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THE ANGULAR DEPENDENCE OF THERMAL NEUTRON SPECTRA IN LATTICES*

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ABSTRACT

Techniques are developed for computing the angular thermal flux spectrum emerging from a lattice. These methods are applied to water lattices and it is found that the computed spectra are sensitive to the geometrical description of the lattice, but insensitive to the anisotropic scattering by water. Comparisons with measured spectra indicate that corrections for gradients and the finite size of the experimental lattice are important.

INTRODUCTION

The thermal flux spectrum in heterogeneous lattices has been studied by Takahashi (1) using first flight collision probability theory developed by Chernick (2) for the calculation of resonance absorption in lattices. Honeck (3,4) has used a more complete form of the integral transport equation to compute spectra in lattices. Both of these studies were concerned with the scalar flux spectrum which determines reaction rates. The results of these calculations have been successfully compared with experimental foil activations in lattices. The directional spectra emerging from a water lattice have been measured by Campbell and Poole (5) and Mostovoi et. al. (6). Campbell has measured the spectrum in a direction both parallel and perpendicular to the rods in the lattice and observes an anisotropy in the spectrum. The purpose of this paper is to relate the directional spectra measured in the lattice to the scalar flux in the lattice.

In the first section of the paper we formulate general methods for computing the spectra emerging from a lattice and

show that a very simple technique can be used if the anisotropic scattering of neutrons can be ignored. The methods are applied to a simple slab lattice and estimates of the spectral anisotropy are obtained. It is further shown that this anisotropy vanishes in the homogeneous limit. We next investigate the validity of ignoring anisotropic scattering and find that for water lattices the spectra are insensitive to anisotropic scattering. The spectra emerging from equivalent slab and cylindrical lattices are then compared and we find that the spectra are sensitive to the geometrical configuration of the lattice. The spectra are found to be much more sensitive to geometry than to anisotropic scattering and further justify the isotropic scattering approximation.

Finally, the preceding methods applied to an infinite lattice are used to predict the experimental spectra measured by Campbell and Mostovoi. The agreement is generally poor indicating that the measured spectra are not characteristic of an infinite lattice.

GENERAL THEORY

The directional spectrum in or emerging from a lattice is given by the transport equation.

$$\Omega \cdot \nabla \phi(\underline{x}, E, \Omega) + \Gamma(\underline{x}, E, \Omega) \phi(\underline{x}, E, \Omega) = H(\underline{x}, E, \Omega) \quad (1)$$

$$H(\underline{x}, E, \Omega) = S(\underline{x}, E, \Omega) + \int_0^\infty d\Omega' \int_0^\infty dE' \Sigma_s(\underline{x}, E' \rightarrow E, \mu_0) \phi(\underline{x}, E', \Omega')$$

where $(\underline{x}, E, \Omega)$ are the coordinates of a neutron at $\underline{x}$ with energy E moving in the direction Ω . ϕ is the neutron flux, S is an external source, Γ is the total cross section, Σ_s is the scattering kernel, and μ_0 is the cosine of the angle between Ω and Ω' . The quantity H is called the emission density and is the sum of all neutrons introduced into the system externally and as a result of a collision. Eqn. (1) could be solved by expanding the angular dependent quantities in spherical harmonics and solving the resulting integro-differential equations numerically. This technique is called the P_N method and is useful if the expansion coefficients of the flux converge rapidly. The rapid convergence of the expansion coefficients of the flux is simply another way to say that the angular flux is well behaved and can be represented by a few polynomials in angle.

We will show that the coefficients do not converge rapidly so that we will use an alternate technique.

In making the spherical harmonics expansion of eqn. (1), it is necessary to also expand H and Σ_s and use the addition theorem of Legendre polynomials. The result is that the isotropic component of H depends only on the isotropic components of S , Σ_s , and ϕ ; the first anisotropic component of H depends only on the first anisotropic components of S , Σ_s , and ϕ ; and so forth. Thus, if ϕ converges, H will converge more rapidly since ϕ is folded in with the rapidly convergent Σ_s . We will show later that a very good approximation is obtained by using only the isotropic component of H for water lattices. If H is known (or can be easily computed) then the complete angular flux is given by the integral form of eqn. (1).

$$\phi(\underline{r}, E, \underline{\Omega}) = \int_0^s ds e^{-\int_0^s \Sigma(\underline{r}-s'\underline{\Omega}, E) ds'} H(\underline{r}-s\underline{\Omega}, E, \underline{\Omega}) \quad (2)$$

This equation states that the flux in direction $\underline{\Omega}$ is obtained by looking backward along the line $\underline{\Omega}$ and summing the product of the sources along the line and the probability that the neutron survives the flight along s . If we further assume that the scattering

is isotropic, then

$$\psi(\underline{r}, E, \underline{\Omega}) = \int_0^\infty ds e^{-\int_0^s \Sigma(\underline{r}-\underline{s}', \underline{\Omega}, E) ds'} H_0(\underline{r}-s\underline{\Omega}, E) \quad (3)$$

where H_0 is the isotropic component of H . The usefulness of eqn. (3) is that, regardless of the complexity of the lattice, if we can map the isotropic sources in the lattice, we can compute the complete angular flux emerging from any beam tube inserted in the lattice. The presence of the beam tube is assumed not to perturb the flux in the lattice.

Eqn. (3) can be put in another form if we define

$$Y = Y(\underline{r}, E, \underline{\Omega}) = \int_0^\infty ds \Sigma(\underline{r}-s\underline{\Omega}, E) \quad (4)$$

$$\psi(\underline{r}, E, \underline{\Omega}) = \int_0^\infty dy e^{-y} \psi_0(y, E) \quad (5)$$

where

$$\psi_0(y, E) = H_0(y, E) / \Sigma(y, E). \quad (5)$$

The quantity y is the optical depth measured backward along the flight path from x to the source at $x=s_0$. The quantity τ is the emission density divided by the total cross section. The variable y and the function τ are the two natural functions which determine the emergent spectra. The definition of τ is (again for isotropic scattering denoted by subscript zero)

$$\tau_0(y, E) = \left[S_0(y, E) + \int_0^\infty dE' \Sigma_s(y, E' \rightarrow E) \phi_0(y, E') \right] / \Sigma(y, E). \quad (7)$$

In a metal fuel region (denoted by subscript F) the slowing down source can be taken to be zero and the scattering kernel can be taken as a delta function (no energy transfer).

$$\tau_{0F}(y, E) = \frac{\Sigma_f(y, E)}{\Sigma(y, E)} \phi_0(y, E) = P_{SF}(y, E) \phi(y, E) \quad (8)$$

The quantity P_{SF} is the probability of scattering by the fuel and is equal to the scattering cross section divided by the total cross section.

If τ does not vary in the direction y , the emergent flux is just equal to τ . The flux measured parallel to a slab lattice or a rod lattice in the fuel rod is proportional to the scalar flux in the fuel times $P_{SF}(E)$.

It is also evident from eqn.(5) that if v_0 is uniform throughout the lattice, the emergent flux is isotropic. In general, v in the moderator is greater than in the fuel since the flux in the moderator is greater than in the fuel, and P_{SF} is less than one while the factors operating on the moderator flux are, on the average, greater than one. If we look parallel to the lattice at the center of the fuel we will observe the smallest flux and at the center of the moderator the largest flux. At any other position or angle we will observe a flux intermediate to these extremes.

We will now consider the special case of a slab lattice shown in Figure 1 for which eqn.(2) becomes

$$\begin{aligned} \phi(x, E, \mu) &= \int_{-x}^x \frac{dx'}{\mu} e^{-(\lambda - \lambda')/\mu} H(x', E, \mu) & \mu \geq 0 \\ &= \int_x^0 \frac{dx'}{-\mu} e^{(\lambda - \lambda')/\mu} H(x', E, \mu) & \mu < 0 \end{aligned} \quad (9)$$

where

$$\lambda = \lambda(x, E) = \int_0^x dx \Sigma(x, E). \quad (10)$$

We need consider only the case $\mu \geq 0$ because of the lattice symmetry. In terms of the quantities y and v , eqn.(9) becomes

$$\dot{\epsilon}(y, E, \mu) = \int_{-b}^y dy' e^{-(y-y')} \nu(y', E, \mu) \quad (11)$$

$$y = \lambda/\mu . \quad (12)$$

The integral over y' can be replaced by an integral from 0 to $y_0 = \lambda(a+b, E)/\mu$ and a sum over the image cells lying to the left of the reference cell. This summation can be done in closed form and eqn. (11) becomes

$$\dot{\epsilon}(y, E, \mu) = \frac{1}{1-e^{-y_0}} \left\{ \int_0^y dy' e^{-(y-y')} \nu(y, E, \mu) + \int_y^{y_0} dy' e^{-(y-y'+y_0)} \nu(y, E, \mu) \right\} . \quad (13)$$

The total collision density in the cell can be obtained by integrating (13) with respect to the variable y .

$$\int_0^{y_0} dy \dot{\epsilon}(y, E, \mu) = \int_c^{y_0} dy \nu(y, E, \mu) \quad (14)$$

or

$$\int_0^{a+b} dx \Sigma(x, E) \dot{\epsilon}(x, E, \mu) = \int_0^{a+b} dx H(x, E, \mu) \quad (15)$$

The integration of the terms on the right hand side of (13) is straight forward if the strip of integration is first changed. The result of eqn. (15) is that if H is isotropic then the total collision density is isotropic. Unless the total cross section is independent of x , the total flux in the cell will not be isotropic.

We will make an estimate of the spatial behavior of ψ in order to examine the angular flux as the lattice approaches the homogeneous limit.

$$\begin{aligned}\psi(x, E) &= \psi_F(E) & 0 \leq x < a \\ &= \psi_M(E) & a \leq x \leq a+b\end{aligned}\tag{16}$$

Where F and M denote the fuel and moderator respectively.

Substitution of this ψ into eqn. (13) leads to

$$\begin{aligned}\psi_F(y, E, \mu) &= \psi_F(E) + (\psi_M(E) - \psi_F(E)) \left[\frac{1 - e^{-y_b}}{1 - e^{-y_0}} \right] e^{-y} \\ \psi_M(y, E, \mu) &= \psi_M(E) - (\psi_M(E) - \psi_F(E)) \left[\frac{e^{y_a} - 1}{1 - e^{-y_0}} \right] e^{-y}\end{aligned}\tag{17}$$

where $y_a = \Sigma_F(E)a/\mu$, and $y_b = \Sigma_M(E)b/\mu$. It is clear from (17) that the anisotropy of the spectrum is due to the gradients of ν .

The average flux in each region is given by

$$\bar{\nu}_M(E, \mu) = \nu_M - (\nu_M - \nu_F) \frac{1}{y_b} F \quad (18)$$

$$\bar{\nu}_F(E, \mu) = \nu_F + (\nu_M - \nu_F) \frac{1}{y_a} F \quad (19)$$

$$F = (1 - e^{-\theta \Sigma_F a / \mu}) (1 - e^{-\Sigma_F a / \mu}) / (1 - e^{-(\theta+1) \Sigma_F a / \mu}) \quad (20)$$

$$\theta = \frac{\Sigma_M b}{\Sigma_F a} \quad (21)$$

We will investigate some limiting case of eqn. (18).

First, let μ tend to zero keeping a and θ fixed and non-zero,

$(\mu/\Sigma_F a \rightarrow 0)$.

$$\bar{\nu}_M(E, \mu) \rightarrow \nu_M - \frac{\mu}{\Sigma_F a} \frac{1}{\theta} (\nu_M - \nu_F) \quad (22)$$

$$\bar{\nu}_F(E, \mu) \rightarrow \nu_F + \frac{\mu}{\Sigma_F a} (\nu_M - \nu_F) \quad (23)$$

These equations predict a linear variation of the angular distri-

bution near $\mu = 0$. A second limiting case is obtained by letting the lattice dimensions tend to zero faster than μ tends to zero ($\Sigma_F a / \mu \rightarrow 0$).

$$\bar{\phi}_M(E, \mu) \rightarrow \phi_H(E) \quad (24)$$

$$\bar{\phi}_F(E, \mu) \rightarrow \phi_H(E) \quad (25)$$

where $\phi_H(E)$ is the isotropic flux in an infinite homogeneous medium

$$\phi_H(E) = \frac{v_F + \theta v_M}{1 + \theta} \quad (26)$$

From these limiting cases we can write the inequality

$$v_M(E) \geq \bar{\phi}_M(E, \mu) \geq \phi_H(E) \geq \bar{\phi}_F(E, \mu) \geq v_F(E) \quad (27)$$

This inequality and equations (18) and (19) are illustrated in Figure 2 for the case $\theta=4$.

It is apparent from Figure 2 that a polynomial representation for $\phi(\mu)$ is poor for $\Sigma_F a$ small. The P_n expansion would converge slowly and should not be used to predict the angular

distribution. If the P_n must be used it should be used to predict $\psi(E)$ and then eqn. (2) used to compute $\phi(E, \mu)$. The poor convergence of the P_n expansion has also been discussed by Chernick (8) and Davison (9).

A possible application of these results is the following. Suppose that we wish to measure the infinite medium spectrum in a poisoned solid moderator. We construct a block of material by alternating thin layers of moderator and absorber. We measure foil activation in each layer and find that the flux is uniform throughout the block. We then assume that from a neutron standpoint the assembly is homogeneous and the flux is isotropic and given by (26). If we measure the spectrum at any $\mu \geq \Sigma_F a$, then from (24) we have measured ϕ_H . Had we measured the spectrum at $\mu=0$ (which would be experimentally easier) we would measure

$$\bar{\phi}_H(E) = \psi_H = \phi_H \frac{1}{\bar{\phi}} [0 + 1 - P_{SF}] \quad (28)$$

$$\bar{\phi}_F(E) = \psi_F = P_{SF} \phi_H \quad (29)$$

if our detector were very small. If our detector extends over many cells, we would measure the volume average of eqns. (28) and (29).

$$\bar{\epsilon}_{\text{cell av.}}(E) = \epsilon_H \left[1 + \left(\frac{a}{a+b} \right) \left(\frac{\epsilon_F - \epsilon_M}{\epsilon_M} \right) (1 - p_{SF}) \right] \quad (30)$$

The error term does not vanish as a tends to zero (b/a fixed).

This result is true for all $\mu < \epsilon_F a$.

METHODS FOR COMPUTING $\psi(E)$

In this section we will review the two methods we use to compute the neutron flux in a lattice. Corngold (7) has expanded in Legendre polynomials both the space and angular variation of the flux in a slab-lattice. The resulting set of coupled integral equations can then be truncated and solved numerically. Another method, and the one we will use, is to again expand the angular flux in Legendre polynomials but represent the spatial variation with step functions.

In order to investigate the importance of anisotropic scattering, we retain three anisotropic scattering terms in the Legendre expansion, but restrict the spatial variation to be flat in each half region. This approximation is roughly equivalent to using the $P_0(x)$ spatial behavior for the even angular components and the $P_1(x)$ spatial behavior for the odd angular components in the Corngold development. The resulting set of eight coupled integral equations are then solved numerically.

The second method is to use only isotropic scattering and many step functions to describe spatial behavior. This method leads to a single integral equation in both space and energy and has been discussed by Honeck (3). The integral equation is solved numerically using the THERMPS code (10).

ISOTROPY OF THE BIRTH RATE DENSITY, H

We have indicated that if H is isotropic, a very simple equation can be used to compute the angular distribution of the emergent flux. We will now show that H is nearly isotropic for the water lattices considered here. The calculation method used is the first one described in the previous section where three Legendre scattering moments are included. The scattering kernel developed by Nelkin (11) is used for the water. The fuel is assumed to scatter isotropically with no energy transfer. The slab lattice considered is equivalent (same fuel surface to volume ratio and same water to metal ratio) to the 1.3 C_0 uranium rod lattice investigated by Campbell. The parameters used are listed in Table I.

The spectra both parallel and perpendicular to the lattice in the fuel and water are shown in Figure 3. The solid curves were obtained using only isotropic scattering. The dashed curves were obtained using three anisotropic scattering terms. The scattering anisotropy produces a small change (5%) in the perpendicular flux and less than 1% change in the parallel flux. The qualitative behavior of the flux is completely predicted by using

only isotropic scattering. Throughout the remainder of this paper we will then use only isotropic scattering.

These conclusions are valid only for the infinite lattice where the high energy flux is uniform across the lattice. The anisotropic flux then vanishes exponentially at high energy. In any lattice where spatial gradients exist at all energies, the anisotropic flux has a $1/E$ behavior at high energies and introduces an anisotropic source into the thermal group.

GEOMETRICAL CONSIDERATIONS

The ~~THERMOS~~ code was used to compute the spatial distribution of $\nu(E)$ in the 1.3 C₀ uranium water lattice used by Campbell. Both slab and cylindrical (Wigner-Seitz) cells were used. The function $\nu(E)$ is shown in Figure 4. Both the slab cell and cylindrical cell calculations give about the same average $\nu(E)$ in the fuel indicating that the definition of equivalence is correct. The $\nu(E)$ at the center of the fuel are quite different for slabs and cylinders. The average $\nu(E)$ in the water for slab and cylinder cells differ by about 5% while the $\nu(E)$ at the center of the moderator differ by at most 20%. The Wigner-Seitz cell approximation leads to an overestimate of the moderator flux so that the flux in the real cell would probably lie near the middle of the group of moderator curves in Figure 4.

The $\nu(E)$ computed in the previous section using isotropic scattering but only two space regions are also shown in Figure 4. The two region approximation is good in the fuel but some 17% low in the moderator.

The curves presented in Figure 4 have been normalized to the same uniform slowing down source ($1/E$ high energy flux). The

$\nu(E)$ in the moderator are still 6% above this $1/E$ tail at 1 ev and $\nu(E)$ in the moderator is 20% lower than the asymptotic $1/E$ tail. This is consistent with the fact that $P_{SF}(1 \text{ ev}) = 0.84$. Since $1/\nu$ absorption cross sections were used, the three curves will merge at a higher energy (about 5 ev). When the actual non $1/\nu$ cross sections are used in the fuel, the $\nu_f(E)$ will always lie below the asymptotic $1/E$ tail.

Finally, it should be noted that the spread of results shown in Figure 4 is much greater than in Figure 3 so that the spectra are much more sensitive to geometrical approximations than to the isotropic scattering approximation.

CALCULATION OF THE SPECTRA IN A REAL LATTICE

The discussion in the previous sections has shown that the spectra emerging from a lattice can be computed by first mapping the isotropic birth rate density, H_0 , and the computing the uncollided flux due to the sources H_0 which enters the beam tube. The H_0 can be accurately computed for an infinite lattice from the integral transport equation.

The real lattice in which experiments are done differs from the ideal lattice in that gradients are always present. The cell under consideration must be far enough from boundaries so that all transients have died out and an asymptotic flux exists. In such an asymptotic region the material buckling $B_m^2 (= (k_{\infty} - 1)/M^2)$ equals the geometric buckling B_g^2 . The first correction is then to increase the absorption cross sections by an amount DB_g^2 in an attempt to simulate the net leakage out of a unit cell. This correction is usually quite small for most lattices.

A common experimental setup for measuring the spectra from a lattice is a small subcritical assembly with a source at one end. If the leakage from the sides of the assembly is given by B^2 , the flux along the source axis varies like $e^{-\kappa x}$, where κ^2

is computed from $B_m^g = B_g^g - \kappa^g$. Even if B_m^g is small, the B^g and κ^g can be large. If we were computing the spectra parallel to the source axis and κ is comparable to $\Sigma(E)$, the gradient will tend to exaggerate the high energy end of the spectrum with respect to the low energy end. Similarly the spectra measured perpendicular to the source axis at the center of the assembly will be depressed at the high energy end. In order to compensate for these gradient effects the fine structure H_0 computed for the cell should be multiplied by the overall shape of the flux in the assembly.

As was shown in the preceeding section the computed spectra are quite sensitive to the spatial variation of ν so that the corrections for gradients across the lattice will also be important.

COMPARISON WITH EXPERIMENTAL RESULTS

The spatial and energy distribution of ν was computed for the lattices investigated by Campbell (5) and Mostcvoi (6). The parameters are given in Table I. The Nelkin kernel was used for water and a mass 238 gas kernel for uranium. The aluminum cladding was included. The best available absorption cross sections were used. The angular fluxes were then computed from eqn. (5).

Campbell measured spectra both parallel and perpendicular to the lattice using uranium rods with 1.0, 1.3, and 1.6 times the U^{235} content of natural uranium. The variation of the flux with enrichment should be smooth and the thermal flux should decrease (with respect to the high energy flux) as the enrichment increases. This trend is not always shown by the experiment. The variations between the experimental spectra are then a measure of the reproducibility of the experiment, hence the experimental data are then plotted as bands in Figures 5 and 6. The theoretical curves are for an enrichment of 1.3 C_0 . The curves for 1.0 C_0 are about 6% higher at $E=0.025$ ev, and those for 1.6 C_0 are about 6% lower.

The spectra in the fuel are shown in Figure 5. All curves have been normalized to 0.5 at 1 ev. The dip near 0.3 ev is due to the U^{235} resonance. The shapes of the experimental curves are much more characteristic of moderator spectra than of fuel spectra. The position of the peak of $\phi_{||}$ at 0.05 ev agrees well with theory but the low energy behavior is quite different. The level of the $\phi_{\perp}$ is about the same as given by the theory but the peak of the experimental curve is at 0.028 ev compared to 0.042 ev for the theory. The theoretical calculation conserves neutrons. This conservation sets the level of the thermal flux in the fuel to the high energy flux since most absorptions occur in the fuel. The experimental $\phi_{||}$ in the fuel is just $P_{SF}(E)$ times the scalar flux. If we then reconstruct the scalar flux from the experimental curves using the calculated distributions as a guide, we find that only about 75% of the neutrons are accounted for. The leakage in this lattice is, then not small.

The spectra in the moderator are shown in Figure 6. All curves have been normalized to unity at 1 ev. Here, the theoretical spectra are again higher than measured but a 10% error due to the Wigner-Seitz cell approximation would bring the theory down to the

band for $\phi_{\perp}$. A definite anisotropy is measured with the $\phi_{\perp}$ higher than $\phi_{\parallel}$. Diffusion theory (which should work well deep in the moderator) would predict the same spectra for both $\phi_{\perp}$ and $\phi_{\parallel}$ since the gradient is zero at the moderator center. The transport calculation shows less than 3% difference between the shape of $\phi_{\parallel}$ and $\phi_{\perp}$ and is not distinguishable on the figure. The experiment shows about a 40% difference with the $\phi_{\perp}$ greater than $\phi_{\parallel}$. The small theory trend is for $\phi_{\parallel}$ to be greater than $\phi_{\perp}$ at $E=0.025$ ev and equal at both high and low energies.

The spectra measured by Mostovoi were all parallel to the lattice and are shown in Figures 7 and 8. Mostovoi considers the spectra in the 50 mm pitch lattice (Figure 8) more reliable because of higher neutron intensity and better resolution. We will also disregard the spectra in Figure 7 since, if one were to construct a neutron balance from these curves, three times as many neutrons are absorbed in the cell as slow down from high energy. The only observation from Figure 7 is that the fuel flux drops off more rapidly at low energies than a Maxwellian and at about the same rate as given by the theory.

The spectra shown in Figure 8 are for a 50 mm pitch lattice. The shape of the spectra in the fuel are very similar but the experimental spectrum is 30% lower than the theoretical spectrum

at the peak. The water spectra are quite different both in level and shape. A neutron balance constructed from the experimental curves indicates that only 70% of the neutrons which slow down from high energy are absorbed in the cell.

We conclude that the spectra measured in these experiments are not characteristic of an infinite lattice.

CONCLUSIONS

We have found that a very simple equation can be used to compute the angular spectra emerging from a lattice and that the two natural variables are η , the emission density divided by the total cross section, and y , the optical depth measured backward along the direction at which the spectrum is to be measured. We have shown that while a polynomial expansion of the angular flux may be satisfactory for predicting η , the angular flux should be computed from the general eqn. (5) rather than from the polynomial representation. Calculations of the flux in water moderated lattices indicate that the spectra are much more sensitive to geometrical approximations than to the neglect of anisotropic scattering.

The spectra measured parallel to the lattice at the fuel center and moderator center are the minimum and maximum spectra in the lattice. All spectra measured at any other angle or position will fall between these extremes. As the dimensions of the lattice become small, the measured spectrum at a particular angle approaches the homogeneous spectrum for that mixture provided that the angle μ is greater than the thickness of the lattice in mean free paths.

Comparisons of measured spectra and computed spectra indicate that the measured spectra do not represent spectra in an infinite lattice. The spectral distortions due to gradients in the finite lattice are large.

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REFERENCES

1. H. Takahashi, Nuclear Sci. & Eng. 5, 338 (1959).
2. J. Chernick, Proc. 1st Intern. Conf. Peaceful Uses of Atomic Energy, Geneva P/603 (1955).
3. H. C. Honeck, Nuclear Sci. & Eng. 3, 193 (1960).
4. H. C. Honeck & I. Kaplan, Nuclear Sci. & Eng. 3, 203 (1960).
5. C. Campbell et. al., Proc. 2nd Intern. Conf. Peaceful Uses of Atomic Energy, Geneva P/10 (1958).
6. V. Mostovoi et. al., Proc. 2nd Intern. Conf. Peaceful Uses of Atomic Energy, Geneva P/2152 (1958).
7. N. Corngold, J. Nuclear Energy 4, 293 (1957) and BNL-445 (T-93), Brookhaven National Laboratory, Upton, N. Y.
8. J. Chernick, BNL-622, Brookhaven National Laboratory, Upton, N.Y.
9. B. Davison, Canadian J. Phys. 36, 784 (1958).
10. H. C. Honeck, BNL-5826, Brookhaven National Laboratory, Upton, N.Y.
11. M. Nelkin, Phys. Rev. 119, 791 (1960).

TABLE I - DIMENSIONS AND CROSS SECTIONS USED IN THE ANALYSIS

CAMPBELL LATTICES

Rod Diameter = 3.048 cm	$N_H = 6.689 \times 10^{-22}$ atoms/cc
Aluminum Clad = 0.0508 cm	$N_{OX} = 3.344 \times 10^{-22}$ atoms/cc
$\begin{Bmatrix} 1.0 \\ 1.3 \\ 1.6 \end{Bmatrix} C_o$ uranium	$N_{25} = \begin{Bmatrix} 3.377 \\ 4.390 \\ 5.403 \end{Bmatrix} \times 10^{-20}$ atoms/cc
Pitch = 3.81 cm (square)	$N_{28} = 4.700 \times 10^{-22}$ atoms/cc
$V_{water}/V_{uranium} = 0.9217$	$N_{AL} = 6.020 \times 10^{-22}$ atoms/cc
Water Thickness (cyclindrical cell with cladding) = 0.5743 cm	
Water Thickness (cylindrical cell without clad) = 0.5836 cm	
Water Thickness (equivalent slab cell, no clad) = 1.405 cm	
Uranium Thickness (equivalent slab cell, no clad) = 1.524 cm	

MOSTOVOI LATTICES

Rod Diameter	$N_H = 6.689 \times 10^{-22}$ atoms/cc
No Cladding	$N_{OX} = 3.344 \times 10^{-22}$ atoms/cc
Natural Uranium	$N_{25} = 3.377 \times 10^{-20}$ atoms/cc
Pitch = 5.00 cm (triangular)	$N_{28} = 4.700 \times 10^{-22}$ atoms/cc
& 6.00 cm (triangular)	
$V_{water}/V_{uranium} = 1.250$ (5 cm pitch)	
$= 2.240$ (6 cm pitch)	
Water Thickness, cyclindrical cell, 5 cm pitch = 0.875 cm	
Water Thickness, cyclindrical cell, 6 cm pitch = 1.400 cm

FIGURE 1 - Schematic of a simple slab lattice.

812 REF. 10. 1-25-62

FIGURE 2 - The function $\bar{\phi}(E, \mu)$ at a given energy for the case $\theta=4$.

BNC Rep. No. 1-29-62

FIGURE 3 - The angular flux spectra in a slab lattice equivalent to the rod lattice used by Campbell.

ANL Rep No 1-30-62

FIGURE 4 - The function $\psi_0(E)$ in the fuel and moderator of a rod lattice.

BNL Rep. No. 13162

FIGURE 5 - The flux $\phi(E)$ in the fuel rod of the Campbell lattice.

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FIGURE 6 - The flux $\phi(E)$ in the center of the water moderator of the Campbell lattice.

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FIGURE 7 - The flux viewed parallel to the fuel rods in the 60 mm Mostovoi lattice.

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FIGURE 8 - The flux viewed parallel to the fuel rods in the 50 mm Mostovoi lattice.

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END