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UNITED STATES ATOMIC ENERGY COMMISSION

IV. PRODUCTION AND HEATING

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ENGINEERING NOTES

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We define $k_\infty = \eta\epsilon pf$ as the infinite reproduction factor where f is thermal utilization, p resonance capture escape probability, ϵ the fast effect coefficient, and η the number of neutrons emitted in thermal fission from the capture of a thermal in U. If ν is the number of neutrons per thermal fission of 25, we have $\frac{\eta}{\nu} = \frac{\Sigma^{25}_{\text{fission}}}{\Sigma^{25}_{\text{total}} + \Sigma^{28}}$ where Σ represents the macroscopic thermal cross section involved, $1 + \alpha = \frac{\Sigma^{25}_{\text{total}}}{\Sigma^{25}_{\text{fission}}}$ so $\frac{\eta}{\nu} = \frac{\Sigma_t^{25}/1+\alpha}{\Sigma_t^{25} + \Sigma^{28}}$ and $\frac{\Sigma^{28}}{\Sigma_t^{25} + \Sigma^{28}} = 1 - (1+\alpha)\frac{\eta}{\nu}$.

Using a three-group model with resonance capture the diffusion equations are;

$$\frac{\lambda_1}{3}\nabla^2\phi_1 - \frac{1}{\Lambda_1}\phi_1 + \frac{k}{p}\frac{\phi_3}{\Lambda_3} = 0$$

$$\frac{\lambda_2}{3}\nabla^2\phi_2 - \frac{1}{p\Lambda_2}\phi_2 + \frac{1}{\Lambda_1}\phi_1 = 0$$

$$\frac{\lambda_3}{3}\nabla^2\phi_3 - \frac{1}{\Lambda_3}\phi_3 + \frac{1}{\Lambda_2}\phi_2 = 0$$

RESTRICTED DATA

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In regions where the curvature of the thermal flux is small (everywhere except near the front face and then only if there is a large leakage there) $\nabla^2\phi_3$ is small, and $\int\frac{\phi_3 dx}{\Lambda_3} \sim \int\frac{\phi_2 dx}{\Lambda_2}$ is a very good approximation because the difference is $\frac{\lambda_3}{3}\left[\frac{\partial\phi_3}{\partial x}\Big|_0 - \frac{\partial\phi_3}{\partial x}\Big|_\infty\right]$. If the leakages are small both $\frac{\partial\phi}{\partial x}\Big|_0$ and $\frac{\partial\phi}{\partial x}\Big|_\infty$ are near 0.

The total number of neutrons captured in the lattice per incident neutron is by definition $1 + k_\infty + k_\infty^2 + \ldots = \frac{1}{1 - k_\infty}$. This number is also equal to the total resonantly captured in group 2 plus those thermally absorbed in group 3. The total leaving the resonance group is $\int\frac{\phi_2}{p\Lambda_2}dx$ of which a fraction $1 - p$ is by resonant capture. Thus:

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$$\frac{1}{1-K_\infty} = \int_0^\infty \left[\frac{\phi_2}{\Lambda_2}\left(\frac{1-p}{p}\right) + \frac{\phi_3}{\Lambda_3}\right] dx$$ in an infinite lattice with no leakage.

$$\frac{1}{1-K_\infty} = \left[\int_0^\infty \frac{\phi_3}{\Lambda_3} dx\right]\left[\frac{1-p}{p} + 1\right] = \frac{1}{p}\int_0^\infty \frac{\phi_3}{\Lambda_3} dx$$

The total production of 49 in the lattice is

$$\int \frac{\phi_2}{\Lambda_2}\frac{1-p}{p} dx$$ in the resonance region

$$+\int \frac{\phi_3}{\Lambda_3}\frac{\Sigma^{28}}{\Sigma_{tot}(\text{lattice})} dx.$$ The second term is equal to

$$\int \frac{\phi_3 f}{\Lambda_3}\frac{\Sigma^{28}}{\Sigma_t^{25} + \Sigma^{28}} dx = \left[\int_0^\infty \frac{\phi_3}{\Lambda_3}\right](f)(1 - [1+\alpha]\frac{\eta}{\nu})$$

$$P_{49} = \left[\int_0^\infty \frac{\phi_3}{\Lambda_3} dx\right]\left(\frac{1-p}{p} + f\left[1 - (1+\alpha)\frac{\eta}{\nu}\right]\right)$$

$$= \frac{p}{1-K_\infty}\left(\frac{1-p}{p} + f\left[1 - (1+\alpha)\frac{\eta}{\nu}\right]\right)$$

$$= \frac{1}{1-K_\infty}\left(1 - p + fp\left[1 - (1+\alpha)\frac{\eta}{\nu}\right]\right)$$

The number of fissions F in an infinite lattice with no leakage resulting from an incident neutron is

$$\int_0^\infty \frac{\phi_3}{\Lambda_3}\frac{\Sigma_f^{25}}{\Sigma_{tot\ lattice}} dx$$ thermal fissions multiplied by $\frac{\nu\epsilon - 1}{\nu - 1}$ to correct for fast fissions

$\nu\epsilon$ = number of neutrons from a thermal fission including following fast fissions

ν = number before fast fissions

$\nu\epsilon - 1$ and $\nu - 1$ are respective excesses.

$$F = \frac{1}{1-K_\infty}\left[pf\frac{\eta}{\nu}\left(\frac{\nu\epsilon - 1}{\nu - 1}\right)\right]$$

In a finite lattice with leakage at the front and back faces

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$$P = \left(\int_0^b \frac{\phi_3}{\Lambda_3}\,dx\right) f\left(1 - [1 + \alpha]\,\frac{\gamma}{\nu}\right) + \left(\int_0^b \frac{\phi_2}{\Lambda_2}\,dx\right) \frac{1 - B}{P}$$

and $$F = \left(\int_0^b \frac{\phi_3}{\Lambda_3}\,dx\right) \frac{\gamma}{\nu}\, f\left(\frac{\nu/\epsilon - 1}{\nu - 1}\right)$$

P and F are in moles of product and fissions per mole of incident neutron, respectively, if ϕ_2 and ϕ_3 are expressed in units of the incident fast flux Q_0.

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