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Eddy Transport in Liquid-Metal Heat Transfer

by

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ABSTRACT

There are certain semi-empirical equations for estimating heat transfer coefficients for liquid metals flowing through regular shapes. These contain the factor $\bar{\psi}$, which is the average, effective value of the ratio of the eddy diffusivity of heat transfer to that for momentum transfer. The main problem in liquid metal heat transfer has been to evaluate this quantity. It is here proposed that this be done by an equation of the type

$$\bar{\psi} = 1 - \frac{\frac{a}{Pr} - c}{(\epsilon_M/\nu)_{\max}^m}$$

There are insufficient experimental results available to evaluate all three constants, but there is good reason to believe that c is small compared to the a/Pr term and can be neglected. On this basis, it is recommended that a equal 1.82 and m , 1.4. With these constants, the above equation is found to give values of $\bar{\psi}$, which, when incorporated in the semi-empirical equations for estimating liquid-metal heat transfer coefficients, bring theoretical predictions and experimental results into good agreement for flow through circular tubes, concentric annuli, and rod bundles.

INTRODUCTION

In the cooling and heating of turbulently flowing fluids, in a circular pipe, heat is transferred in the radial direction by two mechanisms: molecular conduction and eddy transport. Small particles of fluid having velocity components in the y direction, i.e., perpendicular to the wall, transport sensible heat from their points of origin to their points of rest, or disappearance. These eddies also, of course, transport momentum.

In liquid-metal heat transfer analysis, the real problem has been to assess properly the eddy transport contribution in the total rate of heat transfer. In early theoretical studies of liquid-metal heat transfer, the eddy diffusivity of heat transfer was either assumed equal to that for momentum transfer or simply treated as a parameter, whose values were unknown. Neither of these approaches led to theoretical relationships which could be relied upon to predict liquid-metal heat transfer rates with desired accuracy. There have been several attempts (1,2,3,4) to produce a rational method of evaluating the contribution of eddy diffusivity transport in liquid-metal heat transfer; but these have not wholly succeeded, thus far, in reconciling the differences which are usually found between theoretical predictions and experimental results.

This paper represents yet another attempt to reconcile these differences by a still different attack on the eddy diffusivity problem. The purpose is to

develop a practical method of evaluating the eddy diffusivity of heat transfer for liquid metals, so that it can be incorporated into the more or less standard theoretical methods to give heat transfer coefficients which are in reasonable agreement with the experimental facts.

THEORETICAL CONSIDERATIONS

In the following treatment, we shall consider the component of the eddy velocity in the mean-flow, or x, direction, u' ; and that perpendicular to the mean flow, or y direction, v' . Next, we employ Prandtl's (5) mixing-length theory which defines the mixing length by the equation

$$u' = l \frac{du}{dy} \quad (1)$$

where l is the instantaneous mixing length. And next, we write the definition of eddy diffusivity of heat transfer, which is

$$\epsilon_H = f_H l v' \quad (2)$$

where the term f_H represents the ratio of the heat which the eddy actually transported through the distance l compared to what it could have transported if it did not lose any in transit. Since liquid metals have high molecular thermal conductivities, the eddies may lose a good portion of their heat to the surrounding liquid by both conduction and convection. The fraction

lost $(1-f_H)$ will, of course, vary with, among other things, the speed at which the eddy travels and the thermal conductivity of the liquid metal.

For circular conduits, the author estimates that at the low end of the turbulent flow regime, the eddies appear to lose essentially all their heat while in transit, whereas at Peclet numbers above 5000, which would correspond to a Reynolds number of about 250,000 for mercury and 700,000 for sodium, the eddies appear to lose an insignificant fraction of their heat.

For fully-developed flow in a round tube, the heat flux across an imaginary cylindrical surface, coaxial with the axis of the tube, by eddy conduction would be

$$q = f_H l v' C_p \rho \frac{dt}{dr} \quad (3)$$

The lost flux at that particular radius would then be

$$(1-f_H) l v' C_p \rho \frac{dt}{dr} \quad (4)$$

The rate of heat loss by an individual eddy would be

$$h(t_e - t)(\text{surface area of eddy}) \quad (5)$$

where t_e and t are the temperatures of the eddy and surrounding fluid, respectively. It can be assumed that the number of eddies per unit volume of fluid is proportional to the tube Reynolds number, or the eddy Reynolds

number, ϵ_M/ν , raised to some power. Making this assumption, we can combine equations (4) and (5) to give

$$(1-f_H) f v' C_p \rho \frac{dt}{dr} = (\text{a constant}) h (t_e - t) (\epsilon_M/\nu)^n \quad (6)$$

Drawing an analogy between velocity and temperature, as far as eddy properties are concerned, we can write the equation

$$t_e - t = f \frac{dt}{dr} \quad (7)$$

which is analogous to equation (1). Also, we can write the equation

$$h = \frac{k}{b} \quad (8)$$

where k is the molecular conductivity, and b represents the effective thickness of the film, around the eddy, through which the heat must be transferred. Equation (8) assumes that the eddy itself moves in stream-line motion with respect to the surrounding medium. Stream-line conditions were assumed, not because all the eddies move in this fashion, but because, for all practical purposes, it seems logical to assume that those which lose their heat are the ones which are moving more slowly and most probably in the viscous flow regime.

Combining equations (6), (7), and (8) now gives

$$(1-f_H) C_p \rho v' = (\text{a constant}) \frac{k}{b} (\epsilon_M/\nu)^n \quad (9)$$

The definition of Nusselt number implies that the film thickness b is proportional to the particle diameter, and in turbulence, the eddy diameter is generally assumed to be proportional to l .

It is now necessary to write the definition of eddy diffusivity for momentum transfer. It is

$$\epsilon_M = l_M l v' \quad (10)$$

With these relationships, equation (9) can be reduced to

$$l_H = \frac{1}{a(\epsilon_M/\nu)^{n-1} + \frac{(Pr)\psi}{1}} \quad (11)$$

where a = a constant, and

$$\psi = \frac{\epsilon_H}{\epsilon_M} = \frac{l_H}{l_M} \quad (12)$$

An analogous treatment will now be given for the eddy diffusivity of momentum transfer.

The shearing stress, due to momentum transfer by eddy diffusion, along an imaginary cylindrical surface at any radius in turbulent pipe flow is given by the equation

$$\tau = \frac{l_M l v' \rho}{g_0} \frac{du}{dr} \quad (13)$$

The momentum which is lost in transit is then

$$\frac{(1-f_M) \rho v' r}{g_O} \frac{du}{dr} \quad (14)$$

This can be related to the drag on the particle of fluid, which for viscous flow of the surrounding fluid is

$$F = \frac{3\pi d_O \mu v'}{g_O} \quad (15)$$

The shearing stress on the particle becomes proportional to F/d_O^2 , or to

$$\frac{3\pi \mu v'}{g_O d_O} \quad (16)$$

Equations (14) and (16) can now be combined, and the term $(\epsilon_M/\nu)^n$ incorporated as in equation (6), to give the equation

$$1-f_M = (\text{a constant}) \frac{\mu}{\rho v' d_O} (\epsilon_M/\nu)^n \quad (17)$$

Remembering that the actual particle Reynolds number

$$\frac{d_O v' \rho}{\mu} = (\text{a constant}) \frac{\epsilon_M/\nu}{f_M} \quad (18)$$

equation (17) can then be reduced to

$$f_M = \frac{1}{1 + c(\epsilon_M/\nu)^{n-1}} \quad (19)$$

where c equals a constant. Finally, substituting equations (11) and (19) into (12), and simplifying, we get

$$\psi = 1 - \frac{\frac{a}{Pr} - c}{(\epsilon_M/\nu)^m} \quad (20)$$

This equation has three constants, a , c , and m . For a given Prandtl number, however, it simplifies to

$$\psi = \left[1 - \frac{A}{(\epsilon_M/\nu)^m} \right] Pr = \text{constant} \quad (20a)$$

The two constants, A and m , in this equation must be evaluated from experimental data. They can be evaluated from experimental determinations of either ψ or h , the heat transfer coefficient.

There are available three sets of results on the experimental determination of ψ , which might be used to evaluate A and m . They, shown in Figure 1, were all obtained with mercury, were based on measurement of velocity and temperature profiles, and were all supposed to be obtained under conditions of turbulent flow where both velocity and temperature profiles were fully established. Isakoff and Drew (6) made their measurements in a 1.5 in. I.D. stainless steel electrically-heated round tube in the Reynolds number range 36,700 to 373,000. Later Brown et al (7) made

measurements in a 1.61 in. I.D. nickel steam-heated round tube in the Reynolds number range 250,000 to 800,000. And finally, Mizushima and Sasano (4), made measurements in a 25×150 mm stainless steel rectangular channel in the Reynolds number range 10,000 to 130,000. The channel was heated on one side and cooled on the other by flowing water.

It is immediately apparent from Figure 1 that the agreement between the three sets of measurements is poor; but this is not too surprising when one realizes how difficult it is to achieve high accuracy in making such measurements. The coordinates in Figure 1 were chosen for their practical utility, as explained below.

Owing to the great differences which exist between the $\bar{\eta}$ measurements of the different investigators, it was decided that it would be better to evaluate the constants in equation (20a) from h measurements, instead. For this, it is convenient to rewrite the equation as

$$\bar{\eta} = \left[1 - \frac{A'}{[\epsilon_M/\nu]_{\max}^m} \right]_{Pr = \text{constant}} \quad (20b)$$

which correlates the average or effective value of the eddy diffusivity with the maximum value of ϵ_M/ν , the former being easier to use and the latter¹

¹The quantity $[\epsilon_M/\nu]_{av}$ could also have been used.

being easier to obtain. The quantity $\bar{\psi}$ represents the average value of ψ in the flow cross section for use in equations (28), (32), and (33). In this paper it was taken as equal to

$$\frac{1}{2} \int_0^1 \psi(r/R) d(r/R)$$

for flow in circular tubes.

Values of $[\epsilon_M/\nu]_{\max}$ used in this paper are shown plotted against the Reynolds number in Figure 2.

For flow in pipes, values of ϵ_M/ν were calculated from the following equation, which is due to Deissler (2).

$$\epsilon_M/\nu = 0.72 S^* (1 - \frac{y}{R}) (1 - \sqrt{1 - y/R}) \quad (21)$$

where $S^* = R/\nu \sqrt{\tau_{E0}/\rho}$. The quantity ϵ_M/ν reaches a maximum at about $y/R = 0.6$.

For flow in annuli, ϵ_M/ν was obtained by first calculating ϵ_M by use of the standard shear-stress equation

$$\frac{\tau_{E0}}{\rho} = \left[\frac{\mu}{\rho} + \epsilon_M \right] \frac{du}{dr} \quad (22)$$

The velocity distribution results of Rothfus (8) were used, and the method of calculation was the same as that used by Dwyer and Tu (9). Values of $[\epsilon_M/\nu]_{\max}$ were taken for that portion of the annulus lying between r_1 and

r_m , because only the case of heat transfer to (or from) the inner wall of an annulus has been considered here. If heat transfer from the outer wall were considered, then $[\epsilon_M/\nu]_{\max}$ should be for that portion of the annulus lying between r_m and r_2 .

For the case of in-line flow through rod bundles, ϵ_M was also calculated by means of equation (22), using the annulus-velocity-profile results of Rothfus and the method of Dwyer and Tu.

Referring to Figure 2 again, it will be noticed that, for the various geometries, there is a large spread in the values of $[\epsilon_M/\nu]_{\max}$ for a given Reynolds number, with pipes giving the highest values and bundles the lowest. This means that for a given Reynolds or Peclet number, values of $\bar{\psi}$ are greatest for flow in pipes and least for flow through bundles. Thus, in the latter case, considerable error can be introduced by assuming that $\bar{\psi}$ equals unity, as is often done for flow in pipes, particularly at the lower Peclet numbers. It is found that equation (20b) gives a type of $\bar{\psi}$ vs. $[\epsilon_M/\nu]_{\max}$ curve similar to the dashed curve in Figure 3. Moreover, on the basis of available heat transfer results for turbulent flow of liquid metals through pipes, annuli, and rod bundles, it is found that such a curve brings theory and experimental results into very good agreement. After evaluating the constants, equation (20b) becomes

$$\bar{\psi} = 1 - \frac{91.0}{[\epsilon_M/\nu]_{\max}^{1.4}} \quad \text{Pr} = 0.02 \quad (20c)$$

for a Prandtl number of 0.02.

Figure 3 presents a comparison of three theoretical equations for estimating the eddy diffusivity ratio. Jenkins' (1) method calls for solving the following two equations simultaneously:

$$\frac{\epsilon_M}{fv'} = \frac{fv'}{\nu} \left[\frac{2}{15} - \frac{12}{\pi} \cdot \frac{fv'}{\nu} \sum_{n=1}^{n=\infty} \frac{1}{n^6} (1 - e^{-n^2 \pi^2 \nu / fv'}) \right] \quad (23)$$

$$\psi = \text{Pr} \frac{\frac{2}{15} - \frac{12}{\pi} \cdot \frac{fv'}{\alpha} \sum_{n=1}^{n=\infty} \frac{1}{n^6} (1 - e^{-n^2 \pi^2 \alpha / fv'})}{\frac{2}{15} - \frac{12}{\pi} \cdot \frac{fv'}{\nu} \sum_{n=1}^{n=\infty} \frac{1}{n^6} (1 - e^{-n^2 \pi^2 \nu / fv'})} \quad (24)$$

In solving these equations to estimate $\bar{\psi}$, it was assumed that $\bar{\psi} = \psi$ at $y/R = 0.2$ and that

$$[\epsilon_M/\nu]_{y/R=0.2} = 0.6 [\epsilon_M/\nu]_{\max} \quad (25)$$

Both of these assumptions appear to be reasonably well borne out by the available facts. It is interesting to note that equations (23) and (24) have no constants which have been evaluated from experimental data.

Azer and Chao (3) have proposed the equation

$$\psi = \frac{1 + 135 \text{Re}^{-0.45} \exp[-(y/R)^{0.25}]}{1 + 380 \text{Pe}^{-0.58} \exp[-(y/R)^{0.25}]} \quad (26)$$

for estimating values of ψ . Here again $\bar{\psi}$ was assumed equal to ψ at $y/R = 0.2$. The constants in this equation are based on eddy diffusivity measurements made on turbulent flow of air in pipes by Sleicher (10). Equation (26) is represented by curve B in Figure 3.

Finally, Mizushina and Sasano (4), based upon their experimental studies described above, have proposed the following equation:

$$\psi = 0.263(\epsilon_M/\nu)\text{Pr} \left[1 - \exp \frac{-1}{0.242(\epsilon_M/\nu)\text{Pr}} \right] \quad (27)$$

For Figure 3, $\bar{\psi}$ was assumed equal to 0.9ψ at $[\epsilon_M/\nu]_{\max}$, which appears to be generally true, based on the results of Brown et al (7) and Azer and Chao (3) for flow inside pipes. In any case, the uncertainty of the location of curve A in the figure is very small compared to the differences between it and the other curves.

Curves B and C are in surprisingly close agreement, but they are both far from curve A and the now-proposed dashed curve. In fact, all three theoretical curves would obviously fail to reconcile theoretical predictions and supposedly dependable experimental results in liquid-metal heat transfer.

Equation (20c) is shown plotted in Figure 4. For a given Peclet number, it appears that $\bar{\psi}$ is independent of Pr in the fully turbulent regime where $\bar{\psi} \geq 0$. This is tantamount to saying that the constant c in equation (20) is negligible compared to the a/Pr term. Under these conditions, the equation becomes

$$\bar{\psi} = 1 - \frac{1.82}{Pr[\epsilon_M/\nu]_{\max}^{1.4}} \quad (20d)$$

which was then used to calculate the curves for Pr = 0.01 and 0.03 in Figure 4.

Values of the parameter $\bar{\psi}$, evaluated by equation (20d) will now be used to predict new Nu vs. Pe curves for turbulent flow of liquid metals through circular tubes, concentric annuli, and rod bundles. These curves will then be compared with available experimental results.

Care was exercised in choosing experimental results for comparison with the new theoretical curves. No results published prior to 1958 were used. Before that date, results were for the most part obtained on mercury and lead-bismuth alloy, under conditions where "wetting" probably did not exist. This meant that entrained gas and/or particulate matter, which are often present, probably collected at the interface between the flowing metal and heated surface, contributing to low results. Results

obtained in recent years, under improved experimental conditions, tend to give higher coefficients, particularly at the higher Peclet numbers.

In the comparisons below, the experimental results which are used were obtained, for the most part, under wetting conditions or where relatively high contact resistances were not suspected.

FLOW IN CIRCULAR TUBES

The standard semi-theoretical, semi-empirical equation for this case is the so-called Lyon-Martinelli (11) equation,

$$Nu = 7.0 + 0.025(\bar{\psi} Pe)^{0.8} \quad (28)$$

in which the parameter $\bar{\psi}$ is often taken as unity at all values of the Peclet number. Combining this equation with (20d) gives

$$Nu = 7.0 + 0.025 \left| Pe - \frac{1.82 Re}{(\epsilon_M/\nu)_{\max}^{1.4}} \right|^{0.8} \quad (29)$$

which will now be tested against experimental results.

Equation (29) applies when the Peclet number is in excess of 400.

High Peclet Number Range

Figure 5 shows the experimental results of Brown et al (7) and Kirillov et al (12), both for mercury flowing in nickel tubes. For this case, particularly in this range of Peclet numbers, there is little difference between equations (28) and (29). Nevertheless, equation (29) appears to fit the data

points better. The two sets of experimental results are in good agreement. The Hg-Ni system is a "wetting" system. Therefore, there should have been no contact resistances to heat transfer between solid and liquid metals.

Intermediate Peclet Number Range

In this range, three sets of experimental data are available for comparison with equations (28) and (29). Please refer to Figure 6. Due to lack of agreement between the results of the different investigators, a clearcut comparison between theory and experiment is not possible. However, the curve for equation (29) seems to represent a compromise between the differing results. The results of Kirillov et al (12) are believed to be too high. It is difficult to see, for example, how in the Peclet number range 400 to 1000 their results could be consistent with the idea that $\bar{\psi}$, in that range, is equal to unity. On the other hand, the results of Novikov et al (14) would appear to be too low. In each of the three sets of data, the solid-liquid metal system was such as to expect "wetting" to exist.

Low Peclet Number Range

Calculated Nusselt numbers from equations (28) and (29) will now be compared with the limited experimental results available. In the low Peclet number range, two sets of experimental results are compared with the theoretical curves; but here again, as seen in Figure 7, wide disagreement

exists between the experimental results. Pirogov's (15) results are clearly too high, for in the stream-line region they fall about 40% above the correct line. On the other hand, the results of Petukhov and Yushin (16), while correct in the stream-line region, fall lower and lower as the flow gets more and more into the turbulent region. One possible explanation of these results is that, in all probability, "wetting" did not exist and consequently an appreciable contact resistance may have been present, which would have tended to reveal itself more as the flow rate increased.

Again, the presently-proposed curves represent a compromise between the conflicting experimental results.

Comparison of Different Equations for ψ

Referring to Figure 3 again, it will be seen that the equation of Mizushina and Sasano, of the three represented there, comes the closest to the required relationship between $\bar{\psi}$ and $[\epsilon_M/\nu]_{\max}$ as represented by the dashed curve. For that reason, values of $\bar{\psi}$ calculated by that equation were used to solve equation (28) to see how the Nusselt numbers would compare with those calculated from equation (29). The results are shown in Figure 8. Values of $\bar{\psi}$ by the equation of Mizushina and Sasano give results which, over most of the Peclet number range, are quite low. At the very high Peclet numbers, however, they fall above the Lyon-Martinelli curve, and to an unexpectedly high degree.

There is another semi-empirical equation for estimating values of $\bar{\psi}$ which has not been discussed thus far, and that is the equation of Deissler (2), i.e.,

$$\bar{\psi} = B(Pe)[1 - e^{-1/B(Pe)}] \quad (30)$$

This equation was not plotted in Figure 3, because its constant B, has to be determined by arbitrarily setting $\bar{\psi}$ at some particular value for a given value of the Peclet number. The other equations plotted in Figure 3 did not have this restriction. To see how well equation (30) compared with equation (20d), B was evaluated at a point on the curve of equation (20d) which would produce the best agreement between the two. The result is also shown in Figure 8.

When evaluating $\bar{\psi}$ by Deissler's equation, the reduction of the Nusselt number from the Lyon-Martinelli equation varies from about one-half to twice that obtained when evaluating $\bar{\psi}$ by equation (20d).

Moreover, both the equation of Mizushina and Sasano and that of Deissler for evaluating $\bar{\psi}$ produce curves, which, at the lower end of the turbulent regime, come far from merging with the Nu vs. Pe line for streamline flow. Finally, these two equations for evaluating $\bar{\psi}$ would show even wider discrepancies for the cases of flow through annuli and rod bundles;

for, as mentioned earlier, the differences in the Nusselt number between taking $\bar{\psi}$ equal to one and evaluating it properly are much greater in these cases than for the case of flow in pipes.

The equations of Jenkins (1) and Azer and Chao (3), if used to calculate $\bar{\psi}$, would give Nu vs. Pe curves which would fall much lower than the other curves in Figure 8.

Recommended Curves

Figure 9 shows the suggested curves for representing heat transfer conditions in the transition region and in the lower portion of the turbulent region. Curve AB represents equation (29). At point B, which corresponds roughly to the point in Figure 3 where the dashed curve intercepts the abscissa, curve AB branches into a family of curves with Prandtl number as parameter. These curves, in turn, terminate at the viscous-flow line represented by the equation

$$Nu = 48/11 \quad (31)$$

Finally, curves ABC, ABD, ABE, and ABF represent heat transfer rates for liquid metals having Prandtl numbers 0.005, 0.01, 0.02, and 0.03, respectively. These curves were simply drawn in by suitably connecting their established terminal points.

FLOW IN CONCENTRIC ANNULI

Heat transfer to liquid metals flowing under conditions of constant heat flux, fully-established turbulent flow, and heat transfer through the inner wall only, is represented by the following semi-empirical equation of Dwyer and Tu (17).

$$Nu_1 = \alpha_1 + \beta_1 (\bar{\psi} Pe)^{\gamma_1} \quad (32)$$

where

$$\alpha_1 = 4.63 + 0.886y,$$

$$\beta_1 = 0.02154 - 0.000043y,$$

and

$$\gamma_1 = 0.752 + 0.01657y - 0.000883y^2.$$

This equation with $\bar{\psi}$ taken as unity corresponds to the Lyon-Martinelli equation for flow inside circular tubes. It will now be compared with experimental results both ways, i.e., where $\bar{\psi}$ is taken as unity and where it is evaluated by equation (20d). Figure 10 shows the results of Subbotin et al (18) for flow of mercury through an annulus having a y value of 1.09, which closely approximates the geometry of parallel plates. And Figure 11 shows the results of Petrovichev (19) for flow of mercury through annuli having y values of 1.55 and 1.67. These y values are sufficiently close together to be represented by a single theoretical curve. In both of these figures it is seen that

equation (32) with $\bar{\psi}$ evaluated according to equation (20d) represents the experimental results decidedly better than if $\bar{\psi}$ were taken as unity at all values of the Peclet number.

IN-LINE FLOW THROUGH ROD BUNDLES

Dwyer and Tu (9) have recommended the following semi-empirical equation for this case.

$$N_{Nu} = 0.93 + 10.81 \frac{P}{D} - 2.01(P/D)^2 + 0.0252(P/D)^{0.273} (\bar{\psi} Pe)^{0.8}. \quad (33)$$

It is based on the conditions of fully-established turbulent flow, constant heat flux, and equilateral triangular pitch. Following the same procedure as before equation (33) is plotted in Figure 12 two ways: with $\bar{\psi}$ assumed equal to unity and with $\bar{\psi}$ evaluated according to equation (20d). The agreement between the former curve and the experimental results of Friedland et al (20) is very poor, whereas that between the latter curve and the experimental results is excellent. The experimental points shown in Figure 12 are the same as those given in Figure 5 of reference (20), except that slight corrections for variation in heat loss from the test section have been applied. The results were obtained on the central rod of a 13-rod bundle. It was chromium-plated and therefore unwetted by the mercury. The results shown are believed to be reliable for they have since been duplicated

several times (21) by results on other elements, both wetted and unwetted, at the Brookhaven National Laboratory.

It will be noticed that the difference between the two curves in Figure 12 is quite large compared to the difference between the two curves in Figure 5, for example. This is due to the fact that, at a given Reynolds number, the value of $[\epsilon_M/\nu]_{\max}$ is much less for flow through rod bundles than for flow through pipes. This leads to a situation in the lower portion of the turbulent flow regime which is of considerable interest, as far as heat transfer to liquid metals flowing in-line through rod bundles is concerned.

Figure 13 presents what the author believes to be the situation for heat transfer to rod bundles, covering the whole flow spectrum, from stream-line flow to very turbulent flow. Three curves are shown, each for a different P/D ratio. The interesting thing is that the Nusselt number, for a given P/D ratio, remains essentially constant at the stream-line value for a long distance into the turbulent regime, indicating that in this region the eddy contribution to heat transfer is negligible. As the flow rate is increased, a point is reached where eddy transfer begins rapidly to assert itself, and soon thereafter the Nusselt number increases almost proportionately with the 0.5 power of the Peclet number.

It will be noticed that, in the case of flow in tubes, the Nusselt number begins to rise above the stream-line value as soon as the flow becomes turbulent; whereas in the case of in-line flow through bundles, the Nusselt number does not rise above the stream-line value until a Reynolds number of about 25000 (for $Pr = 0.02$) is reached. Again, the reason for this is found in Figure 1, i.e., for a given Reynolds number, values of ϵ_M/ν for pipes are about a factor of four greater than those for bundles. In other words, for bundles, below $Re = 25000$, the eddies do not travel fast enough to prevent losing essentially all their sensible heat while in transit.

ALTERNATIVE EQUATIONS FOR $\bar{\psi}$

By making somewhat different assumptions, it is possible to derive a number of equations, besides (20) and (20a), for evaluating the quantity ψ , all based upon the basic idea used in deriving equation (20).

For example, if the $(\epsilon_M/\nu)^n$ term is omitted from equations (6) and (9) which is simply to assume that the heat and momentum undelivered by the eddy particle is proportional to the heat and momentum, respectively, lost by a representative single particle in transit, then we get

$$\psi = 1 - \frac{(a \text{ constant}) + \frac{a \text{ constant}}{Pr}}{\epsilon_M/\nu} \quad (34)$$

which is the same as equation (20), except there is no exponent on the ϵ_M/ν term.

Again, if in deriving equation (34), we had assumed that ϵ_M is proportional to bv' , we would have obtained the equation

$$\psi = \frac{1 - \frac{\text{a constant}}{\text{Pr}(\epsilon_M/\nu)}}{1 - \frac{\text{a constant}}{\epsilon_M/\nu}} \quad (35)$$

If we had assumed a model where the flow was turbulent rather than stream-line, instead of equation (8) for h and (15) for F , we would have to use their turbulent counterparts. For h , we can use the following equation, proposed by Ranz and Marshall (22)

$$\frac{hd_o}{k} = 2.0 + 0.8(d_o v'/\nu)^{1/2} (C_p \mu/k)^{1/3} \quad (36)$$

and for F , the equation would be

$$F = \frac{(\text{a constant}) \rho (v')^2 d_o^2}{g_o} \quad (37)$$

Using these equations, and the same general procedure employed in deriving equation (20), a comparable relationship for ψ , on the basis of a turbulent model, can be obtained. This relationship, however, is complex. It can be simplified by making assumptions similar to those made in obtaining equations (34) and (35). For example, if it is assumed that the particle Reynolds number is proportional to ϵ_M/ν , one obtains

$$\bar{\psi} = \frac{1 - \frac{(\text{a constant})(\epsilon_M/\nu)^{n-1}}{\text{Pr}} - \frac{(\text{a constant})(\epsilon_M/\nu)^{n+0.5}}{\text{Pr}^{2/3}}}{1 - (\text{a constant})(\epsilon_M/\nu)^n} \quad (38)$$

This equation, having four constants, could be made to fit almost any curve, and therefore it does not prove very much. Making the further simplifying assumption of dropping the $(\epsilon_M/\nu)^n$ term from equation (6), as was done to obtain equation (34), we end up with

$$\bar{\psi} = (\text{a constant}) - \frac{(\text{a constant}) - (\text{a constant})\text{Pr}^{1/2}[\epsilon_M/\nu]^{1/3}}{\text{Pr}[\epsilon_M/\nu]} \quad (39)$$

It was subsequently found that the first constant in this equation was so close to unity that it was set at that, the two constants in the numerator then being evaluated from experimental data.

Equation (34) actually has two constants, but one of them was dropped as was done in equation (20). This was done for two reasons: first, there are not sufficient experimental results available for different values of the Prandtl number to evaluate the second constant, and second, this constant is most probably negligible compared to the other term in the numerator of the fraction. This latter assumption is consistent with the idea that in the higher part of the Peclet number range the Nusselt number is independent of Prandtl number, for liquid metals.

The constants for equations (34), (35), and (39) were evaluated from the dashed curve in Figure 3, the final equations becoming

$$\bar{\psi} = 1 - \frac{0.38}{\text{Pr}[\epsilon_M/\nu]_{\max}} \quad (34a)$$

$$\bar{\psi} = \frac{1 - \frac{0.524}{\text{Pr}[\epsilon_M/\nu]_{\max}}}{1 - \frac{12.75}{[\epsilon_M/\nu]_{\max}}} \quad (35a)$$

and

$$\bar{\psi} = 1 - \frac{0.700 - 0.157 \text{Pr}^{1/2} [\epsilon_M/\nu]_{\max}^{1/3}}{\text{Pr}[\epsilon_M/\nu]_{\max}} \quad (39a)$$

As before, $\bar{\psi}$ and $[\epsilon_M/\nu]_{\max}$ were used instead of ψ and ϵ_M/ν , when determining the constants. The three equations are plotted in Figures 14, 15, and 16, respectively.

The striking similarity of the curves for these three equations, as well as those for equation (20d) shown in Figure 4, is immediately apparent. This is so, even for equation (34a) which has only a single constant. The similarity of these equations, as far as their curves are concerned, shows that any of them could be used to evaluate $\bar{\psi}$, without causing much displacement in the final Nu vs. Pe curves; which tends to give added support to the theoretical approach which has been used.

Another point is worthy of mention. The spacings of the three curves in Figures 15 and 16 are such that they indicate the Nusselt number to be independent of the Peclet number in the higher Peclet number range. This was, in part, the justification for dropping one of the two constants in equations (20) and (34).

Equation (39a), with respect to the other three equations, is unique in that it gives values of $\bar{\psi}$ which can exceed unity, but only slightly. There is no theoretical reason why $\bar{\psi}$ cannot exceed unity. In fact, this author suspects that it may well do so, at very high Reynolds numbers. This then raises a question concerning the assumption of stream-line flow conditions

at the boundary layers surrounding the individual eddy particles, which was made in deriving equation (20). It may well be that this assumption is not entirely valid at very high degrees of turbulence. If so, an equation such as (20d) would be the one to use in the lower portion of the turbulent regime, and one such as (39a) in the upper portion.

In liquid metal heat transfer, as the flow rate is increased, a point is reached where essentially all of the heat in convective transfer would be transferred by eddy transport, as is the case with ordinary fluids in the turbulent flow regime. Since values of $\bar{\psi}$ for such fluids appear to be slightly above unity at high flow rates, there is reason to expect that $\bar{\psi}$ values for liquid metals would also be slightly above unity. However, the departures from unity would be small and therefore have little effect on the Nu vs. Pe curve. Before such fine distinctions are possible, more experimental results of high accuracy are required.

CONCLUSIONS

Based upon the present study, the following are recommended for estimating heat transfer rates for liquid metals flowing under conditions where the heat flux is constant and the flow is turbulent and fully established.

Flow in circular tubes:

Above $Pe = 400$, equation (28) [Lyon-Martinelli (11)].

Below $Pe = 400$, see Figure 9.

Flow in concentric annuli (heat transfer from inner wall only):

Above $Pe = 400$, Equation (32) [Dwyer and Tu (17)].

Flow through rod bundles (equilateral triangular spacing, and not closely packed:

Above $Pe = 1000$, Equation (33) [Dwyer and Tu (9)].

Below $Pe = 1000$, see Figure 13.

In each of the above equations, $\bar{\psi}$ should be evaluated by equation (20d). values of $[\epsilon_M/\nu]_{\max}$ for the various geometries being given in Figure 2. For circular tubes, equation (29) combines equations (20d) and (28).

The fact that $\bar{\psi}$, for use in equations (28), (32), and (33), can be evaluated by a single relationship is taken as an indication of its fundamental soundness.

The constants in equations (20d), (34a), (35a), and (39a) have been evaluated with the most dependable experimental results available at this time. As additional and more accurate results become available, these constants can be expected to change. However, they can change considerably, as seen by comparing equations (20d) and (34a), without greatly affecting the final results, as seen by comparing Figures 4 and 14.

NOTATION

a = a constant in Eq. (11)

A = a constant in Eq. (20a)

A' = a constant in Eq. (20b)

b = effective film thickness around eddy particle, ft

B = a constant in Eq. (30)

c = a constant in Eq. (19)

C_p = specific heat, Btu/(lb-mass)(°F)

d_o = diameter of eddy particle, ft

D = outside diameter of rod or tube, ft

D_e = 4(cross-sectional flow area)/wetted perimeter, ft

f_H = fraction of heat energy, originally possessed by an eddy particle,
which is delivered at point where particle comes to rest with
respect to rest of fluid

f_M = same as f_H except for momentum transfer

F = drag force on eddy particle, lb force

g_o = conversion factor, (lb-mass)(ft)/(lb-force)(hr)²

h = heat transfer coefficient, Btu/(hr)(ft)²(°F)

k = thermal conductivity, Btu/(hr)(ft)(°F)

l = eddy mixing length, defined by Eq. (1), ft

m = exponent in Eq. (20)

n = exponent in Eq. (6)

$Nu = hD_e/k$ = Nusselt number, dimensionless

Nu_1 = Nusselt number for heat transfer to liquid metals flowing through concentric annuli, where heat is transferred from the inner wall only, dimensionless

P = rod pitch, or distance between rod centers in bundle, ft

$Pe = \frac{D_e v_{av} \rho C_p}{k}$ = Peclet number, dimensionless

$Pr = C_p \mu / k$ = Prandtl number, dimensionless

q = heat flux, Btu/(hr)(ft)²

r = radius, ft

r_1 = inner radius of concentric annulus, ft

r_2 = outer radius of concentric annulus, ft

r_m = radius of maximum velocity in concentric annulus, ft

R = radius of circular tube, ft

$Re = \frac{D_e v_a \rho}{\mu}$ = Reynolds number, dimensionless

$S^* = (r/\nu)(R/\rho)^{1/2}$ = tube-radius parameter, dimensionless

t = temperature, °F

t_e = temperature of eddy particle, °F

u = linear velocity at radius r in x direction, ft/hr

u' = measure of turbulence intensity, or root mean square of difference between particle velocity and that of the rest of the fluid, in the x direction, ft/hr

v' = same as u' except in y direction, ft/hr

v_{av} = average linear velocity through pipe, annulus, or rod bundle, ft/hr

x = axial distance, ft

y = radial distance measured from tube wall, ft. Also with reference to concentric annuli, $y = r_2/r_1$ dimensionless

Greek Letters

$\alpha_1, \beta_1, \gamma_1$ = quantities defined in Eq. (32) dimensionless

ϵ_H = eddy diffusivity for heat transfer, ft²/hr

ϵ_M = eddy diffusivity for momentum transfer, ft²/hr

μ = viscosity, lb-mass/(ft)(hr)

ν = kinematic viscosity, ft²/hr

ρ = density, lb-mass/ft³

τ = shear stress at radius r , lb-force/ft²

$\psi = \epsilon_H/\epsilon_M$, dimensionless

$\bar{\psi}$ = average value of ψ for use in Eqs. (28), (32), and (33), dimensionless

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FIGURE CAPTIONS

- Figure 2 Values of $[\epsilon_M/\nu]_{\max}$ for fully-established turbulent flow of liquid metals through circular tubes, concentric annuli, and rod bundles with equilateral triangular spacing.
- Figure 1 Comparison of the results of the experimental determination of the eddy diffusivity ratio by various investigators, showing the effect of turbulence on $\bar{\psi}$.
- Figure 3 Comparisons of theoretical equations for estimating the eddy diffusivity ratio for liquid metals.
- Figure 4 Graphical representation of Equation (20d).
- Figure 5 Heat transfer to mercury flowing in round tubes in the high Peclet number range, under condition of constant heat flux. Comparison between semi-empirical equations and experimental results.
- Figure 6 Heat transfer to liquid metals flowing in round tubes in the intermediate-Peclet-number range under condition of constant heat flux. Comparison between semi-empirical equations and experimental results.
- Figure 7 Heat transfer to liquid metals flowing in round tubes in the low-Peclet-number range under condition of constant heat flux.

Figure 7
(cont.) Comparison between semi-empirical equations and experimental results.

Figure 8 Comparison of semi-empirical equations for predicting heat transfer rates to liquid metals flowing in circular tubes under conditions of constant heat flux and fully-established flow.

Figure 9 Recommended curves for predicting heat transfer rates to liquid metals flowing in circular tubes under conditions of constant heat flux and fully-established flow.

Figure 10 Heat transfer rates to mercury flowing in a concentric annulus under conditions of constant heat flux and heat transfer through inner wall only. Comparison between semi-empirical equations and experimental results of Subbotin et al. (18).

Figure 11 Heat transfer rates to mercury flowing in concentric annuli under conditions of constant heat flux and heat transfer through inner wall only. Comparison between semi-empirical equations and experimental results of Petrovichev (19).

Figure 12 Heat transfer to mercury flowing in-line through unbaffled rod bundle under conditions of constant heat flux and fully-established flow. Comparison between semi-empirical equations and experimental results of Friedland et al. (20) for mercury.

Figure 13 Recommended curves for predicting heat transfer rates to liquid metals flowing in-line through unbaffled rod bundles under constant heat flux and fully-established flow.

Figure 14 Graphical representation of Equation (34a).

Figure 15 Graphical representation of Equation (35a).

Figure 16 Graphical representation of Equation (36a).

Fig 1

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Fig 4

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Fig 8
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Fig 13

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Fig 15

BNI N/29 N/2 8-239-6.7

Fig. 16

END

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