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Diffusion Parameters of Water for Various Scattering Kernels*

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Proposed running head:

Diffusion Parameters of Water

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Abstract

Some diffusion parameters of water are computed with various thermal neutron scattering laws. It is found that the diffusion cooling coefficient, in particular, is reasonably sensitive to the scattering law, but that the diffusion cooling coefficients predicted by the Radkowsky and Nelkin kernels are in fortuitious agreement. The coefficients computed for the Nelkin kernel, when treated in a manner consistent with the way in which experimental data are treated, are in reasonable agreement with the results of a recent experiment.

I. Introduction

The experimental determination of such thermal diffusion parameters of water as the asymptotic thermal diffusion length (1, 2, 3) and the coefficients of the usual expansion in powers of the geometrical buckling, of the time decay constants of neutron fluxes in pulsed assemblies (4-7) is a problem that has attracted considerable interest. The reasons for the interest are several, perhaps the most important being that the diffusion parameters are measurable parameters which models of the thermal neutron scattering law should predict correctly. A converse reason has been the hope that the diffusion parameters are sensitive enough to the models of the neutron scattering law to permit the experiments to distinguish between the various models.

A comparison of the diffusion parameters implied by the Radkowsky and Nelkin thermal neutron scattering kernels has been made by Gelbard et al (8). The calculation indicated that, at least for the two kernels considered, the diffusion parameters were not sensitive to the kernels, but it was not clear whether the general agreement in the parameters was fortuitious, or whether the agreement reflected their genuine insensitivity to the scattering kernels. It is the purpose of this paper to

exhibit the results of computations, similar to those of Gelbard et al but utilizing a greater number of energy exchange kernels, performed in an effort to shed additional light on the last problem; that is, performed in order to investigate the sensitivity of some of the diffusion parameters to the scattering law. The method used to calculate the parameters is described in Section II. The results of the computations are exhibited in Section III, and discussed in Section IV.

II. Computational Method

The method used to calculate the diffusion parameters of interest is based directly on Nelkin's (4) general solution for the fundamental time decay constant of the decaying neutron flux in a pulsed, moderating assembly. The solution $\varphi(B, E, \mu, \tau)$ of the (spatially) Fourier transformed, homogenous Boltzmann equation for a non-multiplying medium in plane geometry, viz,

$$\left[\frac{1}{v} \partial_{\tau} + iB^2 + \Sigma_r(E) \right] \varphi(B, E, \mu, \tau) = \int_{-1}^1 \int_0^{\infty} \Sigma_s(E', \mu' \rightarrow E, \mu) \varphi(B, E', \mu', \tau) dE' d\mu' \quad (1)$$

is assumed to be of the form

$$\varphi(B, E, \mu, \tau) = \varphi(B, E, \mu) \exp(-\lambda \tau) \quad (2)$$

with B the Fourier Transform variable, E the neutron energy,
 μ the angular variable, and λ the time decay constant.
 The angular dependence of φ is expanded in Legendre polynomials,

$$\varphi(B, E, \mu) = \sum_m \frac{1}{2} (2m+1) P_m(\mu) \varphi_m(B, E) \quad (3)$$

and the m^{th} spherical harmonics moment, $\varphi_m(B, E)$, subsequently expanded in a power series in B

$$\varphi_m(B, E) = (iB)^m \sum_{n=0}^{\infty} \varphi_{2n,m}(E) (iB)^{2n} \quad (4)$$

The decay constant, λ , is similarly expanded in a power series in B , viz.;

$$\lambda = \alpha_0 - \sum_{j=1}^{\infty} (iB)^{2j} \alpha_{2j} \equiv \alpha_0 + D_0 B^2 - C B^4 + E B^6 \dots \quad (5)$$

The insertion of eqs. (2) through (5) into (1), the expansion of the scattering kernel in Legendre polynomials,

$$\sum_s (E', \mu' \rightarrow E, \mu) = \sum_L \frac{1}{2} (2L+1) P_L(\mu_0) \sum_{sL} (E' \rightarrow E) \quad (6)$$

(with μ_0 the cosine of the angle between a neutron's initial and final directions), and an appeal to the orthogonality of the

Legendre polynomials yields, upon equating the coefficients of $(iB)^{m+2n}$, the coupled set of equations (eq. (37) in Nelkin's paper (4))

$$\begin{aligned} \left[\Sigma_{s0} + \Sigma_a - \frac{\alpha_0}{v} \right] \varphi_{2n,m} + \sum_{j=1}^m \frac{\alpha_{2j}}{v} \varphi_{2(n-j),m} - \frac{m+1}{2m+1} \varphi_{2n-2,m+1} \\ - \frac{m}{2m+1} \varphi_{2n,m-1} = \int_0^\infty \Sigma_{s,m}(E' \rightarrow E) \varphi_{2n,m}(E') dE' \end{aligned} \quad (7)$$

For a general absorption cross section $\Sigma_a(E)$, the system (7) constitutes an infinite and rather tightly coupled set of equations. If, however, the absorption is inversely proportional to the velocity, the coupling of the equations is relaxed, and the $\varphi_{2n,m}$ and the α_{2j} may be found rather simply, following the procedure suggested by Nelkin (4). The starting point for the calculation is the determination of α_0 and $\varphi_{0,0}(E)$. These quantities are the eigenvalue and eigenfunction, respectively, of the $n=m=0$ equation of the set (7). With $\Sigma_a(E) = \Sigma_0/v$, the solution of the problem is the well known result (4)

$$\alpha_0 = \Sigma_0 \quad \varphi_{0,0}(E) = M(E) \quad (8)$$

with $M(E)$ the Maxwellian flux spectrum.

The succeeding steps of the computation are as follows:

a) The $n=0, m=1$ equation

$$\sum_{s_0}(E) \varphi_{0,1}(E) - \int_0^{\infty} \sum_{s_1}(E' \rightarrow E) \varphi_{0,1}(E') dE' = \frac{1}{3} M(E)$$

is solved for $\varphi_{0,1}(E)$. The solution is effected by writing the equation at discrete energy points E_i , and replacing the integral by a numerical quadrature formula, to obtain

$$\sum_{s_0}(E_i) \varphi_{0,1}(E_i) - \sum_j \sum_{s_1}(E_j \rightarrow E_i) \varphi_{0,1}(E_j) \Delta_j = \frac{1}{3} M(E_i)$$

with Δ_j the appropriate weight coefficient for the numerical integration. The equation, which in matrix form is

$$\sum_m T_m \varphi_{0,1} = \frac{1}{3} M$$

is solved by a matrix inversion routine, to obtain directly

$$\varphi_{0,1} = \frac{1}{3} \sum_m T_m^{-1} M \quad (9)$$

$D_0 (= \alpha_2)$ is then determined by numerically integrating the $n=1, m=0$ equation over energy. D_0 is then (4)

$$D_0 = \frac{\int \varphi_{0,1}(E) dE}{\int \frac{1}{v} M(E) dE} \quad (10)$$

b) The $n=1, m=0$ equation

$$\frac{D_0}{\nu} M(E) - \varphi_{0,1}(E) = \int_0^\infty \sum_{S_0} (\epsilon' \rightarrow E) \varphi_{2,0}(\epsilon') d\epsilon' - \sum_S(E) \varphi_{2,0}(E)$$

is solved for $\varphi_{2,0}(E)$. The equation is not solvable by the apparently straight forward method of writing it in matrix form as

$$D_0 \underline{\nu}^{-1} \underline{M} - \underline{\varphi}_{0,1} = \underline{S}_0 \underline{\varphi}_{2,0} \quad (11)$$

and inverting $\underline{S}_0$, because the equation

$$\underline{S}_0 \underline{\varphi}_{2,0} = 0$$

has the non-trivial solution, which follows from detailed balance

(4, 9) $\varphi_{2,0}(E) = M(E)$, indicating that $\underline{S}_0$ is singular. In practice, however a numerical solution may be forced by the strategem of rewriting (11) as

$$\underline{S}_d \underline{\varphi}_{2,0} = \underline{\varphi}_{0,1} - D_0 \underline{\nu}^{-1} \underline{M} + \underline{S}_R \underline{\varphi}_{2,0}$$

where $\underline{S}_d$ is a diagonal matrix whose elements are the diagonal

elements of $\underline{S}_0$, and $\underline{S}_R = \underline{S}_0 - \underline{S}_d$. A small number, ϵ ,

is added to the element of $\underline{S}_d$ corresponding to the highest

energy, E_c , retained in the calculation, to form $\underline{S}_d + \epsilon \delta_{E_c, E_c}$.

The resulting non-singular equation

$$\underline{\varphi}_{2,0} = \left(\underline{S}_d + \epsilon \underline{S}_{E,E_c} \right)^{-1} \left[\underline{\varphi}_{0,1} - D_0 \underline{\kappa}^{-1} \underline{M} + \underline{S}_R \underline{\varphi}_{2,0} \right] \quad (12)$$

is then solved by power iteration (10). With an ϵ of 10^{-4} to 10^{-6} $\underline{S}_{s0}(E_c)$, $\underline{\varphi}_{2,0}$ was found to converge (laboriously!) to the same five significant figures, except for the high energy element, regardless of ϵ .

c) The $n=0, m=2$ equation, with the assumption that $\underline{S}_{sn}(E' \rightarrow E)$ is zero for $m \geq 2$

$$\underline{S}_{s0} \underline{\varphi}_{0,2} = \frac{2}{5} \underline{\varphi}_{0,1}$$

is solved for $\underline{\varphi}_{0,2}$ by matrix inversion,

$$\underline{\varphi}_{0,2} = \frac{2}{5} \underline{S}_{s0}^{-1} \underline{\varphi}_{0,1} \quad (13)$$

d) The $n=1, m=1$ equation

$$\underline{S}_{s0}(E) \underline{\varphi}_{2,1}(E) - \int_0^\infty \underline{S}_{s1}(E' \rightarrow E) \underline{\varphi}_{2,1}(E') dE' + \frac{D_0}{\underline{\kappa}} \underline{\varphi}_{0,1}(E) - \frac{2}{3} \underline{\varphi}_{0,2}(E) - \frac{1}{3} \underline{\varphi}_{2,0}(E) = 0$$

written in matrix form as

$$\underline{S}_{s1} \underline{\varphi}_{2,1} + D_0 \underline{\kappa}^{-1} \underline{\varphi}_{0,1} - \frac{2}{3} \underline{\varphi}_{0,2} - \frac{1}{3} \underline{\varphi}_{2,0} = 0$$

is solved by matrix inversion for $\underline{\varphi}_{2,1}$,

$$\underline{\varphi}_{2,1} = \underline{\Sigma}^{-1} \left\{ \frac{1}{3} \underline{\varphi}_{2,0} + \frac{2}{3} \underline{\varphi}_{0,2} - D_0 \underline{\nu}^{-1} \underline{M} \right\} \quad (14)$$

e) Finally, the $n=2, m=0$ equation is integrated over E , to yield the diffusion cooling constant C ($= \alpha_T$)

$$C = C_D + C_T$$

with the diffusion cooling part, C_D , given by

$$C_D = \frac{\int \left(\frac{1}{3} \underline{\Sigma}^{-1} - \frac{D_0}{\nu} \right) \varphi_{2,0}^{(E)} dE}{\int \frac{1}{\nu} M(E) dE} \quad (15)$$

and the transport correction, C_T , given by

$$C_T = \frac{\int \underline{\Sigma}^{-1} \left[\frac{4}{15 \underline{\Sigma}_{3,0}(E)} - \frac{D_0}{\nu} \right] \underline{\Sigma}^{-1} \frac{1}{3} M(E) dE}{\int \frac{1}{\nu} M(E) dE} \quad (16)$$

Once α_0 , D_0 , and C are found, the asymptotic thermal diffusion length, L , may be found by noting that the development of L^{-2} as a series in powers of the concentration of $1/\nu$ poisons present in the system is in fact the inverse of eq. (5), with $B = (i/L)$ (11). Correct to order B^4 , then, L may be found from eq. (5) by setting $\lambda = 0$ and solving for L , with the result (6)

$$L = \frac{D_0}{2\alpha_0} \left(1 + \left(1 + \frac{4\alpha_0 C}{D_0^2} \right)^{\frac{1}{2}} \right) \quad (17)$$

Equation (17) is of the form of the Maxwellian averaged diffusion length, D_0/α_0 , multiplied by a correction for diffusion hardening (1).

The system of equations (8)-(17) was solved sequentially with the aid of RAMP, a matrix manipulation program for the IBM-704. 32 thermal energy points, distributed as in the SLOP-1 program, (12) were used, and Simpson's rule, generalized for uneven mesh spacings, was used to perform the numerical integrations. The sequence of operations described above was tested by computing D_0 and C for Nelkin's kernel (13), employing in the calculations the cross sections used by Hobson and Daitch (14). The results were $D_0 = 36,810 \text{ cm}^2\text{-sec}^{-1}$ and $C = 2931 \text{ cm}^4\text{-sec}^{-1}$, in good agreement with Hobson and Daitch's² values $D_0 = 36,883$ and $C = 2924$.

III. Results

The method of Section II was employed to compute D_0 , C , and L for various scattering kernels. Examination of eqs. (8) - (17) shows that α_0 is determined entirely by the amount of $1/v$ absorption present in the system, and not at all on the scattering law. D_0 and C_T depend on the P-1 component, $\sum_s (\epsilon' \rightarrow \epsilon)$, of the scattering law, but not on the P_0 component, and C_D depends on both the P_0 and P_1 components. In all the computations, $2\sigma_a$, the microscopic absorption cross section per water molecule, was chosen as 652.4 mb, whence $\alpha_0 = 4785 \text{ sec}^{-1}$, independent of the kernels.

Since C_D is the only pulsed diffusion parameter considered in the present paper which depends on the P-0 component of the scattering law, the effect of the P-0 component alone was studied by fixing $\sum \kappa(E)$ as that given by the Radkowsky effective mass prescription (15), and computing the diffusion parameters with various P-0 energy exchange laws. The laws used were modifications³ of the free gas kernels of masses 1, 2, 4, and 18 obtained from the formulae given by Clendenin (16), and Goldman's (17) modification of Nelkin's kernel (13), which will be referred to as the GIN⁴ kernel. The results exhibited in Table I, and the spectra $\varphi_{2,0}(E)$ (which appear in the expression (15) for C_D) shown in figure 1, show that C_D , and the spectrum $\varphi_{2,0}$ which is the primary contribution to the diffusion cooling effect, are reasonably sensitive to the P-0 component of the scattering law. The various P-0 kernels, on the other hand, have only a small effect on the asymptotic diffusion lengths as

given by eq. (17), which are also listed in Table I.

The sensitivity of the diffusion parameters to the P-1 component of the scattering law was studied in a similar manner. The isotropic component of the scattering kernel was fixed to be the GIN kernel, and the computations of the diffusion parameters were performed by choosing the P-1 component of the scattering law to be: (1) The Radkowsky Prescription (15), (2) a Selengut - Goertzel approximation to the P-1 component of the GIN kernel, viz:

$$\Sigma_{s1}^{sg}(E' \rightarrow E) = \delta(E' - E) \int_0^\infty \Sigma_{s1}(E' \rightarrow E) dE' \quad (18)$$

(3) A constant Σ_{s1} of the form (18), chosen so that D_0 is given choice (2), and (4) a Σ_{s1} of the form (18), chosen so that

$$D(E) = K \sqrt{\frac{E}{.0253}} \text{ cm} \quad (19)$$

(E in ev); i.e., $D(E)$ proportioned to the velocity, a not unreasonable choice for water (18). Two values of K were used, one ($K=0.104118$) adjusted so that D_0 is that given by choice (2), and the other ($K=0.107969$) adjusted so that D_0 is approximately the "best" (19) experimental value $35,500 \text{ cm}^2 \text{ sec}$.

The results of the calculations of the diffusion parameters with the GIN P-0, and the various P-1 kernels, are shown in Table II. The rows labelled "coarse mesh" and "fine mesh" require some explanation. For a fixed final energy E, $\Sigma_{s1}(E' \rightarrow E)$ is a rapidly varying function of E' in the GIN kernel (20, 21), hence a numerical integration in eq. (18) performed with too coarse an energy mesh will yield inaccurate results. The row labelled "fine mesh" used the results of a careful integration performed

by Ohanian (20); the row labelled "coarse mesh" used a numerical integration in which the energy mesh points were those of the 32 group SLOP-1 code (12). The coarse mesh results agree substantially with the calculation of Honeck (11) and Gelbard et al (8). The fine mesh results, however, are believed to reflect more accurately the properties of the GIN kernel (17). As expected, changes in the P-1 kernel produce changes in all the diffusion parameters considered here.

In a final calculation, an allowance for energy exchange collisions between the neutrons and the oxygen present in water was made by adding to the GIN and mass 1 energy exchange matrices a free gas kernel with mass 16 and $\sigma_s = 4.26$ barns, a procedure previously used by Honeck (11). The results, which represent the "best" calculated values of this paper, are presented in Table III.

IV. Discussion

a) It has earlier been noted that, excluding α_0 , all the diffusion parameters considered here depend on the P-1 component of the scattering law. Equations (9) and (10) show that the parameter D_0 depends in a direct way on both the magnitude and details of the Selengut - Goertzel approximation to the P-1 component of the scattering law.⁵ The Radkowsky, the Selengut-Goertzel GIN, and the $1/v$ P-1 kernels, are not very different in detail (14) however, hence the differences in the D_0 values of Table II are to be regarded as reflecting the different magnitudes of the P-1 laws.⁶ The parameters C_D , C_T , and C , however, depend

on the P-1 kernel in a less straight forward way, and are seen in Table II to depend on the details, as well as on the magnitude, of the P-1 component. That C is reasonably sensitive to the details of the P-1 kernel is suggested by the fact that the not too violent change of detail from the fine mesh GIN to the $D(E) \propto \sqrt{E}$ case, with no change in D_0 , produced a 10% increase in C (entries 3 and 5 in Table II). The drastic change to constant $D(E)$ produced an even more violent change in C (4th entry in Table II). On the other hand, keeping the details of $D(E)$ fixed as the $\sqrt{E}$ case, and increasing D_0 by 3.7%, produced only a 5.4% increase in C (entries 5 and 6 in Table II), indicating that the sensitivity of C to the magnitude of the P-1 kernel is considerably smaller than the sensitivity to its detail. The differences in the diffusion parameters produced by changing the P-1 component from the Radkowsky to the GIN kernels are due both to changes in magnitude and in detail of the P-1 component.

b) The C values of Table III, obtained with the Radkowsky and GIN kernels with allowances for oxygen scattering, are seen to be reasonably close, differing only by some ten per cent. In view of the demonstrated sensitivity of C to the P-0 kernel for fixed P-1 kernel (Table I), however, and in view of the comments in (a), this agreement of the C values must be regarded as fortuitious, and not as reflecting a genuine insensitivity of C to the scattering law.

c) In spite of the inherent sensitivity of C to the P-0 kernel, it is doubtful whether conventional die-away and diffusion length measurements can shed much light on the P-0 component of the scattering law in water, because of the smallness of C compared to D_0 . Assuming for the moment that α_0 is 4785, that D_0 is known to be that given in Table I, and assuming further that a truncation of eq. (5) at the B^4 term is exact, then at $B^2=1$, λ ($=40,619 \text{ sec}^{-1}$) as given by the Mass 1 kernel which is known to be incorrect, and λ ($=39,639 \text{ sec}^{-1}$) as given by the GIN kernel (which is very likely a reasonably good approximation to the scattering law in water), differ by only 2.5%. This small difference would be very difficult to detect in the presence of experimental errors. The wide variation in the C values quoted in the literature⁷, with substantial agreement in the D_0 values, supports this observation. The situation is of course further complicated by the presence of the higher order terms in the series (5). It therefore seems not unreasonable to regard diffusion length and die away experiments in water as determining only α_0 and D_0 with any accuracy, and as yielding unreliable information about the P-0 component of the scattering law.

d) The previous comment aside, it is interesting to compare results of the present calculations with experiment, but in order to do this a correction is necessary. The correction arises because the experimental points cannot, because of experimental errors, be fit meaningfully with a polynomial of the

form (5), containing powers of B greater than B^4 (6, 14, 22), while the data of Table III presupposes an exact fit of (5) to the data. An approximate correction to the results of Table III may be obtained by computing the coefficient, E , of B^6 in the expansion (5), and subsequently fitting the curve

$$\lambda = \alpha_0 + D_0 B^2 - C B^4 + E B^6 \quad (20)$$

with the quadratic

$$\lambda = \alpha + "D" B^2 - "C" B^4 \quad (21)$$

by the method of least squares. Using E values of $500 \text{ cm}^6\text{-sec}^{-1}$ and $200 \text{ cm}^6\text{-sec}^{-1}$ for the Radkowsky and GIN kernels respectively (8, 23), and fitting (20), with the parameters of Table III, by (21) using eleven equally spaced values of B^2 over the range $B^2 = 0$ to $B^2 = 1$ with uniform weighting (as used by Starr and Koppel (22)), the values of " D " and " C ", presented in Table IV together with some recent experimental results, were obtained. It should be noted that the " C " values for the Radkowsky and GIN kernels differ by some 25%, in contrast to the smaller 8% difference in the uncorrected values of Table III.

The GIN parameters of Table IV are in reasonable agreement with the recent measurements of Starr and Koppel (22). The " C " value agrees very well indeed; the " D " value is, however, too small, leading to a value for the thermal diffusion length that is too small. It should be noted that by choosing $D(E)$ as given by eq. (19), with $K=0.107969$, and using this $D(E)$ in conjunction

with the GIN P-0 kernel, excellent agreement with Starr & Koppel's data can be obtained. A similar technique does not yield a fit to the data of Lopez and Beyster: D_0 can be matched, but the corresponding C value is approximately 3900.

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FOOTNOTES

1. There are some minor typographical errors in eq. (37) of reference (4).
2. The differences in the fourth significant figure of D_0 and the third of C are not significant, because of the calculational procedure (least squares fitting) used by Hobson and Daitch.
3. The free gas kernels have the difficulty that $\Sigma_s(\epsilon) = \int_0^\infty \Sigma_{s0}(\epsilon \rightarrow \epsilon') d\epsilon'$ is not the experimental scattering cross section for water. The difficulty may be circumvented by adding to the diagonal elements of the P-0 matrix the difference between the experimental and heavy gas cross section at each energy. It can be shown that this modification does not affect the energy exchange properties of the matrix. In addition, it guarantees that eq. (16) for C_T will yield the value implied by the experimental cross section.
4. For Goldman - Improved - Nelkin
5. Nelkin, in eqs. (42ff) of reference (13), gives arguments that the Selengut-Goertzel approximation should yield good results for D_0 . This seems to be borne out by the calculations of Gelbard et. al. (8).
6. With the obvious exception of the constant D case
7. See, for example, Table III in reference (22)

TABLE I

DIFFUSION PARAMETERS OF WATER WITH
THE RADKOWSKY P-1 KERNEL, AND
VARIOUS P-0 KERNELS AT 68° F

P-0 Kernel	D_0 $\text{cm}^2\text{-sec}^{-1}$	C_D $\text{cm}^4\text{-sec}^{-1}$	C_T $\text{cm}^4\text{-sec}^{-1}$	L cm
Mass 1 ^(a)	38,421	2977	-390	2.845
Mass 2	"	3915	"	2.850
Mass 4	"	5951	"	2.858
Mass 18	"	18,644	"	2.912
GIN ^(b)	"	3930	"	2.850

(a) Free gas kernels, modified as in footnote 3.

(b) See text

TABLE II

DIFFUSION PARAMETERS OF WATER WITH
THE GIN^(a) P-0 KERNEL AT 68°F
AND VARIOUS P-1 KERNELS

P-1 Kernel	D_0 $\text{cm}^2\text{-sec}^{-1}$	C_D $\text{cm}^4\text{-sec}^{-1}$	C_T $\text{cm}^4\text{-sec}^{-1}$	C $\text{cm}^4\text{-sec}^{-1}$	L cm
Radkowsky	38,421	3930	-390	3540	2.850
GIN ^(b)	37,274	3354	-362	2992	2.805
GIN ^(c)	34,234	3009	-115	2894	2.691
D Constant	34,234	1000	-722	278	2.676
$D \propto v^{(d)}$	34,234	3243	-62	3181	2.692
$D \propto v^{(e)}$	35,500	3486	-134	3352	2.741

(a) See reference (17)

(b) Coarse Mesh Selengut-Goertzel approximation - See text

(c) Fine Mesh Selengut-Goertzel approximation - See text

(d) From eq. (19) with $K = 0.104118$

(e) From eq. (19) with $K = 0.107969$

TABLE III

DIFFUSION PARAMETERS OF WATER AT 68°F
WITH VARIOUS KERNELS. O¹⁶ ENERGY EXCHANGE INCLUDED

Kernel	D_0 cm ² -sec-1	C_D	C_T cm ⁴ -sec-1	C	L cm
Radkowsky (a)	38,421	3357	-390	2967	2.847
GIN	34,234	3317	-115	3202	2.692
GIN; $D \propto v$ (b)	35,500	3486	-134	3352	2.741

(a) Mass 1 P-0 kernel, Radkowsky P-1 kernel

(b) GIN P-0 kernel, P-1 kernel from eq. (19), with $K=0.107969$

TABLE IV

COMPARISON OF CORRECTED DIFFUSION PARAMETERS WITH EXPERIMENT

	Radkowsky Kernel	GIN Kernel	GIN-P _O Dav (a)	Starr & Koppel	Lopez & Beyster
"D" (cm ² -sec ⁻¹)	38,140	34,120	35,390	35,850±100	37,503 ± 366
"C" (cm ⁴ -sec ⁻¹)	2218	2902	3052	2900±351	5116±776
L-cm	2.847 ^(b)	2.692 ^(b)	2.741	2.754±.008	2.749±0.016 ^(c)
T (°C)	20	20	20	21	26.5

(a) D(E) from eq. (19), with K = 0.107969

(b) Values from Table III

(c) 2.795 cm at 22° C

PHI vs ENERGY

Figure Caption

Figure 1. The spectra $\gamma_{2,0}$, as defined by equation (12), computed for the Radkowsky P-1 kernel, with the Mass 1, Mass 4, and the GIN P-0 kernels.

END