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Equations for Bilateral Heat Transfer to Fluids Flowing in Concentric Annuli*

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Bilateral Heat Transfer in Annuli

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ABSTRACT

Theoretical equations have been derived for calculating heat transfer coefficients for fluids flowing through concentric annuli for the following two cases: (A) constant and equal heat fluxes from both walls, and (B) constant, but unequal, heat fluxes from the walls, with equal wall temperatures at a given axial position along annular channel. In the derivations, the conditions of fully-established flow, and independence of physical properties with temperature variation across the flow channel, were assumed.

The only geometrical parameter in this general case is the radius ratio r_2/r_1, and in the study it was varied from 1.0 to 10.0.

INTRODUCTION

This paper is the second in a series of three on the subject of heat transfer to liquid metals flowing in concentric annuli. The first[1] dealt with the case of unilateral heat transfer for the conditions of constant heat flux and fully-established temperature and velocity profiles. This paper presents the derivations of the various equations used in the third paper.[2] These equations, although applied to liquid metals, are applicable to any fluid.

Two cases of bilateral heat transfer are considered: (A) heat transfer through both walls under conditions of constant and equal heat fluxes, and (B) heat transfer through both walls under conditions of constant, but unequal, heat fluxes and equal wall temperatures at a given axial position.

CASE A: EQUAL HEAT FLUXES FROM BOTH WALLS

Figure 1 shows a graphical representation of this case, where r represents radius; t, temperature; and the subscripts 1, 2, m, and t refer to the inner wall, outer wall, point of maximum velocity, and point of minimum temperature, respectively.

We can consider the annulus as being divided into two concentric portions, the imaginary boundary between the two being a cylindrical surface of radius r_t.

The heat transferred from the inner wall will be picked up by the fluid flowing in the inner portion; and that transferred from the outer wall will be picked up by that flowing in the outer portion. For the inner portion, we can write the heat transfer equation

$$q_1 = h_1'(2\pi r_1)(t_1 - t_{b1}) \tag{1}$$

and for the outer portion

$$q_2 = h_2'(2\pi r_2)(t_2 - t_{b2}) \tag{2}$$

where t_{b1} and t_{b2} represent the average or bulk temperatures in the inner and outer portions, respectively, of the annulus.

The objective is to develop two equations, one for evaluating h_1, and the other for h_2, where these coefficients are defined by the equations

$$h_1 = \frac{q_1}{2\pi r_1(t_1 - t_b)} \tag{3}$$

and

$$h_2 = \frac{q_2}{2\pi r_2(t_2 - t_b)} \tag{4}$$

The plan of attack has three parts: (a) determination of r_t; (b) determination of h_1' and h_2'; and (c) determination of h_1 as a function of h_1' and h_2', and h_2 as a function of h_1' and h_2'.

Determination of r_t

A heat transfer-transport balance on the inner portion of the annulus, assuming physical properties independent of temperature, gives

$$q_1 dx = v_{a1} \rho C_p \pi (r_t^2 - r_1^2) dt_{b1} \tag{5}$$

A similar balance on the outer portion gives

$$q_2 dx = v_{a2} \rho C_p \pi (r_2^2 - r_t^2) dt_{b2} \tag{6}$$

Since the heat fluxes on both walls of the annulus are equal,

$$q_1 = \frac{r_1}{r_2} q_2 \tag{7}$$

Then, substituting Eqs. (5) and (6) into (7) gives

$$\frac{r_2}{r_1} v_{a1} (r_t^2 - r_1^2) = v_{a2} (r_2^2 - r_t^2) \tag{8}$$

Since

$$v_{a1} = \frac{2 \int_{r_1}^{r_t} v r dr}{[r_t^2 - r_1^2]}$$

and similarly for v_{a2}, Eq. (8) becomes

$$\frac{r_2}{r_1} \cdot \int_{r_1}^{r_t} vrdr = \int_{r_t}^{r_2} vrdr \qquad (9)$$

This equation can be solved for r_t, as long as v is known as a function of r.

For the case of stream-line flow under isothermal conditions, Lamb's[3] equation for the linear velocity distribution in an annulus is

$$v = \frac{(\Delta p)g_o}{4\mu L} \left[r_1^2 - r^2 + \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)} \ln(r/r_1) \right] \qquad (10)$$

When this equation is substituted into (9), and the resulting equation integrated, we get

$$\frac{r_2}{r_1} \left[c_1(r_t^2 - r_1^2) - \frac{r_t^4 - r_1^4}{2} - c_2\left(\frac{r_t^2 - r_1^2}{2} - r_t^2 \ln r_t + r_1^2 \ln r_1 \right) \right]$$

$$= c_1(r_2^2 - r_t^2) - \frac{r_2^4 - r_t^4}{2} - c_2\left(\frac{r_2^2 - r_t^2}{2} - r_2^2 \ln r_2 + r_t^2 \ln r_t \right) \qquad (11)$$

where,

$$c_1 = r_1^2 - \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)} \ln r_1$$

and

$$c_2 = \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)}$$

Eq. (11) is cumbersome and must be solved for r_t by trial. Calculated results for r_2/r_1 varying between 1.0 and 10.0 are given in Table I.

For the case of turbulent flow, the velocity profile information is not available in equation form; for that reason Eq. (9) must be solved graphically. It must also be solved by trial. However, using the velocity-profile relationships of Rothfus et al.,[4] the present author found that the radius of minimum temperature for turbulent flow, in a given situation, was the same as that for stream-line flow.

Determination of h'_1 and h'_2

Equations for calculating these coefficients are analogous to those for unilateral heat transfer to fluids flowing in concentric annuli. The latter are found in the recent paper of Dwyer and Tu.[5] The modified equations are

$$\frac{1}{h'_1} = \frac{4r_1}{(r_t^2 - r_1^2)^2 v_{a1}^2} \int_{r_1}^{r_t} \left[\int_{r_1}^{r} \frac{\int_r^{r_t} vrdr}{rk_{eff1}} \right] vrdr \qquad (12)$$

and

$$\frac{1}{h'_2} = \frac{4r_2}{(r_2^2 - r_t^2)^2 v_{a2}^2} \int_{r_t}^{r_2} \left[\int_{r}^{r_2} \frac{\int_{r_t}^{r} vrdr}{rk_{eff2}} \right] vrdr \qquad (13)$$

Table I

Values of $\frac{r_t - r_1}{r_2 - r_1}$ for Conditions of Stream-line Flow

and Equal Heat Fluxer from Both Walls

r_2/r_1	$\frac{r_t - r_1}{r_2 - r_1}$
1.0	0.500
2.0	0.415
3.0	0.367
4.0	0.334
5.0	0.310
6.0	0.290
7.0	0.275
8.0	0.262
9.0	0.250
10.0	0.240

Determination of h_1

We shall start with Eq. (3) to develop a generalized equation for calculating h_1. The most difficult term to represent mathematically is t_b, the bulk temperature of the fluid, averaged over the total annulus cross section, at some particular axial location.

If we neglect the effect of radial temperature differences on fluid density, we can write the equation

$$t_b = \frac{t_{b1} v_{a1} [r_t^2 - r_1^2]}{v_a [r_2^2 - r_1^2]} + \frac{t_{b2} v_{a2} [r_2^2 - r_t^2]}{v_a [r_2^2 - r_1^2]} \tag{14}$$

Now, substituting t_{b1} and t_{b2} from Eqs. (1) and (2), respectively, into this equation, gives

$$t_b = \beta \left[t_1 - \frac{q_1}{2\pi r_1 h_1'} \right] + (1-\beta) \left[t_2 - \frac{q_2}{2\pi r_2 h_2'} \right] \tag{15}$$

where

$$\beta = \frac{v_{a1} (r_t^2 - r_1^2)}{v_a (r_2^2 - r_1^2)} = \frac{\int_{r_1}^{r_t} vr dr}{\int_{r_1}^{r_2} vr dr}$$

$$1-\beta = \frac{v_{a2} (r_2^2 - r_t^2)}{v_a (r_2^2 - r_1^2)} = \frac{\int_{r_t}^{r_2} vr dr}{\int_{r_1}^{r_2} vr dr}$$

Next, we can express t_2 in terms of t_1 by writing the equation

$$t_2 = t_1 - (t_1 - t_t) + (t_2 - t_t) \tag{16}$$

This equation can be rewritten as

$$t_2 = t_1 - C_p \rho \frac{dt}{dx} \left[\int_{r_1}^{r_t} \frac{\int_r^{r_t} vrdr}{k_{eff1} r} dr - \int_{r_t}^{r_2} \frac{\int_{r_t}^{r} vrdr}{k_{eff2} r} dr \right] \tag{17}$$

Now, combining Eqs. (15) and (17), gives

$$t_b = \beta(t_1 - \frac{q_1}{2\pi r_1 h_1'}) + (1-\beta) \left\{ t_1 - \frac{q_2}{2\pi r_2 h_2'} - C_p \rho \frac{dt}{dx} \left[\quad \right] \right\} \tag{18}$$

where the missing terms inside the square brackets are the same as those inside the square brackets in Eq. (17).

Eliminating q_2 from Eq. (18) by making use of Eq. (7), simplifying, and rearranging gives

$$t_1 - t_b = \frac{\beta q_1}{2\pi r_1 h_1'} + \frac{(1-\beta) q_1}{2\pi r_1 h_2'} + C_p \rho \frac{dt}{dx} (1-\beta) \left[\quad \right] \tag{19}$$

where again the terms inside the square brackets are the same as before.

Next, substituting Eq. (19) in Eq. (3) gives

$$\frac{1}{h_1} = \frac{\beta}{h_1'} + \frac{1-\beta}{h_2'} + \frac{2\pi r_1 C_p \rho \frac{dt}{dx}(1-\beta)}{q_1}\left[\quad\right] \qquad (20)$$

But,

$$q_1 = \pi v_{a1}(r_t^2 - r_1^2)\rho C_p \frac{dt}{dx} \qquad (5a)$$

Since q_1 is independent of x, dt/dx is constant.

Finally, combining Eqs. (20) and (5a), yields

$$\frac{1}{h_1} = \frac{\beta}{h_1'} + \frac{1-\beta}{h_2'} + \frac{2r_1(1-\beta)}{v_{a1}(r_t^2 - r_1^2)}\left[\int_{r_1}^{r_t} \frac{\int_r^{r_t} vrdr}{k_{eff1}r} dr - \int_{r_t}^{r_2} \frac{\int_{r_t}^{r} vrdr}{k_{eff2}r} dr\right] \qquad (21)$$

This is the equation for calculating values of h_1, remembering that h_1' and h_2' must be first calculated from Eqs. (12) and (13), respectively.

The corresponding equation for calculating values of h_2 is

$$\frac{1}{h_2} = \frac{\beta}{h_1'} + \frac{1-\beta}{h_2'} + \frac{2r_2\beta}{v_{a2}(r_2^2 - r_t^2)}\left[\int_{r_t}^{r_2} \frac{\int_{r_t}^{r} vrdr}{k_{eff2}r} dr - \int_{r_1}^{r_t} \frac{\int_r^{r_t} vrdr}{k_{eff1}r} dr\right] \qquad (22)$$

CASE B: EQUAL WALL TEMPERATURES AT A GIVEN AXIAL POSITION, AND CONSTANT BUT UNEQUAL HEAT FLUXES FROM THE WALLS

Again, the objective is to develop equations for h_1 and h_2, where these coefficients are defined, as before, by Eqs. (3) and (4). Also, the same general procedure will be followed.

Determination of r_t

Since, at a given axial position along the annulus channel, the temperatures at the walls are equal, we can write the equation

$$t_1 - t_t = t_2 - t_t \tag{23}$$

And since

$$t_1 - t_t = \int_{r_1}^{r_t} -(\partial t/\partial r)dr \tag{24}$$

and the radial heat flux, q_r, at any radius r is

$$q_r = -2\pi r\, k_{eff1} \frac{\partial t}{\partial r} = \int_r^{r_t} 2\pi v \rho C_p \frac{dt}{dx}\, rdr \tag{25}$$

for the inner portion of the annulus, we can write

$$t_1 - t_t = \int_{r_1}^{r_t} \frac{\int_r^{r_t} \rho C_p \frac{dt}{dx}\, vrdr}{k_{eff1}\, r}\, dr \tag{26}$$

Similarly, for the outer portion we have

$$t_2 - t_t = \int_{r_t}^{r_2} \frac{\int_{r_t}^{r} C_p \rho \frac{dt}{dx} vrdr}{k_{eff2}\, r} \cdot dr \qquad (27)$$

Combining Eqs. (26) and (27) and simplifying gives

$$\int_{r_1}^{r_t} \frac{\int_{r}^{r_t} vrdr}{k_{eff1}\, r} dr = \int_{r_t}^{r_2} \frac{\int_{r_t}^{r} vrdr}{k_{eff2}\, r} dr \qquad (28)$$

In the case of stream-line flow, this equation can be solved numerically for r_t. Substituting Eq. (10) into it, integrating, and rearranging, gives

$$\alpha \ln \frac{r_2}{r_1} + c_3(r_2^2 - r_1^2) + \frac{r_2^4 - r_1^4}{16} = \frac{c_2}{4}(r_2^2 \ln r_2 - r_1^2 \ln r_1) \qquad (29)$$

where

$$\alpha = \frac{c_1}{2} r_t^2 - \frac{r_t^4}{4} + \frac{c_2}{2} r_t^2 \ln r_t - \frac{c_2 r_t^2}{4}$$

$$c_1 = r_1^2 - c_2 \ln r_1$$

$$c_2 = \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)}$$

$$c_3 = \frac{c_2 - c_1}{4}$$

This equation, like Eq. (11) is unwieldy and must be solved for r_t by trial. Calculated results for r_2/r_1 varying from 1.0 to 10.0 are given in Table II. It will be noticed that for annuli having r_2/r_1 ratios less than 5.0, r_t agrees with r_m to within less than 1%. For such annuli, v_{a1} is approximately equal to v_{a2}, for as r_2/r_1 approaches 1.0, r_t/r_m approaches 1.0. If it is assumed that $v_{a1} = v_{a2}$, then Eq. (28), upon integration, reduces to

$$r_t^2 = \frac{r_2^2 - r_1^2}{\ln(r_2/r_1)^2} \qquad (30)$$

which is the same equation as that for r_m^2, for both stream-line and turbulent flows.

For turbulent flow, Eq. (28) must be solved graphically and by trial. It must be solved graphically owing to the fact that the velocity distributions are not available in equation form. When using the velocity distribution data of Rotbfus et al.,[4] it was found that for r_2/r_1 values up to 6.0 (the highest investigated), r_t was, within the precision of the method of calculation, equal to r_m. Thus, just as in case A discussed above, values of r_t are the same for both stream-line and turbulent flows.

Determination of h_1' and h_2'

The equations for calculating these coefficients are the same as those

Table II

Values of $\frac{r_t - r_1}{r_2 - r_1}$ for Conditions of a) Stream-line Flow, b) Constant but Unequal Heat Fluxes from the Annulus Walls, and c) Equal Wall Temperatures at a Given Axial Position Along Annulus

r_2/r_1	$\frac{r_t - r_1}{r_2 - r_1}$	r_t/r_m
1.0	0.500	1.000
2.0	0.467	0.997
3.0	0.447	0.994
4.0	0.432	0.992
5.0	0.420	0.989
6.0	0.412	0.986
7.0	0.406	0.983
8.0	0.401	0.981
9.0	0.396	0.978
10.0	0.392	0.975

for case A, i.e., Eqs. (12) and (13).

Determination of h_1 and h_2

We can start with Eq. (15), replacing t_2 by t_1

$$t_b = \beta \left[t_1 - \frac{q_1}{2\pi r_1 h_1'}\right] + (1-\beta)\left[t_1 - \frac{q_2}{2\pi r_2 h_2'}\right] \tag{15a}$$

which gives

$$t_1 - t_b = \frac{\beta q_1}{2\pi r_1 h_1'} + \frac{(1-\beta) q_2}{2\pi r_2 h_2'} \tag{31}$$

Combining this equation with Eq. (3) then gives

$$\frac{1}{h_1} = \frac{\beta}{h_1'} + \frac{(1-\beta) q_2 r_1}{h_2' q_1 r_2} \tag{32}$$

But

$$q_1 = 2\pi \int_{r_1}^{r_t} v\rho C_p \frac{dt}{dx}\, r dr \tag{5b}$$

and

$$q_2 = 2\pi \int_{r_t}^{r_2} v\rho C_p \frac{dt}{dx}\, r dr \tag{5c}$$

Finally, substituting Eq. (5b) and (5c) into (32) and simplifying gives

$$\frac{1}{h_1} = \frac{\beta}{h_1'} + \frac{(1-\beta)^2 r_1}{\beta\, h_2'\, r_2} \tag{33}$$

For the outer portion of an annulus, an analogous derivation gives

$$\frac{1}{h_2} = \frac{1 - \beta}{h'_2} + \frac{\beta^2 r_2}{(1-\beta)h'_1 r_1} \tag{34}$$

The equations, which have been derived here, have been used to calculate heat transfer coefficients for liquid metals for the two cases which have been treated. The results, correlated in the form of semi-empirical equations, will be presented in a subsequent paper.[2]

NOMENCLATURE

c_1 = constant, defined in Eq. (11)

c_2 = constant, defined in Eq. (11)

c_3 = constant, defined in Eq. (29)

C_p = specific heat, Btu/(lb-mass)(°F)

g_o = conversion factor, (lb-mass)(ft)/(lb-force)(hr)^2

h_1 = heat transfer coefficient, defined by Eq. (3), Btu/(hr)(ft)^2(°F)

h_2 = heat transfer coefficient, defined by Eq. (4), Btu/(hr)(ft)^2(°F)

h_1' = heat transfer coefficient, defined by Eq. (1), Btu/(hr)(ft)^2(°F)

h_2' = heat transfer coefficient, defined by Eq. (2), Btu/(hr)(ft)^2(°F)

k_{eff1} = $k + k_{e1}$ = effective thermal conductivity of fluid flowing between r_1 and r_t, Btu/(hr)(ft)(°F)

k = molecular thermal conductivity, Btu/(hr)(ft)(°F)

k_{e1} = eddy thermal conductivity of fluid flowing between r_1 and r_t, Btu/(hr)(ft)(°F)

k_{eff2}, k_{e2} = comparable conductivities for fluid flowing between r_t and r_2

L = length of annular conduit, ft

Δp = pressure drop across L, lb-force/ft^2

q_1 = radial heat flow rate from inner wall, per linear foot of annulus, Btu/hr

q_2 = same as q_1, except for outer wall, Btu/hr

r = any value of the radius between r_1 and r_2, ft

r_1 = inner radius of annulus, ft

r_2 = outer radius of annulus, ft

r_m = radius of maximum velocity, ft

r_t = radius of minimum temperature, ft

t = temperature of fluid at any radius r, °F

t_1 = surface temperature of inner wall, °F

t_2 = surface temperature of outer wall, °F

t_b = average bulk temperature of fluid flowing between r_1 and r_2, °F

t_{b1} = average bulk temperature of fluid flowing between r_1 and r_t, °F

t_{b2} = average bulk temperature of fluid flowing between r_t and r_2, °F

t_t = minimum temperature in annular flow channel, °F

v = local linear velocity at radius r, ft/hr

v_a = average linear velocity of fluid flowing between r_1 and r_2, ft/hr

v_{a1} = average linear velocity of fluid flowing between r_1 and r_t, ft/hr

v_{a2} = average linear velocity of fluid flowing between r_t and r_2, ft/hr

x = any axial distance along annular channel, ft

Greek Letters

α = quantity defined in Eq. (32), ft^4

β = fraction of total flow passing between r_1 and r_t

μ = dynamic molecular viscosity, (lb-mass)/(ft)(hr)

ρ = fluid density, (lb-mass)/ft^3

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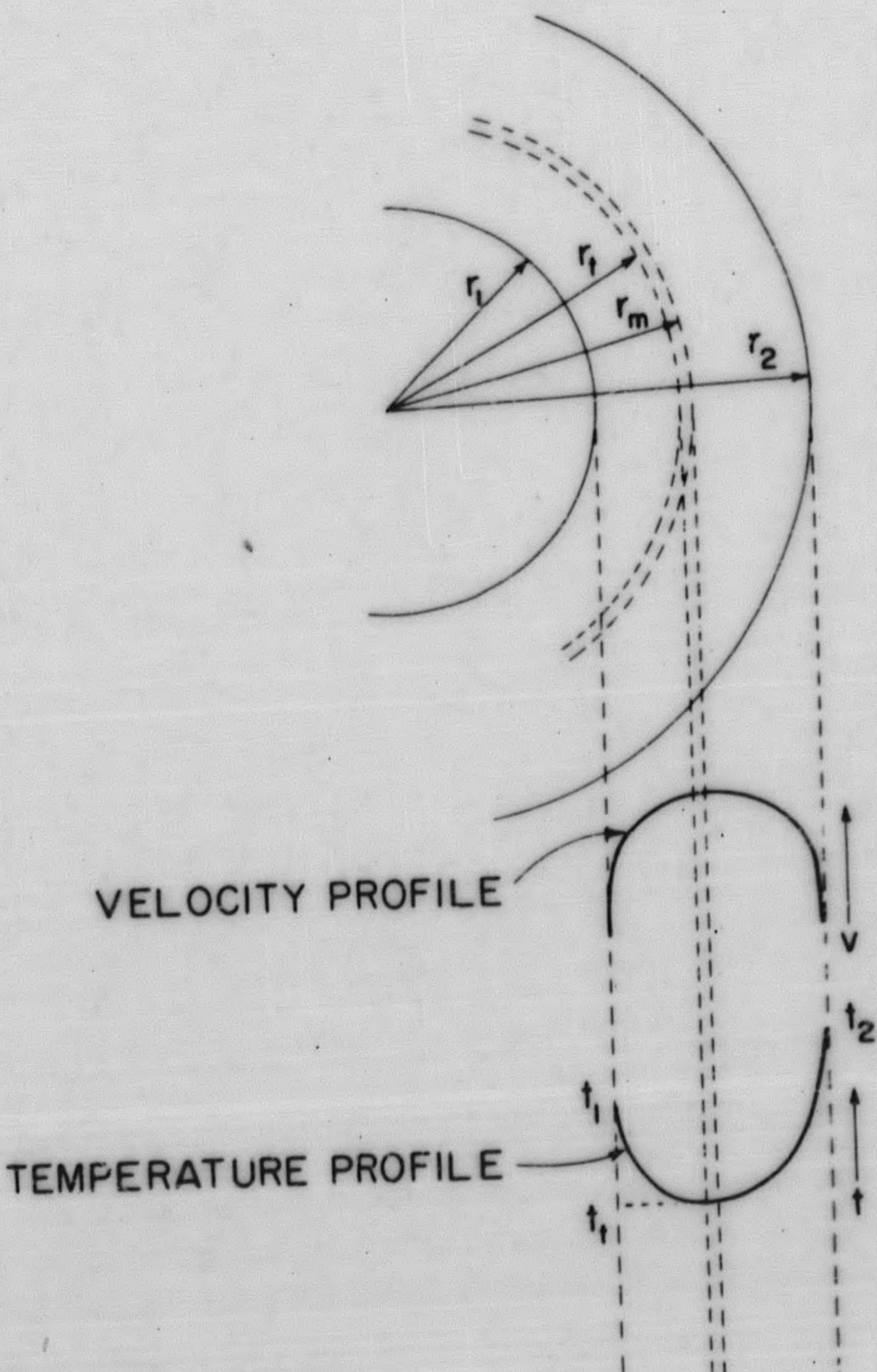
FIGURE CAPTION

Figure 1   Graphical representation for case of equal heat fluxes from both walls of concentric annulus.



r_1
r_t
r_m
r_2
VELOCITY PROFILE
v
t_2
t_1
TEMPERATURE PROFILE
t_t
t