SEP 3 1963

GMELIN REPRESENTATIVE
AED-Conf- 63-124-1

Conference Paper No. F-1

CONF-176-2

CURRENT-CARRYING CAPACITY AND TRANSITION STATE
OF SUPERCONDUCTING SOLENOIDS

W. F. Gauster and D. L. Coffey
Oak Ridge National Laboratory*
Oak Ridge, Tennessee

MASTER

Facsimile Price \$ 1.60
Microfilm Price \$.80

Available from the
Office of Technical Services
Department of Commerce
Washington 25, D. C.

1963 Cryogenic Engineering Conference
August 19, 20 and 21, 1963
Boulder, Colorado

All rights reserved for publication
in Advances in Cryogenic Engineering,
Vol. 9

* Operated by Union Carbide Corporation for
the U. S. Atomic Energy Commission.

Cryogenic Engineering
Conference
Boulder, Colorado
August 19-21, 1963

LEGAL NOTICE
This report was prepared as an account of Government sponsored work. Neither the United States, nor the Commission, nor any person acting on behalf of the Commission, makes any warranty or representation, expressed or implied, with respect to the accuracy, completeness, or usefulness of the information contained in this report, or that the use of any information, apparatus, method, or process disclosed in this report may not infringe on privately owned rights; or
B. Assumes any liabilities with respect to the use of, or for damages resulting from, the use of any information, apparatus, method, or process disclosed in this report.
As used in the above, "person acting on behalf of the Commission" includes any employee or contractor of the Commission, or employee of such contractor, to the extent that such employee or contractor of the Commission, or employee of such contractor prepares, disseminates, or provides access to, any information pursuant to his employment or contract with the Commission, or his employment with such contractor.

CURRENT-CARRYING CAPACITY AND TRANSITION STATE
OF SUPERCONDUCTING SOLENOIDS

W. F. Gauster and D. L. Coffey
Oak Ridge National Laboratory*
Oak Ridge, Tennessee

I. Discussion of the Models of Montgomery
and of Chandrasekhar and Hulm

Solenoids wound with high-field superconducting wire (for instance, NbZr) display maximum currents I_m and maximum fields H_m which are in general considerably less than the critical current I_c of a short piece of the same wire exposed to a homogeneous transversal field of equal strength H_m ("degradation effect").¹ Montgomery² and Chandrasekhar and Hulm³ suggested models for predicting I_m and H_m of high-field superconducting solenoids. Both models consider a linear relation between the field H and the coil current I ("load line") which is assumed to be identical with that of a nonsuperconducting solenoid with the same dimensions:

$$H = \Phi I. \quad (1)$$

For the case of a rectangular coil cross section (inside radius a_1 , outside radius $a_2 = \alpha a_1$, and length $2b = 2\beta a_1$) and uniform turn density (n turns per cm^2), Eq. (1) can be written as

$$H = a_1 \mathcal{F}(\alpha, \beta) I. \quad (2)$$

Another assumption of both models is the relation

$$I_m = I(H_m) = I_c(H_m) - I_d(H_m) \quad (3)$$

between the critical values of the coil current I and a current I_d

*Operated by Union Carbide Corporation for the
U. S. Atomic Energy Commission.

induced in the solenoid ("diamagnetic" or "shielding" current), and the "short sample quenching current" I_c at the critical field value H_m .

For the shielding current I_d in a wire of radius r , Montgomery assumes

$$I_d(H) = \frac{\pi r^2}{f'(\alpha, \beta)} H = f(\alpha, \beta) H \quad (4)$$

(ref. 2, Eq. (2)), whereas Chandrasekhar and Hulm's assumption is

$$I_d(H) = KG(H)H \quad (5)$$

(ref. 3, Eq. (1)). The term $G(H)$ varies monotonically from unity at low fields to zero at high fields, reflecting the shape of the magnetization curve for a hard superconductor (Fig.(1) of ref. 3), and K is a constant determined by the requirement that at very low fields the flux is completely excluded from the wire. This leads to the value

$$K = \frac{10 d}{4\pi} \quad (6)$$

where d is the distance between two successive turns of one layer. If the solenoid is very tightly wound and if the insulation thickness of the wire is much smaller than its radius r , then $d = 2r$.

From Montgomery's model it follows that high-field superconductor coils of similar geometrical shape (α and β constant) wound with identical wire type and equal distances between turns, but with different inside radii a_1 , should display for any field H the same shielding current I_d (see Eq. (4)) and therefore there should exist a unique coil quenching characteristic $I_m(H_m)$ for α and β constant (Fig. 1). If the short sample characteristic $I_c(H)$ of the wire and one quenching point (H_{mI}, I_{mI}) of a solenoid with the inside radius a_{1I} are known, the maximum current I_{mII} of a geometrically similar coil at a critical

field H_{mII} can be found (Eqs. (3) and (4)) by

$$\begin{aligned} I_{mII} &= I_c(H_{mII}) - I_d(H_{mII}) \\ &= I_c(H_{mII}) - I_{dI} \frac{H_{mII}}{H_{mI}} \end{aligned} \quad (7)$$

The inside radius of the second coil is (Eq. (2))

$$a_{III} = a_{II} \frac{I_{mI} H_{mII}}{I_{mII} H_{mI}} \quad (8)$$

Equations (7) and (8) allow a check of Montgomery's model with experimental data. Experiments with a set of geometrically similar coils did not verify the expected data.⁴

Chandrasekhar and Hulm's model leads to a unique $I_m(H_m)$ curve (Fig. 2) for all coils wound with identical wire type and insulation dimensions, since K and $G(H)$, and therefore $I_d(H)$ (see Eq. (5)), are the same for all these coils. Solenoids with identical load factor $\bar{\Phi} = a_1 \Phi(\alpha, \beta)$ should display identical critical currents I_m and critical fields H_m . This yields a criterion for an experimental check of that model.

II. Currents and Fields in an Infinitely Long Ideal Superconductor Solenoid

The following ideal case shall be considered:⁵ an infinitely long solenoid is wound with ideal superconducting wire (p layers, each layer has n turns/cm). The wire cross section is rectangular, and the insulation between turns is infinitely small (Fig. 3). In the wire, only surface currents are possible (Meissner effect). In the outside layer I flows only at the inner surface ($I_{10} = 0$, $I_{11} = I$). In the second layer $I_{20} = -I$ and $I_{21} = +2I$. In the inside layer (number p) $I_{p0} =$

$-(p-1)I$ and $I_{pi} = +pI$. The field inside the solenoid is

$$H_o = \frac{4\pi}{10} pnI \quad (9)$$

Therefore the field inside an ideal superconducting long solenoid is the same as that on a nonsuperconducting coil having the same number of turns.

In this case the coil quenching current is determined by the condition

$$H_m = H_c \quad (10)$$

and therefore the quenching current is

$$I_m = \frac{10}{4\pi pn} H_c \quad (11)$$

For an infinitely long ideal superconductor solenoid with circular wire cross section, Eq. (9) holds with good approximation if the insulation between adjacent turns of each layer is thin and if the wire radius r is small compared with the coil radius. The quenching current I_m , however, will be somewhat smaller than indicated by Eq. (11) because of the distortion of magnetic field lines (Fig. 4). An analytical solution is possible but complicated.

III. Experiments with Lead Solenoids

To verify the considerations of the previous section, several experiments were made. A solenoid 3 in. long with a diameter of $2/3$ in. was wound with a single layer of 45 turns of 0.060-in.-diam Pb wire (0.999 pure). In order to produce a practically homogeneous field inside this coil, the solenoid was extended on both ends to an overall length of 8 in. by means of NbZr windings. Their number of turns per unit length was the

same as that of the lead winding. Since the experiments were performed at the temperature of liquid helium at atmospheric pressure, the critical field strength of the lead wire was close to 550 gauss. Equation (11) yields a critical current $I_m = 75$ amperes if rectangular wire cross section (Fig. 3) is assumed.

The normal resistance R_n at 4.2°K of the lead coil was measured in an external magnetic field of about 1 kilogauss to be 156 microhms, which corresponds to a ratio of about 1:1700 to the resistance at room temperature. Figure 5 shows the value R/R_n as a function of the current. The first trace of resistance was found at 69 amperes indicating a short tail of the resistance curve. This curve smoothly extrapolated to zero, yields a coil quenching current of 70 amperes. The influence of the circular wire cross section and of the insulation thickness ($d/2r = 0.066/0.060$) on the quenching current is apparently small. The experimental value for H_m is approximately 510 gauss.

Other experiments were made with the same arrangement, however, by leaving the NbZr windings floating and energizing only the lead winding. On each turn of this winding thin copper wires were soldered for measuring the individual turn voltages. At 70 amperes only five turns on each end of the lead winding showed voltage drops, indicating that these two parts went into intermediate state; 35 turns of the lead winding remained superconducting. With increased currents, partial resistance was restored to all turns. Figure 6 shows the individual turn resistances at a current of 85 amperes.

Finally, another solenoid was wound with lead wire without NbZr end compensation. It was 8 inches long and consisted of two layers, each with 120 turns of 0.060 inch diameter wire (0.9999 pure).

The diameter of the inside layer was $2/3$ inches, that of the outside layer 1 inch. Forty-two taps on each layer allowed to measure voltages across individual turns and groups of turns in the intermediate state.

Experiments showed that the first traces of resistance across the total inner layer appeared at 34 amperes, corresponding to H_m approximately equal to 500 gauss. Across the outside layer resistance appeared at 66 amperes. An extrapolation of the intermediate state resistance curve of the outside layer to zero yielded a current of 69 amperes.

With $H_c = 550$ gauss, the critical current of the corresponding infinitely long ideal two-layer solenoid (Eq. 11) is 37.5 amperes. The differences to the observed 34 amperes can be easily explained by the circular wire cross section, the insulation separation between adjacent turns and the fact that no end compensation was used. It is remarkable that the critical current of 69 amperes of the outside layer was so little affected by the heat production of the inner layer at that current level.

It is of interest to apply Chandrasekhar and Hulm's model to these experiments. For the short sample quenching current of a soft superconductor in a transversal homogeneous field, Silsbee's rule yields

$$I_c = 5r(H_c - 2H) \quad (12)$$

Considering for this case $G(0) = 1$, from Eqs. (3), (5), (6), (9) and (12) it follows:

$$H_m = \frac{H_c}{2 + \frac{d(p+1)}{2\pi r p}} \quad (13)$$

The resulting numerical values of 204 gauss for a single layer and 218 gauss for a double layer coil are much smaller than the observed maximum

field strengths. It is suggested that Eq. (3) on which the models of Montgomery, and Chandrasekhar and Hulm are based cannot be used in the described ways.⁵

IV. Conclusion

Montgomery's model for predicting the degradation effect of superconducting solenoids leads to a unique coil quenching characteristic if geometrically similar solenoids are considered. Experiments do not verify these predicted results. Chandrasekhar and Hulm's model leads to one unique coil quenching characteristic for all solenoids with identical wire type and turn distance; coils with identical load factor should display identical values I_m and H_m . An analysis of the surface currents in an ideal superconducting infinitely long solenoid demonstrates possible forms of shielding currents.⁵ Experiments with Pb coils with and without NbZr end compensation agree with results expected from this analysis, but contradict Chandrasekhar and Hulm's model. Measurements of individual turn resistances show the behavior of a soft superconductor solenoid in the intermediate state.

REFERENCES

1. J. K. Hulm, B. S. Chandrasekhar, and H. Riemersma, "High Field Superconducting Magnets," Advances in Cryogenic Engineering, 8 17 (1963).
2. D. B. Montgomery, Appl. Phys. Letters 1, 41 (1962) and National Magnet Lab. Rept. AFOSR-3015 (July 1962).
3. B. S. Chandrasekhar and J. K. Hulm, "Current-Carrying Capacity of High-Field Superconducting Solenoids," Appl. Phys. Letters 2, 43 (1963).
4. Z. J. J. Stekly, oral communication.
5. B. S. Chandrasekhar, W. F. Gauster, and J. K. Hulm, "Degradation Factor and Diamagnetic Currents in Supermagnets," Appl. Phys. Letters 2, 228 (1963).

UNCLASSIFIED
ORNL-DWG 63-1408

Fig. 1. Short Sample Characteristic and Coil Quenching Characteristic Based on Montgomery's Model.

UNCLASSIFIED
ORNL-DWG 63-1409

Fig. 2. Short Sample Characteristic Based on Chandrasekhar and Hulm's Model.

Fig. 3. Infinitely Long Soft Superconductor Solenoid (Ideal Case).

UNCLASSIFIED
ORNL-DWG 63-1411

Fig. 4. Distortion of Magnetic Field Lines Due to Circular Wire Cross Section.

Fig. 5. Resistance of a Single Layer Pb Solenoid in Intermediate State.

UNCLASSIFIED
CRNL-DWG 63-1413

Fig. 6. Turn Resistances of a Single Layer Pb Solenoid in Intermediate State ($I = 85$ amperes).

END