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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT

for Period Ending January 31, 1952

A. S. Householder, Chief

EDITED BY:

C. L. Perry

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ORNL-345	December, January, February, 1948-1949
ORNL-408	Period Ending July 31, 1949
ORNL-516	Period Ending October 31, 1949
ORNL-634	Period Ending January 31, 1950
ORNL-726	Period Ending April 30, 1950
ORNL-818	Period Ending July 31, 1950
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MATHEMATICS PANEL QUARTERLY PROGRESS REPORT

SUMMARY

New additions to the Mathematics Panel this quarter are Virginia C. Carlock, Alabama Polytechnic Institute; and Elizabeth N. Lawson, Florence (Alabama) State Teachers College. Also, V. A. Hoyle, University of North Carolina, is spending the winter quarter as Research Participant with the Panel.

The course on the preparation of problems for an automatic digital computer was held as planned during the first two weeks of December (cf. the previous quarterly progress report, ORNL-1151). Seventeen persons were registered for the course, some from colleges and some from industrial laboratories. More than a hundred auditors were present for the opening lecture by Prof. John von Neumann, Institute for Advanced Study, and thereafter the attendance at the lectures remained fairly constant at figures above fifty.

The principal lecturer was D. A. Flanders, Argonne National Laboratory, who gave a series of lectures on the arithmetic operations of the machine, the basic operations, the flow chart, and the design of programs. Professor von Neumann's lecture was entitled "Some General Principles on the Balance of Characteristics of High-Speed Computing Machines." A. H. Taub and J. P. Nash, University of Illinois, lectured on general principles of programming and on the logical design of the computer, respectively. (As they spoke the ORDVAC was successfully undergoing acceptance tests at the University of Illinois.) C. V. L. Smith, Office of Naval Research, gave a comparative survey of digital computers.

Mathematics Panel speakers were C. L. Perry, on subroutines; J. W. Givens, on matrix computations; N. M. Dismuke, on Tchebycheff polynomials and the representation of functions; B. M. Drucker, on a system of nonlinear differential equations computed on the SEAC; and A. S. Householder, on the solution of algebraic and transcendental equations.

Except for minor variations in schedule, the afternoons were devoted to practice sessions in coding for such problems as extracting square roots, the evaluation of trigonometric functions, and multiple precision computations.

Fabrication of the Oak Ridge computer is under way at Argonne National Laboratory and at the Technitrol Company, but the work has been retarded somewhat by procurement difficulties. The arithmetic frame has been completed at Argonne; the *A* and *Q* registers have been fabricated and shipped by Technitrol; the *S* register is partially finished; the adder is being constructed. The clear and transfer drivers and the shift counters have been designed.

Further work is being done to improve the memory system. The problem of "read-around" seems to be essentially solved with the discovery of an efficient tube, the 3BP1; J. R. Klein (ORNL) and B. Norris (ANL) reported on this work at the symposium on the Williams tube held at the Bureau of Standards in Washington. The difficulties caused by impurities in the phosphor of the memory system are now being studied.

The list of basic operations to be built into the machine is now fixed, and thus it has been possible to begin the actual coding of problems. Standard subroutines are being prepared, along with detailed and precise error analyses, to be included in a permanent library. They are constructed in such a way that they can be included in arbitrary special routines as automatically as possible. The subroutines will include computation of elementary, trigonometric, logarithmic, rational power functions, and the like; multiple precision operations; roots of equations; matrix operations; and integrations; and other computations of common occurrence. The library being prepared can contribute greatly to the versatility of the machine and to the speed with which special problems can be coded. Coding groups have been organized for this project, which will be a continuing one, and each group is responsible for the preparation, analysis, and checking of a class of routines. The coding sessions follow the weekly seminars. W. Givens has resumed his series of lectures on the fundamental paper on the inversion of matrices by von Neumann and Goldstine. These lectures were begun in the fall but were interrupted by the course.

Two large computing projects were completed during the quarter: (1) the calculation of the Fermi functions, φ_e (to be issued as an ORNL report), and (2) the Monte Carlo estimate of the collision distribution in tissue for normally incident beams of collimated neutrons of high energy. The neutron histories for problem 2 will be saved for possible use in other problems.

Panel members presented the following lectures as a part of the ORINS Traveling Lecture Program:

A. S. Householder

"Matrices," Alabama Polytechnic Institute, January 18, 1952.

A. W. Kimball

"Statistical Design and Analysis of Experiments," Washington University, St. Louis, January 30, 1952.

C. L. Perry

"Programming for Automatic Computing Machines," University of North Carolina, November 28, 1951.

In addition, the following papers were presented or published during the quarter:

J. Moshman

"Testing a Two-way Classification Using the Range," presented at the American Statistical Association and the Biometrics Society in Boston, 1951.

"A Simple Procedure for Determination of the Approximate Lymph Space," published in Vol. 114 of *Science*.

A. S. Householder

"Polynomial Iterations to Roots of Algebraic Equations," published in the *Proceedings of the American Mathematical Society*, Vol. 2, No. 5, 718-719 (1951).

A. W. Kimball

"On Dependent Tests of Significance in the Analysis of Variance," published in Vol. 22 of the *Annals of Mathematical Statistics*.

W. Givens

"Field of Values of a Matrix," presented at the American Mathematical Society meeting in Auburn, Alabama.

UNCLASSIFIED PROJECTS

PREPARATION AND ANALYSIS OF SUBROUTINES FOR THE DIGITAL COMPUTER

Summary. The basic machine operations and expected speeds are as follows:

1. Summation, eight distinct operations for adding or subtracting the numbers of magnitude with or without prior clearing of the accumulator. 50 μs.

2. Shift, eight distinct right shifts and eight left shifts an arbitrary number of times up to 48. In nonzero shifts the *A* register always shifts and the *Q* register may or may not shift. 10 μs plus 20 μs for each set of four or less.

3. Multiplication, with or without the addition of 2^{-40} for roundoff. A minimum of 210 μs and a maximum of 410 μs.

4. Division, with or without the arbitrary insertion of a unit in the last place to remove the biased roundoff and with either a single- or a double-precision dividend. 410 μs.

5. Memory to *Q*, a single order. 50 μs.

6. Substitution, total or partial transfer of the contents of *A* or *Q* into a specified memory position. In a partial substitution the first, second, third, and fourth sets of ten consecutive digits from a given register can be transferred independently, any one or more sets without the others, or even none at all. 50 μs.

7. Transfer of control, unconditional or conditional. If conditional, the transfer of control is dependent upon the sign of the contents of the *A* register, or upon the occurrence or nonoccurrence of overflow. 30 μs.

8. Stop.

The subroutines, in effect, extend the list of basic operations, although at the expense of some storage space in the memory. For example, if a computation requires the extraction of square roots and a square-rooting subroutine has been prepared, then this will be inserted once in the main routine. Therefore, when the computation requires a square root, the radicand will be transferred to a fixed location in the memory, an "exit order" placed to show where the main routine is to be picked up again, and the control sent to the subroutine. Thereupon the root will be extracted and placed in a fixed location in the memory and the control transferred back to the main routine by the exit order. This operation can be repeated any number of times in the course of the entire computation, and thus any number of square roots can be extracted with only a single insertion of the routine itself. It is only necessary that the main routine provide for the insertion of the radicand and the removal of the previous root before each use. In like manner, routines for multiple precision and others for floating decimal operations, can be prepared once and for all. As a somewhat more elaborate example, a single routine can be prepared for solving a system of linear algebraic equations of arbitrary order by inserting the order as one of the parameters.

Usually there are more ways than one of carrying out a particular computation, and three factors enter into the selection of any particular method. These are the space, or the number of words occupied in the memory by the routine; time, or the length of time required for the machine to go through the computation; and precision, or the effect of roundoff. A proposed routine might compare favorably with another on one criterion and unfavorably on some other criterion, and the extent to which a given factor is critical will depend upon the over-all problem in hand. Consequently if no single routine appears optimal on all factors, different routines may be needed in the library. For each routine, the error limits must be ascertained as precisely as possible. Thus, for $\sqrt{a}$, where a is given to 39 binary digits, one can arrive by Newton's method at a number x, satisfying

$$-2^{-40} < x - \sqrt{a + 2^{-80}} \leqq 2^{-40} .$$

This is achieved by a routine of eight words, which requires three auxiliary storage positions and takes a maximum of 36.02 ms, if it is not thought necessary to test the sign of a to make sure the computation will be meaningful. Incidentally the process yields $x = 2^{-39}$ when $a = 0$. If, as a precaution, it is desired that the machine stop or take remedial steps should a turn out to be given as a negative number and yield $x = 0$ when $a = 0$, then 11 words are necessary and the time is 36.24 ms. By a double-precision computation, if a is given to 78 binary digits, it is possible to obtain x accurately to 76 binary places with 21 additional words and an extra operating time of 2.72 ms.

The floating decimal arithmetical operations have been coded at Argonne, but they require final checking and error analysis. Multiple-precision arithmetic operations including square-rooting are nearly complete. Routines for single-precision square-rooting, obtaining sines and cosines, evaluation of a polynomial of arbitrary degree and for an arbitrary number of values of the argument, and transformation of variables in a polynomial are essentially complete with error analysis. Routines are being prepared for other elementary functions, including rational powers; for operations with complex numbers; for numerical integration and interpolation; for certain statistical computations; and for certain operations with matrices and vectors.

BASIC STUDIES IN THE MONTE CARLO METHOD

Participating Member of Panel. G. E. Albert.

References. (1) G. E. Albert, "Basic Studies in the Monte Carlo Method," *Mathematics Panel Quarterly Progress Reports for the Periods Ending July 31, 1951, and October 31, 1951*, ORNL-1091 and ORNL-1151. (2) G. E. Albert, Memorandums to A. S. Householder on "A General Approach to the Monte Carlo Estimation of the Solutions of Certain Fredholm Integral Equations," Parts I, II, and III.

Background and Status. As explained in reference (1), Parts I and II of reference (2) contain a mathematical exposition of the theory underlying various statistical techniques that have been used in computing centers over the country for the estimation of solutions of certain types of integral equations and the estimation of weighted integrals of such solutions. In addition, Part II of reference (2) contains an innovation, created by the participant, in the introduction of a certain type of simple stratified sampling into these estimation problems.

Part III of reference (2), issued during this quarter, describes a continued study of the use of stratified sampling begun in Part II and gives special consideration to the possibilities of designing estimation procedures to make efficient use of the advantages of high-speed automatic computing machinery. Specifically, Part III contains descriptions of sample spaces and probability distributions in which repeated sampling for chains of fixed length and stratified sampling for both chain lengths and chain points may be used. Suggestions are made for the choice of probability distributions that are simple enough so that complex sampling methods might be practical on high-speed machinery, with which efficiency is gained by computing methods that involve many repetitions of simple operations.

The participant proposes to continue this project in two ways. First, applications of the techniques suggested in reference (2) to specific integral equations should be tried. Second, the estimation techniques proposed give estimates of the solution of an integral equation at a single point. Application of any technique to the estimation of the values of the solution at many points, either through independent or correlated samples, would lead to a discrete random series. Smoothing and interpolation for such a series will be studied at some length.

METHODS OF COMPUTATION FOR USE WITH A HIGH-SPEED AUTOMATIC-SEQUENCED COMPUTER

Participating Member of Panel. W. Givens.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1151.

Problem 2. Numerical Inverting of Matrices of High Order.

Status. After an interruption for the Computer Course in December and some reports on coding, lectures on the von Neumann-Goldstine papers are being continued before the Mathematics Panel Seminar. When it is necessary, the error estimates are being modified to make them applicable to the Oak Ridge computer. Work on this problem will be continued.

Problem 3. Numerical Computation of Eigenvalues and Eigenvectors.

Status. The method described in the last quarterly report is being studied for the effect of roundoff error on the accuracy of the computation. The work of Lidskii[1] permits more efficient estimates by allowing one to replace the sum of squares of the eigenvalues. An upper bound for the maximum eigenvalue of a symmetric (error) matrix with two nonzero rows and columns has been found in terms of those bounds for the elements that are appropriate for the computation under study. Error bounds given by Goldstine for his proposed solution of this problem have been studied in a manuscript obtained from him. This work is to be continued.

EXPECTED VALUE OF RELIABILITY OF A TEST

Participating Member of Panel. J. Moshman.

Background and Status. Let y be a measured value of some true parameter x. It is assumed that

$$y = x + \epsilon ,$$

where ϵ is some error term uncorrelated with x, and that x and ϵ are each normally distributed with means μ and

[1] V. B. Lidskii, "Eigenvalues of the Sums and Products of Symmetrical Matrixes" (translated), *Doklad. Akad. Nauk. SSSR* **75**, 769-772 (1950).

0 and variances σ_z^2 and σ_ϵ^2, respectively. The reliability of the measure y is defined as

$$\mathrm{P} = \frac{\sigma_z^2}{\sigma_y^2} = \frac{\sigma_z^2}{\sigma_z^2 + \sigma_\epsilon^2} = \frac{1}{1 + \zeta} ,$$

where P (capital rho) is the reliability and $\zeta = \sigma_\epsilon^2/\sigma_z^2$.

If σ_z^2 and σ_ϵ^2 can be estimated by s_z^2 and s_ϵ^2, the usual root-mean-square sample estimate, the expected value of

$$R = \frac{1}{1 + z} ,$$

where

$$z = \frac{s_\epsilon^2}{s_z^2} ,$$

can be determined and some concept gained of any existing bias.

It is well known that s_ϵ^2 and s_z^2 each have a χ^2 distribution with, say, m degrees of freedom. That is,

$$f_1(s_\epsilon^2) = K_1(s_\epsilon^2)^{(m/2)-1} \exp\left[\frac{-ms_\epsilon^2}{2\sigma_\epsilon^2}\right]$$

and

$$f_2(s_z^2) = K_2(s_z^2)^{(m/2)-1} \exp\left[\frac{-ms_z^2}{2\sigma_z^2}\right]$$

are the respective probability density functions of s_ϵ^2 and s_z^2, where K_1 and K_2 are appropriate normalizing constants. The distribution of z is then[2]

$$f(z) = \int_0^\infty s_z^2 \, f_1(zs_z^2) \, f_2(s_z^2) \, ds_z^2$$

$$= K_1K_2\int_0^\infty s_z^2(zs_z^2)^{(m/2)-1} \exp\left[\frac{-mzs_z^2}{2\sigma_\epsilon^2}\right] (s_z^2)^{(m/2)-1} \exp\left[\frac{-ms_z^2}{2\sigma_z^2}\right] ds_z^2 .$$

After some reduction,

$$f(z) = Kz^{(m/2)-1} (z + \zeta)^{-m} .$$

To determine K, the following condition is imposed:

$$\int_0^\infty f(z) \, dz = K\int_0^\infty z^{(m/2)-1} (z + \zeta)^{-m} \, dz = 0$$

and it is found that

$$K = \frac{\zeta^{m/2}}{B\left(\frac{m}{2}, \frac{m}{2}\right)} ,$$

where

$$B(r, s) = \frac{\Gamma(r)\,\Gamma(s)}{\Gamma(r + s)} = \frac{(r - 1)!\,(s - 1)!}{(r + s - 1)!} .$$

[2] M. G. Kendall, *The Advanced Theory of Statistics*, 2d ed., rev., Griffin, London, 1945.

To find $\epsilon(R)$, the expected value of R,

$$\epsilon(R) = \int_0^\infty R\, f(z)\, dz$$

$$= \frac{\zeta^{m/2}}{B\left(\frac{m}{2}, \frac{m}{2}\right)} \int_0^\infty \frac{z^{(m/2)-1}}{(1 + z)(\zeta + z)^m}\, dz \quad .$$

After considerable calculation,

$$\epsilon(R) = \frac{\zeta^{n+1}}{B(n + 1, n + 1)} \left\{ \frac{(-1)^n}{(1 - \zeta)^m} \ln \xi + \sum_{k=0}^{m-2} \frac{\sum_{i=0}^{k} \binom{k}{i} \prod_{j=0}^{k} \frac{n - j}{n - i} (-\zeta)^{n-i}}{k!\, (1 - \zeta)^{k+1}\, (m - k - 1)\, \zeta^{m-k-1}} \right\} ,$$

where $n = (m/2) - 1$.

Some sample hand calculations were made for $\zeta = 0.2$ and $m = 4$, 6, and 8. The true value, P, is 0.833 and $\epsilon(R) = 0.787$, 0.802, and 0.812, respectively. R is a biased estimate of P, but it is conjectured that for large m, the order of magnitude used in most practical applications, the bias is negligible. It is planned to employ automatic calculating machines to investigate further the behavior of $\epsilon(R)$ for various combinations of ζ and large values of m.

ANALYSIS OF TEMPERATURE EFFECTS ON HAPLOID AND DIPLOID FORMS OF *SACCHAROMYCES CEREVISIAE* EXPOSED TO X-RADIATION

Origin. S. Pomper, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. Haploid and diploid forms of the yeast *Saccharomyces cerevisiae* were exposed to 30,000 and 120,000 r, respectively, of x radiation at three temperatures. An equal number of controls were run at each temperature. The problem was to determine whether temperature affects the ability of the organisms to survive after exposure to radiation.

Four independent counts were obtained for both types of organisms at each temperature in both the control and the treated groups. The means are shown in Table 1. Control counts are expressed in 10^6 units, and treated counts are expressed in 10^5 units.

TABLE 1

Average Counts

	TEMPERATURE		
	30°C	18°C	12°C
Haploid			
Controls	152.3	145.2	145.0
Treated	30.0	25.5	20.7
Diploid			
Controls	100.4	103.8	100.2
Treated	134.1	162.0	156.2

The analyses in Table 2 were computed for the controls.

It is clear that temperature has no appreciable effect on survival among the control organisms. In view of this result, analyses for the treated organisms could be computed without correcting for a temperature effect among the controls. These analyses are shown in Table 3.

TABLE 2

Analyses of Variance

(based on square roots)

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE	
		HAPLOIDS	DIPLOIDS
Among temperatures	2	0.1181	0.0397
Within temperatures	9	0.3307	0.2762
Total	11		

TABLE 3

Analyses of Variance

(based on square roots)

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE	
		HAPLOIDS	DIPLOIDS
Among temperatures	2	0.8570	1.4812
Within temperatures	9	0.03080	0.6770

A definite temperature effect is indicated in the haploid organisms, but the differences among the diploid means are not large enough to be statistically significant. There seems to be some indication that haploids and diploids react in opposite ways to changes in temperature, but in this experiment excessive variability among the diploid counts (a standard error almost five times as great as for the haploids) makes it impossible to draw a definite conclusion.

ESTIMATION OF VARIANCE COMPONENTS IN A STERILITY EXPERIMENT WITH MICE

Origin. Louis Wickham, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. As part of the analysis of the experiment described in detail in ORNL-1167,[3] components of variance estimates were computed. From each of several lines of mice, several males were selected and each male was mated to four different females. Several litters were obtained from each female. This was done with semisterile males and with fertile males. As a mathematical model, it was assumed that the litter size for the mth line is

$$x_{ijkm} = \mu + a_i + \beta_{ij} + \gamma_{ijk} + \epsilon_{ijkm} ,$$

where

$$i = 1, \ldots, r ,$$

$$j = 1, \ldots, s_i ,$$

$$k = 1, \ldots, t_{ij} ,$$

$$m = 1, \ldots, n_{ijk} ,$$

and

$$a_i \sim N(0, \sigma_a^2) ,$$

$$\beta_{ij} \sim N(0, \sigma_\beta^2) ,$$

$$\gamma_{ijk} \sim N(0, \sigma_\gamma^2) ,$$

$$\epsilon_{ijkm} \sim N(0, \sigma^2) .$$

This notation is conventional for indicating a random variable having

(3) R. A. Boloney, *The Preparation of Thin and of Thick Targets to be Bombarded by Positive Particles*, ORNL-1163, p. 55-59 (Jan. 9, 1952).

TABLE 4

Analyses of Variance

SOURCE OF VARIATION	DEGREES OF FREEDOM	EXPECTED VALUE OF SUM OF SQUARES
Among lines	$(r - 1)$	$\sigma_\alpha^2 \sum_i n_i^2 \left[\frac{1}{n_i} - \frac{1}{N}\right] + \sigma_\beta^2 \sum_{i,j} n_{ij}^2 \left[\frac{1}{n_i} - \frac{1}{N}\right] + \sigma_\gamma^2 \sum_{i,j,h} n_{ijh}^2 \left[\frac{1}{n_i} - \frac{1}{N}\right] + \sigma^2 (r - 1)$
Among males within lines	$\sum_{i=1}^{r} (s_i - 1)$	$\sigma_\beta^2 \sum_{i,j} n_{ij}^2 \left[\frac{1}{n_{ij}} - \frac{1}{n_i}\right] + \sigma_\gamma^2 \sum_{i,j,h} n_{ijh}^2 \left[\frac{1}{n_{ij}} - \frac{1}{n_i}\right] + \sigma^2 \sum_i (s_i - 1)$
Among matings within males	$\sum_{i=1}^{r} \sum_{j=1}^{s_i} (t_{ij} - 1)$	$\sigma_\gamma^2 \sum_{i,j,h} n_{ijh}^2 \left[\frac{1}{n_{ijh}} - \frac{1}{n_{ij}}\right] + \sigma^2 \sum_{i,j} (t_{ij} - 1)$
Among litters within matings	$\sum_{i=1}^{r} \sum_{j=1}^{s_i} \sum_{h=1}^{t_{ij}} (n_{ijh} - 1)$	$\sigma^2 \sum_{i,j,h} (n_{ijh} - 1)$

a normal distribution with a specified mean and variance. It is also assumed that all of the α_i, β_{ij}, γ_{ijk}, and ϵ_{ijkm} are mutually independent. With this model it can be shown that the analysis of variance in Table 4 has expected sums of squares as indicated. As before, n_{ijk} is the number of litters from the kth female mated to the jth male from the ith line, and

$$n_{ij} = \sum_k n_{ijk} ,$$

$$n_i = \sum_j n_{ij} ,$$

$$N = \sum_i n_i .$$

In any experimental situation these expected values may be used to obtain estimates of σ^2, σ_γ^2, σ_β^2, and σ_α^2 by a stepwise method of computation starting with the last sum of squares in the analysis of variance table.

In the mice data, the analyses were based on litter sizes and were performed separately on fertile and semisterile animals. The results are shown in Table 5.

Apart from the variation in litter sizes among litters from the same mating, only one component seems to contribute much to the total error. There is some indication of a component due to males from the same line, but its relative magnitude renders it unimportant. In both fertiles and semisteriles, then, there is evidence of homogeneity among the various factors studied. The variance for the fertiles is about twice the variance for the semisteriles, but the respective average litter sizes are 8.4 and 3.5. In view of this relationship, it is quite possible that the variance is proportional to the mean litter size, at least in the range covered by this experiment.

TABLE 5

Variance Analyses Based on Litter Sizes

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE
FERTILES		
Among lines	4	3.2729
Among males within lines	5	6.3665
Among matings within males	30	3.3321
Among litters within matings	180	3.3317
Total	219	

$\hat{\sigma}^2 = 3.3317$

$\hat{\sigma}_\gamma^2 = 0.0001$

$\hat{\sigma}_\beta^2 = 0.1365$

$\hat{\sigma}_\alpha^2 = 0$

SOURCE OF VARIATION	DEGREES OF FREEDOM	MEAN SQUARE
SEMISTERILES		
Among lines	5	1.1050
Among males within lines	6	4.0385
Among matings within males	36	1.6013
Among litters within matings	178	1.6016
Total	225	

$\hat{\sigma}^2 = 1.6016$

$\hat{\sigma}_\gamma^2 = 0$

$\hat{\sigma}_\beta^2 = 0.1302$

$\hat{\sigma}_\alpha^2 = 0$

ANALYSIS OF RECOMBINATION PERCENTAGES FOR SEVERAL EXPERIMENTS WITH BACTERIOPHAGE T2H

Origin. A. H. Doermann, Biology Division.

Participating Members of Panel. A. W. Kimball and G. J. Atta.

Background and Status. With bacteriophage T2H, a large number of experiments involving many different genetic crosses were performed. In each experiment the number of recombinants and the total number of phage particles were counted. The total numbers ranged from 450 to 4300, and the number of experiments per cross varied from one to nine. In some cases experiments from the same cross checked very well, and in other cases there was considerable divergence. An analysis of the average recombination percentages evaluated with respect to the crosses was desired.

Let x_{ij} be the observed proportion in the jth experiment for the ith cross ($i = 1, \ldots, k_j$; $j = 1, \ldots, n_i$). Further, let

$$x_{ij} = p_i + p_{ij} + \epsilon_{ij} ,$$

where p_i is the true average proportion for the ith cross, p_{ij} is the deviation of the jth experiment from the true proportion for the ith group caused by such extraneous factors as temperature, media, etc., and ϵ_{ij} is the usual binomial variation. p_{ij} and ϵ_{ij} are assumed to have zero expectations and to be mutually independent so that

$$\epsilon(x_{ij} - p_i)^2 = V(p_{ij}) + V(\epsilon_{ij})$$

$$= \sigma_i^2 + \frac{(p_i + p_{ij})(1 - p_i - p_{ij})}{n_{ij}} ,$$

where n_{ij} is the total number of particles counted in the jth experiment of the ith group, and σ_i^2 represents the extrareous variation from experiment to experiment within a group. If weights are chosen equal to the reciprocal variances, the weight for x_{ij} would be

$$w_{ij} = \frac{1}{V(x_{ij})} = \frac{n_{ij}}{n_{ij}\,\sigma_i^2 + \pi_{ij}(1 - \pi_{ij})} ,$$

where $\pi_{ij} = p_i + p_{ij}$. If, further,

$$\sigma_b^2 = \frac{\pi_{ij}(1 - \pi_{ij})}{n_{ij}} ,$$

then

$$w_{ij} = \frac{1}{\sigma_i^2 \left[1 + \dfrac{\sigma_b^2}{\sigma_i^2}\right]}$$

The least that can be expected is the computation of estimates of σ_b^2 and σ_i^2 and the use of them to determine approximate weights. If a χ^2 is computed for each cross in the usual fashion these χ^2's are pooled, it can be shown that

$$\frac{E(\chi^2 \text{ pooled})}{f} = 1 + \frac{\overline{\sigma_i^2}}{\overline{\sigma_b^2}} , \qquad (1)$$

where f is the number of degrees of freedom for the pooled χ^2 and

$$\overline{\sigma_i^2} = \sum_{i=1}^{k} \frac{\sigma_i^2}{k} ,$$

$$\overline{\sigma_b^2} = E\left\{\frac{\pi_{ij}(1 - \pi_{ij})}{n_{ij}}\right\} .$$

From Eq. 1 an estimate of the percentage a of the total variation that is due to extraneous variation can be obtained

$$a = \frac{\dfrac{\chi^2 \text{ pooled}}{f} - 1}{\dfrac{\chi^2 \text{ pooled}}{f}} .$$

For this type of problem, Cochran[4] gives the following set of rules for determining the kind of weights to use:

a	WEIGHTS
> 70%	equal
< 20%	binomial
between 20 and 70%	exact or partial

In the present case, it was found that χ^2 pooled = 153.26, f = 69, a = 55%. Accordingly, it was decided that exact weights would be used.

For the exact weights more information is required. It can be shown that if $x_{ij} = a_{ij}/n_{ij}$,

$$\frac{a_{ij}(n_{ij} - a_{ij})}{n_{ij}(n_{ij} - 1)}$$

is an unbiased estimate of $\pi_{ij}(1 - \pi_{ij})$. Since

$$\overline{\sigma_b^2} = E\left\{\frac{\pi_{ij}(1 - \pi_{ij})}{n_{ij}}\right\}$$

an unbiased estimate of $\overline{\sigma_b^2}$ is given by

$$\widehat{\sigma_b^2} = \left[\frac{1}{k \sum_i n_i}\right] \sum_{i,j} \left[\frac{a_{ij}(n_{ij} - a_{ij})}{n_{ij}^2(n_{ij} - 1)}\right] .$$

This estimate can then be used in conjunction with Eq. 1 to provide an estimate of a_i^2. In most cases, one estimate of a_i^2 is sufficient, since a_i^2 is approximately proportional to $p_i(1 - p_i)$; and if $0.3 < p_i < 0.7$, there is very little variation among the a_i^2. In the present set of experiments, however, several recombination percentages are well below 0.3, so that an estimate of σ_i^2 was actually determined for each cross. Where only one experiment per cross was available, the over-all estimate σ_i^2 was used.

One final complication arose. In some experiments it was possible to identify only one-half the recombinants. These proportions had to be doubled to be comparable with the crosses in which all recombinants were identified. Ordinarily such a procedure would simply increase the variance by a factor of 4, but since the final analysis of variance was based on transformed proportions, the effect on the variance had to be computed.

If p is a bionomial variate with $E(p) = \pi$, then the variance of

$$y = \sin^{-1} \sqrt{p}$$

is given approximately by

$$V(y) = V(p) \left[\left(\frac{dy}{dp}\right)_{p=\pi}\right]^2 ,$$

$$= \frac{\pi(1 - \pi)}{n} \left[\frac{1}{2\sqrt{\pi}\,\sqrt{1 - \pi}}\right]^2 ,$$

$$= \frac{1}{4n} .$$

Similarly, if $z = \sin^{-1} \sqrt{2p}$,

[4] W. G. Cochran, "Analysis of Variance for Percentages Based on Unequal Numbers," *J. Amer. Stat. Assoc.* **38**, 287-301 (1943).

$$V(z) = V(p)\left[\left(\frac{dz}{dp}\right)_{p=\pi}\right]^2 ,$$

$$= \frac{\pi(1-\pi)}{n}\left[\frac{1}{\sqrt{2\pi}\,\sqrt{1-2\pi}}\right]^2 ,$$

$$= \frac{1-\pi}{\frac{1}{2}-\pi}\left(\frac{1}{4n}\right) .$$

Thus $V(z)$ exceeds $V(y)$ by a factor

$$\frac{1-\pi}{\frac{1}{2}-\pi} .$$

This means that the weights corresponding to proportions which have been doubled must be multiplied by the factor

$$\frac{\frac{1}{2}-\pi}{1-\pi} ,$$

where π is the observed proportion.

The weights determined in this fashion were used to compute a weighted analysis of variance, and with the results it was possible to evaluate the differences among the recombination percentages for the several crosses. A paper containing the details of the experiment and the analysis has been prepared and will be submitted to an appropriate journal.

DETERMINATION OF A PROBABILITY DISTRIBUTION FOR A GENETICS EXPERIMENT

Origin. K. C. Atwood, Biology Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. It is postulated that in a diploid organism two neighboring genes each have n compartments of equal size, as illustrated in Fig. 1. It is also assumed that one mutation is induced independently in each gene and will appear at random in one of the compartments. The problem is to determine the probability distribution for the distance x between the compartments containing the mutations. In particular, the mean and the variance of the distribution are desired.

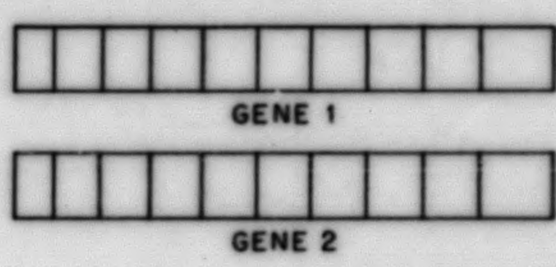

Fig. 1. n **Compartments.**

Clearly there are n^2 ways in which the event can happen. In order for x to be zero, the mutations must occur in adjacent compartments. There are n ways in which this can happen. By simple induction it can be shown that there are $2(n - x)$ ways in which the distance x (for $x > 0$) can be obtained. Thus,

$$\text{Prob. } (x = 0) = \frac{n}{n^2} = \frac{1}{n}$$

and

$$\text{Prob. } (x = a) = \frac{2(n-a)}{n^2} \quad (a > 0) .$$

If these expressions are correct, the probabilities must add to one. Then

$$\frac{1}{n} + \sum_{a=1}^{n-1} \frac{2(n-a)}{n^2} = \frac{1}{n} + \frac{2}{n^2}\sum_{a=1}^{n-1}(n-a) ,$$

$$= \frac{1}{n} + \frac{2}{n^2}\cdot\frac{n(n-1)}{2} ,$$

$$= 1 ,$$

and the condition is satisfied.

By definition the mean and variance of the distribution are given by

$$\mu = \sum_{a=0}^{n-1} p_a x_a ,$$

$$\sigma^2 = \sum_{a=0}^{n-1} p_a (x_a - \mu)^2 = \sum_{a=0}^{n-1} p_a x_a^2 - \mu^2 ,$$

respectively, where

$$x_a = a$$

and

$$p_a = \text{Prob. } (x_a = a) .$$

Hence,

$$\mu = \sum_{a=1}^{n-1} \frac{2(n - a)}{n^2} a$$

$$= \frac{2}{n^2} \left[n \sum_{a=1}^{n-1} a - \sum_{a=1}^{n-1} a^2 \right]$$

$$= \frac{2}{n^2} \left[n \frac{n(n - 1)}{2} - \frac{n(n - 1)(2n - 1)}{6} \right]$$

$$= \frac{n^2 - 1}{3n} .$$

Likewise,

$$\sigma^2 = \sum_{a=1}^{n-1} \frac{2(n - a)}{n^2} a^2 - \mu^2 = \frac{2}{n^2} \left[n \sum_{a=1}^{n-1} a^2 - \sum_{a=1}^{n-1} a^3 \right] - \mu^2$$

$$= \frac{2}{n^2} \left[n \frac{n(n - 1)(2n - 1)}{6} - \frac{n^2(n - 1)^2}{4} \right] - \mu^2 = \frac{(n^2 - 1)(n^2 + 2)}{18 n^2} .$$

For large n, the coefficient of variation σ/μ is approximately $1/\sqrt{2}$ or about 71%.

IRRADIATION EFFECT ON CELL EXTINCTION

Origin. M. E. Gaulden, Biology Division.

Participating Members of Panel. J. Moshman and G. J. Atta.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-1151, 1091, and 1029.

Background and Status. Previous experiments have revealed a highly significant effect of the presence of heat on the cell extinctions. In the present experiment, 25 grasshopper cells were subjected to a dose of 12,000 r of x irradiation. These cells were matched against 25 control cells that received no irradiation. All cells were stained with methyl green and cell extinctions, E, were measured by the formula

$$E = \frac{C^2 R^3}{R^3 - (R^2 - C^2)^{3/2}} \left[\log_{10} I_0 - \log_{10} I_n \right] ,$$

where

C = aperature radius of the lens,

R = radius of cell nucleus,

I_0 = light absorption of background,

I_n = light absorption of nucleus.

On the basis of a pooled variance estimate, a t test revealed that there was no significant ($P > 0.10$) difference between the treated mean ($\overline{E}_t = 0.903$) and the control mean ($\overline{E}_c = 1.045$). Ninety-five per cent confidence limits for the treated mean are (0.507, 1.299) and for the control mean (0.603, 1.487).

A PROBLEM IN MIXING

Origin. A. J. Weinberger, Chemistry Division.

Participating Member of Panel. J. Moshman.

Background and Status. A volume of a given material, tagged with some radioactive molecules, is fed into a mixing tank of known volume, exchanging with the contents of the tank. Assuming that the tank is full, the fraction of radioactive molecules in the tank when a given amount of the material is entering it is determined by

$$df_v^* = \frac{dn^* - f_v^* \, dn}{V} \tag{1}$$

where

f_v^* = the fraction of radioactive moles in the tank,

n^* = the number of radioactive moles entering the tank,

n = the total number of moles entering and hence leaving the tank,

V = the volume of the tank in moles.

Now

$$n^* = n^{k^*/k} , \tag{2}$$

where

$$0 \leqq k^* \leqq k .$$

Letting

$$a = \frac{k^*}{k} , \tag{3}$$

then

$$0 \leqq a \leqq 1 .$$

From Eqs. 2 and 3, it is determined that

$$dn^* = an^{a-1} \, dn ; \tag{4}$$

and substituting in Eq. 1,

$$df_v^* = \frac{[an^{a-1} - f_v^*] \, dn}{V} ,$$

for which the solution is

$$f_v^*(n) \, e^{n/v} = f_v^*(0) + \frac{a}{V}\int_0^n n^{a-1} e^{n/v} \, dn . \tag{5}$$

The integral in Eq. 5 would be an incomplete Γ-function if $a - 1 > 0$, but this is not so.

If $a = 1$ in the integrand of Eq. 5, then

$$f_v^*(n) \, e^{n/v} = f_v^*(0) + ae^{n/v} . \tag{6}$$

If $a = 0$ in the integrand of Eq. 5, then

$$f_v^*(n) \, e^{n/v} = f_v^*(0) + \frac{a}{V}[Ei(n) - Ei(0)] , \tag{7}$$

where

$$Ei(x) = \int_{-\infty}^{x} \frac{e^t}{t}\, dt \;,$$

which may be found in Jahnke and Emde's *Funktionentafeln*.

Then

$$f_v^*(0) + \frac{a}{V}\,[Ei(n) - Ei(0)] \leqq f_v^*(n) \leqq f_v^*(0) + ae^{n/v} .$$

KINETICS OF THE $HBrO_3$ REACTION

Origin. O. E. Myers, Chemistry Division.

Participating Members of Panel. J. H. Fishel and C. L. Perry.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1151.

Background and Status. The numerical integration of the systems of nonlinear differential equations described in the previous report has been continued. Whitehurst of Arnold Research Organization, Inc. will use this system of differential equations as a test problem for the Arnold Research Organization MADDIDA. The numerical integration using IBM computing machines will be continued next quarter.

ION PRODUCTION IN AN IONIZATION CHAMBER

Origin. M. Slater and R. H. Ritchie, Health Physics Division.

Participating Members of Panel. J. H. Fishel and C. L. Perry.

Background and Status. The transcendental function

$$X(x,y) = \int_0^x \frac{\theta}{\cos\theta}\, e^{-(\theta/y)^2}\, d\theta$$

was used in a theoretical evaluation of the ionization current produced in an ionization chamber by electrons scattered from a foil. X is now being tabulated for the grid

$$x = \frac{5\pi}{18} \left(\frac{\pi}{18}\right) \frac{8\pi}{18}$$

and

$$y = \frac{\pi}{12} \left(\frac{\pi}{12}\right) \pi \;.$$

The tabulation will be completed early in the next quarter.

EVALUATION OF THE LATERAL DISTRIBUTION OF ENERGY DISSIPATED BY A MOVING ION

Origin. J. Neufeld and W. S. Snyder, Health Physics Division.

Participating Members of Panel. V. C. Carlock and K. P. Graw.

Background and Status. Calculations were performed for the determination of the energy loss per centimeter of path of an ion traveling in a homogeneous medium because of the interaction of the ion with particles of the medium at a distance x from the path of the ion. The mediums were hydrogen, carbon, nitrogen, oxygen, and tissue. The energy loss was tabulated as a function of energy and distance x. This project was completed.

MONTE CARLO ESTIMATE OF COLLISION DISTRIBUTIONS IN TISSUE

Origin. W. S. Snyder and J. Neufeld, Health Physics Division.

Participating Members of Panel. K. P. Graw and C. L. Perry.

References. *Mathematics Panel Quarterly Progress Reports*, ORNL-345, 408, 561, 634, 726, 818, 888, 979, 1029, 1091, and 1151.

Background and Status. The computations and tabulations for this problem were completed during the quarter. The use of the Monte Carlo neutron histories to simulate an isotropic source is being considered.

AMPLIFIER RESPONSE TO PROPORTIONAL COUNTER PULSES

Origin. G. S. Hurst, Health Physics Division.

Participating Member of Panel. C. Perhacs.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1151.

Status. The computations started last quarter have been completed.

X-RAY CRYSTAL ANALYSIS

Origin. G. P. Smith, Jr., Metallurgy Division.

Participating Member of Panel. J. H. Fishel.

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1151.

Background and Status. The computations described in the previous report were completed in November.

FERMI FUNCTIONS

Origin. M. E. Rose and P. R. Bell, Physics Division.

Participating Members of Panel. C. L. Perry and N. M. Dismuke.

Status. A table of $\varphi_0(p, z)$ was issued as ORNL-1222.[5] The tabulated function was defined as follows:

$$\varphi_0(p, z) = \frac{1 + s_0}{2} \frac{p}{W} F_0(p, z) ,$$

where

$$F_0(p, z) = 4(2pr_0)^{2(s_0 - 1)} e^{\pi y} \frac{|\Gamma(s_0 + iy)|^2}{\Gamma^2(2s_0 + 1)} ,$$

$$s_0 = \sqrt{1 - \alpha^2 z^2} ,$$

$$y = \alpha z \frac{W}{p} ,$$

$$p = \sqrt{W^2 - 1} \geqq 0 ,$$

$$r_0 = \frac{1}{2} \alpha A^{1/3} ,$$

$$\alpha = \frac{1}{137.03} .$$

Table values were compared and found to agree with results obtained by E. Greuling and by the Bureau of Standards.

CALCULATION OF INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

Participating Member of Panel. M. R. Arnette.

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[5] N. Dismuke, C. L. Perry, M. E. Rose, and P. R. Bell, "Fermi Functions for Allowed Beta Transitions," ORNL-1222 (Feb. 22, 1952).

Reference. *Mathematics Panel Quarterly Progress Report*, ORNL-1151.

Status. The coding of the problem for the SEAC and checking of the code on the SEAC has been continued. It is now predicted that the coding and computation will be completed during the next quarter.

CALCULATION OF RACAH COEFFICIENTS FOR THE ANGULAR DISTRIBUTION IN NUCLEAR REACTIONS

Origin. M. E. Rose and L. C. Biedenharn, Physics Division.

Participating Members of Panel. S. L. Hull and V. C. Carlock.

References. *Physics Division Quarterly Progress Report*, ORNL-1005, *Mathematics Panel Quarterly Progress Reports*, ORNL-1029, 1091, and 1151.

Background and Status. The computations described in the previous reports have been completed and checked. The values of $W(l_1, j_1, l_2, j_2; 4L)$ are now being calculated. In addition, tables containing four decimal digit approximations to $W(l_1, j_1, l_2, j_2; s\ L)$ are being prepared for publication in a physics journal.

BETA DECAY (Field Factors)

Origin. M. E. Rose and P. R. Bell, Physics Division.

Participating Members of Panel. C. I. Perry, V. A. Hoyle and N. M. Dismuke.

Status. The approximations used to compute the field factors L, M, N[6] were found to be invalid for part of the ranges of the independent variables p and z. These calculations will therefore be repeated using valid approximations. Plans at present are to perform the calculations on the MIT Whirlwind. Coding of the problem for this computer is in progress. In addition to L, M, N, the factors P, Q, R[6] will be tabulated.

ESTIMATION OF ERROR AND COUNTING PERIODS IN A PILE EXPERIMENT

Origin. M. K. Hullings, Physics Division.

Participating Member of Panel. A. W. Kimball.

Background and Status. In one pile experiment four neutron counts are taken under different experimental conditions and combined to provide one statistic. Let x_1 and x_2 be the total counts with the shutter open and closed, respectively, under the first condition, and let y_1 and y_2 be the corresponding total counts under the second condition. Further, let t_{x_1}, t_{x_2}, t_{y_1}, and t_{y_2} be the counting periods in minutes. One problem is to find the variance of

$$R = \frac{\bar{x}_1 - \bar{x}_2}{\bar{y}_1 - \bar{y}_2}$$

where the bar notation is used to indicate counts per minute (e.g., $\bar{x}_1 = x_1/t_{x_1}$). The second problem is to find some method of estimating optimum counting periods.

The exact probability distribution of R cannot be obtained in a workable form. An approximate variance for R

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(6) D. L. Pursey, "Interaction of the Theory of Beta Decay," *Phil. Mag.* **42**, 1206 (1951).

may be computed by means of the so-called "delta" method described in ORNL-1029.[7] In general

$$V\left(\frac{x}{y}\right) \cong \left(\frac{\mu}{\nu}\right)^2 \left[\frac{V(x)}{\mu^2} + \frac{V(y)}{\nu^2} - 2\,\frac{\mathrm{cov}(x,\,y)}{\mu\nu}\right] , \qquad (1)$$

where $\mu = E(x)$, $\nu = E(y)$. If x and y are independent, the last term in the brackets vanishes. In practice μ and ν are replaced by their sample estimates.

In the present problem $x = \bar{x}_1 - \bar{x}_2$ and $y = \bar{y}_1 - \bar{y}_2$. Since $\bar{x}_1$ is a sample mean from a Poisson distribution and the variance of a Poisson distribution is equal to the mean, an estimate of the variance of $\bar{x}_1$ is given by $\bar{x}_1/t_{x_1}$. Variance estimates for $\bar{x}_2$, $\bar{y}_1$, and $\bar{y}_2$ are obtained in a similar fashion. Since x and y are independent, from Eq. 1 there is obtained

$$V\left(\frac{x}{y}\right) \cong \left(\frac{x}{y}\right)^2 \left[\frac{\dfrac{\bar{x}_1}{t_{x_1}} + \dfrac{\bar{x}_2}{t_{x_2}}}{x^2} + \frac{\dfrac{\bar{y}_1}{t_{y_1}} + \dfrac{\bar{y}_2}{t_{y_2}}}{y^2}\right] \qquad (2)$$

which is based on the well-known theorem that the variance of the difference of two independent random variables is the sum of the separate variances.

The problem of determining optimum counting periods is also complicated by the fact that the distribution of R is available only in integral form. A rough method for determining counting periods can be obtained, however, by incorporating certain restrictions on the respective components of error. If it is required that x and y have the same absolute error, it is necessary that

$$\frac{\bar{x}_1}{t_{x_1}} + \frac{\bar{x}_2}{t_{x_2}} = \frac{\bar{y}_1}{t_{y_1}} + \frac{\bar{y}_2}{t_{y_2}} = k \; . \qquad (3)$$

If it is required further that open and closed shutter readings have the same absolute error, then

$$\frac{\bar{x}_1}{t_{x_1}} = \frac{\bar{x}_2}{t_{x_2}} \quad \text{and} \quad \frac{\bar{y}_1}{t_{y_1}} = \frac{\bar{y}_2}{t_{y_2}} \; . \qquad (4)$$

From Eqs. 3 and 4 the following relations can be derived:

$$t_{x_1} = \frac{2\bar{x}_1}{k} , \qquad t_{y_1} = \frac{2\bar{y}_1}{k} ,$$
$$(5)$$
$$t_{x_2} = \frac{2\bar{x}_2}{k} , \qquad t_{y_2} = \frac{2\bar{y}_2}{k} .$$

If the specified error in x/y is ϵ, k may be computed from Eq. 2 by placing

$$V\left(\frac{x}{y}\right) = \epsilon^2$$

[7] *Mathematics Panel Quarterly Progress Report for the Period Ending April 30, 1951*, ORNL-1029, p. 28.

and solving for k. This gives

$$k = \frac{\epsilon^2}{\left(\frac{x}{y}\right)^2 \left[\frac{1}{x^2} + \frac{1}{y^2}\right]} .$$

Once k is estimated, estimates of the counting periods may be obtained from Eq. 5.

It should be noted that preliminary estimates of x and y are needed in order to compute k. Frequently these may be available from previous experiments or from short trial runs in the experiment under consideration.

CALIBRATION OF A CALORIMETER

Origin. J. B. Trice and A. B. Lewis, Solid State Division.

Participating Member of Panel. N. D. Given.

Background and Status. The experimental values of the temperatures inside and outside a calorimeter as functions of time were smoothed, differenced, differentiated, and graphed. This project was completed.

PROJECTS

DETERMINATION OF FAST NEUTRON FLUX IN X-10 GRAPHITE PILE

Origin. D. K. Holmes, Physics Division.

Participating Members of Panel. J. Moshman and G. J. Atta; E. B. Carter, Central Statistical Laboratory, K-25.

Background and Status. The IBM procedure for the Monte Carlo calculations of neutron histories has been perfected and histories are being calculated.

To estimate the flux in a given region Fig. 2 may be considered. Of interest is the flux through the small cylinder of cross-sectional area A and height δt. The jagged line represents the reflection of the flight path of the ith neutron in the z-y plane; all neutrons are arbitrarily started at $z = 0$. If the starting point of the path were in the interval (z_{i_1}, z_{i_2}), the neutron would have contributed to the flux through the cylinder of volume $\delta t\, A$. The flux φ is then proportional to

$$\varphi \propto \sum_i \frac{\delta t \int_{z_{i_1}}^{z_{i_2}} p(z)\, dz}{\cos \theta_i\, \delta t\, A}$$

where θ_i is the angle of incidence of the path with the cylinder and $p(z)\, dz$ is the probability of a neutron originating in the interval $(z, z + dz)$.

Now

$$p(z) = \frac{\pi}{2L} \cos\left(\frac{\pi z}{L}\right)$$

and

$$\int_{z_{i_1}}^{z_{i_2}} p(z)\, dz = \frac{1}{2}\left[\sin \frac{\pi z_{i_2}}{L} - \sin \frac{\pi z_{i_1}}{L}\right]$$

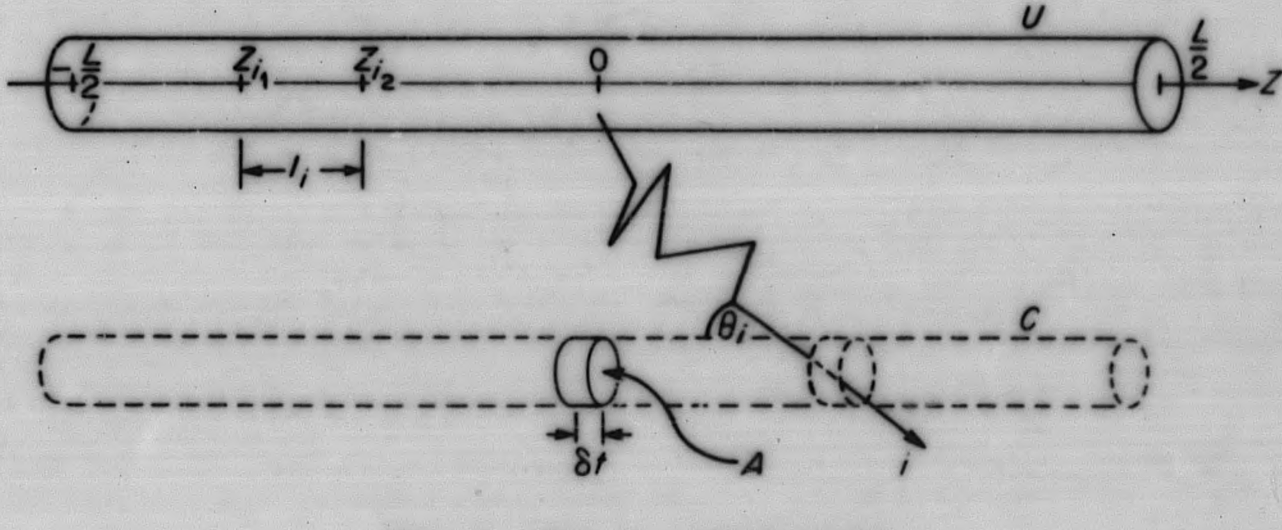

Fig. 2. Flux in a Given Region.

Let

$$z_{i_2} = z_{i_1} + l_i ,$$

where l_i is the reflection of the actual flight path through the cylinder C along the z axis. Then, by dropping subscripts,

$$\frac{1}{2} \sin \left[\frac{\pi z_{i_2}}{L} - \sin \frac{\pi z_{i_1}}{L}\right] = \frac{1}{2}\left[\sin \frac{\pi(z_{i_1} + l_i)}{L} - \sin \frac{\pi z_{i_1}}{L}\right]$$

$$= \frac{1}{2}\left[\sin \frac{\pi z}{L} \cos \frac{\pi l}{L} + \cos \frac{\pi z}{L} \sin \frac{\pi l}{L} - \sin \frac{\pi z}{L}\right] = N .$$

If N were to be estimated by

$$\frac{\pi l}{2L} \cos \frac{\pi z}{L} = \tilde{N} ,$$

then the error would be

$$E = N - \tilde{N} = \frac{1}{2}\left\{\sin \frac{\pi z}{L}\left(\cos \frac{\pi l}{L} - 1\right) + \cos \frac{\pi z}{L}\left(\sin \frac{\pi l}{L} - \frac{\pi l}{L}\right)\right\} .$$

But if $(l/L) < < 1$, $\cos (\pi l/L) \sim 1$, and the $\sin (\pi l/L)$ may be replaced by the angle. By assuming $\tilde{N}$ as the numerator, then

$$\varphi \propto \sum \frac{\frac{\pi l}{2L} \cos \frac{\pi z}{L}}{\cos \theta A} = \sum \frac{p}{A} \frac{\pi}{2L} \cos \frac{\pi z}{L} ,$$

where p is the actual path length in the cylinder C.

The effect of the approximations given above are being investigated to establish statistical estimates of the average error in view of the distributions of z, p, and θ.

KINETICS OF HRE

Origin. T. A. Welton, Physics Division, HRP.

Participating Members of Panel. N. D. Given and W. C. Sangren.

References. *Mathematics Panel Quarterly Progress Report*, ORNL-1151, and *Homogeneous Reactor Project Quarterly Progress Report*, ORNL-1221.

Background and Status. In the reports indicated in the references, the set of nonlinear differential equations that are thought to represent the kinetics of the HRE have been presented. In these reports two general methods of obtaining approximate solutions are noted, one of which uses numerical integration and the other analytical techniques. The systems, which were numerically integrated by the SEAC, have now been analyzed. Additional analytical techniques have been used and various short and iterative formulas been developed. The results of the formulas have been compared with the SEAC results. A separate report will contain the results of this investigation.

CIRCULATION IN BOILING REACTORS

Origin. P. C. Zmola, Reactor Experimental Engineering Division.

Participating Member of Panel. E. N. Lawson.

Background and Status. By using formulas derived by P. C. Zmola, various lumped variables are being calculated and graphed.

INVESTIGATION OF SOLUTIONS OF THE BOLTZMANN EQUATION FOR THE DIFFUSION OF NEUTRONS

Origin. W. K. Ergen, Reactor Physics Division, ANP.

Participating Member of Panel. N. Edmonson.

Background and Status. It is possible to obtain solutions of the Fermi age diffusion equation under some conditions. There has occurred the question: Is it possible to construct a mode of solving the Boltzmann equation under these conditions by utilizing the solutions of the age diffusion equation as initial values? This investigation has just started.

A MULTIGROUP METHOD FOR COMPUTING THE NEUTRON DISTRIBUTION IN A CYLINDRICAL REACTOR WITH REFLECTED CONVEX SURFACE AND BARE ENDS

Origin. N. M. Smith, Reactor Physics Division, ANP.

Participating Member of Panel. N. Edmonson.

Background and Status. Trial computations are now being set up by Y-12 IBM group.

KINETICS OF INTERMEDIATE AND LARGE HOMOGENEOUS REACTORS

Origin. R. B. Briggs, Research Director's Division.

Participating Members of Panel. N. D. Given and W. C. Sangren.

Background and Status. The short and iterative formulas, which were developed for finding power and pressure maximums under step changes in reactivity for the HRE have been used to find power and pressure maximums for larger homogeneous reactors. The results were graphed and submitted to R. B. Briggs.

TWO-GROUP THREE-REGION COMPUTATIONS FOR A PRODUCTION REACTOR

Origin. J. R. Parks, Long Range Planning Group.

Participating Member of Panel. H. B. Goertzel and V. S. Carlock.

Background and Status. The calculations by matrix methods of the core multiplication constant, the radial buckling constant, and fast and slow neutron fluxes and currents for four reactor designs have been started. The calculations will be completed during the next quarter.

OPERATIONS GREENHOUSE

Project 1. Twenty-eight Day Lethality Study.

Origin. G. V. Leroy, University of Chicago, U. S. Navy Task Force 3.

Participating Members of Panel. J. Moshman and G. J. Atta.

Background and Status. Previous studies (ORNL-1151) indicated the possibility of an age effect on the median lethal dose of the mice at stations 70 and 71 for the Easy shot. To investigate this apparent phenomenon further, the original data were broken down by age within weight groups. The slopes and LD_{50}'s were computed and are reproduced in Tables 6 and 7.

TABLE 6

Median Lethal Doses of Greenhouse Mice Stations 70 and 71

WEIGHT (g)	AGE (weeks)				
	8	9	10	11	12
	LD_{50}				
15-19	749.95	755.96	771.11	763.40	750.74
20-24	730.39	755.85	743.10	754.16	757.72
25-29	757.73	743.88	752.66	756.69	771.24

TABLE 7

Slopes of Dose-Response Curves for Greenhouse Mice Stations 70 and 71

WEIGHT (g)	AGE (weeks)				
	8	9	10	11	12
	SLOPES				
15-19	0 02042	0.01880	0.01723	0.02415	0.01612
20-24	0.02004	0.01914	0.01413	0.01442	0.01625
25-29	0.01410	0.01643	0.01221	0.01316	0.01641

Subsequent analysis revealed no significant regression between mortality or slope on age within the various weight groups.

Previously reported[8] provisional LD_{50}'s and slopes may be taken as final with the exception of tray 3. The LD_{50} should read 749.50 instead of 760.22, and the slope should be 0.01295 instead of the reported 0:01081.

Project 2. Depigmentation Study.

Origin. A. C. Upton, Biology Division.

Participating Member of Panel. J. Moshman.

Background and Status. Among the Greenhouse mice a marked depigmentation effect has been observed. The mouse hairs vary in color from the original dark glossy brown to a pronounced white. A quantification procedure was adopted that grades the degree of depigmentation from 0 (original brown) to 4 (all white) at six selected parts of the anatomy. Some of the mice are being periodically observed and graded.

Analysis of the existing data is under way to determine the effectiveness of differentiated depigmentation as a discriminant of dose received and its possible use as a biological dosimeter.

[8] *Mathematics Panel Quarterly Progress Report for the Period Ending October 31, 1951*, ORNL-1151, Table 7, p. 23.

PENDING PROBLEMS

(Unclassified)

THRESHOLD VALUES OF THE ANGULAR CORRELATION COEFFICIENTS

Origin. M. E. Rose and G. B. Arfken, Physics Division.

MEAN FREE PATH FOR FISSION SPECTRUM NEUTRONS

Origin. I. C. Noderer, Physics Division.

INTERPOLATED VALUES OF THE INTERNAL CONVERSION COEFFICIENTS WITH SCREENING

Origin. M. E. Rose, Physics Division.

EFFICIENCY OF A CRYSTAL (Off-Axis Calculation)

Origin. P. R. Bell, Physics Division.

LAMB SHIFT

Origin. T. A. Welton, Physics Division.

CALCULATION FOR COUNTER CONFIDENCE LIMITS

Origin. G. E. Albert, Mathematics Panel.

NUMERICAL EVALUATION OF TRIGONOMETRIC SERIES

Origin. P. S. Borie, Metallurgy Division.

END