0 cents	690.5 cents
Scram Values	533.3 cents	598.4 cents

* The reactivity values from the analytic methods were decreased by 67.3 cents so that they could be compared with the measured values. The reason for this is that in the actual reactor, the control rod replaces a control rod fuel element whereas the control rod in the analytic approach replaces a fixed fuel element. The difference between replacing a fixed fuel element and a control rod fuel element is 67.3 cents.

5.1.2 Conclusions

From the data on the effect of core absorption on control rod worth, it can be concluded that the rod worth decreases as boron steel strips are added to the core. The experimental values show a decrease of approximately 40% for an addition of 12 strips whereas the Scram and three-group calculations show only slight decrease of 2.2% and 2% respectively.

As can be seen from Figures 5.1 and 5.2 and Table 5.3, the different methods approach each other as the core becomes fully poisoned. It is felt that the difference is due to the interpretation of the measurements and not the theory. Since the rod worth was determined by a calibration of the four-rod bank, a major part of the decrease in rod worth may be due to bank interaction. In the poisoned core, the bank is almost fully withdrawn. In the other cases, the bank position changes from full in to full out and therefore the flux distribution changes continually each time the control rod is calibrated. From this, it is concluded that the only measurements which can be reliably interpreted are those taken with the bank fully withdrawn, and that the rod worth decreases only slightly with an increase in the amount of boron steel added to the core.

Analytic calculations for stainless steel addition show a different trend. As more stainless steel strips are added, the rod worth increases. For an addition of 24.28 strips, the rod worth increased by about 10.4%

From fully poisoned core measurements, the rod is worth more when the core is poisoned with stainless steel strips than with boron steel strips. Scram calculations also show this same effect for the fully poisoned and slightly poisoned cores. Even though the absolute rod worths may on the whole contain errors, they can be considered to give adequate trends.

5.2 Neutron Flux Distribution Associated with Fully Inserted Absorber Sections

Measurements were made in and around the absorber section of rod C in the fully inserted position. The fast and thermal flux distributions, as measured with gold foils, are shown in Figure 5.3. The core was poisoned with 27.25 grams of B-10.

Analytical calculations were performed in order to predict the flux distribution in and around a central control rod. The analytic cases were calculated with 12.491 B-SS strips per element added to the core. This corresponds to 26.924 grams of B-10. Therefore, these cases are not exactly identical to the experimental case but close enough for comparison with the measured values.

The results of a multiregion two-group calculation are shown in Figure 5.4. As can be seen, two-group theory does not predict a rise in fast flux within the control rod, as the bare gold traverse would indicate.

It was thought that three-group theory would correctly predict this rise in the water hole.⁽³⁶⁾ With this in mind, three-group calculations (using Valprod) were made and the resulting flux distributions are plotted in Figure 5.5. Three-group theory does predict a rise in the epithermal flux as can be seen from Figure 5.5.

5.3 Effect of Absorber-Fuel Element Separation

It was discovered in the APPR-1 ZPE-I experiments that the flux peaking within the control rod absorber resulted in a flux peak at the tip of the control rod fuel element. This flux peaking was reduced in the APPR-1 by the addition of flux suppressors to the top of the control rod fuel element.⁽³⁷⁾

There was some question whether a reduction of the gap between the control rod fuel element meat and absorber meat would not result in a reduction of this

peaking. To resolve this point, measurements were made with several different fuel-absorber separations. Axial traverses were taken with gold foils along the centerline of rod C. As an aid in interpreting the gold foil measurements, a power traverse was made with fully enriched uranium-aluminum alloy foils. These measurements indicated that the bare minus cadmium covered gold activity was in close agreement with the uranium power traverse. The effect of fuel-absorber separation is seen in Figure 5.6 where the cadmium differenced gold activity at the fuel tip relative to the cadmium differenced gold activity 2 inches back from fuel tip is plotted versus separation distance.

Figure 5.6 also indicates that a reduction of APPR-1 fuel absorber separation below the 1.3 inches that now exists will not eliminate the flux peaking at the tip of the central fuel plates. Calculations at this time indicate that integral flux suppressors seem to be the best solution to this problem. (38)

6.0 Peak to Average Power Reduction

In the course of the ZPE II experiments, two approaches were followed in an attempt to reduce the peak to average power distribution in APPR type reactors. One approach covered in Section 6.1, entails rod programming in which various configurations of rod insertions are utilized; the other approach, covered in Section 6.2, entails non-uniform distribution of poison (boron steel) within the core.

Figure 6.1 is a cross section of the ZPE II core. The fuel and control rod elements are numbered to indicate the core lattice designations.

6.1 Non-Uniform Poison Additions

In order to study the effects of non-uniform poison distribution on peak to average power release, the neutron flux distributions in the core were determined for different methods of non-uniform poisoning by activation of bare gold foils.

The situations studied were as follows:

1. Radial Non-Uniform Poison Distribution

- (a) Partially poisoned core in which 10.06 gms. of B-10 were added to the central 21 fuel elements. Control rods A and B were withdrawn 20.5 inches and the 5 rod bank was withdrawn 7.04 inches.
- (b) Fully poisoned core in which 26.53 gms. of B-10 were added to the central 21 fuel elements. Control rods A and B withdrawn 15.23 inches and the 5 rod bank was withdrawn 20.5 inches.

2. Longitudinal Non-Uniform Poison Distribution

The bottom half of the core was poisoned with boron steel strips. There were 19.25 gms. of B-10 in the active core and 21.36 gms. below the core mid-plane. Control rods A and B were withdrawn 20.5 inches and the 5 rod bank

FIGURE 6.1

7 x 7 CORE ARRAY

38 fixed fuel elements and 7 control rod elements.

Shaded area indicates the region of the active core.

The numbers indicate the core lattice designations.

was withdrawn 11.48 inches.

In order to compare the gross peak to average reduction for the different methods of poisoning, one may use element 35 as a typical case. For comparison purposes, the gross peak to average value for the "clean" core with the 5 rod bank at 6.3 inches, and for an equivalent bare reactor are included. The results are tabulated in Table 6.1.

Table 6.1 Peak to Average Power Distributions for Non-Uniform Poisoned Core

Method of Poisoning	Peak to Average (Element 35)
"Clean" Core	2.30
Centrally, Partially Poisoned	2.05
Centrally, Fully Poisoned	1.60
Lower Half Poisoned	1.97
Equivalent Bare	2.34

In order to see more clearly what constitutes the peak to average power listed in Table 6.1, the power distributions have been separated into radial and axial peak to averages, the product of which yields the overall peak to average. In the cases of the centrally poisoned core, the axial peak to average was computed from a cosine function and the radial peak to average found by division. In the longitudinally poisoned core, the radial peak to average was computed from a J_0 function and the axial peak to average found by division. Table 6.2 lists these peak to averages together with that for an equivalent bare reactor. The computed values are designated by an asterisk.

Table 6.2 Radial and Axial Peak to Average for Non-Uniform Poisoned Cores.

Method of Poisoning	Peak to Average (Element 35)	
	Radial	Axial
"Clean" Core	1.71*	1.35
Centrally, Partially Poisoned	1.50	1.37*
Centrally, Fully Poisoned	1.17	1.37*
Lower Half Poisoned	1.71*	1.15
Equivalent Bare	1.71*	1.37*

This table indicates a significant reduction in the axial peak to average power for a lower half, poisoned core. However, the volumetric peak to average value is not much lower than for a centrally, partially poisoned core. The latter core has the advantage that both axial and radial peak to average values are relatively low and of the same magnitude while the former has a low axial peak to average value but a relatively high radial value.

6.2 Control Rod Programming

The second approach to reduce the gross peak to average power distribution is by control rod programming. (This approach may not be feasible in APPR-1 since tailored flow is used, but may be applicable to other cores.) Two possible programs were investigated. In the symmetric configuration, control rods A, B, and C were withdrawn 20.5 inches and control rods 1, 2, 3, 4 withdrawn as a bank 2.53 inches. The core was poisoned with 8.62 gms. of B-10 to simulate operation at the design temperature. In the asymmetric configuration, control rods 2, A, and B were withdrawn to 20.5 inches and control rods 1, 3, 4, and C were withdrawn as a bank 0.93 inches. This core was also poisoned with 8.62 gms. B-10.

The results of these experiments, compared to the "clean" core with the 5 rod bank at 6.3 inches and to an equivalent bare reactor, are presented in Table 6.3.

Table 6.3 Gross Power Distribution for Various Configurations

Configuration	Gross Peak to Average Value
"Clean" Core	2.30
Symmetric	2.03
Asymmetric	2.67
Equivalent Bare	2.34

The results in Table 6.3 indicate that with control rods of the APPR-1 type in a small core, there is little advantage of rod programming in reduction of peak to average power. As before, in order to see what constitutes the gross peak to average values, the power distributions have been separated into radial and axial peak to averages. In the case of the "clean" core, the radial peak to average was computed using a J_0 distribution and the axial peak to average found by division. In the symmetric and asymmetric cases, the axial distribution was computed using a cosine function and the radial peak to average by division. Table 6.4 tabulates these results. The computed values are designated by an asterisk.

Table 6.4 Radial and Axial Peak to Averages for Various Configurations

Configuration	Peak to Average	
	Radial	Axial
"Clean" Core	1.71*	1.35
Symmetric	1.48	1.37*
Asymmetric	1.95	1.37*
Equivalent Bare	1.71*	1.37*

These results indicate that a symmetric rod insertion yields a comparatively low gross peak to average value due to a reduction in the radial peak to average value. Non-uniform burnup studies would be required to determine what effect the type of programming has on core life.

A long range symmetric effect could possibly be obtained by a proper sequence of asymmetric insertions. As an example, rods 2, A, and B could be fully withdrawn while 1, 3, 4 and C are withdrawn as a bank to the critical position. The reactor could be operated in this condition for a certain predetermined length of time; then perhaps rods 1, A, and B could be fully withdrawn with rods 2, 3, 4 and C withdrawn as a bank to the critical position. Again the reactor would be operated in this condition for the given time. Similarly, the reactor could be operated using 1, 2, 3, and C as a bank and finally 1, 2, 4 and C as a bank while rods A, B, 4 and A, B, 3 would be fully withdrawn. At the end of this cycle, it is possible that the overall effect would be similar to a constant symmetric insertion. Other asymmetric configuration cycles are also possible. One cannot evaluate any possible benefits of an asymmetric cyclic operation over symmetric rod insertion at this time but is mentioned to suggest a field for further study.

6.3 Comparison of the Two Methods

The results of rod programming cannot be directly compared to those of non-uniform poison additions, since the boron additions for the latter were such that the reactivity that was controlled did not equal that used in the control rod programming experiments. Therefore, the peak to average values could only be compared with each other and no direct comparisons between nonuniform poison additions and control rod programming can be made. However, in general the latter does not produce as desirable an effect as the former and places more restriction upon reactor operation. Non-uniform poison additions allows more flexibility since the amount, type, and distribution of poison or any combination of these parameters may be changed.

In summary, both methods can yield lower peak to average values of the power and can be utilized in future APPR-type reactors if this effect is desired. Further study including heat transfer and non-uniform burnup considerations would be necessary to determine the preferable method. There is evidence that under certain conditions, higher gross peak to average power values may actually extend the core lifetime; this possibility also should not be overlooked. Some studies indicate that non-uniform poison additions will not significantly affect core lifetime but have their principal value in obtaining more uniform fuel burn-up and decreasing reactivity excursions during reactor lifetime.

7.0 Temperature Coefficients

Temperature coefficients were measured in the clean core (5 x 5 and 7 x 7 array) and in several cores which were almost fully poisoned with boron-stainless steel or stainless steel strips. In addition, measurements were taken on several cores in which some of the water had been displaced by addition of aluminum strips.

7.1 Calculation Method

Based on the results obtained from the aluminum strip experiments, it was felt that the primary factor influencing temperature coefficient is the effect of temperature on water density. While this represents an oversimplification of the situation it does not appear to be unreasonable. Using this basic assumption, a procedure was developed for calculating the temperature coefficient.

The various cases calculated are listed in Table 7.1

Table 7.1 Calculational Cases for Temperature Coefficient

Case	Size	Number of Elements	Number Strips/Element
1	5 x 5	21	4.381 B-SS
2	5 x 5	21	6 SS
3	6 x 6	32	0
4	6 x 6	32	11.976 B-SS
5	6 x 6	32	17.0937 SS
6	7 x 7	45	0
7	7 x 7	45	16.712 B-SS
8	7 x 7	45	23.7333 SS

The reactivity of each of these fixed element cores was obtained from an analytical two-region calculation (Fink I) using the two group model⁽²¹⁾. The methods used to determine group constants are described in the following section.

This was first done for a water density, $\rho_w = 0.998232 \text{ gm/cm}^3$ (density at 68°F.). A second calculation was made in which the water density was decreased by 2 percent ($\rho'_w = 0.978268 \text{ gm/cm}^3$). ($\Delta\rho$) was determined for each case.

Where,

$\Delta\rho$ = change in reactivity

$\Delta\rho_w$ = change in water density = 0.019964 gm/cm^3

The temperature coefficient was calculated for each case by the equation,

$$\frac{\Delta\rho}{\Delta T} = \left(\frac{\Delta\rho}{\Delta\rho_w} \right) \left(\frac{\Delta\rho_w}{\Delta T} \right)$$

Values for $\frac{\Delta\rho}{\Delta T}$ were obtained by constructing a curve using published values for water density data. (39) This is shown on Figure 7.1.

7.1.1 Thermal Group Constants

The techniques described in Section 3 for the fully poisoned cores were also used here for the first set of calculations ($\rho_w = 0.998232$). For the second set of calculations, ($\rho'_w = 0.978268$) it was assumed that only K_{th} need be changed. D core and $\sum_{\text{core}}^a$ were assumed to be the same as in the first case.

This neglects changes in thermal leakage. The thermal leakage in all cases is less than 0.6% in K_{eff} . A change of 2% in water density changes the diffusion area L^2 by less than 2%. Consequently, the change in K_{eff} due to a change in thermal leakage is less than 0.001%.

For the first set of calculations, the equation for K_{th} can be expressed as,

$$K_{th} = \frac{\overline{\nu \Sigma_{core}^f}}{\overline{\Sigma_{core}^a}} = \frac{\overline{\nu \Sigma_{core}^f}}{\Sigma_W^a \overline{\Phi}_W + \Sigma_{rest}^a}$$

where,

$$\Sigma_{rest}^a = \overline{\Sigma_{core}^a} - \Sigma_W^a \overline{\Phi}_W$$

For the second set of calculations,

$$K'_{th} = \frac{\overline{\nu \Sigma_{core}^f}}{0.98 \Sigma_W^a \overline{\Phi}_W + \Sigma_{rest}^a}$$

or,

$$K'_{th} = \frac{\overline{\nu \Sigma_{core}^f}}{\overline{\Sigma_{core}^a} - 0.02 \Sigma_W^a \overline{\Phi}_W}$$

Analysis of the intercell flux distribution showed that $\overline{\Phi}_W$ (the weighted average flux for the water in the channel and the dead region) was approximately unity.

Since the thermal absorption cross-section for hydrogen at 0.0331 ev is 0.254164 barns, the equation for K_{th} becomes,

$$K'_{th} = \frac{\overline{\nu \Sigma_{core}^f}}{\overline{\Sigma_{core}^a} - 0.05083 \times N_H}$$

where,

$$N_H = \text{atoms hydrogen/cm}^3 \times 10^{-24} \text{ at } \rho_w = 0.998232$$

Thermal group constants for each of the cores calculated are listed in

Table 7.2.

Table 7.2 Thermal Group Constants
(Temperature Coefficient Calculations)

Case	Σ^a	D	$K_{th} (\rho_w = 0.998232)$	$K'_{th} (\rho'_w = 0.978268)$
1	0.278611	0.170003	1.450154	1.451550
2	0.272803	0.174095	1.478678	1.480062
3, 6	0.266740	0.170539	1.521331	1.522873
4	0.298993	0.169094	1.341081	1.342271
5	0.283709	0.180937	1.405919	1.407049
7	0.311538	0.168540	1.280964	1.282047
8	0.289680	0.185228	1.365158	1.366155

7.1.2 Fast Group Constants

Fast group constants were calculated using the Muft III code (GS, P-1 approximation). Input and output data are shown in Table 7.3

Table 7.3 Fast Group Constants

Input Data for Muft III

Atomic Concentrations (atoms/cm³) x 10⁻²⁴ ($\rho_w = 0.998232$)

Case	B, cm ⁻¹	U-235	O	B	C	H	SS
1	0.10587	0.0004242	0.027306	0.000067542	0.000047	0.052726	0.015096
2	0.10587	0.0004242	0.026025	0.000045947	0.000047	0.050166	0.018301
3	0.09333	0.0004242	0.027477	0.000045947	0.000047	0.053064	0.014653
4	0.09333	0.0004242	0.027010	0.000104980	0.000047	0.052142	0.015779
5	0.09333	0.0004242	0.023340	0.000045947	0.000047	0.044808	0.025046
6	0.08458	0.0004242	0.027477	0.000045947	0.000047	0.053064	0.014653
7	0.08458	0.0004242	0.026825	0.000128325	0.000047	0.051777	0.016224
8	0.08458	0.0004242	0.021734	0.000045947	0.000047	0.041601	0.029083

Atomic Concentrations (atoms/cm³) x 10⁻²⁴ ($\rho'_w = 0.978268$)

Concentrations of U²³⁵, B, C and SS are the same as for ($\rho_w = 0.998232$)

Only O and H concentrations change.

Case	O	H
1	0.026781	0.051671
2	0.025527	0.049163
3	0.026947	0.052003
4	0.026495	0.051099
5	0.022901	0.043912
6	0.026947	0.052003
7	0.026316	0.050741
8	0.021330	0.040769

Output Data

$(P_w = 0.998232)$

Core	τ	D_f	p	K_f
1	32.944255	1.317361	0.737419	1.239430
2	33.349060	1.281290	0.725444	1.228289
3	32.777767	1.319945	0.747230	1.285472
4	32.644175	1.309368	0.722394	1.167887
5	34.108830	1.217003	0.683189	1.135018
6	32.378844	1.321185	0.747790	1.286817
7	32.458093	1.304402	0.713104	1.127432
8	34.748944	1.184041	0.655134	1.086205

$(P'_w = 0.978268)$

Core	τ	D_f	p	K_f
1	34.063060	1.335057	0.733046	1.239500
2	34.473589	1.298306	0.720986	1.228560
3	33.694511	1.339730	0.743214	1.287040
4	33.747974	1.327297	0.717948	1.168480
5	35.171590	1.231184	0.678251	1.135497
6	33.476720	1.339587	0.743689	1.287529
7	33.538754	1.321992	0.708565	1.128075
8	35.777550	1.196750	0.649921	1.086122

7.1.3 Water Reflector Constants

The water reflector constants for $\rho_w = 0.998232$ were taken from Table 2.5. The group constants for $\rho'_w = 0.978268$ were calculated by assuming that,

$$\Sigma^a, \Sigma^{rf} \text{ vary as } \rho_w$$

$$D, D_f \text{ vary as } \frac{1}{\rho_w}$$

$$L^2, \tau \text{ vary as } \frac{1}{\rho_w^2}$$

Reflector properties for both water densities are shown in Table 7.4

Table 7.4 Water Reflector Properties

	$\rho_w = 0.998232$	$\rho'_w = 0.978268$
Fast Group		
$D_f, \text{ cm}$	1.671768	1.705885
$\Sigma^{rf}, \text{ cm}^{-1}$	0.047814	0.046858
$\tau, \text{ cm}^2$	34.963812	36.405440
Thermal Group		
$D, \text{ cm}$	0.14340	0.14633
$\Sigma^a, \text{ cm}^{-1}$	0.01947	0.01908
$L^2, \text{ cm}^2$	7.36518	7.66873

7.2 Comparison of Calculations and Measurements

Table 7.5 shows the calculated values of $\frac{\Delta\rho}{\Delta\rho_w}$ for each of the eight cases calculated.

Table 7.5 Reactivity Changes

Case	Size	Strips/Element		$\frac{\Delta\rho}{\Delta\rho_w}$
1	5 x 5	4.381	B-SS	0.375
2	5 x 5	6.	SS	0.384
3	6 x 6	0		0.206
4	6 x 6	11.976	B-SS	0.309
5	6 x 6	17.0937	SS	0.327
6	7 x 7	0		0.220
7	7 x 7	16.712	B-SS	0.259
8	7 x 7	23.7333	SS	0.292

Using the values in Table 7.4 and Fig. 7.1, temperature coefficients were calculated for each case. A comparison of the measured and calculated temperature coefficients is shown on Figs. 7.2, 7.3 and 7.4.

Measurements indicate that the boron poisoned cores have slightly higher temperature coefficients than the stainless steel poisoned cores but the calculations show just the opposite trend. The measurements also indicate that the clean core temperature coefficients lie between the temperature coefficients for the boron and steel poisoned cores. The calculated temperature coefficients for the clean core are significantly lower than those of the poisoned cores.

The measurements indicate only a slight variation of temperature coefficient

FIGURE 7.2
TEMPERATURE COEFFICIENT
5 X 5 CORE

with core size. The calculations show a significant increase in temperature coefficient with decrease in core size for the poisoned cores, but show only a slight change for the clean core cases.

The calculations and experiments both show an increase in temperature coefficient with increase in temperature. Except for this general trend, the calculations and measurements are not in very good agreement. This may be due in part to the fact that the model used in calculating temperature coefficients only considered the effects of a change in water density.

All of the reactivity comparisons which have been made on the clean and poisoned cores show that the calculation model consistently underestimates the reactivity. Although there is some uncertainty in all of the nuclear parameters, the largest uncertainty lies in the neutron age. Results indicate that the actual age is less than the calculated age.

The calculated temperature coefficients are very sensitive to the neutron age, since the major part of the temperature coefficient is due to the increase in fast neutron leakage with increase in temperature. Since the calculated age appears to be larger than the actual age, the calculation model tends to overestimate the temperature coefficient. This is particularly true for the 5 x 5 core arrays where the amount of fast leakage is quite large.

It is apparent that a more elaborate calculation model is required to properly account for the variation of temperature coefficient with changes in core size, core poisoning, and temperature.

8.0 Steel Reflector Experiments

Stainless steel reflectors (of various compositions) were placed at several distances from one core face and the effect on reactivity and neutron flux distribution was measured. The gamma dose rates were measured at the pressure vessel. One group of experiments were run using a laminated steel-water reflector consisting of ten alternating 1/2 inch steel slabs with 1/8 inch water gaps. Another set of measurements were taken with a solid steel reflector of up to 5 inch thickness.

8.1 Calculations

In an attempt to duplicate the experimental results, a series of multiregion, two group radial calculations were made on a clean core consisting of 45 fixed elements with various reflector regions. Thermal and fast group constants for the core are listed in Tables 2.3 and 2.4 respectively. (For the 7 x 7 core $B = 0.08458 \text{ cm}^{-1}$). Properties for the water reflector are given in Table 2.5.

Thermal properties for the laminated and solid steel reflectors were calculated using an unhardened Maxwell-Boltzmann distribution at 58⁰ F. Fast properties were computed using the Muft III Code. Properties for these materials are shown in Table 8.1.

Table 8.1 Two-Group Constants for Stainless Steel Reflectors

Atomic Concentrations (atoms/cm ³ x 10 ⁻²⁴)	Laminated Steel-Water	Solid Steel
H	0.013346	-----
O	0.006673	-----
SS	0.068089	0.085111
Thermal Group		
D	0.249031	0.303789
Σ^a	0.179638	0.219076
Fast Group		
D_f	0.992558	1.227270
τ	60.011860	150.399400
p	0.452316	0.000029

8.2 Reactivity Changes

In general, all of the reactivity coefficients were measured with control rod B (Fig. 6.1). Control rod A was withdrawn to 20.10 inches, and the 5 rod bank withdrawn 3.3 inches. All measurements were made in the clean core.

In order to make a comparison with calculations, the experimental measurements were multiplied by a factor of 5 to account for steel reflectors on all four sides and at the four corners.

8.2.1 Effect of Distance of Reflector from Core

It is seen from Fig. 8.1 that a solid reflector is more effective in increasing the reactivity of the core than a laminated reflector of the same thickness; as the

FIGURE 8.1
REACTIVITY CHANGE FOR A 5 INCH THICK
STEEL REFLECTOR (LAMINATED AND SOLID)

reflector is moved further from the core, the difference decreases until both types of reflectors have essentially the same effect upon reactivity.

8.2.2 Effect of Reflector Thickness

To see the effect of a solid reflector of various thicknesses placed against the core, theoretical and measured values of reactivity changes were made and are given in Figure 8.2. A relatively thin steel reflector actually decreases the reactivity. However, as the reflector becomes thicker, fewer neutrons are lost from the core and the reactivity increases. It is seen that a 5 inch reflector is nearly optimum since the effect on reactivity rapidly decreases for steel thicknesses greater than about 5 inches.

A positive reactivity coefficient for both the laminated and solid steel reflector was measured. It is apparent that a negative component to the coefficient is very pronounced near the surface of the core. After approximately 1/2 inch of steel is placed against the core, this negative component does not vary greatly. The effect of this negative component can also be seen by comparing the solid reflector measurements made perpendicular to the fuel plates to those made parallel to the fuel plates. The presence of the side plate in the fuel element in the perpendicular case has already contributed significantly to the negative component. The positioning of a steel reflector against the side plate face (perpendicular to fuel plates) does not result in as large a negative component as in the case parallel to plates. If the core were to have a skirt as in the APPR-1, it could be expected that the net positive effect of adding a stainless steel reflector would be larger than those measured, provided the construction of the skirt did not prevent a close positioning of the reflector against the core. It should be pointed out that with an all water reflector essentially no change in core reactivity was found when the fuel elements

FIGURE 8.2
REACTIVITY CHANGE FOR A SOLID
STEEL REFLECTOR OF VARYING
THICKNESS.

were rotated 90 degrees so that the outside fuel elements all had plates perpendicular to the face.

8.3 Flux Distributions

Radial flux distributions in the core and the various reflectors were measured by irradiating bare indium and cadmium covered indium foils. In all of the traverses, control rods A and B were withdrawn 20.5 inches. The five rod bank was withdrawn approximately 3.00 inches. The traverses were made parallel to the fuel plates and perpendicular to the steel reflector. The radial traverse in the core was made at an axial elevation of 9.0 inches and through fuel element column 5 to avoid the discontinuities in the power traverse at the control rod positions. In all cases, the average bare indium activity in the core was normalized to unity.

For the laminated steel-water reflector, neutron flux measurements were made with the reflector against the core, 1 inch away, and 2 inches away from the core. The traverses extended to a distance of 23.75 inches from the core centerline, i. e. to the pressure vessel. These data are presented in Figures 8.3, 8.4, and 8.5.

Traverses similar to the laminated reflector measurements were made for a solid steel reflector. Bare indium traverses were made with 5 inches solid stainless steel against the core and 2 inches away from the core. These traverses are shown in Figures 8.6 and 8.7. For purposes of comparison, the bare and cadmium covered indium traverses for the case of a pure water reflector are shown on Figure 8.8.

8.4 Gamma Dose Rates

To see the effect of various reflectors upon the gamma dose rate at the pressure vessel (23.75 inches from the core centerline), the dose rate was measured using film. In all cases, a 5 inch steel reflector was used. The measurements

FIGURE 8.3

FIGURE 8. 4

FIGURE 8.5

FIGURE 8.6

FIGURE 8.7

FIGURE 8.8

were made at the 11.00 inch axial position and are listed in Table 8.2.

Table 8.2 Gamma Dose at Pressure Vessel

Case	Distance from Core Centerline (inches)	Dose μ /watt-hr
Water reflector no thermal shield	23.75	6.00
Water reflector 2 inch thermal shield	23.75	1.28
Laminated reflector against core	23.75	1.33
Laminated reflector 1 inch from core	23.75	1.19
Laminated reflector 2 inches from core	23.75	1.03
Solid reflector against core	23.75	1.52
Solid reflector 2 inches from core	23.75	1.01

It is seen that any type of steel reflector or thermal shield is effective in reducing the dose rate considerably below the case of a water reflector.

8.5 Summary of Results

The results show that as far as reducing the gamma dose rate is concerned, a laminated reflector is essentially as effective as a solid reflector. The principal difference lies in the effect upon core reactivity as discussed in Section 8.2. For the laminated reflector, it is seen that by increasing the distance between the core and reflector, the neutron flux at the pressure vessel is essentially the same; however, the average thermal flux in the steel reflector decreases so that the

quantity of capture gammas produced decreases. On the other hand, increasing this distance results in a lower reactivity gain in the core. The same trends are true for the solid reflector but the effect on reactivity is more pronounced. It is seen that the optimum distance between the core and reflector depends upon whether increased reactivity or decreased gamma dose rate is desired. In order to make a compromise between both factors, a 1 inch gap is nearly optimum.

9.0 Shielding Measurements

Gamma and neutron flux measurements were made through a mock-up of the APPR-1 core, thermal shield, pressure vessel and shielding rings as reported in APAE 21.⁽¹⁾

9.1 Gamma Attenuation

A comparison between the measured and calculated gamma dose rates from the core through the second shield ring is shown in Fig. 9.1. The calculated gamma dose was based upon BSR data and ORNL Lid Tank data.⁽⁴⁰⁾ A water density correction was made in the following manner. For a given distance Z_1 from the APPR-1 with an equivalent radius R_1 , an effective distance Z_{eff} was used with BSR measurements such that the total water mass per unit area was the same. A water density of unity was used for the BSR data. An inverse square correction was made to account for geometry differences. The attenuation of gammas by the annular steel rings was determined from ORNL Lid Tank data. The curve used for the iron correction is given in APAE-3.⁽⁴¹⁾

An excellent agreement exists between the measured gamma source strength at the surface of the APPR-1 core and at the surface of the BSR. The measured value at the BSR is 50 μ /watt hour while the measured value at the APPR-1 is 54 μ /watt hour. The higher attenuation shown through the thermal shield, pressure vessel and shield rings for the APPR gammas indicates a softer gamma leakage spectrum at the surface of the APPR-1 core.

9.2 Neutron Attenuation

The comparison between the measured and calculated neutron flux from the core through the second shield ring is shown in Figs. 9.2 and 9.3. The measured values were taken with bare and cadmium covered indium foils. Twenty mil cadmium

FIG. 9.2

covers were used, for which the corresponding cadmium cutoff is 0.44 ev. The same procedure and group constants given in Section 8.0 were used in obtaining the calculated neutron flux distribution.

The comparison between the calculated fast flux and the flux measured by cadmium covered indium is shown in Figure 9.2. The calculated fast flux was normalized to the same value as the cadmium covered indium activity at the core reflector interface.

The comparison between the calculated thermal flux and the flux measured by bare indium minus cadmium covered indium activation is shown in Fig. 9.3. The calculated thermal flux was normalized to the measured peak in the reflector.

Excellent agreement exists between the calculated and measured fast flux as shown on Fig. 9.2. The agreement between the calculated and measured thermal flux shown in Fig. 9.3 is also quite good, except that the calculated values show a flux depression in the void between the pressure vessel and the first shield ring. This is probably due to the fact that diffusion theory breaks down for void regions. However, good agreement exists on both sides of the void and in the primary shield itself. It should be noted that two group theory gives a good estimate of the thermal flux at the chamber position.

FIG. 9.3

APPENDIX A

MICROSCOPIC CROSS-SECTIONS

1 Thermal Group

1.1 Absorption Cross-Sections

The average absorption cross-section can be obtained by integrating over the thermal neutron energy spectrum. (7)

$$\bar{\sigma}^a = \frac{\int_0^{0.025 \text{ eV}} \sigma^a(E) \Phi(E) dE}{\int_0^{0.025 \text{ eV}} \Phi(E) dE}$$

By extending the upper limit of integration to infinity and assuming a Maxwell-Boltzmann distribution about the effective neutron temperature, T_n , the average cross-section for a $1/v$ absorber is given by

$$\bar{\sigma}^a = 0.886 \sigma^a(E_n) = 0.886 \sqrt{\frac{0.025}{E_n}} \sigma^a(0.025 \text{ eV})$$

Where $\sigma^a(E_n)$ = absorption cross-section at a neutron energy, E_n

For a non- $1/v$ absorber

$$\bar{\sigma}^a = 0.886 \sigma^a(E_n) \times F(E_n)$$

Where $F(E_n)$ = non- $1/v$ factor.

The only non- $1/v$ absorber in the APPR-1 is ^{235}U . At $E_n = 0.0331 \text{ e.v.}$,
 $F(E_n) = 0.9788$ (42)

1.2 Transport and Scattering Cross-Sections

For all atoms except hydrogen the scattering cross-section and average scattering angle can be assumed constant in the thermal group. Scattering cross-sections were taken from BNL-325 (42). The average scattering angle in the laboratory system is given by

$$\bar{\mu} = \frac{2}{3A}$$

where A = atomic weight of nuclide.

The transport cross-section is given by

$$\bar{\sigma}^T = \bar{\sigma}^S(1 - \bar{\mu}) + \bar{\sigma}^a$$

The transport cross-section of hydrogen is complicated by chemical binding in the water molecule. The effective mass of hydrogen in the water molecule is given by (43)

$$\frac{1}{M_{eff}(E)} = \sqrt{\frac{80}{\sigma_H^S(E)}} - 1$$

where

$\sigma_H^S(E)$ = microscopic scattering cross-section of hydrogen, barns.

The average values of the cosine of the scattering angle in the laboratory system is

$$\bar{\mu}_H(E) = \frac{2}{3M_{eff}(E)}$$

The transport cross-section of hydrogen is given by

$$\sigma_H^T(E) = \sigma_H^a(E) + \sigma_H^S(E) [1 - \bar{\mu}_H(E)]$$

The average transport cross-section can be obtained by integration over the thermal neutron energy spectrum. (7)

$$\frac{1}{\sigma_H^T} = \frac{\int_0^{0.2 \text{ ev}} \frac{1}{\sigma_H^T(E)} \Phi(E) dE}{\int_0^{0.2 \text{ ev}} \Phi(E) dE}$$

The transport cross-section for hydrogen is very nearly $1/v$ between 0.008 ev and 0.2 ev. The flux distribution was assumed to be a Maxwell-Boltzmann about the neutron temperature, T_n . An analytical integration was made by extending the upper limit of integration to infinity.

This gave

$$\bar{\sigma}_H^{tr} = 0.752 \bar{\sigma}_H^{tr}(E_n)$$

where

$$\bar{\sigma}_H^{tr}(E_n) = \text{transport cross-section for hydrogen at a neutron energy, } E_n.$$

The average scattering cross-section for hydrogen was found by

$$\bar{\sigma}_H^s = \frac{\bar{\sigma}_H^{tr} - \bar{\sigma}_H^a}{1 - \bar{\mu}_H(E_n)}$$

The above procedure for treating hydrogen was checked by calculating the diffusion length in pure water. This gave a value of $L = 2.71$ cm. Experimental values range from 2.67 cm to 2.86 cm. (44) (45) (46) (47)

1.3 Stainless Steel Cross-Sections

The major constituent in stainless steel is iron. Addition of Ni, Cr, and Mn tends to increase the absorption cross-section per atom while the scattering cross-section per atom decreases.

A weighted average of the thermal cross-section for the Fe, Ni, Cr and Mn in a clean fixed fuel element was calculated. It was found that the absorption cross-section for S.S. was 1.165 times that of Fe, while the scattering cross-section for S.S. was 0.908 times that of Fe. Accordingly it was assumed that,

$$\bar{\sigma}_{ss}^a = 1.165 \bar{\sigma}_{Fe}^a$$

$$\bar{\sigma}_{ss}^s = 0.908 \bar{\sigma}_{Fe}^s$$

Thermal Cross - Sections at 68°F

(Averaged over a Maxwell-Boltzmann distribution about 0.0331 e.v.)

Atom	$\bar{\sigma}^s$	$1-\bar{\mu}$	$\bar{\sigma}^a$	$\bar{\sigma}^{tr} (*)$
U-235 ***	9.0	0.997165	512.656442	521.630
U-238	9.0	0.997201	2.118036	11.093
H	34.32	0.784	0.254164	27.17
O	3.8	0.958334	0.000000	3.642
B	4.4	0.938420	580.634607	584.764
C	4.8	0.944491	0.002465	4.536
Si	2.25	0.976242	0.100125	2.297
P	3.4	0.978481	0.146337	3.473
S	1.1	0.979206	0.377396	1.454
Mo	6.0	0.993052	1.925488	7.883
Mn	2.9	0.987864	10.166574	13.032
Ni	17.5	0.988641	3.542897	20.844
Cr	4.3	0.987182	2.233566	6.479
Fe	11.5	0.988064	1.948593	13.312
SS(**)	10.44	0.988	2.270275	12.586

Notes * $\bar{\sigma}^{tr} = \bar{\sigma}^s(1-\bar{\mu}) + \bar{\sigma}^a$

** The cross sections given for stainless steel are based upon an average of the cross-sections for Fe, Cr, Ni and Mn weighted by their relative atomic concentrations.

*** $\bar{\sigma}_{15}^+ = 1065.139411$

2.0 Fast Group Cross-Sections

2.1 Materials

Uranium

The microscopic data used in the multigroup calculation were based on the best available data (42) and theory. (48) (49) The values used in the Muft III calculations were obtained from WAPD. (9)

Boron

The absorption cross-section was assumed to be $1/v$ from thermal energy to 10 Mev. The scattering cross section was neglected.

Hydrogen, Oxygen and Carbon

The multigroup cross sections were obtained from the KAPL Lost file. (50) The Muft III cross-section file was obtained from WAPD (9) .

Iron

The total cross section curve for iron indicates that there are many unresolved resonances between 0.008 Mev and 10 Mev. These are believed to be primarily scattering resonances since there are no sharp peaks. However, the measured resonance integral ⁽⁵¹⁾ for iron (2.1 barns) is higher than the value to be expected if the iron absorption cross-section followed simple $(1/v)$ behavior. (1.2 barns) It appears that there must be some resonance absorption cross-section distributed somewhere between 10 Mev. and cadmium cut-off energy.

The iron cross-section was assumed to be $1/v$ from thermal out to 20 e.v. Beyond 20 e.v. the iron cross-section was taken to be 0.082 barns. This procedure gives a value of 2.1 barns for the iron resonance integral.

Inelastic cross-sections for iron were obtained by averaging values given in BNL-325 over the various group widths. Elastic scattering cross-sections were

obtained from the KAPL Lost file. (50)

Stainless Steel

The resonance integral for stainless steel was calculated from the published values for Fe, Cr, Ni and Mn (51). This showed $R.I. (S.S.) = 1.12 R.I. (Fe)$. Within the accuracy with which resonance integrals can be measured, this is practically the same ratio on the thermal value. (1.165) Therefore it was assumed that for each group,

$$\bar{\sigma}_{SS}^a = 1.165 \bar{\sigma}_{Fe}^a$$

Since elastic scattering cross-sections are nearly the same in the thermal group and in most of the fast groups for Fe, Ni, Cr and Mn it was assumed that in every group,

$$\bar{\sigma}_{SS}^s = 0.908 \bar{\sigma}_{Fe}^s$$

It was assumed that the mixture of Fe, Ni, Cr and Mn was effectively equal to Fe as far as inelastic scattering cross-section was concerned. Therefore,

$$\bar{\sigma}_{SS}^{in} = \bar{\sigma}_{Fe}^{in}$$

2.2 Muft III Cross-Section File

The Muft III microscopic cross-section file is listed on the following pages. Some quantities which apparently should not be zero are listed as zero because the code does not use these values in the computation. For example, $\xi \sigma^s$ for hydrogen is listed as zero because the Muft III treats the slowing down density in hydrogen separately from the continuous slowing down density in other materials.

Muft III

Energy - Lethargy Group Structure

(Energy-Lethargy Values Correspond to Bottom of Each Group)

Group	Energy (ev)	Lethargy, u	Δu
0	10×10^6	0.00	----
1	7.79	0.25	0.25
2	6.07	0.50	0.25
3	4.72	0.75	0.25
4	3.68	1.00	0.25
5	2.86	1.25	0.25
6	2.23	1.50	0.25
7	1.74	1.75	0.25
8	1.35	2.00	0.25
9	1.05	2.25	0.25
10	821×10^3	2.50	0.25
11	639.	2.75	0.25
12	498.	3.00	0.25
13	387.	3.25	0.25
14	302.	3.50	0.25
15	235.	3.75	0.25
16	183.	4.00	0.25
17	143.	4.25	0.25
18	111.	4.50	0.25
19	86.5	4.75	0.25
20	67.4	5.00	0.25
21	40.9	5.50	0.50
22	24.8	6.00	0.50
23	15.0	6.50	0.50
24	9.12	7.00	0.50
25	5.53	7.50	0.50
26	3.35	8.00	0.50
27	2.03	8.50	0.50
28	1.23	9.00	0.50
29	750×10^0	9.50	0.50
30	454.	10.00	0.50
31	275.	10.50	0.50
32	167.	11.00	0.50
33	130.	11.25	0.25
34	101.	11.50	0.25
35	78.7	11.75	0.25
36	61.3	12.00	0.25
37	47.8	12.25	0.25
38	37.2	12.50	0.25
39	29.0	12.75	0.25

Group	Energy (ev)	Lethargy, u	Δu
40	22.6	13.00	0.25
41	17.6	13.25	0.25
42	13.7	13.50	0.25
43	10.7	13.75	0.25
44	8.32	14.00	0.25
45	6.50	14.25	0.25
46	5.10	14.50	0.25
47	3.97	14.75	0.25
48	3.06	15.00	0.25
49	2.38	15.25	0.25
50	1.855	15.50	0.25
51	1.440	15.7538	0.2538
52	1.125	16.00	0.2462
53	0.835	16.30	0.3000
54	0.625	16.5884	0.2884
55	0.500	16.8114	0.2230
56	0.400	17.0350	0.2236
57	0.315	17.2730	0.2380
58	0.248	17.5110	0.2380
59	0.196	17.750	0.2390

MUFT III
MICROSCOPIC CROSS SECTIONS

GROUP	$\bar{\mu}$	$\bar{\xi}$	MATERIAL $\bar{\sigma}_{50}$	HYDROGEN $\bar{\sigma}_c$	$\bar{\sigma}_m$	$\bar{\sigma}_f$	ν
1	.666666	.000000	1.02500	.00000	.0000	.0000	.0000
2	.666666	.000000	1.26250	.00000	.0000	.0000	.0000
3	.666666	.000000	1.54250	.00000	.0000	.0000	.0000
4	.666666	.000000	1.83250	.00000	.0000	.0000	.0000
5	.666666	.000000	2.16500	.00000	.0000	.0000	.0000
6	.666666	.000000	2.53500	.00000	.0000	.0000	.0000
7	.666666	.000000	2.93250	.00000	.0000	.0000	.0000
8	.666666	.000000	3.37250	.00000	.0000	.0000	.0000
9	.666666	.000000	3.87000	.00000	.0000	.0000	.0000
10	.666666	.000000	4.43500	.00000	.0000	.0000	.0000
11	.666666	.000000	5.06500	.00000	.0000	.0000	.0000
12	.666666	.000000	5.75000	.00000	.0000	.0000	.0000
13	.666666	.000000	6.52500	.00000	.0000	.0000	.0000
14	.666666	.000000	7.37500	.00000	.0000	.0000	.0000
15	.666666	.000000	8.27500	.00000	.0000	.0000	.0000
16	.666666	.000000	9.28000	.00000	.0000	.0000	.0000
17	.666666	.000000	10.40500	.00000	.0000	.0000	.0000
18	.666666	.000000	11.55000	.00000	.0000	.0000	.0000
19	.666666	.000000	12.75000	.00000	.0000	.0000	.0000
20	.666666	.000000	13.95000	.00000	.0000	.0000	.0000
21	.666666	.000000	15.37500	.00000	.0000	.0000	.0000
22	.666666	.000000	17.25000	.00000	.0000	.0000	.0000
23	.666666	.000000	19.00000	.00000	.0000	.0000	.0000
24	.666666	.000000	19.87500	.00000	.0000	.0000	.0000
25	.666666	.000000	20.00000	.00000	.0000	.0000	.0000
26	.666666	.000000	20.05000	.00000	.0000	.0000	.0000
27	.666666	.000000	20.15000	.00000	.0000	.0000	.0000
28	.666666	.000000	20.35000	.00000	.0000	.0000	.0000
29	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
30	.666666	.000000	20.50000	.00000	.0000	.0000	.0000

GROUP	μ	ξ	σ_{ξ}	σ_{ϵ}	σ_{η}	σ_{ζ}	ν
31	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
32	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
33	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
34	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
35	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
36	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
37	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
38	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
39	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
40	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
41	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
42	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
43	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
44	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
45	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
46	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
47	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
48	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
49	.666666	.000000	20.50000	.00000	.0000	.0000	.0000
50	.666666	.000000	20.55000	.00000	.0000	.0000	.0000
51	.666666	.000000	20.65000	.00000	.0000	.0000	.0000
52	.666666	.000000	20.72500	.00000	.0000	.0000	.0000
53	.666666	.000000	20.87500	.00000	.0000	.0000	.0000
54	.666666	.000000	21.30000	.00000	.0000	.0000	.0000
55	.666666	.000000	21.80000	.06980	.0000	.0000	.0000
56	.666666	.000000	22.20000	.07810	.0000	.0000	.0000
57	.666666	.000000	22.60000	.08760	.0000	.0000	.0000
58	.666666	.000000	23.10000	.08990	.0000	.0000	.0000
59	.666666	.000000	23.70000	.11120	.0000	.0000	.0000

GROUP	μ	ξ	MATERIAL σ_0	NATURAL σ_c	BORON σ_m	σ_4	ν
1	.000000	.000000	.00000	.00000	.0000	0.0000	.0000
2	.000000	.000000	.00000	.04542	.0000	0.0000	.0000
3	.000000	.000000	.00000	.05149	.0000	0.0000	.0000
4	.000000	.000000	.00000	.05835	.0000	0.0000	.0000
5	.000000	.000000	.00000	.06613	.0000	0.0000	.0000
6	.000000	.000000	.00000	.07496	.0000	0.0000	.0000
7	.000000	.000000	.00000	.08488	.0000	0.0000	.0000
8	.000000	.000000	.00000	.09622	.0000	0.0000	.0000
9	.000000	.000000	.00000	.10917	.0000	0.0000	.0000
10	.000000	.000000	.00000	.12364	.0000	0.0000	.0000
11	.000000	.000000	.00000	.13997	.0000	0.0000	.0000
12	.000000	.000000	.00000	.15862	.0000	0.0000	.0000
13	.000000	.000000	.00000	.17979	.0000	0.0000	.0000
14	.000000	.000000	.00000	.20375	.0000	0.0000	.0000
15	.000000	.000000	.00000	.23080	.0000	0.0000	.0000
16	.000000	.000000	.00000	.26159	.0000	0.0000	.0000
17	.000000	.000000	.00000	.29620	.0000	0.0000	.0000
18	.000000	.000000	.00000	.33559	.0000	0.0000	.0000
19	.000000	.000000	.00000	.38278	.0000	0.0000	.0000
20	.000000	.000000	.00000	.43113	.0000	0.0000	.0000
21	.000000	.000000	.00000	.51688	.0000	0.0000	.0000
22	.000000	.000000	.00000	.66364	.0000	0.0000	.0000
23	.000000	.000000	.00000	.85271	.0000	0.0000	.0000
24	.000000	.000000	.00000	1.09518	.0000	0.0000	.0000
25	.000000	.000000	.00000	1.40537	.0000	0.0000	.0000
26	.000000	.000000	.00000	1.81608	.0000	0.0000	.0000
27	.000000	.000000	.00000	2.33325	.0000	0.0000	.0000
28	.000000	.000000	.00000	2.97923	.0000	0.0000	.0000
29	.000000	.000000	.00000	3.82215	.0000	0.0000	.0000
30	.000000	.000000	.00000	4.90235	.0000	0.0000	.0000

GROUP	μ	$\bar{x}$	σ_x	σ_x^2	σ_m	σ_s	ν
31	.000000	.000000	.000000	6.30030	.0000	0.0000	.0000
32	.000000	.000000	.000000	8.09070	.0000	0.0000	.0000
33	.000000	.000000	.000000	9.81330	.0000	0.0000	.0000
34	.000000	.000000	.000000	11.12841	.0000	0.0000	.0000
35	.000000	.000000	.000000	12.61683	.0000	0.0000	.0000
36	.000000	.000000	.000000	14.29431	.0000	0.0000	.0000
37	.000000	.000000	.000000	16.19047	.0000	0.0000	.0000
38	.000000	.000000	.000000	18.34539	.0000	0.0000	.0000
39	.000000	.000000	.000000	20.78607	.0000	0.0000	.0000
40	.000000	.000000	.000000	23.54758	.0000	0.0000	.0000
41	.000000	.000000	.000000	26.67457	.0000	0.0000	.0000
42	.000000	.000000	.000000	30.22673	.0000	0.0000	.0000
43	.000000	.000000	.000000	34.23961	.0000	0.0000	.0000
44	.000000	.000000	.000000	38.78723	.0000	0.0000	.0000
45	.000000	.000000	.000000	43.92687	.0000	0.0000	.0000
46	.000000	.000000	.000000	49.65000	.0000	0.0000	.0000
47	.000000	.000000	.000000	56.16000	.0000	0.0000	.0000
48	.000000	.000000	.000000	63.80000	.0000	0.0000	.0000
49	.000000	.000000	.000000	72.52000	.0000	0.0000	.0000
50	.000000	.000000	.000000	82.18000	.0000	0.0000	.0000
51	.000000	.000000	.000000	93.17000	.0000	0.0000	.0000
52	.000000	.000000	.000000	105.60000	.0000	0.0000	.0000
53	.000000	.000000	.000000	120.91000	.0000	0.0000	.0000
54	.000000	.000000	.000000	140.06000	.0000	0.0000	.0000
55	.000000	.000000	.000000	159.70000	.0000	0.0000	.0000
56	.000000	.000000	.000000	178.60000	.0000	0.0000	.0000
57	.000000	.000000	.000000	200.47000	.0000	0.0000	.0000
58	.000000	.000000	.000000	225.92000	.0000	0.0000	.0000
59	.000000	.000000	.000000	254.37000	.0000	0.0000	.0000
60	.000000	.000000	.000000	.000000	.0000	0.0000	.0000

GROUP			MATERIAL	CARBON			
	$\bar{\mu}$	$\bar{f}$	$\bar{\sigma}_0$	$\bar{\sigma}_c$	$\bar{\sigma}_n$	$\bar{\sigma}_t$	ν
1	.650000	.053000	.64000	.00000	.0000	.0000	.0000
2	.558500	.072900	.67000	.00000	.0000	.0000	.0000
3	.452250	.091750	.73000	.00000	.0000	.0000	.0000
4	.365250	.106650	.85000	.00000	.0000	.0000	.0000
5	.291750	.117800	1.06000	.00000	.0000	.0000	.0000
6	.236250	.126700	1.34750	.00000	.0000	.0000	.0000
7	.202500	.132400	1.72250	.00000	.0000	.0000	.0000
8	.181250	.136300	2.04500	.00000	.0000	.0000	.0000
9	.165000	.139150	2.33500	.00000	.0000	.0000	.0000
10	.150500	.141550	2.69500	.00000	.0000	.0000	.0000
11	.137500	.144100	2.99500	.00000	.0000	.0000	.0000
12	.125250	.146250	3.26500	.00000	.0000	.0000	.0000
13	.114000	.148050	3.57500	.00000	.0000	.0000	.0000
14	.104000	.149600	3.82000	.00000	.0000	.0000	.0000
15	.094500	.151050	4.01000	.00000	.0000	.0000	.0000
16	.083750	.152600	4.21000	.00000	.0000	.0000	.0000
17	.073750	.153700	4.35000	.00000	.0000	.0000	.0000
18	.064000	.154600	4.42500	.00000	.0000	.0000	.0000
19	.049000	.155400	4.47000	.00000	.0000	.0000	.0000
20	.032500	.156150	4.50500	.00000	.0000	.0000	.0000
21	.017500	.156700	4.53500	.00000	.0000	.0000	.0000
22	.000000	.157200	4.57500	.00000	.0000	.0000	.0000
23	.000000	.157700	4.65000	.00000	.0000	.0000	.0000
24	.000000	.157950	4.70000	.00000	.0000	.0000	.0000
25	.000000	.158000	4.70000	.00000	.0000	.0000	.0000
26	.000000	.158000	4.70000	.00000	.0000	.0000	.0000
27	.000000	.158000	4.70000	.00000	.0000	.0000	.0000
28	.000000	.158000	4.70000	.00000	.0000	.0000	.0000
29	.000000	.158000	4.70000	.00000	.0000	.0000	.0000
30	.000000	.158000	4.70000	.00000	.0000	.0000	.0000

GROUP	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_0$	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	$\bar{\nu}$
31	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
32	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
33	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
34	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
35	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
36	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
37	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
38	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
39	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
40	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
41	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
42	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
43	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
44	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
45	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
46	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
47	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
48	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
49	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
50	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
51	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
52	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
53	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
54	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
55	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
56	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
57	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
58	.000000	.158000	4.70000	.000000	.00000	.00000	.00000
59	.000000	.158000	4.70000	.000000	.00000	.00000	.00000

GROUP			MATERIAL	OXYGEN			
	μ	ξ	σ_{ξ_0}	σ_{ξ^*}	σ_{in}	σ_4	γ
1	.718000	.040100	.51500	.27500	.0000	.0000	.0000
2	.668000	.040100	.86500	.22000	.0000	.0000	.0000
3	.618000	.046450	.71500	.18000	.0000	.0000	.0000
4	.569250	.053100	1.59500	.14500	.0000	.0000	.0000
5	.479250	.064550	2.03500	.06000	.0000	.0000	.0000
6	.362500	.081250	1.39500	.00000	.0000	.0000	.0000
7	.275750	.092100	1.62500	.00000	.0000	.0000	.0000
8	.228000	.096550	3.08000	.00000	.0000	.0000	.0000
9	.201000	.099650	4.43000	.00000	.0000	.0000	.0000
10	.176250	.102750	3.89000	.00000	.0000	.0000	.0000
11	.154000	.105750	3.22000	.00000	.0000	.0000	.0000
12	.133950	.108550	5.20500	.00000	.0000	.0000	.0000
13	.115700	.110800	6.26500	.00000	.0000	.0000	.0000
14	.100800	.112350	4.58500	.00000	.0000	.0000	.0000
15	.088800	.113800	3.81000	.00000	.0000	.0000	.0000
16	.073750	.115350	3.65500	.00000	.0000	.0000	.0000
17	.060000	.116400	3.57000	.00000	.0000	.0000	.0000
18	.047500	.117000	3.51500	.00000	.0000	.0000	.0000
19	.032500	.117700	3.50000	.00000	.0000	.0000	.0000
20	.017500	.118300	3.50000	.00000	.0000	.0000	.0000
21	.007500	.118650	3.50000	.00000	.0000	.0000	.0000
22	.002500	.118950	3.50000	.00000	.0000	.0000	.0000
23	.000000	.119250	3.60000	.00000	.0000	.0000	.0000
24	.000000	.119700	3.72500	.00000	.0000	.0000	.0000
25	.000000	.120000	3.77500	.00000	.0000	.0000	.0000
26	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
27	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
28	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
29	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
30	.000000	.120000	3.80000	.00000	.0000	.0000	.0000

GROUP	μ	ξ	σ_{ξ}	σ_{ϵ}	σ_{η}	σ_{ζ}	ν
31	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
32	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
33	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
34	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
35	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
36	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
37	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
38	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
39	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
40	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
41	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
42	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
43	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
44	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
45	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
46	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
47	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
48	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
49	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
50	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
51	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
52	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
53	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
54	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
55	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
56	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
57	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
58	.000000	.120000	3.80000	.00000	.0000	.0000	.0000
59	.000000	.120000	3.80000	.00000	.0000	.0000	.0000

GROUP	MATERIAL		FE				
	$\bar{\mu}$	$\bar{w}$	$\bar{\sigma}_o$	$\bar{\sigma}_c$	$\bar{\sigma}_m$	$\bar{\sigma}_f$	ν
1	.010640	.035290	1.72000	.08200	1.3800	.0000	.0000
2	.010640	.035290	2.03000	.08200	1.3700	.0000	.0000
3	.010640	.035290	2.25000	.08200	1.3500	.0000	.0000
4	.010640	.035290	2.29000	.08200	1.3100	.0000	.0000
5	.010640	.035290	2.12000	.08200	1.1800	.0000	.0000
6	.010640	.035290	2.20000	.08200	1.0000	.0000	.0000
7	.010640	.035290	2.20000	.08200	.8000	.0000	.0000
8	.010640	.035290	2.18000	.08200	.6200	.0000	.0000
9	.010640	.035290	2.03000	.08200	.4700	.0000	.0000
10	.010640	.035290	2.59700	.08200	.2000	.0000	.0000
11	.010640	.035290	2.39600	.08200	.0000	.0000	.0000
12	.010640	.035290	3.09600	.08200	.0000	.0000	.0000
13	.010640	.035290	4.04400	.08200	.0000	.0000	.0000
14	.010640	.035290	3.04500	.08200	.0000	.0000	.0000
15	.010640	.035290	3.09600	.08200	.0000	.0000	.0000
16	.010640	.035290	3.54500	.08200	.0000	.0000	.0000
17	.010640	.035290	3.39500	.08200	.0000	.0000	.0000
18	.010640	.035290	3.44600	.08200	.0000	.0000	.0000
19	.010640	.035290	4.39400	.08200	.0000	.0000	.0000
20	.010640	.035290	3.24500	.08200	.0000	.0000	.0000
21	.010640	.035290	5.29300	.08200	.0000	.0000	.0000
22	.010640	.035290	4.64300	.08200	.0000	.0000	.0000
23	.010640	.035290	4.84300	.08200	.0000	.0000	.0000
24	.010640	.035290	6.96500	.08200	.0000	.0000	.0000
25	.010640	.035290	6.99000	.08200	.0000	.0000	.0000
26	.010640	.035290	6.09100	.08200	.0000	.0000	.0000
27	.010640	.035290	5.89100	.08200	.0000	.0000	.0000
28	.010640	.035290	6.34100	.08200	.0000	.0000	.0000
29	.010640	.035290	7.14000	.08200	.0000	.0000	.0000
30	.010640	.035290	8.28800	.08200	.0000	.0000	.0000

GROUP	$\bar{\mu}$	$\bar{\xi}$	$\bar{\sigma}_{sc}$	$\bar{\sigma}_c$	$\bar{\sigma}_m$	$\bar{\sigma}_4$	γ
31	.010640	.035290	9.07700	.08200	.0000	.0000	.0000
32	.010640	.035290	10.44400	.08200	.0000	.0000	.0000
33	.010640	.035290	10.85200	.08200	.0000	.0000	.0000
34	.010640	.035290	10.83500	.08200	.0000	.0000	.0000
35	.010640	.035290	10.93600	.08200	.0000	.0000	.0000
36	.010640	.035290	11.00300	.08200	.0000	.0000	.0000
37	.010640	.035290	11.08400	.08200	.0000	.0000	.0000
38	.010640	.035290	11.01000	.08200	.0000	.0000	.0000
39	.010640	.035290	11.01700	.08200	.0000	.0000	.0000
40	.010640	.035290	11.22200	.08200	.0000	.0000	.0000
41	.010640	.035290	11.39400	.08960	.0000	.0000	.0000
42	.010640	.035290	11.27300	.10160	.0000	.0000	.0000
43	.010640	.035290	11.04000	.11500	.0000	.0000	.0000
44	.010640	.035290	10.88700	.13030	.0000	.0000	.0000
45	.010640	.035290	10.83800	.14760	.0000	.0000	.0000
46	.010640	.035290	11.39700	.16680	.0000	.0000	.0000
47	.010640	.035290	11.66200	.18870	.0000	.0000	.0000
48	.010640	.035290	11.53600	.21440	.0000	.0000	.0000
49	.010640	.035290	11.07600	.24370	.0000	.0000	.0000
50	.010640	.035290	11.24400	.27610	.0000	.0000	.0000
51	.010640	.035290	11.18600	.31310	.0000	.0000	.0000
52	.010640	.035290	11.64600	.35480	.0000	.0000	.0000
53	.010640	.035290	11.39000	.40670	.0000	.0000	.0000
54	.010640	.035290	10.87500	.47110	.0000	.0000	.0000
55	.010640	.035290	10.87500	.53530	.0000	.0000	.0000
56	.010640	.035290	10.92600	.59850	.0000	.0000	.0000
57	.010640	.035290	10.91200	.67180	.0000	.0000	.0000
58	.010640	.035290	10.89500	.75710	.0000	.0000	.0000
59	.010640	.035290	10.87500	.85240	.0000	.0000	.0000

GROUP	MATERIAL		STAINLESS STEEL				
	μ	$\bar{\epsilon}$	$\bar{\sigma}_0$	$\bar{\sigma}_e$	$\bar{\sigma}_m$	$\bar{\sigma}_f$	ν
1	.010640	.035290	1.56200	.09550	1.3800	.0000	.0000
2	.010640	.035290	1.84300	.09550	1.3700	.0000	.0000
3	.010640	.035290	2.04300	.09550	1.3500	.0000	.0000
4	.010640	.035290	2.07900	.09550	1.3100	.0000	.0000
5	.010640	.035290	1.92500	.09550	1.1800	.0000	.0000
6	.010640	.035290	1.99800	.09550	1.0000	.0000	.0000
7	.010640	.035290	1.99800	.09550	.8000	.0000	.0000
8	.010640	.035290	1.97900	.09550	.6200	.0000	.0000
9	.010640	.035290	1.84300	.09550	.4700	.0000	.0000
10	.010640	.035290	2.35800	.09550	.2000	.0000	.0000
11	.010640	.035290	2.17600	.09550	.0000	.0000	.0000
12	.010640	.035290	2.81100	.09550	.0000	.0000	.0000
13	.010640	.035290	3.67200	.09550	.0000	.0000	.0000
14	.010640	.035290	2.76500	.09550	.0000	.0000	.0000
15	.010640	.035290	2.81100	.09550	.0000	.0000	.0000
16	.010640	.035290	3.21900	.09550	.0000	.0000	.0000
17	.010640	.035290	3.08300	.09550	.0000	.0000	.0000
18	.010640	.035290	3.12900	.09550	.0000	.0000	.0000
19	.010640	.035290	3.99000	.09550	.0000	.0000	.0000
20	.010640	.035290	2.94500	.09550	.0000	.0000	.0000
21	.010640	.035290	4.80600	.09550	.0000	.0000	.0000
22	.010640	.035290	4.21600	.09550	.0000	.0000	.0000
23	.010640	.035290	4.39700	.09550	.0000	.0000	.0000
24	.010640	.035290	6.32400	.09550	.0000	.0000	.0000
25	.010640	.035290	6.34700	.09550	.0000	.0000	.0000
26	.010640	.035290	5.53100	.09550	.0000	.0000	.0000
27	.010640	.035290	5.34900	.09550	.0000	.0000	.0000
28	.010640	.035290	5.75800	.09550	.0000	.0000	.0000
29	.010640	.035290	6.48300	.09550	.0000	.0000	.0000
30	.010640	.035290	7.52500	.09550	.0000	.0000	.0000

GROUP	μ	$\bar{x}$	$\bar{\sigma}_{\bar{x}}$	$\bar{\sigma}_{\bar{x}}^2$	$\bar{\sigma}_m$	$\bar{\sigma}_f$	ν
31	.010640	.035290	8.24200	.09550	.0000	.0000	.0000
32	.010640	.035290	9.48300	.09550	.0000	.0000	.0000
33	.010640	.035290	9.85300	.09550	.0000	.0000	.0000
34	.010640	.035290	9.83800	.09550	.0000	.0000	.0000
35	.010640	.035290	9.93000	.09550	.0000	.0000	.0000
36	.010640	.035290	9.99100	.09550	.0000	.0000	.0000
37	.010640	.035290	10.06400	.09550	.0000	.0000	.0000
38	.010640	.035290	9.99700	.09550	.0000	.0000	.0000
39	.010640	.035290	10.00300	.09550	.0000	.0000	.0000
40	.010640	.035290	10.18900	.09550	.0000	.0000	.0000
41	.010640	.035290	10.34600	.10440	.0000	.0000	.0000
42	.010640	.035290	10.23600	.11840	.0000	.0000	.0000
43	.010640	.035290	10.02400	.13400	.0000	.0000	.0000
44	.010640	.035290	9.88500	.15180	.0000	.0000	.0000
45	.010640	.035290	9.84100	.17200	.0000	.0000	.0000
46	.010640	.035290	10.34800	.19430	.0000	.0000	.0000
47	.010640	.035290	10.58900	.21990	.0000	.0000	.0000
48	.010640	.035290	10.47400	.24980	.0000	.0000	.0000
49	.010640	.035290	10.05700	.28390	.0000	.0000	.0000
50	.010640	.035290	10.20900	.32170	.0000	.0000	.0000
51	.010640	.035290	10.15700	.36480	.0000	.0000	.0000
52	.010640	.035290	10.57400	.41340	.0000	.0000	.0000
53	.010640	.035290	10.34200	.47380	.0000	.0000	.0000
54	.010640	.035290	9.87400	.54890	.0000	.0000	.0000
55	.010640	.035290	9.87400	.62370	.0000	.0000	.0000
56	.010640	.035290	9.92100	.69730	.0000	.0000	.0000
57	.010640	.035290	9.90800	.78270	.0000	.0000	.0000
58	.010640	.035290	9.87400	.88210	.0000	.0000	.0000
59	.010640	.035290	9.87400	.99310	.0000	.0000	.0000

GROUP	MATERIAL U-235						
	μ	ξ	$\bar{\sigma}_0$	$\bar{\sigma}_e$	$\bar{\sigma}_m$	$\bar{\sigma}_4$	ν
1	.002830	.008486	2.59500	.03100	2.0000	2.1999	3.5725
2	.002830	.008486	3.34000	.05000	2.0000	1.6750	3.3275
3	.002830	.008486	4.06000	.06000	2.0000	1.2574	3.1335
4	.002830	.008486	4.41990	.08000	2.0000	1.2724	2.9835
5	.002830	.008486	4.46990	.10300	1.9800	1.2825	2.8675
6	.002830	.008486	4.26990	.13400	1.9300	1.2899	2.7775
7	.002830	.008486	4.25990	.14100	1.8400	1.2924	2.7075
8	.002830	.008486	4.50500	.15600	1.6100	1.2874	2.6500
9	.002830	.008486	4.85990	.17300	1.2300	1.2825	2.6075
10	.002830	.008486	5.40990	.19200	1.0300	1.2774	2.5750
11	.002830	.008486	6.13000	.20900	.9200	1.2825	2.5495
12	.002830	.008486	6.96000	.22400	.8100	1.2950	2.5290
13	.002830	.008486	7.88990	.24800	.6700	1.3150	2.5100
14	.002830	.008486	8.71990	.27100	.5000	1.3549	2.4980
15	.002830	.008486	9.37000	.30000	.3200	1.4149	2.4900
16	.002830	.008486	10.00990	.35500	.1500	1.4899	2.4825
17	.002830	.008486	10.63000	.37700	.0000	1.5775	2.4760
18	.002830	.008486	11.20500	.41800	.0000	1.6725	2.4705
19	.002830	.008486	11.70500	.46800	.0000	1.7350	2.4670
20	.002830	.008486	12.17990	.52290	.0000	1.9100	2.4640
21	.002830	.008486	12.86000	.62090	.0000	2.1199	2.4615
22	.002830	.008486	13.42490	.78800	.0000	2.4700	2.4595
23	.002830	.008486	13.38000	.98890	.0000	2.9100	2.4585
24	.002830	.008486	12.85500	1.25000	.0000	3.4700	2.4580
25	.002830	.008486	12.33500	1.59000	.0000	4.1699	2.4580
26	.002830	.008486	11.96000	2.03000	.0000	5.0499	2.4580
27	.002830	.008486	11.37500	2.59000	.0000	6.1750	2.4580
28	.002830	.008486	10.23000	3.25990	.0000	7.5749	2.4580
29	.002830	.008486	8.14990	4.13990	.0000	9.3999	2.4580
30	.002830	.008486	7.52490	5.25990	.0000	11.6749	2.4580

GROUP	μ	ξ	$\bar{\sigma}_0$	$\bar{\sigma}_c$	$\bar{\sigma}_m$	$\bar{\sigma}_f$	ν
31	.002830	.008486	8.22490	6.65990	.0000	14.5000	2.4580
32	.002830	.008486	8.09990	8.40990	.0000	18.0499	2.4850
33	.002830	.008486	7.92500	10.10000	.0000	21.5000	2.4850
34	.002830	.008486	7.82490	11.50000	.0000	24.3750	2.4850
35	.002830	.008486	7.74000	13.00000	.0000	27.3750	2.4850
36	.002830	.008486	7.63990	14.60000	.0000	30.7000	2.4850
37	.002830	.008485	7.58990	16.59990	.0000	34.6999	2.4850
38	.002830	.008486	7.56500	18.79990	.0000	39.2500	2.4850
39	.002830	.008486	7.53990	21.20000	.0000	44.0000	2.4850
40	.002830	.008486	7.51490	23.20000	.0000	48.2500	2.4850
41	.002830	.008486	7.47490	24.79990	.0000	51.4499	2.4850
42	.002830	.008486	7.42500	25.59990	.0000	52.9499	2.4850
43	.002830	.008486	7.38990	25.20000	.0000	52.0000	2.4850
44	.002830	.008486	7.36000	23.29990	.0000	48.0000	2.4850
45	.002830	.008486	7.31990	18.20000	.0000	37.5000	2.4850
46	.002830	.008486	7.27490	12.00000	.0000	24.7500	2.4850
47	.002830	.008486	7.22490	8.63990	.0000	17.7500	2.4850
48	.002830	.008486	7.14990	7.41990	.0000	15.4500	2.4850
49	.002830	.008486	7.09990	9.39990	.0000	15.2500	2.4850
50	.002830	.008486	7.09990	8.79990	.0000	16.7999	2.4850
51	.002830	.008486	7.05000	8.39990	.0000	20.7500	2.4850
52	.002830	.008486	7.05000	14.50000	.0000	30.2500	2.4850
53	.002830	.008486	7.34990	10.69990	.0000	50.0000	2.4850
54	.002830	.008486	7.69990	2.69990	.0000	73.5000	2.4850
55	.002830	.008486	8.00000	11.00000	.0000	74.0000	2.4850
56	.002830	.008486	8.00000	18.00000	.0000	100.0000	2.4850
57	.002830	.008486	8.00000	30.00000	.0000	154.0000	2.4850
58	.002830	.008486	8.00000	59.00000	.0000	189.0000	2.4850
59	.002830	.008486	8.00000	51.00000	.0000	178.0000	2.4850

2.3 Prod II Cross-Section File

The Prod II (multigroup) microscopic cross-section file is listed on the following pages. For fuel materials $\Sigma \sigma^s$ and σ^{tr} are listed as zero since $\Sigma \sigma^s$ is not used and σ^{tr} is set equal to σ^a for fuel materials.

Energy-Lethargy Group Structure

(Energy-Lethargy Values Correspond to Bottom of Each Group)

Group	Energy (ev)	Lethargy, u	Δu
0	10×10^6	0.00	-----
1	6.06	0.50	0.50
2	3.68	1.00	0.50
3	1.35	2.00	1.00
4	0.498	3.00	1.00
5	0.183	4.00	1.00
6	3350×10^0	8.00	4.00
7	167.	11.00	3.00
8	37.3	12.50	1.50
9	10.7	13.75	1.25
10	3.06	15.00	1.25
11	1.13	16.00	1.00
12	0.414	17.00	1.00
13	0.196	17.75	0.75

PROD 11

MICROSCOPIC CROSS SECTIONS

MATERIAL - HYDROGEN

GROUP	$\bar{\sigma}_0$	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	$\bar{\sigma}_n$
1	1.123563	.36694	.000018	.000000	.000000
2	1.651363	.54128	.000024	.000000	.000000
3	2.671317	.86281	.000034	.000000	.000000
4	4.645381	1.50919	.000057	.000000	.000000
5	7.663053	2.50083	.000093	.000000	.000000
6	15.954918	5.07786	.000353	.000000	.000000
7	19.979062	6.66219	.002087	.000000	.000000
8	20.000000	6.67105	.006009	.000000	.000000
9	20.000000	6.68021	.011864	.000000	.000000
10	20.000000	6.69172	.022134	.000000	.000000
11	20.160599	6.87261	.038599	.000000	.000000
12	21.147327	8.01317	.063457	.000000	.000000
13	23.389624	11.68823	.097946	.000000	.000000

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MATERIAL - STAINLESS STEEL

GROUP	$\bar{\sigma}_0$	$\bar{\sigma}_1$	$\bar{\sigma}_2$	$\bar{\sigma}_3$	$\bar{\sigma}_n$
1	.093067	3.25316	.095537	.000000	1.380000
2	.093861	3.23884	.095537	.000000	1.320000
3	.087402	2.78287	.095537	.000000	.890000
4	.085132	2.26929	.095537	.000000	.200000
5	.108299	2.89219	.095537	.000000	.000000
6	.111804	1.82668	.095537	.000000	.000000
7	.258915	6.93425	.095537	.000000	.000000
8	.347795	9.98016	.095537	.000000	.000000
9	.348801	10.02311	.100551	.000000	.000000
10	.348937	10.10065	.190067	.000000	.000000
11	.349017	10.22614	.336886	.000000	.000000
12	.349300	10.40201	.554906	.000000	.000000
13	.349300	10.64399	.847599	.000000	.000000

MATERIAL - FE

GROUP	$\bar{\epsilon}_{\sigma}$	$\bar{\epsilon}_{\tau}$	$\bar{\epsilon}_{\sigma}$	$\bar{\epsilon}_{\tau}$	$\bar{\epsilon}_{\sigma}$
1	.102499	2.06300	.082000	.000000	.000000
2	.103373	2.11330	.082000	.000000	.000000
3	.096261	2.08470	.082000	.000000	.000000
4	.093760	2.27900	.082000	.000000	.000000
5	.119274	3.18530	.082000	.000000	.000000
6	.123135	2.01180	.082000	.000000	.000000
7	.285155	7.63700	.082000	.000000	.000000
8	.383042	10.99160	.082000	.000000	.000000
9	.384150	11.03890	.086304	.000000	.000000
10	.384300	11.12430	.163136	.000000	.000000
11	.384388	11.26250	.289152	.000000	.000000
12	.384700	11.45620	.476280	.000000	.000000
13	.384700	11.72270	.727500	.000000	.000000

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MATERIAL - CARBON

GROUP	$\bar{\epsilon}_{\sigma}$	$\bar{\epsilon}_{\tau}$	$\bar{\epsilon}_{\sigma}$	$\bar{\epsilon}_{\tau}$	$\bar{\epsilon}_{\sigma}$
1	.195890	1.16210	.000000	.000000	.000000
2	.233647	1.33985	.000000	.000000	.000000
3	.305273	1.82065	.000000	.000000	.000000
4	.437350	2.57138	.000001	.000000	.000000
5	.599168	3.57074	.000001	.000000	.000000
6	.726195	4.34246	.000004	.000000	.000000
7	.741600	4.43900	.000020	.000000	.000000
8	.741600	4.43900	.000059	.000000	.000000
9	.741600	4.43900	.000116	.000000	.000000
10	.741601	4.43908	.000216	.000000	.000000
11	.741601	4.43904	.000377	.000000	.000000
12	.741600	4.43904	.000621	.000000	.000000
13	.741600	4.43900	.000959	.000000	.000000

MATERIAL - OXYGEN

GROUP	$\overline{\sigma}_s$	$\overline{\sigma}_a$	$\overline{\sigma}_u$	$\overline{\sigma}_f$	$\overline{\sigma}_m$
1	.128259	.75230	.000120	.000000	.000000
2	.184646	1.00460	.000120	.000000	.000000
3	.196094	1.11920	.000120	.000000	.000000
4	.455593	3.03600	.000120	.000000	.000000
5	.576000	4.08320	.000120	.000000	.000000
6	.480000	3.83000	.000120	.000000	.000000
7	.479950	3.82950	.000120	.000000	.000000
8	.475100	3.79400	.000120	.000000	.000000
9	.468393	3.74480	.000120	.000000	.000000
10	.454509	3.62730	.000120	.000000	.000000
11	.451823	3.60510	.000124	.000000	.000000
12	.451313	3.60040	.000187	.000000	.000000
13	.456000	3.64000	.000292	.000000	.000000

MATERIAL - BORON

GROUP	$\overline{\sigma}_s$	$\overline{\sigma}_a$	$\overline{\sigma}_u$	$\overline{\sigma}_f$	$\overline{\sigma}_m$
1	.327150	1.47400	.042900	.000000	.000000
2	.309710	1.40500	.055100	.000000	.000000
3	.321880	1.50900	.080800	.000000	.000000
4	.389790	2.03300	.133200	.000000	.000000
5	.556610	3.22300	.219500	.000000	.000000
6	.737820	4.90100	.891600	.000000	.000000
7	.748200	8.19500	4.785600	.000000	.000000
8	.748200	16.86300	13.754000	.000000	.000000
9	.748200	30.19100	27.144000	.000000	.000000
10	.748200	51.29900	50.727000	.000000	.000000
11	.748200	89.71900	88.452000	.000000	.000000
12	.748200	143.92500	145.880000	.000000	.000000
13	.748200	219.30800	224.990000	.000000	.000000

MATERIAL - U-238

GROUP	$\overline{\sigma}_s$	$\overline{\sigma}_a$	$\overline{\sigma}_e$	$\overline{\sigma}_f$	$\overline{\sigma}_n$
1	.000000	.000000	.004000	.000000	.000000
2	.000000	.000000	.008000	.000000	.000000
3	.000000	.000000	.050000	.000000	.000000
4	.000000	.000000	.105000	.000000	.000000
5	.000000	.000000	.150000	.000000	.000000
6	.000000	.000000	.640000	.000000	.000000
7	.000000	.000000	4.720000	.000000	.000000
8	.000000	.000000	14.650000	.000000	.000000
9	.000000	.000000	84.320000	.000000	.000000
10	.000000	.000000	102.930000	.000000	.000000
11	.000000	.000000	.322000	.000000	.000000
12	.000000	.000000	.531000	.000000	.000000
13	.000000	.000000	.819000	.000000	.000000

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MATERIAL - U-235

GROUP	$\overline{\sigma}_s$	$\overline{\sigma}_a$	$\overline{\sigma}_e$	$\overline{\sigma}_f$	$\overline{\sigma}_n$
1	.000000	.000000	1.280000	1.280000	2.000000
2	.000000	.000000	1.280000	1.280000	2.000000
3	.000000	.000000	1.290000	1.290000	1.860000
4	.000000	.000000	1.340000	1.240000	1.000000
5	.000000	.000000	1.540000	1.340000	.400000
6	.000000	.000000	3.840000	2.940000	.000000
7	.000000	.000000	15.900000	10.900000	.000000
8	.000000	.000000	42.000000	28.000000	.000000
9	.000000	.000000	82.200000	54.800000	.000000
10	.000000	.000000	78.800000	36.200000	.000000
11	.000000	.000000	36.400000	23.500000	.000000
12	.000000	.000000	82.800000	75.000000	.000000
13	.000000	.000000	207.000000	169.000000	.000000

2.4 Inelastic Scattering Matrices

The inelastic scattering matrix for scattering from j energy groups into i energy groups was calculated by the Weisskopf statistical model. ⁵² (52) The matrix elements were calculated from ⁵³ (53)

$$X_{ji} = \text{const} \left[e^{-\frac{E_i}{\theta}} (E_i + \theta) - e^{-\frac{E_{i+1}}{\theta}} (E_{i+1} + \theta) \right] \quad E_{i+1} \geq E'$$

$$X_{ji} = \text{const} \sqrt{\theta E'} \left[\sqrt{\frac{E_i}{\theta}} e^{-\frac{E_i}{\theta}} - \sqrt{\frac{E_{i+1}}{\theta}} e^{-\frac{E_{i+1}}{\theta}} \right. \\ \left. + \frac{\sqrt{\pi}}{2} E' \sqrt{\frac{E_i}{\theta}} - \frac{\sqrt{\pi}}{2} E' \sqrt{\frac{E_{i+1}}{\theta}} \right] \quad E_{i+1} < E'$$

where

$$\theta = \sqrt{\frac{\Delta \mathcal{E}_{ji}}{\alpha}}$$

$$\Delta \mathcal{E}_{ji} = \frac{1}{2} [E_j + E_{j+1} - E_i - E_{i+1}]$$

$E' =$ error integral

$\alpha =$ value dependent on atomic number

$$= 12.6 \text{ for U235} \quad (52)$$

$$= 1.4 \text{ for S.S.} \quad (52)$$

$$E' = 0.498 \text{ Mev} \quad (53)$$

The values of X_{ji} are normalized such that,

$$\sum_{i=j+1}^I X_{ji} = 1$$

The tables which follow list the values of $g_{j \rightarrow i} = \sigma_{ji}^{\text{in}} X_{ji}$ for both the Muft III and Prod II (multigroup) files. For each group, i , the values of $g_{j \rightarrow i}$ are listed in order from $g_{1 \rightarrow i}$ through $g_{i-1 \rightarrow i}$. In the Muft III listing for U^{235} , two lines are required for some of the groups.

MUFT III
INELASTIC SCATTERING MATRIX
MATERIAL - FE AND SS

-178-

GROUP														
2	.0210	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3	.1238	.0357	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4	.2092	.1529	.0542	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5	.2372	.2304	.1831	.0773	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	.2126	.2344	.2382	.2024	.0942	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7	.1706	.2013	.2279	.2388	.2021	.1018	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
8	.1296	.1592	.1911	.2198	.2229	.1894	.1037	.0000	.0000	.0000	.0000	.0000	.0000	.0000
9	.0900	.1136	.1412	.1706	.1873	.1856	.1578	.0947	.0000	.0000	.0000	.0000	.0000	.0000
10	.0599	.0769	.0977	.1215	.1390	.1471	.1418	.1227	.0787	.0000	.0000	.0000	.0000	.0000
11	.0404	.0525	.0676	.0857	.1005	.1100	.1119	.1076	.0947	.0370	.0000	.0000	.0000	.0000
12	.0260	.0340	.0444	.0569	.0678	.0757	.0792	.0797	.0767	.0389	.0000	.0000	.0000	.0000
13	.0177	.0234	.0307	.0398	.0478	.0541	.0576	.0593	.0592	.0323	.0000	.0000	.0000	.0000
14	.0125	.0165	.0218	.0284	.0343	.0392	.0421	.0440	.0447	.0251	.0000	.0000	.0000	.0000
15	.0089	.0118	.0157	.0206	.0250	.0287	.0311	.0327	.0336	.0191	.0000	.0000	.0000	.0000
16	.0062	.0083	.0111	.0145	.0178	.0205	.0223	.0235	.0243	.0139	.0000	.0000	.0000	.0000
17	.0043	.0058	.0077	.0101	.0124	.0143	.0156	.0166	.0172	.0099	.0000	.0000	.0000	.0000
18	.0031	.0041	.0055	.0073	.0089	.0103	.0113	.0120	.0125	.0072	.0000	.0000	.0000	.0000
19	.0021	.0028	.0038	.0050	.0061	.0071	.0079	.0083	.0087	.0050	.0000	.0000	.0000	.0000
20	.0015	.0020	.0026	.0035	.0043	.0050	.0054	.0058	.0061	.0035	.0000	.0000	.0000	.0000
21	.0017	.0023	.0031	.0041	.0050	.0058	.0064	.0069	.0072	.0042	.0000	.0000	.0000	.0000
22	.0008	.0011	.0015	.0020	.0024	.0028	.0031	.0033	.0034	.0020	.0000	.0000	.0000	.0000
23	.0004	.0005	.0007	.0009	.0012	.0013	.0015	.0016	.0017	.0010	.0000	.0000	.0000	.0000
24	.0002	.0002	.0003	.0004	.0005	.0006	.0007	.0007	.0008	.0005	.0000	.0000	.0000	.0000
25	.0001	.0001	.0002	.0002	.0003	.0003	.0003	.0004	.0004	.0002	.0000	.0000	.0000	.0000
26	.0000	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0001	.0000	.0000	.0000	.0000
27	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0000	.0000	.0000	.0000

GROUP

MATERIAL - U 235

3 .0018 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
4 .0226 .0044 .0001 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
5 .0868 .0395 .0096 .0004 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
6 .1751 .1182 .0611 .0182 .0010 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
7 .2474 .2071 .1513 .0874 .0308 .0024 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
8 .2841 .2714 .2400 .1885 .1195 .0487 .0054 .0000 .0000 .0000 .0000 .0000 .0000 .0000
9 .2685 .2797 .2778 .2581 .2141 .1464 .0683 .0096 .0000 .0000 .0000 .0000 .0000 .0000
10 .2241 .2473 .2647 .2717 .2603 .2242 .1622 .0780 .0120 .0000 .0000 .0000 .0000 .0000
11 .1794 .2062 .2321 .2543 .2657 .2596 .2292 .1622 .0751 .0161 .0000 .0000 .0000 .0000
12 .1317 .1559 .1820 .2084 .2306 .2431 .2395 .2021 .1321 .0744 .0214 .0000 .0000 .0000
13 .0996 .1205 .1444 .1707 .1964 .2177 .2291 .2127 .1624 .1229 .0810 .0279 .0000 .0000
14 .0755 .0929 .1134 .1371 .1621 .1857 .2039 .2005 .1662 .1434 .1214 .0852 .0326 .0000
15 .0574 .0715 .0885 .1088 .1311 .1538 .1737 .1772 .1543 .1427 .1349 .1175 .0833 .0334
16 .0421 .0529 .0662 .0824 .1007 .1202 .1385 .1448 .1302 .1256 .1260 .1208 .1031 .0705
17 .0302 .0382 .0482 .0605 .0747 .0902 .1055 .1122 .1031 .1021 .1061 .1069 .0989 .0791
18 .0222 .0282 .0359 .0453 .0564 .0688 .0813 .0876 .0816 .0823 .0875 .0909 .0878 .0752
19 .0155 .0198 .0252 .0321 .0402 .0493 .0587 .0638 .0602 .0614 .0663 .0702 .0695 .0617
19 .0465 .0250 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
20 .0109 .0140 .0179 .0229 .0288 .0355 .0425 .0466 .0442 .0456 .0497 .0533 .0536 .0486
20 .0377 .0213 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
21 .0130 .0167 .0215 .0275 .0348 .0432 .0521 .0574 .0549 .0571 .0629 .0682 .0696 .0642
21 .0508 .0296 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
22 .0063 .0081 .0105 .0135 .0171 .0214 .0259 .0287 .0277 .0290 .0322 .0352 .0364 .0340
22 .0273 .0162 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
23 .0030 .0039 .0051 .0065 .0083 .0104 .0126 .0141 .0136 .0143 .0160 .0176 .0183 .0172
23 .0139 .0083 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000

GROUP

24	.0014	.0019	.0024	.0031	.0039	.0049	.0060	.0067	.0065	.0059	.0077	.0085	.0088	.0083
24	.0068	.0041	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
25	.0007	.0009	.0012	.0015	.0019	.0024	.0029	.0032	.0031	.0033	.0037	.0041	.0043	.0041
25	.0033	.0020	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
26	.0003	.0004	.0005	.0007	.0009	.0011	.0014	.0015	.0015	.0016	.0018	.0020	.0021	.0019
26	.0016	.0010	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
27	.0002	.0002	.0003	.0003	.0004	.0005	.0007	.0007	.0007	.0008	.0008	.0009	.0010	.0009
27	.0008	.0005	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
28	.0001	.0001	.0001	.0002	.0002	.0003	.0003	.0003	.0003	.0004	.0004	.0004	.0005	.0004
28	.0004	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
29	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0002	.0002	.0002	.0002	.0002	.0002	.0002
29	.0002	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
30	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
30	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
31	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001

PROD II
INELASTIC SCATTERING MATRIX
MATERIAL - U-235

GROUP

2	.0089	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3	.7677	.3468	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4	.9055	.1378	.0381	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5	.2631	.4203	.6508	.7105	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	.0548	.0950	.1711	.2894	.3997	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
7	.0000	.0000	.0000	.0001	.0002	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

MATERIAL - FE AND SS

GROUP

2	.2785	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
3	.8096	.8545	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
4	.2418	.3804	.7036	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
5	.0427	.0724	.1576	.1666	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
6	.0073	.0127	.0287	.0324	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

3.0 Multigroup Cross-Sections for Indium and Gold Foils

Unshielded Capture Cross-Sections (barns)

Group	Indium	Gold
1	-	0.04
2	-	0.05
3	0.08	0.09
4	0.20	0.16
5	0.33	0.31
6	0.74	1.42
7	3.1	6.82
8	9.9	31.8
9	15.5	3.57
10	78.7	1179.8 (**)
11	3590.0 (*)	11.7
12	37.2	10.2
13	57.3	29.6
Thermal	130.	76.5

Notes (*) A self-shielding factor of 0.446 was later used to account for depression of group 11 flux in the indium foil.

(**) A self-shielding factor of 0.405 was later used to account for depression of group 10 flux in the gold foil.

APPENDIX B

MATERIAL CONTENT OF APPR-1 FUEL ELEMENTS

Specification for the APPR-1 fuel elements are given in ORNL-2225⁽⁵⁴⁾.

A breakdown for the materials in each element is detailed in the following sections.

1.0 Fixed Fuel Elements

A schematic drawing of a fixed fuel element is shown on Fig. 1.3. Each element occupies a volume 2.9375 in. x 2.9375 in. x 22.0 in. It contains 18 fuel plates having a meat thickness of 0.020 in. and an active width of 2.500 in. The plates are clad on both sides with 0.005 in. of stainless steel. The total width of the clad fuel plate is 2.780 inches. There are two side plates each having a width of 2.863 in. and a thickness of 0.030 in. Each side plate contains 18 grooves, 0.036 in. wide x 0.025 in. deep, into which the fuel plates are inserted. The fuel plates are brazed to the side plates.

A breakdown of each of the materials in the fuel element is given in the following sections.

1.1 Meat (per plate)

UO₂ - 35.09 grams

87.63 wt % U in UO₂

93.07 wt % U-235 in U

B₄C - 0.1880 grams

75.9 wt % B in B₄C

18.8 atom % B-10 in B

Stainless Steel - 99.8 grams type 302B

Analysis	Wt%
Mn	0.85
Si	2.37
Ni	8.19
Cr	18.11
P	0.019
S	0.008
C	0.08
Fe (Bal)	<u>70.373</u>
	100.000

1.2 Clad, Fuel Plate Dead Edges, and Side Plates - Type 304L Stainless Steel

Density	7.90 gm/cm ³
Analysis	Wt %
Mn	2.0
Si	1.0
Ni	9.0
Cr	19.0
P	0.04
S	0.03
C	0.028
Fe (Bal)	<u>68.902</u>
	100.000

1.3 Braze

Density	7.56 gm/cm ³
32.84 grams/element	
Analysis	Wt %
Mn	0.72
Si	11.30
Ni	49.46
Mo	7.55
P	3.98
Cr	0.41
Fe (Bal)	26.58
	100.00

2. Control Rod Fuel Elements

A sketch of a control rod fuel element is shown on Fig. 4.2. Each element occupies a volume 2.9375 in. x 2.9375 in. x 22.0 in. There are 16 fuel plates having a meat thickness of 0.020 in. and an active width of 2.281 in. The plates are clad with 0.005 in. of stainless steel. The total width of the clad fuel plate is 2.560 in. There are two side plates each having a width of 2.619 in. and a thickness of 0.050 in. Each side plate contains 16 grooves, 0.036 in. wide x 0.025 in. deep into which the fuel plates are inserted. The fuel plates are brazed to the side plates. There is a control rod basket, 0.050 in. thick with outside dimensions 2.78125 in. x 2.78125 in.

A breakdown of each of the materials in the fuel element is given in the following sections.

2.1 Meat (Per Plate)

UO₂ - 32.01 grams

B₄C - 0.1717 grams

Stainless Steel - 91.56 grams, type 302B

2.2 Clad, Fuel Plate Dead Edges, Side Plates and Basket

Type 304 L Stainless Steel

2.3 Braze

29.19 grams

3.0 Atomic Concentrations in Fuel Elements

Both the fixed fuel elements and control rod elements occupy a cell volume of 2.9375 in. x 2.9375 in. x 22.0 in. A breakdown of volumes of each region of the fuel element and the atomic concentrations in each of the regions is shown in the following tables. The nomenclature followed is that used in Section 1.4.2.2.

Volumetric Breakdown for Fuel Elements (cm³)

Region	Fixed Fuel Elements		Control Rod Elements
Meat	324.464		263.147
Clad	<u>162.232</u>		<u>131.574</u>
Plates		486.696	394.721
Channels		<u>2160.839</u>	<u>1752.483</u>
Active		2647.535	2147.204
Braze	4.344		3.862
Side Plates	91.535		84.036
Dead Edges	54.510		48.280
Basket			196.931
Water Gap	<u>312.930</u>		<u>630.541</u>
Dead		<u>463.319</u>	<u>963.650</u>
Total		3110.854	3110.854

Atomic Densities in Fuel Element Regions

(Given in atoms/cm³ region x 10²⁴)

Atom	Either Element		Fixed Elements		Control Rod Elements	
	Plate	Channel	Dead	Core	Dead	Core
U-235	0.0027114	-----	-----	0.0004242	-----	0.0003440
U-238	0.000199	-----	-----	0.000031	-----	0.0000253
H	-----	.066730	0.045070	0.053064	0.043663	0.051116
O	0.006043	.033365	0.022535	0.027477	0.021831	0.026325
B	0.00029368	-----	-----	0.000045947	-----	0.000037264
C	0.000269	-----	0.000035	0.000047	0.000038	0.000046
P	0.000033	-----	0.000074	0.000016	0.000044	0.000018
S	0.000020	-----	0.000014	0.000005	0.000015	0.000007
Si	0.002442	-----	0.000706	0.000487	0.000652	0.000512
Mo	-----	-----	0.000034	0.000005	0.000014	0.000004
Mn	0.000928	-----	0.000552	0.000227	0.000591	0.000298
Ni	0.005534	-----	0.002660	0.001262	0.002647	0.001522
Cr	0.013535	-----	0.005482	0.002934	0.005939	0.003556
Fe	0.047578	-----	0.018706	0.010230	0.020143	0.012276
SS*	0.067575	-----	0.027400	0.014653	0.029320	0.017652

* SS = Mn+Ni+Cr+Fe

4.0 Effect of Poison Strips on Atomic Concentrations

The sections which follow show the change in atomic concentrations produced by the addition of one poison strip per element. The composition, weight, dimensions and volume of the various poison strips are given in the following tables.

Composition of Poison Strips

Stainless Steel		(Allegheny-Ludlum heat #335249)
		Weight Per Cent
C		0.066
Mn		1.54
P		0.030
S		0.010
Si		0.70
Cr		18.25
Ni		9.08
Fe		Balance

1.01% Boron - Stainless Steel		(Superior Steel Corporation)
		Weight Per Cent
C		0.09
Mn		0.66
Si		0.70
Cr		17.88
Ni		10.50
B		1.01
Fe		Balance

1.16% Boron - Stainless Steel

(Superior Steel Corporation)
Weight Per Cent

C	0.07
Mn	0.77
Si	0.62
Cr	18.57
Ni	11.17
B	1.16
Fe	Balance

Dimensions, Volume and Weights

	S.S.	1.01% B-S.S.	1.16% S.S.
Length, in.	46.0	22.0	22.0
Width, in.	0.500	0.500	0.500
Thickness, in.	0.0625	0.020	0.020
Volume, cm ³	23.556	3.605	3.605
Weight, gms.	183.0	27.27	28.18

4.1 Equivalence Factor Between 1.01% Boron Strips and 1.16% Boron Strips

The relative worth of each strip was compared by four methods.

Method	$\frac{1.16\% \text{ Boron Strip}}{1.01\% \text{ Boron Strip}}$
Boron content	1.187
Thermal absorption cross-section	1.177
Fully poisoned core analysis	1.170
Importance function measurements	1.19

For calculational purposes each 1.16% Boron strip was assumed equal to 1.177 strips having a 1.01% boron content.

4.2 Effect of Addition of One Boron Strip (1.01% Boron) per Element)

Addition of one boron strip changes the atomic densities in the channel between the fuel plates and in the whole fuel element as shown in the following table.

Change in Atomic Density (atoms/cm³ x 10⁻²⁰) by One Boron Strip (1.01%B) per Element

Atom	Fixed Fuel Element		Control Rod Fuel Element	
	Channel	Core	Channel	Core
H	-1.11	-0.77	-1.37	-0.77
O	-0.56	-0.39	-0.69	-0.39
SS	+1.35	+0.94	+1.67	+0.94
B	+0.0710	+0.0493	+0.0876	+0.0493

4.3 Effect of Addition of One Stainless Steel Strip

In the ZPE-2 experiments, 46 inch length stainless steel strips were doubled over to form 23 inch length strips. Since the active core is only 22 inches long it was assumed that only 22/23 of the strip was effective in poisoning out the core. Each double strip has a volume of 23.556 cm³ and a weight of 183 grams. Each strip adds 175.043 grams stainless steel to the core and displaces 22.532 cm³ water from the core.

The addition of one double strip per element changes the atomic densities in the channel and in the whole fuel element as shown in the following table.

Change in Atomic Density (atoms/cm³ x 10⁻²⁰) by One S.S. Strip per Element

Atom	Fixed Fuel Element		Control Rod Fuel Element	
	Channel	Core	Channel	Core
H	-6.96	-4.83	-8.58	-4.83
O	-3.48	-2.42	-4.29	-2.42
SS	+8.76	+6.08	+10.80	+6.08

APPENDIX C

GEOMETRY AND BUCKLING FOR VARIOUS CORE ARRAYS

1.0 Lateral Buckling

1.1 Cylindrical Arrays

Various arrays were approximated by cylinders having the same cross-sectional area. Equivalent bare radial buckling was estimated using a radial reflector savings of 6.13 cm. For these cases,

$$1/B_{radial}^2 = \left(\frac{2.4048}{R + 6.13} \right)^2$$

Array	No. Elements	Cylinder Radius, R	Radial Buckling, B_r^2
5 x 5	21	19.29 cm	0.008950 cm ⁻²
6 x 6	32	23.81 "	0.006451 "
7 x 7	44	27.92 "	0.004988 "
7 x 7	45	28.24 "	0.004895 "

1.2 Square Arrays

Measurements have been taken on 4 x 4 and 5 x 5 arrays. Assuming a lateral reflector savings of 6.13 cm,

$$B_x^2 = B_y^2 = \left[\frac{\pi}{T + 12.26} \right]^2$$

Array	No. Elements	Slab Thickness, T	$B_x^2 = B_y^2$
4 x 4	16	29.845 cm	0.005567 cm ⁻²
5 x 5	25	37.306 "	0.004024 "

2.0 Axial Buckling

2.1 Full Core Height - Control Rod Absorbers in Upper Reflector

With control rod absorbers in the upper reflector, the best estimates of axial reflector savings are 6.59 cm for the bottom reflector and 3.62 cm for the top reflector. For a 22.0 inch core height the axial buckling is 0.002259 cm^{-2} .

2.2 Full Core Height, Water Reflector Top and Bottom

In the special experiment on the 4 x 4 array, only one control rod was required. Under these conditions, the upper reflector is nearly identical with the lower reflector and both reflector savings should be about 6.59 cm. Therefore, for this special experiment the axial buckling is 0.002069 cm^{-2} .

2.3 Critical Water Height Experiments

In taking critical water height measurements the upper reflector consisted of a mixture of fuel elements, control rods, and air. The axial reflector savings under these circumstances can be expected to differ from the cases in which the upper reflector consisted of a mixture of water and control rods. Analysis of the critical water height measurements indicated that the total axial reflector savings for these experiments was 11.70 cm. Therefore,

$$B_{AXIAL}^* = \left[\frac{\pi}{h + 11.70} \right]^2$$

where h = core height, cm

Core Height (in.)	$B_{\text{axial}}^2, \text{cm}^{-2}$
8.64	0.008716
8.82	0.008488
10.12	0.007056
12.05	0.005513
12.33	0.005335
12.41	0.005284
14.20	0.004323
15.02	0.003972
15.30	0.003861

APPENDIX D

"MEASURED" REACTIVITY FOR FIXED ELEMENT CORES

In comparing ZPE-2 reactivity measurements with theoretical calculations it was more convenient to calculate cores consisting of all fixed elements. In order to relate measured reactivities on mixed element cores to the calculations, it was necessary to increase measured reactivities by the worth of the substituted elements. "Measured" reactivity for the fixed element core was calculated from the formula, ⁽⁵⁵⁾

$$\rho = 1 - e^{-0.000073 (\bar{I}_{EX} + \bar{I}_{SUBST})}$$

where $\bar{I}_{EX}$ = excess reactivity in mixed element core, cents.

$\bar{I}_{SUBST}$ = reactivity gained by substituting fixed elements for control rod elements, cents.

"Measured" reactivities for the various fixed element cores are shown in the following table.

Measured Reactivities for Fixed Element Cores

Condition	Array	No Elements	Core Height (In.)	I_{EX} cents	I_{SUBST} cents	%f
Clean	7 x 7	45	22.0	2280	315	17.22
Clean	5 x 5	21	22.0	700	392	7.66
Clean	4 x 4	16	22.0	317	56	2.67
Clean	5 x 5	21	14.20	0	392	2.82
Clean	5 x 5	25	12.41	0	379	2.72
Clean	6 x 6	32	10.12	0	357	2.57
Clean	7 x 7	44	8.82	0	318	2.30
Clean	7 x 7	45	8.64	0	315	2.28
24.0923 SS Strips	7 x 7	45	22.0	21	315	2.42
17.2814 "	6 x 6	32	22.0	6	357	2.61
6.6667 "	5 x 5	21	22.0	16	392	2.93
16.7577 "	7 x 7	45	22.0	690	315	7.08
11.0000 "	6 x 6	32	15.02	0	357	2.57
11.0000 "	7 x 7	44	12.05	0	318	2.30
24.2666 "	7 x 7	45	22.0	0	315	2.28
17.340 B Strips	7 x 7	45	22.0	22	315	2.43
12.123 "	6 x 6	32	22.0	37	357	2.83
4.715 "	5 x 5	21	22.0	21	392	2.97
12.491 "	7 x 7	45	22.0	578	315	6.51
8.000 "	6 x 6	32	15.30	0	357	2.57
8.000 "	7 x 7	44	12.33	0	318	2.30
17.494 "	7 x 7	45	22.0	0	315	2.28

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