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MATHEMATICS PANEL

QUARTERLY PROGRESS REPORT
for the Period Ending January 31, 1951

A. S. Householder, Chief

DATE ISSUED: MAR 30 1951

OAK RIDGE NATIONAL LABORATORY
operated by
CARBIDE AND CARBON CHEMICALS COMPANY
A Division of Union Carbide and Carbon Corporation
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SUMMARY

Construction of the Oak Ridge digital computer is now under way at Argonne. J. W. Woody and R. J. Klein have been sent to Argonne from this Laboratory to assist Earl Burdette in the construction. Meanwhile members of the Panel are studying circuit diagrams for control and arithmetic units, and consulting with Dr. Flanders and Dr. Chu, of Argonne, on aspects of design. Some discussion of this will be given below.

A joint meeting of the Institute of Mathematical Statistics and the Biometric Society is being planned for March 15, 16, and 17, to be held at Oak Terrace. A. S. Householder is the local member of the program committee for both organizations, and Jack Moshman is Assistant Secretary for the Institute. Dr. A. M. Weinberg has agreed to speak at the banquet, March 16. The tentative program is as follows:

PROGRAM

Thursday, March 15, 10:00 A.M.

CONTRIBUTED PAPERS FOR BIOMETRIC SOCIETY

Thursday, March 15, 2:00 P.M.

PUBLIC HEALTH STATISTICS

Chairman: Dr. Paul Densen, University of Pittsburgh

"Statistical Studies in Problems of Health"
Evelyn Fix and J. Neyman, University of California

"An Elementary Stochastic Process for a Syphilis Population"
B. G. Greenberg, University of North Carolina

Friday, March 16, 9:00 A.M.

BIOASSAY WITH QUANTAL RESPONSES

Chairman: Dr. Alexander Hollaender, Oak Ridge National Laboratory

"Why I Prefer Logits to Probits and Sinits"
J. Berkson, Mayo Clinic

"How Much Does the Choice of Metameter Matter?"
J. W. Tukey, Princeton University

"Extensions of Elementary Methods in Bioassay"
Irwin Bross, The Johns Hopkins University

"Problems in Biological Assay on A.C.T.H."
W. Weiss, Food and Drug Administration

Discussant: Jermone Cornfield, National Institute of Health

Friday, March 16, 2:00 P.M.

EXPERIMENTAL DESIGN

Chairman: Dr. Cyril Comar, University of Tennessee

"Incomplete Block Designs"
R. C. Bose, University of North Carolina

"Fractional Replication"
Oscar Kempthorne, Iowa State College

"The Analysis of Long Term Experiments"
A. M. Dutton, Iowa State College

"Testing-for-preference Experiments"
Lyle D. Calvin, Searle Pharmaceutical Company, Chicago

Saturday, March 17, 9:00 A.M.

MULTIVARIATE ANALYSIS

Chairman: Dr. E. E. Cureton, University of Tennessee

"On the Properties and Statistical Purposes of Some Well-known and Some New Tests in Multivariate Analysis"
S. N. Roy, University of North Carolina

"Some Applications of Compound Symmetry Tests"
A. W. Kimball, Oak Ridge National Laboratory

"Some Preliminary Results of Multivariate Discriminant Analysis"
D. V. Tiedemann, Harvard University

Saturday, March 17, 11:00 A.M.

CONTRIBUTED PAPERS FOR INSTITUTE OF MATHEMATICAL STATISTICS

Dr. J. W. Tukey, Consultant for the Mathematics Panel, spent January 22 and 23 in Oak Ridge. His advice was sought on Monte Carlo problems by Jack Moshman and by Dr. Walter Snyder; on biometrics problems by Dr. Alan Conger and by Dr. Mary Esther Gaulden; on a dosage problem by Dr. Marshall Brucer of ORINS; and on an ecology problem by Dr. L. A. Krumholz. On Tuesday evening he spoke on the analysis of variance at the first meeting of a statistical seminar organized by Dr. A. W. Kimball, Dr. Osmer Carpenter of K-25, and Mr. D. Chambers of the University of Tennessee.

Preliminary arrangements have been made for having the calculations for the L-shell internal conversion coefficients made on the Bureau of Standards Eastern Automatic Computer in Washington. Ruth Arnette is spending several weeks at the Bureau to assist in the preparation and coding of the problem for this machine. Since the machine is to be moved to new quarters in March and will therefore be out of commission for perhaps two months, considerable effort is being made for completing as much of the work as possible before the move takes place.

The Panel seminar is now meeting on Monday afternoons for the discussion of programming and coding of problems for the various electronic digital computing machines.

Two papers were presented at Mathematical meetings during the quarter by A. S. Householder: *Polynomial Iterations to Roots of Algebraic Equations* (contributed), American Mathematical Society, Northwestern University, November 30, 1950; *A Class of Iterative Methods for Solving Equations* (invited), Mathematical Association of America, University of Florida, December 30, 1950. Also, the paper *Some Numerical Methods for Solving Systems of Linear Equations* appeared in the August-September issue of the American Mathematical Monthly.

SPECIAL PROBLEMS AND PROJECTS

Problem: **CIRCUIT ANALYSIS FOR THE DESIGN OF THE ELECTRONIC DIGITAL COMPUTER**

Origin: Electronic Computer Project

Participating members of Panel: A. S. Householder, C. L. Perry, M. L. Nelson, N. M. Dismuke, J. Moshman

Background: The contract with Argonne National Laboratory for the construction of the Oak Ridge computer calls for some design research in order to make best use of the lessons learned during the construction of the prototype machines at Princeton and at Argonne. Partly in order to participate and perhaps contribute, partly in order to reach a decision on the ultimate design, proposed circuits are being studied by members of the Panel, and periodic conferences with the Argonne group are under way.

The digital (on-off) character of the response of the elements permits a simple representation of the structure and properties of the network in terms of Boolean algebra. Consider any line, whether branching or not, leading from a gate or toggle to gates or toggles or both; then, in principle, the line may lead from a point of maximum potential or of minimum potential and there is no other possibility. Consequently, if we assign any symbol x to this line, x may also represent the assertion that the line leads from a point of minimum potential while x' represents the contrary assertion. Toggles and gates then represent logical operators: If x and y are inputs to an "and" gate, the output is xy ("x and y"); if they are inputs to an "or" gate the output is $x \vee y$ ("x or y"); if they are inputs on the right and left, respectively, to a toggle, the output on the left after the toggle has had time to change over is

$$z(t + 1) = z(t)x(t) \vee y(t),$$

where t and $t + 1$ represent the times just before and just after the change-over, if any. The output on the other side is $z'(t + 1)$. (Hence the output of every toggle in a network at time t can be expressed as a logical function of the states at time $t - 1$ and of the signals which enter the network from the outside at that time.) Also, by successive substitution, the output at time t can be expressed as a logical function of the states of the toggles at any prior time $t - r$, and of the incoming signals at all times from $t - r$ until $t - 1$. Conversely, the requirements of a network can be expressed in the same terms and the network can be drawn up from the description.

Status: Argonne is providing proposed wiring diagrams of the various constituents of the machine, and the analysis is just beginning.

Problem: ITERATIVE METHODS FOR SOLVING EQUATIONS

Participating member of panel: A. S. Householder

Origin and Background: Since the numerical solution of an equation of some form or other is the objective of almost any computation, the development and improvement of methods for solving equations is an ever-present problem for any computation laboratory. We summarize here some observations and results on the methods of Bernoulli and of Newton for solving an equation in one unknown. Newton's method is generalizable to a system of equations and even to functional equations — in fact, the generalization includes such classical methods as that of Picard for solving differential equations. Both methods are self-correcting.

Bernoulli's method is generally stated for an algebraic equation $f(x) = 0$ only. Its application to transcendental equations does not seem to appear in the literature. However, a theorem due to König [*Math. Ann.* **9**, 530-40 (1876)] can be applied to show that if

$$f(x) = a_0 + a_1 x + a_2 x^2 + \dots ,$$

and if

$$g(x) = b_0 + b_1 x + b_2 x^2 + \dots .$$

is any function, analytic in a circle about the origin which contains in its interior the root α of smallest modulus of $f = 0$, assuming such to exist, then, provided only $g(\alpha) \neq 0$, the coefficients h in the expansion

$$g/f = h_0 + h_1 x + h_2 x^2 + \dots$$

are such that

$$\lim h_i / h_{i+1} = \alpha.$$

Also, these coefficients h are given by

$$a_0 h_0 = b_0,$$

$$a_0 h_1 + a_1 h_0 = b_1,$$

$$a_0 h_2 + a_1 h_1 + a_2 h_0 = b_2,$$

$$\cdots$$

which reduces to Bernoulli's algorithm when f is a polynomial and g a polynomial of lower degree. Moreover, by forming determinants of the h's in the usual way the products of roots of smallest modulus are obtainable, a fact which permits the computation of two or more roots of equal modulus.

Newton's method is one of a class in which from the function f one constructs a function ϕ such that whenever x_0 is in a sufficiently small neighborhood of a root α the sequence defined by the recursion

$$x_{i+1} = \phi(x_i)$$

converges to α. Hence $\phi(\alpha) = \alpha$ and ϕ must have the form $\phi = x - f\psi$ for some ψ. Newton's method utilizes the specific choice $\psi = 1/f'$, and its rapid convergence depends upon the fact that $\phi'(\alpha) = 0$, so that in the case of analyticity

$$\phi(x) = \alpha + a_2(x - \alpha)^2 + \ldots .$$

There are, however, other choices of ψ with the same property, and, in fact, one can choose ψ so that $a_1 = a_2 = a_3 = \ldots = a_{r-1} = 0$ for any r. In this case the iteration defined by $\phi(x)$ is said to be of order r. Several methods are known for obtaining iterations of arbitrary order, one of which is deducible from the same theorem of König mentioned above. In particular, when f

is a polynomial it is possible to choose ϕ to be a polynomial, whatever order may be required for the iteration. A method has been developed for constructing such iterations much more simply than by any method previously known. The possible usefulness of this lies in the fact that with some computing machines division is a process to be avoided if possible, but more in the fact that it permits an extension of the method to number fields of much more general type, where division may even be impossible. An application to the equation

$$f \equiv ax - 1 = 0$$

where a is a matrix and x, its inverse, yields a class of iterative methods for inverting the matrix a, one method of the class being classical.

This subject is elaborated in the papers presented at the meetings of the American Mathematical Society and of the Mathematical Association of America, mentioned above.

Problem: **COUNTER STATISTICS**

Participating members of panel: G. E. Albert, M. L. Nelson

Origin and Background: Each participant in this problem has separately had inquiry from Laboratory personnel and others in regard to the statistics of counting. Since the problem is of undoubted importance and since it attracts the interest of each participant, it seemed natural to give it joint study. We have found a large and unsatisfactory literature. To us it appears highly desirable to attempt:

1. Unification of the theory and literature.
2. Careful formulation of the important problems which need solving.
3. Solution of these problems.

Comment and assistance as to the kind of analysis and type of result most needed by those who engage in counting radioactivity is invited. It may be worth observing that, as usual, the problem considered is mathematically similar to others arising from otherwise dissimilar origins.

Status:

References

(1) W. Feller, *On Probability Problems in the Theory of Counters*, Studies and Essays presented to R. Courant on his 60th Birthday, 1948.

(2) S. Malmquist, "A Statistical Problem Connected with the Counting of Radioactive Particles," *Ann. Math. Stat.* **18**, 255-264 (1947).

(3) C. R. Blyth, "Statistical Problems in the Theory of Counters" (abstract), *Ann. Math. Stat.* **20**, 464 (1949).

(4) H. Cramér, *Mathematical Methods in Statistics*, pp. 510-511, Princeton University Press, Princeton, N. J., 1946.

1. Statement of Problems. It is assumed that radioactive particles impinge on a counter at the mean rate of *a counts per second* where a is a constant, and that the actual number, $N_p(t)$, of particles arriving in an *arbitrary time interval* of length t is a chance variable with the Poisson distribution with mean at, i.e.,

$$P_r\{N_p(t) = x\} = e^{-at}\,\frac{(at)^x}{x!} \tag{1}$$

Owing to a dead time effect the counter does not necessarily register all particles that impinge upon it. References 1 and 2 distinguish two types of dead time effect:

> Type I. A registration by the counter at an instant t' causes the counter to lock for a time interval $u > 0$ so that it cannot register any further particles that arrive during the time interval $(t', t' + u)$, but it will register the next particle after $t' + u$.
>
> Type II. A registration by the counter at an instant t' causes it to lock for a time interval $u > 0$, and any particle impinging upon the counter at an instant t'' while the counter is locked will extend the dead interval to $t'' + u$.

To distinguish the two types physically, note that if the rate a is very large, Type I will register a particle *every u seconds* while Type II will register only *one* particle.

In reference 1 it is indicated that the dead time effect in an actual counter is likely somewhere between the two idealizations introduced above. The authors of this report are soliciting concrete authoritative information on this matter.

Feller considers the calculation, for both types of counter, of the population mean and standard deviation of the count $N_r(t)$ registered in a time interval of length t which begins at an instant when the counter is unlocked. In addition, he gives some indications on how to proceed with similar calculations under more general assumptions than Eq. (1). Malmquist considers only the Type I counter. He drops the assumption of Eq. (1) almost entirely and attempts to remove the restriction that the counter be unlocked at the beginning of the counting interval. The authors of this report are not reconciled with certain distribution assumptions that Malmquist introduces in the development of his theory.

Neither of references 1 or 2 considers the practical problem of estimating the source rate a from an actual count $N_r(t)$. The first consideration of this problem seems to be in reference 3 where the maximum likelihood method of estimating the rate a is studied for a Type I counter. One of the authors of this report has attacked the problem by the method of confidence intervals. The results will be reviewed in paragraph 2 below.

The assumption that the distribution of the number of particles arriving at the counter is given by Eq. (1) with a constant appears to be tantamount to assuming that the source has a very long radioactive half-life. The authors propose to study the statistical problems arising in the counting of particles from a source with a short half-life. In this case it appears to be quite difficult to formulate the problems. This will be discussed in paragraph 3 below.

2. *Type I Counter: Confidence Intervals for a.* Proceeding from formulas derived by Feller in reference 1, it is shown that for any integer m the probability that the count $N_r(t)$ registered in an interval of length t beginning at an instant when the counter is unlocked is given by

$$P_r\{N_r(t) \geqslant m\} = \begin{cases} 0, \text{ if } m \geqslant (t/u) + 1, \\ \sum_{k=m}^{\infty} e^{-\lambda_m} \lambda_m^k / k!, \text{ if } m < (t/u) + 1, \end{cases} \qquad (2)$$

where $\lambda_m = a\{t - (m - 1)u\}$. The summation is easily recognized as the upper tail of the Poisson distribution with the mean λ_m and so can be computed from tables of that distribution. Alternatively, the summation may be computed from tables of, or approximate formulas for, the incomplete gamma function since

$$\Gamma(m) \sum_{k=m}^{\infty} e^{-\lambda}\lambda^{k}/k! = \int_{0}^{\lambda} x^{m-1} e^{-x}\, dx. \tag{3}$$

For any fixed values of the observation time t and the dead time u, confidence intervals for the source rate a may be obtained from Eq. (2) by a well-known statistical method (see reference 4). The procedure is rather involved for cases in which the count $N_r(t)$ is small. It is planned that confidence interval charts will be prepared for easy use by experimenters. When the count is large an approximation for the integral in Eq. (3) simplifies the procedure materially. For any assigned probability p, $0 < p < 1$, let x_p be such that for a normal variate x, $P_r(|x| \leq x_p) = p$. For example, if $p = 0.95$, $x_p = 1.96$. Then a 100 percent confidence interval for the rate a is given by

$$\frac{N_r(t) \pm x_p\sqrt{N_r(t)}}{t - [N_r(t) - 1]u} \tag{4}$$

approximately. $N_r(t)$ is the observed count in time t.

This result indicates that the center of the interval

$$\frac{N_r(t)}{t - [N_r(t) - 1]u}$$

may be the best point estimate for a. In addition, the length

$$\frac{2x_p\sqrt{N_r(t)}}{t - [N_r(t) - 1]u}$$

of the interval provides a hitherto unknown indication of the reliability of the estimate. Caution: the interval of Eq. (4) should not be used unless $N_r(t)$ exceeds 100.

The corresponding result for Type II counter has not been achieved at the present date.

3. *Basic Probability Assumptions of Radioactive Decay.* In this paragraph we set down the results of some computations of which use has not yet been made. They come out of an effort to study closely the general sample which a "counter" is asked to "count."

(*a*) The Exponential Law of Decay. It is generally assumed

(i) that the probability $p(t)$, for $t > 0$, that a radioactive atom disintegrates during the time interval $(\tau, \tau + t)$ is independent of τ.

We have, where the random varible ξ is the time of disintegration,

$$P(\xi > t \mid \xi > t - \tau)\, P(\xi > t - \tau) = P(\xi > t - \tau, \xi > t), \qquad \tau < t.$$

Application of (i) and the condition $p(0) = 0$ leads to

$$t < 0,\ p(t) = 0; \qquad 0 < t,\ p(t) = 1 - \exp(-\lambda t) \tag{5}$$

where the constant λ is the reciprocal of the inverse *e*-life.

When more than one similarly radioactive atoms are present, it is further assumed

(ii) that the individual atoms decay independently.

Thus if there are n active atoms present, the probability P_k of exactly k disintegrations in time t is

$$P_k = \binom{n}{k} p^k (1 - p)^{n-k}, \qquad k = 0, 1, \ldots, n$$

The *mean number* of atoms remaining at time t is accordingly $n \exp(-\lambda t)$.

(b) Distribution of Time Between Disintegrations. For $i = 1, 2, \ldots, n$, let t_i denote the time of the ith disintegration, supposing n atoms are active at time zero. These t_i are random variables and with them we consider the random variables ξ_j defined thus:

$$t_{j+1} = t_j + \xi_j, \qquad j = 0, 1, 2, \ldots, n-1, \qquad t_0 = 0.$$

If we assume that t_j and ξ_j are independent random variables, we find

$$t \leqslant 0, \qquad P(\xi_j \leqslant t) = 0;$$

$$0 < t, \qquad P(\xi_j \leqslant t) = 1 - \exp[-(n - j)\lambda t].$$

(c) Distribution of Time Until Next Disintegration. Still supposing n active atoms are present at time zero, let M and N be the random variables: M the number of disintegrations in $(0, \tau)$; and N, the number of disintegrations in $(\tau, \tau + t)$. It follows that for $k = 0, 1, 2, \ldots, n$,

$$P(N = k) = \binom{n}{k} P^k Q^{n-k}$$

where

$$Q = 1 - P \text{ and } P = \exp(-\lambda\tau)[1 - \exp(-\lambda t)].$$

Denote by ξ the time, measured from $\tau > 0$, until the first disintegration later than τ, or to the one next preceding τ in case all n atoms disintegrated prior to τ. The distribution of ξ is

$$t \leq -\tau, \qquad P(\xi \leq t) = 0$$

$$-\tau \leq t \leq 0, \qquad P(\xi \leq t) = \{1 - \exp[-\lambda(\tau + t)]\}^n$$

$$0 \leq t, \qquad P(\xi \leq t) = 1 + [1 - \exp(-\lambda\tau)]^n +$$

$$- \{1 - \exp(-\lambda\tau) + \exp[-\lambda(\tau + t)]\}^n.$$

Problem: BASIC STUDIES IN THE MONTE CARLO METHOD

Participating Member of Panel: G. E. Albert

Status:

Reference

A. S. Householder, "Basic Studies in the Monte Carlo Method," *Mathematics Panel Quarterly Progress Report for the Period Ending July 31, 1950*, ORNL-818, 10 (Sept. 11, 1950).

An informal memorandum on the solution of Fredholm integral equations by the Monte Carlo method is being prepared. Two related papers, (1) *Quota Sampling and Importance Functions in Stochastic Solution of Particle Problems* by G. Goertzel, NDA, June 21, 1949, and (2) *A Monte Carlo Method for Solving a Class of Integral Equations* by R. E. Cutkosky, National Bureau of Standards at Los Angeles, will be compared and redone to fit them into the general pattern of the theory of stochastic processes as presented by J. L. Doob and others. This procedure leads to a clear understanding of the contents of the papers and opens up possibilities for a systematic study and comparison of various sampling techniques and distributions for the solution of certain types of integral equations.

Problem: THERMAL-NEUTRON FLUX DISTRIBUTIONS WHEN AIR VOIDS ARE PRESENT

Origin: Dr. R. L. Echols, NEPA

Participating member of panel: D. W. Whitcombe

Background: The report, ORNL-899, which contains the thermal-neutron flux distributions for a variety of shield materials that contain air voids will soon be issued. Problems are solved where the shield material contains one, two, or three air voids and for a straight-through air duct. Most problems are solved for finite as well as infinite geometries. A problem is included to indicate the effect of shifting a void. Most of the solutions are accompanied by transmission plots which show the increased flux transmission due to the presence of the voids. The majority of the problems are solved for two-dimensional regions, but one problem is included for a three-dimensional region with two air voids.

In all cases the diffusion equation was reduced to linear difference equations which were solved on the Fairchild linear equation solver. The method of setting up the difference equations is explained in detail in the preceding quarterly progress report (ORNL-888). A plane source of thermal neutrons (isotropic or collimated) is used with the boundary condition that the return neutron current shall vanish.

A more exact replacement for the Laplacian in the diffusion equation than the first-order difference approximation was also made. This permitted calculations of the streaming as well as an estimate of the error to be made. The error in the replacement for the Laplacian was found to be of the order of 20 to 25 percent.

Problem: **STEADY-STATE HEAT TRANSFER DUE TO SLUG FLOW IN A SECTOR OF A CIRCLE**

Origin: H. C. Claiborne, Reactor Technology Division

Participating member of panel: D. W. Whitcombe

Background: This is an extension of the work described in the previous quarterly report (ORNL-888) which pertained to a 60° sector of a circle, and which has been issued as ORNL Central Files No. 50-11-77. The purpose of the present work is to find a solution when the angle of the sector is $p\pi/q$ where p and q are positive integers. It is clear that this case will include the case where the angle of the sector is 60°.

Problem: AN INTEGRAL EVALUATION

Origin: L. C. Noderer, Physics Division

Participating member of panel: D. W. Whitcombe

Background: The integral

$$I(\alpha) = \int_0^\infty x^2 e^{-x^2-\alpha/\sqrt{x}}\, dx$$

occurs in studies on the hardening of neutrons. The integral can be approximately evaluated using the method of steepest descent. Mr. Noderer was interested in the case in which α was a small positive number and in this case the method of steepest descent requires some slight modification as follows:

Let

$$I(\alpha) = \int_0^\infty e^{-E(x)}\, dx,$$

where

$$E(x) = x^2 + \frac{\alpha}{\sqrt{x}} - 2 \log x.$$

The method of steepest descent works well only if $E(x)$ has a steep minimum and including the "-2 log x" term in the exponent increases the positive root and gives a more accurate result. If x_0 is defined by

$$E'(x_0) = 2x_0 - \frac{1}{2}\frac{\alpha}{x_0^{3/2}} - \frac{2}{x_0} = 0$$

then I is, approximately,

$$I = e^{-E(x_0)}\sqrt{\frac{2\pi}{E''(x_0)}}.$$

Problem: EVALUATION OF $I(a) = \int_0^{\pi/2} \cos^2\theta\, e^{-a/\cos\theta}\, d\theta.$

Origin: J. W. Webster, ANP (NEPA)

Participating member of panel: D. W. Whitcombe

Background: If in I the replacement is made

$$\cos\theta = 1/x,$$

then we may write

$$I = \int_1^\infty \frac{e^{-ax}}{x^4}\left[1 - \frac{1}{x^2}\right]^{-1/2} dx.$$

Now $(1 - 1/x^2)^{-1/2}$ can be replaced by the series

$$\sum_{n=1}^{\infty} \frac{\left(\frac{1}{2}\right)_n}{n!}\,\frac{1}{x^{2n}}$$

where

$$\left(\frac{1}{2}\right)_n = \left(\frac{1}{2}\right)\left(\frac{1}{2}+1\right)\left(\frac{1}{2}+2\right)\cdots\left(\frac{1}{2}+n-1\right).$$

Then we can write

$$I = \sum_{n=1}^{\infty} \frac{\left(\frac{1}{2}\right)_n}{n!} E_{2n+4}(a)$$

where

$$E_n(a) = \int_1^\infty \frac{e^{-ax}}{x^n}\, dx.$$

Since the exponential integrals E_n are tabulated, the latter expression for I serves as the result. The result for I can be made easier to compute by putting E_n in the form

$$E_n(a) = P_n + Q_n e^{-a}$$

where P_n and Q_n are polynomials. This simplification was made in the paper sent to Mr. Webster.

Problem: DETAILED THERMAL-FLUX DISTRIBUTIONS

Origin: Dr. R. L. Echols, NEPA

Participating Member of Panel: D. W. Whitcombe

Background: The thermal-neutron distribution for a block of concrete with an air void is being found for a plane or isotropic plane source of neutrons. This problem differs from preceding problems in that it is much larger. The shield material is divided up into 312 cells, i.e., the linear-difference-equation approximation to the diffusion equation has 312 unknowns. It was found that the Fairchild linear equation solver would have to be modified before the solution could be obtained. The modification has been made but the solution is not yet obtained because of the backlog of problems awaiting the Fairchild machine.

A series of problems has been prepared for the Fairchild machine where the shield material is variable both as to size and shield composition. The configurations all contain continuous air voids with two right-angle bends. These problems have been coded for the machine and should all be solved some time during February. The first three of the series have been completed with no difficulty.

Problem: SAMPLE CRITERION FOR TESTING OUTLYING OBSERVATIONS

Origin: J. H. Smith, Physics Division

Participating members of panel: D. W. Whitcombe, J. Moshman

Background: Mr. Smith is performing an experiment which involves the recording of counter data. A foil was counted about eight times and the average of the counts was then used to indicate a more accurate value. Mr. Smith wanted a criterion for eliminating a count that differed appreciably from the other counts in a given set. This problem is a favorite one with statisticians, and Mr. Smith was referred to an article by F. E. Grubbs on the subject which appeared in the March, 1950 issue of the *Annals of Mathematical Statistics*.

Problem: DETERMINATION OF SAMPLE SIZE FOR LONG-TERM RADIATION EXPERIMENT

Origin: Dr. J. H. Rust, Dr. J. L. Wilding, UT-AEC Agricultural Research Program

Participating member of panel: A. W. Kimball

Background and Status: A long-term radiation experiment employing large animals is being planned for the purpose of investigating the effects of X radiation on longevity. Since the experiment is expensive and time-consuming, the experimenters wanted to have a reliable estimate of the number of animals which would be required for each experimental group to achieve the desired degree of accuracy in the final comparisons. On the basis of preliminary data, it was determined that if about 290 animals are used in each group, the probability is 0.95 that a time difference (between the control group and any irradiated group) of 10 percent or more will be detected if the final test of significance is made at the 5 percent level.

The method of estimation [Harris, Horvitz, and Mood, *J. Amer. Stat. Assoc.* 43, 391 (1948)] requires some preliminary estimate of the amount of variability to be expected in the survival data. It is believed that there is a 50-50 chance that the coefficient of variation will lie between 25 and 35 percent. By the methods described in the reference, this leads to a preliminary estimate of the standard deviation, $s_1 = 30$ percent of the mean, with 8-2/3 degrees of freedom. Further computation aided by the table given in the reference led to a sample size of 289 animals per group for the degree of accuracy specified above.

It was thought for a while that some sequential sampling plan might be used to help reduce the total number of animals required. Since, however, the contemplated analysis includes both tests of hypothesis and estimation of group differences, no workable sequential method seems applicable.

Problem: **TESTS OF HOMOGENEITY AND CORRELATION IN A RADIATION EXPERIMENT WITH *PARAMECIUM AURELIA***

Origin: Dr. R. F. Kimball, Biology Division

Participating Member of Panel: A. W. Kimball

Background and Status: Two stocks of *Paramecium aurelia*, isogenic except for a single gene, differ in that one produces a substance which kills other paramecia and the other is a nonkiller, sensitive to the killing substance. Twenty-eight conjugate pairs were obtained between the two stocks; the two members of each pair were kept separately and allowed to divide, and one product of the first division from each was discarded. The other product of each was allowed to divide again and these two products were kept separately for several days as daily isolation lines. Three days before autogamy was to be obtained two products of the division of the single animal, isolated the day before, were put into separate containers and allowed to multiply to form small mass cultures. From each culture 25 autogamous animals were isolated and checked for survival. Thus from each original pair eight groups of 25 autogamous animals were obtained.

The general statistical problem was to determine to what extent the various subdivisions could be considered homogeneous with respect to the proportion of animals surviving. Chi-square tests at the final stage indicated no differences between the two groups of 25 animals obtained from the two products of the division of a single animal. Further chi-square tests were applied to the groups of 50 animals, and the pooled chi-squares for both the killer and sensitive conjugates were significant at the 1 percent level. Thus it was apparent that the two daily isolation lines obtained at the second division could not be considered samples from the same population. The next problem was to determine whether the two lines obtained from a single pair (killer and sensitive conjugates being considered separately) were more alike than the lines from pair to pair.

Statistically the problem may be formulated as follows: We have a sample of 28 pairs from a bivariate population and want to estimate the correlation. In the sample, however, it is not possible to identify the variates, and therefore the familiar product-moment correlation coefficient cannot be used. Since the observations are in the form of proportions of animals surviving, the arcsine transformation was employed and the resulting variates were assumed to have a bivariate normal distribution with equal variances. It can be shown that in this case the maximum likelihood estimate of the correlation is given by

$$\hat{\rho} = \frac{B - A}{B + A}$$

where

$$A = \sum_{i=1}^{n} d_i^2,$$

$$B = \sum_{i=1}^{n} (s_i - 2\hat{\mu})^2,$$

$$d_i = x_i - y_i,$$

$$s_i = x_i + y_i,$$

$$2\hat{\mu} = \frac{1}{n}\sum_{i=1}^{n} s_i,$$

n = number of pairs of observations.

This statistic is most often referred to as the "intraclass correlation coefficient."

Two methods are available for testing whether the population correlation coefficient ρ differs from zero. We may take (R. A. Fisher, *Statistical Methods for Research Workers*, 10th ed., p. 215, Oliver and Boyd, London, 1948)

$$z = \tfrac{1}{2}\{\log(1 + \hat{\rho}) - \log(1 - \hat{\rho})\}$$

as a normal variate with zero mean and variance

$$\sigma_z^2 = \frac{1}{n - 3/2}$$

On the other hand, we may use the likelihood ratio test, which in this case amounts to taking

$$-2 \log_e \left[\frac{2\sqrt{AB}}{A + B}\right]^n$$

as an approximate chi-square with one degree of freedom. When these tests were applied to the paramecium data, the correlations were found to be significant at the 5 percent level in both the killer and the sensitive conjugates.

Problem: **COMPARISON OF MUTATIONS IN YEAST INDUCED BY ULTRAVIOLET IRRADIATION**

Origin: Dr. S. Pomper, Biology Division

Participating members of Panel: A. W. Kimball, B. S. McGill

Background: Ultraviolet irradiation experiments are being conducted using marked haploid and diploid yeasts. The haploid cultures are similar in that they both require adenine and uracil, and the diploid was synthesized from them in such a way that it was homozygous recessive for adenine and uracil and heterozygous for trypotophane and methionine. The present experiments are focusing attention on the adenine and uracil mutation frequencies.

The data from a single experiment are derived from three sets of plates. One set has complete agar, and the other two sets have agar from which adenine and uracil, respectively, have been omitted. Each of the three sets consists of plates which have come from cultures that have been exposed to ultraviolet irradiation for lengths of time varying from 0 to 16 min. Uracil and adenine mutation frequencies are computed by comparing the plate counts for deficient agar sets with the complete agar plate counts. The problem is complicated by the fact that at higher doses some killing results in the complete-agar set.

Status: A previous analysis has established that the counts follow the Poisson law, so that the square root transformation was employed before subjecting any of the data to analysis of variance. The first two analyses based on seven diploid and six haploid experiments, respectively, showed that killing begins to have a significant effect after 8 min in the diploid organism and 4 min in the haploid organism. The apparent increased sensitivity of the haploid is due probably to the tendency of the haploids to aggregate in small clusters.

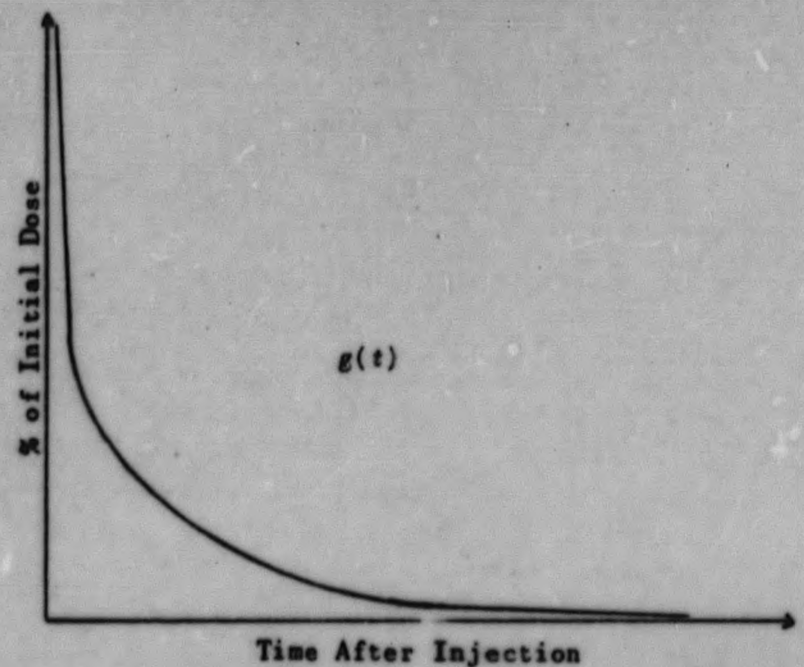
It is believed that this difficulty can be overcome by subjecting all cultures to sonic or ultrasonic treatment before irradiation. A few experiments of this type have been conducted, and an analysis of variance of the mutation frequencies over a total of 16 experiments is now being computed. The data are nonorthogonal with some subclasses containing no observations, so that the analysis must be performed by the lengthy least-squares procedure. In this example it requires the inversion of two 20 by 20 matrices, and work is being done on the NEPA digital computer with the assistance of J. J. Stone, who is the designer of the machine. When completed the analysis will enable us to evaluate the effect of ultrasonic treatment in haploids and diploids and also provide a test for interaction.

Problem: ESTIMATION OF TOTAL ABSORPTION BY THE BLOOD OF RADIOCALCIUM ADMINISTERED ORALLY IN DAIRY CATTLE

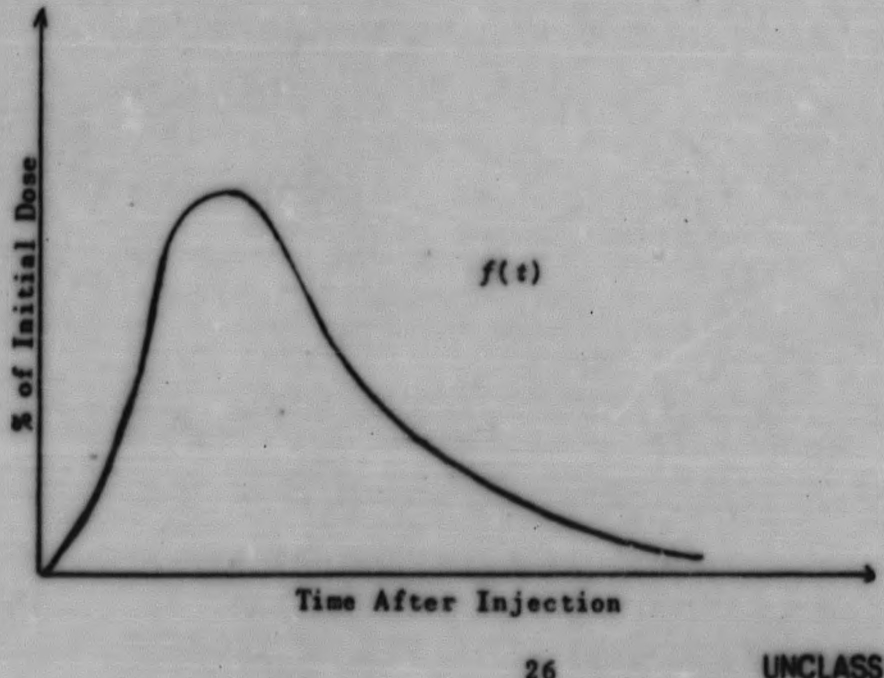
Origin: Dr. S. L. Hansard, Dr. C. L. Comar, UT-AEC Agricultural Research Program

Participating member of Panel: A. W. Kimball, C. L. Perry, J. Moshman, B. S. McGill, G. J. Atta

Background: When radiocalcium is administered intravenously to a cow and blood specimens are taken at periodic intervals after administration, the amount of radiocalcium present in total blood (expressed as a percentage of the initial dose) when plotted against time after initial dose behaves as shown in Fig. 1.



This behavior has been found in different animals, at about the same levels. When radiocalcium is administered orally, the response looks about like that shown in Fig. 2.



After oral administration the radiocalcium is at the same time being absorbed by and released from the blood stream through relatively complicated physiological processes. The experimenters would like to know what the amount of radiocalcium in the blood (as a function of time after initial oral dose) would be if none of the calcium were ever released from the blood.

Status: The most simple formulation of the problem would begin with the rate relation

$$\text{(Rate of accumulation)} = \text{(rate of absorption)} - \text{(rate of release)} \qquad (1)$$

It may be assumed that the rate of release is a function of the amount present, and that this rate determined from $g(t)$, i.e.,

$$[100 - g(t)]' = \phi\ \text{(amount present)},$$

is applicable to any amount present in the blood regardless of the method of administration of the initial dose. In Eq. (1), which involves units of time after oral administration, the amount present at time t is $f(t)$. Accordingly, if we let $h'(t)$ be the rate of absorption, we must have

$$h'(t) = f'(t) + \phi[f(t)], \qquad (2)$$

or, upon integration,

$$h(t) = f(t) + \int_0^t \phi[f(t)]\, dt.$$

In order to obtain approximations to $f(t)$ and ϕ, calculations involving the experimental data are being performed. If satisfactory expressions are found, it will be possible to determine a reasonably accurate representation for $h(t)$.

Problem: BETA DECAY (FERMI FUNCTIONS)

Origin: Dr. M. E. Rose, P. R. Bell, Physics Division

Participating members of panel: N. M. Dismuke, M. R. Arnette

Background: See previous quarterly reports.

Status: Work has been resumed on coefficients and correction factors to be calculated for forbidden transitions. It is expected that machine time will be available during the next quarter for these calculations.

Values of the allowed Fermi functions for $p = 0$ were calculated by hand to incorporate in the table of F_0's completed earlier. Screening corrections will be calculated after the problem of forbidden transitions is finished.

Problem: RaE BETA SPECTRUM

Origin: Dr. M. E. Rose, Physics Division

Participating Member of Panel: N. M. Dismuke

Background: This calculation is an attempt to fit the RaE Beta Spectrum.

Status: The ratios F_ν/F_0 (available from the Fermi function calculations, above) needed for this calculation must first be checked by differencing. When this has been done the factors to be used in the fitting can be calculated. Since the number of operations is large compared with the number of times a given operation is performed, the computations will be done on IBM machines, using a general board for the arithmetic operations.

Problem: RABBIT HEMATOCRIT RATIOS USING Fe^{59} and P^{32} TRACERS

Origin: Dr. J. B. Kahn, Jr., Biology Division

Participating Members of Panel: J. Moshman, B. S. McGill

Background: The ratio of average body to peripheral hematocrits was determined for two series of rabbits, one using Fe^{59} as a tracer element in determining cell and plasma volumes, the other P^{32}. The problem involved the comparability of the two series.

A cursory examination of the data revealed that for each of the six Fe^{59} rabbits, the first of the three determinations was lower than the other two. This was traced to a faulty instrument calibration and discarded from the analysis.

Analysis of variance showed that variation of readings within different rabbits did not differ significantly from the variation between rabbits.

The difference between both groups was then found to be highly significant by the "t" test ($P < 0.001$) and 95 percent confidence limits were finally computed for each group separately.

Status: Completed.

Problem: **OPERATION GREENHOUSE**

Origin: Dr. J. Furth, Biology Division

Participating Member of Panel: J. Moshman

Background: In a forthcoming test of atomic explosives in the Pacific, groups of mice will be exposed at different levels of expected radioactivity and neutron bombardment. After a 30-day period, surviving mice will be flown to Oak Ridge for extended study. This may be the first successful planned investigation of the effects of an atomic bomb on mammals. The investigation will concentrate on the effects on longevity, neoplasm incidence, and cataract formation.

Status: Preliminary methods of statistical analysis have been formulated. A punch card protocal suitable for recording, tabulation, and analysis of the data has been prepared and tentatively approved by the other participants concerned in the project. A complete "preoperations" report has been submitted to the Navy for final approval.

Problem: **EFFECT OF DINITROPHENOL ON MITOSIS**

Origin: Dr. M. E. Gaulden, M. Nix, Biology Division

Participating Members of Panel: J. Moshman, B. S. McGill

Background: Eight grasshopper embryos from four egg cases were given a dose of 0.1×10^{-6} μg of dinitrophenol and their mitotic activity was compared with eight other sister embryos from the same egg cases.

The analysis took the form of a three-way analysis of variance with proportional subclasses since unequal numbers of embryos were chosen from each egg case. In order to stabilize the variance, the transformation

$$y = \sqrt{x} + \sqrt{x + 1}$$

was employed, where x represented the number of cells counted, the index of mitotic activity.

It was found that the F ratio between treated and control was only 0.03, but that between egg cases was 10.81 which, with 3 and 173 degrees of freedom, was significant on the 0.005 level.

Status: The lack of significance due to treatment was expected by the biologists in view of the extremely small dose, but there was no immediate explanation of the heterogeneity between egg cases. The latter is being checked for reproducibility in a series of experiments currently in progress.

Problem: **EFFECT OF RADIATION ON PEANUT PLANTS BY DOSE AND SOURCE**

Origin: Dr. G. R. Noggle, Biology Division

Participating Members of Panel: J. Moshman, B. S. McGill

Background: An experiment was performed to investigate the effect on a peanut plant of six dosages each of X and γ radiation. The variables studied were: (index 1) number of expanded leaves on the main stem; (index 2) number of expanded leaves on the cotyledonary stems; and (index 3) height of the main stem.

The data were placed on punch cards and a three-way analysis of variance was run partitioning the variance by source of irradiation, dose, and greenhouse bench on which the plants were raised. There were two replications. It was found that for all three indices there was a significant ($P < 0.005$) variation between dosages and benches. The difference between X and γ radiation was not significant for index 2 above. Source × dose and dose × bench interactions were significant for the first index only.

Major interest was concentrated on dose effects. Following the procedure of Tukey (*Biometrics* 5, 99-114) for comparing individual means in the analysis of variance, the doses were partitioned into homogeneous groups as follows:

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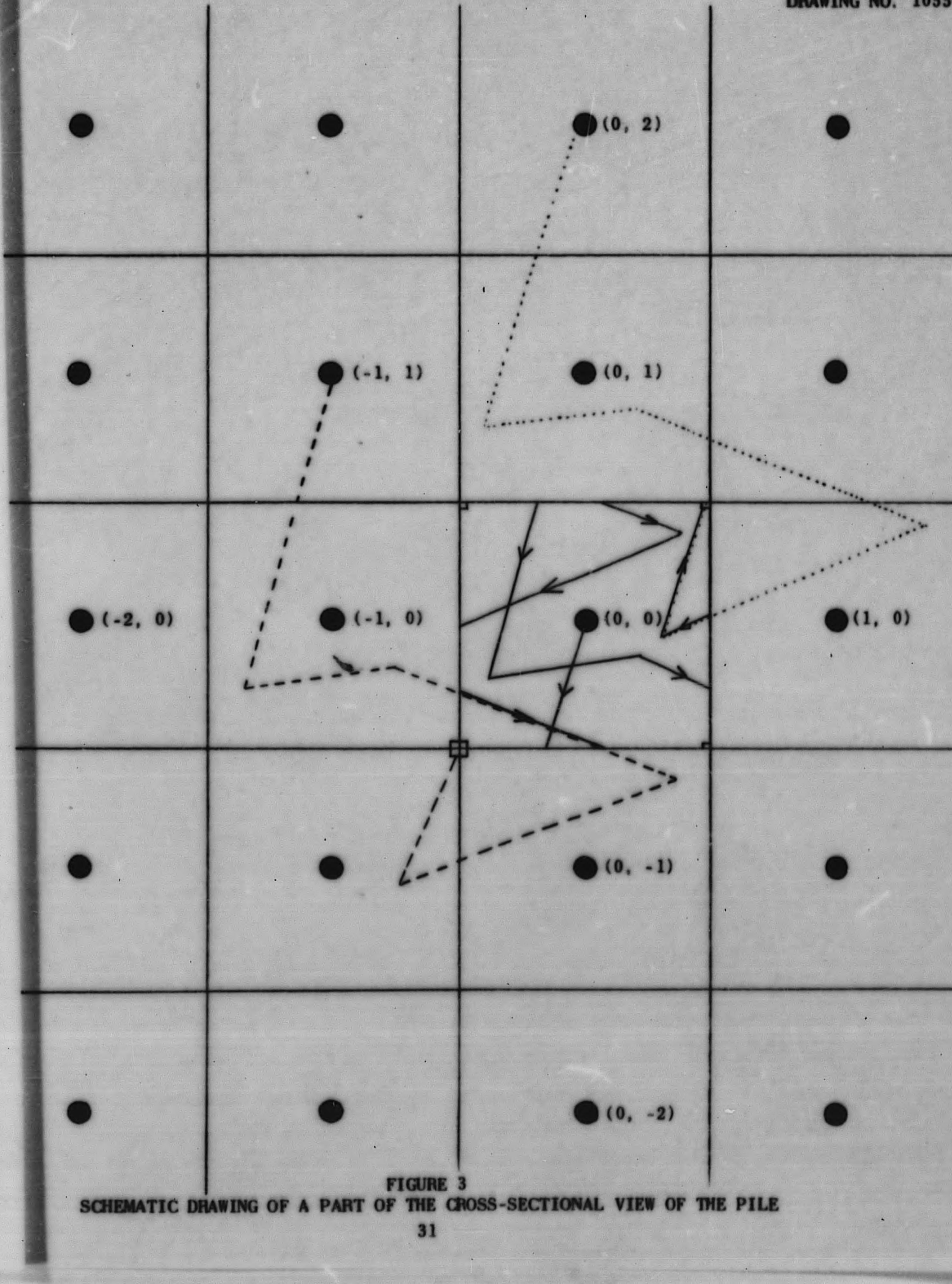

FIGURE 3

SCHEMATIC DRAWING OF A PART OF THE CROSS-SECTIONAL VIEW OF THE PILE

INDEX 1	INDEX 2	INDEX 3
0 r	0 r	0 r
2500 r	2500 r	2500 r
5000 r	5000 r	5000 r
10,000 r	10,000 r	
20,000 r		10,000 r}
	20,000 r}	
40,000 r}		20,000 r}
	40,000 r}	
		40,000 r}

where each bracket encloses a group of doses whose effects were not significantly different from each other.

Problem: (continuation): DETERMINATION OF NEUTRON FLUX IN ORNL PILE

Origin: Dr. D. K. Holmes, Physics Division

Participating Member of Panel: J. Moshman

Background: As reported in ORNL-888, programming for this problem was in progress at the time of issuance of that report. It developed that a straightforward Monte Carlo procedure, maintaining the cross-sectional accuracies demanded, overtaxed the IBM program capacity or involved a prohibitively long time period for the calculations.

Recourse has now been made to consideration of a unit cross-sectional cell of the pile lattice. Consider a schematic drawing of a part of the cross-sectional view of the pile (Fig. 3).

Each neutron would originate in the rod located at (0, 0). Whenever it strikes a boundary of the unit cell, it is continued through from the opposite side. We may follow a sample neutron flight, for example, one like that in the figure given by the solid line, where the arrows indicate the direction of light. A change of direction results from a collision. The sample flight, for instance, passes through the small square near (0.5, 0.5).

This flight, in a sense, may be unwound. We call the initial leg of the flight (0, 0). Then, in general, if the neutron strikes a boundary, the following formulas are followed:

BOUNDARY	CONVERSION
Left	$(x, y) \longrightarrow (x + 1, y)$
Upper	$(x, y) \longrightarrow (x, y - 1)$
Right	$(x, y) \longrightarrow (x - 1, y)$
Lower	$(x, y) \longrightarrow (x, y + 1)$

In this manner, as indicated in the diagram by the dotted line, the neutron is unwound to (0, 2). By translation, a neutron whose path takes the same form would originate at the (-1, 1) rod and proceed into the (-0.5, -0.5) corner square. This is indicated by the dashed line. Hence, solely by a consideration of the unit cell, we may find the contribution to the flux through any region from all rods in the pile.

By carrying along the "pseudo-rod-coordinates" we can at any time eliminate contributions from any desired collection of fuel rods, such as would be desired when finding the flux near a corner of the cube.

The problem of determining whether or not the neutron flight is intercepted by a rod if it originates in the moderator and, if so, the coordinates at the initial point of contact, has been resolved as follows:

Let the coordinates of the ith collision be represented by (x_i, y_i, z_i). It is found that the neutron would then have direction cosines l_i, m_i, n_i and a flight path ρ_i. Then the equation of the reflection of the line of flight in the xy plane is

$$\frac{x - x_i}{y - y_i} = \frac{x_i - (x_i + f_i l_i \rho_i)}{y_i - (y_i + f_i m_i \rho_i)} = \frac{l_i}{m_i}$$

where

$$f_i = \sqrt{1 - n_i^2}.$$

Then

$$(m_i x) + (-l_i y) - (m_i x_i - l_i y_i) = 0$$

or

$$\left[\frac{m_i}{f_i}\right]x + \left[\frac{l_i}{f_i}\right]y - \left[\frac{m_i x_i - l_i y_i}{f_i}\right] = 0.$$

But this is the normal form of the equation of a straight line and therefore the distance from (0, 0), the only rod admissible, is simply

$$p = \frac{m_i x_i - l_i y_i}{f_i}. \tag{1}$$

If r is the radius of the rod and $p > r$, the neutron does not enter a rod; if $p < r$, the neutron does enter a rod; and, if $p = r$, the neutron strikes a rod tangentially.

If $p \leqslant r$, we wish to know the coordinates of the point of nearest contact. This results from the neutron going a distance $\delta \leqslant \rho_i$ and

$$(x_i + l_i\delta)^2 + (y_i + m_i\delta)^2 = r^2,$$

which is a quadratic in δ,

$$(l_i^2 + m_i^2)\delta^2 + 2(l_i x_i + m_i y_i)\delta + (x_i^2 + y_i^2 - r^2) = 0. \tag{2}$$

Hence

$$\delta = \frac{-2(l_i x_i + m_i y_i) \pm \sqrt{4(l_i x_i + m_i y_i)^2 - 4(l_i^2 + m_i^2)(x_i^2 + y_i^2 - r^2)}}{2(l_i^2 + m_i^2)}$$

$$= \frac{-(l_i x_i + m_i y_i) \pm \sqrt{r^2(l_i^2 + m_i^2) - (l_i y_i - m_i x_i)^2}}{l_i^2 + m_i^2}$$

where the smaller value of δ is chosen.

From a machine viewpoint, instead of extracting the root in a straightforward manner we may take Eq. (2) and set

$$\delta = \frac{r^2 - (x_i^2 + y_i^2)}{(l_i^2 + m_i^2)\delta} - \frac{2(l_i x_i + m_i y_i)}{l_i^2 + m_i^2}. \tag{3}$$

The coefficient of $1/\delta$ is always negative, since $l_i^2 + m_i^2 \geqslant 0$ and outside the rod $x_i^2 + y_i^2 > r$ and the constant in Eq. (3) is always positive, since, to start at (x_i, y_i) and head toward (0, 0), l_i and x_i, and m_i and y_i must, pairwise, be of opposite sign. We may then write Eq. (3) as

$$\delta = -\frac{A}{\delta} + B \tag{4}$$

and solve iteratively. If δ' is a solution of Eq. (4), another solution is $B - \delta'$ and the desired solution is

$$\delta^* = \min\,[\delta',\ B - \delta'].\tag{5}$$

The coordinates of the point of entry or contact are $(x_1 + l_1\delta^*,\ y_1 + m_1\delta^*)$.

If the neutron is in a rod and it is desired to find the point of exit, if it exists, that is, if $p \leq r$, where p is as defined in Eq. (1), an entirely analogous procedure is followed, with minor modifications.

A δ^* as in Eq. (5) is undesirable since only one δ is admissible, which may be either δ' or $B - \delta'$; we may consider ourselves at $(x_i + \rho_i l_i,\ y_i + \rho_i m_i)$ rather than $(x_i,\ y_i)$ and then find δ^* as in Eq. (5).

In the event that

$$l_i^2 + m_i^2 = 0,$$

then $l_i = m_i = 0$, and $n_i = 1$, and consequently the neutron is traveling vertically in the z direction. We then let

$$x_{i+1} = x_i$$

$$y_{i+1} = y_i$$

$$z_{i+1} = z_i + \rho_i$$

and we have a collision at $(x_{i+1},\ y_{i+1},\ z_{i+1})$ which necessitates finding a new energy level, new direction cosines l_{i+1}, m_{i+1}, n_{i+1}, a new flight length ρ_{i+1}, and then proceeding as before.

Status: The modifications described above to the procedure are being incorporated into the IBM programming with the aid and cooperation of E. B. Carter of the K-25 Central Statistical Laboratory.

Problem: NUCLEAR DATA ON PUNCHED CARDS

Origin: Dr. Katharine Way, National Bureau of Standards

Participating members of panel: N. M. Dismuke, C. Perhacs

Background: See last quarterly report. (ORNL-888)

Status: *Proposal for Putting Nuclear Data on Punched Cards*, ORNL-883, has been issued.

Among the data to be included on punched cards are log *ft* values, which indicate the type β spectrum for a given β emitter. E. Feenberg and G. L. Trigg,[1] and L. W. Nordheim[2] have listed some of these values. To complete these lists as nearly as possible and to revise old values in case newer half lives, *t*, are known, a revised list of log *ft* values was calculated. Log *f* values were read from the Feenberg and Trigg[1] curves; *t* values were taken from K. Way's *Nuclear Data.*[3]

References

(1) E. Feenberg and G. L. Trigg, *Tables of Comparative Half Lives of Radioactive Transitions*, Washington University, St. Louis, Missouri.

(2) L. W. Nordheim, *Tables for β-Decay Systematics*, Los Alamos Scientific Laboratory, Los Alamos, New Mexico, May 1950.

(3) Katharine Way, Lilla Fano, Millicent R. Scott, and Korin Thew, *Nuclear Data*, NBS Circular No. 499, January 1950.

Problem: **REVIEWING THE MATHEMATICS IN *A STUDY OF STRESS-STRAIN-TIME FUNCTIONS OF METALS*, ORNL CF-50-10-132 and CF-50-10-132A.**

Origin: Dr. A. G. H. Andersen, Metallurgy Division

Participating Member of Panel: C. L. Perry

Background and Status: See *Mathematics Panel Quarterly Progress Report for Period Ending October 31, 1950*, ORNL-888. Dr. Andersen has modified parts of the reports ORNL CF-50-10-132 and ORNL CF-50-10-132A. These revised reports have been reviewed by the Panel.

Problem: **HEAT TRANSFER BY A FLUID FLOWING WITH A SLUG FLOW IN A HOT PIPE**

Origin: H. C. Claiborne, Reactor Technology Division

Participating Member of Panel: C. L. Perry

Background: See *Mathematics Panel Quarterly Progress Report for the Period Ending July 31, 1950*, (ORNL-818).

Status: Mr. Claiborne is now finishing the write-up of his investigations. The Mathematics Panel reviewed part of his writings.

Problem: **MONTE CARLO ESTIMATE OF AGE AND COLLISION DISTRIBUTION IN TISSUE FOR 10-Mev SOURCE NEUTRONS**

Origin: Dr. W. S. Snyder and Dr. J. Neufeld, Health Physics Division

Participating Member of Panel: M. R. Arnette

Background: See the following Mathematics Panel Quarterly Progress Reports: ORNL-345 (period ending February 1949), ORNL-408 (period ending July 31, 1949), ORNL-726 (period ending April 30, 1950), ORNL-818 (period ending July 31, 1950), ORNL-888 (period ending October 31, 1950).

Status: The IBM results for the collision densities appeared to have two tabulation errors. The source of these errors has been found and the tabulation corrected.

Problem: **RADIOACTIVITY OF ALIGNED AND UNALIGNED SAMPLES**

Origin: C. P. Stanford, Low Temperature Physics

Participating Members of Panel: J. Moshman, C. L. Perry

Background: See *Mathematics Panel Quarterly Progress Report for Period Ending October 31, 1950*, ORNL-888.

Status: The Low Temperature Physics Group and Mathematics Panel have discussed the method of calculation (see reference given above) proposed by the Mathematics Panel in a joint seminar. The Low Temperature Physics Group are performing the calculations.

Problem: **INTERPOLATION ON COMPUTED VALUES OF INTERNAL CONVERSION COEFFICIENTS (K-shell)**

Origin: Dr. M. E. Rose, Physics Division

Participating Members of Panel: C. Perhacs, E. A. Forbes, M. L. Nelson, N. M. Dismuke, C. L. Perry

Background and Status: See the following Quarterly Progress Reports: Physics Division, ORNL-228 (for period ending November 1948); Mathematics Panel, ORNL-345 (for period ending February 1949), ORNL-408 (for period ending July 31, 1949), ORNL-726 (for period ending April 30, 1950), ORNL-818 (for period ending July 31, 1950), ORNL-888 (for period ending Oct. 31, 1950).

Let $E_\mu(Z, k)$ and $M_\mu(Z, k)$ (where μ = 1, 2, 3, 4, 5) represent the K-shell internal conversion coefficients for electric (E_μ) and magnetic (M_μ) multipoles of order 2^μ, where Z and k are atomic number and gamma-ray energy in mc^2 units, respectively.

The *key values* of the K-shell internal conversion coefficients were calculated on the Automatic Sequence Relay Calculator (Mark I) from the relativistic formulas (using Dirac wave functions) for

Z = 10, 20, 30, 40, 54, 64, 72, 78, 83, 88, 92, 96

and

k = 0.3, 0.5, 1.0, 1.8, 3.0, 5.0.

The values at k = 0.3 were obtained for the first eight values of Z only.

Dr. M. E. Rose of the Physics Division, ORNL, wants to expand these tables. This will involve an interpolation on the *key values*. His expanded table will be for the values

Z = 10, 15, 20, 25, 30, 35, 40, 44, 48, 51, 54, 58, 61,
64, 67, 70, 72, 75, 78, 81, 83, 86, 88, 90, 92, 96,

and

k = 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.2, 1.5, 1.8, 2.1,
2.4, 2.7, 3.0, 3.5, 4.0, 5.0.

For the values at k = 0.3 and 0.4, Z must be less than or equal to 78 (see preceding paragraph).

The following paragraph explains the scheme of interpolation (proposed by Dr. B. Spinrad, Argonne National Laboratory).

Approximate values for the internal conversion coefficients will be calculated using the Dancoff and Morrison approximate formulas for the above 402 combinations of (Z, k). The ratio $[r_\mu(Z, k) = E_\mu(Z, k)/E_\mu(Z, k)$ approx.] of the exact value (Mark I calculation) to the approximate value (Dancoff and Morrison*) will then be found for the 68 key combinations of (Z, k). The interpolation will be made on the ratio $r_\mu(Z, k)$. This will give values of the ratio at the 402 combinations of (Z, k). The values of $E_\mu(Z, k)$ for the table will be found by multiplying $E_\mu(Z, k)$ approx. by $r_\mu(Z, k)$. The same process is used for $M_\mu(Z, k)$.

The interpolation for $r_\mu(Z, k)$ at the 402 combinations of (Z, k) will first be done in the Z direction. The reason for this is that there are more key values in the Z direction and Z dependence is simpler than the k dependence.

Problem: **ANALYSIS OF β-RAY SPECTROSCOPY**

Origin: P. R. Bell, Physics Division

Participating Members of Panel: J. H. Fishel, M. R. Arnette, A. S. Householder, G. E. Albert, C. L. Perry

Background: See Mathematics Panel Quarterly Progress Reports for periods ending July 31, 1949 (ORNL-408) and July 31, 1950 (ORNL-818).

Status: In ORNL-818 it was proposed that the β spectrum be found by solving a system of algebraic equations. The solutions to two of these systems have been found, the calculations being performed on the NEPA digital computer. The Mathematics Panel will continue the calculations. The Panel will also consider changing the present system for replacing the integral equation (ORNL-818) by a system of algebraic equations.

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*The Dancoff and Morrison (*Phys. Rev.* 55, p. 122) approximation formulas are:

$$E_\mu(Z, k) = \frac{2Z^3\alpha^4}{k^3}\left[\frac{k+2}{k}\right]^{\mu-\frac{1}{2}}\left[k^2 + \frac{4\mu}{\mu+1}\right]$$

and

$$M_\mu(Z, k) = \frac{2Z^3\alpha^4}{k}\left[\frac{k+2}{k}\right]^{\mu+\frac{1}{2}} ,$$

where

$$\alpha = 1/137.03.$$

Problem: EVALUATION OF AN INTEGRAL

Origin: Dr. S. Tamor, Physics Division

Participating members of panel: D. W. Whitcombe, C. Perhacs, J. H. Fishel, C. L. Perry

Background and Status: Dr. Tamor wants the integral

$$\theta(A, E) = \pi - x_0(A, E)/A + 2 \int_0^{x_0} \frac{dx\ [A^2 - \phi(x, A, E) - (x_0 - x)\frac{d\phi}{dx}]}{[A^2 - \phi(x)]^{3/2}}$$

where

$$\phi(x, A, E) = \frac{A^2}{E} xe^{-1/x} + x^2,$$

$$\phi(x_0, A, E) = 0,$$

$$x_0 > 0,$$

evaluated for 27 pairs of A and E. He has investigated the behavior of the integrand. Mr. Whitcombe continued these investigations (see, for example, memorandum to Dr. Tamor, ORNL Central Files No. 51-2-10). Mr. Fishel calculated the value of the integrand on IBM equipment. The value of the integral will be found from the IBM tabulation of the integrand by hand machine calculation.

Problem: CHECKING OF GRAPHS FOR HEALTH PHYSICS

Origin: Dr. K. Z. Morgan, Health Physics

Participating Member of Panel: C. Perhacs

Background and Status: The Mathematics Panel participated in checking some calculations from which graphs were drawn. This work is being completed by Health Physics.

Problem: HARMONIC ANALYSIS

Origin: B. S. Borie, Jr., Metallurgy Division

Participating Member of Panel: C. L. Perry

Background and Status: Mr. Borie asked the Mathematics Panel to evaluate 155 Fourier integrals. These integrals were

$$\frac{1}{\pi}\int_0^2 F_i(x) \cos 2\pi\, nx\, dx,$$

$$\frac{1}{\pi}\int_0^2 F_i(x) \sin 2\pi\, nx\, dx,$$

where

$i = 1, 2, \ldots, 5$

$n = 0, 1, \ldots, 15$

$F_i(x)$ are five functions measured experimentally.

The integrals were approximated by a sum, and the sums were computed on the NEPA digital computer. In future evaluations of the above integrals six hundred subdivisions will be used. The sine and cosine functions for $n = 0, 1, \ldots, 10$ will be stored on a long tape for the NEPA digital computer's library. When this has been completed the integrals can be evaluated and the values returned to the Metallurgy Division in two days or less, provided there is not a backlog of problems for the NEPA digital computer.

Problem: DETERMINATION OF ELECTRONIC STOPPING POWERS FOR IONS

Origin: Drs. W. S. Snyder and J. Neufeld, Health Physics

Participating Members of Panel: C. Perhacs, K. A. Pflueger

Background and Status: A nucleon (Z_1, M_1) moving in a medium (Z_2, M_2) undergoes energy losses owing to the effect of the electrons of the medium at the rate

$$\frac{1}{N}\frac{dE}{dx} = \frac{4\pi e^4}{mV^2}(Z_1^{eff})^2 \sum_i f_i \ln \frac{mV^3}{2\pi\nu_i e^2 (Z_1^{eff})} \qquad (11)$$

where

(Z_1^{eff}) denotes an average degree of ionization of the ion,

m = 9.107×10^{-28} (electron rest mass),

e = 4.803×10^{-10} (electronic charge in esu),

f_i = oscillator strength corresponding to the ith electron of the stopping atom (Z_2, N_2).

The Mathematics Panel is calculating the values of $\frac{1}{N}\frac{dE}{dx}$ for the range $0.2\ e/\hbar \leq V \leq 90\ e/\hbar$ and all combinations of Z_1 = 1, 6, 7, 8 and Z_2 = 1, 6, 7, 8. In addition, the Panel will compute the stopping power for hydrogen (Z = 1), nitrogen (Z = 6), carbon (Z = 7), and oxygen (Z = 8) ions moving in tissue.

Problem: DETERMINATION OF ATOMIC STOPPING POWERS OF RECOIL NUCLEI

Origin: Drs. W. S. Snyder and J. Neufeld, Health Physics Division

Participating member of Panel: C. Perlace

Background and Status: The Mathematics Panel has completed calculations similar to those mentioned above (Determination of Electronic Stopping Powers for Ions) for the determination of the energy loss due to the effect of atoms on recoil nuclei. The computations are for the same Z_1 and Z_2 combinations as in the above report for Z_1 in tissue and for the same range in V.

Problem: ANP COMPUTATION

Origin: Dr. N. M. Smith, ANP, Physics Division

Participating members of Panel: R. R. Coveyou, E. A. Forbes, A. R. Carrey, V. M. Gordon

Background and Status: R. R. Coveyou has joined the Mathematics Panel computers at Y-12. He is supervising their computing for the ANP group.

END