INFLUENCE OF THE DOPPLER EFFECT ON THE MELTDOWN ACCIDENT

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By
B. WOLFE
N. FRIEDMAN
D. RILEY

U.S. ATOMIC ENERGY COMMISSION
CONTRACT AT(04-3)-189
PROJECT AGREEMENT 10

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B. Wolfe
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United States Atomic Energy Commission
Under
Contract No. AT(04-3)-189. Project Agreement No. 10

Printed in U. S. A. Price $7.50. Available from the
Office of Technical Services, Department of Commerce,
Washington 25, D. C.

ATOMIC POWER EQUIPMENT DEPARTMENT
GENERAL ELECTRIC
SAN JOSE, CALIFORNIA
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SECTION I

ABSTRACT

The influence of the Doppler effect in the core disassembly process following a meltdown accident is examined with a Bethe-Tait type model in which the Doppler effect, as well as core disassembly, is considered in the reactor shutdown process.

By means of a digital computer, the power excursion and reactor shutdown are followed starting at the time that a threshold energy density $Q^*$ is achieved at the core center and the reactor is above prompt critical by an excess reactivity $k_0'$. As in the standard Bethe-Tait treatment, pressures tending toward disassembly are assumed proportional to the energy density above $Q^*$. The Doppler effect is taken to have either a $T^{-1/2}$ or a $T^{-3/2}$ temperature dependence. The reactor power, Doppler reactivity, and the reactivity reduction produced by disassembly are followed in time.

In the Bethe-Tait treatment with no Doppler effect, the energy release is a function of one parameter, $X$, which depends upon the core properties and is proportional to

$$\frac{k_0'}{\lambda^2}$$

where $\lambda$ is the neutron lifetime.

In the presence of a Doppler effect, the energy release is dependent not only on the parameter $X$ but also directly on $k_0'$. It is shown that as the Doppler effect is increased for a given $k_0'$, the mechanism determining energy release passes through a transition, being determined at one extreme by the core disassembly process and at the other extreme solely by the Doppler effect. In the latter case, significant core disassembly occurs only after the power is reduced to a low value by the Doppler effect. When the Doppler effect is determining, the energy release depends primarily on $k_0'$, not on the reactor parameters. As one result, it is shown that for the case of a constant reactivity insertion rate, in the presence of a strong Doppler coefficient, the energy release is reduced as the neutron lifetime is reduced.

In the absence of a Doppler effect, strong pressures are required to overcome the inertia of the core, whereas a strong Doppler effect provides time for core disassembly by much lower pressures. Thus, with the Doppler effect, attention must be focused on regions of low pressure buildup which can normally be ignored, to good accuracy, when no prompt temperature coefficient is present.
As a quantitative example, the worst hypothetical accident described in the Fermi Hazards Report is recalculated as a function of the Doppler effect. Maximum explosive energy releases are reduced from about 650 lbs. of high explosive energy with no Doppler effect to less than 20 lbs. in the case of a Doppler coefficient with

$$ T \frac{dk}{dT} = -0.01 $$

at the vaporization point.
SECTION II

INTRODUCTION

This report discusses a serious fast reactor accident in which the ultimate shutdown of the reactor is accomplished by destructive disassembly of the core. This is an accident of the type generally classed as a "worst conceivable" or "worst hypothetical" accident. We will show that a strong negative Doppler effect can radically reduce the explosive energy released in such an accident.

The Doppler effect influences the course of an accident in two important ways. First, in an accident involving reactivity insertion, the Doppler effect removes reactivity prior to the point of fuel damage. In many cases, the presence of a strong negative Doppler effect can prevent entirely major damage which would otherwise result from a given accident. Such situations have been discussed by Cohen, et al. In the author's opinion, this ability of the Doppler effect to prevent otherwise serious core damage is the major attribute of a strong negative Doppler coefficient.

In the present report, it is assumed that due to core melting or other reactivity insertion mechanisms, the reactor is still above prompt critical at the time that substantial pressure generation in the core begins. Further, it is assumed that all control mechanisms will be inoperative so that, in fact, the ultimate shutdown of the reactor must come from core disassembly.

The accident is considered from the time at which initial pressure is generated in the core. Thus, the reactivity present at this time has already been reduced by the negative Doppler effect and heat generation up to the point where pressure is generated.

Under the most pessimistic circumstances, however, there is some justification for ignoring reactivity reduction by the Doppler effect prior to pressure buildup. One can, for example, imagine a reactivity addition which occurred after the molten state had been reached and the fuel temperature had risen to a value just below the threshold value for pressure generation. Secondly, one might consider a reactivity ramp of unlimited extent. In this case, if the effect of delayed neutrons is ignored, the presence of a strong negative Doppler effect produces a series of power peaks with the maximum power and maximum reactivity attaining identical values on each peak. Thus, one can imagine that on the Nth peak the threshold temperature for pressure generation is reached. On the N + 1st power pulse the excess reactivity is the same as that which would be present if there were a reactivity ramp initiated after the reactor were already at the threshold temperature. (In fact, the delayed neutrons will damp the power oscillations.) Both of the situations discussed above may be considered as limiting and
pessimistic. In general, a given excess reactivity at the time of pressure buildup is equivalent, in the case of a strong negative Doppler coefficient, to a more serious accident if there were no negative Doppler coefficient.

In the analysis to follow, the Bethe-Tait formalism as modified by Jankus (2, 3, 4) is used. The Bethe-Tait approximation involves assumptions on the hydrodynamics of the core during the disassembly process. These assumptions have been discussed by McCarthy, et al. (2), Straton, et al. (5), Nicholson (4), and in more detail by Jankus (3). However, on the basis of the Jankus work (3), it appears that the method is adequate for the work described here.

A second assumption which is generally made in the Bethe-Tait analysis is that of a threshold equation of state for the materials in the core. Here it is assumed that the pressure buildup is proportional to energy density above a threshold energy density Q*, i.e., \( P = (\gamma - 1) (E - Q^*) \) where \((\gamma - 1)\) is a proportionality constant and \(E\) has units of energy per unit mass. For reasons discussed below, this threshold equation of state will be retained in the analyses discussed here.

The only published work on the influence of the Doppler effect in a meltdown accident is that of Nicholson (4), who found, as one might hope, that the Doppler effect does indeed greatly reduce the energy release in the meltdown accident. To increase the accuracy of his results, Nicholson abandoned the threshold equation of state and the analytical approach of the Jankus-Bethe-Tait method. Instead, he allowed the pressure to fall off smoothly as the energy density fell below the threshold value and utilized numerical techniques to follow the excursion. However, as Nicholson noted, this attempt at greater accuracy causes the results to depend upon each of the many parameters of the theory individually. It complicates the treatment so much that physical insight is lost and the functional dependence of the results on the parameters of the problem is obtained only by performing many individual calculations.

The present study is aimed at a general understanding of the action of the Doppler effect during the core disassembly process rather than being an investigation restricted to the details of a particular reactor. By retaining the threshold equation of state and modifying only slightly the standard Bethe-Tait analysis, it is possible to express results in terms of a small number of general parameters. It is believed that for the purposes of this study, this advantage outweighs the over-estimate in energy release obtained for cases having small energy releases in the presence of small Doppler coefficients.
Although the study is intended to produce results of general interest, the Fermi reactor has been chosen as a point of reference. The study has utilized two different threshold equations of state. A reactor loaded with metallic uranium fuel is assumed. The threshold equation which was suggested for the worst hypothetical accident in the Fermi Hazards Analysis, \( \gamma - 1 = 0.98 \) and \( Q^* = 0.98 \times 10^{10} \text{ ergs/gm} \) has been utilized. This equation of state (hereafter referred to as Equation A) assumes that there is no pressure buildup until the temperature reaches about 14,000°K in the fuel. All energy densities (i.e., \( Q^* \)) are measured from the liquid state at the melting point. As Nicholson has shown (and as will be shown here), in the absence of a Doppler effect pressures corresponding to these high temperatures generally are achieved in serious accidents, since the inertia of the core prevents rapid disassembly until extremely high pressures are achieved. On the other hand, there will be some initial pressure generation at the vaporization temperature, or shortly thereafter, and in the presence of a strong Doppler effect, it is these low pressures which eventually disassemble the core. Thus, a second threshold equation of state, Equation B, is utilized. Equation B takes the vaporization temperature of uranium as the threshold temperature. The pressure energy slope (\( \gamma - 1 \)) in Equation B was chosen so that the pressure in the two equations of state are identical at an energy one-and-one-half times the threshold energy density of Equation A, i.e., at \( E = 1.5 \times 0.98 \times 10^{10} \text{ ergs/gm} \). For Equation B, \( \gamma - 1 = 0.192 \) and \( Q^* = 3 \times 10^9 \text{ ergs/gm} \). The need to consider the lower threshold arises physically from the fact that, with a strong Doppler effect, the excursion is essentially terminated by the Doppler effect, which then provides time for core disassembly by relatively low pressures. It is these relatively low pressures which, in the presence of a strong Doppler coefficient, ultimately disassemble the core. To good accuracy the low pressure buildup can be ignored for serious accidents when there is no prompt temperature coefficient.

The various equations of state are shown in Figure 1.
Equations of state discussed in this report. Equation A is that used in Fermi Hazards Analysis. Equation B approximates the fact that some pressure will be generated from the start of vaporization. Nicholson's (analytic) equation attempts to approximate both the saturation pressures at low energy density and the rapid buildup of pressure after the fuel forms a single phase occupying all available space.
SECTION IV

MATHEMATICAL FORMULATIONS

Doppler Reactivity Reduction

The rate of change of reactivity due to the Doppler effect can be expressed as

\[ \frac{dk}{dt} = \left( \frac{dk}{dT} \right)_{T_0} \left( \frac{T_0}{T} \right)^n \frac{dT}{dt} \]  \hspace{1cm} (1)

where \( \left( \frac{dk}{dT} \right)_{T_0} \) is the Doppler temperature coefficient at temperature \( T_0 \). The Doppler coefficient is assumed to decrease in magnitude inversely as the \( n \)th power of the temperature. \( T \) is temperature measured from absolute zero, \( t \) is time.

This work will consider the course of an accident from the time that the energy density \( Q^* \) is achieved at the core center. If \( T_0 \) is taken as the temperature when the energy density \( Q^* \) is achieved, then (1) can be rewritten

\[ \frac{dk}{dt} = \left( \frac{dk}{dT} \right)_{T_0} \left( \frac{T_0}{T_0 + \frac{Q-Q^*}{C_y}} \right)^n \frac{d}{dt} \left( \frac{Q-Q^*}{C_y} \right) \]  \hspace{1cm} (2)

In deriving (2), changes in the volume of the core during the excursion are ignored. In addition, use is made of the heat capacity at constant volume. This is in the spirit of the Bethe-Tait approximation although in principle, it would cause an overestimation of the Doppler reactivity reduction. Actually, where the Doppler effect is of major significance, it will be shown that the important Doppler reactivity reduction does occur before significant core motion occurs. Thus, the error in the use of \( C_y \) will not be due primarily to a question of principle but rather to the fact that there are no good values of \( C_y \) available at the temperatures of interest. Equation (2) can be rewritten

\[ \frac{dk}{dt} = - \alpha \frac{A^{n-1}}{(A + y)^n} \frac{dy}{dt} \]  \hspace{1cm} (3)
where \( y = \frac{Q - Q^*}{Q^*} \)

\[ A = \frac{C_v T_0}{Q^*} \]

\[ \sigma = -T_0 \left( \frac{dk}{dT} \right) T_0 \]

Finally, taking \( k_0 \) as the reactivity above prompt critical when \( Q^* \) is achieved, expression (3) can be put in the following form:

\[ \frac{d\Gamma}{d\tau} = -\eta \frac{A^{n-1}}{k_0} \frac{d\eta}{(A + y)^n} \]

(4)

where \( \eta = \frac{k}{k_0} \)

\( \tau = \frac{k_0}{\lambda} \) t

\( \lambda = \) prompt neutron lifetime

The reason for this choice of the form of (4) will become apparent below.

The Bethe-Tait Equations

Because there are several (2, 3, 4) fairly complete treatments of the Bethe-Tait analyses as modified by Jankus, the work is not repeated here. However, throughout this report the notation of reference (4) has been used so that the reader may examine the Bethe-Tait analysis and the present work with the convenience of a consistent notation.

The Bethe-Tait equations have been written by Nicholson (4) in the form
\[ \frac{d^2 \kappa}{d \tau^2} = - \left( \frac{y+1}{X} \right) \left( \frac{y}{y+1} \right)^{5/2} \quad 0 < y < \frac{q}{1-q} \]  

(5)

\[ \frac{d^2 \kappa}{d \tau^2} = - \frac{5q}{2X} \left[ (1 - 3/5q) y - 3/5q \right] \quad y > \frac{q}{1-q} \]

Similarly, other boundary conditions are \( y = 0 \) at \( \tau = 0 \); \( \frac{dy}{d\tau} = 1 \) at \( \tau = 0 \); and \( \frac{dx}{d\tau} = 0 \) at \( y = 0 \).

Equation (5) describes the change in reactivity due to disassembly of the core while equation (6) is the standard equation (in Nicholson's notation) relating rate of change of power to reactivity. Equations (5) and (6) depend only upon the parameter \( X \) and, weakly, on \( q \).

In deriving (5), the assumption was made that the reactor power shape in the (spherical) core is of the form

\[ P(r) = 1 - q \frac{r^2}{a^2} \]

(7)

where \( a \) is the core radius and \( q \) is a constant. Similarly, the reactivity worth function of core material was also assumed to be parabolic and of the form

\[ W(r) = W(0) - \frac{dr^2}{2a^2} \]

(8)

The expression for \( X \) is given by

\[ X = \frac{5q^{3/2}}{8 \pi a d (\gamma - 1) Q^*} \frac{k_{o0}^3}{1} \]

(9)
Thus, for a given reactor $X$ is proportional only to $k_0^4$. The value of $y$ achieved at the end of the excursion (i.e., the energy density) increases monotonically with $X$.

Jankus (2,3) has shown that according to one group theory, the dependence of $d$ on core size is such that $X$ increases with increasing core size. Thus, the energy release for a given $k_0$ will increase more rapidly than the core mass as the core is made larger assuming $q$, 1, $(\gamma -1)$, and $Q^*$ remain unchanged.

The Doppler effect can be included in the above framework by modifying equation (6) to read

$$\frac{d^2 y}{d\tau^2} - \left(\kappa + \Gamma\right) \frac{dy}{d\tau} = 0$$

where $\Gamma$ is obtained from equation (4). Equations (4), (5), and (6a) then constitute the Bethe-Tait equations including a prompt temperature (Doppler) effect. For the convenience of the reader, the equations are repeated below:

$$\frac{d\Gamma}{d\tau} = -\frac{\alpha}{k_0} \frac{A^{n-1}}{(A+y)^n} \frac{dy}{d\tau}$$

$$\frac{d^2 x}{d\tau^2} = -\left(\frac{y+1}{X}\right) \left(\frac{y}{y+1}\right)^{5/2} \quad 0 < y < \frac{q}{1-q}$$

$$\frac{d^2 x}{d\tau^2} = -\frac{5q^2}{2X} \left[ (1-3.5q) y - 3.5q \right] \quad y > \frac{q}{1-q}$$

$$\frac{d^2 y}{d\tau^2} - \left(\kappa + \Gamma\right) \frac{dy}{d\tau} = 0$$

Equations (4), (5), and (6a), are to be solved with the initial conditions

$$y(0) = 0 \quad dy(0)/d\tau = 1$$

$$x(0) = 0 \quad dx(0)/d\tau = 0$$

$$\Gamma(0) = 0 \quad d\Gamma(0)/d\tau = 0$$

4-4
For the work described in this report, the above equations have been solved simultaneously on a
digital computer. Starting with the initial conditions, the equations are numerically integrated
in short-time steps. The integrations continue until the reactor power falls below a preset value,
after which the computation is terminated. In all cases discussed herein, the criterion for
terminating the calculation was set at $\frac{dy}{d\tau} = 1 \times 10^{-6}$, i.e., the calculation terminates when
the power drops to one-millionth of the initial power level. In all cases it was observed that the
energy density, $y$, was not changing appreciably during the period prior to termination of the
calculation. The value of $y$ at the end of the calculation measures the energy density achieved
at the core center and is algebraically related to the total energy release in the excursion.

To make the points of this report in concrete terms, the Fermi reactor was used as a model and
the parameters given in the Fermi Hazards Report (6) were used. Thus, in all cases, except
as noted, the following values have been assigned to the various parameters:

\[ a = 45 \text{ cm} \]
\[ d = 5.6 \times 10^{-6} \text{ cm}^3 \]
\[ l = 1 \times 10^{-7} \text{ second} \]
\[ q = 0.8 \]

For Equation A

\[ Q_1 = 0.98 \times 10^{10} \text{ ergs/gm} \]
\[ b_{1} - 1) = 0.98 \]
\[ A = 1.6 \]

For Equation B

\[ Q_1 = 0.3 \times 10^{10} \text{ ergs/gm} \]
\[ b_{1} - 1) = 0.192 \]
\[ A = 1.9 \]
A number of calculations have been performed to illustrate the effect of a Doppler coefficient on a meltdown accident. Calculations have been performed as a function of the initial reactivity above prompt critical at the threshold energy density and as a function of the Doppler coefficient. The Doppler coefficient has been characterized by the quantity $\alpha$, which is equal to $-T \frac{dk}{dT}$ at the threshold temperature $T_0$.

In the large (several thousand liter) cores presently proposed for fast power reactors the Doppler coefficient exhibits a temperature behavior approximately proportional to $1/T$ so that $T \frac{dk}{dT}$ is independent of temperature. Greebler and others have calculated values of $T \frac{dk}{dT}$ in excess of 0.01 for power reactors of interest. On the other hand, for smaller reactors and for very high temperatures $\frac{dk}{dT}$ is proportional to $\left(\frac{1}{T}\right)^{3/2}$. A number of parallel calculations were performed for the two situations ($n = 1$ and $n = 3/2$ of equation (4)). The calculations indicate that the important quantity is $\frac{dk}{dT}$ at $T_0$ and that the temperature dependence has a significant influence only for cases of an intermediate Doppler coefficient. That is, for very small coefficients there is, in any event, little effect on energy release, while for very large coefficients the excursion is terminated before the temperature rises a significant fraction of $T_0$. Thus, the temperature dependence of the Doppler coefficient plays its major role in determining the value of the Doppler coefficient at the temperature of initial pressure generation.

For the case in which the Doppler coefficient is zero, $\alpha = 0$ the present analysis is equivalent to the normal Bethe-Tait analysis and the energy release depends only upon the Bethe-Tait parameter $X$. On the other hand, in the presence of the Doppler coefficient, the energy release depends not only upon $X$ but also upon the initial reactivity $k_0$. Results of a set of calculations are shown in Figure 2. The parameter $y = \frac{(Q - Q^*)/Q^*}{Q^*}$ was plotted against the Doppler coefficient for different values of $X$. For a given reactor, $X$ is a function only of $k_0^3$. However, in order to gain physical insight, for each $X$ value $k_0^3$ was changed by a factor of ten in each direction, while at the same time keeping $X$ constant by making a corresponding change in one of the other parameters in $X$ (it matters not which). The solid line in Figure 1 corresponds to the parameters of the Fermi reactor, the dashed lines to parameters modified so that with $k_0^3$ increased (or decreased) by a factor of 10, $X$ remained unchanged. The important result shown in Figure 2 is:

For low values of $\alpha$, the energy released depends primarily upon $X$; however, for high values of $\alpha$ it may make the point clearer to note that in practice when $k_0^3$ was increased by a factor of 10 $d$ was also increased to keep $X$ unchanged. The results would have been identical if we had increased $d^2$. 

\[ \text{5-1} \]
Figure 2. Energy Density

The energy density achieved at the end of the excursion as a function of the Bethe-Tait parameter $X$, and the Doppler effect. The solid line represents the Fermi reactor model as given in the Fermi Hazards Report. $X$ is proportional to $k_3$. The dashed lines were obtained for cases where $k_3$ was increased or decreased by a factor of ten while at the same time one of the reactor parameters was also changed to leave $X$ unchanged.
the energy released becomes independent of $X$ but depends only upon the initial reactivity $k_0$. This can be seen by noting that for $\alpha = 0.01$ the various curves with the same $k_0$ asymptotically approach each other. Physically what is happening is that for high values of $\alpha$ the Doppler coefficient terminates the excursion and reduces the power level to a low value. The pressures generated up to this time, although small, now have time to overcome the inertia of the core and "gently" blow the reactor apart. Thus, in the presence of a strong Doppler coefficient, the energy generated becomes independent of the details of the pressure-energy relationship provided only that enough pressure is generated to eventually disassemble the core. (In the case of a continuing reactivity ramp, enough pressure would have to be generated to disassemble the core and terminate the ramp before another power pulse would occur.)

This point is further illustrated in Table I which shows a typical case at the time that the excursion has been turned around and the power level has returned to the power level at the threshold condition. It can be easily shown that if the Doppler coefficient were temperature independent and were alone responsible for the reactor shutdown, then at the time when the power level had returned to the initial power level, the Doppler effect would have reduced reactivity by an amount equal to twice the initial excess reactivity. In the case where the Doppler coefficient decreases with temperature more energy is released, but less reactivity reduction occurs. In Table I it can be seen that for $\alpha = 0$, the reactivity reduction due to disassembly of the core is about 5 times the initial reactivity for the case considered. As the Doppler coefficient increases in magnitude, the reactivity reduction by disassembly gets smaller while the reduction caused by the Doppler effect gets larger. For the high Doppler coefficient, the power reduction can be attributed almost entirely to the Doppler effect.

To summarize, in the case of a strong Doppler coefficient, the energy release becomes essentially independent of the parameters of the problem and depends primarily on the reactivity at the start of pressure buildup and the magnitude of the Doppler effect.

One other point of interest may be noted from Figure 2. Whereas, with a strong negative Doppler effect, the inserted reactivity, and not the reactor parameters, determines the energy release, the situation is reversed in the case of a positive prompt temperature coefficient. Here the governing reactivity is not that at the start of the excursion but rather the reactivity inserted during the excursion. The energy release then depends not so much on $k_0$, as on the other parameters in $X$. Thus, in Figure 2 where $k_0^2$ was decreased and $X$ was kept constant by changing other parameters, the energy release increases for a positive temperature coefficient. With a positive coefficient, a short neutron lifetime, a low rate of pressure buildup with energy, or any other condition which causes a high temperature before disassembly, obviously will increase the energy release.
TABLE I

Reactivity reduction when power level is reduced to initial power level

\[ k_0 = 0.0013 \]
\[ Q^* = 3 \times 10^9 \]
\[ A = 1.47 \]
\[ n = \frac{3}{2} \]

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<td>( \alpha )</td>
<td>( (1-\alpha) )</td>
<td>( \Gamma )</td>
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<td>5.23</td>
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<tr>
<td>0.01</td>
<td>0.0044</td>
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\(^*\) Measured in units of \( k_0 \)
ENERGY RELEASE IN A HYPOTHETICAL ACCIDENT

To further explore the effect of the Doppler coefficient on the meltdown accident, the worst hypothetical accident described in the Fermi Hazards Report as a function of an assumed Doppler coefficient was calculated. In this case, the initial reactivity above prompt critical was taken to be 0.0013 and again the parameters of the reactor described in the hazards report have been used. The calculations were performed with the two different threshold equations of state, A and B, and the results have been cast in several different forms.

Figure 3 shows the excess energy above threshold for the two equations of state as a function of the Doppler coefficient. It can be seen that for a zero coefficient there is actually greater energy release above threshold for the case of the lower threshold energy. As previously noted, this is due to the fact that with no Doppler coefficient, high pressures must be generated to overcome the inertia of the core. The equation with the high threshold (Equation A) involves more rapid pressure generation above the threshold and thus causes the disassembly of the core at a lower value of excess energy. With a large Doppler coefficient on the other hand, the energy release depends only upon the Doppler coefficient for a given initial reactivity. For a given value of $\alpha$, the Doppler coefficient is smaller for the high threshold case than for the low threshold case. This explains the cross-over of the curves as $\alpha$ is increased.

The excess energy above threshold does not measure directly the damage potential of the accident since much of the energy produced cannot be converted into work. Jankus (3) has estimated the fractional energy available for work for a threshold equation of state by assuming an isentropic expansion to the threshold energy. Jankus’ method has been used to convert the results shown in Figure 3 into curves giving the maximum explosive energy as shown in Figure 4. It must be noted that these are maximum values which in an actual case will be reduced since the expansions will not be isentropic.

The curves of Figures 3 and 4 measure, in each case, the energy above the assumed threshold values. To assess the situation more realistically, in both cases the energy should be measured from the same value. In Figure 5 the total energy release is measured from the vaporization temperature for both equations of state. It is expected from Nicholson’s work that for a zero Doppler coefficient the energy release will be close to that of the upper curve. On the other hand, with a strong Doppler coefficient, it is expected that the actual energy release will be closer to the lower curve. To illustrate this point, the true situation has been approximated by the dashed curve shown in the figure.
Figure 3. Energy Release vs. Doppler Effect
Excess energy release above threshold for equation A and B with $k_0 = 0.0013$, $n = 3/2$. In each case, the excess energy is measured from the threshold appropriate to the equation used in the calculation. For small $a$, equation A is more effective in terminating the excursion because pressure buildup is more rapid once the threshold is exceeded. At large $a$, equation A is less effective because its higher threshold temperature means a lower Doppler coefficient for the same value of $a$. 

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Figure 4. Explosive Energy Release vs. Doppler Effect

The maximum explosive energy release as a function of the Doppler effect for equations A and B. In each case, the work available from the excess energy shown in Figure 3 has been estimated by assuming an isentropic expansion with the Jankus model.
Figure 5. Total Energy Release vs. Doppler Effect

Total energy release measured from the melting point of uranium for the Fermi worst hypothetical accident, $k_0 = 0.0013$. The upper curve results with equation A, the lower with the equation of state B. The dashed curve is estimated to be close to the true situation.
Brout \(^{(9)}\) has estimated the maximum work available from vaporized uranium under an isentropic expansion and his results have been interpreted in the form of an analytical curve by the APDA group \(^{(6)}\). The APDA analytical curve has been applied to the results of Figure 5 to obtain an upper estimate of the explosive energy which could be produced in the accident. The results are shown in Figure 6; again a dotted line indicates an approximation of the true situation.

It should be noted that the Brout results were obtained for the very high temperatures resulting for the case with no Doppler coefficient. An unwarranted liberty was taken in extending them to the much lower temperatures resulting when there is a strong Doppler coefficient. In this case, it is expected that the Jankus method would have more validity so that the maximum explosive energy release would be reduced further than indicated by the dashed line shown in Figure 6. Thus, the calculations indicate that the maximum explosive energy release would be reduced from a figure of about 1200 MW-sec with no Doppler coefficient to a figure close to 35 MW-sec for a case with a Doppler coefficient having an \(\alpha\) value at threshold of 0.01. Again it is noted that the actual explosive release would be reduced further by the less-than-ideal expansions involved.
Figure 6. Explosive Energy vs. Doppler Effect

The explosive work available from the total energy of Figure 5 has been estimated using Brout's calculations as interpreted by APDA. For the high values of \( \alpha \) and equation B, the Jankus estimate (Figure 4) is probably more accurate.
SECTION VII

CONCLUSIONS

The Fermi (6) and EBR-II (10) hypothetical meltdown accident studies indicated that (maximum) explosive energies equivalent to several hundred pounds of high explosives could be produced. These reactors have indicated the feasibility of designing reactor structures to contain such energy releases. However, as indicated in the EBR-II analysis, an energy release in excess of several hundred pounds of high explosives makes difficult a design which will assure the integrity of the reactor containment. The Fermi containment is designed to withstand 1,000 lbs. of high explosive release. Since with no Doppler effect the energy release increases faster than the core mass, containment of large power reactors 10 to 20 times larger than the Fermi reactor poses a serious problem. However, in the present work it has been shown that a negative Doppler effect of the magnitude calculated for these large power reactors can reduce the energy release by factors of 10 or more so that the containment problem again becomes manageable.

Other conclusions are:

1. In the presence of a strong negative Doppler effect, the energy release per unit mass of core becomes essentially independent of the reactor parameters and depends only on the inserted reactivity at the point where substantial pressure buildup is initiated.

2. In the presence of a strong Doppler effect, the energy release from the meltdown accident can be terminated by a relatively low pressure buildup. The pressure-energy relation must, therefore, be understood for temperatures well below those normally of importance in the absence of a Doppler coefficient.

3. If one relies on the Doppler effect to reduce the energy release, it is advantageous to have a short neutron lifetime. This is because the energy release depends only on the inserted reactivity at the threshold of pressure generation. The inserted reactivity, on the other hand, (assuming a reactivity ramp insertion) will increase with increasing neutron lifetime as shown by McCarthy, et al. (2)

4. In the presence of a strong Doppler effect, the energy release goes linearly with reactor mass. The energy per unit mass does not increase as (McCarthy, et al., indicate) would be the case with no Doppler coefficient.

Finally, the authors wish to point out that the analysis does not consider the effect of a positive sodium coefficient. If, in the disassembly process, it were possible to preferentially blow out sodium from regions where positive reactivity would result, this would, in some respects, resemble the situation of an accident with a positive Doppler coefficient and might lead to a greatly increased energy release.
REFERENCES


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