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(M-3679, 25th Ed.)FLOW CHARACTERISTICS OF A CIRCULAR WEIR
IN A CENTRIFUGAL FIELD

by

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ABSTRACT

The mixing and settling required for solvent extraction between liquid phases can be accomplished in a relatively small volume by a centrifugal contactor in which intense mixing is followed by separation of phases in a centrifugal field. Circular weirs, concentric with the axis of rotation, are used to set the overflow radii of the two liquid phases at positions that will cause the emulsion to float evenly in the disengaging section. For accurate control the flow characteristics of the circular weirs must be known. An equation was developed for circular weirs by analogy to the relations for a simple weir, and the appropriate weir coefficients were evaluated empirically by experiments with both a steel contactor and a transparent plastic contactor. A correlation of the experimental coefficients was developed to allow calculation of general weir performance.

CONTENTS

	<u>Page</u>
Introduction	4
Summary	5
Discussion	5
Preliminary Relations	5
Pressure Balance in the Disengaging Section	5
Liquid Rise over a Circular Weir	6
Experimental Procedures and Results	8
Stainless Steel Unit	8
Plastic Unit	12
Correlation of Results	16
Nomenclature	22
Bibliography	23

LIST OF TABLES AND FIGURES

Table

I	Coefficients for Aqueous Weir of the Steel Unit	12
II	Coefficients for Aqueous Weir of the Plastic Unit	16
III	Comparison of Coriolis Rise to Maximum Rise at Weir	18
IV	Correlation of Aqueous Weir Coefficients	19

Figure

1	Disengaging Section and Control Weirs of Typical Centrifugal Contactor	7
2	Stainless Steel Contactor	9
3	Displacement of Organic Phase by Rise over Aqueous Weir - Steel Contactor	11
4	Interface Radius as Function of Organic Volume Displaced - Steel Contactor	11
5	Plastic Contactor	13
6	Direct Observation of Liquid Surfaces - Plastic Contactor	15
7	Comparison of Coriolis Rise to Maximum Rise at Aqueous Weir - Plastic Contactor	18
8	Correlation of Aqueous Weir Coefficients	20

FLOW CHARACTERISTICS OF A CIRCULAR WEIR IN A CENTRIFUGAL FIELD

INTRODUCTION

The purpose of the centrifugal contactor program at the Savannah River Laboratory is the development of a device that mixes and settles two liquid phases in a volume that is small relative to that of conventional mixer-settlers operating at the same flow rates. In addition, the design must not allow nuclear incidents or require bearings that are in contact with the liquids. Such a contactor would be suitable for solvent extraction of either enriched or natural uranium fuel elements in a remotely maintained plant.

In the type of unit that has been developed, the two phases are drawn into the bottom of a cylindrical chamber by a rotating paddle, which also intimately mixes the phases. The pumping action of the paddle is directed so as to force the mixture through an axial hole at the bottom end of a rotating cylinder, which is equipped with radial vanes to keep the liquid mass turning at constant angular velocity. The two phases are centrifugally separated in the cylinder. The light phase is displaced inward and leaves the cylinder by flowing "over" the weir formed by the inside edge of an annular baffle that partially closes the upper end of the cylinder, and being thrown from the rotating assembly to a surrounding collector ring. The heavy phase passes "under" the annular baffle at the cylinder wall, then inward through a duct with radial vanes and over a circular weir of greater radius (i.e., "lower") than the weir that controls the light phase. The heavy phase is also thrown outward to a surrounding collector ring; both liquid streams flow from their respective rings to adjacent units in a bank.

For such a unit to achieve maximum capacity, the emulsion in the rotating cylinder should float so that it nearly fills the space between the inner and outer edges of the annular baffle, but does not leave at either of these exits. A few observations have shown that an emulsion floats so that roughly equal volumes lie on either side of the "interface" radius. The latter is defined to be the radius of the interface that would exist between the two clear phases if no emulsion were present. The problem of emulsion position then resolves into the more precise problem of "interface" position, which can be readily calculated for two known phase densities if the radii of the liquid surfaces over the two control weirs are known. The radius of such a liquid surface is equal to the radius of the weir less the depth of flow that is required to pass the liquid over the weir. The purpose of this study is to develop equations that will allow prediction of the depth of flow, and hence of the interface position.

SUMMARY

A pressure balance in the disengaging section of a centrifugal contactor gives the radius of the interface, between aqueous and organic phases of known density, as a function of the radii of the liquid surfaces over the aqueous and the organic control weirs:

$$r_1^2 = \frac{\rho_a r_a^2 - \rho_o r_o^2}{\rho_a - \rho_o}$$

The radius of the liquid surface over each weir is related to the radius of the weir (r_{a*} or r_{o*}) through an equation derived by analogy to simple weirs. The equation involves the phase flow rate, q , the angular velocity, ω , and an undetermined weir coefficient, k . For the aqueous phase the relation is

$$r_a^2 = r_{a*}^2 - \left(\frac{q_a/\omega}{k_a f_a \pi} \right)^{2/3}$$

in which f_a is a factor close to unity. These equations were used with measurements of the variables in two centrifugal contactors to evaluate k_a under a variety of conditions. The resulting coefficients were related to the fixed coefficient for a weir in a gravitational field, k_* , by

$$\ln \frac{k}{k_*} = - \frac{4}{bn} (k_* f \pi)^{2/3} (q/\omega)^{1/3}$$

where b is the width of the duct leading inward to the weir and n is the number of radial vanes in the duct.

DISCUSSION

PRELIMINARY RELATIONS

The general equations that determine interface location were known before the experimental work was undertaken. As a result, the experimental work is followed more readily if these relationships are presented first, as background information.

PRESSURE BALANCE IN THE DISENGAGING SECTION

The centrifugal acceleration that acts on a fluid element at any radius (r) in a mass of liquid rotating at constant angular velocity (ω) is

$$a = r\omega^2 \quad (1)$$

Consequently the differential pressure (dp) across a differential thickness (dr) of liquid with density ρ is

$$dp = \rho \omega^2 r dr = \rho \omega^2 r dr \quad (2)$$

Integration of equation 2 between r_1 , p_1 and the larger r_2 , p_2 gives⁽¹⁾

$$p_2 - p_1 = \frac{1}{2} \rho \omega^2 r_2^2 - \frac{1}{2} \rho \omega^2 r_1^2 \quad (3)$$

Figure 1 shows the essential elements in the disengaging section of a centrifugal contactor. To make the description specific, the light phase is considered to be an organic liquid and the heavy phase to be an aqueous solution. The radii of the corresponding circular weirs are then designated as r_{o*} and r_{a*} , and the radii of the surfaces of the liquids flowing over these weirs are r_o and r_a . In Figure 1 the emulsion band is very small and located at the interface radius, r_1 . Under these conditions the pressure exerted by the layer of organic phase lying between r_o and r_1 in the disengaging section must be balanced by the aqueous phase extending from r_a to r_1 in the aqueous phase duct. By use of equation 3 this pressure balance becomes⁽¹⁾

$$\rho_o(r_1^2 - r_o^2) = \rho_a(r_1^2 - r_a^2) \quad (4)$$

(Because of the constant angular velocity, the factor $\frac{1}{2} \omega^2$ cancels out of each term.) An alternative statement of equation 4² can be made in terms of weir radii by defining, for any weir and the liquid surface over it,

$$F = \left(\frac{r}{r_*}\right)^2 \quad (5)$$

One alternative form is

$$F_a r_{a*}^2 = (1 - \rho_o/\rho_a) r_1^2 + (\rho_o/\rho_a) F_o r_{o*}^2 \quad (6)$$

It is apparent from equation 6 that the interface position is not only dependent upon the ratio of phase densities and size of weirs, but also upon the extent to which the liquids rise over the weirs. A change in the rise, caused by a change in rate of flow over the weir, must be balanced by an appreciably greater change in position of the interface; in this, the action is similar to the exaggeration of movement observed with a two-phase manometer. Consequently, a relationship for rise over the weir must be obtained next.

LIQUID RISE OVER A CIRCULAR WEIR

For a plain weir in a gravitational field, the rate of flow is proportional to the cross-sectional area of liquid flowing across the weir, and to the linear velocity of flow across the weir. The cross-sectional area in turn, is proportional to the weir length, L , and to the height

FIG. 1 DISENGAGING SECTION AND CONTROL WEIRS OF TYPICAL CENTRIFUGAL CONTACTOR

of liquid, ΔH , above the weir. The velocity across the weir is proportional to the velocity attained during a fall of height, ΔH , at an acceleration of g , that is, to $\sqrt{2g\Delta H}$. Thus the flow rate (q) is in consistent units⁽²⁾

$$q = k_*(L\Delta H)\sqrt{2g\Delta H} = k_*\sqrt{2g} L(\Delta H)^{3/2} \quad (7)$$

The value of the proportionality constant k_* for a sharp-edged weir is 0.415 and for a broad-crested weir is about 0.33⁽²⁾.

An analogous expression can be derived for flow over the inner edge of a circular weir in a centrifugal field. In this situation the height of liquid "above" the weir is Δr , and the mean radius of this liquid is $(r_* - \frac{\Delta r}{2})$. The "length" of the weir is essentially the perimeter of the inner edge, but the radial vanes block a small portion of the perimeter, depending on their number and thickness, leaving a large fraction (f) across which flow occurs. As a result, the cross-sectional area of liquid flowing across the weir is proportional to $f2\pi(r_* - \frac{\Delta r}{2})\Delta r$. The linear velocity across the weir is proportional to the velocity attained during a fall of height Δr at a centrifugal acceleration of $(r_* - \frac{\Delta r}{2})\omega^2$, that is, to $\sqrt{2(r_* - \frac{\Delta r}{2})\omega^2\Delta r}$. Thus, in consistent units, the flow rate is

$$q = kf\pi\omega \left[2\Delta r \left(r_* - \frac{\Delta r}{2} \right) \right]^{3/2} \quad (8)$$

A more convenient form of equation 8 results from substitution of $\Delta r = r_* - r$:

$$q = kf\pi\omega (r_*^2 - r^2)^{3/2} \quad (9)$$

A final modification can be made by using equation 5 to obtain

$$q = kf\pi\omega r_*^3 (1 - F)^{3/2} \quad (10)$$

The values of q , f , ω , and r_* will, in the general case, be known for both weirs in a centrifugal contactor. Experimental evaluation of k will then allow calculation of F_a and F_o by equation 10, and these factors in turn can be used in equation 6 to calculate the position of the interface. Consequently, experimental evaluation of k is the next subject of this report.

EXPERIMENTAL PROCEDURES AND RESULTS

STAINLESS STEEL UNIT

The first experiments to determine the relation between liquid flow over a weir and the position of the interface were made with the centrifugal extractor shown in Figure 2. The operation was essentially as described in the Introduction. The aqueous phase was 1.2M $Al(NO_3)_3$ + 0.5M HNO_3

FIG. 2 STAINLESS STEEL CONTACTOR

with a density of 1.21 g/cc. The organic phase consisted of 6% tri-n-butyl phosphate in "Ultrasene" and had a density of 0.795 g/cc at the temperature of operation (20 to 25°C). Each liquid was pumped from a tank through a rotameter to one of the two inlet pipes; the effluent stream from each collector ring was returned to the proper tank. Since the interior could not be observed, it was necessary to deduce the position of the interface from the displacement experiment described below, and to calculate the corresponding rise of liquid over the weir from the interface position and the pressure balance of equation 6.

For the displacement experiments, both phases were admitted to the unit at rates that were gradually reduced to zero. Under these conditions the two liquid surfaces ended flush with the weirs, so that the interface radius attained was that calculated by using $F_a = F_o = 1.0$ in equation 6. After liquid stopped dripping from the collector rings, the aqueous flow was increased from 0 to 1 gpm. The resulting slight rise over the aqueous weir diminished the radius of that liquid surface and consequently pushed the interface inward, displacing some of the organic phase. The latter was collected in a graduate and its volume recorded. The aqueous flow was then increased to 2 gpm and the displaced organic phase was collected again. This procedure was repeated at flow increments of about 1 gpm until a total rate of about 10 gpm was reached, at which point the flow was again turned off. With the bowl still rotating at full speed to keep its contents in place, liquid was drained from the bottom of the inlet section, the side of the mixing chamber, and from the bottom of the space between the bowl and the casing (none of these drain points are shown on Figure 2). The rotation was stopped, the contents of the bowl were allowed to drain into a vessel, and the volume of organic phase was recorded. This volume represented organic phase remaining in the bowl at the highest aqueous rate.

The cumulative volume of organic phase displaced during five runs is presented on Figure 3 as a function of aqueous flow rate. The consistency of the data was checked by also plotting the difference between the measured volume of organic phase remaining in the bowl at the end of the run and the calculated volume that was present at the start of the run. This difference represents an alternative, but indirect, measurement of displacement at the highest flow rate, and is distinguished by symbol from the direct measurements shown on Figure 3. Agreement between direct and indirect measurements is reasonably good.

From the dimensions given on Figure 2 it is possible to calculate the organic volume that is present, radially, from the organic phase weir to any interface location and, longitudinally, from the deflection baffle at the bottom to the conical dispersion baffle at the top. The results of such calculations are given on Figure 4, with volume shown as a function of r_1^2 . The initial radius of the interface (at the time when both flows have been reduced to zero), determined from equation 6 and the phase densities and weir radii, is $r_{1*} = 1.90$ inches. The corresponding volume at the start of the runs, found on Figure 4 at $r_{1*}^2 = 3.61$ sq. in., is

568 cc. This volume was used to make the complementary plot shown, relating interface radius to organic phase volume displaced. It was then possible to translate the experimental volume displacements into values of r_i^2 , for the calculations of k_a shown below.

FIG. 3 DISPLACEMENT OF ORGANIC PHASE BY RISE OVER AQUEOUS WEIR
Steel Contactor

FIG. 4 INTERFACE RADIUS AS FUNCTION OF ORGANIC VOLUME DISPLACED
Steel Contactor

Calculations were performed for data at 4, 9, and 15.7 gpm of aqueous flow; data at the highest rate will be used for illustration. From Figure 3 it is seen that 465 cc of organic phase were displaced at 15.7 gpm, equivalent (by Figure 4) to $r_1^2 = 1.66$ sq. in. The value of F_o in equation 6 is unity, since there was no sustained flow of organic phase, and $\rho_o/\rho_a = \frac{0.795}{1.21}$, $r_{o*}^2 = (1.125)^2$ sq. in., $r_{a*}^2 = (1.44)^2$ sq. in. Equation 6 yields $F_a = 0.675$, which can now be used in equation 10. The units in the latter must be consistent, so the cgs system was chosen. Thus $q_a = 15.7$ gpm = 990 cc/sec, $r_{a*}^3 = (1.44 \times 2.54)^3 = 48.9$ cc, and $\omega = 1730$ rpm = 181 radians/sec. The four radial vanes leave a fraction $f_a = 0.93$ for flow of liquid over the aqueous weir. Substitution of these numbers into equation 10 yields a value of 0.183 for k_a . The results of similar calculations for the other two flow rates are shown in Table I.

TABLE I

Coefficients for Aqueous Weir of the Steel Unit

$$r_{o*} = 1.125 \text{ in.} \quad \rho_o = 0.795 \text{ g/cc} \quad f_a = 0.93$$

$$r_{a*} = 1.44 \text{ in.} \quad \rho_o = 1.21 \text{ g/cc}$$

q_a , gpm	Displaced Vol., cc	r_1^2 , sq. in.	F_a	rpm	k_a
4	143	2.97	0.891	1775	0.264
9	280	2.39	0.795	1750	0.234
15.7	465	1.66	0.675	1730	0.183

It is apparent that these observed weir coefficients are considerably lower than the 0.415 cited for rectangular weirs in a gravitational field, and that they show a variation with flow rate. In order to see what was causing this behavior, experiments were made with the transparent unit described in the next section.

PLASTIC UNIT

The simple device shown in Figure 5 was built of transparent plastic so that with the aid of a stroboscope the flow patterns at the aqueous phase weir and the position of liquid surfaces could be observed directly. Since the bottom of the cylinder was closed, to allow a bottom drive for rotation at 1200 rpm, the incoming liquids were introduced axially from the top and fell to the bottom of the bowl. Here they were accelerated by the radial vanes, thrown outward, and separated into two layers of clear liquid, with essentially no emulsion layer. The separated phases left the bowl by the same routes as in

FIG. 5 PLASTIC CONTACTOR

the steel unit and were caught in a surrounding collector not shown in Figure 5. Marks were scored on the underside of the emulsion baffle so that the radius of the interface could be estimated to within 1/32 inch by looking down through the transparent aqueous phase duct. Rise of liquid "above" the aqueous weir was determined by estimating the fraction of the 0.15-inch gap filled with liquid. Each liquid was pumped from a tank through a rotameter to the entry pipe. The mixture that discharged into the collector was passed to a decanter, and the separated streams were returned to their respective tanks. The aqueous stream was water, and the organic stream was "Ultrasene", with a density of 0.78 g/cc at the operating temperature of about 25°C.

Initial experiments with water alone flowing through the unit showed an unexpected behavior at the aqueous weir. The water, flowing inward through the aqueous phase duct and over the weir, was seen to build up at the back of each radial vane instead of being distributed uniformly along the weir. The interpretation was that as the water moved in toward smaller radii there was a tendency for it to increase in angular velocity and to overtake the vane ahead of it. Opposition by the vane to this motion resulted in a pressure increase at the back of the vane and a corresponding elevation of liquid. (This general action is called the Coriolis effect.) The appearance of the buildup at a typical vane is shown in the two quadrant sketches at the bottom of Figure 6. A wedge of water seemed to be flowing over the weir at only the leading portion of each quadrant. The flow pattern was confirmed by cementing one end of a couple of threads to the outer periphery of the aqueous phase duct, where water enters from the settling section, and letting the remainder of each thread be dragged in the direction of flow. The pattern, also included in the quadrant sketches, shows that a portion of the flow is directed forward toward the wedge, while another portion starts toward the rear of the quadrant but reverses its direction before reaching the edge of the weir. The surface of water in the disengaging section was seen to lie at the same radius as the maximum rise on the weir, indicating that pressure developed in the aqueous duct was characterized by the height of the maximum rise.

In the next experiments, both phases were admitted to the unit at rates that were gradually reduced to zero, after which the aqueous flow alone was increased in increments. At each flow rate the position of the interface in the disengaging section was observed and the maximum rise over the aqueous weir was estimated. For reasons described in the next section, it was surmised that if more vanes were used in the aqueous phase duct the maximum rise for a given flow rate would be diminished. Consequently, four more vanes were installed and the two-phase experiments were repeated. The results obtained with both four and eight vanes are given in the upper part of Figure 6, with positions plotted to the size at which they were observed. It can be seen that there is less rise over the weir and less displacement of the interface at a given flow rate for the eight-vane arrangement than for the four-vane arrangement.

Data plotted at size observed with stroboscope
 Lines calculated from final correlation
 Aqueous phase, water; organic phase, "Ultrasene"

View of Typical Quadrant from Above

FIG. 6 DIRECT OBSERVATION OF LIQUID SURFACES
 Plastic Contactor

Since movement of the interface is a magnified response to rise of the liquid surface over the weir, the observed interface radius was used to calculate k_a , rather than the observed rise over the weir. As before, F_a was obtained from equation 6, using $F_o = 1.0$, $\rho_o/\rho_a = 0.78$, $r_{o*}^2 = (1.31)^2$ sq. in., and $r_{a*}^2 = (1.65)^2$ sq. in. The resulting value of F_a was then used in equation 10 to calculate k_a , with q_a in cc/sec, $\omega = 1200$ rpm = 126 radians/sec, $r_{a*}^3 = (1.65 \times 2.54)^3 = 71.7$ cc, and $f_a = 0.9$ for 4 vanes and $f_a = 0.8$ for 8 vanes. The results are shown in Table II.

TABLE II

Coefficients for Aqueous Weir of the Plastic Unit

$r_{o*} = 1.31$ in. $\rho_o = 0.78$ g/cc $f_a = 0.9$ for $n = 4$

$r_{a*} = 1.65$ in. $\rho_a = 1.0$ g/cc $f_a = 0.8$ for $n = 8$

<u>n,</u> <u>no. of vanes</u>	<u>q_a, gpm</u>	<u>r₁, in.,</u> <u>observed</u>	<u>F_a</u>	<u>rpm</u>	<u>k_a</u>
4	1	2.35	0.939	1200	0.165
4	2	2.25	0.901	1200	0.159
4	3	2.13	0.859	1200	0.140
4	4	2.00	0.815	1200	0.125
8	1.5	2.40	(too high - not used)		
8	2	2.30	0.920	1200	0.247
8	3	2.21	0.887	1200	0.219
8	4	2.13	0.859	1200	0.209

A variation of k_a with flow rate is again apparent. Since it is also apparent, from the observations described, that the Coriolis effect is interfering with the expected simple weir flow, an empirical correlation of k is developed in the next section that involves the ratio of the rise calculated to be caused by the Coriolis effect to the rise calculated to occur at a simple weir.

CORRELATION OF RESULTS

When an element of a rotating liquid flows toward positions of smaller radii, the speed of rotation of the element tends to increase. If this tendency is opposed, a force develops that can be characterized by an acceleration called the Coriolis acceleration. For complete suppression of the increase in angular velocity (that is, for constant ω), the Coriolis acceleration of a fluid moving with a radial velocity V_r is⁽³⁾

$$a_{Cor} = 2V_r\omega \quad (11)$$

The radial velocity of flow at any radius in an aqueous duct of width b (Figure 1) and free area f is

$$V_r = \frac{q}{b(f2\pi r)} \quad (12)$$

so that equation 11 may be restated as

$$a_{Cor} = \frac{2q\omega}{b(f2\pi r)} \quad (13)$$

This acceleration is acting in such a direction on the liquid between adjacent pairs of vanes as to produce a force perpendicular to the leading vane of each pair. The length of the "column" being acted on between adjacent vanes at any radius is $\frac{f2\pi r}{n}$ for a duct with n vanes, so that the pressure exerted against the leading vane by a fluid of density ρ is

$$p_{Cor} = \rho a_{Cor} \frac{(f2\pi r)}{n} \quad (14)$$

Substitution of equation 13 for a_{Cor} converts equation 14 to

$$p_{Cor} = \frac{2\rho q\omega}{bn} \quad (15)$$

The fact that the radius has disappeared from equation 15 means that the Coriolis pressure developed at the vane is the same at all radii, including the radius of the weir edge. The Coriolis pressure should then be able to produce an additional rise of $(\Delta r)_{Cor}$ above the weir edge, the extent of rise depending upon the centrifugal acceleration opposing the rise. The centrifugal acceleration is that occurring at the mean radius of liquid over the weir, $r_w = r_* - \frac{\Delta r}{2}$, so that the Coriolis pressure required to produce the additional rise is

$$p_{Cor} = (\Delta r)_{Cor} \rho (r_w \omega^2) \quad (16)$$

When equations 15 and 16 are solved for $(\Delta r)_{Cor}$ the result is

$$(\Delta r)_{Cor} = \frac{2q}{bnr_w\omega} \quad (17)$$

The data labeled "maximum rise over aqueous weir" in Figure 6 presumably include, as well as $(\Delta r)_{Cor}$, some rise caused by simple weir action, and in consequence should be larger than $(\Delta r)_{Cor}$, as calculated by equation 17. The expectation was checked by calculating $(\Delta r)_{Cor}$ as illustrated below and comparing with the corresponding values of $(\Delta r)_{max}$. The quantities in equation 17 must be dimensionally consistent, and again the cgs system was chosen. At 4 gpm for the unit with 8 vanes in the

aqueous duct, $(\Delta r)_{\max}$ was visually estimated to be 0.08 inch. In addition, $r_w = r_{a*} - \frac{\Delta r}{2} = 1.65 \text{ inches} - \frac{0.08}{2} = 1.61 \text{ inches} = 4.09 \text{ cm}$; $b = 0.42 \text{ inch} = 1.067 \text{ cm}$; $q = 4 \text{ gpm} = 252 \text{ cc/sec}$; $\omega = 1200 \text{ rpm} = 126 \text{ radians/sec}$; $n = 8$. Substitution of these quantities into equation 17 yields 0.114 cm, or 0.045 inch, for $(\Delta r)_{\text{Cor}}$, which represents about half of the observed maximum rise over the aqueous weir. Similar values calculated from the data of Figure 6 are shown in Table III. The only changes are in n , q , $(\Delta r)_{\max}$, and $(\Delta r)_{\text{Cor}}$.

TABLE III

Comparison of Coriolis Rise to Maximum Rise at Weir

n	q , gpm	$(\Delta r)_{\max}$, in.	$(\Delta r)_{\text{Cor}}$, in.
4	1	0.06	0.022
4	2	0.11	0.045
4	3	0.14	0.069
8	2	0.03	0.022
8	3	0.05	0.033
8	4	0.08	0.045
8	5	0.10-0.11	0.057

FIG. 7 COMPARISON OF CORIOLIS RISE TO MAXIMUM RISE AT AQUEOUS WEIR
Plastic Contactor

The observed values of $(\Delta r)_{\max}$ are plotted against the calculated values of $(\Delta r)_{\text{Cor}}$ in Figure 7, and can be seen to be proportional to the latter but about twice as great. Because of the small scale (0.14 inch is the largest reading) and difficulty in making these measurements, few conclusions are warranted other than that the behavior caused by the postulated Coriolis effect appears to agree with the observations, and that the effect is probably involved in the shifts of k_a with flow, aqueous duct width, and number of vanes.

Trial of various equations involving the Coriolis effect indicated that, the most satisfactory correlation of k_a could be obtained by using the ratio of the calculated additional rise caused by Coriolis acceleration to the calculated rise that would

occur at the weir in the absence of the Coriolis effect. The latter will be called $(\Delta r)_{\text{weir}}$ and is given by equation 8 with r_w substituted for $(r_* - \frac{\Delta r}{2})$, and with $k_* = 0.415$, the coefficient for sharp-edged rectangular weirs in a gravitational field:

$$(\Delta r)_{\text{weir}} = \frac{1}{2r_w} \left[\frac{q}{k_* f \pi \omega} \right]^{2/3} \quad (8a)$$

The ratio is obtained by dividing equation 17 by equation 8a:

$$\frac{(\Delta r)_{\text{Cor}}}{(\Delta r)_{\text{weir}}} = \frac{4}{b n} (k_* f \pi)^{2/3} (q/\omega)^{1/3} \quad (18)$$

A consistent set of units is required in equation 18 for the dimensional quantities q , ω , and b . Equation 18 was evaluated with the data in Tables I and II to obtain the ratio $\frac{(\Delta r)_{\text{Cor}}}{(\Delta r)_{\text{weir}}}$ for each of the points listed. The effective width of the sloped aqueous duct in the steel unit was taken as the arithmetic mean of the greatest (1.3 inches) and the least (0.7 inch) widths taken perpendicular to the radius. The results are given in Table IV.

TABLE IV

Correlation of Aqueous Weir Coefficients

<u>Unit</u>	<u>No. of Vanes</u>	<u>b_a, in., aqueous duct width</u>	<u>q_a, gpm</u>	<u>k_a</u>	<u>$\frac{(\Delta r)_{\text{Cor}}}{(\Delta r)_{\text{weir}}}$</u>
Steel	4	1.0	4	0.264	0.497
			9	0.234	0.657
			15.7	0.183	0.79
Plastic	4	0.42	1	0.165	0.83
			2	0.159	1.05
			3	0.140	1.20
			4	0.125	1.31
Plastic	8	0.42	2	0.247	0.48
			3	0.219	0.55
			4	0.209	0.61

The logarithm of k_a for each entry in Table IV is plotted as a function of $\frac{(\Delta r)_{\text{Cor}}}{(\Delta r)_{\text{weir}}}$ on Figure 8, and a straight line is drawn through the

FIG. 8 CORRELATION OF AQUEOUS WEIR COEFFICIENTS

resulting points which has the simple mathematical representation

$$\ln \frac{k}{k_*} = - \frac{(\Delta r)_{\text{Cor}}}{(\Delta r)_{\text{weir}}} \quad (19)$$

or, with equation 18,

$$\ln \frac{k}{k_*} = - \frac{4}{bn} (k_* f \pi)^{2/3} (q/\omega)^{1/3} \quad (20)$$

Given the dimensions and speed of a centrifugal contactor, and the phase densities and flow rates, it is now possible to calculate the interface position in the unit.

(a) The value of k is determined for each weir with

$$\ln \frac{k}{k_*} = - \frac{4}{bn} (k_* f \pi)^{2/3} (q/\omega)^{1/3} \quad (20)$$

For a broad-crested weir such as the long organic weirs in the units described, the proper k_* is 0.33. For all organic weirs the "width of the duct", b , is the height of the disengaging section.

(b) The value of F is determined for each weir with

$$q = k f \pi \omega r_*^3 (1 - F)^{3/2} \quad (10)$$

(c) The value of r_1 is determined with

$$F_a r_{a*}^2 = (1 - \rho_o/\rho_a) r_1^2 + (\rho_o/\rho_a) F_o r_{o*}^2 \quad (6)$$

(d) If the rise over a weir is of interest, it can be obtained from

$$\Delta r = r_*(1 - \sqrt{F}) \quad (21)$$

a relation that follows from the definitions that $\Delta r = r_* - r$ and that $F = \frac{r^2}{r_*^2}$.

The procedure outlined was used to calculate the single line drawn corresponding to the data obtained with the steel unit, and the several lines on Figure 6 for the plastic unit. The agreement of calculated lines with data is considered satisfactory.

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NOMENCLATURE

a	- centrifugal acceleration, $r\omega^2$
a_{Cor}	- Coriolis acceleration, $2V_r\omega$
b	- width of duct through which liquid approaches a weir; for the organic weir the width is equal to the height of the disengaging section
F	- the square of the ratio (radius of liquid surface flowing over a weir) to (radius of the solid weir), $\frac{r^2}{r_*^2}$
f	- fraction of weir perimeter available for flow, $f = 1 - \frac{nt}{2\pi r_*}$
g	- acceleration due to gravity
ΔH	- height of liquid above a rectangular weir in a gravitational field
k	- coefficient for a circular weir in a centrifugal field
k_*	- weir coefficient in the absence of the Coriolis effect (That is, coefficient for a rectangular weir in a gravitational field, or for a circular weir in a centrifugal field as the flow rate approaches zero.)
L	- length of a simple rectangular weir
n	- number of radial vanes in duct through which liquid approaches a weir
p	- pressure in liquid
p_{Cor}	- additional pressure caused by Coriolis acceleration against back of a radial vane
q	- flow rate of liquid
r_1	- radius of interface between clear phases in disengaging section with no emulsion present
r	- radius of liquid surface flowing over a circular weir
r_*	- radius of circular weir, or value of r as the flow rate approaches zero
r_w	- average radius of the liquid flowing over a circular weir $= r_* - \frac{\Delta r}{2}$
Δr	- rise of liquid over circular weir $= r_* - r$
$(\Delta r)_{Cor}$	- additional rise of liquid over circular weir caused by Coriolis effect
$(\Delta r)_{max}$	- maximum rise of liquid over circular weir, occurring at back of vanes (Is dependent on both $(\Delta r)_{Cor}$ and $(\Delta r)_{weir}$. Is equivalent to above Δr for use in a pressure balance.)

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- $(\Delta r)_{\text{weir}}$ - rise of liquid over a circular weir expected in absence of Coriolis effect
- t - thickness of radial vanes
- V_r - radial velocity (inward) of liquid flow at any radius
- ρ - density of liquid
- ω - angular velocity (radians/sec = $\frac{2\pi}{60}$ rpm)

Subscripts

- a - pertains to aqueous phase, or "heavy phase"
- o - pertains to organic phase, or "light phase"
- $*$ - refers to the limiting value of a variable as the flow approaches zero

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END

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