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Relationships between states of nuclei with
constant neutron excess[†]

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Abstract: Level schemes of light nuclei with constant T_5 are correlated by means of the nuclidic mass excess in order to show the $ld_{5/2}$ subshell to have an almost linear trend in the mass excess or total nuclear binding energy of excited states and ground states as a function of the mass number.

The systematics suggests a Ne^{20} 0.65 MeV, 0^+ level for which only weak experimental evidence exists. This state would form a linear relation with the ground states of Mg^{24} and Si^{28} , analogous to established linear trends for the first 2^+ and 4^+ states of these 3 nuclei.

The O^{16} 6.052 MeV, 0^+ state appears to be the beginning of this trend, where the $ld_{5/2}$ subshell is empty, but with a rearranged O^{16} ground state. The rearrangement energy seems to be independent of additional $2s_{1/2}$ nucleons.

Strict validity of this systematics would also require a S^{32} 0.41 MeV, 0^+ level as an additional exception to the empirical rule of an energy gap in even nuclei.

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The suspected separation energy for 4 nucleons from the $2s_{\frac{1}{2}}$ subshell decreases linearly with decreasing mass number to Ne^{20} 8.71 MeV, 0^+ and is assumed to continue so to C^{12} 7.656 MeV, 0^+ in analogy to the behaviour of certain 1^+ states in odd self-conjugate nuclei and $\frac{1}{2}^+$ states of odd-mass nuclei with $A = 4n + 1$ and $I_{\frac{1}{2}} = \pm \frac{1}{2}$.

In conclusion, 9 additional requirements are tabulated not as predictions, but as possibilities for checking to what extent additional effects, such as level shifts, selection rules or configuration mixing may destroy such simple regularities.

1. Introduction

Total nuclear binding energies and nuclidic masses have been plotted and discussed in the past mainly for the ground states. On the other hand, the level schemes of nuclei have been compared generally independently of their ground state energy; the ground states were normalized to the same height — except for isobars.

In order to locate systematic trends of states with a certain angular momentum and parity J^π in nuclei having constant neutron excess, both the ground state and the level energies will be considered simultaneously: We plot the total nuclear binding energy or nuclidic mass ^{excess} of excited states, whereby the ground states appear as a special case. Such a natural correlation of level schemes has been employed before by Inglis¹⁾ already in 1953.

The question treated in the present work is: Can the build-up of the $1d_{\frac{5}{2}}$ subshell be accounted for by a smooth trend? And does the occupation of the $2s_{\frac{1}{2}}$ subshell affect such a trend?

A short preliminary note on the existence of such linear relations between nuclear states has already been given²⁾.

Smooth trends in the total nuclear binding energy as a function of the mass number for constant neutron excess have been found by Cappeller³⁾ for the ground states of nuclides with mass number $A \leq 100$; the lowest $J^\pi = 0^+$, $I = 1$ states of odd selfconjugate

nuclei were already included. The trends were related to the subshells of the shell model.

2. Experimental Data Used

The level schemes of Ajzenberg-Selove and Lauritsen^{4,5)} and Endt and van der Leun⁶⁾ have been adopted, revised according to recent literature. References giving changes essential for this work in the regions of excitation which appear in the following graphs are: B⁹: ref. 7); see sect. 6. C¹²: ref. 8). N¹³: refs. 9-11). F¹⁸: refs. 12-14). Ne²⁰: refs. 15-19); doubtful levels at 0.65 and 2.53 MeV (ref. 20)) have been added; the uncertainty of the broad 0⁺ level^{4,5)} reported at (8.71 ± 0.01) MeV in ref. 20) is assumed to be 0.05 MeV. S³²: refs. 21-22); a doubtful 0.46 MeV level^{23,24)} has been added.

In addition, Nuclear Data Sheets²⁵⁾ and sect. 2 of the Landolt-Börnstein cumulation²⁶⁾ have been of considerable help and were particularly used for Ne²¹.

The empirical masses of the nuclides treated can be found in the 1961 Mass Table²⁷⁾ (or, with an older value of Na²¹ and Cl³⁴, in the previous Mass Table²⁸⁾). Corresponding separation energies were taken from a table of Q-values²⁹⁾.

3. Method

In most of the following figures the sum of the nuclidic mass excess Δ with a straight line, $\Delta + pA$, has been plotted versus the mass number A , essentially as Inglis¹⁾ and Cappeller³⁾ did, who used the total nuclear binding energy instead.

$\Delta = M - Am_u$ is the mass excess in MeV (in the unified scale where $\Delta(C^{12}) = 0$) for the ground state (Δ_0) or any excited state ($\Delta = \Delta_0 + E_x$, where E_x is the excitation energy).

M is the nuclidic mass in u (unified mass units, $M(C^{12}) = 12$) in the same generality as Δ .

A is the dimensionless mass number or number of neutrons and protons $N + Z$.

m_u is the unified mass constant which has the value 1u (unified mass unit) $\equiv \frac{1}{12} M(^{12}\text{C})$.

p is a constant in MeV, chosen to get a convenient graph $\Delta + pA$ versus A .

The nuclides for each plot have constant excess of neutrons over protons $N - Z$ or I_z -component of the isobaric spin $I_z = \frac{1}{2}(N - Z)$.

Any linear relation (or deviation from it) in $\Delta + pA$ means a linear relation in the mass excess Δ itself as well as in the total mass M . It also means a linear relation in the total nuclear binding energy $B(Z, N) = N \Delta(n^1) + Z \Delta(H^1) - \Delta(Z, N)$, because $N = \frac{1}{2}A + I_z$ and $Z = \frac{1}{2}A - I_z$ lead to (see, e.g., ref. 30)

$$B(A, I_z) = \frac{1}{2}A [\Delta(n^1) + \Delta(H^1)] + I_z [\Delta(n^1) - \Delta(H^1)] - \Delta(A, I_z)$$

which shows that for constant I_z the transformation of Δ into B is linear. It involves no distortion, but just the addition of a straight line with slope $\frac{1}{2} [\Delta(n^1) + \Delta(H^1)]$ and a constant. B increases in the opposite direction as Δ . As a matter of convenience we prefer to use the mass excess Δ here instead of the total nuclear binding energy B which is a derived quantity. As such it is also generally a little less precise.

The transformation contains the following numerical values:

$$\Delta + pA = -B + p'A + q'$$

$$p' = p + \frac{1}{2} [\Delta(n^1) + \Delta(H^1)] = p + 7.6800 \pm 0.0004 \text{ MeV}$$

$$q' = I_z [\Delta(n^1) - \Delta(H^1)] = I_z (0.7826 \pm 0.0004) \text{ MeV}$$

4. Results for Selfconjugate Even Nuclei

This group of nuclei with $I_z = 0$ and $A = 4n$ (n integer) should show subshell effects clearly if they are present at all. It is expected that at least the ground states are generally symmetric for protons and neutrons with regard to their structure, so that proton and neutron subshells fall together.

Fig. 1 implies that the ground states, connected by dotted lines, do not show any clear relation to the shell model subshell structure as indicated at the bottom, although the $1p_{3/2}$ and $1d_{5/2}$ subshells are

long enough for a sequence of 3 and 4 points, respectively. Apparently the ground states at magic numbers $N, Z = 2$ and 8 must not be considered as belonging to the following subshell. The first 2^+ and 4^+ states of Ne^{20} , Mg^{24} and Si^{28} lie on almost straight lines. Therefore one tends to assume the same for the 0^+ basis of them. This leads to the connection of the ground states of Si^{28} and Mg^{24} with the doubtful state Ne^{20} (0.65 ± 0.04) MeV (ref. ²⁰) — which happens to be in the right position — and to the assumption that it has $J^\pi = 0^+$.

This Ne^{20} (0.65 MeV) level was only observed by Morita and Takeshita²⁰⁾ in $\text{F}^{19}(\text{d}, \text{n}) \text{Ne}^{20}$ with $E_d = 2.17$ MeV at 12 different angles. Other experimentors^{4,5,31)}, using the same reaction at different bombarding energies, did not resolve such a level from the ground state. The level did not show up in the many other reactions investigated^{4,5)}. However, in the reaction $\text{C}^{12}(\text{C}^{12}, \alpha) \text{Ne}^{20}$ studied by Gove, Litherland and Clark¹⁵⁾ the tail of the ground state α -group appears to be enlarged.

Table 1 shows that the partial lifetime of a gamma transition from the Ne^{20} 1.632 MeV, 2^+ level to the doubtful 0.65 MeV, 0^+ level would be expected to be ≈ 20 ps, if the transition strength is taken to be an average of Mg^{24} and Si^{28} . This lifetime would be ≈ 36 times larger than the value observed for the transition to the ground state, so that only $\approx 3\%$ branching may be expected from the 1.632 MeV, 2^+ level to this state.

The experimental upper limit for a 0.98 MeV gamma transition to this state is in the order of $\approx 2\%$ as can be seen from the scintillation spectrum by Scharff-Goldhaber, Goodman and Silbert³³⁾. Therefore the existence of a 0.65 MeV state is not ruled out experimentally at present.

The conclusion that Ne^{20} g.s. and the 1.632 MeV, 2^+ state seem to have different configurations was drawn earlier by Clegg, Jones and Wilkinson³⁴⁾ and by Kane, Pixley and Wilkinson³⁵⁾ who found a strength of only $|M|^2 \approx 2 \times 10^{-4}$ Weisskopf units for the transition from the Ne^{20} , 13.482 MeV, 1^+ level to the ground state compared to $|M|^2 = 0.065$ for the one to the 1.632 MeV, 2^+ level, assuming no additional low-lying level. If a 0.65 MeV level exists, it seems difficult to distinguish a 12.83 MeV gamma ray to this level from a 13.48 MeV transition to the ground state in fig. 2 of ref.³⁵⁾. Especially with regard

to the rotational features mentioned in sect. 7.4 below, the 12.83 MeV transition to the 0.65 MeV level may even have a greater strength than the ground state transition.

A further example of an almost straight 0^+ line connects the origin, He^4 g.s., Be^8 g.s. and C^{12} 7.656 MeV.

Fig. 2 is an enlargement of this relation. It represents the known fact that Be^8 is slightly α -unstable by 0.094 MeV and that the binding energy of an α -particle in C^{12} is only 0.285 MeV below the 7.656 MeV state.

Fig. 3 gives the heavier selfconjugate nuclei with a slope constant $p = 1.7$ MeV, different from fig. 1 for convenient display.

The extension of the line Si^{28} g.s., Mg^{24} g.s. and Ne^{20} (0.65 MeV) leads with a very slight break to the 0^+ first excited state of O^{16} at 6.052 MeV. A line similar to this can be found by extending the connection between the 0^+ levels Si^{28} 4.975 MeV and Mg^{24} 6.432 MeV to Ne^{20} 8.71 MeV, 0^+ and, into the other direction, to a doubtful level in S^{32} at 0.46 MeV.

This level has been observed only in the $\text{P}^{31}(\text{d},\text{n})\text{S}^{32}$ reaction with poor statistics by Snowdon²³⁾ at (0.5 ± 0.1) MeV and by Singh²⁴⁾ at 0.43 MeV. El Bedewi and El Wahab³⁶⁾ did not find this state in the same reaction, but there is a slight asymmetry in their ground state group. Since other states near 1.5, 2.7 and 3.3 MeV, reported by Snowdon²³⁾ and Singh²⁴⁾, and a state at 1.7 MeV reported by Singh²⁴⁾ could not be confirmed and the statistics were not sufficient, these states including the 0.46 MeV level are generally not believed to exist.

A recent $\text{P}^{31}(\text{p},\gamma)\text{S}^{32}$ experiment by Nelson, Carlson and Schlenker²¹⁾ shows an enlarged tail of the ground state gamma transition from the 9.267 MeV, 1^+ level which perhaps may in part correspond to a transition to the proposed 0.46 MeV level. In careful studies of the P^{32} beta decay related to the spectral shape no additional branch has been found⁶⁾.

Such low lying 0^+ states as adopted here tentatively for Ne^{20} (0.65) and S^{32} (0.46) have not been observed in other light nuclei and could be interpreted only as remarkable exceptions to the empirical rule that the ground state configuration is separated by an energy gap³⁷⁾ from the following configuration.

The S^{32} (0.46 MeV) level would also form a straight line with the ground states of Ar^{36} and Ca^{40} where the $1d_{5/2}$ subshell is expected to become occupied. This would mean a configuration $(1d_{5/2})^{12}(2s_{1/2})^4$ for S^{32} (0.46 MeV). Therefore apparently the line leading over the 0^+ excited states of Si^{28} and Mg^{24} to Ne^{20} , 8.71 MeV, represents the repeated removal of 2 neutrons and 2 protons from the $1d_{5/2}$ subshell while the $2s_{1/2}$ subshell remains occupied. The Ne^{20} 8.71 MeV, 0^+ level would then be " 0^{16} 6.05 MeV + α " with the " α -particle" being in the $(2s_{1/2})^4$ state.

The $1f_{7/2}$ shell also shows a smooth trend from Ti^{44} to Fe^{52} . There is a step at Ca^{40} , less pronounced than at 0^{16} .

Returning to the occupation of the $1d_{5/2}$ subshell while the $2s_{1/2}$ subshell is full, one may ask where the 2^+ and 4^+ levels are which pertain to this configuration. For the 2^+ levels there seems to be only one choice, the dashed line in fig. 3. These 2^+ excitation energies in Mg^{24} and S^{32} , counted from the 0^+ level below it, are almost the same as those of the corresponding ones in Ne^{20} and Si^{28} where the $2s_{1/2}$ subshell is suspected to be empty. In Si^{28} an additional state at (6.31 ± 0.10) MeV with $J^\pi = 2^+$ may be expected, assuming the 3^+ assignment of the known state 6,22 as correct.

The degree of straightness of the lines in fig. 3 can be checked in enlargements by employing suitable slope constants p .

Fig. 4 shows the line through the doubtful Ne^{20} (0.65 MeV) level. The break is assumed real here and also in analogous examples although the trend may be bent as well. However, a subshell completely empty may give rise to special conditions. In Ca^{40} such a corresponding break would be still larger (fig. 3), if there is no additional, as yet undetected level at 2.68 MeV.

Fig. 5 gives the relative position of the masses of the 2^+ and 4^+ excited states for Ne^{20} , Mg^{24} and Si^{28} . The 2^+ states are linear within the experimental uncertainty of about 5 keV or 20 ppm (part per million) of the total nuclear binding energy. The 4^+ states represent the more frequent case of a deviation from linearity of the order of 100 keV.

Fig. 6 has been drawn as if there were a similar break at Mg^{24} as the one found in Ne^{20} (fig. 4). The possible linearity in the $1d_{3/2}$ subshell is exhibited in the same graph.

Fig. 7 is an enlargement of the 2^+ excitations as discussed above (fig. 3).

Fig. 8 repeats two parts of fig. 3 by normalizing in the left part the $(2s_{1/2})^0, 0^+$ states and in the right part the supposed $(2s_{1/2})^4, 0^+$ states.

Fig. 9 compares the 2^+ excitation energies of fig. 8. The difference is only about 50 keV. This may perhaps indicate that the $(2s_{1/2})^4$ "α-particle" does not get involved into the 2^+ excitation or at least does not influence its energy.

Fig. 10 is an extract of fig. 1. Since the $2s_{1/2}$ "α-particle" seemed to be rather independent of the 2^+ excitation of the core, one wonders whether the same applies to the 0^{16} 6.052 MeV, 0^+ excitation. In fact, these 6 Mev occur in neighbouring nuclides as energy difference between states of the same J^π , as already reported in a short note³⁸⁾. In Ne^{20} there is a doubtful level at (2.53 ± 0.07) MeV, 6.18 MeV below the 8.71 MeV, 0^+ state. This is, in addition to the 0.65 MeV level, the second previously unknown and not confirmed nor generally accepted level found by Morita and Takeshita²⁰⁾. From our point of view, it would correspond to the 0^{16} g.s. with a $2s_{1/2}$ "α-particle" attached. Fig. 10 continues this analogy further to " $\text{C}^{12} + \alpha$ " and " $\text{Be}^8 + \alpha$ ", by assuming the latter structure to belong to the level C^{12} 7.656 MeV, 0^+ . Obviously the upper curve must not be drawn through the 0^{16} 6.052 MeV, 0^+ state, because the $2s_{1/2}$ subshell can not be empty (lower curve) and occupied (upper curve) simultaneously. Such an additional 0^+ state at (6.20 ± 0.10) MeV, close to the 6.134 MeV, 3^- level, has not been found experimentally^{4,5)}, although such state has been looked for in connection with the α-particle model. If this state is not forbidden at all by unknown selection rules, its production rate may be very low in most reactions due to its complicated structure, which requires both the magic neutron and proton configurations to be destroyed.

Figs. 11 and 12 show the difference between these states of fig. 10. It is remarkable that the slope of the $1d_{5/2}$ subshell continues to the left side just with a small change to the experimental point of C^{12} in its 7.656 MeV state. Since the O^{16} 6.20 MeV level is hypothetical, this continuation requires further support which will be given below.

5. Results for Selfconjugate Odd Nuclei.

Fig. 13 exhibits the odd nuclei from H^2 to Cl^{34} . The results of the former section suggest looking for the states where the $2s_{1/2}$ subshell is half occupied, i.e. $\pi(2s_{1/2})^1 \nu(2s_{1/2})^1$, where π refers to protons and ν to neutrons. There should be one state with parallel spins, $J^\pi = 1^+$, $I = 0$ and one with antiparallel spins, $J^\pi = 0^+$, $I = 1$ in each case. The former can be expected for P^{30} g.s., 1^+ . It could also be a low lying 1^+ state in analogy with S^{32} . If P^{30} g.s., 1^+ is connected with the lowest 1^+ state in Al^{26} , however, the prolongation leads to a position just 6 MeV above F^{18} g.s., 1^+ . It is indeed somewhat likely that the F^{18} g.s. has $\pi(2s_{1/2})^1 \nu(2s_{1/2})^1$ and that the O^{16} core undertakes the 6 MeV excitation almost independently as suspected in Ne^{20} (fig. 10).

The line for the states with parallel spins can not be located yet, although the $I_\pi = 1$ nuclei are of considerable help in such attempts.

It should also be possible to remove one neutron and one proton with opposite spins from the $1d_{5/2}$ subshell in the sequence Ne^{20} (0.65 MeV), Mg^{24} g.s. and Si^{28} g.s. In Al^{26} this $J^\pi = 0^+$, $I = 1$ state seems to be at 0.229 MeV, but again the complete sequence can not be located yet. It is also not clear where the corresponding $J^\pi = 5^+$ states are. In Na^{22} such a state is missing.

Fig. 14 enlarges the 1^+ trend. The Na^{22} 3.066 MeV level has the right position, but its J^π is not known yet. The F^{18} 6.139 MeV state has been chosen somewhat arbitrarily; maybe the F^{18} 6.247 MeV, $1^{(+)}$ state should be used instead.

Fig. 15 shows the zig-zag of the 1^+ trend to be about parallel to the upper 0^+ trend of the even nuclei, fig. 10.

The existence of the N^{14} , 3.945 MeV, 1^+ level may be taken as weak support for the hypothetical O^{16} 6.20 MeV, 0^+ level in which the $2s_{\frac{1}{2}}$ subshell is thought to be occupied completely by 2 neutrons and 2 protons instead of only by one of each as in N^{14} 3.945 MeV, 1^+ .

The level B^{10} 0.7174 MeV, 1^+ (or perhaps the one at 2.153 MeV) appears as $\pi(2s_{\frac{1}{2}})^1 \nu(2s_{\frac{1}{2}})^1$ here instead of a p-shell configuration. A corresponding low-lying state with only the odd neutron, $\nu(2s_{\frac{1}{2}})^1$, $J^\pi = \frac{1}{2}^+$, may exist in Be^9 (see next section).

6. Results for Odd-Mass Nuclei

It was shown above that apparently the $2s_{\frac{1}{2}}$ subshell can be occupied by 2 neutrons and 2 protons (or by one of each) while the $1d_{\frac{5}{2}}$ subshell is being filled in a regular manner. Therefore similar trends may be expected for one neutron or one proton.

Fig. 16 gives the $I_{\frac{1}{2}} = \frac{1}{2}$ nuclei. The $\frac{1}{2}^+$ line certainly is supposed to meet Si^{29} g.s. because of its configuration $\pi(1d_{\frac{5}{2}})^6 \nu(1d_{\frac{5}{2}})^6 2s_{\frac{1}{2}}$. Similarly, the first O^{17} level is doubtless $\nu 2s_{\frac{1}{2}}$, so the line from Si^{29} g.s. should meet a $\frac{1}{2}^+$ level in O^{17} about 6.0 MeV above this state. This is indeed so if one uses the first $\frac{1}{2}^+$ level in Mg^{25} and assumes a new level in Ne^{21} . The Be^9 1.75 MeV, $\frac{1}{2}^+$ level tentatively used here has been interpreted by Spencer, Phillips and Young³⁹⁾ as being no state in the usual sense. Tang et al.⁴⁰⁾ pointed out that the empirical evidence for positive parity is very weak and found negative parity in a cluster model calculation.

Fig. 17 shows even the usual break as in $I_{\frac{1}{2}} = 0$, $A = 4n$ nuclei. In Ne^{21} one may assume an upward shift of 0.39 MeV for unknown reasons instead of a new state at 2.41 MeV.

Fig. 18 gives the mirror nuclei, $I_{\frac{1}{2}} = -\frac{1}{2}$. The 1 MeV wide peak^{4,5)} in $Be^9(p,n)B^9$ centered around $E_x = (1.43 \pm 0.05)$ MeV (the uncertainty is our estimate) as observed by Takayanagi et al.⁷⁾ and earlier by Marion and Levin⁴¹⁾ was tentatively assumed to be a level here.

Fig. 19 is a little more convincing than the corresponding fig. 17 because of the known J^π in F^{17} . Again, the missing Na^{21} level may only be shifted for reasons not known.

Fig. 20 extracts the $\frac{1}{2}^+$ trends from figs. 18 and 21 and may be compared with fig. 15. The average of the mirror trends has roughly the same slope as the trends in fig. 15.

7. Discussion

7.1. Existence of almost linear trends

A fairly smooth increase of the total nuclear binding energy or nuclidic mass^(excess) within subshells in sequences of constant $I_{\frac{1}{2}}$ appears to be consistent with a simple independent particle model. In these sequences with constant neutron excess the symmetry energy gives no distortion. When adding 2 neutrons and 2 protons several times into the same subshell, an exactly straight line would mean that the energy gain were the same each time, independent of the number of pairs already there. The expected deviation from linearity caused by residual interactions may be cancelled in part by the opposite effect of the Coulomb interaction. Both effects are small enough as not to bend the lines considerably over the range of the $1d_{\frac{5}{2}}$ subshell of only 12 mass units.

It appears that configuration mixing, although essential in the calculation of level schemes⁴²⁾, does not necessarily destroy such regularities, similarly as it seems to be the case for beta-decay energies including excited states⁴³⁾.

In the $1f_{\frac{7}{2}}$ shell such trends are generally bent upward; in each set of 3 points the middle one deviates by ≈ 50 keV from a straight connection of the outer ones.

7.2. Step phenomenon at certain magic numbers

Perhaps the most surprising result is the fact that the $1d_{\frac{5}{2}}$ subshell seems to be built up not from the ground state of O^{16} , but from its 6.05 MeV state. This seems to imply a rearrangement for the accommodation of the $1d_{\frac{5}{2}}$ subshell in a regular manner. Then the $(1s_{\frac{1}{2}})^4(1p_{\frac{3}{2}})^8(1p_{\frac{1}{2}})^4$ configuration would belong to both states, a fact hard to understand in the framework of the shell model. The step presumably was noted earlier in the ground state trend of O^{16} , Ne^{20} , Mg^{24} and Si^{28} , but if the Ne^{20} (0.65 MeV) level exists, the

conviction to use it for the $ld_{\frac{5}{2}}$ trend would enlarge this step from ≈ 3 MeV to as much as 6 MeV. Of course the approximate coincidence of the O^{16} 6.05 MeV, 0^+ level with the excitation energy obtained from extrapolation of the suspected $ld_{\frac{5}{2}}$ trend to zero occupation number may be purely accidental; the rearrangement may be possible only if there are particles already in the new subshell. Indeed this question is still open, although such an interpretation is not favoured by observations we made at higher mass numbers, not subject to this paper. Incidentally, the nature of this O^{16} 6.05 MeV, 0^+ state is not yet known. We may quote only the most recent attempt of an explanation here⁴⁴⁾.

A similar step of 7.46 MeV at He^4 occurs when the ground state connection of C^{12} and Be^8 is extrapolated to $A = 4$. But apparently He^4 can not assume a corresponding rearranged state with an empty $lp_{\frac{3}{2}}$ subshell.

The corresponding step at Ca^{40} is 2.68 MeV. No 0^+ level at this energy has been found, but a 0^+ level exists at 3.351 MeV (ref.⁶⁾).

7.3. Repetition rule for the O^{16} 6.05 MeV, 0^+ excitation

Apparently the rearrangement just mentioned (or whatever excitation it may be) can take place also when the $2s_{\frac{1}{2}}$ subshell is occupied completely or partly.

The fact that this O^{16} 6.05 MeV, 0^+ excitation energy occurs in neighbouring nuclei (and in the own level scheme) approximately as energy difference of states with the same spin and parity, was presented already in a short note³⁸⁾.

If the Ne^{20} 0.65 MeV, 0^+ level exists, it would be interesting to see whether or not this excitation energy occurs in Ne^{21} and Na^{21} , so that about 0.65 MeV below the required Ne^{21} 2.41 MeV, $\frac{1}{2}^+$ level there were another $\frac{1}{2}^+$ state Ne^{21} (1.76 ± 0.10) MeV and correspondingly in Na^{21} at (1.53 ± 0.10) MeV. In Na^{21} a state at 1.47 MeV has been observed indeed as neutron threshold by Marion et al.⁴⁵⁾. Measurements of neutron spectra by Benenson and Lidofsky⁴⁶⁾ and Ajzenberg-Selove et al.⁴⁷⁾, however, revealed only a state at 1.73 MeV with $J^\pi = \frac{7}{2}^+$ or $\frac{9}{2}^+$ (ref.⁴⁷⁾). Should really the population depend so much on the bombarding energy here and the Na^{21} 1.47 MeV level exist in addition and be $J^\pi = \frac{1}{2}^+$,

the known Ne^{21} 1.74 MeV state should be a close doublet.

If these additional $\frac{1}{2}^+$ states exist (we can by no means predict them!), there would be a linear relationship in the neutron and proton separation energies as described earlier⁴⁸⁾, which involve the ground state of Ne^{20} . If they do not exist, the separation energy systematics would have to use the Ne^{20} (0.65 MeV) level, provided that this exists.

7.4. Independence rule for 2^+ excitations

The independence of 2^+ excitations exhibited in figs. 8 and 9 is still very doubtful until a Si^{28} 6.31 MeV, 2^+ level can be shown to exist. A similar independence may be expected for the 4^+ excitations.

7.5. Rotational bands in Ne^{20}

If the Ne^{20} (0.65 MeV, 0^+) level exists, the lowest 2^+ and 4^+ levels may be described as a rotational band based on it rather than on the ground state, as the comparison with Mg^{24} and Si^{28} suggests (see, e.g., fig. 3). Several rotational bands have been discovered by Litherland, Kuehner, Gove, Clark and Almqvist⁴⁹⁾. The plot E_J versus $J(J+1)$, fig. 2 of ref.⁴⁹⁾, would become still more linear for the lowest band if a 0.65 MeV level could be used instead of the ground state. The nature of Ne^{20} g.s. would then remain unexplained.

From the close relation of Ne^{20} , Mg^{24} and Si^{28} (fig. 3) it would seem likely that all 3 nuclei could be explained on the same basis. The most recent attempt of a collective interpretation of Mg^{24} and Si^{28} by Cohen and Cookson⁵⁰⁾ may be quoted here, as well as the summary on d,s-shell nuclei by Gove²²⁾.

7.6. Relations to the cluster model

It is obvious that most of the results can be interpreted in terms of the cluster model^{51,52)}, where the $2s_{\frac{1}{2}}$ nucleons represent a relatively independent cluster. The Ne^{20} 8.71 MeV, 0^+ level, e.g., considered here as 0^{16} 6.05 MeV + $(2s_{\frac{1}{2}})^4$, has a large width for α -decay $\Gamma_\alpha > 800$ keV in accordance with such model. However, disagreement occurs in the parity of Be^9 1.75 MeV, $\frac{1}{2}^{(+)}$ (ref.⁴⁰⁾).

8. Conclusions

From various examples we conclude that the nuclidic mass^{excess} and therefore the total nuclear binding energy of certain ground states or excited states with the same angular momentum and parity increase linearly within ≈ 0.1 MeV while the $1d_{5/2}$ subshell is being filled 3 times by 2 neutrons and 2 protons. Possible exceptions occur at Ne^{20} and S^{32} , however, the excellent fit of the levels Ne^{20} (0.65 MeV) and S^{32} (0.46 MeV), for which only weak experimental evidence exists, as well as apparently analogous trends for a partly or completely occupied $2s_{1/2}$ subshell, make such states a little more likely and predict their angular momentum as $J^\pi = 0^+$.

The nature of the ground states of Ne^{20} and S^{32} remains unexplained, however.

Such low-lying 0^+ levels would be exceptions to the empirical rule of an energy gap in even nuclei.

The lowest 2^+ and 4^+ levels in Ne^{20} would then form a rotational band with the 0.65 MeV, 0^+ level rather than with the ground state.

In case these low-lying 0^+ levels do not exist, the peculiarity of the pattern is noteworthy.

A step phenomenon at magic number 8 exists. Assuming the states mentioned above to exist, its height has the considerable size of 6 MeV. Occupation of $2s_{1/2}$ states appears to be independent of this excitation which is probably a rearrangement for the accommodation of the $1d_{5/2}$ subshell in a regular manner. Also, 2^+ excitations seem to be independent of the occupation of the $2s_{1/2}$ subshell.

The systematics points to a relationship of Be^8 g.s., 0^+ , Be^9 1.75 MeV, $\frac{1}{2}^+$, B^{10} 0.717 MeV, 1^+ and C^{12} 7.656 MeV, 0^+ , although in the corresponding parentage relation 4 mass units higher an O^{16} 6.20 MeV, 0^+ level has not been found.

Table 2 points out instances in which the extent of validity of such a systematics can be checked, rather than creating predictions. Configuration mixing or undiscovered effects may shift or forbid certain levels expected from such simple considerations.

On the other hand, some states are hard to detect because they can be sufficiently populated only by suitably chosen means of production. Therefore the most convincing proof of the absence of a state once observed may come from a repetition of the same reaction with the same bombarding energy and angle of observation under better elimination of contaminants, etc., and with better resolution.

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Table 1
Mean lifetimes of $2^+ \rightarrow 0^+$ gamma transitions

	Ne ²⁰		Mg ²⁴	Si ²⁸
	1.632 $\rightarrow$ 0 MeV	1.632 $\rightarrow$ 0.65 MeV		
E_γ (E2) (MeV)	1.632	0.98	1.3676	1.772
$\tau_{m, \text{exp.}}$ (ps)	$0.56^{+0.28}_{-0.12}$ a)	[≈ 20] b)	1.7 ± 0.3 ⁶⁾	0.87 ± 0.16 ⁶⁾
$\tau_{m, \text{Weissk.}}$ (ps)	20	400	50	10
$ M ^2 = \frac{\tau_{m, \text{Weissk.}}}{\tau_{m, \text{exp.}}}$	36	[≈ 20]	29	12

a) The most recent and precise value ³²⁾ has been adopted. For two slightly higher values, compatible with it, see ref. ⁴⁾.

b) This value is not experimental, but estimated by choosing a transition strength $|M|^2 \approx 20$, the average of the values for Mg²⁴ and Si²⁸.

Table 2
Excitation energies, angular momenta and parities
required by this systematics

Nucleus	Required by syst.		Experimental		Ref.	Fig.
	E_x (MeV)	J^π	E_x (MeV)	J^π		
B^9	as observed	$\frac{1}{2}^+$	$(1.43 \pm 0.05)^a)$	$(\frac{1}{2}^+)^b)$	7)	<u>18</u> , 20
O^{16}	$6.20 \pm 0.10^c)$	0^+				<u>10</u> , 11, 12, 15
O^{17}	as observed	$\frac{1}{2}^+$	$6.869 \pm 0.014^d)$		4)	16, <u>17</u> , 20
F^{18}	as observed	1^+	$6.139 \pm 0.015^e)$		4)	13, <u>14</u> , 15
Ne^{20}	as observed	0^+	(0.65 ± 0.04)		20)	1, 3, <u>4</u> , 8, 10
Ne^{20}	as observed	0^+	(2.53 ± 0.07)		20)	1, 3, 8, <u>10</u> , 11, 12, 15
Ne^{21}	$2.41 \pm 0.05^f)$	$\frac{1}{2}^+$				16, <u>17</u> , 20
Na^{21}	$2.18 \pm 0.05^g)$	$\frac{1}{2}^+$				18, <u>19</u> , 20
Na^{22}	as observed	1^+	3.065 ± 0.005		6)	13, <u>14</u> , 15
Si^{28}	$6.31 \pm 0.10^h)$	2^+				1, 3, <u>7</u> , 8, 9
S^{32}	0.41 ± 0.05	0^+	$(0.46 \pm 0.07)^i)$		23)	1, 3, <u>6</u> , 8, 10, 11, 12, 15

a) The wide neutron peak seen ⁷⁾ in $Be^9(p,n) B^9$ has been taken to be the mirror state of Be^9 1.75 MeV. The uncertainty is estimated.

b) $(\frac{1}{2}^+)$ from analogy with $\frac{1}{2}^+$ of Be^9 1.75 MeV. See, however, refs. ^{39,40)}.

c) If such level is not forbidden by reasons not yet known, it should exist in addition to the 0^+ state at (6.052 ± 0.004) MeV.

Table 2 (continued)

- d) One of the neighbouring states may belong to this systematics instead.
- e) The $1^{(+)}$ level at 6.247 MeV may belong to this systematics instead.
- f) For reasons not yet known this level may be shifted by 0.39 MeV to (2.796 ± 0.006) MeV, where a $\frac{1}{2}^{+}$ state exists²⁵⁾.
- g) For reasons not yet known this level may be shifted by 0.24 MeV to (2.42 ± 0.02) MeV, where a $\frac{1}{2}^{+}$ state exists⁶⁾.
- h) The 2^{+} level is required in addition to the known 3^{+} level at (6.272 ± 0.006) MeV (ref. ⁶⁾).
- i) Average of two doubtful measurements (0.05 ± 0.1) MeV (ref. ²³⁾) and (0.43 ± 0.10) MeV (ref. ²⁴⁾). The uncertainty of the latter is our estimate.

Figure Captions

- Fig. 1. Mass excess Δ versus mass number A , where a straight line $0.9A$ MeV has been added to Δ for convenient display. Δ refers to ground states (circles) and excited states (dashes). Uncertainties are given by shorter dashes (negligable in this figure for all ground states). For excitation energies they are shown independent of the ground state uncertainty, which actually contributes to the total uncertainty of a level. Parentheses indicate doubtful empirical data, square brackets indicate unobserved levels or angular momenta and parities J^π as required by the present systematics. Full lines and dashed (doubtful) lines connect states apparently related. Dotted lines just connect ground states.
- Fig. 2. The line from the origin to C^{12} , 7.656 MeV (fig. 1) in an enlarged scale with a straight line of different slope added.
- Fig. 3. Right part of fig. 1 up to the $1f_{7/2}$ shell nuclei with a different slope constant.
- Fig. 4. Enlargement of the trend through the measured $^{20}_{10}\text{Ne}$, but doubtful $^{20}_{10}\text{Ne}$ level at 0.65 MeV as shown in figs. 1 and 3. This state has been found in $F^{19}(d,n)^{20}_{10}\text{Ne}$ only.
- Fig. 5. Enlargement of the trends in the masses of 2^+ and 4^+ excited states as shown in figs. 1 and 3.
- Fig. 6. Enlargement of the trend of certain 0^+ states as shown in fig. 3. Different slope constants are applied below and above $A = 32$ and the two parts are matched by a shift of the upper one. The measured $^{32}_{16}\text{S}$, but very doubtful 0.46 MeV level in S^{32} has not been found by other experimentors.

Fig. 7. Enlargement of the trend in the masses of certain 2^+ states as shown in figs. 1 and 3. The circle represents the required 2^+ state in the neighbourhood of a state known^{6,22)} as 3^+ .

Fig. 8. Excitation energies above certain 0^+ states shown in figs. 4 and 6. The states emphasized at the right side apparently differ from those at the left side by the fully occupied $2s_{\frac{1}{2}}$ subshell.

Fig. 9. Comparison of the 2^+ excitations emphasized in the former fig. 8. For the circle see caption of fig. 7.

Fig. 10. Certain 0^+ states extracted from fig. 1. The lower, heavy connected states are interpreted as having empty $2s_{\frac{1}{2}}$ subshells. The upper, thin connected states are interpreted here as having these subshells occupied. However, no such state has been found in O^{16} at the required position (6.20 ± 0.10) MeV.

Fig. 11. Separation energy of an alpha-particle from the upper states in fig. 10 leading to the lower states. The separation energy for Be^8 and the trivial value zero for He^4 have been included.

Fig. 12. Enlargement of fig. 11 for $A \geq 12$ by subtraction of a straight line.

Fig. 13. Odd selfconjugate nuclei. For legend see fig. 1. The cross at $A = 2$ indicates where the total nuclear binding energy B is zero. If the isobaric spin is known as $I = 1$, it has been given following the parity.

Fig. 14. Enlarged 1^+ trend of fig. 13 as expected from analogy with fig. 4. (The slope has been arbitrarily adjusted to be the same as in fig. 4 by choosing the additive term 2.2455 A.)

Fig. 15. Extracted 1^+ trend of fig. 13 together with the upper 0^+ trend of fig. 10.

Fig. 16. Odd-neutron nuclei for $I_{\frac{1}{2}} = \frac{1}{2}$. For legend see fig. 1.

Fig. 17. Enlarged $\frac{1}{2}^+$ trend of fig. 16 as expected from analogy with fig. 4. (The slope has been arbitrarily adjusted to be the same as in fig. 4 by choosing the additive term 2.3065 A.)

Fig. 18. Odd-proton nuclei for $I_{\frac{1}{2}} = -\frac{1}{2}$. For legend see fig. 1.

Fig. 19. Enlarged $\frac{1}{2}^+$ trend of fig. 18.

Fig. 20. Extracted $\frac{1}{2}^+$ trends from figs. 16 and 18.

Fig. 1. Mass excess Δ versus mass number A , where a straight line $0.9A$ MeV has been added to Δ for convenient display. Δ refers to ground states (circles) and excited states (dashes). Uncertainties are given by shorter dashes (negligable in this

Caption Fig. 1 (continued)

figure for all ground states). For excitation energies they are shown independent of the ground state uncertainty, which actually contributes to the total uncertainty of a level. Parentheses indicate doubtful empirical data, square brackets indicate unobserved levels or angular momenta and parities J^π as required by the present systematics. Full lines and dashed (doubtful) lines connect states apparently related. Dotted lines just connect ground states.

Fig. 2. The line from the origin to C^{12} , 7.656 MeV (fig. 1) in an enlarged scale with a straight line of different slope added.

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Fig. 4. Enlargement of the trend through the measured $^{20}_{0^{+}}$, but doubtful Ne^{20} level at 0.65 MeV as shown in figs. 1 and 3. This state has been found in $F^{19}(d,n)Ne^{20}$ only.

Fig. 5. Enlargement of the trends in the masses of 2^+ and 4^+ excited states as shown in figs. 1 and 3.

$$\Delta + 2.2375 A \quad \text{for } A = 20 \text{ to } 32$$

$$\Delta + 1.25 A + 31.6 \quad \text{for } A = 32 \text{ to } 40$$

Fig. 6. Enlargement of the trend of certain 0^+ states as shown in fig. 3. Different slope constants are applied below and above $A = 32$ and the two parts are matched by a shift of the upper one. The measured^{23,24)}, but very doubtful 0.46 MeV level in S^{32} has not been found by other experimentors.

Fig. 7. Enlargement of the trend in the masses of certain 2^+ states as shown in figs. 1 and 3. The circle represents the required 2^+ state in the neighbourhood of a state known^{6,22)} as 3^+ .

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Fig. 13. Odd selfconjugate nuclei. For legend see fig. 1. The cross at $A = 2$ indicates where the total nuclear binding energy B is zero. If the isobaric spin is known as $I = 1$, it has been given following the parity.

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Fig. 15. Extracted 1^+ trend of fig. 13 together with the upper 0^+ trend of fig. 10.

Fig. 16. Odd-neutron nuclei for $I_\pi = \frac{1}{2}$. For legend see fig. 1.

Fig. 17. Enlarged $\frac{1}{2}^+$ trend of fig. 16 as expected from analogy with fig. 4. (The slope has been arbitrarily adjusted to be the same as in fig. 4 by choosing the additive term 2.3065 Δ .)

Fig. 18. Odd-proton nuclei for $I_\pi = -\frac{1}{2}$. For legend see fig. 1.

Fig. 19. Enlarged $\frac{1}{2}^+$ trend of fig. 18.

Fig. 20. Extracted $\frac{1}{2}^+$ trends from figs. 16 and 18.

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