50

Considerations of a Nonhomogeneous Fluid in the Deep Groundwater Flow System at Hanford

R. W. Nelson

November 1988

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THE DEEP GROUNDWATER FLOW SYSTEM AT HANFORD

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Pacific Northwest Laboratory
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PREFACE

The Basalt Waste Isolation Project (BWIP) managed by Rockwell Hanford Operations for the U.S. Department of Energy, provided support to the author during late FY 1984 and early FY 1985 to describe physical and mathematical theoretical descriptions of the flow of nonhomogeneous fluids in the deeper Hanford groundwater system. The nonhomogeneities (spatially changing fluid properties) result from the temperature, pressure, or chemically induced variations in fluid densities, viscosities, and related fluid properties as they occur in the deeper system.

This report of that study was originally prepared in late FY 1986 and was submitted in draft form to BWIP for comments and technical review prior to publication. Subsequent funding limitations and later closeout of the BWIP prevented official review and formal publication. However, Rockwell Hanford Operations authorized the author to proceed with technical review of the document according to Pacific Northwest Laboratory (PNL) procedures and then to publish the report as a PNL document.

Mr. R. L. Skaggs, Manager of PNL's Hydrology Section, encouraged the author and provided ways to complete the review and publish, for which the author expresses appreciation. Accordingly, the review comments have been incorporated in the report, but there has been no attempt to change the overall setting for the presentation or the recommendations even though the BWIP has since been terminated and direct application at Hanford likely will never occur. The theoretical results are relevant to other applications as a general framework and the actual Hanford BWIP setting detracts little from the detailed results presented.

EXECUTIVE SUMMARY

The Basalt Waste Isolation Project (BWIP) is measuring baseline geohydrologic characteristics of the deeper groundwater system. This characterization effort is an integral part of the Hanford Site assessment for a deep nuclear waste repository. The initial baseline measurements show variations in fluid density and other fluid properties caused by temperature variations or dissolved gases within the deeper groundwater system. These and additional variations in properties, when the fluids are nonhomogeneous, collectively contribute to the overall flow system performance, although each property variation may contribute differently. Such varying influences and contributions arising from fluid nonhomogeneities are in marked contrast to the usual flow conditions in traditional shallower hydrologic systems where the fluids are effectively homogeneous.

The challenge is to provide a theoretical approach broad and versatile enough to consider the many possible fluid property variations caused by subsurface temperature, pressure, and chemical composition changes in the fluid. As each of these variations impact the overall flow system performance, the magnitude of those impacts need to be carefully monitored for significance. Then, as the significance of any particular fluid property variation is found negligible, the approach should allow the modeler to readily drop that property from continuing analysis. In this way, the analysis remains tractable and yet the variety of fluid nonhomogeneities can be considered, providing the basic theory is general enough.

This report presents such a general theory capable of describing the flow of nonhomogeneous fluids in porous media, a theory that is a composite from several disciplines including groundwater hydrology, soil physics, civil engineering, petroleum reservoir engineering, mechanics, and mathematical physics. The report discusses the conceptual basis for considering the flow of nonhomogeneous fluids. From this conceptual basis emphasis shifts to providing complete definitions and then appropriately describing those definitions in mathematical terms. Throughout the report, the necessary assumptions are stated in detail because the limitations of any theory are best assessed

through careful scrutiny of the assumptions. From the mathematical definitions with appropriate functional dependence the results and constraints needed are derived to provide the general theory necessary to describe the flow of non-homogeneous fluids in porous media.

Particular attention is given to comparing the general theory with the classical theory of flow for a homogeneous fluid. Such comparison provides significant insight to the effects of variable fluid properties on subsurface flow systems. The comparisons also indicate the importance of carefully formulating subsurface flow models within the more general theoretical framework describing the flow of nonhomogeneous fluids in porous media.

The mathematical theory describing the flow of nonhomogeneous fluids in porous media demonstrates some results that are both more general and considerably different than the usual results from situations involving homogeneous fluids. Analysis shows that the impelling-force field in general cannot be defined as the gradient of a scalar energy potential, when the fluid is nonhomogeneous.

In more detail, an energy conserving potential can exist only when there are no rotational force components associated with the impelling-force field. If the fluid is nonhomogeneous with respect to the fluid mass density, then in general the density gradients cause rotational force components; so no scalar energy potential can be defined. Specifically, the scalar potential definition is not possible in general in any of the three forms of the impelling force descriptions, i.e., either when described as force per unit mass, per unit volume or per unit weight of fluid as in Equations (7), (8), and (9), respectively. For these reasons, the advantages and convenience of an energy potential, e.g., the potential head, that are so beneficial when the fluid is homogeneous, are not in general applicable when significant density gradients are present.

A further detailed assessment is made of all those special conditions when the fluid is nonhomogeneous but where no rotational components are present in the impelling-force field; so an energy potential may be defined. All but one of the conditions are essentially trivial, so they warrant no further consideration. One special case is particularly important to the BWIP because it

so closely resembles the conditions found in the deep groundwater system at Hanford. This special case is investigated in detail. When the impelling force is described on a unit volume basis and the density gradient vector is parallel with the gravitational vector, i.e., only vertical fluid density variations occur, then a scalar energy potential can be defined. That is, with only a vertical density change, the lateral density gradient components are zero; so an energy potential can be defined. Also, if the lateral density gradients were small enough to be ignored, then an approximate scalar energy potential can be defined. However, in the repository licensing arena, a significance expression likely would be required that gives the error expected in the approximate potential, as induced by any lateral density gradients that were neglected.

Getting the approximate energy potential for actual use is more involved than just definition, because it contains an integral of the vertical density variation. When the vertical density variation is not known a priori, the integral cannot be evaluated, as will occur in the modeling of nonhomogeneous fluid flow. In this case, the potential, although certainly defined, turns out to be very cumbersome if not essentially impossible to use. But if the density changes are known as a function, as numerical values, or as an adequately measured data set from the actual problem, then the integral can be evaluated (numerically, if necessary) to provide usable potential results.

An error significance expression for the approximate energy potential is provided in Equation (60). The expression for the error significance ratio comes from careful consideration of the important flow kinematics present when nonhomogeneous fluids are involved. The expression describes the significance of the two lateral density gradient components as an interactive force between the rotational and translational impelling forces. Such an interactive force is shown to be unique, present only when the fluid is nonhomogeneous. Accordingly, a significance measure is provided that gives a basis for estimating the inaccuracy of an energy potential applied when the fluid is nonhomogeneous.

An example of the kinds of actual flow system results that can occur is found in the comparison between a homogeneous and nonhomogeneous fluid flow system as described in the last section titled, "Illustration of Some Non-homogeneous Fluid Effects in an Example Subsurface Flow System."

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CONTENTS

PREFACE	iii
EXECUTIVE SUMMARY	v
ACKNOWLEDGMENTS	ix
INTRODUCTION	1
THEORETICAL BASIS FOR DESCRIBING FLOW	5
FLUID PROPERTIES	6
Nonhomogeneous Fluid Defined	6
A Homogeneous Fluid	8
HETEROGENEOUS POROUS MEDIA DEFINED	8
DYNAMIC EQUATIONS	11
THE IMPELLING-FORCE VECTOR FIELD	11
THE FLUID FLUX FIELD	15
THE NONHOMOGENEOUS FLUID LIMITATIONS ON THE EXISTENCE OF A SCALAR ENERGY POTENTIAL	19
NONHOMOGENEOUS FLUID EFFECTS ON THE MACROSCOPIC VELOCITY FIELD	31
THE MACROSCOPIC SEEPAGE VELOCITY	31
THE MACROSCOPIC VELOCITY FIELD	33
OVERVIEW OF THEORETICAL RESULTS AND SOME RECOMMENDATIONS FOR THEIR USE	43
ILLUSTRATION OF SOME NONHOMOGENEOUS FLUID EFFECTS IN AN EXAMPLE SUBSURFACE FLOW SYSTEM	49
EXAMPLE FLOW SYSTEM WITH VERTICAL TEMPERATURE EFFECTS	50
ILLUSTRATION OF SOME NONHOMOGENEOUS FLUID INDUCED FLOW AFFECTS	53
REFERENCES	59
APPENDIX A	A.1
APPENDIX B	B.1

FIGURES

1	The Differential Potential, $d\phi$, as a Line Integral of the Impelling-Force Field	21
2	Fluid Pressure Contours and Some Values, in Kilo-Pascals, When There is a Vertical Temperature Profile	51
3	The Fluid Temperature, Density, and Viscosity Changes with Elevation in the Example Flow System	52
4	Point-by-Point Comparison of Impelling-Force Fields a. Homogeneous Fluid at 40°C b. Nonhomogeneous Fluid at 55°C to 25°C	54
5	Point-by-Point Comparison of Fluid Flux Fields a. Homogeneous Fluid at 40°C b. Nonhomogeneous Fluid at 55°C to 25°C	56
6	Comparison of Steady Fluid Pathlines or Streamlines a. Homogeneous Fluid at 40°C b. Nonhomogeneous Fluid at 55°C to 25°C	57

TABLES

1	The Expressions of the Fluid-Impelling-Force Vectors and Associated Hydraulic Conductivities That Provide the Darcy Equations for the Macroscopic Fluid Flux	16
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INTRODUCTION

The deeper groundwater flow system considered by the Basalt Waste Isolation Project (BWIP) as a component of the potential site for a nuclear waste repository has special features. These special features are more involved than those usually considered in the shallower groundwater systems. In short, the deeper systems require more complex analysis involving the flow of nonhomogeneous fluids in heterogeneous porous media, while the shallower systems often are evaluated adequately by considering the fluid essentially homogeneous.

Field measurements of the deeper Hanford groundwater flow system suggest that it is important to consider the special features associated with the flow of nonhomogeneous fluids in porous media. Significant thermal changes in the deeper system have been observed both vertically, as a result of the geothermal gradient, and laterally (US DOE 1982). These thermal variations translate into significant fluid density and viscosity variations in the deeper system. The largest effects are generally observed at greater depths and gradually diminish toward the land surface. Measurements in parts of the deeper system also show that considerable methane gas may be dissolved in the deeper waters (US DOE 1982). The dissolved methane primarily affects the fluid compressibility, although small changes are caused in other properties. Variations in the water quality have also been measured in the deeper system and could cause spatial variations of the fluid properties; however, to date such measured changes in water quality have been too small to cause more than very minor fluid density and viscosity variations.

The challenge is to provide a theoretical approach broad and versatile enough to consider the many possible fluid properties variations caused by subsurface temperature, pressure, and chemical composition changes in the fluid. As the effect of any one fluid property variation on the flow system is found insignificant, the approach should allow the modeler to readily drop the negligible effect from continuing analysis. Providing reliable expressions and values that measure the significance on flow of individual fluid property variations, and having methods that are easily reducible as effects appear negligible, are central to efficiently meeting this challenge. Such a broad

and versatile approach may be particularly important to the BWIP since further subsurface characterization and measurements are continuing over the next few years, and a particular property variation believed important today could be shown less significant by later measurements. Conversely, less significant effects today could gradually need to be considered in more detail as additional field measurements are obtained.

In addition, providing a technically defensible yet practical approach is critical in establishing the fundamental credibility of both the modeling and measurements analysis used by the BWIP. A broad-based approach that is easily reducible, when justified, for simpler problems would meet such a criterion.

This report has four purposes. First, it provides a broad-based and extended theory capable of adequately describing the flow of nonhomogeneous fluids in porous media. Second, it considers some of the special kinematic effects induced into the flow by nonhomogeneous fluid variations. Third, it points out those specific theoretical approaches obtained that are likely most pertinent to BWIP applications. Last, this report illustrates, insofar as possible by using one 2-dimensional example, the effects induced in a flow system by a vertical temperature change of the general magnitude measured in the deeper Hanford materials.

To accomplish these four purposes, this report discusses the basis for deriving the extended theory needed to describe the flow of nonhomogeneous fluids in porous media. This discussion is followed by detailed consideration of the flow kinematics when nonhomogeneous fluids are involved. Special consideration is next given to highlighting those theoretical results that are most useful for application by the BWIP. Finally, the results are presented from an example flow system to illustrate some of the nonhomogeneous fluid flow affects induced by a vertical temperature distribution similar to that observed by BWIP findings in the deeper Hanford groundwater flow system.

The section that follows begins with a discussion of the conceptual basis for considering the flow of nonhomogeneous fluids in porous systems. This discussion involves providing complete definitions and then describing those definitions mathematically. The general results are derived from the formalized mathematical definitions with appropriate functional dependence. In

this way, the extended analysis necessary to describe flow of nonhomogeneous fluids in porous media was developed. The necessary assumptions are stated in detail because the limitations of any theory are always best assessed through careful scrutiny of the assumptions involved.

THEORETICAL BASIS FOR DESCRIBING FLOW

The flow of fluids in porous materials may be considered from two points of view, the microscopic and the macroscopic. The microscopic viewpoint considers flow in the fine or detailed scale through individual pores; the macroscopic describes the larger or average effects.

The bases for microscopic analysis of flow in porous media are essentially the classical expressions of hydrodynamics, which impose dynamic equilibrium between inertial, body, pressure, and viscous forces over a velocity field. A rigorous use of these expressions would require determination of flow behavior around each microscopic particle of porous material. All particles must then be placed together randomly to obtain a mathematical model. Formulating such a mathematical model would be complicated, if not impossible, to say nothing of solving the resulting equations. Consequently, we would expect theories of flow in porous media to be of a statistical nature. Until quite recently, however, most of the work done was based on the macroscopic methods of continuum mechanics, in which the statistical nature of the problem is incorporated implicitly by postulates. The validity of the postulates then rests on careful experimental verification.

The more recent interest and work in statistical formulations of microscopic flow as a basis for macroscopic analysis are gradually further verifying and strengthening the previously postulated basis (Sposito and Chu 1981; Baveye and Sposito 1984, 1985; Cushman 1984). Although the statistical approach should be studied further, a general pattern is evolving that tends to unify the macroscopic and microscopic treatment of flow in porous media.

The more complete analysis technique that is evolving has two steps. Macroscopic equations are formulated and solved to give the average or mean velocity, macrostreamlines, macropathlines, distributions of average travel times, and the like. The micro-aspects are then treated as a variation or deviation about the previously obtained macroscopic averages.

The result of this developing approach is a re-emphasis of realistic macroscopic description, based on Darcian-type equations as the dynamic relationships describing flow in porous media. Then basic macroscopic equations

describing the flow of a nonhomogeneous fluid in a medium that is heterogeneous with respect to the intrinsic permeability can begin to be formulated. Precise statements of the properties of the fluid and porous media, along with a characterization of flow, provide the equations of state and dynamics, and are discussed in the following subsections.

FLUID PROPERTIES

The fluid is assumed to be effectively continuous in structure throughout some appropriate control volume, implying that the properties of the smallest subdivisions are the same as those of larger differential samples. Strictly speaking, this continuity is not the case, because matter is ultimately made up of subatomic building blocks. However, in this application the dimensions concerned are large compared with molecular structure, and the assumed continuum throughout some differential control volume is both acceptable and useful. Accordingly, gross measurements of physical properties, for example, of density, viscosity, compressibility, and surface tension, are appropriate.

Nonhomogeneous Fluid Defined

"Nonhomogeneous" as used here implies "not of the same kind or structure"; a nonhomogeneous material is composed of dissimilar parts or of nonidentical elements or of elements of differing quality. Accordingly, a nonhomogeneous fluid is one that is not of the same compositional characteristics or is one having differing characteristics at different locations. More precisely, if a fluid is nonhomogeneous with respect to some property, then that property is dependent on the spatial location at any instant in time. Specifically considering the fluid density as one physical fluid property, the expression for fluid of nonhomogeneous density could be written in the functional dependent form:

$$\rho = F_1[T(x,y,z,t), p(x,y,z,t), c(x,y,z,t)] \quad (1)$$

where

ρ = the fluid mass density	M/L^3
F_1 = a function of the subsequently shown variables	M/L^3
T = the temperature of the fluid that is dependent upon the spatial location coordinate (x,y,z) and time, t	Degrees
p = the fluid pressure at location x,y,z at time t	F/L^2
c = the composition of dissolved solutes, other liquids, or gases contained in the fluid at the location denoted by x,y,z at time t.	M/L^2

and where M is mass, L is length, and F is force. The dependence indicated in Equation (1) is general. Numerous simpler combinations of dependence can occur representing appropriate reduced or special cases. One such example would be a barotropic fluid in an isothermal flow system with no dissolved constituents in the fluid, which would be of the reduced functional form

$$\rho = F_1[p(x,y,z,t)] \quad (2)$$

Similarly, the dynamic viscosity, μ , of a nonhomogeneous fluid has a variety of combinations of intermediate dependence leading to various spatial and temporal patterns. A rather general functional dependence represented by F_2 for the viscosity is

$$\mu = F_2[T(x,y,z,t), p(x,y,z,t), c(x,y,z,t)] \quad (3)$$

Other fluid properties such as compressibility and surface tension could be considered under the broader category of nonhomogeneous fluids using appropriate spatial and temporal dependence forms of the type illustrated by Equations (1) and (3). The equations can be reduced further to consider the homogeneous case commonly used in shallow groundwater hydrology systems.

A Homogeneous Fluid

When the fluid is homogeneous, then fluid properties are neither spatially nor temporally dependent in the flow. In other words, the fluid is the same throughout the flow system. This similarity provides the reduced cases where the mass density, ρ , is

$$\rho = \text{constant} \quad (4)$$

and the dynamic viscosity, μ , is

$$\mu = \text{constant} \quad (5)$$

Similarly, for a homogeneous fluid, all of the other physical and chemical properties are implied to be effectively constant in time and space.

Before proceeding to the dynamic equations, it is useful to characterize and describe the features of a heterogeneous porous media.

HETEROGENEOUS POROUS MEDIA DEFINED

"Heterogeneous" as used here implies "differing in kind, having unlike quantities, or possessing different characteristics." Accordingly, a heterogeneous porous medium has unlike quantities or differing characteristics at different locations. If a porous medium is heterogeneous with respect to some property, then that property is functionally dependent on the spatial location. This definition is in contrast to the definition often used in physical chemistry, where heterogeneous implies a mixture of discrete homogeneous phases that form regions large enough to be regarded as continuums on a microscale. To avoid confusion, the prefix "micro" or "macro" might be added to "heterogeneous." In such a case, "microheterogeneous" would imply the physical chemistry connotation as used by Frisch et al. (1961, 1962), Frisch and Hammersley (1963), and Prager (1961), whereas "macroheterogeneous" would denote the system defined and discussed here. Specifically considering the intrinsic permeability, k , as heterogeneous, then

$$k = F_3(x,y,z) \quad (6)$$

where

k = the intrinsic permeability	L^2
F_3 = a function of the variables in parenthesis	L^2
x,y,z = the Cartesian spatial coordinates.	L

Spatial variation of several other properties, of which porosity and medium compressibility are but two, is implied by the broad title of "heterogeneous porous media." In this work, however, the phrase will be used to imply primarily heterogeneity of the intrinsic permeability.

Equation (6) is a scalar function; hence isotropy with respect to permeability is implicit. Some may believe anisotropy should be incorporated through considering k a tensor. This incorporation could easily be done; however, such treatment may be premature, because in-place measurement of such properties is currently very limited if not impossible. In fact, research now under way suggests that several major research accomplishments must be realized before rational in-place measurement of permeability in anisotropic and heterogeneous media is likely to be realized (Baveye and Sposito 1984, Cushman 1984). There are also indications that much of the reported experimental evidence on anisotropy may have resulted from interactions with heterogeneity and the sample size involved in measurement rather than true anisotropy [see bottom of page 1756 in Nelson (1960)].

This completes the discussion of permeability. The needed definition is incorporated in the dependence in Equation (6). When combined with the definitions of nonhomogeneous fluid in the preceding section, the specific dependences of Equations (1) and (3) are available for use in the dynamic equations.

DYNAMIC EQUATIONS

A statement of the physical principles governing motion is needed to develop the required basic equations. The usual working hypothesis that Darcy's Law is valid is the starting point here. Although Darcy's Law is widely accepted for macroscopic description of flow of nonhomogeneous fluids in heterogeneous media, and the author believes it to be valid, it must be introduced here as a postulate assuming nonviscous flow because no generalized statistical treatment for nonhomogeneous fluids in heterogeneous porous media is now available. In fact, only in the last 5 years has the postulate basis for Darcian description of flow of homogeneous fluids in heterogeneous porous media been supported on theoretical grounds through statistical treatment starting from the relativist concept (Sposito and Chu 1981; Baveye and Sposito 1984, 1985). Because the very recent relativist concepts in their present form have not been applied for statistical consideration of nonhomogeneous fluids in porous media, this discussion accepts this postulate based on an experimentally demonstrated basis and from intuitive or physical arguments (see, for example, Rose 1945; Wooding 1959; Yih 1965). Postulating the validity of Darcy's Law, the dynamic constitutive relationship involves two parts, the driving-force terms and the fluid conductance part of the expressions. The driving force is often more descriptively called "the fluid-impelling-force vector field" and is of fundamental importance in considering porous media flow.

THE IMPELLING-FORCE VECTOR FIELD

At any macroscopic point in a porous media flow system, there is an impelling force or driving force that acts on the fluid, if movement is occurring. It is convenient to think of the fluid-impelling forces at all points in the macroscopic space as a whole, rather than fix attention on forces at isolated macroscopic points. When the impelling force is defined at a given instant at every macroscopic point in space, one has an impelling-force field. Analytically, the general impelling-force field at that instant amounts to three functions (the components of the driving force) of the three variables (the coordinates of the point). The driving force is most easily dealt with by recognizing that at each macroscopic point in space capable of being occupied

by that fluid, a macroscopic force-intensity vector will exist. The aggregate of those intensity vectors is the fluid-impelling-force vector field, which may be expressed in three forms depending upon the unit measure of the fluid that is used. Specifically, when expressed in a Lagrangian sense as the impelling force per unit mass of fluid, $\bar{E}$, then

$$\bar{E} = - \frac{1}{\rho} \nabla p - g \nabla z \quad (7)$$

where $\bar{E}$ = the vector representing impelling force per unit mass	$F/M = L/T^2$
ρ = the fluid density	M/L^3
∇ = the "del" or gradient mathematical operator	$1/L$
p = the scalar fluid pressure	F/L^2
g = the gravitational scalar	L/T^2
z = the vertical Cartesian coordinate oriented parallel but in the opposite direction to the gravitational vector	L

or expressing the impelling force, $\bar{G}$, per unit volume of fluid gives in a Eulerian control volume

$$\bar{G} = - \nabla p - \rho g \nabla z \quad (8)$$

where $\bar{G}$ = the impelling force per unit volume of fluid	F/L^3
--	---------

and expressed as the force per unit weight of fluid, $\bar{H}$, yields:

$$\bar{H} = - \frac{1}{\rho g} \nabla p - \nabla z \quad (9)$$

where $\bar{H}$ = the impelling force per unit weight of fluid,	F/F
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and the other terms are as already defined.

Equations (7), (8), and (9) represent, respectively, the impelling force per unit mass, volume, or weight of fluid that is acting upon the macroscopic

element of the fluid at that point. In each of the equations, the special implication of the fluid being nonhomogeneous enters the analysis through the spatial dependence of the fluid density.

The spatial dependence of the fluid density (under conditions of the fluid being nonhomogeneous) may raise a question on the correctness of the functional form used in Equations (7), (8), and (9) for the variable fluid density. Specifically, should the fluid density, ρ , which is spatially dependent when the fluid is nonhomogeneous, appear outside the gradient operator, ∇ , as written above or should it be inside the gradient operator. When the fluid is homogeneous so ρ is constant, it may equivalently be inside or outside of the "del" operator. But when the fluid is nonhomogeneous so ρ is spatially dependent then, for example, some may suggest the impelling force per unit volume, $\bar{G}$, should be written in the form:

$$\bar{G} = - \nabla p - g \nabla(\rho z) \quad (10)$$

where ρ is inside the gradient operator rather than outside as in Equation (8), i.e., as in the more usual fluid mechanics form for the gravitational body force.

There are at least two ways to infer the appropriate functional form, i.e., Equation (8) or (10), for the spatially dependent fluid density. The first begins with an arbitrary control volume, applies conservation of momentum (Currie 1974) and then, through dropping the usual terms that lead to Darcy's Law, shows that Equation (8) is the appropriate form. Second, insight concerning the correct form may also be provided by using the two possible equation forms to interpret a thought experiment in which one of the equations leads to a counter example. Consider the case of static groundwater so that all of the impelling force components were zero, i.e., $G_x = G_y = G_z = 0$. Such a stable density variation with the fluid remaining static would occur when there was a temperature variation only in the vertical z direction, with the water temperature coolest at the bottom and always becoming warmer at higher elevations. The impelling force components of $\bar{G}$ from Equation (8) are:

$$\begin{aligned}
G_x &= - \frac{\partial p}{\partial x} \\
G_y &= - \frac{\partial p}{\partial y} \\
G_z &= - \frac{\partial p}{\partial z} - \rho g
\end{aligned} \tag{11}$$

Setting all three of the components equal to zero since the fluid is static, rearranging, and integrating we obtain the primary result from the z component that

$$p = - \int_{z_0}^z \rho g \, dz' \tag{12}$$

which is immediately recognized as the valid expression for the vertical pressure distribution for the static fluid in the porous media thought experiment. Accordingly, use of Equation (8) to interpret the thought experiment has provided a consistent result.

Similarly, use of the second and different expression for $\bar{G}$ from Equation (10) leads to a case in which the components are

$$\begin{aligned}
G_x &= - \frac{\partial p}{\partial x} \\
G_y &= - \frac{\partial p}{\partial y} \\
G_z &= - \frac{\partial p}{\partial z} - \rho g - gz \frac{\partial \rho}{\partial z}
\end{aligned} \tag{13}$$

because p in the thought experience was only a function of z . Again, since the fluid is static in the experiment, all three components in Equation (13) are set equal to zero, rearranged, and integrated to obtain the result:

$$p = - \int_{z_0}^z \rho g \, dz' - \int_{z_0}^z gz \frac{d\rho}{dz} \, dz' \tag{14}$$

but knowledge of fluid statics in porous media indicates that only the first integral is correct, so this equation is a counter example; hence, Equation (10) with the variable density inside the gradient operator is discarded as incorrect. In a similar way, the variable density experiment can be considered for the $\bar{E}$ and $\bar{H}$ force expressions, demonstrating that the functional dependency forms for the fluid mass density in Equations (7), (8), and (9) are the correct forms of the force fields.

All three of the descriptions of the force vector [$\bar{E}$, $\bar{G}$, and $\bar{H}$ in Equations (7), (8), and (9)] depend on the spatial changes of 1) the fluid density and 2) the fluid pressure. If the scalar variations of both the fluid density and the fluid pressure are known, then any of the three representations of the force vector is known or can be determined.

The appropriate conductivity expressions may now be combined with the impelling-force vector field expressions defined above to provide the complete expressions for Darcy's Law. The results are the dynamic flux equations needed for the extended evaluation.

THE FLUID FLUX FIELD

From Darcy's Law, the product of the impelling-force vector and the associated appropriate conductance expression provides the macroscopic flux, $\bar{q}_v$, or the volume of the fluid moving across a unit area normal to the flow direction in unit time. The conductance expressions contain both properties of the porous media (intrinsic permeability, k) and the fluid properties. Each expression of the conductance is different depending on which of the force vector fields are involved.

In Table 1, the various impelling force and conductance expressions are tabulated with the first three entries (i.e., the first three rows in the table provide the vector fluid flux, $\bar{q}_v$, expressed on a volume basis). The last three rows in Table 1 express the macroscopic fluid flux, $\bar{q}_m$, on a mass flux basis (i.e., as the mass of fluid moving across a unit area normal to the flow direction in unit time). The flux expressed on a mass basis is a more concise measure or characterization of flow under conditions of variable density

TABLE 1. The Expressions of the Fluid-Impelling-Force Vectors and Associated Hydraulic Conductivities That Provide the Darcy Equations for the Macroscopic Fluid Flux

Impelling Force Vector			Associated Conductance		Darcian Fluid Flux Vector		
Expressed as Force Per: Col. 1	Equation Col. 2	Units Col. 3	Equation Col. 4	Units Col. 5	Equation Col. 6	Units Col. 7	Expressed as Fluid: Col. 8
Unit Mass of Fluid	$\bar{E} = -\frac{1}{\rho} \nabla p - g \nabla z$	$\frac{F}{M} = \frac{L}{T^2}$	$K_{mv} = \frac{k\rho}{\mu}$	T	$\bar{q}_v = K_{mv}(\bar{E})$	$\frac{L^3}{TL^2} = \frac{L}{T}$	Volume per Unit Time per Unit Area
Unit Volume of Fluid	$\bar{G} = -\nabla p - \rho g \nabla z$	$\frac{F}{L^3}$	$K_{vv} = \frac{k}{\mu}$	$\frac{L^4}{FT}$	$\bar{q}_v = K_{vv}(\bar{G})$	$\frac{L^3}{TL^2} = \frac{L}{T}$	Volume per Unit Time per Unit Area
Unit Weight of Fluid	$\bar{H} = -\frac{1}{\rho g} \nabla p - \nabla z$	$\frac{L}{L}$	$K_{wv} = \frac{k\rho g}{\mu}$	$\frac{L}{T}$	$\bar{q}_v = K_{wv}(\bar{H})$	$\frac{L^3}{TL^2} = \frac{L}{T}$	Volume per Unit Time Unit Area
Unit Mass of Fluid	$\bar{E} = -\frac{1}{\rho} \nabla p - g \nabla z$	$\frac{F}{M} = \frac{L}{T^2}$	$K_{mm} = \frac{k\rho^2}{\mu}$	$\frac{TM}{L^3}$	$\bar{q}_m = K_{mm}(\bar{E})$	$\frac{M}{TL^2}$	Mass per Unit Time per Unit Area
Unit Volume of Fluid	$\bar{G} = -\nabla p - \rho g \nabla z$	$\frac{F}{L^3}$	$K_{vm} = \frac{k\rho}{\mu}$	T	$\bar{q}_m = K_{vm}(\bar{G})$	$\frac{M}{TL^2}$	Mass per Unit Time per Unit Area
Unit Weight of Fluid	$\bar{H} = -\frac{1}{\rho g} \nabla p - \nabla z$	$\frac{L}{L}$	$K_{wm} = \frac{k\rho^2 g}{\mu}$	$\frac{M}{TL^2}$	$\bar{q}_m = K_{wm}(\bar{H})$	$\frac{M}{TL^2}$	Mass per Unit Time per Unit Area

fluids. If the volume-based expression is used in variable density systems, either the fluid density distribution must accompany the unadjusted volumetric fluxes, q_v , for the more complete description, or the volumetric fluxes must be used adjusted to standard fluid conditions. The mass basis is also preferred by those encouraging emphasis of the International System of Units (SI) in terms of length, L; mass, M; and time, T, over the MKS system units (Rose 1979, Campbell and Van Schilfagrade 1981). When L, T, and the force, F, combination is more convenient or physically meaningful, then the author believes the latter may be used with equal propriety. As will become apparent in the material to follow, it is not always possible to use only the mass-based flux expressions. Specifically, when the fluid pore velocity and related kinematic aspects of flow are considered, one must return to the volumetric flux form.

Those forms of Darcy's Law appropriate to describing the fluid flux in porous materials are listed in Table 1. The entries as rows in the table represent those combinations for describing the flux either on a volumetric or mass basis, and each of the fluxes are further extended by the three bases for describing the impelling-force vector acting on the fluid. The result is a set of six equations in Column 6 of Table 1 that are all valid forms of Darcy's Law. Each is an appropriate dynamic equation for further analysis. The choice of a specific form should be based upon the utility and convenience of applying that form in the particular system being analyzed.

One of the conveniences commonly used in the groundwater hydrology and soil physics disciplines with homogeneous fluids is the scalar energy potential. The gradient of that scalar potential yields the impelling-force field, which was already discussed in the Darcian flux expressions. The possibility of such scalar potentials existing and any particular implications for further analysis is considered in the next section.

THE NONHOMOGENEOUS FLUID LIMITATIONS ON THE EXISTENCE OF A SCALAR ENERGY POTENTIAL

The utility and convenience of a scalar energy potential in any analysis where such a potential can be defined have special significance in at least three ways. First, the gradient of the scalar potential directly yields the fluid driving force or the impelling-force field, which is basic to describing fluid flow in porous media. Second, the scalar energy potential itself is one single scalar function of space and time that can be easily conceptualized and directly measured when it exists. Third, when appropriate, setting up boundary and initial conditions based on a potential formulation is easier and more direct. Accordingly, it is useful to consider the impelling force expressions in some detail to see under what conditions the energy potential scalar can be defined. This definition capitalizes on the several desirable features that a potential provides to analysis and measurement. For completeness of this presentation, the derivation here is presented in detail, after Hubbert (1956) and Nelson (1966).

Assuming the existence of a scalar potential, the conditions necessary for the assumed scalar to exist or be defined as a useful expression are derived. The conditions are then carefully interpreted in terms of the special properties of the fluids or flow system conditions under which the scalar potential is appropriate. Letting the assumed scalar potential be represented by ϕ and using the impelling force per unit mass of fluid, $\bar{E}$, from Equation (7), the equation

$$-\nabla\phi = -\frac{1}{\rho} \nabla p - g\nabla z \quad (15)$$

is investigated to determine if the scalar potential, ϕ , can be appropriately defined. Either of the other two expressions [Equations (8) or (9)] for the impelling-force field could be considered in a similar way.

The right-hand side of Equation (15) is the vector $\vec{E}$ having a particular value of force at each point in space and the ensemble of such values comprise the force vector field to be represented, if possible, by the gradient of the scalar potential, ϕ . The field of force, $\vec{E}$, is illustrated by lines of force and an arbitrary path of integration between two particular selected points is shown in Figure 1. If $\overline{ds}$ is defined as the differential vector length along the arbitrary path between the two points in Figure 1, then taking the dot product of both sides of Equation (15) with $\overline{ds}$ gives

$$\nabla\phi \cdot \overline{ds} = + \frac{1}{\rho} \nabla p \cdot \overline{ds} + g\nabla z \cdot \overline{ds} \quad (16)$$

However, the left-hand side of Equation (16) is the total derivative of ϕ , and similarly $\nabla p \cdot \overline{ds}$ is dp in any curvilinear, orthogonal-coordinate system where z alone enters into the gravitational component; therefore, along the line selected,

$$d\phi = \frac{1}{\rho} dp + g dz \quad (17)$$

Equation (17) can be integrated to give a relationship defining ϕ if and only if the integral is shown to be independent of the path of integration (see Figure 1). The classical requirement for path independence in evaluating the integral requires that the right-hand side of Equation (15) has a vanishing curl (Lass 1950, pp. 103-107).

$$\text{curl} \left(- \frac{1}{\rho} \nabla p - g\nabla z \right) = 0 \quad (18)$$

FIGURE 1. The Differential Potential, $d\phi$, as a Line Integral of the Impelling-Force Field

Expanding the left-hand side and considering ρ for a nonhomogeneous fluid as defined in Equation (1) and with g constant gives

$$\left[-\frac{1}{2} (\nabla \rho) \times (\nabla p) \right] + \frac{1}{\rho} (\nabla) \times (\nabla p) = 0 \quad (19)$$

The second term in Equation (19) vanishes; that is the curl of every gradient of a scalar such as p is zero. Equation (19) becomes

$$(\nabla \rho) \times (\nabla p) = 0 \quad (20)$$

Expanding Equation (20) in Cartesian coordinates and equating each component to zero gives, on rearranging and reducing:

$$\begin{aligned}
\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial z} &= \frac{\partial \rho}{\partial z} \frac{\partial p}{\partial y} \\
\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial z} &= \frac{\partial \rho}{\partial z} \frac{\partial p}{\partial x} \\
\frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} &= \frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x}
\end{aligned}
\tag{21}$$

Equation (21) must be satisfied if the integration of Equation (17) is to be independent of the path and hence a useful scalar potential ϕ be defined. Recalling Equation (1), which provides the dependence of ρ for a nonhomogeneous fluid, and differentiating gives:

$$\begin{aligned}
\frac{\partial \rho}{\partial x} &= \frac{\partial F_1}{\partial T} \frac{\partial T}{\partial x} + \frac{\partial F_1}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial F_1}{\partial c} \frac{\partial c}{\partial x} \\
\frac{\partial \rho}{\partial y} &= \frac{\partial F_1}{\partial T} \frac{\partial T}{\partial y} + \frac{\partial F_1}{\partial p} \frac{\partial p}{\partial y} + \frac{\partial F_1}{\partial c} \frac{\partial c}{\partial y} \\
\frac{\partial \rho}{\partial z} &= \frac{\partial F_1}{\partial T} \frac{\partial T}{\partial z} + \frac{\partial F_1}{\partial p} \frac{\partial p}{\partial z} + \frac{\partial F_1}{\partial c} \frac{\partial c}{\partial z}
\end{aligned}
\tag{22}$$

Substituting the equations in (22) into (21) and rearranging, obtains

$$\begin{aligned}
\frac{\partial F_1}{\partial T} [(\nabla T) \times (\nabla p)] + \frac{\partial F_1}{\partial p} [(\nabla p) \times (\nabla p)] \\
+ \frac{\partial F_1}{\partial c} [(\nabla c) \times (\nabla p)] = 0
\end{aligned}
\tag{23}$$

This expression is the expanded form of Equation (20), required if Equation (17) is to be integrable when the density of the nonhomogeneous fluid is as defined in Equation (1).

If the fluid is homogeneous as in Equation (4) then

$$\frac{\partial F_1}{\partial T} = \frac{\partial F_1}{\partial p} = \frac{\partial F_1}{\partial c} = 0
\tag{24}$$

and Equation (23) is also zero. Therefore, the force field is conservative and Equation (17) can be integrated or

$$\phi = gz + \int_{p_1}^{p_2} \frac{1}{\rho} dp \quad (25)$$

which is the valid energy potential for a homogeneous fluid. Suppose also that the flow system is isothermal, then

$$\frac{\partial F_1}{\partial T} = 0 \quad (26)$$

and if the composition of the fluid is constant, then

$$\frac{\partial F_1}{\partial c} = 0 \quad (27)$$

After substituting Equations (26) and (27) into (23), then only the middle term remains. But, by definition, the cross product of the vector ∇p on itself is always zero, so the most general condition required for the left-hand side of Equation (23) to reduce identically to zero is that of Equation (26) and (27). We note that this condition is precisely the dependence represented by Equation (2). Therefore, the scalar potential expressed as energy per unit mass of fluid in Equation (25) is defined, if the density is at most a function of only the pressure (barotropic as, for example, a gas under isothermal or adiabatic conditions). Therefore, Equation (2) represents the most general functional dependence for the density for which it is still possible to define a potential.

A few other very special cases perhaps should be mentioned in the interest of completeness of this discussion [i.e., where the left-hand side of Equation (18) reduces to zero]. These cases include those where very special combinations of conditions in the flow system occur. For example, if the vectors ∇T and ∇p are in the same direction or are colinear and either Equation (27) occurs or the ∇c and ∇p vectors are colinear, then the left-hand side of

Equation (23) is zero and the potential in Equation (25) is valid. Also, one observes that if the ∇c and ∇p vector are colinear in Equation (23) and if Equation (27) is satisfied then again a very special case is noted where the potential is valid as defined by Equation (25).

However, the equality in Equation (15) is valid only under the very general conditions where flow involves homogeneous fluids and for a few additional special situations as discussed in the preceding two paragraphs. The scalar energy potential for the broad class of flow of nonhomogeneous fluids involving density variations cannot be defined. The inability to correctly define such a scalar potential results because the impelling-force field is nonconservative. Accordingly, the line integration of Equation (17) required to define such a potential is, in general, path dependent when non-homogeneous fluids are involved. Hence, a single unique energy potential cannot be defined.

The above considerations of the limitation imposed by fluid density variations arising from the flow of nonhomogeneous fluids are based on using the impelling force expressed as force per unit mass. Although the overall conclusions vary little from our discussion, for completeness the other two force vector fields described in Equations (9) and (8) will be considered respectively below.

The equation that must be satisfied for the energy scalar potential to exist when expressed as energy per unit weight of fluid requires that the curl of $\bar{H}$ defined in Equation (9) be zero or

$$\text{curl} \left(-\frac{1}{\rho g} \nabla p - \nabla z \right) = 0 \quad (28)$$

or expanding

$$-\frac{1}{\rho g} (\nabla) \times (\nabla p) + \frac{1}{g \rho^2} (\nabla \rho) \times (\nabla p) - (\nabla) \times (\nabla z) = 0 \quad (29)$$

But, the curl of every gradient of a scalar [as for example $(\nabla) \times (\nabla p)$ and $(\nabla) \times (\nabla z)$ in Equation (29)] are identically zero, leaving only the middle term, i.e.,

$$(\nabla \rho) \times (\nabla p) = 0 \quad (30)$$

which is identical to Equation (20) and must be satisfied for a scalar energy potential per unit weight of fluid to exist. Accordingly, all of the conditions derived earlier for the unit mass basis are equally applicable for the basis of impelling force per unit weight of fluid because Equation (30) is identical to Equation (20).

The requirement for an energy scalar to be defined or derived from the third and last equation form for the impelling force expressed, i.e., on the basis of unit volume, requires the curl of $\bar{G}$ from Equation (8) to be zero or

$$\text{curl } (-\nabla p - \rho g \nabla z) = 0 \quad (31)$$

Expanding Equation (31) and again assuming the gravitational scalar, g , constant gives

$$-(\nabla) \times (\nabla p) - \rho g (\nabla) \times (\nabla z) - g (\nabla \rho) \times (\nabla z) = 0 \quad (32)$$

Simplifying because the curl of every gradient of a scalar is zero, the requirement reduces to only the last term or

$$-g (\nabla \rho) \times (\nabla z) = 0 \quad (33)$$

Equation (33) is satisfied when the gradient in fluid density or $\nabla \rho$ vector is parallel with the gravitation vector, $-g \nabla z$, in which case:

$$\begin{aligned}\frac{\partial \rho}{\partial x} &= 0 \\ \frac{\partial \rho}{\partial y} &= 0\end{aligned}\tag{34}$$

Substituting the first two expressions from Equation (22) into (34) yields the general condition for a potential to exist for a nonhomogeneous fluid on a unit volume basis:

$$\frac{\partial F_1}{\partial T} \frac{\partial T}{\partial x} + \frac{\partial F_1}{\partial p} \frac{\partial p}{\partial x} + \frac{\partial F_1}{\partial c} \frac{\partial c}{\partial x} = 0\tag{35}$$

and

$$\frac{\partial F_1}{\partial T} \frac{\partial T}{\partial y} + \frac{\partial F_1}{\partial p} \frac{\partial p}{\partial y} + \frac{\partial F_1}{\partial c} \frac{\partial c}{\partial y} = 0\tag{36}$$

which is the expanded complete condition required for an energy potential to exist when the impelling force is expressed on a unit fluid-volume basis. If only a homogeneous fluid is involved, then both Equations (35) and (36) are satisfied by Equation (24). Again the single energy scalar potential is completely defined only for homogeneous fluids.

All of the special cases, when the impelling force per unit volume is used, are contained in Equation (33) where the gradient of the fluid density vector is parallel with the gravitational vector. Equation (33) includes the very special case of a static nonhomogeneous fluid, when the vectors are parallel, equal, and opposite in direction. However, one should note that the barotropic case involving only pressure dependent density as in Equation (2), and which was valid for both the fluid mass and unit weight impelling force expressions, does not hold as a special case for the impelling force per unit volume expression.

It is important to realize that the special case for the impelling force per unit volume of fluid, $\bar{G}$, is less restrictive than both the $\bar{E}$ and $\bar{H}$ force fields, i.e., force per unit mass and per unit weight of fluid, respectively. Specifically, when using $\bar{G}$ in Equation (8), the expression

$$d\phi' = dp + \rho g dz \quad (37)$$

defines an energy potential, ϕ' , to be path independent and hence integrable. The two conditions in Equation (34) on density variations are also required. However, three conditions or one more than in Equation (31) are needed to satisfy the required integrability condition when using the impelling force both for a unit mass and weight basis as for $\bar{E}$ and $\bar{H}$ in Equations (7) and (9). The additional condition required after substituting Equation (34) into (21) is:

$$\frac{\partial \rho}{\partial z} = 0 \quad (38)$$

This condition suggests a special advantage in using $\bar{G}$, the impelling force per unit volume, rather than either $\bar{E}$ or $\bar{H}$, for approximate calculations in specialized systems with predominantly vertical variations and rather small lateral variations in fluid density. This condition is usually found when geothermal or predominantly depth-dependent salinity or dissolved gas is the only significant change that occurs. Usually under these conditions, the lateral components of density change are very small, because only the slow groundwater transport rates of heat, salinity, or lateral pressure changes are causing minuscule lateral density variations.

The predominance of the vertical geothermally induced effects in the deeper Hanford system suggests the advantage of using the force field, $\bar{G}$, as in Equation (8) in specific evaluations in the remainder of this report. It further suggests the BWIP might be well advised to capitalize on the advantages of the $\bar{G}$ force field as the basis for both model formulation and in the analysis of measurements. Specifically, by using the volume-based impelling-force formulation in Equation (8), large vertical or z component density variations may be included and yet, if the approximation is appropriate, the lateral components may also be set to zero, thereby providing rather good approximations to the actual deeper Hanford situation. As will become clearer as further progress is described, this emphasis on the vertical components that

have been specifically demonstrated just for the existence of a scalar potential could beneficially carry over into the other facets of analysis.

The implications of using the approximation that the lateral variations of the fluid densities are negligibly small, so Equation (34) is satisfied approximately, will be considered in more detail. Within that approximation, the scalar potential ϕ' exists, the integration is independent of the path, and Equation (37) may be evaluated to yield

$$\phi' = p + g \int_{z_0}^z \rho \, dz \quad (39)$$

where the mass density ρ remains inside the integral because it is dependent upon z . The impossibility of being able to carry out the integration in Equation (39) may severely limit the utility of the approximate potential and may in some cases negate the advantages of such an approximate potential existing.

If the integration of Equation (39) cannot be evaluated, then the potential, ϕ' , is not known as a single scalar; hence Equation (39) requires the pair p and ρ to both be known or determined together so p and ρ might as well be used as the direct variables. Further, if Equation (39) cannot be integrated, then boundary conditions cannot be set realistically in terms of ϕ' , because ρ is not known a priori. In effect, though a potential has been derived using the approximation satisfying Equation (34), it really may be of very limited direct utility because the specific dependence of ρ upon z must be known a priori in order to perform the integration necessary to define that approximation potential.

This section then conclusively demonstrated that when nonhomogeneous fluids are involved in porous media flow, a scalar energy potential, in general, is not usable for analysis. In fact, in all but a few very restricted situations, i.e., in the general case, a single scalar energy potential can neither be defined nor realistically measured as a meaningful evaluation tool when nonhomogeneous fluid flow is involved.

When nonhomogeneous fluids are involved in the analysis, two scalar functions are required to determine the impelling-force vector field rather than only one scalar. The first of the two required scalars is the fluid pressure distribution at the boundaries and throughout the flow system of interest. The second required scalar is the fluid density distribution, also needed at the boundaries and throughout the flow system being considered. Given the fluid pressure and fluid density variation, any of the three impelling-force vector fields can be determined [see Equations (7), (8), or (9)].

Measurements in nonhomogeneous fluid systems under general conditions should concentrate on measuring the fluid pressure and other variables necessary to accurately define the fluid density variations throughout the system of interest. For example, the analysis should include measurements of temperature variations, variations in dissolved constituents, salinity, and any other factors required to adequately define the fluid density variation throughout the system.

NONHOMOGENEOUS FLUID EFFECTS ON THE MACROSCOPIC VELOCITY FIELD

In any system where the evaluation of flow in subsurface systems concerns contaminant transport, the Darcian fluid flux expressions discussed previously and summarized in Table 1 need to be extended to provide the macroscopic fluid velocity field. An appropriate expression is needed to relate the macroscopic fluid velocity and the fluid flux, as defined previously, on a macroscopic basis.

THE MACROSCOPIC SEEPAGE VELOCITY

The macroscopic flux expressed either as $\bar{q}_v$ or as $\bar{q}_m$ are bulk flow rates or fluxes through a unit area normal to the flow. The bulk unit area or total coordinate area has not been reduced to account for the fact that the porous solid occupies part of that bulk area. For example, in a bulk or overall coordinate volume, the actual voids or maximum volume of void space that could be saturated with fluid can be no greater than the porosity, which represents the fraction of the total or bulk volume occupied by the fluid under saturated conditions.

By contrast, the macroscopic velocity, $\bar{v}$, sometimes called the mean fluid pore or seepage velocity, is needed if travel times and actual traversed distances are to be interrelated. The mean seepage velocity is also needed for further kinematic analysis. Let the mean or macroscopic velocity be $\bar{v}$ and the proportionality scalar be σ ; then

$$\bar{v} P \sigma = \bar{q}_v \quad (40)$$

where P is the porosity in the volume of pore space per unit of bulk or total volume in L^3/L^3 . From this equation, the defining relationship for σ for saturated systems is

$$\sigma = \frac{|\bar{q}_v|}{|\bar{v}|P} \quad (41)$$

where $\bar{v}$ = the meaningful macroscopic (pore area based)	
velocity vector	L/T
σ = the scalar pore velocity conversion term	
defined by Equation (41)	Dimensionless

The term, σ , as defined by Equation (41) is used again to avoid making the unsubstantiated assumption that the traditionally defined seepage velocity based on the volumetric porosity is the appropriate average or reference velocity (Nelson 1966). If the traditional seepage velocity is found to be the valid basis for subsequent analysis then σ simply reduces to unity. Meanwhile for those who by some heuristic manner are able to determine the so-called "effective porosities" then the product of P and σ can be construed as representing such parameters. Present evidence suggests that Equation (4D) is the best and most useful form now available and should be used until more complete resolution and description of the needed interrelationship to dispersion is provided. Considerable research is under way and many significant publications are becoming available in this area (for example Sposito, Gupta, and Bhattacharya 1979; Gelhar, Gutjahr, and Naff 1979; Gelhar and Axness 1983; Simmons 1982, 1983; Dagan 1982; Matheron and De Marsily 1980; Molz, Guven, and Melville 1983). Among those issues requiring resolution are the interrelationship of porous media heterogeneity and dispersion. Characterization and analysis of heterogeneous systems are only now being addressed. Accordingly, considerable progress may be expected in this area in the immediate years ahead.

Both the fluid volume and fluid mass forms of Darcy's Law were presented previously in Table 1. Some investigators and groups (Rose 1979, Campbell, and Van Schilfagrade 1981) believe that the mass flux form of Darcy's Law as presented in the bottom three rows of Table 1 should always be used. Although such emphasis may have seeming pedagogical merits and the author certainly concurs with the desirability of commonality among reported units, when the macroscopic velocity field is involved, the physical system requires that the mass flux expressions be converted back to the volume flux basis before proceeding with further flow kinematics, that is:

$$\bar{q}_v = \frac{1}{\rho} \bar{q}_m \quad (42)$$

If fluid transport or flow kinematics is part of an analysis sequence, volumetric flux must be used. If only the mass flux form, $\bar{q}_m$, is used, Darcy's Law requires that mass form must always be converted using Equation (42) to the volumetric flux, $\bar{q}_v$. For this reason, any of the three expressions for mass flux appearing in the bottom three entries in Column 6 of Table 1 are converted through Equation (42) to the volumetric flux basis appearing as the upper three entries in the same column. But all of the top three volumetric flux are essentially the same with the second expression being most direct. Substituting this more direct form into Equation (40) yields

$$\bar{v} = - \frac{k}{\rho \sigma \mu} (\nabla p + \rho g \nabla z) \quad (43)$$

which defines the macroscopic velocity vector field, $\bar{v}$. The more direct form in Equation (43) is used for the advantage already discussed for describing the impelling force per unit volume.

Some of the more general properties and flow characteristics pertinent to the flow of nonhomogeneous fluids in porous media are considered in the following section.

THE MACROSCOPIC VELOCITY FIELD

The kinematics of the macroscopic velocity field [Equation (43)] is examined briefly using special terms from classical mechanics (Truesdell 1954; Truesdell and Toupin 1960). Through such kinematic relationships, considerably more insight into the flow fields involving nonhomogeneous fluids flowing in heterogeneous porous media is available. The purposes here, rather than to treat the kinematics exhaustively, are to gain greater insight on the interdependence of nonhomogeneous fluid related interactions. From such interaction terms may come measures of significance useful in better determining the importance in the flow systems of specific nonhomogeneous fluid effects.

Specifically sought are expressions that can efficiently describe the approximation error that would be involved if particular fluid property variations are assumed negligibly small.

The velocity characterized by Equation (43) is a rotational vector field arising from the nonhomogeneous fluid and heterogeneous permeability dependence; that is, the rotation or vorticity vector, $\bar{W}$, of the macroscopic velocity field is

$$\bar{W} = \text{curl } \bar{v} = - \left[\left(\nabla \left(\frac{k}{P\sigma\mu} \right) \right) \times (\nabla p + \rho g \nabla z) \right] - \frac{k}{P\sigma\mu} \left[(\nabla p) \times (g \nabla z) \right] \quad (44)$$

where $\bar{W}$ = the vorticity vector of the macroscopic velocity field,

1/T

$() \times ()$ = the vector cross product operator.

The first term to the right of the second equality sign in Equation (44) represents the rotational components induced by three effects: 1) the heterogeneity of the porous media (represented by the gradient component in terms of k), 2) the heterogeneity of the effective porosity (represented by the $P\sigma$ term), and (3) the rotational effect induced by the viscosity variation of the nonhomogeneous fluid as it interacts with the force field. The last term in Equation (44) involves primarily the fluid rotation induced by the density variations in the gravitational field arising directly from the fluid nonhomogeneity.

Of interest also, for comparison purposes, is the rotation or vorticity vector, $\bar{W}$, for the simplest reduced flow systems, for example where both the porous media and the fluid are homogeneous so k , $P\sigma$, μ , and ρ are all constants. Then in Equation (44), the first term or $\nabla(k/P\sigma\mu)$ is zero when the permeable material and if the fluid is homogeneous. The second term containing ∇p is also zero; hence the vorticity vector, $\bar{W}$, is identically zero or the velocity field is irrotational for conditions when both the media and fluid are homogeneous. Note what effect various combinations have upon the rotationality. If the porous media is heterogeneous and the fluid is homogeneous then

part of the first term remains, but the second term vanishes from the rotational vector. However, when the porous media is homogeneous, but the fluid is nonhomogeneous, then both the first and second terms in Equation (44) contribute to the vorticity. The retention of this second term in the vorticity expression whenever the fluid properties are nonhomogeneous or vary in space will be found hereafter as an important distinguishing feature of such velocity fields.

The classical type of velocity field occurring when a nonhomogeneous fluid flows in a heterogeneous porous media is shown by investigating the interdependence between the macroscopic velocity in translation $\bar{v}$ and the rotational velocity components or vorticity vector, $\bar{W}$, by forming:

$$\begin{aligned} \bar{W} \cdot \bar{v} = & \left\{ \left(\nabla \left(\frac{k}{p\sigma\mu} \right) \right) \times (\nabla p + \rho g \nabla z) + \frac{k}{p\sigma\mu} [(\nabla \rho) \times (g \nabla z)] \right\} \\ & \cdot \left[\frac{k}{p\sigma\mu} (\nabla p + \rho g \nabla z) \right] \end{aligned} \quad (45)$$

Expanding and rearranging the first triple scalar product in Equation (45) gives

$$\begin{aligned} \bar{W} \cdot \bar{v} = & \frac{k}{p\sigma\mu} \left\{ \left[\nabla \left(\frac{k}{p\sigma\mu} \right) \right] \cdot [(\nabla p - \rho g \nabla z) \times (\nabla p + \rho g \nabla z)] \right\} \\ & + \frac{k}{p\sigma\mu} [(\nabla \rho) \times (g \nabla z) \cdot (\nabla p + \rho g \nabla z)] \end{aligned} \quad (46)$$

But in the rearranged first triple scalar product on the right-hand side, the cross product of the impelling force with itself is zero so all of the effects of media heterogeneity and the viscosity variation caused by fluid nonhomogeneity vanish from the vector product, $\bar{W} \cdot \bar{v}$. But the density effect or variation remains because of the fluid nonhomogeneity. Specifically:

$$\overline{\mathbf{W}} \cdot \overline{\mathbf{v}} = \frac{k^2 g}{\rho^2 \sigma^2 \mu^2} \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right) \quad (47)$$

This remaining effect demonstrates again the special implications when the fluid density is spatially dependent or the fluid is effectively nonhomogeneous. If the fluid were homogeneous so that

$$\nabla \rho = 0 \quad (48)$$

then Equation (47) would reduce to

$$\overline{\mathbf{W}} \cdot \overline{\mathbf{v}} = 0 \quad (49)$$

which demonstrates that the vorticity vector, $\overline{\mathbf{W}}$, is orthogonal to the velocity, $\overline{\mathbf{v}}$, when the fluid is homogeneous. Equation (49) is precisely the necessary and sufficient condition for Equation (37) to describe a classical complex-lamellar velocity field (Truesdell 1954; Truesdell and Toupin 1960). The complex-lamellar velocity field is the classical field type that occurs when porous media is heterogeneous (Nelson 1966), even when the flow involves two immiscible fluids. However, if fluid nonhomogeneity is introduced, then from Equation (47) the vorticity vector is no longer orthogonal to the velocity, and in fact, instantaneous vortex lines and streamlines are not orthogonal; there is an interaction between them. The very special interaction caused by the fluid nonhomogeneity warrants further careful investigation and consideration.

This special interaction introduced by the nonhomogeneity of the fluid needs to be explained and its physical significance appreciated. This is done by considering Equation (47) and those previous expressions leading to it in greater detail. Using the definition of the dot vector product of the two vectors, $\overline{\mathbf{W}}$ and $\overline{\mathbf{v}}$, on the left-hand side of Equation (47), then:

$$|\overline{\mathbf{W}}| |\overline{\mathbf{v}}| \cos \theta = \frac{k^2 g}{\rho^2 \sigma^2 \mu^2} \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right) \quad (50)$$

where the angle θ is defined as the angle between the rotational or angular velocity vector, $\bar{W}$, and the translational velocity vector, $\bar{v}$. Equation (50) is more useful by rearranging to the form:

$$\cos \theta = \frac{g \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right)}{\left| \frac{\bar{W}}{\frac{k}{\rho \sigma \mu}} \right| \left| \frac{\bar{v}}{\frac{k}{\rho \sigma \mu}} \right|} = \frac{g \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right)}{|\bar{W}| |\bar{G}|} \quad (51)$$

To obtain the last equality of Equation (51), Equation (43) was rearranged and Equation (8) then used to give

$$\frac{\bar{v}}{\frac{k}{\rho \sigma \mu}} = -(\nabla p + \rho g \nabla z) = \bar{G} \quad (52)$$

or in other words the second term in the denominator of Equation (51) is the magnitude or absolute value of the impelling force per unit volume. Similarly, if $\bar{W}$ is defined and Equation (44) is used then:

$$\bar{W} = \frac{\bar{W}}{\frac{k}{\rho \sigma \mu}} = -[(\nabla(\ln \frac{k}{\rho \sigma \mu})) \times (\nabla p + \rho g \nabla z) + g(\nabla \rho) \times (\nabla z)] \quad (53)$$

where $\bar{W}$ is the rotational impelling force that appears as the first term in the denominator of the final expression in Equation (51). The entire denominator is the product of the magnitudes of the impelling rotational and translational forces at any particular point in the system where the fluid flow is being considered.

The numerator of Equation (51) is the square of the magnitude of the interaction between the angular and purely translational impelling forces. This interpretation comes from physically interpreting, in sequence, Equations (34), (44), (46), (50), (52), and (53). Beginning from Equation (34),

the nongravitational aligned or lateral components of the density gradient, i.e., $\frac{\partial \rho}{\partial x}$ and $\frac{\partial \rho}{\partial y}$, must both be zero in order for the impelling-force field (expressed on a unit volume basis) to be conservative. The identical condition of zero-lateral density gradients is precisely the requirement necessary to reduce the numerator of Equation (51) to zero. The density variation in the z-coordinate direction, which is aligned with the gravity, need not be zero to have a force field that is energy conserving. But the nonaligned or lateral x and y components of the fluid density gradient are vital to the conservance of the translational force field, whereas the z or gravitational aligned component has no effect.

The total rotation of the velocity field in Equations (44) and (46) is induced by the combination of nonconservative force effects arising from heterogeneous media effects and the variable fluid viscosity contribution. But in Equation (46) and as commented immediately thereafter, the major rotational increments arising from media heterogeneity, and the fluid viscosity variations as contained in the term, $\nabla(k/P\alpha\mu)$, go out of the equation as discussed previously. Only the small nonvanishing impelling-force field contributes to the rotational term, i.e., $(\nabla\rho) \times (g\nabla z)$ remains in Equation (46) and subsequently becomes the right-hand side of Equation (47) and (50). The last velocity related effects may be removed from Equation (50) through use of Equations (52) and (53); thus leaving in the numerator of Equation (51) only an impelling force. That impelling force has special significance in two ways. First, the numerator of Equation (51) represents physically the square of the magnitude of the small interaction force between the angular and purely translational impelling forces. Secondly, it is precisely this small interaction component in the numerator of Equation (51) that is set to zero by assuming the lateral components of the density gradient are negligibly small. This strongly suggests that Equation (51) is an appropriate expression to measure the significance of the lateral density gradient components.

Before considering the use of Equation (51) further as a significance measure, it is instructional to see what interaction terms result when the velocity is expressed in terms of the impelling force per unit mass of fluid.

Specifically, using the results from Column 6, Row 1 of Table 1 along with Equation (40) and also recalling Equation (7) yields for the fluid pore velocity, $\bar{v}$

$$\bar{v} = - \frac{k\rho}{p_{\sigma u}} \left(\frac{1}{\rho} \nabla p + g \nabla z \right) = \frac{k\rho}{p_{\sigma u}} (\bar{E}) \quad (54)$$

where the impelling force is now on a basis of per unit mass of fluid.

The rotational or vorticity vector designated by $\bar{W}_m$ is:

$$\bar{W}_m = \text{curl } \bar{v} = (\nabla) \times \left[- \frac{k\rho}{p_{\sigma u}} \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \right] \quad (55)$$

or expanding

$$\bar{W}_m = - \nabla \left(\frac{k\rho}{p_{\sigma u}} \right) \times \left(\frac{1}{\rho} \nabla p + g \nabla z \right) - \frac{k}{p_{\sigma u}} (\nabla \ln \rho) \times (\nabla p) \quad (56)$$

which is the vorticity vector of the velocity vector. The interaction term or effect between rotational and translational components is examined by forming the vorticity-velocity dot product, i.e.,

$$\begin{aligned} \bar{W}_m \cdot \bar{v} &= - \nabla \left(\frac{k\rho}{p_{\sigma u}} \right) \times \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \cdot - \frac{k\rho}{p_{\sigma u}} \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \\ &\quad - \frac{k\rho}{p_{\sigma u}} (\nabla (\ln \rho)) \times (\nabla p) \cdot - \frac{k\rho}{p_{\sigma u}} \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \end{aligned} \quad (57)$$

But rearranging the order of the first triple dot products and collecting terms yields:

$$\begin{aligned} \overline{W}_m \cdot \overline{V} = & \frac{k\rho}{\rho^2 \sigma_\mu^2} \left[\nabla \left(\frac{k\rho}{\rho \sigma_\mu} \right) \cdot \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \times \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \right] \\ & + \frac{k^2 \rho^2}{\rho^2 \sigma_\mu^2} \left[\left(\nabla (\ln \rho) \times (\nabla p) \right) \cdot \left(\frac{1}{\rho} \nabla p + g \nabla z \right) \right] \end{aligned} \quad (58)$$

The first term on the right hand side of Equation (58) drops out because the cross product of any vector with itself is always zero, and then expanding the second term gives

$$\overline{W}_m \cdot \overline{V} = \frac{k^2 g}{\rho^2 \sigma_\mu^2} \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right) \quad (59)$$

which is identical to Equation (47), so the interaction of the rotational and translational velocity expressions is the same when the fluid-pore impelling force is expressed either on a basis of unit volume or mass. This same fundamental interaction term is also obtained when the impelling force is expressed as the force per unit weight of fluid and further suggests its use as a basic significance ratio for the lateral density gradient components.

A dimensionless significance measure of fluid density variations is provided through use of the commonly based interaction term [Equation (47) or (59)] in combination with the volumetric impelling force description. This significance measure ratio is from Equations (51), (52), and (53):

$$S_d = \cos \theta = \frac{g \left(\frac{\partial \rho}{\partial y} \frac{\partial p}{\partial x} - \frac{\partial \rho}{\partial x} \frac{\partial p}{\partial y} \right)}{\left| \left(\nabla \left[\ln \left(\frac{k}{\rho \sigma_\mu} \right) \right] \right) \times (\nabla p + \rho g \nabla z) + g (\nabla \rho) \times (\nabla z) \right| \left| \nabla p + \rho g \nabla z \right|} \quad (60)$$

where S_d is the dimensionless significance ratio for the lateral density gradient effects on flow as induced by fluid nonhomogeneity. When the lateral fluid density gradient components are zero, i.e., $\frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial x} = 0$, then the significance measure ratio is zero. If the lateral density gradients are not zero, then S_d directly reflects the effects of those density gradient

magnitudes present. As described previously, the numerator of the significance ratio represents physically the square of magnitude of the interaction force between the rotational and translational impelling forces. The interaction force is present only when lateral density gradients are present. The denominator or scaling forces in the significance ratio is the product of the magnitudes of the total rotational and translational impelling forces. Therefore, the significance ratio is really the square of the fluid density interaction force between the rotational and translational components expressed as a fraction of the total combined magnitudes of the rotational and translational impelling forces.

It will be necessary through computational experiments to gain experience concerning at what magnitude of the significance ratio does the lateral density gradient component become important. For example, the needed experience could be obtained if several problems with a wide range of lateral density gradients were solved, the significance ratios calculated throughout, and then the same problems solved again but this time with the lateral gradient components set equal to zero. By comparing the flow system results with and without lateral density gradients as the significance ratios increased, tolerance limits on the ratio could be set for later use on a much wider range of problems.

Additional kinematic results could perhaps be derived here that would be instructional, though the most of those primary theoretical results have been obtained. Accordingly, it is believed most useful to summarize the main results, suggest how those findings may be pertinent to the BWIP, and illustrate some of the effects the nonhomogeneous fluids have on flow in porous media.

OVERVIEW OF THEORETICAL RESULTS AND SOME RECOMMENDATIONS FOR THEIR USE

The previous sections have provided a broad and rather complete framework for considering theoretically the special features introduced into flow in porous media by nonhomogeneous fluids. The discussion has tended to involve considering in minute detail many individual facets in order to ensure a thorough theoretical framework. Though no apology is meant for this intentional pursuit of thoroughness and completeness, certainly not all that has been discussed is of the same importance in understanding the flow of nonhomogeneous fluids.

The purpose of this section is to present an overview of the theoretical pathway followed and to highlight the important findings and pertinent results. Where appropriate, suggestions and recommendations will be made concerning the findings and the best ways of utilizing them.

The background was first considered in detail, which led to the six general Darcian expressions for the macroscopic fluid flux vectors provided in Table 1. Specifically, the Darcian flux could be expressed in two forms, either as the volumetric fluid flux per unit area or as the mass of fluid flux per unit area. Each of these two expressions for the fluid flux have in turn three different equation forms for describing the impelling-force field that is acting on the fluid. The force field may be described in any one of the three forms, either as a force per unit mass, force per unit volume, or as force per unit weight of fluid. Since each of these three impelling force descriptions can be used to build the two fluid flux expressions, the total is the six Darcian fluid flux equations as shown in Table 1.

Only the three impelling force equations that all lead to the volumetric flux were considered further, i.e., those in the first three rows of Table 1. Only the volumetric flux expressions enable direct evaluation for contaminant flow paths and travel times in subsurface pollution problems. Accordingly, further evaluation was limited to the three Darcian equations expressed in terms of either a force per unit mass, volume, or unit weight of fluid as in Equations (7), (8), and (9), respectively.

The impelling force parts of the three Darcian volumetric flux equations were next carefully considered to see if an energy potential could be shown to exist and to be valid when the fluid is nonhomogeneous. An energy conserving potential can exist only when there are no rotational force components in the force field. When the fluid is nonhomogeneous, the density gradients present were found in the general case to cause rotational components in the force field. Accordingly, in general no energy potential exists when the fluids are nonhomogeneous. This in-depth investigation of all three of the impelling-force fields conclusively demonstrated that when nonhomogeneous fluids are involved in porous media flow, a scalar energy potential is not usable for analysis. In fact, in all but a few very restricted special situations, a single scalar energy potential can neither be defined nor be realistically evaluated as a meaningful analysis tool for flow when a nonhomogeneous fluid is involved.

When the fluids are nonhomogeneous, i.e., the fluid properties change with location in the flow system, there are, in general, not one but two scalar functions required to determine the impelling-force vector field. They are the scalar fluid pressure and the scalar fluid density distributions. With both the fluid pressure and the density distribution available, any of the three impelling force field descriptions are appropriate.

During consideration of those very special situations where an energy potential might be defined, one of the impelling-force forms was found potentially more useful for an approximate class of problems. When the impelling force is described as the force per unit volume of fluid as in Equations (8) and/or (43), and further if the lateral gradients component of the fluid density are small enough that they may be assumed to be zero, i.e.,

$$\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = 0 \quad (61)$$

then a special scalar potential, ϕ' , was shown to exist from Equation (39), i.e.:

$$\phi' = p + g \int_{z_0}^z \rho(z) dz' \quad (62)$$

where ϕ' is the special energy scalar that exists when Equation (61) is satisfied and $\rho(z)$ is the vertical distribution of the density when the z axis is parallel with the gravitational field.

With the special potential shown to exist, the next question is under what practical conditions can Equation (62) be evaluated and used, i.e., what is required for the integration to be performed to provide the complete and useful scalar energy potential. The integration may be performed (numerically if necessary) if the vertical density variation $\rho(z)$ is known either as a function, as numerical results, or as a measured data set from the actual problem. If the vertical density variation is not known in one of these forms, the approximate potential exists as an integral but cannot be evaluated so still is of very limited utility.

One of the important situations where the integral may be evaluated is in the interpretation of field measurements. If the vertical density variation were adequately measured, then numerical integration could likely provide the approximate scalar energy potential; provided, of course, the lateral density gradient components were small enough and could be demonstrated small enough to be considered negligible. Perhaps then in the interpretation of field data, if density variation is essentially vertically dominant, i.e., lateral components are negligible, and the vertical variation is measured, then a useful approximate scalar energy potential may be obtained.

Unfortunately, attempting to use the approximate energy potential to formulate predictive models is generally unsuccessful. The needed density variation to allow integration of the vertical term in Equation (61) is not available or known a priori. The vertical variation in density is actually one of the results to be determined in the modeling process if the fluid is non-homogeneous. Without the approximate energy potential known in integral form, some arbitrary functional forms must be selected for the vertical density variation, and later in the solution by some means the coefficients must be

determined or some iterative procedure must be performed in order to accomplish the solution. All such procedures are very cumbersome and time consuming. More important, after all of these special and intricate procedures are implemented, a flow model capable of solving only one small class of flow problems involving nonhomogeneous fluids still may be solved. Specifically, the model could study only those special systems in which the lateral density gradient components are small enough to be realistically ignored. Further, when faced with modeling a particular problem, it would be difficult, if not impossible, to determine if the lateral fluid density gradients are small enough to be ignored. Such restrictions on the modeling process are both unrealistic and unacceptable in view of the moderately simple remedy.

A recommended remedy long followed in the petroleum reservoir engineering field and completely consistent with the above theory for modeling the flow of nonhomogeneous fluids should be adopted and used. It involves replacing the single energy scalar potential, which, though very useful in groundwater analysis when the fluids are homogeneous, must be replaced in analysis, with two scalar functions when modeling flow systems with nonhomogeneous fluids. The two scalar functions required are the fluid pressure used as a scalar potential and the spatial density distribution. Both the fluid pressure and the fluid density distribution are obtained during solution of the coupled general set of boundary value problems describing the flow of nonhomogeneous fluids in porous media. This recommended approach provides much more versatility in modeling by allowing the model formulation to use any one of the impelling force descriptions. It is not restricted in terms of the variation in fluid density in any of the coordinate directions and provides maximum flexibility in treating the variety of problems encountered in terms of boundary conditions and in handling the many effects associated with nonhomogeneous fluids.

Although the approximate energy potential is in general inappropriate for use as the direct solution variable, when the flow model involves nonhomogeneous fluids, that approximation can be helpful in some cases. To illustrate this helpful interpretive use of the approximate energy potential, the following analysis sequence is recommended.

Suppose that the general pressure-density formulation recommended above for handling the flow of nonhomogeneous fluids is the basis for the model being used. Suppose also that the description of the impelling force used in the model formulation was expressed upon the basis of unit volume of fluid so that Equations (43), (52), (53), and (60) are all compatible with the model formulation being used to solve the particular problem of interest. The user of the model also expects that the predominant density variations may be induced by the vertical or z-coordinate-oriented geothermal gradient. Using the general pressure-density based code, the problem of specific interest is solved yielding, among other things, the fluid pressure and density variations throughout the flow region. Also known as part of the original input to the problem or obtained as part of the model solution of the problem are the hydraulic conductivity variation, k ; the variation in effective porosity, P_0 ; and the spatial distribution of the fluid viscosity, μ . Using the just mentioned spatially varying system parameters and fluid properties as required in Equation (60), the significance ratio, S_d , could be calculated at selected points or perhaps more conveniently on a regular grid over the entire modeled region. If the calculated lateral density gradient significance ratios over the modeled region were generally greater than the tolerance values of the calibrated significance ratio,^(a) then the lateral density gradient components would be expected to significantly affect the overall flow system so no approximate energy potential should be calculated. However, for the opposite condition, i.e., if the ratios over the modeled region were consistently smaller than the value of the calibrated significance tolerance ratio,^(a) then neglecting the small lateral density gradients components is likely a good approximation. Since the lateral gradients are small enough to be ignored, i.e., $\frac{\partial \rho}{\partial x} = \frac{\partial \rho}{\partial y} = 0$, which satisfies Equation (61), an approximate scalar energy potential, as in Equation (62), can be approximately used. More important, the approximate scalar energy potential, ϕ' , can be realistically evaluated because the vertical dependence, $\rho(z)$, is known from the numerical model solution.

(a) See the first and second paragraph following Equation (60) suggesting how the needed calibrated significance tolerance ratio values may be obtained.

Conceptually, in the way just outlined, the energy potential could be provided whenever the lateral density gradients are found to be small enough that providing an approximate energy potential is appropriate.

The approximate potential obtained from the method just outlined could then be used in all of the beneficial ways a traditional energy potential is utilized, although the approximate potential is strictly valid only when the fluid is homogeneous. If the use of the approximate potential is found appropriate for Hanford, by continued measurements at the site that show the lateral density gradient components are small, then the significance ratio concept should be developed further. Specifically, the calibration and numerical experiments to establish tolerance levels as outlined previously should be carried out.

The significance ratio for the lateral density gradient components was only derived for the first time and its potentiality discussed in this report. As already suggested in the discussion following Equation (60), numerical experiments involving flow systems with a wide range of lateral density gradients will be required to provide a usable range of significance ratio tolerance values. Once sufficient tolerance values have been obtained, there will also need to be considerable experience gained in the actual application before the complete utility will be fully evaluated.

In view of the likely usefulness of these significance ratios at Hanford, the author recommends that the ratio tolerance limit values be obtained through planned numerical experiments. Unfortunately, the one flow system that will be considered in some detail in the next section has only a vertical density and viscosity gradient induced by a geothermal gradient. Accordingly, the lateral components of the density gradient are identically zero so that flow system cannot provide even an initial value of the significance ratios.

ILLUSTRATION OF SOME NONHOMOGENEOUS FLUID EFFECTS IN AN EXAMPLE SUBSURFACE FLOW SYSTEM

This section shows the influence of nonhomogeneous fluid on subsurface flow, under conditions similar to those being observed in the deeper Hanford groundwater system. This is done by considering an example vertical, two-dimensional flow system. In the example flow system there is a vertical, geothermally induced temperature profile similar to the magnitudes observed to date by the BWIP on the Hanford Site. The vertical temperature profile introduces both density and viscosity fluid nonhomogeneities into the example flow system that will be used to illustrate such effects on flow.

It was possible to obtain an analytical closed-form solution describing flow in the example subsurface system containing the typical vertical temperature profile. The solution equations describing the nonhomogeneous fluid density and viscosity effects are presented in Appendix A. Evaluation of the solution equations provides a variety of specific flow system results, which include the effects of fluid density and viscosity nonhomogeneities. Liberal use of graphical displays of the various evaluation results helps in interpreting the flow results and in later comparisons.

A second analytical closed-form solution was also obtained and is presented in Appendix B. This second solution describes simpler fluid conditions but involves the same porous media and flow region as in the first solution. The second solution is for a constant average temperature throughout the flow system so there are no fluid density and viscosity nonhomogeneity effects. Other than the temperature induced effects, the second solution involved the same porous material, same flow region, and same pressure boundary conditions as were used in the first analytical solution. Accordingly, comparison of the flow results from the two analytical solutions is a good basis for illustrating the influence of nonhomogeneous fluid effects in the original example flow system.

In the material to follow, the example flow system with the geothermal vertical temperature profile is considered to indicate the magnitudes of fluid

nonhomogeneities involved. This is followed with comparisons of the impelling force, fluid flux, and some steady flow paths of evaluations from the two analytical flow system results.

EXAMPLE FLOW SYSTEM WITH VERTICAL TEMPERATURE EFFECTS

The first analytical solution involves a vertical, geothermal temperature profile selected to resemble somewhat the deeper Hanford groundwater flow system. The flow is steady in the vertical region with predominantly horizontal flow but with some vertical movement. The pressure boundary conditions have a small but consistent lateral variation and large vertical pressure changes that are consistent for a vertical flow system in the gravity field.

Figure 2 shows the contours of pressure obtained from evaluating the first analytical solution. The contours in the figure appear as essentially horizontal lines because of the much larger gravitational-related pressure changes with elevation. Actually, a consistent horizontal pressure variation is present as is observed by considering the numerical pressure values, which are the fluid pressures in kilopascals on a grid appearing in the figure. These seemingly small horizontal pressure differences become the predominantly horizontal flux components that will be observed in later results. The very large vertical pressure changes are opposed by the almost equally large gravity induced fluid body forces with a net result that relatively small vertical flux components occur in the flow system. Also in Figure 2, the largest pressure numbers occur in each horizontal line of numerical values at a distance of $x = 200$ m. A stagnation point may occur in this vicinity and should be watched for in subsequent results. It will be useful to observe the interactions that tend to occur near the stagnation area as the result of fluid nonhomogeneities.

The temperature variations in the example flow system involve a vertical temperature difference of almost 30°C . Along the bottom of the flow system, the temperature is 55°C or corresponding roughly to the temperature at 1100 to 1200 m below the land surface at Hanford. Along the top elevation of 800 m in the example problem, the water temperature is 25.65°C . There are no

FIGURE 2. Fluid Pressure Contours and Some Values in Kilo-Pascals When There Is a Vertical Temperature Profile

temperature variations horizontally in the example flow system. Only gradually varying temperature changes with the elevation occur, as is shown in Figure 3. The geothermal gradient ranges from a value of -0.030°C/m at the bottom, to -0.035°C/m at an elevation of 400 m, to a highest value of -0.050°C/m at the top elevation of 800 m.

Also shown in Figure 3 are the temperature-induced vertical variations of the fluid density and viscosities for the example problem obtained from the first analytical solution. The maximum change in fluid density from 986.48 kg/m^3 at the bottom to 997.70 kg/m^3 at the top represents a 1.13% change over the entire problem height. Also shown in Figure 3 is the vertical variation in fluid viscosity from bottom to top of 5.042 to 8.771 Pa·s respectively, or a change of 42.5%. The viscosity then, in general, is observed to involve changes larger than the density. Caution, however, should be exercised in considering only the relative changes between the fluid viscosity and density because their overall affect on flow is drastically different.

FIGURE 3. The Fluid Temperature, Density, and Viscosity Changes with Elevation in the Example Flow System

The fluid viscosity enters the analysis as an inverse scalar multiplier so it can only enlarge (as the viscosity decreases) or contract (as viscosity increases) the magnitude of the final fluid flux at any given point in the flow system. In contrast, the density only contributes in a minor way as a slight scalar multiplier, but its predominant and important effect on flow occurs within the impelling-force vector. In particular, the density variation first tends to be amplified by the gravitational scalar constant, but far more important are changes in the flow direction induced by the density variations. It is just such directional changes of the force vector field that make the stable or unstable patterns of density variations so important to the fluid flow pattern which will occur.

Having gained some insight concerning the nature of the example problem results in terms of fluid pressure, temperature, density, and viscosity, attention is next directed to the comparisons. The results from the first

analytical solution involving nonhomogeneous fluid results will be compared to results of the second solution for a homogeneous fluid where the temperature is a constant average value throughout the system.

ILLUSTRATION OF SOME NONHOMOGENEOUS FLUID-INDUCED FLOW EFFECTS

To provide some initial insight to the flow system effects introduced by nonhomogeneous fluids, comparisons are made between the results obtained from evaluating the two analytical solutions. The results of the second analytical solution show the flow system when no temperature effects are present so the fluid is homogeneous. The results from evaluating the first solution involve nonhomogeneous fluid effects of the amounts already presented in Figures 2 and 3.

Fluid nonhomogeneity effects on the flow will be considered on two sets of local or point by point sets of results and one more global or integrating set of results. Specifically compared will be:

- the impelling-force vector fields
- the fluid flux vector fields
- the steady fluid pathlines or streamlines

The first two comparisons are made at a typical number of points over the example flow problem while the last comparison is an integrated comparison of nonhomogeneity effects on the flow system.

Figure 4 enables comparing the impelling-force fields with each other, expressed on a basis of force per unit volume of fluid. Any differences observed between Figure 4a and 4b are induced by the fluid density variation that tends to alter the vector direction. In the figure, the best agreement tends to occur at the middle elevations or where the temperatures are both at about 40°C. Below or toward the bottom, in the case of the nonhomogeneous fluid, the buoyance effects of the warmer, less dense fluid are observed as the larger vertical force components that occur in the nonhomogeneous fluid system.

a. Homogeneous Fluid at 40°C

b. Nonhomogeneous Fluid at 55°C to 25°C

FIGURE 4. Point-by-Point Comparison of Impelling-Force Fields

A downward, density-induced effect is observed near the top of the non-homogeneous fluid system. Greater downward movement of the components of the impelling-force field occurs because the denser heavy fluid tends to settle. One also recalls from the form of volume-based flux, Equation (43), used here, that all of the fluid density-induced effects on flow are acting in the impelling force.

The variations in fluid viscosity for nonhomogeneous fluids act only as inverse scalar multipliers so the effects are demonstrated best by comparing the fluid flux results as shown in Figure 5. Specifically, the additional change that occurs beyond that which already was found in the impelling-force field comparison is caused by the fluid viscosity variations arising from fluid nonhomogeneities. Though these increased differences are more difficult to observe, they are present and are easiest to observe when considering the lowermost right-hand vector in Figures 4a, 4b, 5a, and 5b, where the largest differences occur.

The last comparison of pathlines or streamlines is a more global or integrated way of contrasting the two systems to show the effect of nonhomogeneous fluids. The steady pathlines or streamlines are seen in Figure 6. The flow-paths for fluid particles originating around the boundary and a few on the interior at crucial locations provide a rather good indication of the overall effects on flow of fluid nonhomogeneities. Generally, the effects already observed and discussed are seen to prevail. Buoyance and denser settling force components are larger when the fluid is nonhomogeneous. Also a slight tendency toward circulation cell formation is noted. The more gently curved and sweeping flow paths occur when the fluid is homogeneous. Definite shifts in the stagnation point for the flow systems are seen, perhaps again related to the circulation cell development tendency present in nonhomogeneous fluid systems.

This completes the cursory illustration of the significance of some density and viscosity fluid nonhomogeneity effects caused by a vertical temperature profile. Some insight and experience toward better understanding the flow effects caused by nonhomogeneous fluids has been provided.

a. Homogeneous Fluid at 40°C

b. Nonhomogeneous Fluid at: 55°C to 25°C

FIGURE 5. Point-by-Point Comparison of Fluid Flux Fields

a. Homogeneous Fluid at 40°C

b. Nonhomogeneous Fluid at 55°C to 25°C

FIGURE 6. Comparison of Steady Fluid Pathlines or Streamlines

a. Temperature dependence of phase behavior

b. Composition dependence of phase behavior

Figure 2. Comparison of the phase behavior of the polymer blend at 25°C and 40°C.

REFERENCES

- Baveye, P., and G. Sposito. 1984. "The Operational Significance of the Continuum Hypothesis in the Theory of Water Movement Through Soils and Aquifers." Water Resour. Res. 20(5):520-530.
- Baveye, P., and G. Sposito. 1985. "Macroscopic Balance Equations in Soils and Aquifers: The Case of Space- and Time-Dependent Instrumental Response." Water Resour. Res. 21(2):1116-1120.
- Campbell, G. S., and J. Van Schilfagrade. 1981. "Use of SI Units in Soil Physics." J. of Agron. Educ. 10:73-74.
- Currie, L. G. 1974. Fundamental Mechanics of Fluids. Sections 1.4, 1.5 and 1.7. McGraw-Hill, Minneapolis, Minnesota.
- Cushman, J. H. 1984. "On Unifying the Concepts of Scale, Instrumentation, and Stochastics in the Development of Multiphase Transport Theory." Water Resour. Res. 20(11):1668-1676.
- Dagan, G. 1982. "Stochastic Modeling of Groundwater Flow by Unconditional and Conditional Probabilities. 2. The Solute Transport." Water Resour. Res. 18(4):835-848.
- Frisch, H. L., and J. M. Hammersley. 1963. "Percolation Processes and Related Topics." J. Soc. Ind. Appl. Math. 11(4):894-918.
- Frisch, H. L., V. A. Vysotsky, S. B. Gordon, and J. M. Hammersley. 1961. "Critical Percolation Probabilities (Bond Problem)." Phys. Rev. 123(5):1566-1567.
- Frisch, H. L., S. B. Gordon, V. A. Vysotsky, and J. M. Hammersley. 1962. "Monte Carlo Solution of Bond Percolation Processes in Various Crystal Lattices." Bell Telephone Tech. J. 41:909-920.
- Gelhar, L. W., and C. L. Axness. 1983. "Three-Dimensional Stochastic Analysis of Macrodispersion in Aquifers." Water Resour. Res. 19(1):161-180.
- Gelhar, L. W., A. L. Gutjahr, and R. L. Naff. 1979. "Stochastic Analysis of Macrodispersion in a Stratified Aquifer." Water Resour. Res. 15(6):1387.
- Hubbert, M. K. 1956. "Darcy's Law and the Field Equations of the Flow of Underground Fluids." Tran. Am. Inst. Mining Met. Petrol. Engrs. 207:222-239.
- Lass, H. 1950. Vector and Tensor Analysis. McGraw-Hill, New York, New York.
- Matheron, G., and G. De Marsily. 1980. "Is Transport in Porous Media Always Diffusive? A Counterexample." Water Resour. Res. 16(5):901-917.

- Molz, F. J., O. Guven, and J. G. Melville. 1983. "An Examination of Scale-Dependent Dispersion Coefficients." Groundwater 21(6):715.
- Nelson, R. W. 1960. "In-Place Measurement of Permeability in Heterogeneous Media. I. Theory of a Proposed Method." J. Geophys. Res. 65(6):1753-1758.
- Nelson, R. W. 1966. "Flow in Heterogeneous Porous Mediums. 1. Darcian-Type Description of Two-Phase Systems." Water Resour. Res. 2(3):487-495.
- Prager, S. 1961. "Viscous Flow Through Porous Media." Phys. Fluids 4(12):1477-1482.
- Rose, D. A. 1979. "Soil Water: Quantities, Units, and Symbols." J. of Soil Sci. 30:1-15.
- Rose, H. E. 1945. "An Investigation into the Laws of Flow of Fluids Through Beds of Granular Materials." Inst. Mech. Engs. 153:141-48.
- Simmons, C. S. 1982. "A Stochastic-Convective Transport Representation of Dispersion in One-Dimensional Porous Media Systems." Water Resour. Res. 18(4):1193-1214.
- Simmons, C. S. 1983. A Stochastic-Convective Ensemble Method for Representing Dispersive Transport in Groundwater. CS-2558, prepared by Battelle, Pacific Northwest Laboratories for the Electric Power Research Institute, Palo Alto, California.
- Sposito, G., V. K. Gupta, and R. N. Bhattacharya. 1979. "Foundation Theories of Solute Transport in Porous Media: A Critical Review." Adv. Water Resour. 2:59-68.
- Sposito, G., and S. Y. Chu. 1981. "The Statistical Mechanical Theory of Groundwater Flow." Water Resour. Res. 17(4):885-892.
- Truesdell, C. 1954. The Kinematics of Vorticity. Indiana University Science Series No. 19, Indiana University, Bloomington, Indiana.
- Truesdell, C., and R. A. Toupin. 1960. "The Classical Field Theories." In Handbook of Physics, Vol. 3(1), pp. 226-793. Springer-Verlag, Berlin, Germany.
- U.S. Department of Energy (DOE). 1982. Site Characterization Report for the Basalt Waste Isolation Project. DOE/RL 82-3, prepared by Rockwell Hanford Operations for the U.S. Department of Energy, Washington, D.C.
- Wooding, R. A. 1959. "The Stability of a Viscous Liquid in a Vertical Tube Containing Porous Material." Royal Soc. London Proc. 252:120-134.
- Yih, Chi-Shun. 1965. Dynamics of Nonhomogeneous Fluids, Chapter 5. McMillan, New York, New York.

APPENDIX A

AN ANALYTICAL SOLUTION USEFUL IN
EVALUATING THE FLOW EFFECTS OF NONHOMOGENEOUS
FLUID FLOW IN POROUS MEDIA

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AN ANALYTICAL SOLUTION USEFUL IN EVALUATING THE FLOW EFFECTS OF NONHOMOGENEOUS FLUID FLOW IN POROUS MEDIA

This appendix presents an analytical solution for subsurface flow in a two-dimensional cross section with a vertical temperature profile. It is somewhat similar to that measured in the deeper Hanford groundwater flow system by the BWIP and discussed in the main body of the report in the section titled, "Illustration of Some Nonhomogeneous Fluid Effects in an Example Subsurface Flow System." To provide such a solution, the approach starts with a simple polynomial fluid pressure distribution that, by inspection, provides the general background flow conditions for the cross section. Darcy's equation for the fluid flux with the impelling force expressed on the basis of a unit volume is next written. Contained within the Darcian expression are the fluid density and viscosity that, in the case of a nonhomogeneous fluid, are functionally dependent on the temperature. The functional expressions for the density and viscosity dependence of water on temperature are readily available.

The Darcian expression is next substituted into the two-dimensional conservation of mass equation yielding a rather complicated overall equation. However, if the temperature is restricted to only vertical variations in the z-coordinate direction, the rather complicated conservation of mass equation can be reduced to a first-order ordinary differential equation in the unknown temperature, T , and the vertical coordinate, z . Being only of first order and an ordinary differential equation even though rather involved, the expression can be integrated term by term thus yielding the final temperature dependence needed. The final thermal equation is the z-coordinate elevation expressed as a transcendental function of the temperature, T , which is not a particularly desired form of dependence. However, numerical evaluation of the expression provides easily used tabular results for the temperature, T , dependence on the elevation, z .

Rather than lead the reader through the extended process of reduction and integration just outlined, a simpler presentation will be followed. The four analytical equations representing the solution will be written. Next the partial differential, which must be satisfied, to describe the nonhomogeneous fluid flow in the vertical rectangular cross section will be obtained. Then it will be demonstrated that the solution equations previously written do indeed satisfy the general equation. Finally, the boundary conditions satisfied around the vertical rectangular flow region will be provided.

Consider a vertical, two-dimensional saturated flow system in porous media within a region defined by a rectangular x z coordinate system. The z -coordinate axis is oriented vertically and parallel with, but directed opposite to, the gravitational vector. The flow system of interest lies in the rectangular region:

$$0 \leq x \leq 1000 \text{ meters}$$

and (A.1)

$$0 \leq z \leq 800 \text{ meters}$$

THE ANALYTICAL SOLUTION EQUATIONS

Within the porous fluid flow region of Equation (A.1), the solution for the fluid pressure, p , is defined by

$$p = \frac{1}{a} x - \frac{1}{2b} x^2 - Cz + D \quad (A.2)$$

The vertical temperature solution, T , but expressed as the elevation, z , as a transcendental function of T , is:

$$\begin{aligned}
z = & 2bg\rho_0\alpha T - Cb \{ \ln[1 + \alpha(T - T_0)] \} + \{ Cb - \rho_0 gb[1 + \alpha(T - T_0)] \} \ln \mu \\
& + \rho_0 gb\alpha \{ \ln(\mu_0) T + 19.993(A' - 19.993B') \ln(T + C') \\
& - (A' - 39.986B') [T - C' \ln(T + C')] - B' \left[\frac{1}{2} (T + C')^2 \right. \\
& \left. - 2C'(T + C') + (C')^2 \ln(T + C') \right] \} + C''
\end{aligned} \tag{A.3}$$

The fluid nonhomogeneity introduced by the vertical temperature profile of Equation (A.3) arises from the fluid mass density, ρ , dependence on temperature which is represented here in the form

$$\rho = \rho_0[1 + \alpha(T - T_0)] \tag{A.4}$$

and is part of the total analytical solution. Similarly, the last part of the solution is provided in the general functional dependence of the fluid viscosity, μ , on the temperature, i.e.,

$$\ln \mu = \ln \mu_0 - \frac{A' (T - 19.993) + B' (T - 19.993)^2}{T + C'} \tag{A.5}$$

where in the solution Equations (A.2) through (A.5):

p = the fluid pressure	F/L^2
a = a parameter in the pressure solution equation	L^3/F
A' = a coefficient in the viscosity Equation (A.5)	$1/\text{Degrees}$
b = a parameter in the pressure solution equation	L^4/F
C = a parameter in the pressure solution equation	F/L^3
D = a parameter in the pressure solution equation	F/L^2
g = the scalar gravitational constant	L/T^2
ρ_0 = the fluid mass density at reference temperature	M/L^3
α = the density temperature coefficient	$1/\text{Degrees}$
T = the temperature	Degrees
$\ln$ = the natural logarithm to the base e	Dimensionless
T_0 = the reference temperature at which ρ_0 is defined	Degrees

μ = the fluid viscosity that is temperature dependent	FT/L ²
μ_0 = the fluid viscosity at a base temperature	FT/L ²
B' = a coefficient in the viscosity Equation (A.5)	1/Degrees ²
C' = a coefficient in the viscosity Equation (A.5)	Degrees
C'' = a constant of integration in Equation (3)	L
ρ = the fluid mass density	M/L ³

The solution and definitions for the vertical rectangular example flow problem are now complete. The basic partial differential that the solution Equations (A.2), (A.3), (A.4), and (A.5) together must satisfy will be obtained next.

THE PARTIAL DIFFERENTIAL EQUATION FOR THE EXAMPLE PROBLEM

The dynamic equation for describing the flow of a nonhomogeneous fluid in porous media with the impelling force expressed as force per unit volume of fluid is the equation from Column 6, Row 2 in Table 1 in the body of the report. That is:

$$\bar{q}_v = - \frac{k}{\mu} (\nabla p + \rho g \nabla z) \quad (A.6)$$

where $\bar{q}_v$ = the Darcian volumetric flux	L/T
k = the intrinsic permeability of the media	L ²
μ = the dynamic viscosity that depends on temperature	FT/L ²
p = the fluid pressure	F/L ²
ρ = the temperature dependent fluid density	M/L ³
g = the gravitational scalar	L/T ²
z = the vertical Cartesian coordinate, and	L
∇ = the "del" or gradient mathematical operator	1/L

Note that the impelling force in Equation (A.6) is expressed on the basis of unit volume.

The conservation of mass equation for the example problem is

$$\nabla \cdot (\rho \bar{q}_v) = 0 \quad (\text{A.7})$$

since the flow is steady. So substituting Equation (A.6) into (A.7) gives:

$$\nabla \cdot \left[-\frac{k\rho}{\mu} (\nabla p + \rho g \nabla z) \right] = 0 \quad (\text{A.8})$$

But the intrinsic permeability is constant so dividing through by $-k$ and expanding gives for Equation (A.8):

$$\nabla^2 p + \frac{1}{\rho} \nabla \rho \cdot \nabla p - \nabla(\ln \mu) \cdot \nabla p + 2g \nabla \rho \cdot \nabla z - \rho g \nabla(\ln \mu) \cdot \nabla z = 0 \quad (\text{A.9})$$

The functional dependence in Equations (A.2), (A.3), (A.4), and (A.5) are:

$$\begin{aligned} \rho &= \rho(x, z) \\ \rho &= \rho(T) \\ \mu &= \mu(T) \\ T &= T(z) \end{aligned} \quad (\text{A.10})$$

Consequently, operating on the dependence in Equation (A.10) gives

$$\begin{aligned} \nabla^2 p &= \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} \\ \nabla \rho \cdot \nabla p &= \frac{\partial \rho}{\partial T} \nabla T \cdot \nabla p = \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial z} \frac{\partial p}{\partial z} \\ \nabla \ln \mu \cdot \nabla p &= \frac{\partial \ln \mu}{\partial T} \nabla T \cdot \nabla p = \frac{\partial \ln \mu}{\partial T} \frac{\partial T}{\partial z} \frac{\partial p}{\partial z} \\ \nabla \rho \cdot \nabla z &= \frac{\partial \rho}{\partial T} \nabla T \cdot \nabla z = \frac{\partial \rho}{\partial T} \frac{\partial T}{\partial z} \\ \nabla \ln \mu \cdot \nabla z &= \frac{\partial \ln \mu}{\partial T} \frac{\partial T}{\partial z} \end{aligned} \quad (\text{A.11})$$

and substituting Equations (A.11) into (A.9) gives, on reducing,

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} + \left[g \left(2 \frac{\partial \rho}{\partial T} - \rho \frac{\partial \ln \mu}{\partial T} \right) + \left(\frac{1}{\rho} \frac{\partial \rho}{\partial T} - \frac{\partial \ln \mu}{\partial T} \right) \frac{\partial p}{\partial z} \right] \frac{\partial T}{\partial z} = 0 \quad (\text{A.12})$$

which is the partial differential equation that must be satisfied in order to have a valid solution for the example flow problem.

THE PARTIAL DIFFERENTIAL EQUATION IS SATISFIED

Recalling Equation (2) in the text for the pressure, and differentiating yields the expressions:

$$\begin{aligned} \frac{\partial p}{\partial z} &= -C \\ \frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial z^2} &= \frac{-1}{b} \end{aligned} \quad (\text{A.13})$$

Substitution of Equation (A.13) into (A.12) gives, on reducing, rearranging, and recognizing that since the temperature, T , is dependent only on z so that $\frac{dz}{dT}$,

$$\frac{dz}{dT} = 2bg \frac{\partial \rho}{\partial T} - \frac{Cb}{\rho} \frac{\partial \rho}{\partial T} + b(C - \rho g) \frac{\partial \ln \mu}{\partial T} \quad (\text{A.14})$$

But differentiation of Equations (A.4) and (A.5) with respect to the temperature, T , gives respectively:

$$\frac{\partial \rho}{\partial T} = \alpha \rho_0 \quad (\text{A.15})$$

and

$$\begin{aligned} \frac{\partial \ln \mu}{\partial T} = & \frac{A'(T-19.993)}{(T+C')^2} - \frac{A'}{(T+C')^2} + \frac{B'(T-19.993)^2}{(T+C')^2} \\ & - \frac{2B'(T-19.993)}{(T+C')} \end{aligned} \quad (A.16)$$

Substitution of Equations (A.4) and (A.15) into Equation (A.14) yields:

$$\frac{dz}{dT} = 2bg\alpha\rho_0 - \frac{Cb\alpha}{[1+\alpha(T-T_0)]} + b \{C-g\rho_0 [1+\alpha(T-T_0)]\} \frac{\partial \ln \mu}{\partial T} \quad (A.17)$$

Rewriting Equation (A.17) as indefinite integrals and performing the more obvious integrals, including one integration by parts, yields:

$$\begin{aligned} z = & 2bg\rho_0\alpha T - Cb \{ \ln | [1+\alpha(T-T_0)] | \} + \{Cb - \rho_0 gb [1+\alpha(T-T_0)]\} \ln \mu \\ & + \alpha\rho_0 gb \int \ln \mu \, dT + C'' \end{aligned} \quad (A.18)$$

And using Equation (A.5) in (A.18) gives

$$\begin{aligned} z = & 2bg\rho_0\alpha T - Cb \{ \ln | [1+\alpha(T-T_0)] | \} + \{Cb - \rho_0 gb [1+\alpha(T-T_0)]\} \ln \mu \\ & + \alpha\rho_0 gb (\ln \mu_0) T - \alpha\rho_0 gb \left[A' \int \frac{(T-19.993)}{(T+C')} \, dT + B' \int \frac{(T-19.993)^2}{(T+C')^2} \, dT \right] \end{aligned} \quad (A.19)$$

Comparison of Equation (A.19) with (A.3) indicates their similarity, and suggests that the difference be taken (or subtracting the latter from the former), and gives on rearranging:

$$\begin{aligned} -A' \int \frac{(T-19.993)}{(T+C')} \, dT - B' \int \frac{(T-19.993)^2}{(T+C')^2} \, dT = & 19.993 (A' - 19.993B') \ln(T+C') \\ & - (A' - 39.986B') [T-C' \ln(T+C')] \\ & - B' [0.5 (T+C')^2 - 2C'(T+C') + (C')^2 \ln(T+C')] \end{aligned} \quad (A.20)$$

Now differentiating both sides with respect to the temperature, T, gives:

$$-A' \frac{(T-19.993)}{T+C'} - B' \frac{(T-19.993)^2}{T+C'} = \frac{19.993(A'-19.993B')}{T+C'} - [A'-2(19.993B')] \left[1 - \frac{C'}{T+C'}\right] - B[(T+C') - 2C' + \frac{(C')^2}{T+C'}] \quad (A.21)$$

Continuing reduction of only the right-hand side gives:

$$= \frac{19.993 A'}{T+C'} - \frac{(19.993')^2 B'}{T+C'} - [A'-2(19.993)B'] \cdot \left[\frac{T+C-C}{T+C}\right] - B \frac{(T+C')^2 - 2C'(T+C') + (C')^2}{T+C'} \quad (A.22)$$

and

$$= - \frac{A'(T-19.993)}{T+C'} - \frac{(19.993)^2 B'}{T+C'} + \frac{2(19.993) B' T}{T+C'} - \frac{B[(T+C')-C']^2}{T+C'} \quad (A.23)$$

So further reducing Equation (A.23) and bringing down the left-hand side of Equation (A.20) gives

$$\frac{-A'(T-19.993)}{T+C'} - \frac{B(T-19.993)^2}{T+C'} \equiv - \frac{A'(T-19.993)}{T+C'} - \frac{B'(T-19.993)^2}{T+C'} \quad (A.24)$$

with the identical equality in Equation (A.24) demonstrating that the solution, namely Equations (A.2), (A.3), (A.4), and (A.5) together, does indeed satisfy the basic partial differential in Equation (A.12).

BOUNDARY CONDITIONS FOR THE EXAMPLE FLOW PROBLEM

The boundary conditions for the rectangular example flow problems are for the fluid pressure:

$$p(0,z) = - Cz + D \quad (\text{A.25})$$

$$p(x,0) = \frac{1}{a} x - \frac{1}{2b} x^2 + D \quad (\text{A.26})$$

$$p(1000,z) = D + \left(\frac{1}{a} - \frac{500}{b}\right) 10^3 - Cz \quad (\text{A.27})$$

$$p(x,800) = \frac{1}{a} x - \frac{1}{2b} x^2 + D - 800C \quad (\text{A.28})$$

And the temperature boundary conditions are

$$T(x,0) = 55^{\circ}\text{C} \quad (\text{A.29})$$

$$T(x,800) = 25.6516^{\circ}\text{C} \quad (\text{A.30})$$

which require that the constant of integration be

$$C'' = 33550.03 \text{ m} \quad (\text{A.31})$$

in Equation (A.3).

SPECIFIC PARAMETERS USED IN EXAMPLE PROBLEM

The specific values for the parameters for the evaluation of the example flow problem for the nonhomogenous fluid are found in Table A.1. Use of these parameter values in the equations indicated enabled providing the numerical results shown in the body of the report for the example flow problem when the fluid was nonhomogenous. The evaluated results are shown in Figures 2, 3, 4b, 5b, and 6b.

TABLE A.1. Specific Parameters Used to Evaluate the Example Flow Problem

Pressure or Variable		Value and Units	Equation Involved	Source Reference
Pressure Coef.	$a =$	$0.03346 \text{ m}^3/\text{Newton}$	A.2	
Viscosity Coef.	$A' =$	$2.6961/^\circ\text{C}$	A.5	Korson 1969
Pressure Coef.	$b =$	$6.69205 \text{ m}^4/\text{Newton}$	A.2	
Viscosity Coef.	$B' =$	$0.0042068/(^\circ\text{C})^2$	A.5	Korson 1969
Pressure Coef.	$C =$	9750 Newton/m^3	A.2	
Viscosity Coef.	$C' =$	89.93°C	A.5	Korson 1969
Integration Const.	$C'' =$	33550.03 m	A.3	
Pressure Coef.	$D =$	$8,605,950 \text{ Pascals}$	A.2	
Grav. Const.	$g =$	9.8067 m/s^2	A.5	CRC 1980
Permeability	$k =$	$1 (10^{-6}) \text{ m}^2$	A.6	
Ref. Temp.	$T_0 =$	40°C	A.4	CRC 1980
Density Coef.	$\alpha =$	$-385.30 (10^{-6}) 1/^\circ\text{C}$	A.4	CRC 1980
Viscosity Coef.	$\mu_0 =$	$10.02 \text{ Pa}\cdot\text{s}$	A.5	Korson 1969
Density Coef.	$\rho_0 =$	992.2187 kg/m^3	A.4	CRC 1980

REFERENCES

- Chemical Rubber Company (CRC). 1980. Handbook of Chemistry and Physics, 60th Edition. CRC Press, Inc., Boca Raton, Florida.
- Korson, L., et al. 1969. "Viscosity of Water at Various Temperatures." J. Phys. Chem. 73(1):34-39.

APPENDIX B

AN EQUIVALENT FLOW PROBLEM WHEN THE FLUID IS HOMOGENEOUS

APPENDIX B

AN EQUIVALENT FLOW PROBLEM WHEN THE FLUID IS HOMOGENEOUS

This appendix provides an analytical closed-form solution that satisfies the same pressure boundary conditions and porous media properties as the example flow system derived in Appendix A; however, in this case, within the rectangular flow system the fluid is homogeneous because there is no vertical temperature profile. The temperature is at a constant average value of 40°C throughout the flow system. In this homogeneous fluid system, it is also convenient to define a scalar energy potential based on a unit volume basis. Defined specifically:

$$\phi = p + \rho g z \quad (B.1)$$

And by Darcy's Law, the fluid flux, $\bar{q}$, is

$$\bar{q} = - \frac{k}{\mu} \nabla \phi \quad (B.2)$$

The conservation of mass equation then is

$$\nabla \cdot \rho \bar{q} = 0 \quad (B.3)$$

when the flow is steady. Substituting Equation (B.2) into (B.3) and recalling that here intrinsic permeability, k ; fluid viscosity, μ ; and fluid density, ρ , are all constant, which for such condition reduces the conservation of mass expression, Equation (B.3), to

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \quad (B.4)$$

or the classical Laplacian expression, which is to be satisfied by the energy potential, ϕ . The boundary conditions to be satisfied [recalling Equations (A.25) through (A.28)] are:

$$p(0,z) = -Cz + D \quad (B.5)$$

$$p(x,0) = \frac{1}{a} x - \frac{1}{2b} x^2 + D \quad (B.6)$$

$$p(1000,z) = D + \left(\frac{1}{a} - \frac{500}{b}\right) 10^3 - Cz \quad (B.7)$$

$$p(x,800) = \frac{1}{a} x - \frac{1}{2b} x^2 + [D - 800C] \quad (B.8)$$

These equations are the same boundary conditions on the pressure that exist in the example problem of Appendix A where the fluid was nonhomogeneous.

The specific boundary value problem to be solved is Equation (B.4), subject to the boundary conditions of Equations (B.5) through (B.8), where:

ϕ = the scalar energy potential	FL/L^3
p = the fluid pressure	F/L^2
ρ = the fluid mass density, which is constant here	M/L^3
g = the gravitational scalar	L/T^2
$\bar{q}$ = the Darcy volumetric flux	L^3/L^2T
k = the constant intrinsic permeability of the media	L^2
μ = the constant dynamic fluid viscosity	FT/L^2

and a , b , C and D are the pressure equation coefficients already defined [see Equations (A.2) through (A.5) and associated definitions].

Since Equation (B.4) is linear, superposition holds, so it will be convenient to superimpose some potential functions in order to satisfy the

boundary conditions in Equation (B.5) through (B.8). Specifically, let us combine Equations (B.1) and (A.2) in the form

$$\phi = \Phi(x, z) + \frac{1}{a}x - \frac{1}{2b}x^2 - Cz + D + \rho gz \quad (\text{B.9})$$

where $\Phi(x, z)$ is an arbitrary new defined scalar potential function of x and z such that ϕ will satisfy Equation (B.4). Solving Equation (B.9) for the new arbitrary potential function, i.e.,

$$\Phi(x, z) = \phi - \frac{1}{a}x + \frac{1}{2b}x^2 - (\rho g - C)z - D \quad (\text{B.10})$$

Now investigate the boundary conditions by evaluating Equation (B.10) using successively Equations (B.5) through (B.8) and Equation (B.1)

$$\Phi(0, z) = -Cz + D + \rho gz - \rho gz + Cz - D = 0 \quad (\text{B.11})$$

$$\Phi(x, 0) = \frac{1}{a}x - \frac{1}{2b}x^2 + D - \frac{1}{a}x + \frac{1}{2b}x^2 - D = 0 \quad (\text{B.12})$$

$$\begin{aligned} \Phi(1000, z) &= D + \left(\frac{1}{a} - \frac{500}{b}\right) 10^3 - Cz + \rho gz \\ &\quad - \left(\frac{1}{a} - \frac{500}{b}\right) 10^3 - \rho gz + Cz - D \end{aligned}$$

$$\Phi(1000, z) = 0 \quad (\text{B.13})$$

$$\begin{aligned} \Phi(x, 800) &= \frac{1}{a}x - \frac{1}{2b}x^2 + D - 800C + \rho g(800) \\ &\quad - \frac{1}{a}x + \frac{1}{2b}x^2 - \rho g(800) + 800C \end{aligned}$$

$$\Phi(x, 800) = 0 \quad (\text{B.14})$$

So if Equations (B.11) through (B.14) are all satisfied then the potential ϕ will meet all the pressure boundary conditions of Equations (B.5) through (B.8). Differentiation of Equation (B.9) gives:

$$\frac{\partial \phi}{\partial x} = \frac{\partial \Phi}{\partial x} + \frac{1}{a} - \frac{1}{b} x$$

$$\frac{\partial^2 \phi}{\partial x^2} = \frac{\partial^2 \Phi}{\partial x^2} - \frac{1}{b} \quad (\text{B.15})$$

and

$$\frac{\partial \phi}{\partial z} = \frac{\partial \Phi}{\partial z} + \rho g - C$$

$$\frac{\partial^2 \phi}{\partial z^2} = \frac{\partial^2 \Phi}{\partial z^2} \quad (\text{B.16})$$

Substitution of Equations (B.15) and (B.16) into (B.4) provides the partial differential equation that the new potential, ϕ , must satisfy or:

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{1}{b} \quad (\text{B.17})$$

For further convenience, let us define the arbitrary function Ψ such that

$$\Psi = -b\phi \quad (\text{B.18})$$

and so using Equation (B.18) in Equation (B.9) and later in (B.17) yields:

$$\phi = -\frac{1}{b} \Psi + \frac{1}{a} x - \frac{1}{2b} x^2 - Cz + D + \rho gz \quad (\text{B.19})$$

and

$$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial z^2} = -1 \quad (\text{B.20})$$

which is the basic expression that determines Ψ . Further, using Equations (B.11) through (B.14) successively in Equation (B.18) gives as transformed boundary conditions

$$\Psi(0,z) = 0$$

$$\Psi(x,0) = 0 \quad (B.21)$$

$$\Psi(1000,z) = 0$$

$$\Psi(x,800) = 0$$

which are completely homogeneous conditions.

Accordingly, the transformed boundary value problem for which a solution is to be found includes Equation (B.20) subject to Equation (B.21). When that solution is found, substitution into Equation (B.19) gives the final scalar energy potential for the flow problem when the fluid is homogeneous. The solution to the transformed boundary value problem in Ψ is available on page 4.23, Equation 4.24 of Collatz (1960), which in our variables is

$$\Psi = \frac{16}{\pi^4} \sum_{\substack{r,s \\ \text{odd}}}^{\infty} \frac{\sin \frac{r\pi x}{1000} \sin \frac{s\pi z}{800}}{rs \left(\frac{r^2}{10^6} + \frac{s^2}{6.4(10^5)} \right)} \quad (B.22)$$

where the summation is a double sum over all odd values of the r and s integers, i.e., $r = 1, 3, 5, \dots$ and $s = 1, 3, 5, \dots$. Then substituting Equation (B.22) into (B.19) gives the solution originally sought for ϕ as:

$$\phi = \frac{1}{a} x - \frac{1}{2b} x^2 + (\rho g - C)z - \frac{16}{\pi^4 b} \sum_{\substack{r,s \\ \text{odd}}}^{\infty} \frac{\sin \frac{r\pi x}{1000} \sin \frac{s\pi z}{800}}{rs \left(\frac{r^2}{10^6} + \frac{s^2}{6.4(10^5)} \right)} + D \quad (\text{B.23})$$

which is the energy potential solution. For subsequent evaluation it is convenient to have available the Darcian fluid flux components, that is, using Equation (B.23) in (B.2) gives for those components

$$q_x = -\frac{k}{\mu} \left(\frac{1}{a} - \frac{1}{b} x \right) + \frac{16k}{b\mu\pi^4} \sum_{\substack{r,s \\ \text{odd}}}^{\infty} \frac{\frac{r\pi}{1000} \cos \frac{r\pi x}{1000} \sin \frac{s\pi z}{800}}{rs \left(\frac{r^2}{10^6} + \frac{s^2}{6.4(10^5)} \right)} \quad (\text{B.24})$$

and

$$q_z = -\frac{k}{\mu} (\rho g - C) + \frac{16k}{b\mu\pi^4} \sum_{\substack{r,s \\ \text{odd}}}^{\infty} \frac{\frac{s\pi}{800} \sin \frac{r\pi x}{1000} \cos \frac{s\pi z}{800}}{rs \left(\frac{r^2}{10^6} + \frac{s^2}{6.4(10^5)} \right)} \quad (\text{B.25})$$

Equations (B.23), (B.24), and (B.25) are the results that were evaluated to provide the homogeneous fluid results shown in the body of the report in Figure 4a, 5a, and 6a. The particular parameters used in the solution for this homogeneous fluid case to obtain the results in the body of the report are given in Table B.1.

TABLE B.1. Parameters Used in the Second Analytical Solution
for the Example Flow Problem

<u>Parameter or Variable</u>	<u>Value and Units</u>	<u>Equation Involved</u>	<u>Source Reference</u>
Pressure Coef. a =	0.03346 m ³ /Newton	B.19	
Pressure Coef. b =	6.69205 m ⁴ /Newton	B.19	
Pressure Coef. C =	9750 Newton/m ³	B.19	
Pressure Coef. D =	8,605,950 Pascals	B.19	
Grav. Const. g =	9.8067 m/s ²		CRC 1980
Permeability k =	1 (10 ⁻⁶) m ²	B.24 & B.25	
Temperature T ₀ =	40°C		
Viscosity μ =	10.02 Pa·s		Korson 1969
Density (40°C) ρ =	9.92.2187 kg/m ³		

REFERENCES

- Collatz, L. 1960. The Numerical Solution of Differential Equations, Third Edition. Springer-Verlag, Berlin, Germany.
- Chemical Rubber Company (CRC). 1980. Handbook of Chemistry and Physics, 60th Edition. CRC Press, Inc., Boca Raton, Florida.
- Korson, L., et al. 1969. "Viscosity of Water at Various Temperatures."
J. Phys. Chem. 73(1):34-39.

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