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**Effect of Eddy Diffusion on Temperatures in a Fixed Bed,
Particulate-Fueled Nuclear Reactor***

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Introduction

There is currently much interest in particulate-fueled reactors. Examples are: 1) The Settled Bed Reactor (1), a sodium-cooled fast reactor which is being developed at Brookhaven National Laboratory; 2) The Pebble Bed Reactor, a graphite-moderated, helium-cooled reactor which in the United States was developed by Sanderson & Porter Inc. (2) and by Oak Ridge National Laboratory (3), and which in Germany is being built by BBC-Krupp (4). In such reactors, in which the coolant flows through interconnected interstices between the fuel particles, eddy diffusion tends to attenuate the temperature distribution which would otherwise result from the power distribution. The degree to which eddy

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diffusion is important depends upon a number of factors, which are developed in the body of this paper. Illustrative results for a wide range of parameters are presented in tables and graphs. Details of the mathematical analysis are given in the Appendix.

Physical System

A typical settled bed reactor is 7.4 ft in diameter by 4.4 ft high, and is filled with spherical fuel particles 0.225 in. in diameter. Sodium coolant flows axially with an average superficial velocity of 2 ft/sec, entering at 550°F and leaving at an average temperature of 1200°F. The axial max/av power ratio is 1.41 and the radial is 1.85.

Preliminary investigations indicated that the Peclet number, ud/E , for radial diffusion would be 8 (5,6), that axial diffusion would be an order of magnitude or more greater than radial diffusion (7,8,9), and that the temperature difference between the interior of the fuel particles and the coolant would be so small that the temperature distribution in the reactor would be fixed by the temperature of the coolant.

Mathematical Model

To assess the effect of diffusion, it is assumed that the specific heat, c , the density, ρ , and the axial velocity, u , of the coolant are constant, that there is no radial component of velocity, and that there is radial symmetry.

A heat balance on a differential element yields:

$$A \frac{\partial^2 T}{\partial x^2} + B \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] + Q(x, r) = \frac{\partial T}{\partial x}, \quad (1)$$

where $A = (E/u)_{\text{axial}}$, $B = (E/u)_{\text{radial}}$, $Q = q/cpu$; q is the volumetric heat generation rate at (x, r) . Eq. (1) is to be solved subject to the boundary conditions: $x = -H$, $T = 0$; $r = 0$ or R , $\partial T / \partial r = 0$. (The case $T = 0$ at $r = R$ was also solved, but it is not presented here because it sheds no additional light on the effects of eddy diffusion.)

Analytical Results

As shown in the Appendix, the problem may be solved by using a delta function formulation for the heat source. The result for the ratio of the maximum temperature, $t(H, 0)$, to the average, $\bar{t}(H)$, at the reactor outlet is [using $q = q_{\text{max}} \cos(\pi x / 2H') J_0(2.405 r / R')$]:

$$t(H, 0) / \bar{t}(H) = 1 - \sum_{n=1}^{\infty} F_n \cdot G_n \quad (2)$$

where

$$F_n = 1 / J_0(\gamma_n R) \{ [\gamma_n R / 2.405(R/R')]^2 - 1 \} \quad (2a)$$

$$G_n = \frac{1}{2} \frac{[\beta_n H / (\pi H / 2H')] [1 - e^{-2\beta_n H}] \cot(\pi H / 2H') + [1 + e^{-2\beta_n H}]}{1 + [\beta_n H / (\pi H / 2H')]^2} \quad (2b)$$

$$J_1(\gamma_n R) = 0 \quad (2c)$$

$$\beta_n = \left[\sqrt{1 + \frac{4AB}{R^2} (\gamma_n R)^2} - 1 \right] / 2A \quad (2d)$$

The diffusivities appear in the G_n , where they are contained in the β_n . When there is no radial diffusion, $B = 0$, and therefore $\beta_n = 0$, and consequently $G_n = 1$, so that $1 - \Sigma F_n$ is the radial max/av of the power as well as of the temperature. Eq. (2d) shows that the radius of the bed plays an important part in determining the effect of diffusivity. For example, if A and B , which are proportional to the diffusivities, remain finite, $\beta_n \rightarrow 0$ as $R \rightarrow \infty$, and therefore diffusion has no effect. Contrariwise, as $R \rightarrow 0$, $\beta_n \rightarrow \sqrt{B/A} (\gamma_n R)/R \rightarrow \infty$, $t(H,0)/\bar{t}(H) \rightarrow 1$, and diffusion has a relatively large effect. For the usual situation, in which R is relatively large, expansion of Eq. (2d) leads to $\beta_n \approx B(\gamma_n R/R)^2 [1 - AB(\gamma_n R/R)^2]$, which shows that the axial diffusivity ($\sim A$) is relatively unimportant, since it enters in a correction term. The effect of bed length is contained in G_n , which lies between the limits:

$$G_n^1 = \frac{1}{2} \frac{(\pi/2)^2 [1 + e^{-2\beta_n H}]}{(\pi/2)^2 + (\beta_n H)^2} \leq G_n \leq \frac{1}{2} \frac{1 - e^{-2\beta_n H}}{\beta_n H} = G_n^0 \quad (3)$$

for $H/H' = 1$ for $H/H' = 0$

As $\beta_n H \rightarrow 0$, $G_n^1 = G_n^0 = 1$. As $\beta_n H \rightarrow \infty$, $G_n^0 = 1/2\beta_n H$, $G_n^1 = 0.5/[1 + (2\beta_n H/\pi)^2]$. Some intermediate values are tabulated below:

$\beta_n H$	G_n^1	G_n^0
0	1	1
0.1	0.906	0.906
0.5	0.621	0.632
1	0.403	0.432
2	0.194	0.245
3	0.108	0.166
4	0.0668	0.125
5	0.0449	0.100
10	0.0120	0.050

This indicates that differences in H/H' are not felt until $\beta_n H$ gets large (> 1).

Computer Results

To show the interrelationships of the parameters, a number of cases have been calculated on a high-speed computer for a value of $B = 0.00234$ ft. Table 1 shows how the outlet temperature distribution along a radius is affected by changes in the axial diffusivity and in R/R' , for a core in which $R = 4$ ft, $H = 2$ ft, and $H = H'$. Table 2 is similar, except that H/H' is varied and $R = R'$. Figure 1 plots the ratio of the maximum to the average temperature at the outlet versus R for various values of H , with $R = R'$ and $H = H'$. Figure 2 plots the temperature ratio versus H , for various values of R , with $R = R'$ and $H = H'$. Figure 3 shows the temperature ratio versus

R/R' , for various values of H/H' , with $R = H = 1$ ft. Figure 4 is a cross-plot of Figure 3. Figure 5 shows the effect of varying the value of the diffusion coefficient on both the axial and radial temperature distributions.

Conclusions

Eddy diffusion has a negligible effect on the temperature distribution in a large reactor with small fuel particles, such as the settled bed reactor, but it could have appreciable effects in small reactors with relatively large fuel particles. With additional refinements, the calculation method is applicable to chemical reactors in which the heat source is a function of temperature as well as of position.

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Appendix

To solve Eq. (1) with the chosen boundary conditions, it is assumed first that the reactor contains a short, annular source of heat only at (x_1, r_1) , and the temperature distribution which would result from such a source is derived. Then a distribution of such sources is assumed throughout the volume of the reactor, and the resulting temperature distribution is obtained by superposition. The details are outlined below for a cylindrical reactor in which $-H \leq x \leq H$ and $0 \leq r \leq R$.

$$x < x_1, \quad T = 0 \quad (A-1)$$

$$x > x_1, \quad A \frac{\partial^2 T}{\partial x^2} + B \left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right] = \frac{\partial T}{\partial x} \quad (A-2)$$

$x = x_1$, $T = 0$ except at $r = r_1$, where

$$\begin{aligned} T = T(x_1, r_1) &= \frac{q(x_1, r_1) dx_1}{u \rho c} = Q(x_1, r_1) dx_1 \\ &= Q(x, r) \delta(x - x_1) \delta(r - r_1) dx_1 \end{aligned} \quad (A-3)$$

Note

$$T(\infty, r) = q(x_1, r_1) 2r_1 dr_1 dx_1 / u \rho c R^2 \quad (A-4)$$

T is separable here, and, since T must decrease as x increases beyond

x_1 , choose

$$T = T(\infty, r) + \sum_{n=1}^{\infty} A_n e^{-\beta_n x} J_0(\gamma_n r) \quad (A-5)$$

[The γ_n are determined by Eq. (2c) and the β_n from Eq. (2d).] The A_n are determined from

$$T(x_1, r) = Q(x_1, r) \delta(r - r_1) dx_1 = T(\infty, r) + \sum_{n=1}^{\infty} A_n e^{-\beta_n x_1} J_0(\gamma_n r) \quad (A-6)$$

through the orthogonal properties of the functions as:

$$A_n = Q(x_1, r_1) dx_1 r_1 J_0(\gamma_n r_1) dr_1 / e^{-\beta_n x_1} (R^2/2) J_0^2(\gamma_n R) \quad (A-7)$$

Therefore the temperature at (x, r) due to a ring source of strength q at (x_1, r_1) is (for $x > x_1$):

$$T(x, r) = q(x_1, r_1) 2r_1 dr_1 dx_1 / u\rho c R^2 + \sum_{n=1}^{\infty} \frac{q(x_1, r_1)}{u\rho c} \frac{2dx_1 r_1 J_0(\gamma_n r_1) dr_1}{R^2 J_0^2(\gamma_n R) e^{-\beta_n x_1}} e^{-\beta_n x} J_0(\gamma_n r) \quad (A-8)$$

and the temperature at (x, r) due to a distribution of ring sources throughout the volume of the reactor is:

$$t(x, r) = \frac{2}{u\rho c R^2} \int_{-H}^x \int_0^R q(x_1, r_1) r_1 dr_1 dx_1 + \frac{2}{u\rho c R^2} \sum_{n=1}^{\infty} \frac{e^{-\beta_n x} J_0(\gamma_n r)}{J_0^2(\gamma_n R)} \int_{-H}^x \int_0^R \frac{q(x_1, r_1) J_0(\gamma_n r_1) r_1 dr_1 dx_1}{e^{-\beta_n x_1}} \quad (A-9)$$

Taking $q(x_1, r_1) = q_{\max} \cos(\pi x_1/2H') J_0(2.405 r_1/R')$, the average temperature at the outlet is

$$\bar{t}(H) = \frac{2Hq_{\max}}{u\rho c} \left[\frac{\sin(\pi H/2H')}{\pi H/2H'} \right] \left[\frac{2J_1(2.405 R/R')}{2.405 R/R'} \right] \quad (\text{A-10})$$

where the quantities in brackets are recognized as the axial and radial av/max ratios of the power distribution, and the temperature distribution in the reactor becomes

$$\begin{aligned} \frac{1}{q_{\max}} \frac{u\rho c}{2H} t(x, r) = & \frac{2H'}{\pi H} \left[\sin\left(\frac{\pi x}{2H'}\right) + \sin\left(\frac{\pi H}{2H'}\right) \right] \frac{J_1(2.405 R/R')}{2.405 R/R'} \\ & + \sum_{n=1}^{\infty} \frac{J_0(\gamma_n r)}{J_0(\gamma_n R)} \left[\frac{(2.405 R/R') J_1(2.405 R/R')}{(2.405 R/R')^2 - (\gamma_n R)^2} \right] \times \\ & \times \left[\frac{\beta_n H \cos\left(\frac{\pi x}{2H'}\right) + \left(\frac{\pi H}{2H'}\right) \sin\left(\frac{\pi x}{2H'}\right) - e^{-\beta_n(H+x)} [\beta_n H \cos\left(\frac{\pi H}{2H'}\right) - \left(\frac{\pi H}{2H'}\right) \sin\left(\frac{\pi H}{2H'}\right)]}{(\beta_n H)^2 + (\pi H/2H')^2} \right] \quad (\text{A-11}) \end{aligned}$$

Nomenclature

A	= $(E/u)_{\text{axial}}$, ft
B	= $(E/u)_{\text{radial}}$, ft
c	= specific heat of coolant, Btu/lb-°F
d	= diameter of particles, ft
E	= eddy diffusivity, ft ² /hr
F_n	= a dimensionless function of the radius, see Eq. (2a)
G_n	= a dimensionless function of the length and the diffusivity, see Eq. (2b)
H	= half-length of reactor, ft
H'	= extrapolated half-length of reactor, ft
J₀	= Bessel function, first kind, zero order
J₁	= Bessel function, first kind, first order
n	= index of term in series
q	= heat generation rate per unit volume, Btu/hr-ft ³
Q	= q/cpu
r	= radius, ft
R	= radius of reactor, ft
R'	= extrapolated radius of reactor, ft
t	= temperature rise of coolant for a distributed source, °F
T	= temperature rise of coolant for a local source, °F

- u = axial velocity of coolant, superficial, ft/hr
- x = axial position, ft, (center = 0)
- β_n = eigenvalue, ft^{-1} , see Eq. (2d)
- γ_n = eigenvalue, ft^{-1} , see Eq. (2c)
- ρ = density of coolant, lb/ft^3

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Table 1

Effect of Radial Reflector and Axial Diffusivity on Outlet Radial Temperature Distribution

($R = 4$ ft, $H = 2$ ft, $H = H'$, $B = 0.00234$ ft)

$R/R' \backslash r/R$	$t(H, r/R) / \bar{t} (H)$							
	$0.1 B \leq A \leq 100 B$					$A \leq B$	$A=10B$	$A=100B$
	0	0.2	0.4	0.6	0.8	1.0	1.0	1.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
0.2	1.0294	1.0270	1.0199	1.0081	0.9917	0.9729	0.9729	0.9729
0.4	1.1249	1.1145	1.0837	1.0332	0.9644	0.8881	0.8881	0.8879
0.6	1.3126	1.2854	1.2056	1.0780	0.9104	0.7330	0.7329	0.7325
0.8	1.6554	1.5947	1.4193	1.1482	0.8110	0.4785	0.4784	0.4776
1.0	2.3124	2.1806	1.8076	1.2566	0.6195	0.0545	0.0543	0.0531

Table 2

Effect of Axial Reflector and Axial Diffusivity on Outlet Radial Temperature Distribution

(R = 4 ft, H = 2 ft, R = R', B = 0.00234 ft)

$\begin{array}{c} \text{H/H}' \\ \text{r/R} \end{array}$		$t(H, r/R) / \bar{t}(H)$							
		$0.1 B \leq A \leq 100 B$					$A \leq B$	$A=10B$	$A=100B$
		0	0.2	0.4	0.6	0.8	1.0	1.0	1.0
0		2.3115	2.1805	1.8075	1.2565	0.6194	0.0454	0.0454	0.0449
0.2		2.3123	2.1806	1.8076	1.2566	0.6195	0.0531	0.0530	0.0518
0.4		2.3123	2.1806	1.8076	1.2566	0.6195	0.0533	0.0531	0.0519
0.6		2.3123	2.1806	1.8076	1.2566	0.6195	0.0535	0.0534	0.0521
0.8		2.3123	2.1806	1.8076	1.2566	0.6195	0.0539	0.0537	0.0525
1.0		2.3124	2.1806	1.8076	1.2566	0.6195	0.0545	0.0543	0.0531

FIG.1 EFFECT OF REACTOR SIZE ON MAX/AVG OUTLET TEMPERATURE

FIG.2 EFFECT OF REACTOR SIZE ON MAX/AVG OUTLET TEMPERATURE

FIG. 3 EFFECT OF REFLECTOR ON MAX/AVG OUTLET TEMPERATURE

FIG. 4 EFFECT OF DIFFUSION ON COOLANT TEMPERATURES IN PACKED BED REACTORS

END