Propagation Characteristics of Capillary Ripples, II. Instrumentation
for Measurement of Ripple Velocity and Amplitude^{*†}

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ABSTRACT

An instrument for measuring the propagation velocity of capillary ripples to within 0.3% and for measuring the amplitude of such ripples to within 1 to 10% (depending upon amplitude) was designed and constructed. The principle components of the instrument were a teflon Langmuir trough (with provision for automatic recording of compression data); generating and receiving probes consisting of a thin rod and a razor blade respectively mounted so that the rod and razor edge were parallel and accurately placed in the surface to be studied. The generator probe was driven sinusoidally by a highly modified audio "tweeter" speaker and the receiving probe was coupled to a crystal phonograph cartridge mounted on a micromanipulator. The standing wave pattern generated between the probes caused a sinusoidal output from the crystal cartridge which was amplified; the signal amplitude was proportional to the amplitude of the

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ripples and the phases of generator and receiver oscillations could be compared. Propagation velocity and damping coefficient dispersion could be readily calculated from this information collected as a function of frequency.

INTRODUCTION

The study of capillary ripple propagation, and particularly of ripple attenuation, has received little experimental study. Brown (1,2) has reviewed work in this area prior to 1936; his reviews list only 20 references concerning velocity of propagation and only four concerning attenuation. A brief recent review on damping coefficients is given by Davies and Rideal (3). Goodrich (4) used Brown's technique in a very recent study of ripple amplitude attenuation. Experimental techniques have heretofore been limited to optical methods. This paper will present simple electronic instrumentation for the accurate and convenient measurement of the propagation velocity and amplitude attenuation of capillary ripples.

THEORY

In order to study the visco-elastic properties of surface film by the technique of capillary ripples (5), the propagation parameters α , the damping coefficient, and λ , the ripple wave length, must be known accurately as functions of frequency. The older techniques for establishing these parameters used traveling plane wave propagation so that, in particular, log amplitude varied linearly with distance perpendicular to the plane wave source. Our use of a sensing probe transforms the problem to that of finding the propagation parameters from standing wave patterns;

the wave reflected from the receiving probe must be taken into account for the proper calculation of λ and α .

The coordinate system for the calculation has the z axis as the origin of the plane waves which oscillate in the y direction and travel in the x direction. The undisturbed fluid surface will be taken as the plane given by $y = 0$. The object of this section is to relate the r.m.s. output signal from the crystal receiving cartridge to the ripple damping coefficient, α , and the phase relationship between generating and receiving probes to the ripple wave length.

It was interesting to find that the treatment of ripple wave motion parallels the analysis of vibration damping in stretched polymer filaments. The work of Hillier and Kolsky (6) was found to be pertinent.

Let the driven probe be situated at $x = 0$ and move according to $\text{Re}\{A \exp(i\omega t)\}$. Let the receiving probe be situated at $x = L$ and move according to $\text{Re}\{B \exp i(\omega t - \theta)\}$ where A and B are constants, ω is the angular frequency ($2\pi\nu$), t is time and θ is the phase angle between the generator and receiver motions. After steady state conditions have been established, the outgoing wave will be given by

$$\vec{y}(x) = y_0 \exp(-\alpha x) \exp i(\omega t - kx)$$

where k is the wave number ($\frac{2\pi}{\lambda}$) and the reflected wave by

$$\overleftarrow{y}(x) = -y_0 \exp(-2\beta + i\phi) \exp -\alpha(2L-x) \exp i(\omega t - k(2L-x))$$

where

$$\frac{\text{complex amplitude of reflected wave}}{\text{complex amplitude of incident wave}} = -\exp(-2\beta + i\phi) .$$

This formulation accounts for both an amplitude change and a phase change on reflection. Change in phase might result from an energy dissipation on reflection; the experiments to be described apparently involve no such dissipation, so we shall set $\phi = 0$.

The displacement of the surface, y , at any point x between the probes will be given by

$$y(x) = y_0 [\exp(-\alpha x) \exp i(\omega t - kx) - \exp(-2\beta) \exp(-\alpha(2L-x)) \exp i(\omega t - k(2L-x))]. \quad (1)$$

When $x = 0$, $y = Ae^{i\omega t}$, hence from Eq. (1) we find

$$y_0 = A[1 - \exp(2\beta) \exp(-2\alpha L) \exp(-2ikL)]^{-1}$$

so that

$$y(x) = \frac{A[\exp(-\alpha x) \exp i(\omega t - kx) - \exp(-2\beta) \exp(-\alpha(2L-x)) \exp i(\omega t - k(2L-x))]}{[1 - \exp(-2\beta) \exp(-2\alpha L) \exp(-2ikL)]} \quad (2)$$

The receiving probe signal is proportional to the acceleration of the blade forced into motion by ripples traveling down the trough; the probe output will be also proportional to the displacement in the case of sinusoidal wave motion. At the probe, $x = L$, we have

$$V(L) = \frac{[1 - \exp(-2\beta)] \exp(-ikL)}{1 - \exp(-2\beta) \exp[-2L(\alpha + ik)]} \bar{A} \exp(-\alpha L) \exp(i\omega t) = c \bar{A} \exp(-\alpha L) \exp(i\omega t) \quad (3)$$

where $V(L)$ is the output voltage from the crystal cartridge with the receiving probe L cm from the generator probe and $\bar{A}$ is the appropriate proportionality constant.

We introduce the phase relationship between the generating and receiving probes by writing the complex constant c as

$$c = |c| \exp - i\theta$$

$$V(L) = \frac{A(1 - \exp(-2\beta))}{1 - 2 \exp[-2(\beta + \alpha L)] \cos 2kL + \exp[-4(\beta + \alpha L)]} \exp(-\alpha L) \exp i(\omega t - \theta) \quad (4)$$

and

$$\tan \theta = \frac{1 + \exp[-2(\beta + \alpha L)]}{1 - \exp[-2(\beta + \alpha L)]} \tan kL \quad (5)$$

Let $\overline{V(L)}$ be the root-mean-square (r.m.s.) value of $V(L)$ as expressed in Eq. (4). Since $\overline{V(L)}$ damps out to a simple exponential decay curve at large L , it is convenient to discuss two cases in terms of the magnitude of αL . If αL is sufficiently large,

$$\overline{V(L)} \approx \overline{V_a(L)} = K \exp(-\alpha L) \quad (6)$$

where $\overline{V(L)}$ and $\overline{V_a(L)}$ are the actual and approximate r.m.s. output voltages and K is a constant. Suppose $\alpha L = 2$ and $0 < \beta < \alpha L$, then

$$\text{Max} \left| \frac{\overline{V(L)} - \overline{V_a(L)}}{\overline{V(L)}} \right| \approx 0.02$$

which implies that the assumptions lead to an error of approximately 2%.

Suppose αL is small so that Eq. (6) is no longer satisfactory, then from Eq. (4) the envelope curves of $\overline{V(L)}$ are given by

$$\overline{E(L)} = \text{Max} (\overline{V(L)}) = \frac{A(1 - \exp(-2\beta)) \exp(\beta)}{2 \sinh(\beta + \alpha L)}$$

$$\underline{E(L)} = \text{Min} (\overline{V(L)}) = \frac{A(1 - \exp(-2\beta)) \exp(\beta)}{2 \cosh(\beta + \alpha L)}$$

so that

$$\tanh^{-1} \frac{\underline{E(L)}}{\overline{E(L)}} = \alpha L + \beta \quad (7)$$

This last equation can be written in the form

$$\log \frac{\bar{E}(L) + \underline{E}(L)}{\bar{E}(L) - \underline{E}(L)} = 2\alpha L + 2\beta . \quad (8)$$

Equation (5) will be used to calculate ripple wave lengths while Eqs. (6) and (8) will be used to calculate damping coefficients. Obviously, Eq. (6) affords the simplest calculation for α , and in many cases αL will be large enough to justify the use of this simple exponential relationship.

EXPERIMENTAL

A. Instrumentation

Figure 1 is a block diagram of the instrumentation which proved sufficient for accurate measurement of the wave amplitudes and phase relationships necessary to calculate velocity and attenuation coefficient dispersion. This section will be divided into several parts: The generator probe design including the circuit, the receiving probe design including the circuit, vibration control, and temperature control.

1. The Ripple Generator

The ripple generator (Fig. 2) was an electro-mechanical transducer, essentially a highly modified audio speaker. The magnet, designated by 1 in Fig. 2, was a small, 2 1/2 in. military voice speaker of conventional design. The voice coil, 2, was firmly attached, by an epoxy resin, to a piston, 3. The paraffin coated Tee bar, 4, actually producing the ripples was coupled to the piston by a wing nut clamping arrangement, 5. The Tee bar was easily released so that height and angular position could be adjusted. The thin hollow tube acting as a piston was supported by two pieces of flexible rubber sheet washers, 6. The voice coil was excited by the output from a 25 watt Heathkit power amplifier fed by a Hewlett

Packard Model 200 JR audio oscillator. The signal level into the voice coil was monitored by a Hewlett Packard 400 H A.C. vacuum tube voltmeter. The signal output passed through a filter circuit and then was led to the x-axis input of a DuMont oscilloscope. A Computer Measurements Co. Model 225c counter or a Hewlett Packard Model 100ER frequency standard oscillator was used to measure frequency to ± 0.1 cps or better.

The probe was assembled from monel rod ~ 1 mm. in diameter into the form of a braced Tee and carefully silver soldered. The horizontal rod was 5 cm long and the vertical rod was 3 cm high. Two angled braces eliminated low frequency flutter, however the assembly did have a sharp resonance frequency at approximately 1540 cps. The side of the horizontal rod in the plane of the vertical rod was ground flat to better than 25μ . The bottom of the horizontal rod was also ground flat to better than 25μ . The horizontal rod was carefully coated with paraffin to render it hydrophobic.

Unfortunately the mass of the Tee bar was sufficiently great that under the circumstances of the measurement the mechanical impedance of the system was mass controlled so that the amplitude attenuated as $\frac{1}{\omega^2}$. The Tee bar amplitude at constant power input rapidly decreased with frequency beyond roughly 300 cps.

2. The Ripple Receiving Transducer

The ripple sensing device, designated by 1 in Fig. 3, was a phonograph crystal cartridge, designated N4-3, made by Astatic Inc. The cartridge was designed for resonance free response in the range of 50 cps to 5,000 cps. The crystal cartridge was mounted on a micro-manipulator distributed by Brinkman, Inc. The mounting arm (2) was

arranged so that the orientation of the cartridge with respect to the micromanipulator could be adjusted and then firmly clamped. The micromanipulator had 10:1 planetary gear attachments on each of the three axis movements. It was possible to move the cartridge over the surface precisely positioning it with respect to the sending Tee bar and with respect to the water surface. The receiving probe was initially a Tee constructed from 30 mil. copper wire. The shank of the Tee was fitted into the screw clamp well on the crystal cartridge in place of the usual phonograph needle. Any vibration of the Tee bar was transmitted to the sodium potassium tartrate piezoelectric wafer enclosed in the cartridge. The voltage out of the crystal was first amplified by a Tektronix low level preamplifier. The filtered output from the preamplifier was fed into the y axis of the oscilloscope and into a second 400H Hewlett Packard A.C. vacuum tube voltmeter. The oscilloscope displayed the phase relationship between the sending and receiving probes while the voltmeters measured the amplitudes of the vibrating probes.

During the initial phase of the experimental effort, we were interested in establishing whether the receiving probe was responding only to surface disturbances or whether mechanical coupling was also occurring through the aqueous substrate or through the air. Thus probe response was studied as a function of vertical displacement. Capillary ripples were generated on a swept water surface at some convenient frequency such as 300 cps. The receiving probe was placed so that the sending and receiving Tee bars were parallel. The receiving probe could be moved in three mutually perpendicular directions by use of the

micromanipulator. The probes were separated about $1\frac{1}{2}$ cm and the output from the crystal cartridge was recorded as a function of vertical displacement from the surface. A rapid increase in signal occurred when the probe just touched the water surface, and a rapid decrease in signal occurred when the probe broke through the surface. Bringing the probe back into the surface region produced the same results; the signal from the probe increased rapidly when the probe just reached the surface and decreased rapidly when the probe broke through the surface into the air. This experiment showed conclusively that the signal was caused by surface disturbances.

A difficult aspect of the experimental technique was the measurement of probe separation. The micromanipulator had vernier scales on each axis allowing measurement of probe displacement to 0.01 cm. To measure the ripple wave lengths directly, probe separation changes must be established to at least 0.001 cm. Mechanically coupling a 10 turn 100 K Helipot, designated by 3 in Fig. 3, through gears to the coarse adjustment on the x axis of the micromanipulator allowed one to establish the magnitude of probe displacements to the required precision. The Helipot was hooked into a bucking potentiometer circuit. Mercury dry cells were used as current sources and a Kintel, Model 203, high impedance micro-voltmeter was used to establish the null. A high resistance potentiometer was used since the resolution of a wire-wound potentiometer increases with the number of turns of wire on the slide wire helix. The probe travel resolution with this circuit was about ± 0.0004 cm.

The calibration of the probe separation circuit was fortunately easy. The scale on the x-axis movement was compared to the scale on a Cambridge Instrument Co., Ltd., universal measuring machine. They were found to agree to within 0.0001 cm. Viewing the scale, replaced on the micromanipulator, through a low power microscope allowed us to calibrate the probe separation circuit to within ± 0.0004 cm.

A number of different Tee designs were tried. A satisfactory arrangement consisted of a single edge razor blade soft soldered to a short piece of high temperature silver solder. Since the needle well on the phonograph cartridge was angled, the blade was soldered to the shank at an angle so that the cutting edge would be parallel and the blade perpendicular to the water surface. As in the case of the generator probe, the mass of the probe should be minimized.

The blade was always coated with a thin layer of paraffin to render it hydrophobic. In use the blade could be brought down to touch the monolayers without radically disturbing the film. The vertical position of the blade was easily kept constant relative to the surface by watching light reflected from surface regions near the blade and adjusting the trough water level or probe height until no meniscus was visible close to the blade.

3. The Air Thermostat

For the purposes of temperature, contamination, vibration and sonic noise control, the sending and receiving probe mechanisms and the film balance were mounted in a constant temperature air bath, approximately

5 ft. x 3 ft. x 2 1/3 ft. The air bath looked much like a deep freeze unit placed on its side and oriented so that the door could open out. The unit was constructed from galvanized sheet metal. The space left between the inner walls of the chamber and the outer walls, roughly 4 in., was insulated with Styrofoam packing. Copper tubing was soldered to the walls of the inside chamber so that water from a constant temperature bath could be circulated exchanging heat with the inner chamber. The door was also constructed from galvanized sheet metal with copper tubing soldered to the inner wall panel. The annular space between the inner and outer walls of the door was filled with insulation. A section of this door, 29 in. x 5 in. x 1 1/2 in., could be independently removed to allow observation and manipulation of equipment in the chamber. The door was held in place against weather stripping glued to the front edge of the inner chamber by 12 spring clamps. At 25°C, the temperature was controlled to $\pm 0.05^\circ\text{C}$. A small fan controlled from outside the chamber circulated the air destroying thermal gradients. To minimize vibrations the fan was turned off during ripple measurements.

4. Vibration and Electronic Noise Control

The receiving probe was very sensitive to vibration, and therefore the isolation of the apparatus from building vibration and sonic noise was important. Decoupling was never completely successful, however, the noise generated at the receiving probe due to building vibration and sonic noise was minimized by the following steps:

- i) Several layers of 1/4 in. rubber sheet were placed under a soap stone 5 ft. x 3 ft. x 3 in. resting on a laboratory bench. The air thermostat was supported at the four corners on this soap stone

by four damped spring shock mounts (Korfund, type LO-A32).

The floor of the inner chamber of the air thermostat was covered with $1/4$ in. rubber strips, 1 in. wide. A flat piece of boiler plate 37 in. x 16 in. x $1/2$ in. was placed on these rubber strips. The micromanipulator and surface balance were mounted on this plate. The ripple generator was mounted on a Dexion structure built over but separated from the steel plate and isolated from the chamber floor by 1 in. x 1 in. x $1/4$ in. rubber squares. The tripod support screws for the teflon Langmuir (7) trough ride in three shallow depressions punched in the steel plate at appropriate positions. The barrier sweep mechanism was positioned behind the trough slightly beyond the edge of the steel plate on rubber supports.

- ii) Lint free wet towels were hung from the Dexion supporting structure close to the inner walls. A piece of acoustic damping composition board was placed across one end of the trough. These precautions appeared to decrease the noise level by a factor of approximately five.
- iii) Matched R.C. rejection filter circuits were built to filter out selectively 60 cps and 20 cps noise. A matched pair of high pass filters was also included in the circuit. The 3 db point for this filter pair was 240 cps. The filters were placed in the circuit symmetrically with respect to the x and y axis of the oscilloscope, see Fig. 1. Thus even though the signal passing through the filter

from the preamplifier side would suffer phase changes as the frequency was brought close to the maximum reject region, the phase shift was exactly matched by the other member of the filter pair placed in the circuit between the oscilloscope and the speaker coils. Switching allowed the six filters to be by-passed or not in any desired combination. The filter circuits were vitally needed above roughly 300 cps.

- iv) Electronic pick-up noise was minimized by carefully shielding sensitive circuits. Particular care was taken with the leads from the crystal cartridge to the preamplifier; low noise Amphenol RG-62/ μ coaxial cable was used in this circuit. All instruments used in this study were eventually grounded through an independent grounding system available in the laboratory.

B. Collection of Data

1. The Wave Number k

Equation (5) implies that, if $\beta + \alpha L$ is sufficiently large

$$\theta \approx kL \quad (9)$$

but for the special cases $\theta = n\pi$, $n = 0, 1, 2, \dots$ then regardless of the magnitude of $\beta + \alpha L$

$$\theta = n\pi = kL \quad (10)$$

By analysis of the Lissajous figure formed on the oscilloscope by beating the signal from the generating probe against the signal from the receiving probe we can establish whether or not the frequency and probe separation are such that the sending and receiving probes are exactly in phase or exactly out of phase, and so establish whether or not an even or odd number of half-waves reside between the probes. The wave number can then be

conveniently determined by one of two techniques. In either case the generating and receiving probes were first carefully positioned so as to just touch the liquid surface, and then adjusted until closely parallel.

- i) The Variable L. Technique. At a given frequency, probe separation was adjusted to a value in the neighborhood of 1 cm such that sending and receiving probes were in phase. Separation was then increased (or decreased) holding the frequency constant by a counted integer number Δn "in phase" + "out of phase" positions; the corresponding difference in separation was measured, and the wave length λ calculated by $\lambda = \frac{2\Delta L}{\Delta n}$ (Δn here refers to half waves). For best precision, n was plotted against L for each frequency studied, examples are shown in Fig. 4. The slope of this line is twice the wave number $\frac{k}{2\pi} = 1/\lambda$, and by repeating this procedure at different frequencies the dependence of wave number on frequency is established.
- ii) The Constant L Technique. At a given fixed probe separation, characteristically approximately 1 cm, the set of frequencies $\{\nu_n\}$ for which sending and receiving probes are exactly in phase or exactly out of phase is established by varying frequency at constant separation until in-phase or out-of-phase relation is indicated by the oscilloscope Lissajous figures.

At these frequencies integer numbers of half waves reside between sending and receiving probes. If the probe separation L is known, the set of frequencies $\{\nu_n\}$ arranged in sequence, and the number of half waves known for one frequency in the sequence, the numbers of half waves for all other frequencies in the sequence

are readily obtained, and hence the wave numbers for these frequencies can be calculated. Table I illustrates the process for a clean distilled water surface.

2. The Damping Coefficient α

Equations (6) and (8) indicate that measurement of damping coefficient separates into cases of small ($\alpha L < 2$) and large ($\alpha L > 2$) probe separations. In the latter case (see Fig. 5) plots of logarithm of cartridge output against probe separation were linear, so that the variable L technique at large probe separations furnished a rapid and simple method for evaluation of damping coefficients. In the former case (see Fig. 6) plots of cartridge output voltage against probe separation showed sequential maxima and minima and it was necessary to use Eq. (8) for evaluation of the damping coefficient.

The output of the crystal cartridge (connected to the receiving probe) was shown to be directly proportional to the amplitude of vibration of the sending probe, and therefore to the amplitude of ripples at the receiving probe, in two steps. First, the amplitude of vibration of the sending probe was proved proportional to the exciting voltage at a given frequency (see Fig. 7a), then the output voltage from the crystal cartridge attached to the receiving probe was proved proportional to the voltage exciting the speaker attached to the sending probe at a given frequency (see Fig. 7b).

To show that the amplitude of the sending probe was proportional to the voltage exciting the sending transducer, the core of a 003-ML Schaevitz Co. Linear Variable Differential Transformer (LVDT) was coupled to the transducer in place of the sending probe, and the piston

so weighted that the transducer load with core in place was the same as the transducer load with sending probe in place. The LVDT was excited by 5 volts at 20 kilocycles and its coil positioned around the core to obtain a secondary output of approximately 1 millivolt peak-to-peak. This signal was monitored by a Tektronix 545A oscilloscope with the low level plug-in unit, Tektronix Type E, as preamplifier. The 20 kilocycle output from the LVDT had been shown by previous calibration to vary linearly with core position; a change in core position of 1 micron produced a change in LVDT output easily measured by the Tektronix oscilloscope with low level plug in. Therefore amplitude of core vibration was directly proportional to modulation level on the 20 kilocycle output from the LVDT secondary. At every frequency investigated in the range 100 - 1400 cps core vibration amplitude was found to be strictly proportional to speaker coil excitation voltage, and therefore the sending probe vibration amplitude must also have been proportional to excitation voltage. Characteristic results for two of the many frequencies checked are shown in Fig. 7a.

Since the LVDT was calibrated, these data also permitted determination of the absolute amplitude of vibration of the sending probe, and its dependence on frequency at constant excitation voltage. The sending probe amplitude could hence always be adjusted so as to make this amplitude very small compared to ripple wave length as required for simple theoretical treatment. Characteristically, sending probe amplitude was adjusted so as to make the ratio of amplitude to wave length approximately 10^{-4} ; the amplitude at $x = 0$ never exceeded 1 micron in ripple propagation experiments.

It was next shown that capillary ripples generated by the sending probe produced a signal voltage from the cartridge attached to the receiving probe which was proportional to the sending transducer excitation voltage (and hence to sending probe and receiving probe vibration amplitude) at constant frequency. This was established at a sufficient number of frequencies to justify its validity in the frequency range 100 cps - 2000 cps; results at two frequencies are shown in Fig. 7b. It was not necessary to establish receiving probe amplitudes absolutely, since only amplitude ratios are needed for establishment of damping coefficients. It should be noted that the ratio between excitation and output voltages is markedly frequency dependent, i.e. the instrumentation does not have a "flat" frequency response. Since the variable L (constant frequency) technique was used for determination of damping coefficients this property of the instrumentation was of little concern.

DISCUSSION

This section will be divided into two parts--determination of ripple wave length and its frequency dependence, and determination of ripple damping coefficient and its frequency dependence. Data collected by means of these techniques will be presented in a subsequent paper (8) and discussed in terms of the theory previously developed by us (5).

A. Determination of Ripple Wave Length and Its Frequency Dependence

If the variable L (constant frequency) procedure is commenced with probes exactly in phase or exactly out of phase at a separation L_0 , then an integer number n_0 of half waves reside between probes at this separation

and frequency. Other separations L at which probes are in phase or out of phase then correspond to different integer numbers n of half waves, such that

$$n - n_0 = \Delta n = \frac{2}{\lambda} (L - L_0) .$$

Now quantities such as L and L_0 can be measured to a precision of somewhat better than $1/2 \lambda$, but the difference $(L - L_0)$, the quantity actually sensed by the potentiometer circuit previously described, can be measured to approximately $\frac{1}{100} \lambda$. Since $n = 0$ when $L = 0$, a plot of Δn against L gives $L_0 = \frac{\lambda}{2} n_0$ for the $L = 0$ axis intercept, knowledge that n_0 must be an integer permits refinement of L_0 , and hence of $L = L_0 + (L - L_0)$, to a precision of $\frac{1}{100} \lambda$. Occasionally (particularly when n_0 is large) an error of ± 1 in n_0 may result from this procedure. The corresponding error in wave number k will be apparent from the resulting anomaly in a plot of k against frequency and from discrepancies between results obtained by variable L and constant L techniques.

The constant L technique requires that the value of n be established at L for one frequency (for example, by the variable L technique). Determination of the frequencies at which the number of half waves between L is increased or decreased by 1, 2, 3, ... is then quickly accomplished, and this method therefore permits a rapid scanning of a frequency region (e.g. in search of anomalous dispersion). The variable L and constant L techniques are hence complementary. The constant L technique is considerably slower for scanning a frequency region, but is unaffected by small phase shifts due to frequency changes so long as the phase shift is independent of probe separation. It was found that phase shifts due to mechanical

coupling at certain resonance frequencies were indeed independent of probe separations. The worst resonance in our system occurred at 1540 cps, and was sufficiently severe to preclude accurate measurements at this frequency, but even in this case the phase shift was found to be independent of probe separation.

Uncertainties in the wave number k resulted chiefly from uncertainties in the wave length λ computed from plots of change in number of half-waves Δn against probe separation L . A probe separation of 1 cm or more can be measured to 1 part per 1000 with present instrumentation; small differences in probe separation can be determined to better than 1 part per 100. It is possible to determine the slope of the Δn vs L plot to approximately 5 parts per 1000. Alternating between the events per 10 sec. and 10^{-5} sec. per 10 events mode of counter operation for high frequency and low frequency measurements respectively, we were able to establish frequencies to 1 part per 10,000. Using the Hewlett Packard frequency standard to beat against the working oscillator, frequencies in convenient multiples of the standard oscillator could be established to better than 1 part per 1,000,000. The stability of the working oscillator limited us to 1 part per 100,000. While resetting the working oscillator to 1 part per 100,000 was difficult, once warmed up for several hours and set to a frequency, the oscillator would not go off frequency by more than 1 part per 10,000 in an hour.

It was hard to estimate objectively an error level for the phase angle measurement necessary for both the vary L and constant L techniques. The

signal to noise ratio* played a very important part in this aspect of the measurement. The phase error was extremely small when the signal to noise ratio was greater than 10:1, a level usually attained except at high frequencies. With present instrumentation precision of probe separation measurement is the limiting factor in wave length determination; this precision would need to be improved by a factor 10 to be comparable to the phase error. With such an improvement propagation velocities could be measured to 1 part in 10,000.

The constant L technique was much faster than the variable L technique. Once L and n had been determined, the whole frequency region available, 50 cps to 2000 cps, could be scanned in frequency increments, $\nu_n - \nu_{n-1}$, of approximately 25 cps in about 30 min. As some reflection on Eq. (5) will show, $\nu_n - \nu_{n-1}$ can be controlled by the choice of L; as L becomes larger $\nu_n - \nu_{n-1}$ will become smaller. It was possible, then, by collecting $\{\nu_n\}$ at various probe separations to search a frequency region as thoroughly as desired. At the high frequency end of the available range, damping and maximum wave amplitudes were such that probe separation had to be reduced in order to realize a sufficient signal to noise ratio. Hence members of $\{\nu_n\}$ at a given L were rather widely separated, so that sets had to be collected at several spacings to obtain a suitable density of frequencies. It was found that a much lower signal to noise level was tolerable in the constant L mode of measurement; a signal to noise ratio

* Using the filter system previously described, the noise level over most of the frequency range investigated was equivalent to 3 to 5 microvolts at the crystal cartridge, and was random with respect to generator signal.

of 1.5:1 was sufficient to find $\{\nu_n\}$. The maximum probable error increased to between 0.5 and 1%. Fortunately the eye must resolve only the transition of Lissajous figure from elliptical to linear form to establish members of the set $\{\nu_n\}$.

The major difficulty with the constant L technique was that the probes were weakly coupled through the air and supporting structure. In general, coupling of this type was of the order of the intrinsic noise level of the circuits; at most frequencies the noise level did not increase beyond $5 \mu v$ compared to the intrinsic noise level, $3 \mu v$, found with the vibrating probe at rest. However, there were a number of resonance frequencies, e.g., 1540 cps. As a result, we found that members of $\{\nu_n\}$ could be shifted in the neighborhood of a resonance frequency or else pseudo members of $\{\nu_n\}$ could appear. This last possibility produced frequency sequences such that a rational, consistent assignment of n could not be made. The previously discussed transducer design along with careful placement of the coil in the speaker magnet made these problems almost negligible except at very low signal to noise ratios. This is attested by agreement of data for dependence of $c = \lambda \nu$ on ν on clean water surfaces obtained from three $\{\nu_n\}$ sets at three different probe separations as shown in Fig. 8. Plots of $(\nu_n - \nu_{n-1})$ against ν_n obtained with clean water surfaces indicate mechanical resonances by anomalies; anomalies in such plots for film covered surfaces not found on clean water would then appear to indicate anomalous dispersion due to film structure.

B. Determination of Ripple Damping Coefficients

When applicable, the variable L procedure at large probe separations using Eq. (6) was the most precise way of finding α . The $\log_{10} V$ vs L

plots usually allowed calculation of the slope, $\frac{-\alpha}{2.303}$, to 1%. Since in this procedure the probe separation must necessarily be large, the signal level was low, and particularly with low surface tension systems at high frequencies it approached the intrinsic noise level of the circuits. However α seemed to be determined within 10% even under poor signal noise conditions.

Clean water surfaces presented a slightly different problem. Damping was so small that very large probe separations, 2 cm or larger, were needed before Eq. (6) could be used. Below 400 cps, the V vs L curves showed maxima and minima even at probe separations of 5 cm. The micromanipulator had an extension of only 6 cm. Under these conditions, it was more convenient to use $V_{\max}$, $V_{\min}$ data to calculate α from Eq. (8).

The small probe separation variable L procedure appeared to determine α to within 10%, however, the method was slow since five or six sets of V_{extrema} were needed to give sufficiently well defined envelope curves which would allow precise calculation of α . The exact positions of the extrema were hard to determine quickly without outlining the V vs L curve as extensively as was done in Fig. 6. A further complication was found in the fact that small deviations from parallel probe orientation had a very significant effect on the positions of the V_{extrema} . Undoubtedly part of the difficulty was due to the fact that at short probe separations, of the order of 1 to 6 millimeters, the surface was distorted due to slight roughness of the paraffin covering the probe surfaces.

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TABLE I

Ripple Propagation Velocity on Monolayer Free Water ($\gamma = 71.8 \text{ d/cm}$)
using Frequency Sets Obtained by Constant L Technique ($t = 25.0^\circ\text{C}$)

$\nu(\frac{1}{\text{sec}})^*$	n	$\lambda = \frac{3.010}{n} \text{ (cm)}$	$c(\frac{\text{cm}}{\text{sec}})$
193.8	13	0.2315	44.86
205.3	13.5	0.2229	45.76
216.7	14	0.2150	46.59
228.6	14.5	0.2076	47.46
240.5	15	0.2007	48.27
253.8	15.5	0.1942	49.29
266.3	16	0.1881	50.09
278.3	16.5	0.1824	50.76
291.0	17.	0.1771	51.53
303.6	17.5	0.1720	52.22
316.7	18	0.1672	52.95
330.1	18.5	0.1627	53.71
343.4	19	0.1584	54.39
355.6	19.5	0.1544	54.90
370.9	20	0.1505	55.82
385.0	20.5	0.1468	56.52
399.2	21	0.1433	57.20
413.4	21.5	0.1400	57.88
428.1	22	0.1368	58.56
442.5	22.5	0.1338	59.21
457.3	23	0.1309	59.86
472.0	23.5	0.1281	60.46
486.5	24	0.1254	61.01
500.7	24.5	0.1229	61.54
515.3	25	0.1204	62.04
529.5	25.5	0.1180	62.48
544.0	26	0.1158	62.99
558.9	26.5	0.1136	63.49
573.9	27	0.1115	63.99
589.5	27.5	0.1094	64.49
606.1	28	0.1075	65.16
622.3	28.5	0.1056	65.71

TABLE I. Continued

$\nu(\frac{1}{\text{sec}})$	n	$\lambda = \frac{3.010}{n}(\text{cm})$	$c(\frac{\text{cm}}{\text{sec}})$
638.7	29	0.1038	66.29
655.5	29.5	0.1020	66.86
672.4	30	0.1003	67.44
689.5	30.5	0.09869	68.04
706.7	31	0.09710	68.62
724.1	31.5	0.09556	69.19
742.0	32	0.09406	69.79
759.9	32.5	0.09262	70.38
777.8	33	0.09121	70.94
795.2	33.5	0.08985	71.45
813.8	34	0.08853	72.05
831.9	34.5	0.08725	72.58
850.3	35	0.08600	73.13
869.1	35.5	0.08479	73.69
888.2	36	0.08361	74.26
907.4	36.5	0.08246	74.82
926.0	37.	0.08135	75.33
945.2	37.5	0.08027	75.87
964.6	38	0.07921	76.41
983.9	38.5	0.07818	76.92
1003.3 [†]	39	0.07718	77.43
1023.0	39.5	0.07620	77.95
1042.4	40	0.07525	78.44
1062.5	40.5	0.07432	78.96
1081.9	41	0.07341	79.42
1102.1	41.5	0.07253	79.93
1122.7	42	0.07167	80.46
1143.3	42.5	0.07082	80.97
1164.7	43	0.07000	81.53
1184.5	43.5	0.06919	81.96
1205.7	44	0.06841	82.48
1224.9	44.5	0.06764	82.85
1246.4	45	0.06689	83.37

* Column lists, in order, $\{\nu_n\}$ found at a probe separation of 3.010 cm.

[†] n was established at this frequency by the variable L technique.

FIGURES AND CAPTIONS

1. Block diagram of apparatus for measurement of ripple wave length and amplitude.
2. The ripple generator.
3. The ripple receiving transducer.
4. Dependence of n on probe separation, monolayer free distilled water at 25°C. Zero of probe separation circuit has corrected according to procedure given in Discussion.
5. Dependence of receiver output on probe separation at large separations.
6. Dependence of receiver output on probe separation at small separations.
7. Demonstration of proportionality between receiving probe vibration amplitude and receiving transducer output.
 - (a) Demonstration that sending probe amplitude is proportional to voltage exciting sending transducer.
 - (b) Demonstration that receiving transducer output voltage is proportional to sending transducer input voltage.
8. Dependence of velocity of propagation of capillary ripples on frequency as obtained by constant L technique at three probe separations.

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